

Volatility trading in options market: How does it affect where informed traders trade?

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Abstract: Although it is widely accepted that options implied volatility is a good estimate of market expectations, very little work has focused on the impact of volatility trading on market microstructure. The present article attempts to fill this gap. We develop a multimarket sequential trades model with asymmetric information in which directional-traders and volatility-traders interact strategically. The major finding is that volatility-traders evict directional-traders from the options market. Indeed, we provide conditions under which volatility trades have a positive impact on options bid-ask spread so that directional-traders choose the spot market. While these results do not confirm that option returns lead spot returns, they are consistent with previous empirical findings.

Keywords: Options market microstructure, volatility trading, informed traders, sequential trades model, asymmetric information.

JEL Classification: C32, G12, G14.

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1 Introduction

For most of the professionals, options markets are volatility markets (Nandi and Wagonner, 2001). As such, considerable attention has been paid in the literature to the forecasting ability of implied volatility. Despite some mixed results (Canina and Figlewski (1993), Day and Lewis (1992), Lamoureux and Lastrapes (1993)), several recent papers (Christensen and Prabhala (1998), Fleming (1998), Lin, Strong and Xu (1998)) report evidence of incremental information.¹ However, very little work has focused on theoretical implications of volatility trading. In the microstructure literature, most of the models with options markets deal only with information about the underlying asset future price. This is the case in Easley, O'Hara and Srinivas (1998) and in John, Koticha, Narayanan and Subrahmanyam (2000). But traders who buy or sell options express a view on the future volatility of the underlying asset without necessarily having an initial view on the future direction of the asset price movement.

We argue that there is an important theoretical contribution in considering the impact of volatility trading on traders strategy that has heretofore gone largely unrecognized. Indeed, due to the existence of -at least- two sources of asymmetric information (about prices and volatility), the market maker's adverse selection problem on the options market is particularly tricky.

The purpose of this paper is to investigate the implications of volatility trading on where (directional) informed traders trade. It is also motivated by previous empirical studies that provide conflicting results about this question (see for instance Manaster and Rendleman (1982), Anthony (1988), Vijh (1988), Kumar and Shastri (1990), Bhattacharya (1987), Stephan and Whaley (1990), Chang *et al.* (1993), Easley *et al.* (1998), Jarnecic (1999), Capelle-Blancard *et al.* (2001)). Despite the inherent advantage of derivatives due to the leverage effect, there is a lack of evidence concerning whether informed traders use or not options. One may argue that leverage effect is offset by illiquidity and higher transaction costs.² But how to explain that the relative bid-ask spread is wider on the options market?

We consider a sequential model where risk-neutral market makers serve market orders placed either by informed or liquidity traders. There are two kinds of informed traders: Directional-traders who have information on the future underlying asset price and volatility-traders who have information on the future underlying asset volatility. To exploit their private information, while avoiding full revelation, directional-traders split their trades between spot and options markets; volatility traders cannot trade in the spot market. The major finding is that volatility-traders evict directional-traders from the option markets. Indeed, we provide conditions under which volatility-trades have a positive impact on options bid-ask spread so that directional-traders choose the spot market, despite the leverage effect of the options. The model explain why options markets should not lead spot market which is consistent with empirical findings (Easley *et al.* (1998), Mayhew (1999)). The model predicts also that the informational role of stock price is great when uncertainty about future volatility is high, which has been recently confirmed by Chakravarty, Gulen and Mayhew (2004).

¹Moreover, in recent years, there is a greater exploitation of the information content of option prices to derive the risk neutral distribution of the underlying asset. See Jackwerth (1999) for a survey of the different methods.

²Note that other effects may affect the price discovery process between spot and derivatives markets. For instance, the use of index derivatives allows investors to easily and rapidly carry out strategies on the basis of their expectations about the general market trends, without having to consider specific changes in each stock that constitutes the index. Further, following Biais, Foucault and Salanié (1998), fully computerized exchanges are expected to exhibit faster price discovery than open outcry.

The remainder of this paper is organized as follows. Section 2 surveys the theoretical literature on the microstructure of options markets. Section 3 describes the basic structure of the model. Section 4 presents the equilibrium and analyzes its properties. Section 5 deals with empirical predictions. Section 6 concludes.

2 The microstructure of options markets: A theoretical review

Despite the growing importance of derivatives markets and the extended literature devoted to the pricing of options, there are relatively few theoretical models that study options market microstructure. In an extension of the continuous-time model of Kyle (1985), Back (1993) demonstrates that the introduction of options that are initially redundant causes the volatility of the underlying stock to become stochastic. Kraus and Smith (1996) also study how trading in a replicatable option can affect the price process of the underlying asset. Brennan and Cao (1996) use a noisy rational expectations model and show that as the number of trading sessions increases (subject to some conditions on the public information flow), the payoff allocation approaches Pareto efficiency which can be achieved with a single round of trading when a quadratic option is introduced. In a static setting, Biais and Hillion (1994) analyze the effect of introducing options on an incomplete market. They show that it can reduce insider profit and mitigates the market breakdown problems caused by asymmetric information, but its consequences on the informational efficiency of the market is ambiguous. None of these authors models really the informational linkages between the underlying and the derivative market.

Easley, O'Hara and Srinivas (1998) develop a sequential trade model wherein informed traders choose whether to trade the stock, a call or a put and determine conditions under which, in equilibrium, informed traders trade in one or both markets. They also test their theoretical predictions by using Granger causality tests apply to 266 firms whose options are traded on the CBOE. Although they report evidence of informed trading in the options market when long and short option trades are distinguished, they also find bi-directional causality between option volumes and stock prices. John, Koticha, Narayanan and Subrahmanyam (2000) also use a sequential trade approach, but examine both the impact of option trading and margin requirements on the behavior of informed traders. They show that, in the absence of margin requirements, informed traders split their trades between the stock and the option, although they exhibit a bias towards the stock because of its greater information sensitivity. But when informed traders do face wealth constraints and differential margin requirements, this bias is enhanced or mitigated depending on the leverage provided by the option. In the first case it leads to wider stock bid-ask spreads while in the latter one, it leads to wider or narrower stock bid-ask spreads. Besides, in all cases, the introduction of option trading improves the informational efficiency of stock prices.

A common deficiency of the above-mentioned models is that they ignore an important feature of options markets: the possibility to trade on volatility informations. A first step is take in this direction by Cherian (1998) and Cherian and Jarrow (1998). In these models, there are two types of informed traders. First, directional-traders who have perfect information about the underlying asset future price movements but imperfect information about volatility, so they buy (sell) options if their estimate of the option value, given their private information, is above (below) the quoted ask (bid) price. Second, volatility-traders who have perfect information about volatility but do not know how underlying price will move, which buy (sell) options when implied volatility is lower (higher) than their estimation. Cherian and Jarrow (1998) show that the Black-Scholes price can arise in equilibrium from self-fulfilling beliefs that it is the correct price. In a related model, Cherian (1998) extend this approach by introducing two

option contracts on different maturities. Nandi (1999) also study the impact of trades on implied volatility. Unlike previous models, he uses a rational expectations model in line with Kyle (1985) and Back (1992). Moreover, unlike Cherian (1998) and Cherian and Jarrow (1998), he considers only volatility traders. He shows that order flows to the market makers has an impact on the volatility smile. He show also that larger volume on options markets is correlated with greater uncertainty.

3 A model of multimarket trading

In this section we present a theoretical model of multimarket trading. This model extends the models of Easley *et al.* (1998) and John *et al.* (2000) with volatility traders in the spirit of Cherian (1998) and Cherian and Jarrow (1996). It is a sequential trade model with risk neutral and competitive market makers wherein traders choose to transact in spot and/or options market.

3.1 The structure

In the model, a stock and a European call option (strike price K and expiration time T) on the stock are traded.³ We consider three dates: 0, 1 and T and we focus on the period between 0 and 1. This is during this period that strategic choices take place and anti-selection problems occur.⁴ At the initial trading date, $t = 0$, the stock price, S is observed by all market participants. This initial stock price can be interpreted as the last recorded transaction price.

The spot price evolution over time is taken as given. The possible values for the stock at time 1 are S_H with probability 1/2 and S_L with probability 1/2 where $S_H > S_L$. In addition, at time 1, the possible values for the volatility of the stock return over the remaining life of the option is σ_H with probability 1/2 or σ_L with probability 1/2 where $\sigma_H > \sigma_L$.⁵ Hence, there are four future states of the world at time 1 $\{S_H, \sigma_H\}$; $\{S_L, \sigma_H\}$; $\{S_H, \sigma_L\}$; $\{S_L, \sigma_L\}$.⁶ In the model, volatility is assumed independent of stock price, otherwise a trader informed of the future price (the volatility) will be able to speculate on the volatility (the future price). This assumption is also adopted by Nandi (1999) and it is necessary for tractability.⁷ We note $\bar{S} = (S_L + S_H)/2$, $\bar{\sigma} = (\sigma_L + \sigma_H)/2$ and $\bar{C} = (\bar{\sigma}, \bar{S})$.

Traders arrive sequentially as in Easley and O'Hara (1987). The game takes place in four steps. 1) At $t = 0$, some traders privately observe a signal about the price at $t = 1$ or about the volatility from $t = 1$ to T .⁸ 2) Between $t = 0$ and $t = 1$ a trader is randomly chosen and submit a buy or a sell order to the market maker and a trade occurs. A trade consists in a single round lot of stock equaling γ shares or a single option contract controlling θ shares of stock.⁹ Traders cannot simultaneously execute multiple trades. 3) Market makers revise their quotations to reflect the information contained in the trade that occurred. Steps 2) and 3) can be repeated between time 0 and time 1. 4) At time 1, the state of the world is realized and publicly observed by all market participants. Stock price is equal to S_H or S_L whereas call price is equal to $C(S_H, \sigma_H)$, $C(S_H, \sigma_L)$, $C(S_L, \sigma_H)$ or $C(S_L, \sigma_L)$. Until T , the game may continue

³Unlike Easley *et al.* (1998), we do not use any specific assumption concerning the option strike price.

⁴Two dates are enough in Easley *et al.* (1998) and John *et al.* (2000) because the date when information is revealed and positions are liquidated coincide with the expiration date of the option.

⁵The assumption of equiprobability has no consequence since parameters $S_H, S_L, \sigma_H, \sigma_L$ are unrestricted.

⁶At time 1, call price can be determined using the traditional risk-neutral method. If we suppose, for instance, that the spot price is lognormally distributed at time T with the parameters revealed at $t = 1$, the call price can be computed by the Merton-Black-Scholes model.

