Who Tames the Celtic Tiger? Portfolio Implications from a Multivariate Markov Switching Model*

Massimo Guidolin
Federal Reserve Bank of St. Louis
Research Division
P.O. Box 442
St. Louis, MO 63166
United States

Stuart Hyde
Manchester Business School
MBS Crawford House
Booth Street East
Manchester M13 9PL
United Kingdom

JEL code: G11, F30, F37, C32.

Abstract

We use multivariate regime switching vector autoregressive models to characterize the time-varying linkages among the Irish stock market, one of the top world performers of the 1990s, and the US and UK stock markets. We find that two regimes, characterized as bear and bull states, are required to characterize the dynamics of excess equity returns both at the univariate and multivariate level. This implies that the regimes driving the small open economy stock market are largely synchronous with those typical of the major markets. However, despite the existence of a persistent bull state in which the correlations among Irish and UK and US excess returns are low, we find that state comovements involving the three markets are so relevant to reduce the optimal mean-variance weight carried by ISEQ stocks to at most one-quarter of the overall equity portfolio. We compute time-varying Sharpe ratios and recursive mean-variance portfolio weights and document that a regime switching framework produces out-of-sample portfolio performance that outperforms simpler models that ignore regimes. These results appear robust to endogenizing the effects of short term interest rates on excess stock returns.

Keywords: international portfolio diversification; multivariate regime switching; Sharpe ratio.

*We thank seminar participants at Dublin City University. All errors remain our own.

1. **Introduction**

Understanding the relationships between small open economies and major international stock markets and how linkages between these markets vary through time is of great importance for portfolio diversification. Standard finance textbooks usually argue that there are significant gains from international portfolio diversification. However, these claims are based on the existence of relatively low, constant correlations between national equity markets. There is now massive evidence that correlations are neither constant nor particularly modest. This feature seems particularly important for emerging markets representative of economies with strong trade and real (e.g. foreign direct investments) ties with economies that happen to host major international stock markets and financial intermediation centers: while the real ties work to make correlations higher, time-variation in such ties and business cycles conjure to make such correlations time-varying. Our paper investigates the case of an interesting small open economy − the Irish stock market as represented by its value-weighted index, the ISEQ − by exploring the ability of a flexible class of nonlinear models − multivariate Markov switching vector autoregressions − to capture dynamic patterns of economic importance because useful to portfolio managers.

Recent empirical findings have shown that not only have correlations increased over time due to expanding capital market deregulation, increasing free trade, globalization, growth in the activities of multinational enterprises, the number of cross-listings, and cross-border merger and acquisitions, but that these correlations are time-varying. Longin and Solnik (1995, 2001) show that correlations between markets increase during periods of high market volatility, with the result that correlations would be higher than average exactly in the moment when diversification promises to yield gains. Such changes in correlations imply that the benefits to portfolio diversification may be rather modest during bear markets (see Butler and Joaquin, 2002, and Baele, 2005).[1] Despite a number of stylized facts regarding correlations, comovement and integration of stock markets over time, much of the extant literature fails to encapsulate these facts in a genuine multivariate setting. Therefore in this paper we investigate the time-varying nature of the relationship between the equity market in a small open economy, Ireland, and the major Anglo-Saxon markets, the UK and the US, using a Multivariate Markov Switching (MMS) model.

Ireland seems to offer the ideal case of a small open economy with long-standing political and economic links with both the UK and US. For instance, prior to Ireland joining the European Monetary System in 1979, the Irish punt was held at parity with the UK pound sterling. Moreover, the majority of Irish firms are listed on the London stock exchange in addition to Dublin's exchange. Ties with the US have been increasing significantly over the past three decades. By 1994, nearly a quarter of the Irish workforce were employed by US owned firms and in 1999 US foreign direct investment accounted for virtually 90% of total investment (capital formation) in Ireland. Moreover, the degree of economic and financial dependence of Ireland on the US and the UK remains non-obvious throughout the entire 1978-2004 sample, in the sense that while Ireland joined the EMU in 1999, the UK (and obviously the US) did not, meaning no structural break affects the commonality of fundamentals in any of the pairs of countries under investigation.[2]

[1] Kim, Moshirian and Wu (2005) and Aggarwal, Lucey, and Muckley (2004) find that linkages among European stock markets inside and outside of the euro-area have strengthened following the currency unification. An earlier paper by Arshanapalli and Doukas (1993) found that similar effects had been produced by the October 1987 crash.

[2] This remark is particularly important for the analysis of the effects of short-term interest rates on stock markets in Section 5. Gottheil (2003) discusses the causes for the "tiger-like" economic growth observed in Ireland between 1995 to 2000; see also The Economist, "Green is Good", May 17 1997, issue 8017.

There are also good reasons arising from financial considerations that suggest that the Irish stock market ought to be strongly comoving with other major international markets, especially the US and UK ones. First, all small size markets are prone to rebalancing-induced effects of large movements in other major markets: when larger markets are bull (bear), diversification considerations suggest buying (selling) in smaller markets as well, thus spreading the bull (bear) state to them. In the specific case of the ISEQ, we have an additional effect caused by the choice of the majority of large Irish firms to be cross-listed on the ISEQ and the London Stock Exchange, which ought to accentuate the influence of the latter on the former. In fact, a number of papers have investigated the relationships between the Irish and UK equity markets (Gallagher, 1995, Kearney, 1998, and Alles and Murphy, 2001) providing evidence of substantial integration and significant spillovers between the UK and Ireland, while Cotter (2004) highlights the influence of the US market on Irish stock returns. However, all of these studies fail to capture the time-varying nature of the relationship beyond simple sub-sample analysis. Using smooth transition regressions, Bredin and Hyde (2005) capture the relationship of the ISEQ with the UK and US through time in a univariate context and demonstrate the significant role that UK stock returns and in particular US stock returns have in determining Irish equity returns when there is an allowance for differing states. Although previous research clearly establishes a relationship between the small open economy, Ireland, and the major economies and markets of the UK and the, US it fails to capture the multivariate nature of the interrelationships and is silent on any commonality in state dependence.

We report a number of interesting results. First, the null of linear (either i.i.d. or vector autoregressive) Gaussian excess stock returns is severely rejected for all the markets under consideration, in the sense that univariate analysis reveals the presence of clearly interpretable regimes.[3] We find that the degree of 'synchronization' across markets is surprisingly high and that this finding also applies to Ireland, whose stock market strongly comoves (both linearly, as measured by pairwise correlation coefficients, and non-linearly) with US and UK equity markets. In fact, when truly multivariate models are considered, we obtain evidence that a simple two-state vector autoregressive model in which the regime is common across the three markets is sufficient to capture the salient properties of the data. Second, a formal (nonlinear) impulse-response analysis uncovers that – despite a VAR component is suggested by the data – the common wisdom that it is difficult to predict excess stock returns simply on the basis of past behavior is fairly accurate, especially when horizons exceeding 3-4 months are considered and for pairs of national equity markets. This result is consistent with the idea that linear patterns of interdependence are difficult to estimate with any accuracy and therefore hard to exploit in portfolio management. However, in a regime switching model, the linear channel is not the only one through which cross-market linkages appear: dynamic associations through commonality in regimes may be more important and can be accurately estimated. Third, we show this is the case by calculating mean-variance portfolio weights and documenting their usefulness in a recursive, pseudo-out-of-sample exercise in which the performances of portfolio decisions based on alternative statistical models are compared. It turns out that using regime switching VAR models produces useful portfolio indications able to maximize (average) realized Sharpe ratios. Fourth, we repeat some of

[3] This is less than surprising in the light of the existing literature, even with reference to ISEQ returns. Among others, Lucey (2001) tests whether there is evidence of long memory – and hence of nonlinear dependence – in daily ISEQ returns using the Fractional Differencing Model of Geweke and Porter-Hudak. Hamill, Opong and Sprevak (2000) use a variety of statistical tools to test whether ISEQ returns are independently and identically distributed over time and reject the hypothesis in favor of fractionally integrated ARMA models.

the exercises when the short-term interest rates from the three countries affect the definition of the regimes and linearly forecast subsequent excess equity returns. Although the analysis is complicated by the higher dimensionality of the estimation problem, we obtain some evidence that implied portfolio indications are not qualitatively different from those that are based purely on excess stock return data.

All in all, our paper shows that the ISEQ parallels the real side of the Irish economy and therefore manifests a high association with returns and especially regimes characterizing US and UK markets. This makes Irish stocks a diversification vehicle that remains certainly useful, but that ought to receive an optimal weight that fails to reflect the "Tiger", emerging-like recent performance of the Irish economy. On the contrary, the negative skewness and fat-tailed properties of ISEQ excess returns contribute in the end to limit the importance of Irish stocks to weights that hardly reach one-quarter of the overall equity portfolio, even under the most favorable configurations of preferences and parameters (see Guidolin and Nicodano, 2005, for similar conclusions with reference to European small capitalization stocks).

A number of papers have employed regime switching techniques to model time-varying linkages among international stock indices. Ramchand and Susmel (1998) examine the relationship between correlation and variance in a regime-switching ARCH model estimated on weekly stock returns data for the US and a few major markets (in pairs). They find that correlations between US and other markers are 2 to 3.5 times higher when the US market is in a high variance state. They also calculate mean-variance portfolio choices and find that their switching framework leads to high Sharpe ratios. Ang and Bekaert (2002) consider bivariate and trivariate regime models that capture asymmetric correlations in volatile and stable markets and characterize a US investor's optimal asset allocation under power utility. Butler and Joaquin (2002) characterize the consequences of asymmetric correlations in bear and bull markets in an international portfolio diversification framework and show that risk averse investors may want to tilt portfolio weights away from stock markets characterized by the highest correlations during downturns.[4] Our focus is distinctly on the dynamic linkages between the market of a small open and major stock markets representative of countries with which the real ties are strong. Sarno and Valente (2005) propose a regime switching vector error correction (VEC) representation that captures international spillovers across the differences futures-spot index prices for the S&P 500, the FTSE 100 and the NIKKEI 225. They show that not only a VEC model with regimes outperforms several benchmarks in-sample, but that it also provides significant gains in terms of density forecasts. In our paper we ignore information from derivatives (futures) markets and instead focus on the spillovers across major Anglo-Saxon markets and the emerging ISEQ.[5]

Finally, there is a literature that has examined the structure of stock returns in Europe and the US asking whether international markets are integrated and whether their degree of integration could be increasing over time as a result of globalization processes (see e.g. the review in Kearney and Lucey, 2004). For instance, Heston, Rouwenhorst, and Wessels (1995) use a large sample of over 4000 US and 1800 European stocks to find evidence that countries share multiple risk factors, although a simple international version of the intertemporal CAPM is rejected. Some of this research has been specifically directed at assessing the linkages between US and UK stock returns: for instance, Engsted and Tanggaard (2004) investigate the nature of their comovements using variance decompositions suggesting that the comovement

[4]Butler and Joaquin (2002) simply define their three regimes (bear, normal, and bull) according to the level of domestic returns. Each regime is exogeneously constrained to collect exactly one-third of the sample. In our paper the regimes are instead data-driven and identified as latent state variable by using Hamilton and Kim's smoothing algorithm.

[5]Notice that a futures contract on the ISEQ is still not traded.

is driven by common real shocks; Becker, Finnerty, Friedman (1995) employ intra-day price movements of stock index futures to show that UK prices react to US news and that the correlation between returns systematically increases on US announcement days. Differently from this literature, we use index data at relatively low (monthly) frequencies and instead of testing precise asset pricing models, we use a statistical model to pin down the dynamic law of the degree of "integration" between the three markets and choose and evaluate out-of-sample portfolio strategies that exploit information on the dynamic linkages.

The paper has the following structure. Section 2 provides a quick primer on multivariate Markov switching models. After an introduction to the data employed in the paper, Section 3 reports the main body of empirical results of the paper, distinguishing between univariate and multivariate findings. Section 4 is devoted to the economic implications of our econometric results, in particular to examining the nonlinear impulse-responses implied by MMS models, to predicting Sharpe ratios useful in portfolio choice, and to calculating and assessing the recursive out-of-sample performance of portfolio strategies that rely on different statistical models. Section 5 performs a robustness check encompassing the case in which short-term interest rates can predict subsequent excess returns. Section 6 concludes.

2. Models of Regimes in the Joint Return Process

A vast literature in finance has reported evidence of predictability in stock market returns, mostly in the context of linear, constant-coefficient models, see e.g. Campbell and Shiller (1988), Fama and French (1988, 1989), Ferson and Harvey (1991), and Goetzmann and Jorion (1993). This evidence on linear predictability and comovements of asset returns has been extended to model dynamic linkages across international equity markets (see e.g. Kim, Moshirian and Wu, 2005). More recently, some papers have found evidence of regimes in the distribution of returns on individual asset returns or pairs of these (e.g., Guidolin and Timmermann 2003, 2005a, Turner, Startz and Nelson, 1989, and Whitelaw, 2000).

To our knowledge, there are very few applications of the class of multivariate Markov switching models studied by Hamilton (1991) and Krolzig (1997) involving relatively large vectors of national stock index returns.[6] On the contrary, and following the expanding macroeconometrics literature, in this paper we model the joint distribution of a vector of n (excess) stock index returns, $\mathbf{x}_t = (x_{1t}, x_{2t}, ..., x_{nt})'$ as a multivariate regime switching process driven by a common discrete state variable, s_t, that takes integer values between 1 and k:

$$\mathbf{x}_t = \mu_{s_t} + \sum_{j=1}^{q} \mathbf{A}_{j,s_t}\mathbf{x}_{t-j} + \varepsilon_t. \tag{1}$$

Here $\mu_{s_t} = (\mu_{1s_t}, ..., \mu_{ns_t})'$ is a vector of intercepts in state s_t, $\mathbf{A}_{j,s_t}$ is an $n \times n$ matrix of autoregressive coefficients at lag j in state s_t and $\varepsilon_t = (\varepsilon_{1t}, ..., \varepsilon_{nt})' \sim N(0, \mathbf{\Sigma}_{s_t})$ is the vector of return innovations that are assumed to be joint normally distributed with zero mean and state-specific covariance matrix $\mathbf{\Sigma}_{s_t}$. Innovations to returns are thus drawn from a Gaussian mixture distribution that is known to provide a flexible approximation to a wide class of distributions, see Timmermann (2000). Importantly, it is well known that mixtures of conditionally Gaussian densities can approximate highly non-Gaussian unconditional multivariate distributions rather well. In our application, $n = 3$ (Ireland, UK, and US).

[6] The only exception is Ang and Bekaert (2002) who model bi- and tri-variate vectors of national stock index returns (US and UK, US, UK, and Germany) although their focus is mainly on optimal asset allocation issues (e.g. the home country bias puzzle) and not on time-varying linkages across small and large equity markets.

Moves between states are assumed to be governed by the $k \times k$ transition probability matrix, $\mathbf{P}$, with generic element p_{ji} defined as

$$p_{ji} \equiv \Pr(s_t = i | s_{t-1} = j), \quad i, j = 1, .., k. \tag{2}$$

Each regime is hence the realization of a first-order Markov chain. Our estimates allow s_t to be unobserved and treat it as a latent variable. This feature corresponds to the common observation that although nonstationarities and regime shifts seem to be pervasive, they remain extremely difficult to predict and even pin down once they take place.

(1) - (2) nest several popular models from the literature as special cases. In the case of a single state, $k = 1$, we obtain a linear vector autoregression (VAR) with predictable mean returns provided that there is at least one lag for which $\mathbf{A}_j \neq 0$. This type of statistical framework has been employed e.g. by Lund and Engsted (1996). In the absence of significant autoregressive terms ($q = 0$), the discrete-time equivalent of the standard IID Gaussian model adopted by much of the mean-variance based literature obtains.

Our model can be extended to incorporate an $l \times 1$ vector of predictor variables, $\mathbf{z}_{t-1}$, comprising variables such as inflation and/or interest rates that have been used in recent studies on predictability of stock returns from macroeconomic variables (e.g. Bossaerts and Hillion, 1999) able to capture the dynamics of the underlying state of the economy. Define the $(l + n) \times 1$ vector of state variables $\mathbf{y}_t \equiv (\mathbf{x}_t' \ \mathbf{z}_t')'$. Then (1) is readily extended to

$$\mathbf{y}_t = \begin{pmatrix} \mu_{s_t} \\ \mu_{zs_t} \end{pmatrix} + \sum_{j=1}^{q} \mathbf{A}^*_{j,s_t} \mathbf{y}_{t-j} + \begin{pmatrix} \varepsilon_t \\ \varepsilon_{zt} \end{pmatrix}, \tag{3}$$

where $\mu_{zs_t} = (\mu_{z_1 s_t}, ..., \mu_{zl s_t})'$ is the intercept vector for $\mathbf{z}_t$ in state s_t, $\{\mathbf{A}^*_{j,s_t}\}_{j=1}^{q}$ are now $(n + l) \times (n + l)$ matrices of autoregressive coefficients in state s_t and $(\varepsilon_t' \ \varepsilon_{zt}')' \sim N(0, \mathbf{\Sigma}^*_{s_t})$, where $\mathbf{\Sigma}^*_{s_t}$ is an $(n + l \times n + l)$ covariance matrix. This model allows for predictability in returns through the lagged values of $\mathbf{z}_t$. It embeds a variety of single-state VAR models that have been considered in recent studies.

MMS models are estimated by maximum likelihood. In particular, estimation and inferences are based on the EM (Expectation-Maximization) algorithm proposed by Hamilton (1993), a filter that allows the iterative calculation of the one-step ahead forecast of the state vector π_t given the information set, and the consequent construction of the log-likelihood function of the data. Maximization of the log-likelihood function within the M-step is actually made faster by the fact that the first-order conditions defining the MLEs may often be written down in closed form. Krolzig (1997) is an excellent introductory reference. Under standard regularity conditions (such as identifiability, stability and the fact that the true parameter vector does not fall on the boundaries) Hamilton (1993) and Leroux (1993) have proven consistency and asymptotic normality of the ML estimator. As a consequence, and with one important exception, standard inferential procedures are available to test statistical hypothesis.

