Basel Requirement of Downturn LGD:
Modeling and Estimating PD & LGD Correlations

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Abstract

Basel II requires that banks use \textit{downturn} loss given default (LGD) estimates in regulatory capital calculations, citing the fact that the probability of default (PD) and LGD correlations are not captured. We show that the lack of correlation can be taken care of by incorporating certain degree of conservatism in cyclical LGD in a point-in-time (PIT) framework. We examine a model which can capture the PD and LGD correlation in its entirety, differentiating the different components of correlations in question. Using historical LGD and default data of a loan portfolio, we calibrate our model and, through the simulation of economic capital, we show the mean LGD needs to be increased by about 35\% to 41\% in order to compensate for the lack of correlations. Our hope is to provide a framework that the banks can use based on their internal data to estimate and justify their LGD choices for different portfolios.

Keywords: Basel II, correlation, probability of default, downturn loss given default, economic capital, Point-in-Time, Through-the-Cycle, credit risk models, calibrations, simulations
Introduction

LGD required for Basel II has been the subject of heavy debate in the industry. Basel document (2004) requires the use of a downturn or bottom-of-the-cycle LGD, which appears to be the maximum of the long-run default-weighted average LGD and the stressed LGD. Industry, concerned about the excessive conservatism and implementation difficulties has taken the view that default-weighted average LGD should be conservative enough for the purpose. A follow up Basel document (2005) elaborated on the so-called “downturn LGD” standard and suggested a principles-based approach. It requires the banks to (a) identify the appropriate downturn conditions and the adverse dependencies, if any, between default rates and recovery rates; and (b) incorporate them so as to produce LGD parameters for the bank’s exposures, which are consistent with the identified downturn conditions. Understanding the extent and manner by which potential dependencies between default rates and recovery rates impact capital estimations is the focal point of the discussion.

A good starting point is to understand the motivation behind the requirement of the bottom of the cycle LGD in Basel document (2004). The more apparent reason for this requirement is that Pillar I regulatory capital formula (as well as off-the-shelf credit risk management systems) does not incorporate the correlations between PD and LGD, therefore potentially underestimates the capital requirement. In order to compensate for this, some degree of conservatism is intended to be incorporated into the mean LGD. Perhaps a less apparent but more important motivation could be to reduce the cyclicity of the capital by adding a not only conservative but also an a-cyclical LGD ingredient to the capital calculation.

By considering the different LGD philosophies, we show that if the reason is to simply introduce some degree of conservatism to account for the lack of correlations, this also can be achieved using cyclical LGD. In other words, not only the a-cyclical through-the-cycle (TTC) framework but also the cyclical point-in-time (PIT) framework can accommodate the required conservatism. We then show that when we use downturn LGD in an a-cyclical fashion as it is commonly interpreted, we effectively increase the risk horizon to beyond one year in calculating the capital requirement, resulting in a larger and less cyclical capital.

We then look at the PD and LGD correlations with one question in mind: How much do we need to increase the mean LGD in order to compensate for the fact that correlations are not modeled? The first thing we realize is that the terminologies of PD and LGD correlations are used rather loosely by both academia and practitioners. A closer look reveals that there are in fact different types correlations in question. Namely, the correlation between PD and LGD risk of the same borrower, and the (pair-wise) correlation in LGD risk among a group of borrowers.

To examine the correlation issue in its entirety, we propose a stylized model and conduct simulations to examine the impact of each element of the correlation structure on the capital requirement. Through this exercise, we provide a framework to quantify Basel bottom-of-the-cycle LGD requirement.

We then go one step further and propose methodologies to estimate the different elements of the correlation structure. We introduce frameworks to estimate the pair-wise LGD correlations among different borrowers and the correlation of systematic PD and LGD risk.
factors. Using these frameworks, we estimate the correlations using historical default data of loan portfolios. Finally, using these estimated parameters, we run simulations to examine the impact on the required economic capital. The main findings are highlighted as follows.