⁷Empirically, when prices go down, volatility is likely to goes up.

⁸Without loss of generality, we assume that an information event occurs before the beginning of the trades.

⁹By avoiding multiple trade sizes we abstract from the informational content of trade size.

under the same scenario with new information arrivals throughout the trading day. The sequence of conditional probabilities is common knowledge.

There are two competitive risk neutral market makers: a first who sets prices to buy or sell the stock, a second who sets prices to buy or sell calls upon the stock. We denote by $b_{i=s,c}$ and $a_{i=s,c}$ the bid and ask prices in the stock and the call. Market makers cannot determine whether the agent is informed or not; they set bid and ask prices competitively and rationally so as to yield zero expected profit on each trade given the information conveyed by the trade history in the two markets. Market makers cope with an adverse selection problem. They offset losses due to informed traders orders with profits from uninformed traders orders. We suppose that the market makers can monitor perfectly and simultaneously orders and trades in both markets, so they share the same information set and there is no role for arbitrageurs.¹⁰

Trades arise from uninformed and informed traders. More precisely, as in Cherian (1996) and Cherian and Jarrow (1996), three kind of risk-neutral traders trade options: Uninformed-traders, directional-traders and volatility-traders. There is a fraction $(1 - \alpha - \beta)$ of uninformed-traders, a fraction α of directional-traders and a fraction β of volatility-traders.

Uninformed-traders are assumed to trade for liquidity based reasons that are exogenous, like portfolio rebalancing or hedging. They do not recognize that their trades affect the prices so they do not have any strategic behavior, unlike informed-traders.¹¹ As usual, their presence is necessary to camouflage the informed trades; in our model of multimarket trading, this assumption is also intended to incorporate hedging behavior. The proportion of uninformed traders who buy stocks, sell stocks, buy calls and sell calls is respectively noted a, b, c, d , with $a + b + c + d = 1$.

With probability α (β), order stems from an informed directional- (volatility-) trader.¹² Though the future state of the world is currently unobservable, directional-traders receive identical information regarding the spot price at $t = 1$, while volatility-traders receive identical information about its future volatility.

Directional-traders see trading opportunities in both the stock and options markets and have the opportunity to choose where they are going to trade based on the profits available. For instance, a trader informed of good news could profit from buying the stock or buying a call.¹³ Further, directional-traders face a trade-off between trading too aggressively in either market and facing larger trading costs (bid-ask spreads) in that market. We denote by η_L and η_H the fraction of directional-traders who choose to trade in the spot market when $S = S_L$ and $S = S_H$ at $t = 1$; η_L and η_H are endogenous.

1. If the directional-trader knows that $S = S_L$ at $t = 1$ then profit is:

$$\pi(S_L, \sigma) = \begin{cases} (b_s - S_L) \gamma & \text{if sell a stock} \\ (b_c - C(S_L, \sigma)) \theta & \text{if sell a call} \end{cases} \quad (1)$$

2. If the directional-trader knows that $S = S_H$ at $t = 1$ then profit is:

$$\pi(S_H, \sigma) = \begin{cases} (-a_s + S_H) \gamma & \text{if buy a stock} \\ (-a_c + C(S_H, \sigma)) \theta & \text{if buy a call} \end{cases} \quad (2)$$

¹⁰As pointed by Easley et al.(1998), in practice such immediacy is unlikely to be available. For instance, in the US, stock and options are usually traded separately and lead or lags can be found. In European markets, the trading venues are often consolidated and Capelle-Blancard and Chaudhury (2003) show that such inefficiency are less pronounced.

¹¹See Biais and Hillion (1994) for an alternative framework.

¹²In a dynamic setting, the repartition between traders who seek information about future price or volatility depend on expected profits, which depend themselves on market characteristics.

¹³In a more general setting, a trader informed of good news could profit from buying the stock, buying a future or a forward, buying a call or writing a put, ...

Volatility-traders has information regarding the future underlying asset volatility; that is they observe σ_H or σ_L but they cannot distinguish whether S_H or S_L occurs. Volatility-traders can exploit their information only in the options markets.¹⁴

1. If the volatility-trader knows that $\sigma = \sigma_L$, then profit is:

$$\pi(S, \sigma_L) = (b_c - C(S, \sigma_L)) \theta \text{ for selling a call} \quad (3)$$

2. If the volatility-trader knows that $\sigma = \sigma_H$, then profit is:

$$\pi(S, \sigma_H) = (-a_c + C(S, \sigma_H)) \theta \text{ for buying a call} \quad (4)$$

Informed-traders are risk neutral and they do not attempt to hedge their position. Directional- and volatility-information are certain.¹⁵ Informed-traders will buy if their valuations exceed the ask price and sell if their valuations are less than the bid price. Information structure and action choices of market participants at $t = 0$ are describe in Figure 1.¹⁶

4 Resolution of the sequential model

Our aim is to examine the localisation of directional-traders and the determinant of bid-ask spreads. As usual, we first solve for the market makers optimal prices given probabilities of informed trade (3.4.1). Then, we derive the condition under which directional-traders choose to trade exclusively on one market (separate equilibrium) or to split their trades (polling equilibrium) (3.4.2). Third, we compute the equilibrium bid-ask spreads (3.4.3). At the equilibrium, the following conditions must be verified: i) bid and ask price are posted in such a way that expected profits are equal to zero, given available informations; ii) informed traders maximise their profit, given information they have received. Finally, we examine the consequences of options introduction on informational efficiency (3.4.4).

¹⁴In fact, volatility traders dispose of different strategies which combine several options. The most common consist in buying a straddle, that is buying simultaneously a call and a put with the same strike and the same time to expiration. See Carr and Madan (1999) for a comparison of different volatility trading strategies.

¹⁵The model can easily be extended by assuming that information are uncertain. As investors are risk neutral, result will be the same with an additional error term. See for instance John *et al.* (2000).

¹⁶The framework developed here is quite general and allows us to study traders behavior and the bid-ask process in several configurations. Some of them have already been studied.

1. Trades are allowed only in the spot market: $c = d = 0$, $\eta_L = \eta_H = 0$
 - (a) Asymmetric information about stock price (Easley and O'Hara, 1987): $\beta = 0$;
2. Trades are allowed only in the options market: $a = b = 0$, $\eta_L = \eta_H = 1$
 - (a) Asymmetric information stock price: $\beta = 0$;
 - (b) Asymmetric information volatility: $\alpha = 0$;
 - (c) Asymmetric information stock price and volatility (Cherian, 1998; Cherian and Jarrow, 1998);
3. Trades are allowed in the stock and in the options markets:
 - (a) Asymmetric information stock price (Easley *et al.*, 1998; John *et al.*,2000): $\beta = 0$;
 - (b) Asymmetric information volatility: $\alpha = 0$;
 - (c) Asymmetric information stock price and volatility.

In the following, we focus on the last case (3-c). Then, Easley and *al.* (1998), John and *al.* (2000) and, at a lower extant, Cherian (1998) may be view as specific cases.

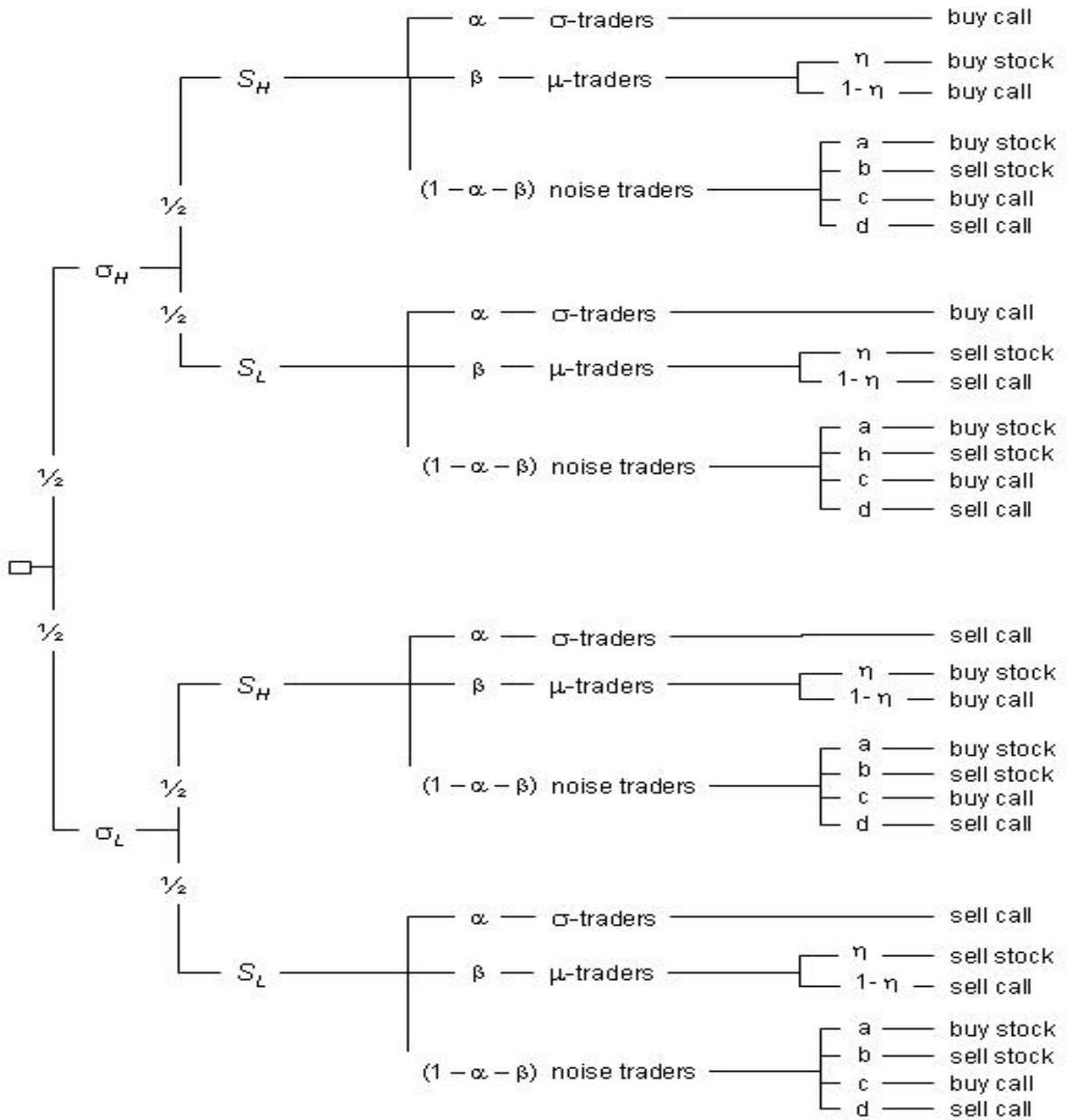


Figure 1: Information structure and action choices of market participants at $t = 0$

4.1 Conditional bid-ask spreads

Whatever the market considered, market maker bid (ask) is equal to the expected value of the asset given a sell (buy) order. On each markets, a sell (buy) order increase the probability associated with a lowering (raise) of the future asset price and induce an adjustment of the market maker beliefs and so, of the bid-ask spread.