3. **Empirical Results**

3.1. *The data*

We use monthly series on Irish, US, and UK nominal stock returns for the period 1978:05-2004:12. In particular, we focus on continuously compounded total (inclusive of dividends and all distributions) returns on the Dublin ISEQ, the US S&P 500, and the UK FTSE 100 stock market indices. These data are supplemented by data on short-term interest rates for the three countries, in particular Irish and UK money market (federal funds) rates and the US FED funds rate. All data series are obtained from Datastream. Table 1 reports summary statistics for all series under consideration. Consistently with the literature on stock return predictability, we investigate the properties of *excess* equity returns. To make the table easy to read, the statistics refer to monthly percentage returns.

The three markets display similar median excess returns, in the order of 8-9 percent a year, i.e. values consistent with the recent debate on the high equity premium.[7] Some structural difference is displayed by the volatility coefficients, a textbook annualized value of 15 percent for the US index, 17 percent for the UK, and more than 18 percent for Irish excess stock returns. As a result the (median-based, annualized) Sharpe ratios range from 0.49 for the UK to 0.58 for the US (the Irish index is 0.53). However, should we add confidence bands around such values, we would fail to find significant differences among these reward-to-risk ratios, which appear to cluster around a 'typical' 0.5 per year value. This feature suggests that in a portfolio logic, an investor might derive substantial benefits from a strategy that diversifies across these three equity portfolios. However, Panel A of Table 1 also shows that such a simplistic approach may be inappropriate, as the three indices also display asymmetric, left-skewed, and fat-tailed distributions. In particular, excess Irish returns show a large and statistically significant negative skewness (-1.5) and a large kurtosis (12.4) that exceeds the Gaussian benchmark (three) with a negligible p-value. The values of skewness and kurtosis for UK and US excess returns are less impressive, but they still bring to stark rejections (using a standard Jarque-Bera test) of the null hypothesis that each of these univariate series may have a Gaussian marginal distribution.[8]

Interestingly, such non-Gaussian features do not appear to be exclusively (or even mainly) driven by the presence of volatility clustering (ARCH effects) in the three excess equity returns series. For instance, we calculate an eight-order Ljung-Box test statistics on squared excess returns and test whether there is any serial correlation structure in volatility. We find high p-values (0.91, 0.95, and 0.84 for Ireland, UK, and US, respectively) and very weak indications of ARCH.[9] On the other hand, while Irish excess returns appear to be highly serially correlated in levels, this is not the case for UK and US excess returns.[10] Section 3.2 to follow considers another possible source of departures from (marginal) normality for the excess returns under consideration, the presence of regimes in the first two moments that equate the unconditional marginal densities to mixtures of normals.

---

[7]Mean (as opposed to median) excess equity returns are surprisingly low (less than 1 percent) for the UK. This is caused by 2 extreme observations (of -19 and -16 percent) that lie more than 3 standard deviations away from the mean.

[8]The Jarque-Bera statistics are 1301, 594, and 231 for Ireland, UK, and US, respectively. The associated p-values are always negligible.

[9]This result is partially explained by the presence of a few influential observations in the 1980s, in particular the large, negative returns of October 17, 1987. In fact, when we test for clustering in squares in a 1988:01 - 2004:12 sample, we find p-values of 0.01, 0.54, and 0.01, i.e. some evidence of ARCH reappears, with the UK exception.

[10]The associated p-values for a eighth order Ljung-Box test on levels are 0.001, 0.41, and 0.51. In fact, Box-Jenkins techniques select a simple AR(1) model for excess Irish returns:

$$x_t^{ISEQ} = \underset{(0.394)}{0.102} + \underset{(0.054)}{0.269} x_{t-1}^{ISEQ} + \epsilon_t^{ISEQ}.$$

Consistent with the discussion in Lo and MacKinlay (1990), such serial correlation structure may be related to spurious, stale price-induced components surviving on the less efficient Irish market. In fact, the evidence becomes weaker (e.g. the LB(8) p-values increases to 0.10) for shorter, more recent samples (like 1992:01 - 2004:12), when the Irish market can be presumed to have become more efficient and better integrated with foreign markets.

Panel B of Table 1 reports summary statistics for short-term interest rates, later used as predictors of excess stock returns ($\mathbf{z}_t = \mathbf{r}_t \equiv (r_{1t}, r_{2t}, r_{3t})'$). Means and medians fit classical values of 7-8 percent a year. Irish and US short-term yields are also highly non-normal, positively skewed (1.5 and 0.9) and fat-tailed (kurtosis coefficients are 8.8 and 4.0). However, in this case we obtain for all series evidence that short yields are highly serially correlated (with near-unit root properties) and present strong ARCH effects. However, as argued by many recent papers (e.g. Gray, 1996, and Guidolin and Timmermann, 2005b), such properties are also potentially consistent with the presence of regimes in the (joint) distribution of short-term yields.

Panel C concludes by showing simultaneous correlation coefficients among excess equity returns and short yields. Excess stock returns are generally positively correlated, with coefficients between 0.54 (Ireland-US) and 0.70 (US-UK). Even such a large value implies the existence of substantial international diversification opportunities. A similar remark applies to correlations among interest rates (generally around 0.7). Finally, short-term yields tend to imply simultaneous movements in opposite direction of excess stock returns, although the correlation coefficients are small (generally around -0.1) and weakly significant from a statistical viewpoint.

3.2. *Regimes in national stock indices*

To assess whether regime switching models are capable to provide an adequate fit of the asset return properties revealed by Table 1, in this Section we search for appropriate multi-state models for the univariate excess equity returns series. In practice, we fit a variety of MSIAH(k, q) models,

$$x_t^i = \mu_{s_t^i}^i + \sum_{j=1}^{q} a_{j,s_t^i} x_{t-j}^i + \varepsilon_t^i \quad \varepsilon_t^i \sim N(0, \sigma_{s_t^i}^i), \tag{4}$$

where $i =$ ISEQ, FTSE100, S&P500 and also the latent, first-order Markov state variable s_t^i is assumed to be specific to each national stock market, as indexed by i. In the acronym MSIAH(k, q) (see Krolzig, 1997), MS stands for 'Markov Switching', 'I' points to the fact that the intercept $\mu_{s_t^i}^i$ is regime-specific, 'A' to the regime-dependence in the AR(p) component, and 'H' to analog structure in the covariance matrix. Clearly, single-state, Gaussian IID models correspond to $k = 1$ and $q = 0$, while simpler MS models can be easily derived by imposing restrictions on (4), e.g. MSIH(k, q) models in which $q \geq 1$ but the AR component of the conditional mean function fails to be regime-dependent or MSI(k) models in which $q = 0$ and the covariance matrix fails to be regime-dependent.

Based on the analysis in Section 3.1, we expect UK and US excess stock returns to display $q = 0$ and possibly require relatively simple MSI(k) models, given the weak evidence of time-variation in second moments; Irish excess returns are likely to imply either a MSI(k, q) or a MSIA(k, q) model, given the evidence of serial correlation. We perform a specification search with reference to each of the excess stock return series using three information criteria (AIC, BIC, and Hannan-Quinn) and two different likelihood-ratio tests. The first type of LR test concerns the appropriate number of regimes k in (4). In particular, we would like to test whether the null of a single-state (also called "linear" in what follows) model ($k = 1$) can be rejected in favor of $k > 1$. As already stressed, when $k = 1$ (4) reduces to a simpler Gaussian AR(p) model. As discussed in Garcia (1998), testing for the number of states in a regime switching framework is complex. Given some $k \geq 2$, the problem is that under any number of regimes smaller than k there are a few parameters of the unrestricted model that can take any values without influencing the maximized likelihood function.[11] The result is that the LR test fails to have a standard chi-square distribution with number of degrees of freedom equal to the number of restrictions imposed. Therefore we also utilize Davies' (1977) correction to the standard LR test who circumvents the problem of estimating the nuisance parameters under the alternative hypothesis and derives instead an upper bound for the significance level of the LR test:

$$\Pr(LR > x) \leq \Pr\left(\chi_1^2 > x\right) + \sqrt{2x}\exp\left(-\frac{x}{2}\right)\left[\Gamma\left(\frac{1}{2}\right)\right]^{-1}.$$

where $\Gamma(\cdot)$ is the standard gamma function. Therefore we systematically test the null of $k = 1$ against $k > 1$ (the exact number of regimes varies with the different models) analyzing the p-values calculated under Davies' upper bound.

The second set of LR tests is of a standard type but $-$ because of the considerations of the previous paragraph $-$ is applied only to models with the same level of k, i.e. the test is applied to standard restrictions that purely involve:[12]

- The autoregressive order q;
- Given $q \geq 1$, the presence of regime-dependence in the AR coefficients $\{a_{j,s_t^i}\}_{j=1}^p$ in MSIA(k, q) and MSIAH(k, q) models;
- The regime-dependence of the variance in the case of MSIH(k, q) and MSIAH(k, q) models.

For ISEQ excess returns, the null of a single-state model is rejected, with LR statistics always in excess of 49 and therefore highly significant, even after applying Davis' (1977) correction. For each value of k = 2, 3, 4, standard LR tests reveal themselves to be scarcely selective, apart from casting some doubts on whether regime-dependence in the variance may be required or not. For instance, the restriction that leads from a MSIAH(2,1) to a MSIA(2,1) implies a p-value of 0.16; such evidence is obviously consistent with the absence of strong time-variation in volatility uncovered in Section 3.1. Finally, the information criteria are split between the MSIA(2,1) (selected by the BIC) and the MSIAH(2,1) (selected by H-Q and AIC) models.[13] In the case of UK excess returns, the information criteria signal some uncertainty as to the appropriate number of regimes, with BIC and H-Q selecting $k = 2$ and the AIC $k = 3$. Within the two-state class, standard LR tests reveal further ambiguity on the need of an autoregressive component with regimes (the p-value is 0.01). For S&P 500 excess returns, standard LR tests are again hardly informative, but the information criteria all favor a relatively parsimonious MSIH(2,0) model.[14]

To provide an idea for the types of regime models estimated on univariate excess return series, Table 2 provides parameter estimates (along with implied significance levels) for a common MSIAH(2,1) model

[11] These are some (or all) elements of the transition probability matrix associated to the rows that correspond to "disappearing states", plus corresponding parameters in the conditional mean and/or variance functions. The presence of these (nuisance) parameters gives the likelihood surface so many degrees of freedom that computationally one can never reject the null that the non-zero values of those parameters were purely due to sampling variation.

[12] The p-values of all these tests are tabulated and available upon request from the authors.

[13] The BIC is well-known to be the most parsimonious among the most widely used information criteria in applied work.

[14] A standard LR test of MSIAH(2,1) vs. MSIH(2,0) gives a p-value of 0.04, which points towards the need of an autoregressive component, although of regime-switching type. As shown in Table 2, this is not inconsistent with the Ljung-Box tests commented in Section 3.1: there is only one regime in which an AR(1) term should appear in the conditional mean function.

estimated for the three markets. Notice that although such a model was required only of ISEQ excess returns (e.g. using the H-Q criterion), also in the case of UK and US excess stock returns we have failed to find clear evidence against specifying a further autoregressive component. For comparison, panel A of Table 2 also reports estimates for a single-state, Gaussian AR(1) model. Consistently with the discussion in Section 3.1, the conditional mean model is 'significant' (precisely, the autoregressive coefficient is) for ISEQ excess returns; for other markets, a simple AR(1) model provides no useful fit to the data.

The picture provided by panel B is radically different. For all markets, the two regimes have a rather natural interpretation as bear and bull market states, although the regime-specific intercept μ^i_{bear} is (weakly) significantly negative only for US data (-1.2 percent per month). The bear states are characterized by negative and large unconditional, state-specific risk premia, with an extreme value of -31 percent in Irish case.[15] This means that the ISEQ is characterized by the possibility of crash states that inflict large losses to investors, as reflected by the large and negative skewness of -1.5 in Table 1. Moreover, the bear states are highly volatile, with annualized, unconditional state-specific volatilities between 2 and 3.4 times the unconditional AR(1) values.[16] In particular, the bear annualized volatility for the ISEQ is a whopping 60 percent. However, while for US and UK excess returns the implied bear state is rather persistent, with average durations between 7 and 11 months, in the Irish case such a state is as extreme as short-lived, with a duration barely in excess of 1 month.

The other regime is a bull state of positive and high unconditional, state-specific risk premia (between 7 percent for the ISEQ and 11 percent for US returns, in annualized terms) with moderate unconditional volatility between 8 and 15 percent. This is also the regime in which Irish returns are positively serially correlated, while also UK and US returns appear to have some mild linear structure, with some degree of mean-reversion built in the boom regime (i.e. the estimated serial correlation coefficients are negative). The bull state is rather persistent in all markets, with average durations of 25, 7, and 14 months for Ireland, UK, and US, respectively.

Such values are consistent with an equilibrium interpretation, since the regimes are unobservable and never perfectly predictable (in particular, the bear state is latent and − even when persistent − never perfectly anticipated) and the one-step predicted risk premium conditional on each of the two states,

$$E_t[x^i_{t+1}|bear] = (\mathbf{e}_1'\hat{\mathbf{P}}^i\mathbf{e}_1)\cdot\frac{\mu^i_1}{1-a^i_1} + (\mathbf{e}_1'\hat{\mathbf{P}}^i\mathbf{e}_2)\cdot\frac{\mu^i_2}{1-a^i_2}$$
$$E_t[x^i_{t+1}|bull] = (\mathbf{e}_2'\hat{\mathbf{P}}^i\mathbf{e}_2)\cdot\frac{\mu^i_2}{1-a^i_2} + (\mathbf{e}_2'\hat{\mathbf{P}}^i\mathbf{e}_1)\cdot\frac{\mu^i_1}{1-a^i_1},$$

is always positive, which is consistent with risk-aversion and standard preference specifications.[17]

Figure 1 shows the ex-post, full-sample smoothed state probabilities for each of three national stock markets under examination. Clearly, the structure and frequency of regime shifts differs when the ISEQ is compared to the UK and US markets. In fact, the bear state is a true crash regime in the case of Ireland: the state appears relatively infrequently and tends to be short-lived; well-known episodes of declining markets are identified, such as October 1987, the Summer of 1990 (the Iraqi invasion of Kuwait), the Russian crisis

[15] Unconditional, state-specific means are calculated as $E_{S^i}[x^i_t] = \mu^i_{s^i_t}/(1-a^i_{s^i_t})$ exploiting the simple AR(1) within-state structure implied by a MSIAH(2,1) model.

[16] Unconditional, state-specific variances are calculated as $Var_{S^i}[x^i_t] = (\sigma^i_{s^i_t})^2/[1-(a^i_{s^i_t})^2]$.

[17] $\mathbf{e}_j$ is defined as a $k\times 1$ vector with zeros everywhere but in its j-th position, where a 1 appears.

of the Summer 1998, and a few months surrounding the September 2001 terror attacks. On the other hand, the bear state is milder and much more persistent when inferred from UK or US data. Although the previous episodes are all identified by prolonged bear episodes also on UK and US data, a few more periods are ex-post fitted using the bear density, e.g. the oil shocks of the early 1980s, and the Spring of 2002 in correspondence to an international recession cycle.

Figure 2 shows that the visual impression provided by Figure 1 is not completely accurate. In Figure 2 we provide scatter plots and correlation coefficients for each pair of national stock markets (hence, smoothed state probabilities).[18] In fact, despite the difference frequency of the switches, the ISEQ smoothed probabilities of a bear/crash state are substantially correlated with the FTSE and (especially) the S&P 500 bear state probabilities, with coefficients of 0.29 and 0.47, both statistically significant at 5 percent. Less surprisingly, the FTSE and S&P 500 smoothed probabilities are also positively correlated (0.50) although we fail to detect any systematic difference vs. the association characterizing ISEQ probabilities with other markets.

In conclusion, the brief analysis of regimes at univariate level shows that excess returns in the three national stock markets under investigation all display overwhelming evidence of recurring non-stationarities, in the form of shifts in risk premia as well as risk (volatilities). Moreover, the national regimes appear to be positively, although imperfectly correlated. Section 3.3 adopts a truly multivariate approach and models the *joint* density of excess stock returns using regime switching techniques. This strategy will allow us to explicitly ask whether bull and bear states are common across national markets.

3.3. *A regime switching vector autoregressive model*

Specification tests are performed for vector regime switching models as in (1) when $\mathbf{x}_t = (x_t^{ISEQ}, x_t^{FTSE}, x_t^{S\&P})'$ becomes the object of interest.[19] The null of a single regime is decidedly rejected in correspondence of all models estimated. Even for the worst fitting MMSI(2,0) model (where MMS stands now for 'Multivariate MS'), the LR statistic takes a value of 60, which is hardly compatible with the null of a single state, even taking into account nuisance parameters issues. Once more, the information criteria appear to be split: while the parsimonious BIC and H-Q criteria select a MMSIAH(2,1) model $-$ which is purely the vector generalization of the univariate models estimated in Section 3.1 $-$ the AIC indicates that modeling three different regimes within a MMSIAH(3,1) framework might improve the fit. While in the former case a standard LR test provides then weak support to the option of increasing the VAR order from $q = 1$ to 2 (to a MMSIAH(2,2) model, the associated p-value is 0.03), in the latter case in principle one may be tempted to estimate a relatively richly parameterized MMSIAH(3,2) model, which implies the estimation of as many as 87 parameters. To keep the saturation ratio (the ratio between the number of useful observations and the parameters estimated, a non-linear analog to the concept of 'degrees of freedom') sufficiently high (25 vs. 11 in the MMSIAH(3,2) case) and for consistency with the battery of MSIAH(2,1) models in Table 2, we entertain in what follows a MMSIAH(2,1).[20]

The weak support for richer models with $k \geq 3$ represents in itself a rather meaningful result: we find

[18]Notice that in this application, the correlation coefficient is an imperfect measure of pairwise statistical association, as by construction smoothed probability series are always bounded in the interval $[0, 1]$.