- Pair-wise LGD correlation is found to be small (5.9% for large corporate obligors).\(^1\)
- Systematic PD and LGD risks are found to be evolving in a synchronized fashion over the 90s, with an estimated correlation of 53%. It is however a noisy estimate given the short time-series of data made available. Moreover, the correlation is found to be much smaller for the LGD risks of mid-market borrowers with predominantly secured loans.
- Since pair-wise LGD correlation is small, economic capital is found to be more sensitive to changes in idiosyncratic PD/LGD correlation (than to those in systematic PD/LGD correlation).
- At a moderate level of idiosyncratic PD/LGD correlation and at a level of systematic PD/LGD correlation comparable with that estimated empirically, we need to increase the mean LGD by about 37% from its unbiased estimate in order to achieve the correct economic capital under a model where LGD correlation is ignored.\(^2\) However, the required mark-up for secured loans, which are common for mid-market borrowers, is likely to be much smaller.
- The required “mark-up” in mean LGD is a decreasing function of the pair-wise PD correlation. As pair-wise PD correlation becomes large, portfolio credit risk is governed by the co-movement in credit risk among obligors rather than the PD/LGD correlation of individual obligors.
- The required “mark-up” becomes less sensitive to idiosyncratic PD/LGD correlation as pair-wise PD correlation increases.

**But What Does Bottom-Of-Cycle Mean?**

In using the advanced internal ratings-based (IRB) approach of Basel II, banks will need to estimate LGDs of their borrowers, as a key input to the formula for the calculation of minimum capital requirement. A good place to start is to discuss the LGD philosophy adopted by the particular bank.

LGD philosophy, analogous to the risk rating philosophy for PD, defines assigned LGD’s expected behavior over a business/economic cycle.\(^3\) Under a cyclical philosophy, LGD is intended to be synchronized with the cycle and thus change with the cycle, whereas, under an a-cyclical philosophy, LGD remains constant over the cycle. Note that under the commonly used PIT philosophy, LGD is an cyclical measure reflecting the expected LGD over the coming, typically, 12 months. Conversely, under the TTC philosophy, LGD is a-cyclical, and may define a cycle-average LGD, which is relatively constant over the business cycle. Figure 1 illustrates a plot of the variation of assigned LGD (and thus the amount of capital assigned) over time for these two measures.

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1 Similar results are documented for mid-market portfolio.
2 An unbiased LGD estimate is likely to be lower than the default-weighted average over time.
3 LGD cycles may be different from PD cycles.
Basel II requires bottom-of-the-cycle LGD, estimated from a sufficiently stressed period during which high LGDs are observed. This is comparable with that of a PIT LGD during a market downturn (see Figure 2).

Does the bottom-of-the-cycle LGD requirement necessitate the banks to adopt an a-cyclical LGD philosophy? It depends. If Basel’s motivation is to prevent cyclicality in regulatory capital requirement, it warrants the adoption of an a-cyclical measure similar to the TTC philosophy. However, if the requirement is solely for the compensation of the potential underestimation of capital due to the lack of consideration of the correlations between PD and LGD in the Basel capital formula, we might as well fulfill this objective with a cyclical LGD philosophy. In this case the problem becomes a calibration problem independent of the particular philosophy chosen. The question becomes: Given the adoption of a certain LGD philosophy, how should one assign a sufficiently conservative LGD value in order to achieve an appropriate capital number despite the fact that correlations are not captured? Figure 3 illustrates how conservatism can be incorporated under the TTC and PIT philosophy respectively.

For example, under a cyclical LGD philosophy, we could consider increasing the PIT cycle-average LGD by a certain amount in order to cater for the lack of modeling of PD and LGD correlation (i.e. to the level of the dotted red line in Figure 3). The resulting capital requirement however will still be cyclical. The amount of mark-up required can be ascertained by considering the impact of the correlations on the resulting capital requirement, for example via the stylized model as outlined in the subsequent section.

On the other hand, imposing a conservative a-cyclical LGD (i.e. the dotted blue line) would result in a more stable but a conservative capital requirement over the business cycle. Interestingly, using an a-cyclical conservative LGD would be equivalent to increasing the risk horizon to beyond one year in calculating capital requirement. Suppose a bank adopts a PIT risk rating system and intends to measure the risk over a one-year risk horizon with respect to the current state of the business cycle. If we use an a-cyclical conservative LGD, we effectively consider the same extreme LGD event irregardless of where we currently are in the cycle (i.e. whether we are one year or five years away from a downturn). Suppose we are at the top of the LGD cycle (e.g. Point “A” in Figure 4, when LGD risk is generally low), and if things turn bad, it may actually take another three to four years to hit the bottom of the cycle. Thus, when we adopt an a-cyclical conservative LGD when we are at Point “A”, we are effectively lengthening our risk horizon to beyond our intended one-year period (see illustration in Figure 4). The net effect would therefore be a less cyclical capital but with a capital cushion which is likely to be above the PIT capital required during the booming stage of the economy. This appears to be consistent with the intention of creating an additional capital buffer for market downturns.