4.1.1 The spot market

Consider a market maker on the spot market who receive a sell order. The sell order may come from i) an uninformed-trader, for exogenous reasons, with a probability $(1 - \alpha - \beta)b$; ii) or from a directional-trader, who knows the future price will be equal to S_L , with a probability $\alpha\eta_L/2$.¹⁷ In the first case, the market maker estimates the spot price to its unconditional expected value ($E[S] = (S_H + S_L)/2$); in the second case, he/she estimates to its value in the “bad state” (S_L). Inversely, consider a market maker on the spot market who receive a buy order. The buy order may come from i) an uninformed-trader, for with the probability $(1 - \alpha - \beta)a$; ii) or from a directional-trader, who knows the future price will be equal to S_H , with a probability $\alpha\eta_H/2$. In the first case, the market maker estimates the spot price to its unconditional expected value ($E[S]$); in the second case, he/she estimates to its value in the “good state” (S_H). Then, conditional bid and ask on the spot market are obtained following the Bayes’ rule.

For market makers, the difference between the bid price and the ask price offset market risk inherent to their professional activity. Given that market markets are risk neutral, we can focus on the problem of adverse selection problem and ignore inventory costs. Market makers balance losses undergone when they trades with informed-traders with profits they make when they trade with uninformed-traders. So, it is interesting to express market makers bid and ask in terms of unconditional prices.

Lemma 1 *Bid and ask prices on the spot market, given informed-traders strategy are the following:*

$$b_s = \bar{S} - \frac{1}{2} \left[\frac{(S_H - S_L)\alpha\eta_L}{2(1 - \alpha - \beta)b + \alpha\eta_L} \right] \quad (5)$$

$$a_s = \bar{S} + \frac{1}{2} \left[\frac{(S_H - S_L)\alpha\eta_H}{2(1 - \alpha - \beta)a + \alpha\eta_H} \right] \quad (6)$$

Proof: See in appendix.

From the previous equations, we can easily see that bid and ask prices enclose the unconditional expected value. Not surprisingly, if there is no signal which may affect the future price, bid and ask prices are equal.¹⁸ On the contrary, if there are only informed-traders ($\alpha + \beta = 1$), bid price is minimum and equal to S_L and ask price is maximum and equal to S_H .

Every factor that increases informed-traders profit increases also market makers adverse selection costs. It is actually a zero-sum game. The spot market bid-ask spread vary therefore in accordance with the fraction of directional-traders who take part in the market ($\partial b_s / \partial \eta_H < 0$ et $\partial a_s / \partial \eta_L > 0$). Unlikely, the more traders trade for liquidity reasons, the more market makers can

¹⁷Recall that volatility-traders cannot profit from their information about volatility to make profit on the spot market.

¹⁸More generally, when information is randomly diffused (Easley *et al.*, 1998) - or when informed traders do not have perfect knowledge (John *et al.*, 2000) - the bid-ask spread is also weighted by the probability that the signal is correct - or the probability of a new information. Formally, if we note $\phi < 1$ the probability that the signal is correct, the bid-ask spread is equal to $\alpha(2\phi - 1)(S_H - S_L)$.

spread their costs and reduce average costs. Ceteris paribus, bid ask spread is negatively related to market depth (parameters a and b) and to the fraction of uninformed-traders $(1 - \alpha - \beta)$. Bid-ask spread is also positively related to the price difference between the good state and the bad state $(S_H - S_L)$, which can be interpreted as the informed traders informational advantage.¹⁹

4.1.2 The options market

The options market bid-ask spread is also equal to conditional expectations but computation gets trickier. Indeed, the adverse selection problem is twofold and make the interpretation of the information content of trades by the market makers more difficult.

The European call option gives the holder the right to buy θ shares of the underlying stock from the writer of the option for the exercise price K at the expiration date T . Suppose that a market maker receives a sell order from i) an uninformed trader who ignore the underlying future price and its volatility, ii) a directional-trader who knows the price will decrease but ignore the volatility level, iii) a volatility-trader who knows the volatility will decrease but ignore the underlying price at $t = 1$. The probability for the market maker to receive a sell order from an uninformed trader is $(1 - \alpha - \beta)d$, from a directional-trader is $\alpha(1 - \eta_L)/2$ and from a volatility-trader is $\beta/2$. In the first case, the market maker assesses the option price to its unconditional expected value $(E[C] = (C(S_H, \sigma_H) + C(S_L, \sigma_H) + C(S_H, \sigma_L) + C(S_L, \sigma_L))/4)$; in the second case, he assesses the option price to its expected value given that the underlying price at time 1 is equal to S_L $(E[C|S = S_L] = (C(S_L, \sigma_H) + C(S_L, \sigma_L))/2)$; in the third case, he assesses the option price to its expected value given that the volatility will be equal to σ_L $(E[C|\sigma = \sigma_L] = (C(S_H, \sigma_L) + C(S_L, \sigma_L))/2)$. The bid price on the options market can be written as follow:

$$b_c = \frac{(1 - \alpha - \beta)dE[C] + \alpha(1 - \eta_L)E[C|S = S_L]/2 + \beta E[C|\sigma = \sigma_L]/2}{(1 - \alpha - \beta)d + \alpha(1 - \eta_L)/2 + \beta/2} \quad (7)$$

In the same way, the ask price can be written:

$$a_c = \frac{(1 - \alpha - \beta)cE[C] + \alpha(1 - \eta_H)E[C|S = S_H]/2 + \beta E[C|\sigma = \sigma_H]/2}{(1 - \alpha - \beta)c + \alpha(1 - \eta_H)/2 + \beta/2} \quad (8)$$

Options bid and ask prices depend on options price at $t = 1$ which depend themselves on the underlying price and its volatility. Then, it is useful to write bid-ask spread directly in terms of those two parameters. It may be reached by linearizing options price around \bar{S} and $\bar{\sigma}$.²⁰ Two new parameters are needed: the *delta* (Δ) and the *vega* (∇). These two parameters measure, respectively, the sensibility of option price to a small variation of the underlying price and to a small variation of its volatility.

Using a second-order Taylor expansion around $(\bar{S}, \bar{\sigma})$, we can write option prices in the different states:

$$C(S_H, \sigma_H) = \bar{C} + (S_H - \bar{S})\Delta + (\sigma_H - \bar{\sigma})\nabla \quad (9)$$

$$C(S_L, \sigma_H) = \bar{C} + (S_L - \bar{S})\Delta + (\sigma_H - \bar{\sigma})\nabla \quad (10)$$

$$C(S_H, \sigma_L) = \bar{C} + (S_H - \bar{S})\Delta + (\sigma_L - \bar{\sigma})\nabla \quad (11)$$

$$C(S_L, \sigma_L) = \bar{C} + (S_L - \bar{S})\Delta + (\sigma_L - \bar{\sigma})\nabla \quad (12)$$

Some algebraic simplifications lead to the following bid-ask prices.

¹⁹The effect is the same whether S_H (S_L) increase (decrease) or the probability associated with the good (bad) state rise. So, to assume that the two states are equiprobable has no consequence on the model.

²⁰An illustration is proposed in appendix.

Lemma 2 *The market maker bid-ask prices on the options market, conditionally to their information, are the following:*

$$b_c = \bar{C} - \frac{1}{2} \left[\frac{(S_H - S_L)\Delta\alpha(1 - \eta_L) + (\sigma_H - \sigma_L)\nabla\beta}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta} \right] \quad (13)$$

$$a_c = \bar{C} + \frac{1}{2} \left[\frac{(S_H - S_L)\Delta\alpha(1 - \eta_H) + (\sigma_H - \sigma_L)\nabla\beta}{2(1 - \alpha - \beta)c + \alpha(1 - \eta_H) + \beta} \right] \quad (14)$$

Proof: See in appendix.

Like on the spot market, if there is no informed-traders ($\alpha + \beta = 0$), bid and ask prices are equal to the option unconditional value. In the spot market, if there is no directional-trader ($\alpha = 0$), the bid-ask spread is nil whatever the number of volatility-traders²¹ whereas in the options market it is only a necessary but not a sufficient condition.

On the options market the bid-ask spread depend negatively on market depth (parameters c and d) and on the fraction of uninformed traders ($1 - \alpha - \beta$). Like on the spot market, the bid-ask spread vary also in accordance with the informed-traders informational advantage. Not only an increase of the futur price difference ($S_H - S_L$) widen the bid-ask spread, but also an increase of the volatility difference in the two states ($\sigma_H - \sigma_L$). In the same way, if we considered η_H and η_L fixed, the bid-ask spread vary with the *delta* and the *vega*.²²

Unlike the bid-ask spread on the stock market which vary in accordance with, the impact of a variation of the percentage of directional-traders (η_H and η_L) who choose the spot market is ambiguous. An increase of the fraction of directional-traders who choose the options market ($1 - \eta_H$ and $1 - \eta_L$) have a positive impact on the bid-ask spread if the two following conditions are verified:

$$\frac{\partial b_c}{\partial \eta_L} > 0 \Leftrightarrow \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} + 2(1 - \alpha - \beta)d \right) > 0 \quad (15)$$

and

$$\frac{\partial a_c}{\partial \eta_H} < 0 \Leftrightarrow \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} + 2(1 - \alpha - \beta)c \right) > 0 \quad (16)$$

These conditions are verified for a broad set of parameters. But, if the options market is far from depth, if the fraction of volatility-traders is large and if the sensibility of options to volatility variation is strong, the option bid-ask spread may vary in accordance with the fraction of directional traders who choose to trade on the spot market. However, in these extreme cases, market makers on the options market will propose a wide bid-ask spread. The bid-ask spread will be so wide that it is no more profitable for directional-traders to trade on the spot market, especially if the *delta* is small and if the price difference in the good and the bad states is small too. This situation is unbearable. It is demonstrated in the following section.