[19]Detailed specification test results are not reported to save space and are available from the authors on request.

[20]Furthermore, this model retains the intuitive interpretation of regimes as bear and bull states.

no evidence that our small-open economy national stock market would command by itself the presence of specific states apt to describe its (less frequent) regime shifts between crash and normal/bull states. On one hand, this is consistent with our finding in Section 3.1 that − despite a few visual differences among the smoothed probability plots of the three markets − the univariate state probabilities display rather high correlations. On the other hand, the fact that a simple two-state, "bull & bear" model is sufficient to capture the nonlinear dynamic properties of $\mathbf{x}_t$ implies that it is easy to find "who tames (drives) the Irish market": the same underlying state variable that seems to characterize many stock markets in the world, and particularly the UK and US markets.[21]

Table 3 reports parameter estimates for the selected model. Also in this case, panel A provides a benchmark by reporting the estimates of a single-state VAR(1) model. Visibly, a single-state model is not only resoundingly rejected in a statistical dimension (the LR statistic in this case exceeds 104), but provides a few puzzling implications: for instance, the ISEQ is significantly affected by lagged S&P 500 returns, but the corresponding coefficient estimate is economically negligible (a one standard deviation shock to lagged S&P 500 excess returns would move ISEQ excess returns in the same direction by 0.02 percent only); on the contrary, the FTSE100 seems to oddly react to lagged Irish excess returns, with a coefficient with p-value below 0.1 and economically important.[22]

Panel B shows the MMS EM-MLE estimates. The table confirms the interpretation of the two regimes provided at the univariate level. Regime 1 remains a bear (or normal), persistent state (average duration is 13 months) in which excess returns are characterized by a negative (albeit not statistically significant) intercept. This is confirmed by the within state unconditional (monthly) risk premia:[23]

$$E[\mathbf{x}_t|bear] = [\text{-1.20 } 0.21 \text{ -0.55}]'.$$

All of the indices display significant (partial) first-order serial correlation coefficients; Irish excess returns are highly serially correlated (i.e. they display aggregate momentum), while UK and US excess returns imply significant (and in the UK case, economically non-negligible) mean reversion. As one would expect, the ISEQ market is heavily influenced by the recent levels of risk remuneration in the major, foreign reference markets, although past positive returns in the UK seem to depress the ISEQ index, while the opposite happens as a reaction to past positive US excess returns. Although the associated coefficients are small, the FTSE100 and the S&P500 show some (delayed) interdependence.[24] At a simultaneous level, the same is true: all shocks to excess returns appear to be significantly and positively correlated in the bear state, with the FTSE-S&P coefficient particularly large (0.68). Finally, volatilities (both for VAR shocks and unconditionally, within state) are sensibly above the overall unconditional levels. As evidenced by scores of earlier papers (e.g. Longin and Solnik, 2001) bear, volatile states imply high contemporaneous correlation among international stock markets.[25]

[21] This finding is consistent with results in Guidolin and Timmermann (2005b) that simple two-state, "bull & bear" models can adequately capture the properties of relatively long vectors of equity index returns (e.g. also including Asian, Japanese, and continental European stock returns).

[22] In Tables 3 and 6 the VAR matrices have to be read by row, i.e. in each row we report the impact on the row variable of the lagged variables in the columns.

[23] These are calculated as $E[\mathbf{x}_t|S_t] = (\mathbf{I}_3 - \mathbf{A}_{s_t})^{-1}\mu_{s_t}$.

[24] In a related exercise, Kim, Moshirian and Wu (2005) interpret this type of interdependencies as a result of the 'globalization' process.

[25] There is now specific empirical evidence of this phenomenon concerning emerging markets, see e.g. Yang, Kolari, and Min

The second regime is instead another relatively persistent (average duration is between 6 and 7 months) bull state in which excess returns all display positive intercept and positive and high within state unconditional (monthly) risk premia:

$$E[\mathbf{x}_t|bull] = [1.71\ 0.90\ 1.64]'.$$

Clearly, these mean excess returns correspond to double-digit annualized risk premia. ISEQ excess returns remain rather extreme in both regimes, switching from an annualized -14 percent in the bear state to +21 percent in the bull state. This fits the general awareness that a small open economy, emerging stock market may often be prone to jumps and sudden corrections. In this regime, only S&P 500 excess returns display a significant negative serial correlation coefficient. The ISEQ remains strongly affected by lagged excess returns in UK and US markets although the coefficients now display signs that run opposite those obtained in the bear state. Although the corresponding VAR coefficient estimates are from being economically negligible, the finding of opposite signs in the two states is consistent with our remark that a simple, single-state VAR(1) would produce statistically significant but rather small interdependence coefficients between the ISEQ and other major Anglo-Saxon markets.[26] For instance, notice that with reference to the lagged dependence of the ISEQ on the FTSE100, the Irish reaction to a past 1 percent FTSE return is

$$0.67 \times (-0.38) + 0.23 \times (0.64) = -0.11$$

(where 0.67 and 0.23 are the long-run, steady-state probabilities of the bear and bull state, respectively), which is not very different from the single-state VAR(1) coefficient estimate of -0.17 in panel in Table 3.

Even more crucially, in the bull state shocks to excess ISEQ returns appear to be weakly correlated at best with similar, contemporaneous shocks to other national stock markets. This fact seems to be rather specific to the small open economy stock market, as the FTSE and the S&P indices anyway have a significantly positive correlation of 0.45. Additionally, the bull volatilities are systematically lower than the unconditional values, generally between one-half and two-thirds of the matching bear volatilities.

Figure 3 shows the full-sample smoothed probabilities implied by the model in Table 3. The sequence of bear periods in which risk premia are negative or negligible accurately match the historical experience of the major world equity markets: 1978-1981 are characterized by two bear spells, coinciding to the oil shocks; 1983 is the period of high and volatile interest rates following the US monetarist, anti-inflationary experiment; the early 1990s identify a worldwide recession and the effects of the first Iraqi war; finally, since 1998 the world equity markets go in and out of the bear state (initially in correspondence to events such as the Russian debt crises), with acute bear episodes of a few consecutive months in 2001 and 2002. Matching recent experience, in 2003-2004 the three equity markets switch to a bull regime of high expected returns.

3.4. *Diagnostic checks*

Even though the specification tests and Table 3 provide strong evidence that a two-state model provides a superior fit to the data than simpler, single-state VAR models, two issues remain open. First, the finding that moving from a VAR(1) to a MMSIAH(2,1) model improves the maximized log-likelihood "enough"

(2003).

[26] For instance, a one standard deviation positive shock to the FTSE100 causes an excess return of +3.1 percent on the ISEQ, in the following month. The corresponding estimate for a one standard deviation shock to the S&P 500 is -2.5 percent.

both in a statistical (as signalled by the formal rejection of the null hypothesis of $k = 1$ using Davies' (1977) LR bounds) and in a logical (i.e. even after taking into account the tighter parameterization of the two-state model in a view that trades-off in-sample fit with potential predictive accuracy) sense, does not per se imply that the MMSIAH(2,1) be correctly specified. For instance, we have noticed that LR tests only weakly fail to reject a MMSIAH(2,1) model vs. a MMSIAH(2,2); similarly, the AIC clearly signalled a preference for a MMSIAH(3,1) or MMSIAH(3,2) model. Second, in any event a good in-sample fit offers weak guarantees of a satisfactory forecasting performance. Since our paper is not primarily concerned with offering good statistical descriptions of the data, but aims instead at characterizing the economic and financial (portfolio) implications of such features, some attention to the predictive performance of our model is advisable.

As shown by Krolzig (1997) standard, residual-based diagnostic checks are made difficult within the MMS class by the fact that in (1), $\varepsilon_t \sim$ i.i.d. $N(0, \mathbf{\Sigma}_{s_t})$ only within a given regime. Since for most times t, the vector of state probabilities $\hat{\pi}_t$ will differ from $\mathbf{e}_s$ ($s = 1, ..., k$, i.e. there is uncertainty as to the current regime), the generalized residuals,

$$\sum_{s=1}^{k}(\mathbf{e}_s'\hat{\pi}_t)\left(\mathbf{x}_t - \hat{\mu}_s - \hat{\mathbf{A}}_s\mathbf{x}_{t-j}\right),$$

will fail to be either i.i.d. or normally distributed. Therefore standard residual-based tests will fail if focussed around testing the i.i.d. properties of the residuals and will anyway run into difficulties when tests rely on their normality. However, Krolzig (1997) shows that under the assumption of correct specification, one important property ought to pin down at least the one-step ahead forecast errors,

$$\eta_{t+1} \equiv \mathbf{x}_{t+1} - \sum_{s=1}^{k}(\mathbf{e}_s'\hat{\mathbf{P}}\hat{\pi}_t)\left(\hat{\mu}_s + \hat{\mathbf{A}}_s\mathbf{x}_t\right)$$

(where $\hat{\pi}_t$ is the vector of real-time, filtered state probabilities and $\mathbf{e}_s'\hat{\mathbf{P}}\hat{\pi}_t$ is the one-step ahead prediction of the probability of state $s = 1, ..., k$): $\{\eta_{t+1}\}$ should define a martingale difference sequence, i.e.

$$E[\eta_{t+1}|\Im_t] = 0.$$

This hypothesis is testable in standard ways, i.e. looking at the ability of elements of the information set at time t, $\Im_t$ (e.g. current excess returns, short-term interest rates, their combinations, etc.), to forecast both elements of η_{t+1} as well as their powers (since $E[\eta_{t+1}|\Im_t] = 0$ is more restrictive than $Cov[\eta_{t+1}, Y_t] = 0$, where Y_t is any variable that belongs to $\Im_t$).

We implement two types of residual-based tests. In each case, we make an effort to provide intuition for what a rejection of the null of the forecast errors being a martingale difference sequence would imply in economic and financial terms. To gain additional insights, we generally apply tests to the each of the elements of $\{\eta_{t+1}\}$ in isolation (i.e. to the univariate series of forecast errors concerning national stock market excess returns). We start by testing whether any lags of excess returns predict current and future forecast errors. Rejections of the null of zero predictive power, would point to misspecification in the conditional mean function implied by our MMSIAH(2,1) model in particular (but not exclusively) in the VAR order (q). While for the ISEQ and the S&P, past excess returns fail to be correlated with current forecast errors, for the FTSE100 we find that at one lag such correlation is 0.24 and with a p-value below 0.05. This is hard to interpret because in Section 3.1 it became clear that at the univariate level (and even within a MSIH(2,0)) FTSE100 excess returns hardly required any AR component. Obviously, similar restrictions apply to the ability of past forecast errors to predict future errors, i.e. on the implied serial correlation structure of the forecast errors themselves. If past forecast errors help predict future errors, clear improvements in the model are possible. Here we find once more that while ISEQ and S&P errors have no appreciable serial correlation structure (e.g. their Ljung-Box order 12 p-values are 0.57 and 0.14, respectively), FTSE100 forecast errors are negatively serially correlated at lag one (-0.12), which is borderline significant. All in all, especially given that FTSE100 excess returns fail to display any autoregressive structure at the univariate level, we interpret this evidence as roughly consistent with the absence of obvious misspecifications in our conditional mean functions.[27]

Next, we examine the ability of variables in the information set to predict squared forecast errors. In case of rejections of the no predictability restriction, this test can be interpreted as a test of omitted volatility clustering and ARCH effects in the model. There is only borderline evidence of some positive and significant first-order serial correlation in squared forecast errors for UK and US excess returns, while both past own and cross-excess returns fail to predict subsequent squared forecast errors. All in all, there is no evidence of a need of specifying ARCH effects on the top of making $\mathbf{\Sigma}_{s_t}$ a function of the state.[28]

4. Economic Implications

4.1. *Dynamic links across markets: impulse response analysis*

Models (1) and (3) are of course amenable to computing impulse-response functions (IRFs) at several horizons of interest. For instance, assuming $q = 1$, in (1) a unit, standardized shock to the excess return of market i, $\mathbf{e}_i$ $(i = 1, 2, 3)$, generates the impulse response function:

$$E[\Delta\mathbf{x}_{t+h}|\Im_t] = \sum_{s=1}^{k}(\hat{\pi}_t'\hat{\mathbf{P}}^h\mathbf{e}_s)\hat{\mathbf{A}}_s E[\Delta\mathbf{x}_{t+h-1}|\Im_t] \quad h \geq 1, \tag{5}$$

with $E[\Delta\mathbf{x}_{t+1}|\Im_t] = \sum_{s=1}^{k}(\hat{\pi}_t'\widehat{\mathbf{P}}\mathbf{e}_s)\hat{\mathbf{\Omega}}_s\mathbf{e}_i$, $\hat{\mathbf{P}}^h \equiv \prod_{i=1}^{h}\hat{\mathbf{P}}$, and $\Im_t = \{\mathbf{x}_q\}_{q=p}^{t}$. $\hat{\pi}_t'\hat{\mathbf{P}}^h\mathbf{e}_s$ is simply the $h-$step ahead prediction of the probability of state $s = 1, ..., k$, where $\hat{\pi}_t$ is the vector of state probabilities. $E[\Delta\mathbf{x}_t|\Im_t] = \sum_{s=1}^{k}(\hat{\pi}_t'\widehat{\mathbf{P}}\mathbf{e}_s)\hat{\mathbf{\Omega}}_s\mathbf{e}_i$ stresses that a unit standardized shock translates into a different return shocks, depending on the current regime, where $\hat{\mathbf{\Omega}}_s$ is the state s Choleski decomposition of the regime-specific covariance matrix, $\hat{\mathbf{\Omega}}_s\hat{\mathbf{\Omega}}_s' = \hat{\mathbf{\Sigma}}_s$.[29] Hats on all the matrices of interest stress that what we can

[27] We also examine the ability of lagged excess returns of market i to predict forecast errors of market j, $i \neq j$. We find that there is some linear (cross-) structure only in FTSE100 errors; in particular, $t-1$ excess S&P returns predict time t FTSE100 forecast errors.

[28] We formally test a regime switching ARCH(1) specification in which

$$\mathbf{\Sigma}_t = \mathbf{\Lambda}_{0s_t} + \mathbf{\Lambda}_{1s_t}\varepsilon_t\varepsilon_t'.$$

This specification implies specifying 18 additional parameters, the elements of the matrix $\mathbf{\Lambda}_{1s_t}$. A LR test resoundingly rejects this specification, consistently with our conclusion of no ARCH at the univariate level in Section 3.1.

[29] This implies that a unit, standardized shock to the excess return of national stock market i will be accompanied by contemporaneous shocks to the other markets, in accordance with the structure of the covariance matrix. The interpretation is that we must take into account that random influences on asset returns rarely appear in isolation, but tend instead to take the form of spreading bull or bear waves.

calculate is purely an estimate of the impulse response function, based on MLE full-sample estimates of the parameters. The formula has a clear recursive structure that reduces to the familiar VAR(1) impulse-response only when $k = 1$:

$$E[\Delta\mathbf{x}_{t+h}] = \hat{\mathbf{A}}^h \hat{\mathbf{\Omega}} \mathbf{e}_i.$$

(5) can be cumulated to record the total deviation of the vector of excess stock returns from their 'baseline' level, absent the assumed standardized, unit shock:

$$\sum_{f=1}^{h} E[\Delta\mathbf{x}_{t+f}|\Im_t] = \sum_{f=1}^{h} \left[\sum_{s=1}^{k} (\hat{\pi}_t' \hat{\mathbf{P}}^f \mathbf{e}_s) \hat{\mathbf{A}}_s E[\Delta\mathbf{x}_{t+f-1}|\Im_t] \right] \quad h \geq 2.$$

These definitions obviously admit an immediate extension to the case of model (3).

An IRF analysis appears particularly justified in our study, as the presence of a VAR component in the model providing the best fit to the data implies in principle that delayed linkages exist across national markets. However, the structure of (1) should make it clear that pure shocks to national stock markets are hard to define and highly counterfactual: our estimates of $\hat{\mathbf{\Sigma}}_s$ in both regimes implies a positive (and in the bear state, substantial) correlation across markets that should be taken into account. Moreover, even though many VAR coefficients turned out to be statistically significant in Table 3, it remains to be seen whether the interactions among significant and insignificant coefficients does deliver IRFs that are 'estimated' accurately enough to deliver economically meaningful results.

Figure 4 starts by showing the IRFs of a one-standard deviation shock to each of the three national stock markets when the initial state is bull. The figure contains nine distinct plots which should be read in the following way: each row of plots shows the effects of a one-standard deviation shock on the three markets. The graphs also report 95 percent confidence bands obtained using (parametric) bootstrapping methods.[30] Shaded areas inside the IRFs highlight horizons h for which the null of no statistically non-zero effect may be rejected (i.e. these are regions over which the confidence bands fails to be so wide to include the zero). In the bull state, very few shocks have persistent, cumulative impact. In particular, a one-standard deviation increase in US excess returns tends to be quickly absorbed by the US market and leave a small cumulative impact of only 0.1 - 0.2 percent after 2-3 months to then decline to an overall effect of approximately -0.2 percent, which is however not significantly different from zero. A US shock has instead a rather small and transitory, but still significant positive effect on UK excess returns (roughly -0.2 percent at 2-3 months). Consistent with the small coefficient in Table 3, a US shock fails to significantly impact the ISEQ. Finally, a UK shock has only significant effect on itself, with an interesting sinusoidal shape in the cumulative effect, that starts with the same sign as the shock and then changes sign at a 2-3 month horizon. As expected, a Irish shock has effect only on the ISEQ, with a cumulative pattern similar to the US one, although the magnitudes are larger (e.g. the long-run effect is -0.5 percent, not statistically significant).