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4 For example by a certain percentage of the current LGD.
What Do We Mean By LGD Systematic Risk?  A Stylized Model of PD and LGD Correlations

It appears that (1) the correlation between PD and LGD risk of the same borrower, and (2) the correlation in LGD risk among a group of borrowers are used almost interchangeably when in fact they are very different concepts. In the previous section, we mention that one of the motivations for Basel to impose a bottom-of-the-cycle LGD is to compensate for the lack of consideration of LGD correlations. Are we talking about the former or latter concept of correlation, or both? It is important to note that neither type of correlation is captured in the Basel capital formula, as well as in most commonly used off-the-shelf models used for economic capital estimation. In this section, we explicitly model the different elements of correlation structure with a stylized credit risk model. Simulations are then conducted using the model to assess the impact of these correlations on the amount of capital required to cushion the credit risk of a portfolio of defaultable assets.

Before we formally introduce our model, we would like to examine some basic intuition.

I- LGDs of different portfolios may be driven by different risk factors thus follow their own cycles. As a result there can be a significant amount of LGD diversification among different portfolios and different countries. If we assume all portfolios hit the LGD bottom-of-the-cycle at the same time and thus ignore the diversification effect in LGD, we may grossly over estimate the capital requirement.\(^5\)

II- PD and LGD may be driven by different risk factors and thus follow their own cycles: There have been reports (e.g. in Araten, Jacobs Jr. and Varshney (2004)) suggesting the correlation between PD and LGD may not always be material in practice.

III- PD and LGD may have a lagging relationship depending upon a bank’s collection practices: Some banks tend to sell collateral before the default, others tend to sell the defaulted loans shortly after the defaults, while others go through a lengthy collection period during which the cycle may have already changed materially. In the last case, it is likely that a high PD may not necessarily be accompanied by a high LGD, which therefore moderates the extreme loss events.

Note that the first intuition is related to correlation of LGD risk factors among different borrowers, whereas the second and third refer to the correlation between PD and LGD risk factors. The suggestion that the amount of correlation between PD and LGD may not always be material does not mean that LGD does not have systematic risk, because we also have to consider the first type of correlation, i.e. the correlation of LGD risk factors among different borrowers.

\(^5\) One of the advantages in using a default-weighed average LGD is that it automatically takes care of this difference in LGD peak times – as it averages over all loans over the different cycles.
Considering all these relationships, we identify four different elements of correlation as follows. Figure 5 presents a schematic representation.

1. Correlations between the systematic risk drivers of PD and LGD for a given borrower
2. Correlations between specific risk drivers of PD and LGD for a given borrower
3. Correlations between the PD risk drivers among different borrowers
4. Correlations between LGD risk drivers among different borrowers

The intuition behind the first two elements of correlation can be explained by considering the asset value of a firm. Both the systematic and specific risk drivers of the asset value of a firm not only affect the default likelihood of a firm but also the value of its assets and thus recovery value in case of default. The third element, which is the commonly used PD correlation, is captured in the Basel capital formula and most structural models. Finally, the last element represents the systematic factors affecting the LGDs of all borrowers, which may be independent of those affecting the firms' default likelihood.

The following examples may help further conceptualize these different elements of correlations. For example, LGDs for unsecured credits are shown to be well correlated with the economic cycle, while LGDs for secured credits are not (e.g. as documented by Araten, Jacobs Jr., and Varshney (2004)). This observation is intuitive considering the different elements of correlations. Security gives the bank priority interest in a source of repayment that may be independent of the state of economy (e.g. a $1M pool of receivables is still a $1M pool of receivables regardless of whether the company that generates the receivables is prospering or failing). Any variations in the value of the receivables, due to defaults among the receivable borrowers, are supposed to be absorbed through the *lending value haircuts*, which is usually applied. Unsecured credits are more dependent on the enterprise value (or asset value) of the borrower's business. If the borrower is in financial distress, it is also likely that the business is not prospering and therefore the enterprise value of the business is reduced. Therefore, we expect that unsecured credits have high first and second elements of correlations. To examine the last type of correlations, consider also a portfolio of loans in a certain country with the same or similar types of collateral. Further assume that value of the collateral follows their specific cycles. Year by year when we examine the LGDs of the defaulted loans, we expect to see high and low LGD years based on the collateral value cycle, which may be different from the default cycle.