4.2 Directional-traders strategy

Directional-traders react towards bid-ask spreads by choosing the market where they can make the higher profit: they may trade only in the spot market or only in the options market (separate equilibrium), but they may also split their trades (pooling equilibrium).²³ Their choice depend on the characteristics of the contracts (lot size, strike price, time to expiration), on market depth,

²¹This is because stock price and future volatility are independent.

²²It should be noted that these parameters do not always vary in the same way (*cf. infra*).

²³To take advantage of volatility information, traders have to trade on the options market.

on the sensibility of option to price or volatility variation and on the relative informational advantages. The model lead to three kind of equilibrium according to the parameters value:

- i) Even though $\eta_L = 1$, the profit of directional-traders on the options market will be less than it is in the spot market. In this case, although transaction costs on the spot market are maximum, due to directional-traders aggressiveness, none of them are incited to move towards the options market.
- ii) Even though $\eta_L = 0$, directional-traders profit will be less on the spot market than on the options market. In this case, although transaction costs on the options market are maximum, none of them are incited to move towards the spot market.
- iii) If $0 < \eta_L < 1$, that is neither spot market nor options market has a strong advantage, directional-traders split their trades in such a way that profits will be identical on the two markets. This repartition is the only one compatible with an equilibrium. Indeed, suppose that profits are different, directional-traders are incited to move from the market where profit is the lowest to the one where the profit is the highest. The fraction of directional-traders who choose to trade on the spot market is derived by equalizing equations (??) and (??). By proceeding in the same way when the spot price at time 1 is equal to S_H , we can determine directional-traders strategy in any case. All the situations are formally presented in the next proposition.

Proposition 1

1. A trader who knows that the stock price at time 1 will be equal to S_L :

(a) buy a stock ($\eta_L = 1$) if

$$\frac{\theta}{\gamma}\Delta < \frac{2(1-\alpha-\beta)b}{2(1-\alpha-\beta)b+\alpha} \times \frac{2(1-\alpha-\beta)d+\beta}{2(1-\alpha-\beta)d+\beta(1-\frac{\nabla\sigma_H-\sigma_L}{\Delta}\frac{\sigma_H-\sigma_L}{S_H-S_L})} \equiv A_L \quad (17)$$

(b) buy an option ($\eta_L = 0$) if

$$\frac{\theta}{\gamma}\Delta > \frac{2(1-\alpha-\beta)d+\alpha+\beta}{2(1-\alpha-\beta)d+\beta(1-\frac{\nabla\sigma_H-\sigma_L}{\Delta}\frac{\sigma_H-\sigma_L}{S_H-S_L})} \equiv B_L \quad (18)$$

(c) and split its trade otherwise (mixed strategy); then, the part of directional trader who buy stocks is equal to:

$$\eta_L = \frac{2(1-\alpha-\beta)b \left[2(1-\alpha-\beta)d(\gamma-\Delta\theta) + \gamma(\alpha+\beta) - \Delta\theta\beta(1-\frac{\nabla\sigma_H-\sigma_L}{\Delta}\frac{\sigma_H-\sigma_L}{S_H-S_L}) \right]}{\alpha \left[2(1-\alpha-\beta)(\gamma b + \Delta\theta d) + \Delta\theta\beta(1-\frac{\nabla\sigma_H-\sigma_L}{\Delta}\frac{\sigma_H-\sigma_L}{S_H-S_L}) \right]} \quad (19)$$

2. A trader who knows that the stock price at time 1 will be equal to S_H :

(a) sell a stock ($\eta_H = 1$) if

$$\frac{\theta}{\gamma}\Delta < \frac{2(1-\alpha-\beta)a}{2(1-\alpha-\beta)a+\alpha} \times \frac{2(1-\alpha-\beta)c+\beta}{2(1-\alpha-\beta)c+\beta(1-\frac{\nabla\sigma_H-\sigma_L}{\Delta}\frac{\sigma_H-\sigma_L}{S_H-S_L})} \equiv A_H \quad (20)$$

(b) sell an option ($\eta_H = 0$) if

$$\frac{\theta}{\gamma}\Delta > \frac{2(1-\alpha-\beta)c+\alpha+\beta}{2(1-\alpha-\beta)c+\beta(1-\frac{\nabla\sigma_H-\sigma_L}{\Delta}\frac{\sigma_H-\sigma_L}{S_H-S_L})} \equiv B_H \quad (21)$$

(c) and split its trade otherwise (mixed strategy); then, the part of directional trader who sell stocks is equal to:

$$\eta_H = \frac{2(1 - \alpha - \beta)a \left[2(1 - \alpha - \beta)c(\gamma - \Delta\theta) + \gamma(\alpha + \beta) - \Delta\theta\beta \left(1 - \frac{\nabla}{\Delta} \frac{\sigma_H - \sigma_L}{S_H - S_L} \right) \right]}{\alpha \left[2(1 - \alpha - \beta)(\gamma a + \Delta\theta c) + \Delta\theta\beta \left(1 - \frac{\nabla}{\Delta} \frac{\sigma_H - \sigma_L}{S_H - S_L} \right) \right]} \quad (22)$$

Proof: See in appendix.

All factors increasing directional-traders profit on one market attract them on this market and push them away from the other. It should be noted that the effect of each parameter is the same whether the point is to examine the conditions under which traders concentrate their orders in one of the two markets or just prefer one market to the other.

Directional-traders profit and the way these traders are distributed between the two markets is related, first, to the characteristics of the contracts. Whatever the signal received by the directional-traders may be (positive or negative), they are more likely to trade on the spot (options) market if:

- i) size lot on the spot market γ is large (small);
- ii) size lot on the option market θ is small (large);
- iii) sensibility of options price to underlying price variations Δ is weak (strong);
- iv) sensibility of options price to volatility ∇ is strong (weak). This last effect, due to the the widening of the options bid-ask spread by market makers in order to limited their losses against volatility-traders, is ignored by Easley *et al.* (1998) and John *et al.* (2000).

Directional-traders strategy depend also on the possibility to hide their trades in order to reduce transaction costs. η_L and η_H depend on spot and options market depth (measure by a , b , c and d) as well as they depend on the fraction of uninformed-traders $1 - \alpha - \beta$. Effects are the same, but in the opposite direction, for the spot or the options market.

A crucial consequences is that derivatives market may play a main role in the information diffusion process only if they are liquid. However, it is just a necessary condition, not a sufficient one.

Proposition 2 *Directional-traders' choice to trade on the spot market depends positively on:*

- i) *the fraction of volatility trader since they force the market maker to wide the options bid-ask ask spread ($\partial\eta_{H,L}/\partial\beta > 0$);*
- ii) *the volatility informational advantage ($\partial\eta_{H,L}/\partial(\sigma_H - \sigma_L) < 0$).*

The first part of the proposition states that depth is only a necessary condition for options market to lead spot market. Say it in a different way, if volatility-traders dominate the order, directional-traders will trade on the spot market or any other market where volatility-traders cannot take advantage of their information, like the future market. The second part means that the informational role of stock price is greater when uncertainty about future volatility is high, which has recently confirmed by Chakravarty, Gulen and Mayhew (2004).

To focus on the strategy of directional-traders, we suppose now that spot and options market depth is the same, that is the fraction of uninformed-traders is identical on both markets: $a = b = c = d = 1/4$.²⁴ Further, we suppose that none of the market has an advantage for good,

²⁴This assumption is also adopted by Easley *et al.* (1998) and John *et al.* (2000).

that is $A < \frac{\theta}{\gamma}\Delta < B$. In this case, directional-traders strategy is the same whatever they receive a good or a bad signal and $A \equiv A_H = A_L$, $B \equiv B_H = B_L$ and $\eta \equiv \eta_H = \eta_L$:

$$\eta = \frac{(1 - \alpha - \beta) \left[\gamma \left((1 - \alpha - \beta)/2 + \alpha + \beta \right) - \theta \Delta \left((1 - \alpha - \beta)/2 + \beta \left(1 - \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} \right) \right) \right]}{\alpha \left[\gamma (1 - \alpha - \beta) + \theta \Delta \left((1 - \alpha - \beta) + 2\beta \left(1 - \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} \right) \right) \right]} \quad (23)$$

4.3 The equilibrium prices

In equilibrium, the following conditions are satisfied. First the bid and ask prices, contingent on market makers information, result in zero expected profit.²⁵ Second, equilibrium require that informed traders select the trade that maximizes their profit given their information. In the rest of the paper, we assume that the four states of the world at time 1 (S_H, σ_H) , (S_H, σ_L) , (S_L, σ_H) , (S_L, σ_L) are equally likely to occur.

Informed-traders correctly anticipates the reaction of the market makers to their trades. Option market makers cannot distinguish between liquidity-, volatility- or directional-traders. Instead, they make inferences about the probability of trading with either group from the observed order flow in cash and option markets. So market makers, in interpreting the information content of the trade, apply Bayes rule and correctly conjecture the trading strategies of the other.

To determine equilibrium prices, we substitute η by its equilibrium value (equation 23) in bid-ask quotation defined in lemmas 1 and 2.

Proposition 3 *Equilibrium market-market bid-ask prices are defined as follow.*

1. *On the spot market:*

$$b_s^* = \bar{S} - \frac{1}{2}(S_H - S_L) \left[\frac{1 + \alpha + \beta}{2} - \frac{\theta \Delta}{\gamma} \left(\frac{1 - \alpha + \beta}{2} - \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} \right) \right] \quad (24)$$

$$a_s^* = \bar{S} + \frac{1}{2}(S_H - S_L) \left[\frac{1 + \alpha + \beta}{2} - \frac{\theta \Delta}{\gamma} \left(\frac{1 - \alpha - \beta}{2} - \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} \right) \right] \quad (25)$$

2. *On the options market:*

$$b_c^* = \bar{C} - \frac{1}{2}(S_H - S_L) \Delta \left[\frac{1 + \alpha - \beta}{2} + \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} - \frac{\gamma}{\theta \Delta} \frac{1 - \alpha - \beta}{2} \right] \quad (26)$$

$$a_c^* = \bar{C} + \frac{1}{2}(S_H - S_L) \Delta \left[\frac{1 + \alpha - \beta}{2} + \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} - \frac{\gamma}{\theta \Delta} \frac{1 - \alpha - \beta}{2} \right] \quad (27)$$

Proof: See in appendix.