Figure 5 repeats the experiments in Figure 4, when the initial state is bear. Results are qualitatively similar, although the magnitudes involved are an order of magnitudes larger. This is fully consistent with

[30] A large number (5,000) of IRFs are generated in correspondence of a given type of shock (and initial state): each IRF is computed by randomly drawing (for $h \geq 1$) both regimes and state-specific shocks from the estimated two-state models in Table 3. The 95 percent bands are obtained by reporting the 2.5 and 97.5 percentiles of the distribution of the responses in correspondence for each h.

the fact that many of the estimated VAR coefficients are larger in the bear state. For instance, a time 0 shock to excess ISEQ returns is persistent and generates a short-term effect of approximately 1.4 percent while the cumulative effect is just below 2.2 percent. We also produced (unreported) IRFs for the case in which the initial state is unknown and the state probabilities are initialized to correspond to their ergodic (long-run) counterparts. In this case the statistically significant responses are limited to the impact of shock in one market to the market itself.[31]

Overall, the result that weak (in the bear state) or ambiguous (in their statistical strength) cumulative responses follow sizeable shocks to excess returns in the three stock markets is consistent with the commonly maintained notion that the linear (vector autoregressive) structure present in international stock returns hardly allows one to find low frequency evidence of systematic, delayed reactions of national stock markets to other markets (see e.g. Solnik and McLeavey, 2004). However, other patterns of comovement have been isolated, patterns that essentially rely on bull and bear states being common across markets. The remaining sub-sections characterize these commonalities and explore the portfolio implications of our multivariate regime switching model.

4.2. *Time-varying predicted risk premia and second moments*

Another obvious way in which we can gauge the economic implications is by calculating the one-step ahead predictions of risk premia and volatilities characterizing the three stock markets. These are obviously crucial pieces of information relevant to portfolio mangers interested in international portfolio diversification when a rampant, high-return emerging market belongs to the menu. To this purpose, we proceed to the recursive estimation of our two-state regime switching VAR(1) model over the period 1995:01 - 2004:12. This means that the first estimation uses data for the interval 1978:05 - 1995:01 (i.e. 201 observations), the second for 1978:05 - 1995:02 (202 observations), etc. This recursive updating of the parameter estimates implied by (1) captures the expanding learning of an investor who uses the model to characterize the dynamic properties of international equity markets.

Figure 6 shows one-step ahead predicted risk premia, volatilities, and correlations resulting from such a recursive updating process.[32] Clearly, risk premia tend to substantially fluctuate over time, and are sometimes negative even only for relatively short periods of time. In particular, two different periods can be easily isolated: during 1995 - 1998 and then again from mid-2003 to 2004, predicted risk premia are generally positive and scarcely volatile, always falling in the narrow range 0 - 3 percent per month; on the contrary, over 1999-2003 (and in particular in 2001 and 2002) risk premia appear extremely volatile and often turn negative, with one-month ahead spikes below -5 percent. Additionally, the predicted risk premia tend to move in a largely symmetric fashion across stock markets. Although the plot allows one to detect a few episodic differences, they never exceed 0.3-0.5 percent per month, in absolute value. This implies that in a two-state VAR(1) model, the specific emerging, small open economy features of the Irish stock market fail to be reflected in systematically higher or different risk premia.

[31] Since the ergodic probability of the bear state exceeds the one for the bull state, the ergodic IRFs are similar to those obtained for the bear state.

[32] While predicted risk-premia are simply calculated as a predicted-probability weighted average of state-specific risk premia, predicted volatilities adjust for possible switches in means between t and $t+1$ as shown by Timmermann (2000). The same applies to the correlations presented later on.

The second panel of Figure 6 offers a similar picture for predicted monthly volatilities of excess returns in each of the three markets. Differently from the risk premium case, volatilities are systematically different across national markets: the ISEQ is always predicted to be most volatile market, with forecasts always between 3.5 and 5.8 percent per month; the FTSE 100 follows and actually displays a considerably wider range of variation, 2.7 to 5.5 percent; the S&P 500 is structurally more stable with modest fluctuations in the range 3 - 4 percent. A few differences appear to be related to time. For instance, the three volatility series start out relatively distinct in the mid-1990s, but after 1998 the UK stock market displays high variation and in some periods seems to mimic the high volatility dynamics of the ISEQ (e.g. 1999 and 2001), while in others it actually settles to the low volatility implied by the S&P 500 (e.g. from mid-2003 to 2004).

The last panel of Figure 6 shows the dynamics of the predicted, one-month ahead correlation between the ISEQ and the two major Anglo-Saxon stock markets.[33] Also in this case, while over 1995-1997 the anticipated correlation between ISEQ and FTSE 100 and S&P 500 were similar and relatively moderate (around 0.4), after 1998 the pairwise correlation with the FTSE 100 becomes systematically higher than the one with the S&P. While the former correlation is generally high and often exceeds 0.5, the latter gravitates around 0.3 for the rest of the sample, and in some months drifts towards zero. This means that for most of the sample the ISEQ appears to offer appreciable hedging benefits vs. the US national stock market and more moderate ones vs. the UK market. Interestingly, our model fails to imply that the correlations between the Irish market and the two major markets would be drifting up in time; to the contrary, the end-of-sample implied correlations of 0.3 - 0.4 are similar to those characterizing the mid-1990s and allow in principle enormous diversification benefits.

4.3. *Sharpe-ratio dynamics*

Once values for predicted risk premia and volatilities are available, it becomes natural to proceed to recursively calculate one-month ahead predicted Sharpe ratios, which give an indication for the recursive behavior over time of the expected reward-to-risk ratio perceived by an investor using the two-state VAR(1) model of Section 3.2. Since computing a Sharpe ratio implies the availability of a riskless interest rate and in our sample we have three such series available, in the following we specialized to the viewpoint of a US investor, i.e. compute the predicted Sharpe ratio as:

$$\widehat{SR}^{i}_{t+1}|\Im_t = \frac{E[x^i_{t+1} + r^i_t|\Im_t] - r^{US}_t}{Var[x^i_{t+1}|\Im_t]}, \tag{6}$$

where $E[x^i_{t+1} + r^i_t|\Im_t]$ and $Var[x^i_{t+1}|\Im_t]$ are computed using $\hat{\pi}_t \in \Im_t$, the vector of recursive state probabilities. Notice that $E[x^i_{t+1} + r^i_t|\Im_t] - r^{US}_t$ converts local currency net stock returns into excess returns in the perspective of a US investor, with both r^i_t and r^{US}_t known at time t (they are short-term interest rates).[34]

Given our finding in Section 4.2 that the predicted risk premia are relatively close to each other over our sample while heterogeneity exists in the dynamics of predicted volatilities, it seems clear that Sharpe ratio forecasts in this Section will be mostly driven by the time variation in the latter. However, since (6) comes

[33] An equivalent plot concerning the pairwise correlation between FTSE and S&P excess returns is omitted to save space and is available upon request. It shows a pairwise correlation that oscillates in the narrow range 0.4 - 0.6.

[34] The 'hat' on the Sharpe ratio ($\widehat{SR}^{i}_{t+1}$) indicates that this is a predicted reward-to-risk ratio.

in the form of a ratio and not of a simpler difference, it is unclear whether heterogeneous volatility dynamics will be sufficient to induce large differences in the evolution of the three Sharpe ratios. Figure 7 gives an answer which is essentially negative: the ratios fundamentally inherit the dynamic behavior of predicted expected returns at their numerators. In practice, between 1995 and mid-1998 the predicted Sharpe ratios remain positive in the three countries and oscillate around an average of approximately 0.4, which seems to be a rather typical value for bull markets. If any, over this period it is the UK market that displays the highest Sharpe ratio, although differences are generally small, between 0.05 and 0.2. From mid-1998 and until the end of the sample (but the peak is reached in 2002-2003), the predicted reward-to-risk ratio becomes highly volatile and often breaks into the negative numbers. Towards the end of the sample (after mid-2003), the Sharpe ratios remain rather unstable but they also tend to structurally increase towards average levels again close to 0.4, mostly as a result not of unusually high forecast of one-month ahead risk premia, but as a consequence of the generalized decline in volatilities (see Figure 6).

These results suggest two preliminary conclusions. First, the ISEQ seems to compensate risk in ways that are perfectly consistent with the ratios that are typical of major, developed markets. This means that the differences in volatilities apparent in Figure 6 are in the end small enough to go unreflected by the corresponding Sharpe ratio. Second, a situation in which correlations are systematically below 1 (between 0.3 and 0.6, see Section 4.2) and in which Sharpe ratios are essentially similar across markets suggests the existence of enormous potential for international portfolio diversification. Section 4.4 tests this conjecture.

4.4. *Implications for optimal portfolio decisions*

In this section we proceed to the recursive calculation of optimal mean-variance portfolio weights and assess the comparative (pseudo) out-of-sample portfolio performance of our two-state regime switching model vs. a few benchmarks of common usage.[35] Assume an investor has preferences described by a simple mean-variance functional:

$$\begin{aligned} V_t &= E_t[W_{t+1}] - \frac{1}{2}\lambda Var_t[W_{t+1}] \\ W_{t+1} &= \omega_t^{ISEQ}(1 + x_{t+1}^{ISEQ} + r_t^{IRL}) + \omega_t^{FTSE}(1 + x_{t+1}^{FTSE} + r_t^{UK}) + \omega_t^{S\&P}(1 + x_{t+1}^{S\&P} + r_t^{US}) \\ &\quad + (1 - w_t^{ISEQ} - w_t^{FTSE} - w_t^{S\&P})(1 + r_t^{US}) \end{aligned} \qquad (7)$$

where λ is interpreted as coefficient of risk aversion that trades-off (conditional) predicted mean and variance of the one-step ahead wealth. At each time t in the sample, the investor maximizes V_t by selecting weights $\omega_t \equiv [\omega_t^{ISEQ}\ \omega_t^{FTSE}\ \omega_t^{S\&P}]'$ when the predicted moments are calculated using some reference statistical model, e.g. our two-state model. Simple algebra shows that:

$$\tilde{\omega}_t = \frac{1}{\lambda}\hat{\mathbf{\Sigma}}_t^{-1}\left(\hat{\mu}_t + \hat{\mathbf{A}}_t\mathbf{x}_t + \mathbf{r}_t - r_t^{US}\iota_3\right),$$

[35] Ramchand and Susmel (1998) fit bivariate regime switching ARCH models to pairs of local currency equity index returns and also proceed to calculate mean-variance portfolio weights. They find that in the high-variance state, when correlations are high, the international diversification benefits are small so that it becomes optimal for a US investor to hold 100% of her portfolio in domestic equities; in the low-variance state, when correlations are modest, diversification benefits would be larger and investing abroad becomes optimal. However, Ramchand and Susmel (1998) restrict the regime-dependent correlations to depend on a state defined by US stock returns only.

where the time index appended to the matrices $\hat{\mathbf{\Sigma}}_t$, $\hat{\mu}_t$, and$\hat{\mathbf{A}}_t$ reflects the possibility that parameters may be a function of the state and hence of current time. $\mathbf{r}_t$ is a 3×1 vector that collects the short-term yields for each of the three markets. As in Section 4.3, we solve the problem from a US viewpoint, which explains why (after calculating predicted one month local returns, $\hat{\mu}_t + \hat{\mathbf{A}}_t\mathbf{x}_t + \mathbf{r}_t$) we are subtracting r_t^{US} from predicted mean returns on all markets. Portfolio weights are calculated recursively using the recursive parameter estimates underlying Sections 4.2-4.3.[36] (7) is solved both without and with restrictions on the admissible range for ω_t; in particular, in what follows we compute and discuss weights that prevent the investor from selling any securities short, i.e. such that $\omega_t'\mathbf{e}_j \in [0,1]$ $\forall t$ and $j = ISEQ$, $FTSE$, $S\&P$.[37]

As hinted at earlier, in this section we extend the calculation of portfolio weights for a range of commonly employed benchmarks:

- A simple, myopic IID model,
$$\mathbf{x}_t = \mu + \varepsilon_t \quad \varepsilon_t \sim N(0, \mathbf{\Sigma}), \tag{8}$$
in which $q = 0$, and μ and $\mathbf{\Sigma}$ are matrices of constant parameters. Clearly, μ and $\mathbf{\Sigma}$ will have to be recursively estimated over time as the vector of sample means and the sample covariance matrix, respectively.

- A single-state, VAR(1) model,
$$\mathbf{x}_t = \mu + \mathbf{A}\mathbf{x}_{t-1} + \varepsilon_t \quad \varepsilon_t \sim N(0, \mathbf{\Sigma}), \tag{9}$$
in which only risk premia are predictable according to the simple law $E_t[\mathbf{x}_{t+1}] = \mu + \mathbf{A}\mathbf{x}_t$. In this case, variance and covariances are restricted to be constant over time. This is the model employed, for instance, by Campbell and Viceira (1999).

Figure 8 starts by showing plots of recursive, one-month ahead predicted Sharpe ratios for each stock market and under each of the three models investigated. Independent of the stock market under investigation, the plots show the existence of striking differences between the IID Sharpe ratios and ratios implied by models that account for predictability. While the former model generates ratios that are small (generally between 0 and 0.15) and that change smoothly over time as a consequence of recursive updating, the predictability models induce substantial variation in Sharpe ratios, which are actually often predicted to be negative. Some differences exist also between regime switching and VAR Sharpe ratios, as the latter tend to be less volatile than the MMS ratios. Moreover, periods can be found in which the two models imply rather heterogeneous ratios and hence potentially different portfolio implications.

Figure 9 compares the evolution of portfolio weights induced by the three different models in the case $\lambda = 0.5$. Table 4 provides summary statistics. Differences are striking, as observed in Figure 8 with reference to one-step ahead Sharpe ratios: the IID model generates almost no demand for stocks, independently of whether short sales are admitted or not; the only positive (but still small) weights are obtained for the S&P 500 index. Clearly, this is less than surprising since equities have (for instance) end-of-sample monthly Sharpe ratios of 0.05 (for the S&P 500) and lower (essentially zero for the FTSE 100). A single-state VAR(1) model generates a higher demand for stocks, although it remains still moderate: 3-4

[36] For instance, $\tilde{\omega}_{1995:01}$ is based on estimated parameters obtained using data for the interval 1978:05 - 1995:01, etc.
[37] When short-sales are restricted, $\tilde{\omega}_t$ has no closed-form solution and is therefore calculated numerically (by grid search).

percent for Ireland and 9 percent for the S&P 500. The bulk of the portfolio remains invested in the (US) riskless asset (85 percent). Removing short-sale possibilities marginally increases the equity weights, to a total of approximately 20 percent. Finally, a regime switching model implies much larger portfolio weights. When short-sales are admitted, the investment in stocks tends generally to be moderate, on average around 40-50%. Although rich temporal dynamics can be detected and periods (e.g. 2001-2002) exist in which the net weight to all stocks ought to be negative (i.e. $w_t^{ISEQ} + w_t^{FTSE} + w_t^{S\&P} < 0$), in general there is a tendency towards a thorough diversification across the three national stock markets. Table 4 provides a more accurate image by reporting mean values of portfolio weights over our recursive exercise. The mean investment in stocks is 43 percent, with a prevalence of US and UK stocks (16 and 15 percent). However, standard deviations are high, as a reflection of the remarkable time variation displayed by Figure 9. There is some evidence that the larger variations in the predicted Sharpe ratio for US stocks (see Figure 8) may often command short position in this market, especially to finance the purchase of UK stocks.

Table 4 in fact reports summary statistics for recursive portfolio weights under regime switching also for other values of λ, i.e. 0.2, 1, and 2. However, it is clear that when short sales are permitted, the optimal weights are simply multiples of those previously obtained under $\lambda = 0.5$, since

$$\frac{\tilde{\omega}_t(\lambda_1)}{\tilde{\omega}_t(\lambda_2)} = \frac{\frac{1}{\lambda_1}\hat{\mathbf{\Sigma}}_t^{-1}\left(\hat{\mu}_t + \hat{\mathbf{A}}_t\mathbf{x}_t + \mathbf{r}_t - r_t^{US}\iota_3\right)}{\frac{1}{\lambda_2}\hat{\mathbf{\Sigma}}_t^{-1}\left(\hat{\mu}_t + \hat{\mathbf{A}}_t\mathbf{x}_t + \mathbf{r}_t - r_t^{US}\iota_3\right)} = \frac{\lambda_2}{\lambda_1}.$$

This is not the case when no-short sale constraints are imposed, although a rough proportionality across values of λ may be preserved. The table shows that for low λs, an investor obviously becomes very aggressive and would invest roughly 100 percent of her wealth in stocks. UK and US equities play equal roles (with weights around 35 percent), while Irish equities enter with a weight between 25 and 30 percent. Higher λs (1 and 2) imply moderate demand for equity. In general, imposing no-short constraints has the effect of increasing the demand for the S&P 500 and to reduce the weight of the other two portfolios, an indication that the more extreme oscillations of the US Sharpe ratio may induce investors to go short in it, to finance positive demands of the two other portfolios.

Table 5 completes the analysis by showing the (pseudo) out-of-sample, one month portfolio performance under different levels of λ and for the three competing models. In particular, we report mean one-month net portfolio return, the lower and upper values of a standard 95% confidence interval (that reflects the volatility of portfolio returns over 1995:01 - 2004:11), and the implied Sharpe ratio that adjusts mean returns to account for risk.[38] The final eight columns of Table 5 report performance measures also for portfolios that fail to include the US riskless asset, i.e. pure equity portfolios in which $w_t^{ISEQ} + w_t^{FTSE} + w_t^{S\&P} = 1$.[39] The table reports in bold the maximum values of mean portfolio returns and of the Sharpe ratio across models. Obviously, the boldface font abounds in the third panel, where performance results for the two-state regime switching VAR appear: mean performance is always superior for all portfolios including the riskless asset,

[38] Clearly, performance evaluation may be performed only over 1995:01 - 2004:11; this implies the loss of one observation, with negligible consequences. Similar to Aslanidis, Osborn, and Sensier (2003), it is vital to test the out-of-sample performance of nonlinear models of stock returns dynamics, as the potential for overfitting the sample observations is an obvious concern.