In Merton’s (1974) structural model of credit risk, the asset value of a firm is the single risk driver for both PD and LGD. While a declining asset value increases the default likelihood,

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6 Note that elements #1, #2 and #4 are components which constitute the correlations we have been describing in this section, while we need to incorporate #3 to complete the PD and LGD correlation structure.
7 Lending value haircut is the discount applied to the “gross” collateral value to estimate the “net” collateral value at the time of default. It is an adjustment for the volatility of the collateral value. The higher the volatility, the higher the haircut, and vice versa. For example, typically cash and marketable securities have no or little haircuts as the value volatility is very low.
once the default threshold is reached, it also drives the level of LGD.\(^8\) PD and LGD risks are therefore structurally related. The concept of credit risk correlation is therefore much restrictive to the framework introduced above. In most of the applications of Merton’s model, correlation typically refers to the correlation of underlying asset values of borrowers under a multi-firm setting.\(^6\)

This characteristic may be too restrictive in practice. Firstly as argued previously, the comovement of PD and LGD is not perfect in reality. Suppose a loan made to a borrower is collateralized on some fixed assets (e.g. a building). Any degeneration of the credit quality of the borrower will have an adverse impact on her PD, but may not have any material impact on LGD, which in this case is directly related to the relatively stable value of the real estate. Secondly, the impact exerted by the systematic risk factor can be very different from that of the idiosyncratic factor. In the previous example, any adverse impact on LGD is likely to be stronger if the degeneration of credit quality is due to market-wide effects rather than borrower-specific effects. We expect both the lowering of credit quality of all borrowers and the lowering of real estate values during an economic down turn, while a borrower-specific credit event (e.g. a cash flow problem) may have nothing to do with the value of the building used as collateral (also consider the previous discussion about the secured credits).

In the rest of this section, we would like to develop a model which is general enough to encompass and differentiate the different elements of correlations mentioned above. A number of comparative tests are then conducted to measure the impact of each of these correlations on the portfolio value-at-risk. In our model, unlike the Merton’s model, PD and LGD risks are not structurally related but follow their own (correlated) random process. Moreover, we decompose these risk drives into their systematic and idiosyncratic components, and define the correlations in PD and LGD separately for the two components. Correlations in credit risks among different borrowers can therefore be derived from the sensitivity of individual’s credit risk on the systematic factors.

Let’s start by describing the systematic credit risks. Suppose there is a single systematic risk driver \(X_t\) affecting the changes in both PD and LGD risks, but can be of different extent.\(^10\) We assume \(X_t\) to be normally distributed with mean zero and unit standard deviation. Market-wide systematic PD and LGD risks at time \(t\) (\(P_t\) and \(L_t\)) are assumed to be driven by \(X_t\) via the following equations.

\[
P_t = \beta_{PD} \times X_t + \varepsilon_{PD,t} \tag{1}
\]

\[
L_t = \beta_{LGD} \times X_t + \varepsilon_{LGD,t} \tag{2}
\]

The coefficients \(\beta_{PD}\) and \(\beta_{LGD}\) therefore govern the degrees of impact of \(X_t\) on \(P_t\) and \(L_t\). The residual changes (\(\varepsilon_{PD,t}\) and \(\varepsilon_{LGD,t}\)) are independent of \(X_t\) and are assumed to be independent

\[^8\] It can be shown that if the normalized risk driver for the PD for obligor \(i\) is \(p_i' \sim N(0,1)\), the risk driver for LGD is \(1 - \text{Exp}(p_i' - DP)\) where DP is normalized default point.

\[^9\] It is also common to be referred to as pair-wise PD correlation.

\[^10\] It can be readily extended to a multi-factor model.
normally distributed and with standard deviations such that both $P_t$ and $L_t$ follow standard normal distribution.

In this economy, borrowers are uniform in terms of their credit risks. Their individual PD risk $p_t$ affected by both the systematic PD risk $P_t$ and the borrower-specific PD risk $e_{PD,t}$. For example, for borrower $i$,

$$p_t^i = R_{PD} \times P_t + \sqrt{1 - R_{PD}^2} \times e_{PD,t}$$  \hspace{1cm} (3)

Individual PD risk $p_t$ therefore follows a standard normal distribution. Under the Merton’s framework, we can interpret $p_t$ as a normalized function of the borrower’s asset value. The borrower defaults when $p_t$ becomes less than some constant default point ($DP$). Thus, the lower the value of $p_t$ (i.e. the closer to $DP$), the higher is the borrower’s PD. The coefficient $R_{PD}$ is uniform across borrowers and measures the sensitivity of individual risks to the systematic PD risk. It is also be shown that $R_{PD}^2$ is the pair-wise correlation in PD risks among borrowers as a result of the systematic risk factor.