As market makers are looking for offsetting losses they made against informed-traders by profit made at the expense of liquidity-traders, the bid-ask spread vary, in any case, with the fraction of directional-traders ($\partial(a_s^* - b_s^*)/\partial\alpha > 0$ or $\partial(a_c^* - b_c^*)/\partial\alpha > 0$) and with the fraction of volatility-traders ($\partial(a_s^* - b_s^*)/\partial\beta > 0$ or $(\partial(a_c^* - b_c^*)/\partial\beta > 0)$). On the contrary, bid-ask spread are lower when the fraction of uninformed-traders $(1 - \alpha - \beta)$ is large. Informed-traders profit - and market makers losses - depend on order size. Spot (options) bid-ask spread widen with γ (θ) and narrow with θ (γ). Bid-ask spread vary also in accordance with informational advantage. But, bid-ask spread in the option markets market reflects both information components. Indeed, spot bid-ask spread widen with an increase of the price informational advantage $(\partial(a_s^* - b_s^*)/\partial(S_H -$

²⁵As usual, the zero expected profit condition for the market makers stems from Bertrand competition between identical, risk-neutral agents.

$S_L) > 0$), whereas options bid-ask spread widen both with an increase of the price informational advantage ($\partial(a_c^* - b_c^*)/\partial(S_H - S_L) > 0$) and with an increase of the volatility informational advantage ($\partial(a_c^* - b_c^*)/\partial(\sigma_H - \sigma_L) > 0$).

Options are all the more useful for informed-traders since their price are sensitive to price and volatility variation. When option prices react strongly to underlying price variations – when *delta* is close to 1 – directional-traders have a stake in the options market and market makers have to wide the bid-ask spread ($\partial(a_c^* - b_c^*)/\partial\Delta > 0$), whereas on the spot market they may narrow it ($\partial(a_s^* - b_s^*)/\partial\Delta < 0$). In the same way, when option prices are sensitive to volatility variation – when *vega* is high – market makers widen the bid-ask spread. Further, as transaction costs increase, some directional-traders move towards the spot market. At the equilibrium, it is not enough to offset the initial bid-ask spread widening ($\partial(a_c^* - b_c^*)/\partial\nabla < 0$) while it has a positive impact on the spot bid-ask spread ($\partial(a_s^* - b_s^*)/\partial\nabla > 0$).²⁶ For a call, the δ is all the closest to 1 as strike price is low and the time-to-expiration is short, while the ∇ is high when strike price is close to the underlying price and time-to-expiration is long (see in appendix for a graphical illustration). Naturally, this lead to the question of knowing the impact of strike price and time-to-expiration on bid-ask spread.²⁷ Ceteris paribus, options bid-ask spread increase with time-to-expiration. Indeed, more the time-to-expiration is later, more the option prices is sensitive to the price variations and to the volatility variations. But the impact of strike price is ambiguous. Bid-ask spread is narrower for at-the-money options than for out-of-the-money options because the latter are less sensitive than the former both to price and volatility variations. But for in-the-money options, the impact of strike price is not ambivalent. When strike price is higher than the spot price of the underlying asset, the option is more sensitive to price variations and market makers widen the bid-ask spread. But, the options is also less sensitive to volatility variations and market makers may narrow their bid-ask spread. Then, the whole effect depend on the distribution of informed-traders ant on their informational advantage. For instance, if volatility-traders dominate the order process, then bid-ask spread is narrow for in-the-money options than for at-the-money option.

4.4 Options and informational efficiency

In this subsection, we examine whether the introduction of options leads to better market efficiency. Like in John *et al.* (2000), market efficiency (ε) is measure by the amount of information revealed by trades. We compare market efficiency in the following situations: with or without options and with or without volatility-traders.

At time 1, all the information is perfectly known. Spot price is equal to S_H with probability 1/2 or to S_L with probability 1/2. Price variance is equal to $(S_H - S_L)^2/4$.

Without options, spot price is equal to $\bar{S} - \frac{1}{2}(S_H - S_L)\alpha$ or $\bar{S} + \frac{1}{2}(S_H - S_L)\alpha$ according to the fact that the market maker received a buy or a sell order. Increase and decrease of the spot price at time 1 is equiprobable and market depth is symmetric, the probability of a buy order or a sell order is the same. Just after a trade, price variance is equal to $\alpha^2(S_H - S_L)^2/4$. Then, market efficiency is measure by α^2 , and it increases with directional-traders.²⁸

When options are listed, the expected spot price is equal to $\bar{S} - \frac{1}{2} \frac{(S_H - S_L)\alpha\eta}{(1 - \alpha - \beta)/2 + \alpha\eta}$, or to $\bar{S} +$

²⁶It may be noted that the impact of *vega* on options bid and ask prices is all the stronger as the fraction of volatility-traders is high ($\partial^2(a_c^* - b_c^*)/\partial\nabla\beta > 0$), but it is independent of the fraction of directional-traders ($\partial^2(a_c^* - b_c^*)/\partial\nabla\alpha = 0$).

²⁷The impact on the spot bid-ask spread is not examined here because, in practice, market makers propose several options with various strike price and various time-to-expiration and the whole impact is nil.

²⁸If directional-traders only received an imperfect signal concerning the futur price, market efficiency depend also positively on the signal quality.

$\frac{1}{2} \frac{(S_H - S_L)\alpha\eta}{(1-\alpha-\beta)/2+\alpha\eta}$, or to $\bar{S} - \frac{1}{2} \frac{(S_H - S_L)\alpha(1-\eta)}{(1-\alpha-\beta)/2+\alpha(1-\eta)+\beta}$ or to $\bar{S} + \frac{1}{2} \frac{(S_H - S_L)\alpha(1-\eta)}{(1-\alpha-\beta)/2+\alpha(1-\eta)+\beta}$ according to the order the market maker received. Again, as the four states are equiprobable and market depth symmetric, the probability of a buy order, a sell order on the spot market or on the options market is the same. To measure spot market efficiency, we have to compute price variance.

Proposition 4 *Informational spot market efficiency is equal to:*

$$\varepsilon = \alpha^2 \left[\frac{2\eta^2}{(1-\alpha-\beta)+2\alpha\eta} + \frac{2(1-\eta)^2}{(1-\alpha-\beta)+2\alpha(1-\eta)+2\beta} \right] \quad (28)$$

Proof: See in appendix.

Spot market efficiency is stronger (the term between bracket is higher than 1) when options are listed. However, we find that market efficiency decrease with the participation of volatility-traders ($\partial\varepsilon/\partial\beta < 0$). This last result is mainly due to an increase in transaction costs.

5 Discussion, extension and previous empirical evidence

5.1 Options, futures and ETF

Informed-trader trades have a crucial impact on bid-ask spread so informed-traders choose to split their trades to hide their informational advantage. For this reason, one may think that the medium set is an important input.

Traders may based their strategies on several assets. Now it is possible to buy or sell, not only stock, but also futures, forwards, a multitude of call and put options with different strike price and different time-to-expiration. But, in fact, to focus on just one call option is not so constraining.

First, consider futures or forward. Unlike options, futures and forwards may be used to profit from information concerning future prices. For that reason, introducing futures and forwards amounts to reduce the relative depth of options market, all the more since it diminishes transaction cost and lessen short-sales constraints. Besides, it is the same thing if we consider Exchange Traded Funds (ETF). Then, even if we add, forwards, futures or ETF to our model, it does not change any conclusion.

Second, consider that several options with different strikes are written on the stock. We have already seen that the characteristics of options contract have a crucial impact on the leverage effect, the informed trader strategies and the bid-ask spread. For instance, trader who expect good news buys for the underlying stock are more likely to buy out-of-the-money call.²⁹

But, it will be also interesting to study other kind of information. It may done by adding several options contracts. Since several years, a lot of improvement has been done in extracting information from options market prices concerning the whole risk-neutral density (see Jackwerth (1999) for instance).

5.2 Bid-ask spread

Our analysis can also be related to empirical studies of the effect of the introduction of options on the bid-ask spread in the stock market. In our model, volatility-traders on options market increase not only options bid-ask spread but also spot bid-ask spread. Indeed, when options bid-ask spread widen, directional-traders move towards the spot market. Then, our model may

²⁹Cherian (1998) examines volatility-traders strategies who may trade two options with different maturity, whereas Bourghelle (1998) examines volatility-traders strategies who may trade two options with different strike price and a call and a put with the same strike price.

explain why empirical studies (Fedenia and Grammatikos, 1992) find that options listing has an ambiguous impact on spot-bid-ask spread.

Our model is also useful to understand why the impact of options listing is less important nowadays (Sorescu, 2000). One answer - it is merely a conjecture - is may be that volatility-trading is much more common now than in the past. Further this assumption may reconcile the first empirical works which suggest that implied volatility is not a good predictor of future volatility (Day and Lewis, 1992; Canina and Figlewski, 1993; Lamoureux and Lastrapes, 1993) and the one who show recently that implied volatility has a significant informational content (Jorion, 1995; Amin and Ng, 1997; Christensen et Prabhala, 1998; Fleming, 1998; Lin, Strong and Xu, 1998). A last empirical result supporting our model: Mayhew and Mihov (2004) examine the factors influencing the selection of stocks for option listing by exchanges. They suggest a shift over time from volume toward volatility as markets have evolved.

Anyway, it will be interesting to compare spot and options bid-ask spread. Nonetheless, for doing so it is necessary to wonder about inventory costs on options market. If these costs are the same on the two markets, the difference between the relative bid-ask spreads may reveals the intensity of adverse selection problem faces by the market maker.

5.3 Lead-lag relationships between spot and options markets

The empirical evidence on lead-lag relationships between spot and options markets is mixed. If directional-traders trade mostly on the spot market, then price or volume changes in the options market may improve the forecast of price changes in the spot market. According to our model, lead-lag relationships depend not only on market depth but they depend also on the presence of volatility-traders. From this point of view, our model is consistent with empirical results which state that options trades have little predictive power about the future evolution of the stock: Easley, O'Hara, and Srinivas (1998) and Pan and Poteshman (2003) find that signed trading volume in the option market can help forecast stock returns.

Easley *et al.* (1998) show that options volumes do not have a significant impact on stock returns. They use the most actively traded equity options listed on the CBOE. This result is somewhat in contradiction with their theoretical model so they push further their analysis. They suggest to distinguish between trades based on "good news" and trades based on "bad news" concerning the future spot price and they find that option volume *informationally-defined* affects stock prices whereas standard measures of option volume do not. Our analysis sheds some light on this issue. In fact, in the second set of tests the authors focus only on directional-informed trades and our model show that option volumes have an impact only if there not a lot of volatility-traders. Put it in another way, only if information concern the future spot price.

When volatility traders are added to the picture, the larger trading costs (bid-ask spread) in options market may offset the leverage provided by options and reduce or eliminate the informed traders' bias towards option trading. Besides, it may be interesting from this point of view to examine directly if volatility-traders have a significant impact on the lead-lag relationship by comparing situations where they dominate, or not, the order flow. One should test empirically this situation. However, the traders motivation are unobservable so we need to impose some additional assumptions (Cherian and Weng (1999)).