[39] Notice that the four columns concerning the pure equity portfolios performance are identical across values for λ. The algebra of mean-variance portfolio optimization implies that when a riskless asset is available a two-fund separation result applies, such that heterogeneous risk preferences only produce different demands for the riskless asset and a homogeneous portfolio of the risky assets.

independently of the assumed value for λ. For most values of λ and in this case quite independently of the fact that portfolios are allowed to include the riskless asset, the MMS framework also produces the best possible Sharpe ratios. For instance, when $\lambda = 0.5$ and a no short sale constraint is imposed, a regime switching asset allocation obtains a 1.26 percent per month average performance (higher than 1.20 for a VAR strategy and 0.86 for an IID one); however, some performance risk should be taken into account, as a 95 percent interval spans $[-0.7, 3.2]$, i.e. covers a negative region. Even adjusting for risk, the Sharpe ratio is 0.72, which is higher than those produced by competing models. In general and in a Sharpe ratio metric, the distance between the MMS model and the benchmarks tends to increase when higher values of λs are considered: for instance, when $\lambda = 2$ the MMS ratio is 1.2, vs. 1 for both VAR and IID models. For pure equity allocations, the implied Sharpe ratios are systematically lower, but the regime switching framework still tends to systematically outperform other models.

In conclusion, not only we have found strong statistical evidence of regimes in the multivariate distribution of the three national stock index (excess) returns under investigation, but we have also evidence that such regimes are economically important insofar as they systematically improve the out-of-sample performance of a mean-variance portfolio optimization strategy. Importantly, Table 4 shows that − despite the good performance of the Irish stock market over the past 25 years celebrated by much of the media during the late 1990s − Irish stocks ought to enter the optimal portfolio, but never with dominant weights and more in virtue of the good linear hedging properties (low correlation) of the ISEQ vs. other stock indices than as a result of exceptional expected mean returns or Sharpe ratios.

5. **The Role of Short-Term Interest Rates**

It is well known (see e.g. Keim and Stambaugh, 1986) that short-term interest rates are accurate and useful predictors of subsequent, realized excess equity returns. Moreover, recent papers by Ang and Bekaert (2002), Bredin and Hyde (2005) and Guidolin and Timmermann (2005b) document that such a relationship between stock returns and interest rates may contain important nonlinear components that ought to be carefully modeled. Therefore this section briefly considers extending the results of Section 3.2 when short-term yields in the three economies under consideration are added to the vector of excess returns to form a 6×1 vector $\mathbf{y}_t$ (i.e. $l = 3$, in the notation of Section 2). The result is a version of (3) in which interest rates both linearly predict subsequent excess stock returns and at the same time contribute to the empirical characterization of the non-linearities (regimes) required to characterize the data.[40]

For simplicity, we refrain here from conducting afresh the model specification search previously undertaken in Section 3.2 and proceed instead to simply estimate a two-state multivariate regime switching VAR(1) model generalized to include interest rates. Parameter estimates are reported in Table 6. Once more, panel A presents a benchmark single-state VAR(1) model, while panel B is devoted to the regime switching estimates. Panel A gives already interesting indications: the (US) short-term interest rate predicts with the expected sign (negative, as implied by a simple version of Gordon's dividend discount model) excess stock returns only in the US case; the coefficient is large (in absolute value) and highly significant. The UK rate also (weakly) influences US stock markets, although the sign is difficult to interpret. Similar to Table 3, Irish excess returns keep displaying a positive serial correlation coefficient, while the opposite

[40]Bredin, Gavin, and O'Reilly (2003) find that monetary policy conditions in the Euro-area and the UK have weak influence on Irish stock returns, while the opposite finding holds for (unanticipated) US monetary conditions.

applies to UK excess returns. All short rates are highly serially correlated, although distant from containing a unit root. While the US monetary policy influences interest rates in all other countries with the expected sign (positive, see Obstfeld and Rogoff (1995), Canova and De Nicoló (2000) and Kim (2001)), there is evidence that also UK policy impacts Irish short-term rates, consistently with Walsh (1993). Shocks to interest rates are only weakly correlated with excess return shocks.

Panel B presents regime switching estimates. Both states are persistent, with an average duration of 30 months for state 1 and 10 months for state 2. Their interpretation is made easy by computing within state unconditional (monthly) means:

$$E[\mathbf{y}_t|S_t = 1] = [0.39\ \text{-}0.40\ 0.51\ 0.33\ 0.47\ 0.32]'$$
$$E[\mathbf{y}_t|S_t = 2] = [\text{-}0.56\ 0.56\ 0.20\ 1.34\ 0.95\ 0.87]'$$

(the first three elements are equity risk premia). It is natural to start the interpretation from the second state, when nominal interest rates are high (double digit, from 10.4 percent in the US to 16.1 percent in Ireland) and risk premia are low in the US (2.4 percent per year) and in Ireland (negative, -6.7 percent); on the opposite, the UK risk premium is high (6.7 percent). State probability plots (unreported) clearly identify this state with the period 1978-1984, although the state episodically reappears for a few months in the late 1980s, early 1990s, and late 1990s. The fact that in an otherwise bear state, the FTSE risk premium is positive and substantial is well explained by the different reaction of the countries involved to the oil shocks of the late 1970s and early 1980s, and with the real-side effects of the exploitation of the North Sea gas reserves by the UK: for instance, while the sample mean of excess equity returns over 1978:05 - 1984:12 has been -9 and -4 percent (in annualized terms) for the ISEQ and the S&P 500, respectively, the figure turns positive (3.2 percent) for the UK. On the other hand, it is well known that nominal interest rates soared to double-digit heights over this very period, as a reaction to the inflationary pressures caused by the supply-side shocks and the subsequent anti-inflationary stance assumed by the FED.

In this state, US monetary policy strongly affects all the stock markets with the expected, negative sign. Interestingly, the corresponding coefficients are rather similar, in the range -4.5 to -5.5 and imply that a one standard deviation expansionary impulse to US short-term rates would cause a stock market reaction that goes from a +1.4 percent in the UK to a +1.8 percent in the US. Also, UK monetary policy responds somewhat to US policy. In this state this is not the case for Irish policy, a finding probably explained by the different degree of accommodation of the inflationary phenomenon in the early part of our sample. The UK rate also influences the stock markets under analysis. Finally, in this state interest rates appear to be somewhat volatile, while the pairwise correlations involving excess equity returns are below their unconditional counterparts.

The first state is a regime of low nominal interest rates (between 4 and 6 percent per year) and high risk premia in Ireland and the US (4.7 and 6.1 percent per year, respectively). However, the UK risk premium is negative.[41] State probability plots confirm that roughly 90% of the period 1985-2004 following the oil shocks is in fact captured by this regime. In this state, international monetary policy has ambiguous effects on stock prices: while in the US the channel work in the traditional way (i.e. higher interest rates cause negative excess stock returns, although with a coefficient that is approximately half the coefficient that is

[41] This fact is unsurprising: after the oil shocks (i.e. 1985-2004) the sample average of the FTSE-100 excess return has been a meager -0.08 percent per month.

estimated in the second state), Irish excess returns seem to react positively to monetary policy tightening in the US and negatively (and significantly) to tightening in the UK, and once more FTSE excess returns positively respond to domestic increases in the interest rate. In this state, interest rates are more highly serially correlated than unconditionally and display low volatility.

Figure 10 shows one-month ahead Sharpe ratios calculated from the two-state VAR(1) model in Table 6. Once more, the reward-to-risk ratios are computed in the perspective of a US investor.[42] Qualitatively, it is difficult to detect visible differences vs. the predicted Sharpe ratios in Figure 7. This is confirmed by pairwise correlations coefficients between the one-month ahead Sharpe ratios in Figures 7 and 10: they are 0.85, 0.83, and 0.87 for the ISEQ, FTSE 100, S&P 500, respectively.[43] For instance, also in a model in which interest rates predict excess stock returns, the period 1995-1998 is characterized by relatively stable and positive Sharpe ratios, while 2000-2003 is marked by high volatility. Also in this case, the Sharpe ratios strongly comove (correlation coefficients range between 0.86 and 0.97), although there is some tendency for the ISEQ first (over 1995-1996) and the S&P 500 later (after 2002) to imply the highest reward-to-risk ratios. Although recursive portfolio calculations could be explicitly performed afresh, it appears difficult that even when the predictive role of interest rates is taken into account, substantially different portfolio choice implications could be found.

6. **Conclusions and Extensions**

In this paper we have documented that − despite the stellar (+15% a year on average, over the period 1994-1999), "Tiger-like" performance of the Irish stock market during the 1990s − the multivariate process of Irish, UK, and US excess equity returns is characterized by substantial nonlinearities − in the form of regimes − that make the long-run, overall 'association' among the ISEQ, the FTSE 100, and the S&P 500 much higher than commonly thought of. In particular, using nonlinear models it turns out that simple analysis of pairwise correlation coefficients may be misleading. In fact, despite moderate correlation coefficient estimates between Irish, UK, and US excess returns, we find that state comovements involving the three markets are so relevant to depress the optimal mean-variance weight carried by ISEQ stocks to at most one-quarter of the overall equity portfolio. In this sense, it seems that international bull and bears shared by the more developed US and UK equity markets involve the Dublin's stock exchange so heavily to greatly reduce the diversification benefits available through indirect equity investments in Ireland.

Several extensions of our framework would be of interest. For instance, Kim, Moshirian and Wu (2005) have reported that standard bivariate ARMA-EGARCH methods would offer evidence of a regime shift in the integration of twelve national stock markets following the creation of the EMU, represented by a dummy variable. Their sample of countries includes also Ireland, Japan and the US, although their analysis is eventually performed for a weighted basket of Euro-zone countries in which the ISEQ plays a minor role. Aggarwal, Lucey, and Muckley (2004) report similar results for European stock markets and also highlight the importance of the EMU in the integration process. Although our focus was dominantly on the portfolio implications of a small open economy stock market displaying time-varying, linear and

[42]Notice that the fact that interest rates are now recognized to be random does not affect Sharpe ratio calculations: the interest relevant for the time $t+1$ prediction is the one known in advance at time t.

[43]Mean forecasts of the ratios are essentially identical with the exception of the UK market, where the forecasts in Figure 10 are 30 percent lower than those in Figure 7.

nonlinear dynamic linkages with two other major international markets, our analysis on a longer sample implies that correlations (and other forms of association) have been simply time-varying, not necessarily increasing over time (see e.g. Figure 6),[44] while our tests have shown that on monthly data a two-state VAR model provides a fit that hardly requires ARCH-type components. Furthermore, Kearney's (1998) study of the (cointegrating) relationships between the ISEQ and the FTSE suggests that the exchange rate (in particular, its volatility) may play a dominant role in determining strength and nature of their linkages. It would be natural to extend Section 5 to incorporate such additional macroeconomic influences.

Finally, although our paper has used models in the multivariate Markov switching class to capture nonlinearities in the joint dynamics of three stock market indices, other choices in terms of nonlinear modeling would have been possible. For instance, Aslanidis, Osborn, and Sensier (2003, with reference to the relationship between UK and US stock markets) and Bredin and Hyde (2005, with reference to the same set of markets analyzed in this paper) use smooth transition regression models. It would be interesting to also track the consequences of the nonlinearities implied by these models for optimal portfolio strategies.

References

[1] Adjaouté, K., and J.-P., Danthine, 2005, "Portfolio Diversification: Alive and Well in Euro-Land!", *Applied Financial Economics,* 14, 1225-1231.

[2] Aggarwal, R., B., Lucey and C., Muckley, 2004, "Dynamics of Equity Market Integration in Europe: Evidence of Changes over Time and with Events", Institute of International Integration Studies Discussion paper No. 19.

[3] Alles, L., and L., Murray, 2001, "An Examination of Return and Volatility Patterns on the Irish Equity Market", *Applied Financial Economics,* 11, 137-146.

[4] Ang, A., and G., Bekaert, 2002, "International Asset Allocation with Regime Shifts", *Review of Financial Studies,* 15, 1137-1187.

[5] Arshanapalli, B., and J., Doukas, 1993, "International Stock Market Linkages: Evidence from the Pre- and Post-October 1987 Period", *Journal of Banking and Finance*, 17, 193-208.

[6] Aslanidis, N., D., Osborn and M., Sensier, 2003, "Explaining Movements in UK Stock Prices: How Important is the US Market?", Centre for Growth and Business Cycles Research, University of Manchester.

[7] Baele, L., 2005, "Volatility Spillover Effects in European Stock Markets", *Journal of Financial and Quantitative Analysis*, 40, 373-401.

[44] This is consistent with recent findings in Goetzmann, Li, and Rouwenhorst (2005) by which international equity correlation have moved dramatically over the last century and half so that diversification benefits to global investing are simply not constant, but not necessarily declining. Brooks and Del Negro (2004) have a similar implication: since the rise in correlations of the 1990s appears to have been mostly due to irrational exuberance induced by the new information technology, the end of the bubble would have marked a decline in correlations from the previous peaks; therefore diversifying across countries may still be effective in reducing risk. Finally, Adjaouté and Danthine (2005) report that low frequency movements in the time series of return dispersions for European stocks are suggestive of cycles and long swings in return correlations that fail to imply the declining importance of international diversification.

[8] Becker, K., J., Finnerty, and J., Friedman, 1995, "Economic News and Equity Market Linkages Between the US and UK", *Journal of Banking and Finance*, 19, 1191 - 1210.

[9] Bossaerts, P., and P., Hillion, 1999, "Implementing Statistical Criteria to Select Return Forecasting Models: What Do We Learn?", *Review of Financial Studies*, 12, 405-428

[10] Bredin, D., C. Gavin and G. O'Reilly 2003, "The Influence of Domestic and International Interest Rates on the ISEQ", *Economic and Social Review*, 34, 249-265.

[11] Bredin, D., and S. Hyde, 2005, "Regime Change and the Role of International Markets on the Stock Returns of a Small Open Economy", Working paper, University of Manchester.

[12] Brooks, R., and M., Del Negro, 2004, "The Rise in Comovement Across National Stock Markets: Market Integration or IT Bubble?", *Journal of Empirical Finance*, 11, 659-680.

[13] Butler, K., and D. Joaquin, 2002, "Are the Gains from International Portfolio Diversification Exaggerated? The Influence of Downside Risk in Bear Markets", *Journal of International Money and Finance*, 21, 981-1011.

[14] Campbell, J., and R., Shiller, 1988, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors", *Review of Financial Studies*, 1, 195-228.

[15] Campbell, J., and L., Viceira, 1999, "Consumption and Portfolio Decisions when Expected Returns are Time Varying", *Quarterly Journal of Economics*, 114, 433-495.

[16] Canova, F., and G., De Nicoló, 2000, "Stock Returns, Term Structure, Inflation and Real Activity: An International Perspective", *Macroeconomic Dynamics*, 4, 343-372.

[17] Cotter, J., 2004, "International Equity Market Integration in a Small Open Economy: Ireland January 1990 - December 2000", *International Review of Financial Analysis*, 13, 669-685.

[18] Davies, R., 1977, "Hypothesis Testing when a Nuisance Parameter is Present only under the Alternative", *Biometrika*, 64, 247-254.

[19] Engsted, T., and C., Tanggaard, 2004, "The Comovement of US and UK Stock Markets", *European Financial Management*, 10, 593-607.

[20] Fama, E., and K., French, 1988, "Dividend Yields and Expected Stock Returns", *Journal of Financial Economics*, 19, 3-29.

[21] Fama, E., and K., French, 1989, "Business Conditions and Expected Returns on Stocks and Bonds", *Journal of Financial Economics*, 25, 23-49.

[22] Ferson, W., and C., Harvey, 1991, "The Variation of Economic Risk Premiums", *Journal of Political Economy*, 99, 385-415.

[23] Gallagher, L., 1995, "Interdependencies among the Irish, British and German stock markets", *Economic and Social Review*, 26, 131-148.

[24] Gottheil, F., 2003, "Ireland: what's Celtic about the Celtic Tiger?", *Quarterly Review of Economics and Finance,* 43, 720-737.

[25] Garcia, R., 1998, "Asymptotic Null Distribution of the Likelihood Ratio Test in Markov Switching Models", *International Economic Review,* 39, 763-788.

[26] Goetzmann, W., and P., Jorion, 1993, "Testing the Predictive Power of Dividend Yields", *Journal of Finance*, 48, 663-679.

[27] Goetzmann, W., L., Li and K., Rouwenhorst, 2005, "Long-Term Global Market Correlations", *Journal of Business*, 78, 1-38.

[28] Gray, S., 1996, "Modeling the Conditional Distribution of Interest Rates as a Regime Switching Process", *Journal of Financial Econometrics*, 42, 27-62.

[29] Guidolin, M., and G., Nicodano, 2005, "Small Caps in International Equity Portfolios: The Effects of Variance Risk", Federal Reserve Bank of St. Louis working paper No. 2005-075.

[30] Guidolin, M., and A., Timmermann, 2003, "Recursive Modelling of Nonlinear Dynamics in UK Stock Returns", *Manchester School*, 71, 381-395.

[31] Guidolin, M., and A., Timmermann, 2005a, "Economic Implications of Bull and Bear Regimes in UK Stock and Bond Returns", *Economic Journal*, 115, 111-143.

[32] Guidolin, M., and A., Timmermann, 2005b, "International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences", Federal Reserve Bank of St. Louis working paper No. 2005-034.

[33] Hamill, P., K., Opong, and S., Sprevak, 2000, "The Behaviour of Irish ISEQ Index: Some New Empirical Tests", *Applied Financial Economics*, 10, 693-700.