We have a similar set up for borrower’s individual LGD risk $l_t$. Again, it can be partitioned into its systematic and idiosyncratic component. Thus, for borrower $i$,

$$l_t^i = R_{LGD} \times L_t + \sqrt{1 - R_{LGD}^2} \times e_{LGD,t}$$  \hspace{1cm} (4)

Individual LGD risk $l_t$ again follows a standard normal distribution. We interpret $l_t$ as a normalized transformed distribution of the empirical distribution of LGD.11 There is a one-to-one monotonic mapping between the value of $l_t$ and the LGD value, which is typically bounded between zero and one. The higher the value of $l_t$, the higher is the borrower’s LGD. The coefficient $R_{LGD}$ is again uniform across borrowers and measures the sensitivity of individual risks to the systematic LGD risk. The parameter $R_{LGD}^2$ therefore becomes the pair-wise correlation in LGD risks among borrowers as a result of the systematic risk factor.

It is interesting to note that equation (3) and (4) together actually imply the degree of correlation between PD and LGD of the same borrower (i.e. the first form of LGD correlation mentioned above) via the systematic risk factors of $P_t$ and $L_t$. It can be shown that

$$\text{Corr}(p_t^i, l_t^i) = \beta_{PD} \beta_{LGD} R_{PD} R_{LGD}$$  \hspace{1cm} (5)

However, correlation between PD and LGD of the same borrower does not only come from the systematic risk drivers. How about risk factors specific to the borrower under consideration? For example, if we are considering an unsecured loan made to a borrower, a borrower-specific credit event can cause an increase in both its PD and LGD. This event can have nothing to do with any market-wide systematic risk. In our model, such a

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11 Beta distribution is commonly used to represent the empirical LGD distribution. We also use Beta distribution in our subsequent comparative analysis.
characteristic can be modeled by ensuring the residuals $\epsilon_{PD,t}$ and $\epsilon_{LGD,t}$ in (3) and (4) are not independent for the same borrowers. Let $x_i$ denotes the borrower-specific credit risk factor, which is assumed to be normally distributed with zero mean and unit standard deviation. Then, for borrower $i$,

$$e_{PD,i}^i = \theta_{PD}^i \times x_i^i + \epsilon_{PD,i}^i$$

$$e_{LGD,i}^i = \theta_{LGD}^i \times x_i^i + \epsilon_{LGD,i}^i$$

The coefficients $\theta_{PD}$ and $\theta_{LGD}$ therefore govern the degrees of impact of $x_i$ on $e_{PD,i}$ and $e_{LGD,i}$. The residual changes ($\epsilon_{PD,i}$ and $\epsilon_{LGD,i}$) are independent of $x_i$ and are assumed to be normally distributed and with standard deviations such that both $e_{PD,i}$ and $e_{LGD,i}$ follow standard normal distribution.

With the introduction of the idiosyncratic risk factor, it can be shown that the correlation between PD and LGD risk drivers of the same borrower becomes

$$Corr(p_i^i, l_i^i) = \beta_{PD} \beta_{LGD} R_{PD} R_{LGD} + \theta_{PD} \theta_{LGD} \sqrt{1 - R_{PD}^2} \sqrt{1 - R_{LGD}^2}$$

The correlation is therefore made up of two parts. The first term represents the correlation due to systematic risk factors, while the second term that of idiosyncratic risk factors. It is interesting to note that the larger the pair-wise correlations of $R_{PD}$ and $R_{LGD}$, the larger (smaller) the impact of the systematic (idiosyncratic) factors. Moreover, no matter the values of $R_{PD}$ and $R_{LGD}$, we have zero correlation between PD and LGD for the same borrower if either one of the $\beta$s and either one of the $\theta$s are both equal to zero. It however does not imply there is zero LGD correlation among different borrowers (i.e. the second form of LGD correlation), which is solely governed by $R_{LGD}$. Our model can therefore differentiate the origins and effects of the two forms of LGD correlation mentioned above. It opens up a window for researchers and practitioners to conduct theoretical or empirical analysis on the different types of LGD correlation.

**A Comparative Static Study**

In this section, we report the comparative static study conducted by using the model outlined in the previous section. The objective is to measure the impacts of the different correlation factors on the value-at-risk of a credit portfolio. Through this exercise and with realistic assumptions in input parameters, we would like to provide information to both the banks and regulators with regards to the appropriate conservatism required in imposing the down turn LGD requirement mentioned previously.