Chakravarty, Gulen and Mayhew (2004) investigate also the contribution of option markets to price discovery. Their results are consistent with theoretical arguments that informed investors trade in both stock and option markets, but they support our model since less price discovery occurs in the options market when the level of uncertainty is high.

6 Conclusion

It is widely accepted that options implied volatility is a good estimate of the market expectations of the assets future volatility, even if some studies (Canina and Figlewski (1993) for instance) refute that view. But this fundamental aspect of the options market has not been explicitly considered in theoretical models, except by Cherian (1998) and Cherian and Jarrow (1998).

To our knowledge, this paper is the first attempt to investigate the implication of volatility trading on where informed traders trade. Moreover, it allows us to develop testable hypotheses about the location at which informed traders trade.

This setup allows us to examine the impact of volatility trading both on option market bid-ask spread and on the directional traders behavior. We show that when volatility traders are presents, directional-informed traders facing wider bid-ask spreads in option market. Hence, in equilibrium, they split their trades in such way that their profits are the same in both markets. Their optimal trading strategy depends on the percentage of liquidity traders and on the percentage of volatility traders in option markets. Consequently, bid and ask prices in both markets will also be functions of these percentage. We also find that the introduction of option trading does not improve unambiguously the efficiency of stock market.

In fact, our result casts doubt on the price discovery argument between stock and option market. When market makers are not subject to volatility trading, options market play an important role in optimal trading strategies of directional-traders. But in a more realistic setting, where traders do face volatility-traders, directional-traders do not use options market. These results are consistent with previous results of Easley et al. (1998) since, they report, on the one hand, evidence of feedback, and, on the other hand, that negative and positive volumes contain information about future stock prices. In doing so, they only focus on directional trade. They find that option volume informationally-defined affects stock prices, while standard measures of option volume do not.

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Appendix

Graphical illustrations

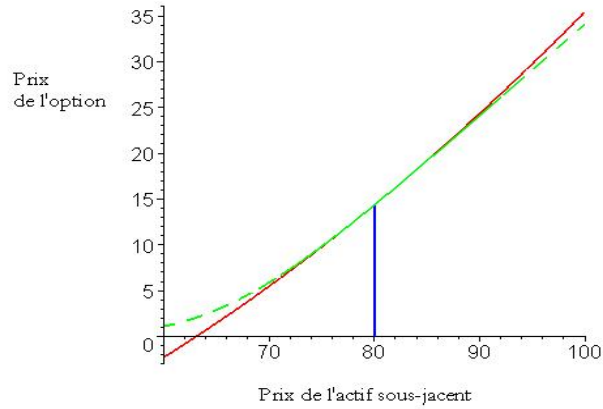


Figure 2: Quadratic approximation of a call price with a Taylor expansion around $S = 80$ with $K = 70$; $\sigma = 18\%$; $r = 12\%$; $\tau = 0,5$; $S = [50; 110]$.

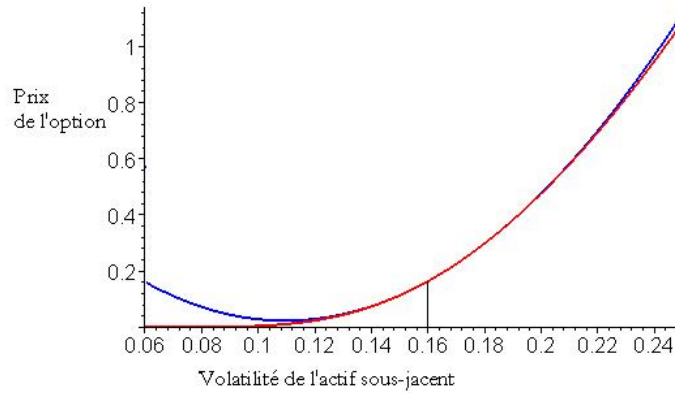


Figure 3: Quadratic approximation of a call price with a Taylor expansion around $\sigma = 16\%$ with $K = 70$; $S = 60$; $r = 12\%$; $\tau = 1$; $\sigma = [0,01; 0,25]$.

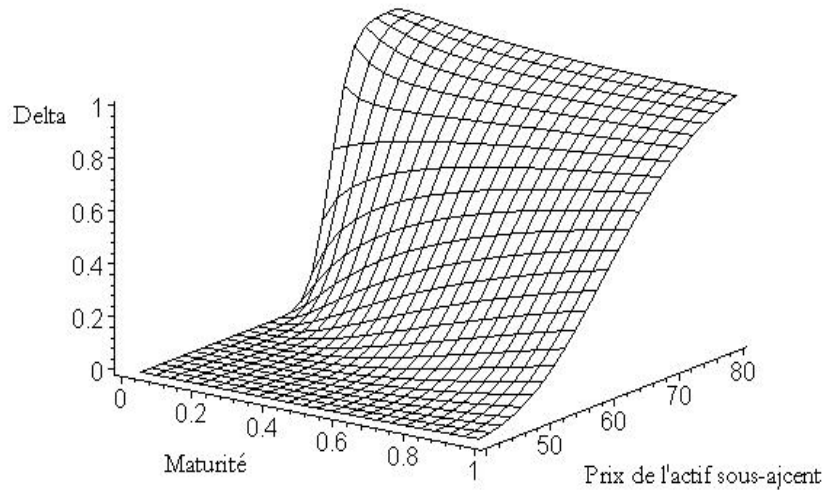


Figure 4: Sensitivity of call option price to underlying price variation (Δ) accordingly to underlying price and time-to-expiration. $K = 70; \sigma = 18\%; r = 12\%; \tau = [0, 1], S = [40, 80]$.

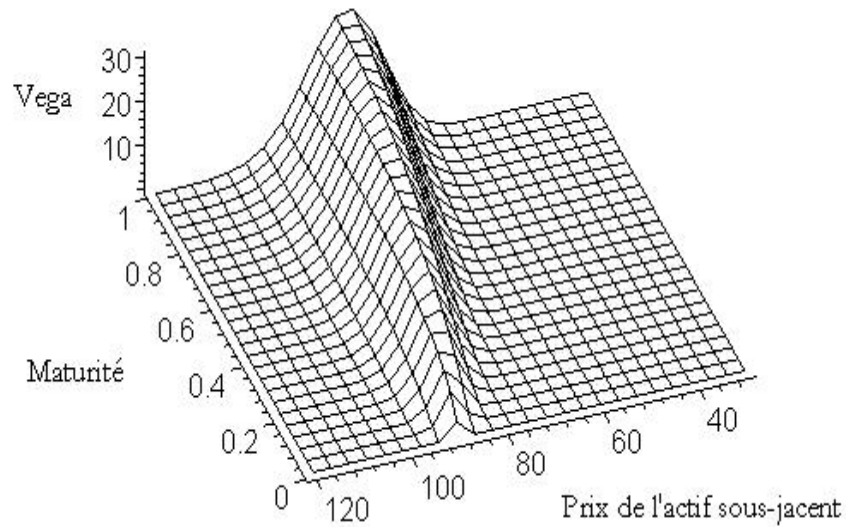


Figure 5: Sensitivity of call option price to volatility (∇) accordingly to underlying price and time-to-expiration. $K = 90; \sigma = 12\%; r = 10\%; \tau = [0, 1], S = [30, 120]$.

Conditional market makers bid-ask spread

The spot market

Bid price is equal to the market maker expected value given a sell order. Applying Bayes' rule:

$$\begin{aligned} b_s &= E[S / \text{sell order}] \\ &= \frac{S_H(1 - \alpha - \beta)b + S_L(\alpha\eta_L + (1 - \alpha - \beta)b)}{(1 - \alpha - \beta)b + (\alpha\eta_L + (1 - \alpha - \beta)b)} \end{aligned} \quad (29)$$

and

$$\begin{aligned} a_s &= E[S / \text{buy order}] \\ &= \frac{S_H\lambda(\alpha\eta_H + (1 - \alpha - \beta)a) + S_L(1 - \lambda)(1 - \alpha - \beta)a}{\lambda(\alpha\eta_H + (1 - \alpha - \beta)a) + (1 - \lambda)(1 - \alpha - \beta)a} \end{aligned} \quad (30)$$

We may rewrite the last equations:

$$b_s = \bar{S} - \frac{1}{2} \left[\frac{(S_H - S_L)\alpha\eta_L}{2(1 - \alpha - \beta)b + \alpha\eta_L} \right] \quad (31)$$

$$a_s = \bar{S} + \frac{1}{2} \left[\frac{(S_H - S_L)\alpha\eta_H}{2(1 - \alpha - \beta)a + \alpha\eta_H} \right] \quad (32)$$

so:

$$a_s - b_s = \frac{1}{2}(S_H - S_L)\alpha \left[\frac{\eta_H}{2(1 - \alpha - \beta)a + \alpha\eta_H} + \frac{\eta_L}{2(1 - \alpha - \beta)b + \alpha\eta_L} \right] \quad (33)$$

with

$$\frac{\partial b_s}{\partial \eta_L} = -\frac{(S_H - S_L)\alpha(1 - \alpha - \beta)b}{(2(1 - \alpha - \beta)b + \alpha\eta_L)^2} < 0 \quad (34)$$

$$\frac{\partial a_s}{\partial \eta_H} = \frac{(S_H - S_L)\alpha(1 - \alpha - \beta)a}{(2(1 - \alpha - \beta)a + \alpha\eta_H)^2} > 0 \quad (35)$$

$$\frac{\partial b_s}{\partial \alpha} = -\frac{(S_H - S_L)\eta_L(1 + \alpha - \beta)b}{(2(1 - \alpha - \beta)b + \alpha\eta_L)^2} < 0 \quad (36)$$

$$\frac{\partial a_s}{\partial \alpha} = \frac{(S_H - S_L)\eta_H(1 + \alpha - \beta)a}{(2(1 - \alpha - \beta)a + \alpha\eta_H)^2} > 0 \quad (37)$$