[34] Hamilton, J., 1991, "A Quasi-Bayesian Approach to Estimating Parameters for Mixtures of Normal Distributions", *Journal of Business and Economic Statistics*, 9, 27-39.

[35] Hamilton, J., 1993, "Estimation, Inference, and Forecasting of Time Series Subject to Changes in Regime", in Maddala, G., Rao, C., and Vinod, H., *Handbook of Statistics*, vol. 11., Amsterdam: North Holland.

[36] Heston, S., K., Rouwenhorst, and R., Wessels, 1995, "The Structure of International Stock Returns and the Integration of Capital Markets", *Journal of Empirical Finance*, 2, 173-197.

[37] Kearney, C., 1998, "The Causes of Volatility in a Small Internationally Integrated Stock Market: Ireland July 1975-June 1994", *Journal of Financial Research*, 21, 85-104.

[38] Kearney, C., and B., Lucey, 2004, "International Equity Market Integration: Theory, Evidence and Implications", *International Review of Financial Analysis*, 13, 571-583.

[39] Keim, D., and R., Stambaugh, 1986, "Predicting Returns in the Stock and Bond Markets", *Journal of Financial Economics*, 17, 357-390.

[40] Kim, S., 2001, "International Transmission of U.S. Monetary Policy Shocks: Evidence from VAR's", *Journal of Monetary Economics*, 48, 339-372.

[41] Kim, S., F., Moshirian, and E., Wu, 2005, "Dynamic Stock Market Integration Driven by the Europe an Monetary Union: An Empirical Analysis", *Journal of Banking and Finance*, 29, 2475-2502.

[42] Krolzig, H.-M., 1997, *Markov-Switching Vector Autoregressions*, Berlin, Springer-Verlag.

[43] Leroux, B., 1992, "Maximum Likelihood Estimation for Hidden Markov Models", *Stochastic Processes and their Applications*, 40, 127-143.

[44] Lo, A., and C., MacKinlay, 1990, "An Econometric Analysis of Nonsynchronous Trading", *Journal of Econometrics*, 45, 181-211.

[45] Longin, F., and B., Solnik, 1995, "Is the Correlation in International Equity Returns Constant", *Journal of International Money and Finance*, 14, 3-26.

[46] Longin, F., and B., Solnik, 2001, "Extreme Correlation and International Equity Markets", *Journal of Finance*, 56, 649-676.

[47] Lucey, B., 2001, "Fractionally Integrated Dynamics in Irish Stock Returns", working paper, Trinity College Dublin.

[48] Lund, J., and T., Engsted, "GMM and Present Value Tests of the C-CAPM: Evidence from the Danish, German, Swedish and UK Stock Markets", *Journal of International Money and Finance*, 1996, 15, 497-521.

[49] Obstfeld, M., and K., Rogoff, 1995, "Exchange Rate Dynamics Redux", *Journal of Political Economy*, 103, 624-660.

[50] Ramchand, L., and R., Susmel, 1998, "Volatility and Cross Correlation Across Major Stock Markets", *Journal of Empirical Finance*, 5, 397-416.

[51] Sarno, L. and G., Valente, 2005, "Modeling and Forecasting Stock Returns: Exploiting the Futures Market, Regime Shifts, and International Spillovers", *Journal of Applied Econometrics,* 20, 345-376.

[52] Solnik, B., and D., McLeavey, 2004, *International Investments*, New York, Pearson Addison Wesley.

[53] Timmermann, A., 2000, "Moments of Markov Switching Models", *Journal of Econometrics,* 96, 75-111

[54] Turner, C., R., Startz, and C., Nelson, 1989, "A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market", *Journal of Financial Economics*, 25, 3-22.

[55] Yang J., J., Kolari, and I., Min, 2003, "Stock Market Integration and Financial Crises: the Case of Asia", *Applied Financial Economics,* 13, 477-486.

[56] Walsh, B., 1993, "Credibility, Interest Rates and the ERM: The Irish Experience", *Oxford Bulletin of Economics and Statistics*, 55, 439-452.

[57] Whitelaw, R., 2000, "Stock Market Risk and Return: An Equilibrium Approach", *Review of Financial Studies*, 13, 521-547.

Table 1

Summary Statistics

This table reports summary statistics for percentage monthly excess stock returns and short-term (money market) nominal interest rates for Ireland, the United States, and the United Kingdom. The sample period is 1978:05 – 2004:12.

	Mean	**Median**	**Minimum**	**Maximum**	**Standard deviation**	**Skewness**	**Kurtosis**
	A. Excess Stock Returns						
ISEQ	0.143	0.816	-39.779	14.378	5.330	-1.492	12.419
FTSE-100	0.007	0.701	-32.742	11.969	4.914	-1.276	9.169
S&P 500	0.216	0.705	-25.241	12.731	4.236	-0.889	6.762
	B. Money Market Nominal Interest Rates						
Ireland	0.774	0.732	0.170	3.667	0.464	1.458	8.783
United Kingdom	0.727	0.708	0.250	1.490	0.308	0.357	2.019
United States	0.568	0.504	0.082	1.592	0.317	0.885	3.996

	C. Correlation Matrix					
	ISEQ	FTSE-100	S&P 500	Ireland – interest rate	UK – interest rate	US – interest rate
ISEQ	1.000					
FTSE-100	0.572	1.000				
S&P 500	0.539	0.698	1.000			
Ireland – mon. mkt. int. rate	-0.101	0.044	-0.034	1.000		
UK – money mkt. int. rate	-0.122	-0.019	-0.021	0.680	1.000	
US – money mkt. int. rate	-0.120	-0.012	-0.105	0.653	0.734	1.000

Table 2

Estimates of Univariate Two-State AR(1) Models for Excess Stock Returns

The table reports the estimation output for the MSIAH(*k*,*p*) model:

$$x_t^i = \mu_{s_t^i}^i + a_{s_t^i}^i x_{t-1}^i + \sigma_{s_t^i}^i \varepsilon_t^i$$

where $\mu_{s_t^i}^i$ is the intercept vector in state S_t^i, $a_{s_t^i}^i$ is the first-order autoregressive coefficient in state S_t^i, and $\varepsilon_t^i \sim \text{I.I.D. } N(0,1)$, *i* = ISEQ, UK, US. S_t^i is governed by an unobservable, discrete, first-order Markov chain that can assume two values (states). Excess stock returns are calculated as the log-difference in the total return indices for ISEQ, FTSE-100, and S&P 500 minus a short-term interest rate. The data are monthly. The sample period is 1978:05 – 2004:12. Volatilities are expressed in annualized terms.

	Panel A – Single State Models		
	ISEQ	FTSE-100	S&P 500
1. Intercept	0.102	-0.003	0.211
2. AR(1) Coefficient	0.269***	-0.022	0.041
3. Volatility	17.837***	17.061***	14.705***
4. Unconditional Mean	0.140	-0.003	0.220
5. Unconditional Volatility	17.812	17.065	14.717
	Panel B – Two State Models		
	ISEQ	FTSE-100	S&P 500
1. Intercept			
Regime 1 (bear)	-4.681	-0.563	-1.209*
Regime 2 (bull)	0.825*	1.045***	1.004***
2. AR(1) Coefficient			
Regime 1 (bear)	0.684	-0.015	0.070
Regime 2 (bull)	0.221***	-0.183**	-0.107*
3. Volatility			
Regime 1 (bear)	37.492***	20.458***	19.900***
Regime 2 (bull)	15.000***	7.648**	10.525***
4. Unconditional Mean			
Regime 1 (bear)	-30.931	-0.555	-1.300
Regime 2 (bull)	0.546	0.883	0.907
5. Unconditional Volatility			
Regime 1 (bear)	60.397	20.460	19.949
Regime 2 (bull)	15.380	7.779	10.586
4. Transition probabilities P[*i*,*i*], *i* = 1, 2.			
Regime 1 (bear)	0.142*	0.914***	0.848***
Regime 2 (bull)	0.960***	0.862***	0.931***

* denotes 10% significance, ** significance at 5%, and *** significance at 1%.

Table 3

Estimates of Multivariate Regime Switching VAR(1) Model for Excess Stock Returns

The table reports the estimation output for the MMSIAH(*k,p*) model:

$$\mathbf{x}_t = \boldsymbol{\mu}_{s_t} + A_{s_t}\mathbf{x}_{t-j} + \Sigma_{s_t}\boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, A_{s_t} is the matrix of autorgressive coefficients associated to lag $j \geq 1$ in state s_t and $\varepsilon_t = [\varepsilon_t^1\ \varepsilon_t^2\ \varepsilon_t^3]' \sim \text{I.I.D. } N(0, I_3)$. s_t is governed by an unobservable, discrete, first-order Markov chain that can assume *k* distinct values (states). Excess stock returns are calculated as the log-difference in the total return indices for ISEQ, FTSE-100, and S&P 500 minus a short-term interest rate. The data are monthly. The sample period is 1978:05 – 2004:12. The data reported on the diagonals of the correlation matrices are annualized volatilities. Asterisks attached to correlation coefficients refer to covariance estimates.

		Panel A – Single State Model	
	ISEQ	FTSE-100	S&P 500
1. Mean excess return	0.009	-0.057	0.185
2. VAR(1) Coefficient			
ISEQ	0.359**	-0.174**	0.004**
FTSE-100	0.416*	-0.284**	0.007**
S&P 500	0.333	-0.183	-0.037
3. Correlations/Volatilities			
ISEQ	17.619**		
FTSE-100	0.523**	15.790**	
S&P 500	0.490**	0.657**	13.809**
		Panel B – Two State Model	
	ISEQ	FTSE-100	S&P 500
1. Mean excess return			
Regime 1 (bear/normal)	-0.573	-0.317	-0.407
Regime 2 (bull)	2.124**	1.073**	1.462**
2. VAR(1) Coefficient			
Regime 1 (bear/normal):			
ISEQ	0.442**	-0.377**	0.135**
FTSE-100	0.050	-0.375**	0.048*
S&P 500	0.037	-0.288**	-0.015**
Regime 2 (bull):			
ISEQ	-0.009	0.636**	-0.589**
FTSE-100	0.086	-0.015	-0.190**
S&P 500	0.152*	0.348**	-0.237**
3. Correlations/Volatilities			
Regime 1 (bear/normal):			
ISEQ	19.049**		
FTSE-100	0.529**	18.136**	
S&P 500	0.486**	0.680**	14.927**
Regime 2 (bull):			
ISEQ	9.060**		
FTSE-100	0.165*	6.967**	
S&P 500	0.098	0.449**	8.391***

4. Transition probabilities	Regime 1 (bear)	Regime 2 (bull)
Regime 1 (bear/normal)	0.924**	0.076
Regime 2 (bull)	0.155	0.845**

* denotes 5% significance, ** significance at 1%.

Table 4

Summary Statistics for Recursive Mean-Variance Portfolio Weights under a Multivariate Two-State VAR(1) Model for Excess Stock Returns

The table reports summary statistics for the weights solving the one-month forward mean-variance portfolio problem:

$$\max_{\mathbf{w}_t} E_t[W_{t+1}] - 1/2\,\lambda\, Var_t[W_{t+1}],$$

where W_{t+1} is end-of period wealth and λ is a coefficient of (absolute) risk aversion that trades-off mean and variance. The problem is solved recursively over the period 1995:01 – 2004:12 using in each month updated parameter estimates (and when appropriate, filtered state probabilities) obtained over an expanding sample that starts in 1978:05. The table shows means and standard deviations for recursive portfolio weights. For the case of $\lambda = 1/2$, the table also reports summary statistics for portfolio weights obtained under two benchmark statistical models: the IID (myopic case) in which there is no predictability and means, variances, and covariances are simply updated over time; the single-state, the VAR(1) case in which only risk premia are predictable. Finally, the problem is solved from the point of view of a perfectly hedged US investor, i.e. the riskless interest rate is a short-term US yield.

	Statistic	Short Sales Admitted				No Short Sales			
		ISEQ	FTSE 100	S&P 500	Riskless	ISEQ	FTSE 100	S&P 500	Riskless
$\lambda = 0.2$		Two-State VAR(1) Model							
	Mean	0.288	0.383	0.403	-0.074	0.264	0.367	0.336	0.033
	Standard dev.	0.295	0.274	0.385	0.456	0.177	0.205	0.214	0.160
$\lambda = 0.5$		IID (Myopic) Model							
	Mean	-0.008	-0.011	0.053	0.965	0	0	0.039	0.961
	Standard dev.	0.009	0.011	0.010	0.011	0	0	0.011	0.011
		VAR(1) Model							
	Mean	0.033	0.035	0.085	0.847	0.056	0.061	0.081	0.802
	Standard dev.	0.073	0.080	0.058	0.190	0.037	0.047	0.053	0.122
		Two-State VAR(1) Model							
	Mean	0.115	0.153	0.161	0.571	0.112	0.125	0.170	0.592
	Standard dev.	0.109	0.088	0.154	0.302	0.103	0.110	0.123	0.299
$\lambda = 1$		Two-State VAR(1) Model							
	Mean	0.057	0.077	0.081	0.785	0.062	0.069	0.085	0.784
	Standard dev.	0.058	0.046	0.079	0.146	0.040	0.044	0.052	0.110
$\lambda = 2$		Two-State VAR(1) Model							
	Mean	0.029	0.038	0.040	0.893	0.018	0.025	0.057	0.900
	Standard dev.	0.035	0.029	0.044	0.074	0.021	0.022	0.026	0.054

Table 5

Summary Statistics for Recursive Mean-Variance Portfolio Performances under a Variety of Models for Excess Stock Returns

The table reports summary statistics for the 1-month portfolio return based on weights that solve the one-month forward mean-variance portfolio problem:

$$\max_{\mathbf{w}_t} E_t[W_{t+1}] - 1/2\,\lambda\, Var_t[W_{t+1}],$$

where W_{t+1} is end-of period wealth and λ is the coefficient of (absolute) risk aversion. The problem is solved recursively over the period 1995:01 – 2004:12 using in each month updated parameter estimates (and when appropriate, filtered state probabilities) obtained over an expanding sample that starts in 1978:05. The table shows means and standard deviations for recursive portfolio weights. The problem is solved from the point of view of a perfectly hedged US investor, i.e. the riskless interest rate is a short-term US yield. Boldfaced values for means and Sharpe ratios indicate the best performing model.

Statistic	Unconstrained				No Short-Sales				Pure Equity				Pure Equity, No Short-Sales			
	λ=0.2	λ=0.5	λ=1	λ=2	λ=0.2	λ=0.5	λ=1	λ=2	λ=0.2	λ=0.5	λ=1	λ=2	λ=0.2	λ=0.5	λ=1	λ=2
	IID (Myopic) Model															
Mean	0.87	0.86	0.86	0.86	0.88	0.87	0.86	0.86	1.56	1.56	1.56	1.56	1.41	1.41	1.41	1.38
95% l.b.	-0.27	0.04	0.19	0.27	-0.21	0.05	0.19	0.27	-11.7	-11.7	-11.7	-11.7	-7.45	-7.45	-7.45	-7.47
95% u.b.	2.00	1.68	1.53	1.44	1.97	1.68	1.53	1.45	14.8	14.8	14.8	14.8	10.3	10.3	10.3	10.2
Sharpe rat.	0.52	0.71	0.86	0.98	**0.57**	0.73	0.86	0.98	0.15	0.15	0.15	0.15	0.19	0.19	0.19	0.18
	VAR(1) Model															
Mean	1.71	1.20	1.03	0.94	1.38	1.07	0.96	0.91	**2.54**	**2.54**	**2.54**	**2.54**	**1.79**	**1.74**	**1.86**	**1.91**
95% l.b.	-2.42	-0.57	-0.00	0.21	-1.54	-0.27	0.09	0.22	-13.1	-13.1	-13.1	-13.1	-4.67	-4.71	-4.36	-4.17
95% u.b.	5.85	2.96	2.06	1.67	4.30	2.41	1.84	1.59	18.2	18.2	18.2	18.2	8.25	8.18	8.07	7.99
Sharpe rat.	0.55	0.71	0.89	1.02	0.56	**0.74**	0.90	0.99	0.25	0.25	0.25	0.25	0.38	0.36	0.41	**0.44**
	Two-State VAR(1) Model															
Mean	**1.88**	**1.26**	**1.06**	**0.96**	**1.57**	**1.14**	**1.00**	**0.93**	2.01	2.01	2.01	2.01	1.67	1.66	1.67	1.66
95% l.b.	-2.82	-0.67	0.01	0.30	-2.32	-0.49	0.08	0.32	-9.22	-9.22	-9.22	-9.22	-3.46	-3.46	-3.45	-3.47
95% u.b.	6.58	3.20	2.11	1.61	5.46	2.77	1.91	1.54	13.2	13.2	13.2	13.2	6.79	6.79	6.78	6.79
Sharpe rat.	**0.56**	**0.72**	**0.94**	**1.19**	0.51	0.70	**0.94**	**1.18**	**0.26**	**0.26**	**0.26**	**0.26**	**0.43**	**0.43**	**0.43**	0.42

Table 6

Estimates of Multivariate Regime Switching VAR(1) Model for Excess Stock Returns and Nominal Short-Term Interest Rates

The table reports the estimation output for the MMSIAH(*k,p*) model:

$$r_t = \mu_{s_t} + A_{s_t} r_{t-1} + \Sigma_{s_t} \varepsilon_t$$

The data are monthly. The sample period is 1978:05 – 2004:12. The data reported on the diagonals of the correlation matrices are annualized volatilities. Asterisks attached to correlation coefficients refer to covariance estimates.