We consider a portfolio consists of a total of 2,000 loans made to 2,000 uniform borrowers. The promised repayment in one year is $1 each. The PD and LGD risk factors governing the credit risks are assumed to follow the descriptions of the model proposed in the previous

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12 By construction, these residuals are however independent among different borrowers.
section. From the perspective of the bank, we are interested to measure the economic capital requirement over a one-year risk horizon and at a confidence level of 99% and 99.9% respectively.

The followings describe the parametric and model assumptions:

**PD risks:** The PD of each borrower is assumed to be 1%, which corresponds to the average PD of the Bank’s large corporate borrowers. We assume the PD risk of each borrower is governed by the random variable $p_t$, as defined in equation (3). Specifically, we interpret $p_t$ as a normalized function of the borrower’s asset value. Borrower defaults within the risk horizon when the realized borrower-specific $p_t$ becomes less than the constant default point of -2.3263, which is the inverse of the cumulative probability of 0.01 (i.e. PD = 1%).

**LGD risks:** If a borrower defaults, we assume the realized LGD follows a Beta distribution with parametric values of $a = 0.630$ and $b = 0.975$. These parametric assumptions together imply a mean LGD of 39% and a LGD standard deviation of 30%, which are found to be consistent with the Bank’s experience with its large corporate loan portfolio (see subsequent section). The LGD value realized is assumed to be governed by the random variable $l_t$ as defined in equation (4). Variables $l_t$ follows a standard normal distribution and is a transformed distribution of the empirical Beta distribution of LGD. That is, there is a one-to-one monotonic mapping between the realized value of $l_t$ and the realized LGD value.

$$LGD_i = B^{-1}(\Phi(l_i), a, b)$$

(9)

where $LGD_i$ denotes the realized LGD of borrower $i$ at time $t$, $B^{-1}(\bullet)$ denotes the inverse of the cumulative Beta distribution with shape parameters $a$ and $b$, and $\Phi(\bullet)$ denotes the cumulative standard normal distribution, which transforms $l_i$ to a uniform distribution.

**Correlation parameters:** To fully define the model, we need to specify:

- The sensitivities ($\beta_{PD}$ and $\beta_{LGD}$) of the systematic PD and LGD risks ($P_t$ and $L_t$) to the single systematic risk driver $X_t$ as described in equations (1) and (2): We assume $\beta_{PD} = \beta_{LGD} \equiv \beta$, and we consider the cases where $\beta^2$ equal to 0.0, 0.2, 0.4, 0.6 and 0.8 (note that $\beta^2$ is estimated to be equal to 0.53 in the subsequent sections of the paper).
- The pair-wise PD correlation as defined in equation (3): We consider different cases where the pair-wise PD correlation ($R_{PD}$) takes on values ranging from 0.0 to 0.5.
- The pair-wise LGD correlation as defined in equation (4): We report the results where the pair-wise LGD correlation is equal to 5.9% as estimated in the subsequent section for the bank’s large corporate borrowers. The corresponding $R_{LGD}$ is therefore equal to the square root of 0.059.
- The sensitivities ($\theta_{PD}$ and $\theta_{LGD}$) of the idiosyncratic PD and LGD risks ($e_{PD,t}$ and $e_{LGD,t}$) to the single borrower-specific credit risk factor $x_t$ as described in equations (6) and (7): We assume $\theta_{PD} = \theta_{LGD} \equiv \theta$, and we consider the cases where $\theta^2$ equal to 0.0, 0.2, 0.4, 0.6 and 0.8 respectively.
The portfolio value-at-risks (i.e. economic capital requirements) are defined as the differences between the mean year-end portfolio value and the year-end 99% and 99.9% critical portfolio values respectively at the loss tail. The distribution of portfolio values is obtained by simulating 20,000 credit scenarios each involving all the 2,000 uniform borrowers.

The followings outline the simulation process:

- **Step 1:** For each scenario, a single set of market-wide risk factors $X_t$, $e_{PD,t}$ and $e_{LGD,t}$ is first drawn from independent normal distributions. The systematic PD and LGD risk factors $P_t$ and $L_t$ are then computed via equations (1) and (2).

- **Step 2:** A total of 2,000 sets of mutually independent borrower-specific credit risk drivers $x_{it}$, $e_{PD,i,t}$ and $e_{LGD,i,t}$ are then drawn from normal distributions. Borrower-specific idiosyncratic risks $e_{PD,i,t}$ and $e_{LGD,i,t}$ can then be constructed for each of the borrowers via equations (6) and (7).