The options market

Like on the spot market:

$$\begin{aligned} b_c &= E[C/\text{sell order}] \\ &= \frac{(1 - \alpha - \beta)dE[C] + \alpha(1 - \eta_L)E[C|S = S_L]/2 + \beta E[C|\sigma = \sigma_L]/2}{(1 - \alpha - \beta)d + \alpha(1 - \eta_L)/2 + \beta/2} \end{aligned} \quad (38)$$

and

$$\begin{aligned} a_c &= E[C/\text{buy order}] \\ &= \frac{(1 - \alpha - \beta)cE[C] + \alpha(1 - \eta_H)E[C|S = S_H]/2 + \beta E[C|\sigma = \sigma_H]/2}{(1 - \alpha - \beta)c + \alpha(1 - \eta_H)/2 + \beta/2} \end{aligned} \quad (39)$$

with

$$E[C] = (C(S_H, \sigma_H) + C(S_L, \sigma_H) + C(S_H, \sigma_L) + C(S_L, \sigma_L))/4 \quad (40)$$

$$E[C|S = S_L] = (C(S_L, \sigma_H) + C(S_L, \sigma_L))/2 \quad (41)$$

$$E[C|S = S_H] = (C(S_H, \sigma_H) + C(S_H, \sigma_L))/2 \quad (42)$$

$$E[C|\sigma = \sigma_L] = (C(S_H, \sigma_L) + C(S_L, \sigma_L))/2 \quad (43)$$

$$E[C|\sigma = \sigma_H] = (C(S_H, \sigma_H) + C(S_L, \sigma_H))/2 \quad (44)$$

We apply a quadratic Taylor expansion around \bar{S} and $\bar{\sigma}$:

$$C(S_H, \sigma_H) = C(\bar{S}, \bar{\sigma}) + (S_H - \bar{S})\Delta + (\sigma_H - \bar{\sigma})\nabla \quad (45)$$

$$C(S_L, \sigma_H) = C(\bar{S}, \bar{\sigma}) + (S_L - \bar{S})\Delta + (\sigma_H - \bar{\sigma})\nabla \quad (46)$$

$$C(S_H, \sigma_L) = C(\bar{S}, \bar{\sigma}) + (S_H - \bar{S})\Delta + (\sigma_L - \bar{\sigma})\nabla \quad (47)$$

$$C(S_L, \sigma_L) = C(\bar{S}, \bar{\sigma}) + (S_L - \bar{S})\Delta + (\sigma_L - \bar{\sigma})\nabla \quad (48)$$

so

$$C(S_H, \sigma_H) + C(S_L, \sigma_H) - C(S_H, \sigma_L) - C(S_L, \sigma_L) = 2(\sigma_H - \sigma_L)\nabla \quad (49)$$

$$C(S_H, \sigma_H) - C(S_L, \sigma_H) + C(S_H, \sigma_L) - C(S_L, \sigma_L) = 2(S_H - S_L)\Delta \quad (50)$$

Some algebraic simplifications lead to:

$$b_c = \bar{C} - \frac{1}{2} \left[\frac{(S_H - S_L)\Delta\alpha(1 - \eta_L) + (\sigma_H - \sigma_L)\nabla\beta}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta} \right] \quad (51)$$

$$a_c = \bar{C} + \frac{1}{2} \left[\frac{(S_H - S_L)\Delta\alpha(1 - \eta_H) + (\sigma_H - \sigma_L)\nabla\beta}{2(1 - \alpha - \beta)c + \alpha(1 - \eta_H) + \beta} \right] \quad (52)$$

Then, options bid-ask spread is:

$$(S_H - S_L)\Delta \left[\frac{\alpha(1 - \eta_H) + \beta \frac{\nabla}{\Delta} \frac{\sigma_H - \sigma_L}{S_H - S_L}}{2(1 - \alpha - \beta)c + \alpha(1 - \eta_H) + \beta} + \frac{\alpha(1 - \eta_L) + \beta \frac{\nabla}{\Delta} \frac{\sigma_H - \sigma_L}{S_H - S_L}}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta} \right] \quad (53)$$

with

$$\frac{\partial b_c}{\partial \eta_L} = - \frac{2(S_H - S_L)\Delta(1 - \alpha - \beta)d + ((S_H - S_L)\Delta - (\sigma_H - \sigma_L)\nabla)\beta}{(2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta)^2} \quad (54)$$

so

$$\frac{\partial b_c}{\partial \eta_L} > 0 \Leftrightarrow \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} + 2(1 - \alpha - \beta)d \right) > 0 \quad (55)$$

and

$$\frac{\partial a_c}{\partial \eta_H} = \frac{2(S_H - S_L)\Delta(1 - \alpha - \beta)c + ((S_H - S_L)\Delta - (\sigma_H - \sigma_L)\nabla)\beta}{(2(1 - \alpha - \beta)c + \alpha(1 - \eta_H) + \beta)^2} \quad (56)$$

so

$$\frac{\partial a_c}{\partial \eta_H} < 0 \Leftrightarrow \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} + 2(1 - \alpha - \beta)c \right) > 0 \quad (57)$$

Directional-traders strategy

Case 1: S_L

Suppose that directional-traders know that the spot price at time 1 will be S_L , they may sell a stock or sell a call (equation (1)).

1. On the spot market the profit will be:

$$\begin{aligned} \gamma(b_s - S_L) &= \gamma \left[\bar{S} - \frac{1}{2} \left[\frac{(S_H - S_L)\alpha\eta_L}{2(1 - \alpha - \beta)b + \alpha\eta_L} \right] - S_L \right] \\ &= \frac{1}{2} \gamma (S_H - S_L) \left(1 - \frac{\alpha\eta_L}{2(1 - \alpha - \beta)b + \alpha\eta_L} \right) \\ &= \frac{1}{2} \gamma (S_H - S_L) \frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b + \alpha\eta_L} \end{aligned} \quad (58)$$

2. On the options market, the profit will be:

$$\begin{aligned}
\theta(b_c - C(S_L, \sigma)) &= \theta \left[\bar{C} - \frac{1}{2} \left[\frac{(S_H - S_L)\Delta\alpha(1 - \eta_L) + (\sigma_H - \sigma_L)\nabla\beta}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta} \right] \right. \\
&\quad \left. - \frac{C(S_L, \sigma_H) + C(S_L, \sigma_L)}{2} \right] \\
&= \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{\alpha(1 - \eta_L) + \beta\frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta} \right) \\
&= \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{2(1 - \alpha - \beta)d + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta} \right) \tag{59}
\end{aligned}$$

First suppose $\eta_L = 1$, that is, any directional-traders choose the options market. The question is: under which conditions profit is the highest on the options market? Formally, we are looking for parameters value for which the following condition is accepted:

$$\begin{aligned}
&\gamma(b_s - S_L) < \theta(b_c - C(S_L, \sigma)) \\
\Leftrightarrow &\frac{1}{2}\gamma(S_H - S_L)\frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b + \alpha} < \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{2(1 - \alpha - \beta)d\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)d + \beta} \right) \\
\Leftrightarrow &\gamma\frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b + \alpha} < \theta\Delta \left(1 - \frac{2(1 - \alpha - \beta)d\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)d + \beta} \right) \\
\Leftrightarrow &\frac{\theta}{\gamma}\Delta > \frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b + \alpha} \times \frac{2(1 - \alpha - \beta)d + \beta}{2(1 - \alpha - \beta)d + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)} \tag{60}
\end{aligned}$$

If this inequality is not verified, all directional-traders trade on the spot market.

Second, suppose $\eta_L = 0$, that is any directional-trader choose the spot market. The question is: under which conditions profit is the highest on the spot market? Formally, we are looking for parameters value for which the following condition is accepted:

$$\begin{aligned}
&\gamma(b_s - S_L) > \theta(b_c - C(S_L, \sigma)) \\
\Leftrightarrow &\frac{1}{2}\gamma(S_H - S_L)\frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b} > \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{2(1 - \alpha - \beta)d\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)d + \alpha + \beta} \right) \\
\Leftrightarrow &\gamma\frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b} > \theta\Delta \left(1 - \frac{2(1 - \alpha - \beta)d\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)d + \alpha + \beta} \right) \\
\Leftrightarrow &\frac{\theta}{\gamma}\Delta < \frac{2(1 - \alpha - \beta)d + \alpha + \beta}{2(1 - \alpha - \beta)d + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)} \tag{61}
\end{aligned}$$

If this inequality is not verified, all directional-traders trade on the options market.

If the two last inequality are verified, at the equilibrium directional-traders profit must be the same on the spot market and on the options market. The fraction of directional traders on the spot market is determined by resolving the following equality:

$$\gamma(b_s - S_L) = \theta(b_c - C(S_L, \sigma))$$

$$\begin{aligned}
&\Leftrightarrow \frac{1}{2}\gamma(S_H - S_L)\frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b + \alpha\eta_L} = \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{2(1 - \alpha - \beta)d\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta} \right) \\
&\Leftrightarrow \gamma\frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b + \alpha\eta_L} = \theta\Delta \left(1 - \frac{2(1 - \alpha - \beta)d\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_L) + \beta} \right)
\end{aligned} \tag{62}$$

and

$$\eta_L = \frac{2(1 - \alpha - \beta)b \left[\gamma(2(1 - \alpha - \beta)d + (\alpha + \beta)) - \theta\Delta \left(2(1 - \alpha - \beta)d + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right) \right) \right]}{\alpha \left[\gamma(2(1 - \alpha - \beta)b) + \theta\Delta \left(2(1 - \alpha - \beta)d + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right) \right) \right]} \tag{63}$$

Case 2: S_H

If $S = S_H$, the logic is the same but directional-trader may buy a stock or a call (equation (2)).