	Panel A – Single State Model					
	ISEQ	FTSE-100	S&P 500	Ireland *r*	UK *r*	US *r*
1. Mean excess return	0.724	-1.051	-0.197	0.016	0.040**	0.007
2. VAR(1) Coefficient						
ISEQ	0.352**	-0.161*	-0.018	0.674	0.300	-2.405
FTSE-100	0.043	-0.300**	-0.001	1.621*	0.962	-1.696
S&P 500	0.034	-0.169**	-0.071	0.672	2.255*	-3.120**
Ireland *r*	-0.008	0.009**	-0.001	0.670**	0.178**	0.192**
UK *r*	0.001	-0.002	0.002	0.018	0.827**	0.124**
US *r*	0.000	0.001	0.001	-0.002	-0.005	0.994**
3. Correlations/Volatilities						
ISEQ	17.515**					
FTSE-100	0.528**	15.609**				
S&P 500	0.487**	0.657**	13.621**			
Ireland *r*	-0.024	0.063	0.036	0.801**		
UK *r*	-0.112	-0.161*	-0.026	0.107*	0.363**	
US *r*	0.028	0.023	-0.015	0.011	0.030	0.190**
	Panel B – Two State Model					
	ISEQ	FTSE-100	S&P 500	Ireland *r*	UK *r*	US *r*
1. Mean excess return						
Regime 1 (bear/normal)	0.619**	-0.793**	-0.080**	-0.005*	0.008*	0.001*
Regime 2 (bull)	-0.702	-0.597**	1.946**	1.061**	0.241**	0.034
2. VAR(1) Coefficient						
Regime 1 (bear/normal):						
ISEQ	0.334**	-0.143*	-0.045	1.517*	-3.087**	1.700*
FTSE-100	0.054	-0.317*	-0.088	-0.212*	0.953*	-0.285*
S&P 500	0.043	-0.179*	-0.118*	-0.125	3.110**	-2.656**
Ireland *r*	-0.000	0.001	-0.000	0.919**	0.043*	0.038*
UK *r*	0.001	0.001	-0.002	-0.015	0.951**	0.065*
US *r*	0.000	-0.000	0.001	0.001	-0.026	1.032**
Regime 2 (bull):						
ISEQ	0.303**	-0.153*	0.082	0.556*	3.859**	-4.624**
FTSE-100	0.095	-0.208*	0.150*	0.690*	4.553**	-4.531**
S&P 500	0.045	-0.120	-0.054	-0.403	3.919**	-5.536**
Ireland *r*	-0.036	0.023	-0.005	0.187*	0.144*	-0.160*
UK *r*	-0.000	-0.006	0.005	0.001	0.563**	0.200**
US *r*	0.001	0.004	0.006	-0.019	-0.001	0.988**

* denotes 5% significance, ** significance at 1%.

Table 6- continued

Estimates of Multivariate Regime Switching VAR(1) Model for Excess Stock Returns and Nominal Short-Term Interest Rates

	ISEQ	FTSE-100	S&P 500	Ireland *r*	UK *r*	US *r*
3. Correlations/Volatilities						
Regime 1 (bear/normal):						
ISEQ	17.904**					
FTSE-100	0.584**	14.771**				
S&P 500	0.552**	0.732**	13.556**			
Ireland *r*	-0.046	0.026	0.018	0.101**		
UK *r*	-0.095	-0.044	0.052	0.143*	0.195**	
US *r*	0.080	0.070	-0.004	0.042	0.034	0.059*
Regime 2 (bull):						
ISEQ	15.528**					
FTSE-100	0.389**	16.507**				
S&P 500	0.305**	0.403**	12.589**			
Ireland *r*	-0.064	-0.003	-0.098	1.276**		
UK *r*	-0.128	-0.283*	-0.112	0.108*	0.595**	
US *r*	-0.025	-0.003	-0.003	-0.050	0.049	0.346**
4. Transition probabilities	Regime 1 (bear/normal)			Regime 2 (bull)		
Regime 1 (bear/normal)	0.967**			0.033		
Regime 2 (bull)	0.100			0.900**		

* denotes 5% significance, ** significance at 1%.

Figure 1

Smoothed State Probabilities from Univariate Two-State Models of Nominal Stock Index Returns

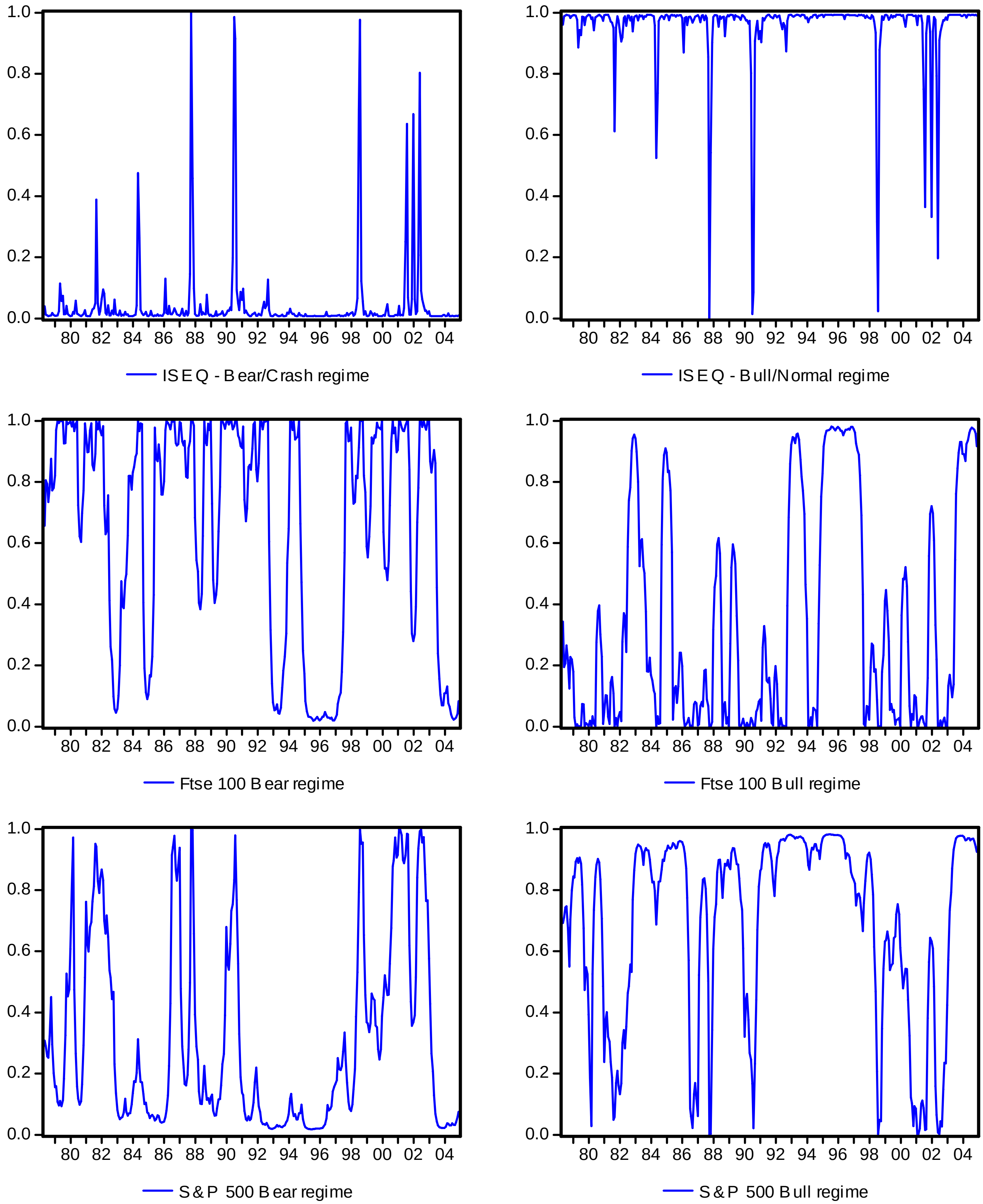


Figure 2

Comparing Smoothed State Probabilities from Univariate Two-State Models of Nominal Index Stock Returns

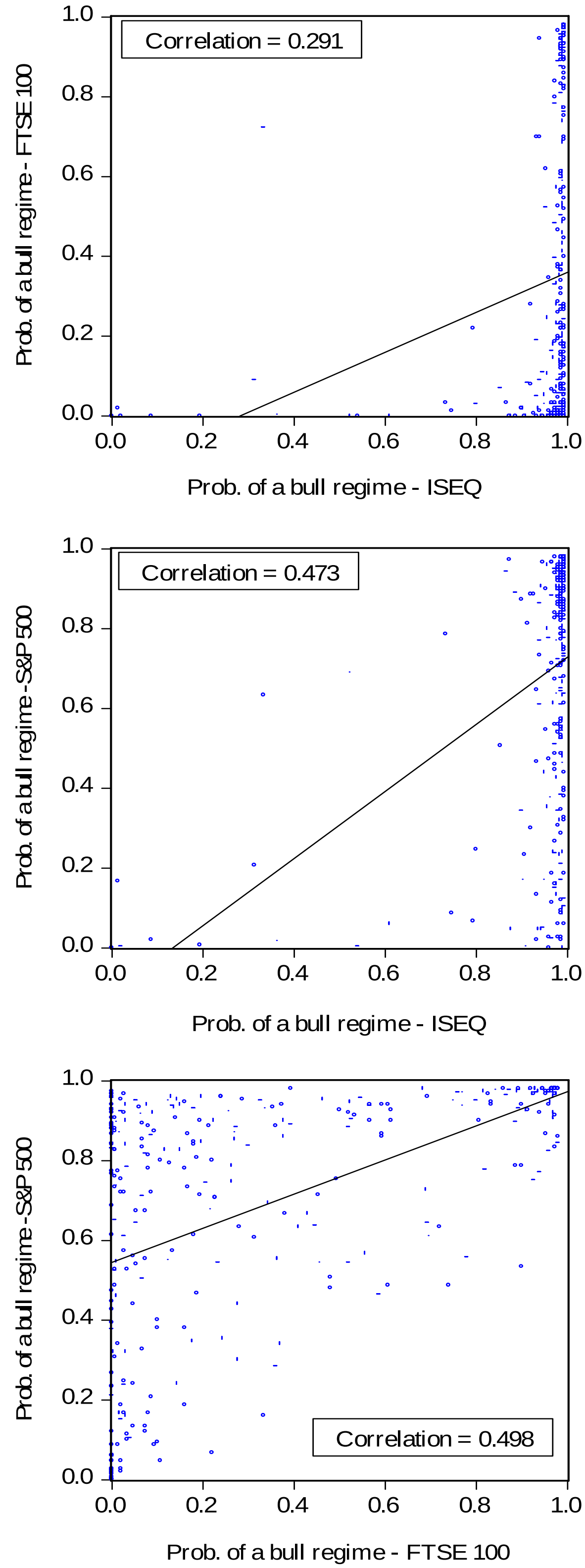
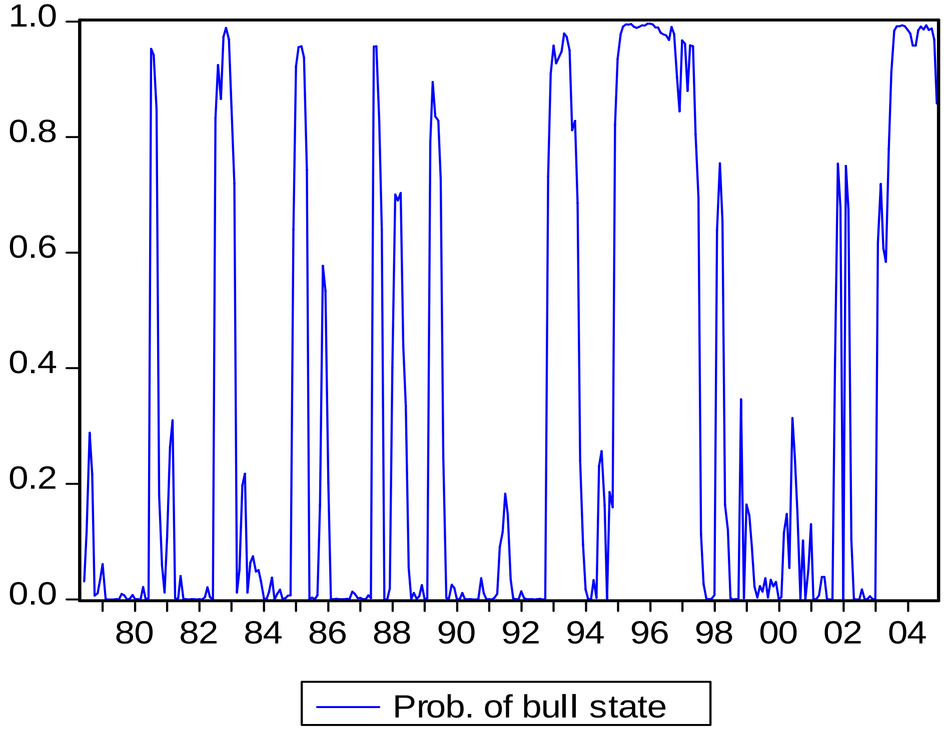


Figure 3

Smoothed State Probabilities from a Multivariate VAR(1) Two-State Model for Excess Stock Index Returns