- **Step 3:** For each borrower, the borrower-specific PD and LGD risks ($p_t$ and $l_t$) can then be computed by combining the systematic and idiosyncratic components obtained in Step 1 and 2, and as defined by equations (3) and (4).

- **Step 4:** We then check for each borrower if there is a default event (i.e. if $p_t < -2.3263$). If it defaults, ending value of the loan is equal to $1$ minus the realized LGD, which is equal to the transformation of the borrower-specific $l_t$ under the above-mentioned Beta distribution as per equation (9). If it does not default, the loan value is simply equal to $1$. The portfolio value is therefore the summation of all the loan values across the 2,000 borrowers.

- **Step 5:** Steps 1 to 4 are then repeated 20,000 times to obtain the distribution of year-end portfolio values for the calculations of economic capitals.

This simulation process is then repeated for each set of the correlation parametric values as described above. The objective is to measure the impact on the resulting economic capital. The results are reported in Table 1 and 3. In Table 1, we consider different scenarios of systematic and idiosyncratic sensitivity factors $\beta$ and $\theta$. Pair-wise PD correlation is set at 0.25 (i.e. $R_{PD}$ equals to 0.5). We use a pair-wise LGD correlation of 0.059 (i.e. $R_{LGD}$ equals to 0.24) from the estimation results of the subsequent section. In Table 3, we consider different scenarios of the idiosyncratic sensitivity factor $\theta$ and the pair-wise PD correlation $R_{PD}^2$. Same as Table 1, we use a pair-wise LGD correlation of 0.059. However, the systematic sensitivity factor $\beta$ is set as the square root of 0.53, which corresponds to the correlation in systematic PD and LGD risks estimated in the last section of this paper.

Table 2 and 4 report the required percentage increase in the mean LGD (from 0.39) when correlations are set to zero. That is, we estimate the mean (zero-correlation) LGDs (by setting $\beta = \theta = R_{LGD} = 0$) required to arrive at the economic capitals reported in Table 1 and 3 respectively. The (percentage) difference between the “zero-correlation-LGDs” and the original LGD of 39% represents the required “mark-up” in mean LGD (from an unbiased LGD estimate) in a model where LGD correlations are ignored or not explicitly modeled.

\^13 For example, in KMV PM or in the Basel regulatory capital formula.
From Table 1, it can be observed that the economic capital is more sensitive to changes in the idiosyncratic sensitivity factor $\theta$ than in the systematic sensitivity factor $\beta$. This is as expected given the low value of pair-wise LGD correlation used in the simulations, which makes the systematic risks (i.e. the first term of equation (8)) a less important component in individual obligors’ PD and LGD correlations. The estimation of the value of $\beta$ is therefore of secondary importance in the calculation of economic capital. We however need an accurate measure of the idiosyncratic factor $\theta$, which is unfortunately more difficult to be estimated. Moreover, as argued previously, this obligor-specific idiosyncratic correlation is likely to be governed by the specifics of the individual loan contracts. Namely, unsecured loans should have a higher $\theta$ than secured ones. From Table 2, even at a moderate level of $\theta$ (say $\theta^2 = 0.2$) and at a level of $\beta$ (say $\beta^2 = 0.5$) comparable with that estimated subsequently for large corporate borrowers, we need to increase the mean LGD by about 37% in order to achieve the correct economic capital under a model where LGD correlation is ignored. However, the required mark-up for mid-market portfolios, with predominantly secured loans, is likely to be much smaller.

The results in Table 3 suggest that economic capital is highly sensitive to pair-wise PD correlation at all levels of the idiosyncratic sensitive factor $\theta$, while only sensitive to $\theta$ when $R^2_{PD}$ is reasonably large. That is, there is a certain leverage effect between these two parameters. To provide some intuitions to the latter finding, let us again consider the decomposition of the PD and LGD correlation as presented in equation (8). When $R^2_{PD}$ is small and close to zero, we would expect the correlation between PD and LGD of a single obligor to be solely governed by the idiosyncratic component and thus the value of $\theta$. This effect however does not translate into a higher sensitivity in the economic capital to $\theta$ given the fact that it is the pair-wise effects rather than the PD and LGD correlation of individual obligors which determine the credit risk of a loan portfolio. For example, if $R^2_{PD}$ and $R^2_{LGD}$ are both equal to zero, the portfolio capital requirement would still be small even if the PD and LGD of each obligor are perfectly correlated.