1. On the spot market, the profit will be:

$$\begin{aligned}
\gamma(-a_s + S_H) &= \gamma \left[-\bar{S} - \frac{1}{2} \left[\frac{(S_H - S_L)\alpha\eta_H}{2(1 - \alpha - \beta)a + \alpha\eta_H} \right] + S_H \right] \\
&= \frac{1}{2}\gamma(S_H - S_L) \left(1 - \frac{\alpha\eta_H}{2(1 - \alpha - \beta)a + \alpha\eta_H} \right) \\
&= \frac{1}{2}\gamma(S_H - S_L)\frac{2(1 - \alpha - \beta)a}{2(1 - \alpha - \beta)a + \alpha\eta_H}
\end{aligned} \tag{64}$$

2. On the options market, the profit will be:

$$\begin{aligned}
\theta(-a_c + C(S_L, \sigma)) &= \theta \left[-\bar{C} - \frac{1}{2} \left[\frac{(S_H - S_L)\Delta\alpha(1 - \eta_H) + (\sigma_H - \sigma_L)\nabla\beta}{2(1 - \alpha - \beta)c + \alpha(1 - \eta_H) + \beta} \right] \right. \\
&\quad \left. + \frac{C(S_H, \sigma_H) + C(S_L, \sigma_H)}{2} \right] \\
&= \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{\alpha(1 - \eta_H) + \beta\frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}}{2(1 - \alpha - \beta)c + \alpha(1 - \eta_H) + \beta} \right) \\
&= \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{2(1 - \alpha - \beta)d + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)c + \alpha(1 - \eta_H) + \beta} \right)
\end{aligned} \tag{65}$$

First suppose $\eta_L = 1$, that is, any directional-traders choose the options market. The question is: under which conditions profit is the highest on the options market? Formally, we are looking for parameters value for which the following condition is accepted:

$$\begin{aligned}
&\gamma(-a_s + S_H L) < \theta(-a_c + C(S_H, \sigma)) \\
&\Leftrightarrow \frac{1}{2}\gamma(S_H - S_L)\frac{2(1 - \alpha - \beta)a}{2(1 - \alpha - \beta)a + \alpha} < \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{2(1 - \alpha - \beta)c\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)c + \beta} \right) \\
&\Leftrightarrow \gamma\frac{2(1 - \alpha - \beta)a}{2(1 - \alpha - \beta)a + \alpha} < \theta\Delta \left(1 - \frac{2(1 - \alpha - \beta)c\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}{2(1 - \alpha - \beta)c + \beta} \right) \\
&\Leftrightarrow \frac{\theta}{\gamma}\Delta > \frac{2(1 - \alpha - \beta)a}{2(1 - \alpha - \beta)a + \alpha} \times \frac{2(1 - \alpha - \beta)c + \beta}{2(1 - \alpha - \beta)c + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta} \right)}
\end{aligned} \tag{66}$$

If this inequality is not verified, all directional-traders trade on the spot market.

Second, suppose $\eta_L = 0$, that is any directional-trader choose the spot market. The question is: under which conditions profit is the highest on the spot market? Formally, we are looking for parameters value for which the following condition is accepted:

$$\begin{aligned}
& \gamma(-a_s + S_H) > \theta(-a_c + C(S_H, \sigma)) \\
\Leftrightarrow & \frac{1}{2}\gamma(S_H - S_L)\frac{2(1 - \alpha - \beta)a}{2(1 - \alpha - \beta)a} > \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{2(1 - \alpha - \beta)c\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}\right)}{2(1 - \alpha - \beta)c + \alpha + \beta}\right) \\
\Leftrightarrow & \gamma \frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)a} > \theta\Delta \left(1 - \frac{2(1 - \alpha - \beta)c\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}\right)}{2(1 - \alpha - \beta)a + \alpha + \beta}\right) \\
\Leftrightarrow & \frac{\theta}{\gamma}\Delta < \frac{2(1 - \alpha - \beta)c + \alpha + \beta}{2(1 - \alpha - \beta)c + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}\right)} \tag{67}
\end{aligned}$$

If this inequality is not verified, all directional-traders trade on the options market.

If the two last inequality are verified, at the equilibrium directional-traders profit must be the same on the spot market and on the options market. The fraction of directional traders on the spot market is determined by resolving the following equality:

$$\begin{aligned}
& \gamma(-a_s + S_H) = \theta(-a_c + C(S_H, \sigma)) \\
\Leftrightarrow & \frac{1}{2}\gamma(S_H - S_L)\frac{2(1 - \alpha - \beta)a}{2(1 - \alpha - \beta)b + \alpha\eta_H} = \frac{1}{2}\theta(S_H - S_L)\Delta \left(1 - \frac{2(1 - \alpha - \beta)c\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}\right)}{2(1 - \alpha - \beta)d + \alpha(1 - \eta_H) + \beta}\right) \\
\Leftrightarrow & \gamma \frac{2(1 - \alpha - \beta)b}{2(1 - \alpha - \beta)b + \alpha\eta_H} = \theta\Delta \left(1 - \frac{2(1 - \alpha - \beta)c\beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}\right)}{2(1 - \alpha - \beta)c + \alpha(1 - \eta_H) + \beta}\right) \tag{68}
\end{aligned}$$

and

$$\eta_H = \frac{2(1 - \alpha - \beta)a \left[\gamma(2(1 - \alpha - \beta)c + (\alpha + \beta)) - \theta\Delta \left(2(1 - \alpha - \beta)c + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}\right)\right) \right]}{\alpha \left[\gamma(2(1 - \alpha - \beta)a) + \theta\Delta \left(2(1 - \alpha - \beta)c + \beta \left(1 - \frac{(\sigma_H - \sigma_L)\nabla}{(S_H - S_L)\Delta}\right)\right) \right]} \tag{69}$$

We can easily set a typology of the different equilibrium; this typology is presented in Table 1. Cases 1 and 3 are quite extreme, so in the following we will focus on case 2 where none of the market has an advantage for good.

Equilibrium prices

To determine equilibrium we just have to substitute η in conditional bid and ask prices defined in lemmas 1 and 2. After some algebraic simplifications, we obtain the following bid and ask prices.

On the spot market:

$$b_s^* = \bar{S} - \frac{1}{2}(S_H - S_L) \left[\frac{1 + \alpha + \beta}{2} - \frac{\theta\Delta}{\gamma} \left(\frac{1 - \alpha - \beta}{2} + \beta \left(1 - \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L}\right) \right) \right] \tag{70}$$

$$a_s^* = \bar{S} + \frac{1}{2}(S_H - S_L) \left[\frac{1 + \alpha + \beta}{2} - \frac{\theta\Delta}{\gamma} \left(\frac{1 - \alpha - \beta}{2} + \beta \left(1 - \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L}\right) \right) \right] \tag{71}$$

Table 1: **Typology of equilibrium**

	$\frac{\theta}{\gamma}\Delta < A$		$A < \frac{\theta}{\gamma}\Delta < B$		$B < \frac{\theta}{\gamma}\Delta$	
	spot	options	spot	options	spot	options
Uninformed	yes	yes	yes	yes	yes	yes
Directional	yes	no	yes	yes	no	yes
Volatility	no	yes	no	yes	no	yes
Situation	1		2		3	

On the options market:

$$b_c^* = \bar{C} - \frac{1}{2}(S_H - S_L)\Delta \left[\frac{1 + \alpha - \beta}{2} + \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} - \frac{\gamma}{\theta\Delta} \left(\frac{(1 - \alpha - \beta)((1 - \alpha + \beta) - 2\beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L})/4}{(1 - \alpha - \beta)/2 + \beta \left(1 - \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L}\right)} \right) \right] \quad (72)$$

so

$$b_c^* = \bar{C} - \frac{1}{2}(S_H - S_L)\Delta \left[\frac{1 + \alpha - \beta}{2} + \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} - \frac{\gamma}{\theta\Delta} \frac{1 - \alpha - \beta}{2} \right] \quad (73)$$

and

$$a_c^* = \bar{C} + \frac{1}{2}(S_H - S_L)\Delta \left[\frac{1 + \alpha - \beta}{2} + \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} - \frac{\gamma}{\theta\Delta} \left(\frac{(1 - \alpha - \beta)((1 - \alpha + \beta) - 2\beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L})/4}{(1 - \alpha - \beta)/2 + \beta \left(1 - \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L}\right)} \right) \right] \quad (74)$$

so

$$a_c^* = \bar{C} + \frac{1}{2}(S_H - S_L)\Delta \left[\frac{1 + \alpha - \beta}{2} + \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} - \frac{\gamma}{\theta\Delta} \frac{1 - \alpha - \beta}{2} \right] \quad (75)$$

At the equilibrium,

1. the spot bid-ask spread is

$$a_s^* - b_s^* = (S_H - S_L) \left[\frac{1 + \alpha + \beta}{2} - \frac{\theta\Delta}{\gamma} \left(\frac{1 - \alpha + \beta}{2} - \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} \right) \right] \quad (76)$$

2. the option bid-ask spread is

$$a_c^* - b_c^* = (S_H - S_L)\Delta \left[\frac{1 + \alpha - \beta}{2} + \beta \frac{\nabla \sigma_H - \sigma_L}{\Delta S_H - S_L} - \frac{\gamma}{\theta\Delta} \frac{1 - \alpha - \beta}{2} \right] \quad (77)$$

with

$$\partial(b_s^* - a_s^*)/\partial(\sigma_H - \sigma_L) > 0 \quad (78)$$

$$\partial(b_s^* - a_s^*)/\partial(S_H - S_L) > 0 \quad (79)$$

$$\partial(b_s^* - a_s^*)/\partial\gamma > 0 \quad (80)$$

$$\partial(b_s^* - a_s^*)/\partial\theta < 0 \quad (81)$$

$$\partial(b_s^* - a_s^*)/\partial\nabla > 0 \quad (82)$$

$$\partial(b_s^* - a_s^*)/\partial\Delta < 0 \quad (83)$$

$$\partial(b_s^* - a_s^*)/\partial\alpha > 0 \quad (84)$$

$$\partial(b_s^* - a_s^*)/\partial\beta > 0 \quad (85)$$

$$\partial(b_s^* - a_s^*)/\partial(1 - \alpha - \beta) < 0 \quad (86)$$

$$\partial(b_c^* - a_c^*)/\partial(\sigma_H - \sigma_L) > 0 \quad (87)$$

$$\partial(b_c^* - a_c^*)/\partial(S_H - S_L) > 0 \quad (88)$$

$$\partial(b_c^* - a_c^*)/\partial\gamma < 0 \quad (89)$$

$$\partial(b_c^* - a_c^*)/\partial\theta > 0 \quad (90)$$

$$\partial(b_c^* - a_c^*)/\partial\nabla > 0 \quad (91)$$

$$\partial(b_c^* - a_c^*)/\partial\Delta > 0 \quad (92)$$

$$\partial(b_c^* - a_c^*)/\partial\alpha > 0 \quad (93)$$

$$\partial(b_c^* - a_c^*)/\partial\beta > 0 \quad (94)$$

$$\partial(b_c^* - a_c^*)/\partial(1 - \alpha - \beta) < 0 \quad (95)$$

Market efficiency

$$\varepsilon = \alpha^2 \left[\frac{2\eta^2}{(1 - \alpha - \beta) + 2\alpha\eta} + \frac{2(1 - \eta)^2}{(1 - \alpha - \beta) + 2\alpha(1 - \eta) + 2\beta} \right] \quad (96)$$

with

$$\left[\frac{2\eta^2}{(1 - \alpha - \beta) + 2\alpha\eta} + \frac{2(1 - \eta)^2}{(1 - \alpha - \beta) + 2\alpha(1 - \eta) + 2\beta} \right] > 1 \quad (97)$$

and

$$\partial\varepsilon/\partial\beta < 0 \quad (98)$$