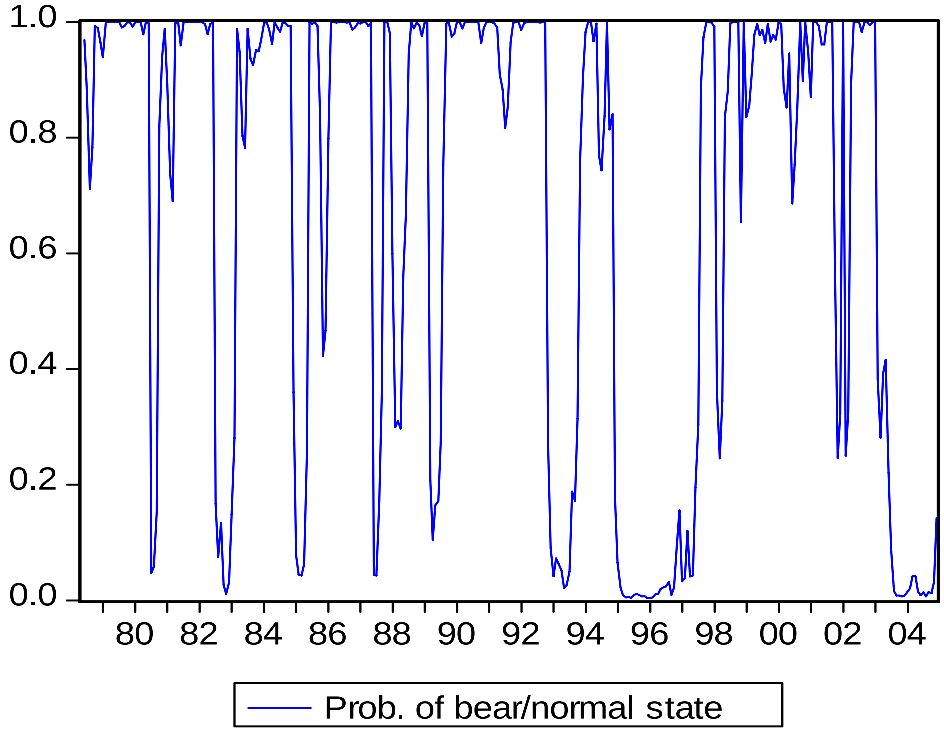


Figure 4

Generalised impulse response function – Bull State

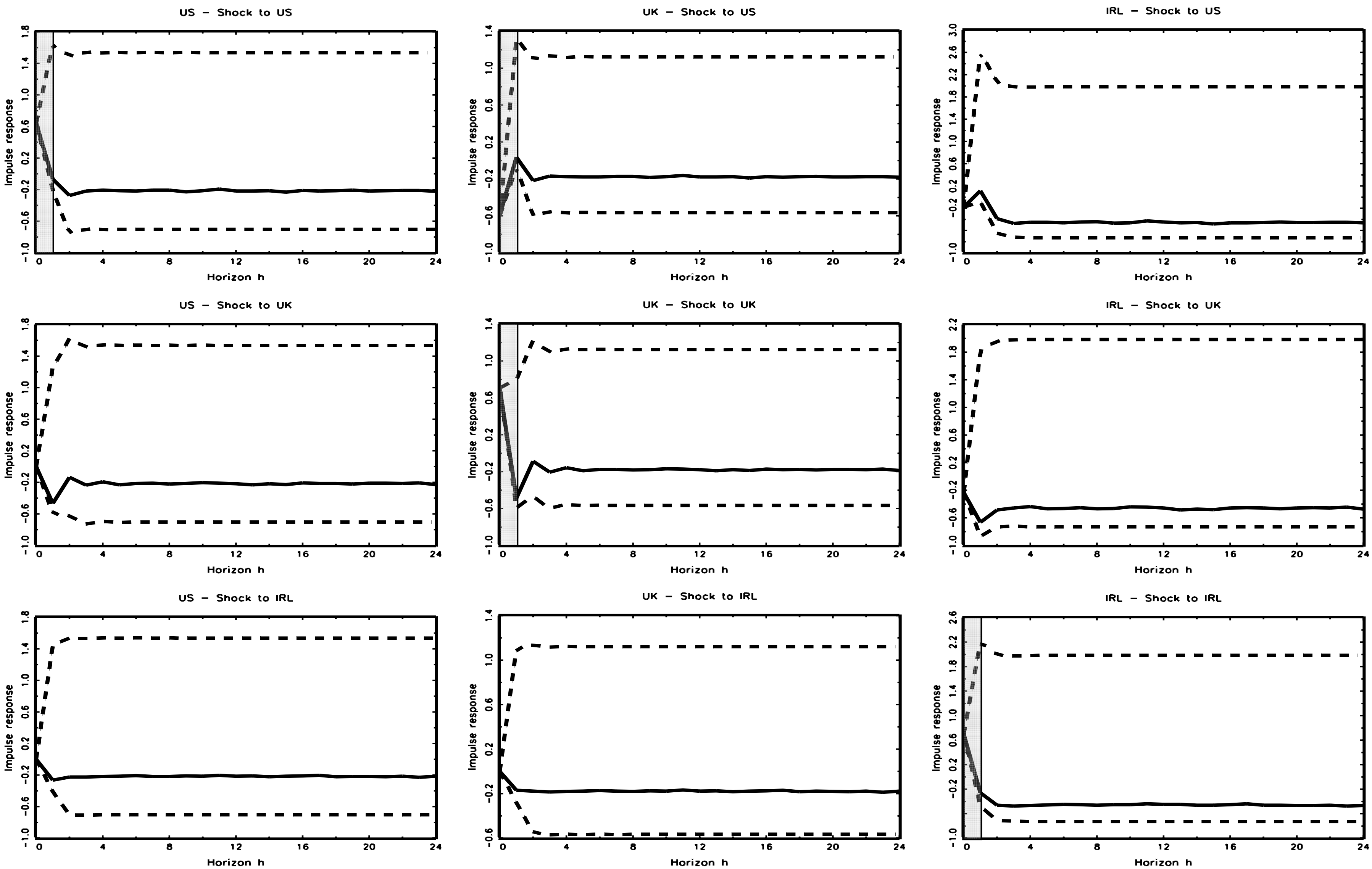


Figure 5

Generalised impulse response function – Bear State

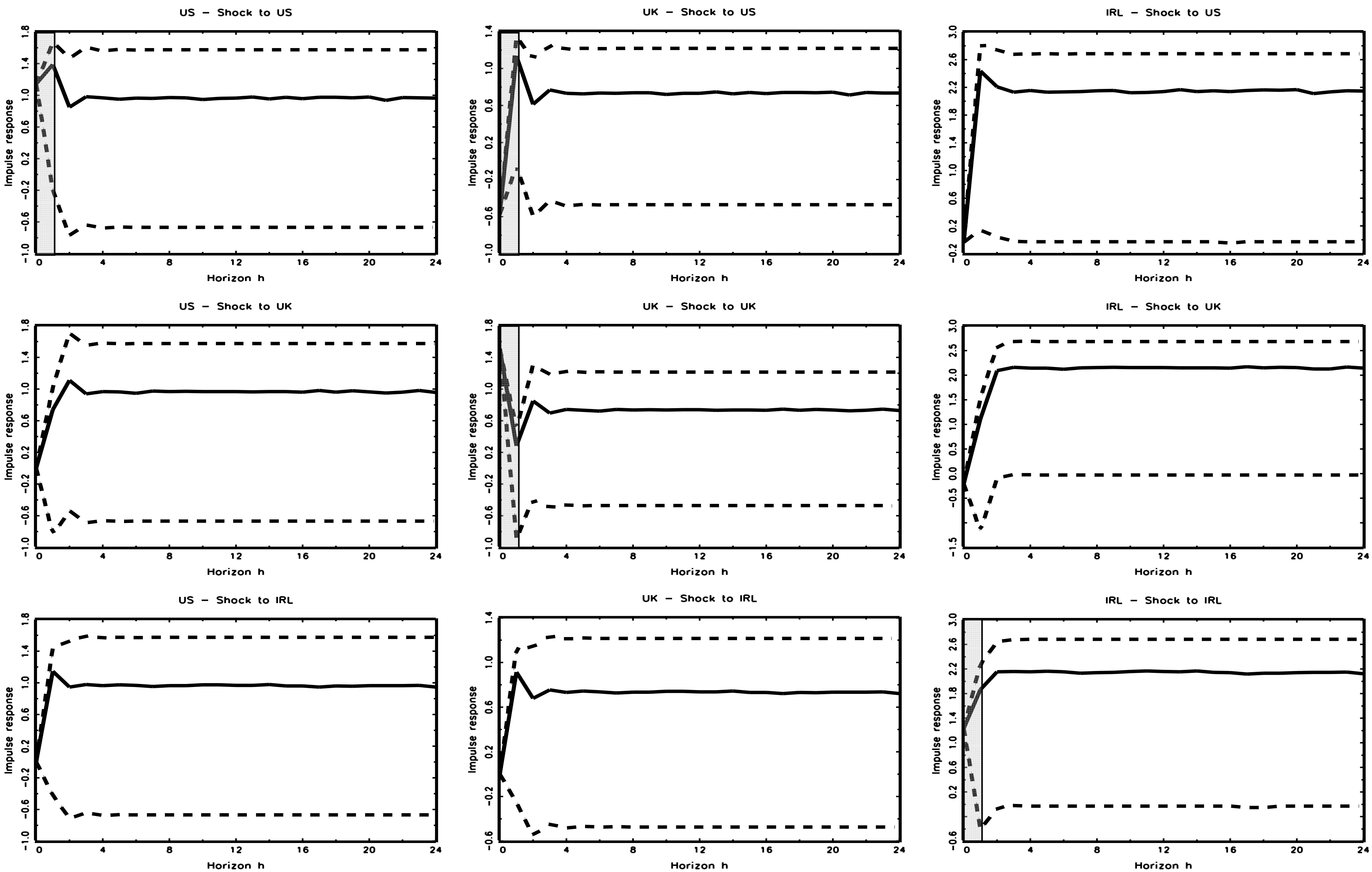
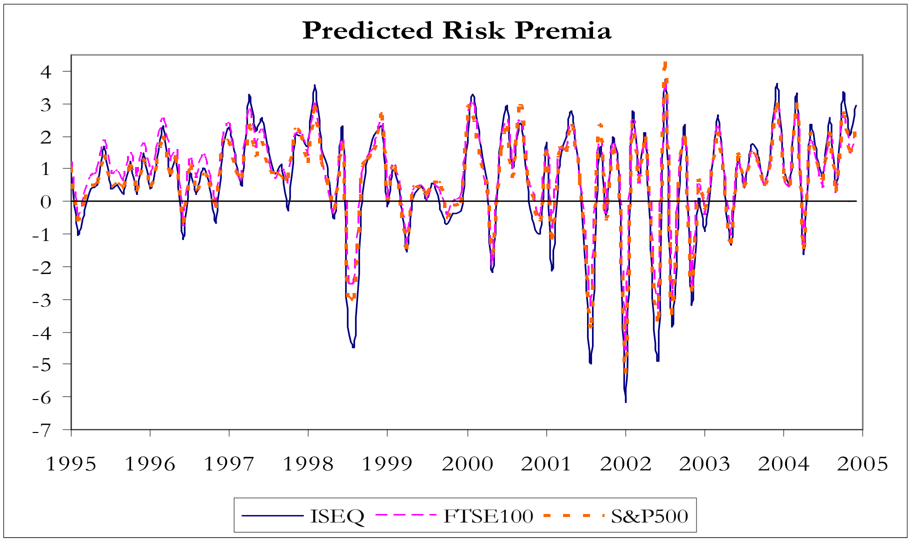
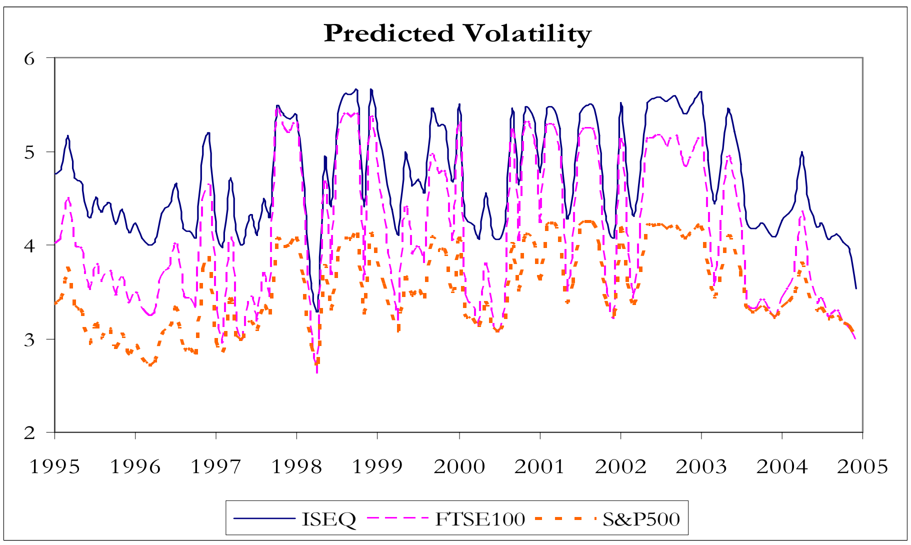


Figure 6

Predicted One-Step Ahead Risk Premia and Volatilities in a Two-State Model





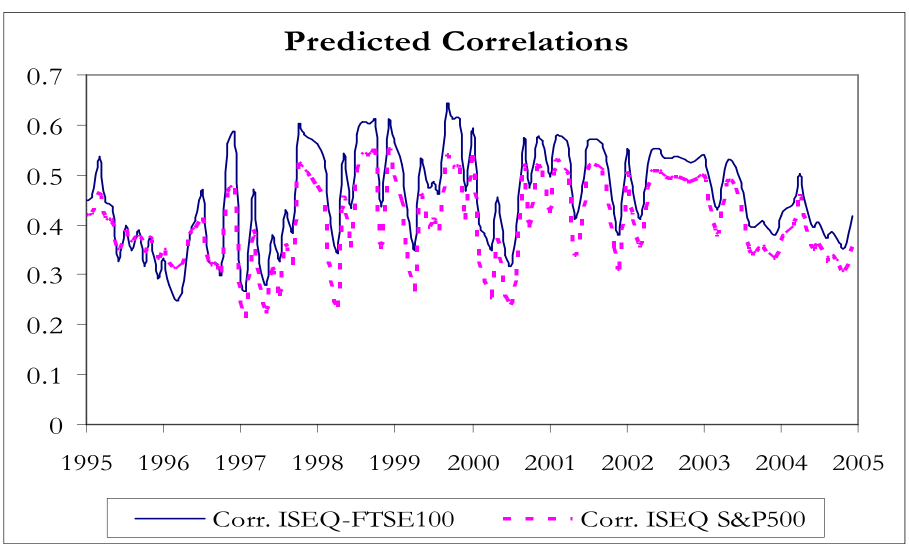


Figure 7

Predicted One-Step Ahead Sharpe Ratios Under a Two-State Model

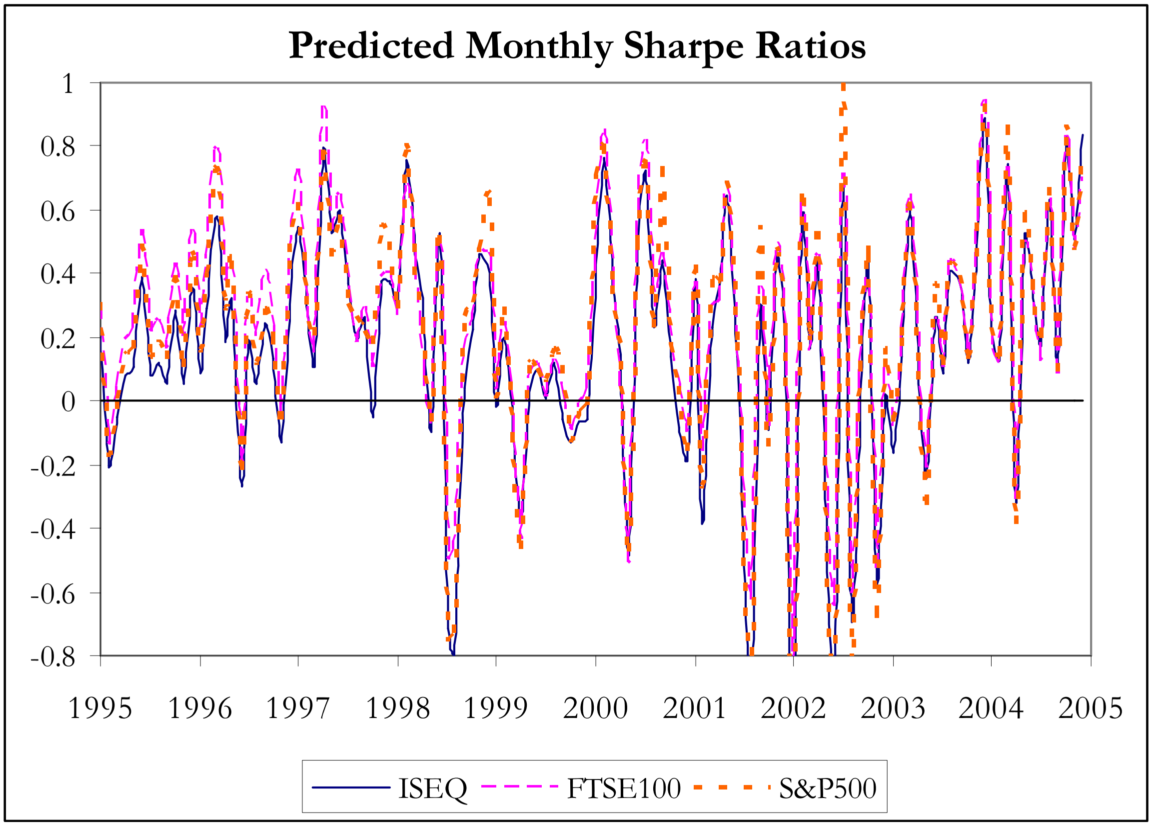
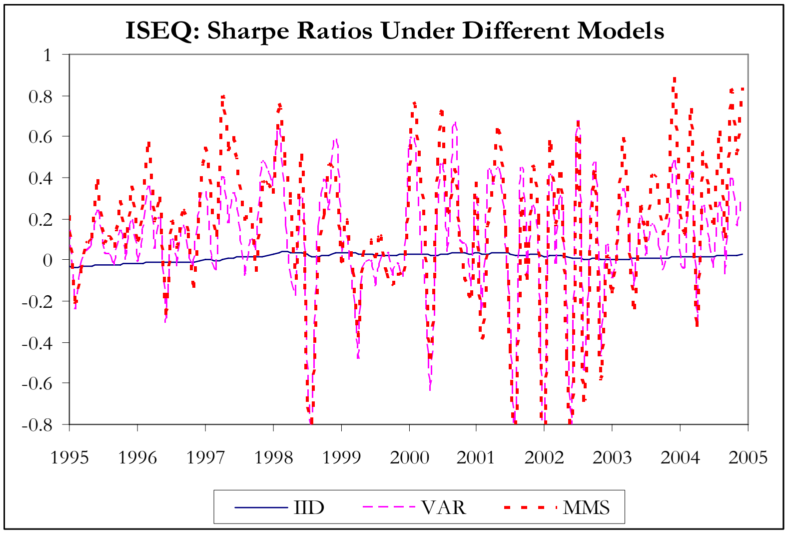
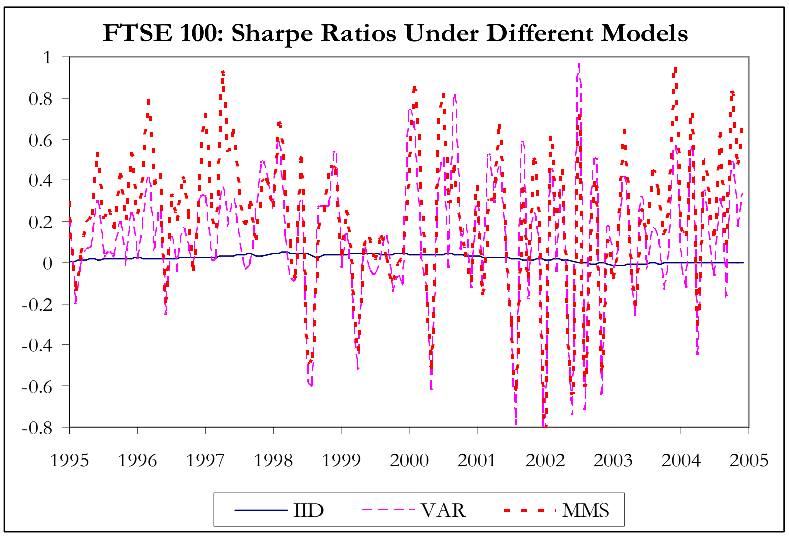


Figure 8

Comparing Predicted One-Step Ahead Sharpe Ratios





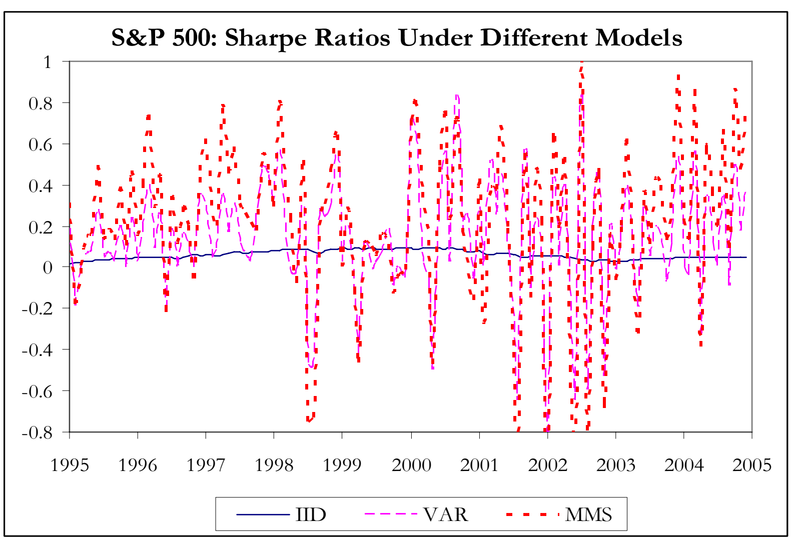


Figure 9

Comparing Optimal Mean-Variance Portfolio Weights

Figure 10

Predicted One-Step Ahead Sharpe Ratios Under a Two-State VAR(1) Model that Account for the Effects of Short-Term Interest Rates