The results in Table 4 suggest the required “mark-up” in mean LGD is a decreasing function of pair-wise PD correlation. As $R^2_{PD}$ increases, although the systematic component of the PD and LGD correlation of individual obligors increases (see equation (8)), the understatement of capital is relatively smaller since the portfolio credit risk becomes mainly governed by pair-wise effects rather than PD and LGD correlation of individual obligors. Finally, the required “mark-up” becomes less sensitive to $\theta$ as $R^2_{PD}$ increases, since the systematic component of correlation together with the pair-wise effects dominates the determination of the portfolio credit risk when $R^2_{PD}$ is large.

---

14 Here, we consider a definition of economic capital at the 99.9% critical level. The required mark-up will be higher if the appropriate critical level is 99% rather than 99.9%.

15 As documented in the last section, LGD risks of mid-market portfolio are found to exhibit insignificant systematic correlation (i.e. $\beta = 0$). At a moderate level of $\theta$ (say $\theta^2 = 0.2$), the corresponding mark-up is therefore only 16% (from Table 2).
An Empirical Study on Pair-Wise LGD Correlations

In this section and the next, we would like to go one step further and attempt to estimate the correlation parameters governing our model using historical default data available to a bank. The ability to assign values (or ranges of values) to these correlation parameters (namely, $\beta_{PD}$, $\beta_{LGD}$, $\theta_{PD}$, $\theta_{LGD}$, $R_{PD}$ and $R_{LGD}$) is crucial to answer those important business questions we have asked in the first section of the paper. Namely, how much conservatism is needed to compensate for the lack of consideration of correlation in Basel II?

Among these correlation parameters, the pair-wise PD correlation $R_{PD}$ is most studied and relatively well understood. Most banks have very good idea of the PD correlation among borrowers of their loan portfolios. The remaining parameters are however not as well documented. In this section, we outline a methodology to estimate pair-wise LGD correlation $R_{LGD}$ by observing historical LGDs. We then consider correlations among systematic PD and LGD (i.e. $\beta_{PD}$ and $\beta_{LGD}$) in the next section.

Here we abstract from the full-fledge model of PD and LGD risks and focus on what’s happening after the borrowers go into default. Essentially, we want to estimate equation (4), which is restated here for easy reference.

$$l_t^{i} = R_{LGD} \times L_t + \sqrt{1-R_{LGD}^2} \times e_{LGD,t}^{i}$$

In equation (4), we partition a borrower’s LGD risk $l_t^{i}$ into its systematic ($L_t$) and idiosyncratic ($e_{LGD,t}^{i}$) component. We assume individual’s LGD risk $l_t^{i}$ follows a standard normal distribution and we interpret $l_t^{i}$ as a normalized transformed distribution of the empirical distribution of LGD, which is assumed to follow a Beta distribution. There is a one-to-one monotonic mapping between the value of $l_t^{i}$ and the realized LGD value, which is bounded between zero and one under the Beta distribution. The coefficient $R_{LGD}$ is assumed to be uniform across borrowers and measures the sensitivity of individual risks to the systematic LGD risk. It is therefore the pair-wise correlation in LGD risks among borrowers as a result of the systematic risk factor.

In this setup, there are three parameters we need to estimate to fully define the LGD risks. Namely, the pair-wise correlation $R_{LGD}$ of equation (4) and the two parameters of the Beta distribution (i.e. $a$ and $b$). We estimate them empirically by observing historical LGDs of loans made to large corporate borrowers by a bank from 1993 to 2003. The number of defaults and their respective LGDs for each quarterly period were recorded during the sample period. The panel data comprises of a total of 165 defaulted loans. Table 5 reports some summary statistics.

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16 Perhaps with the help of credit risk models (e.g. KMV CM and PM).

17 In the subsequent estimation of equation (4), we might need to incorporate an intercept term to accommodate for the fact that the unconditional mean of $l_t^{i}$ can be significantly different from zero.
First of all, we estimate the parameters governing the Beta distribution. From the properties of Beta distribution, parameters $a$ and $b$ are related to the mean and variance of the distribution according the following equations.

$$\mu = \frac{a}{a + b}$$  \hspace{1cm} (10)

$$\text{Var} = \frac{ab}{(a + b + 1)(a + b)^2}. \hspace{1cm} (11)$$

Using the unconditional mean and variance (i.e. 39% and 0.0916) over the full sample period, we estimate $a = 0.630$ and $b = 0.975$. Given these parameter estimates, we can then obtain the normally distributed LGD risk factor $l_i$ by transforming each of the observed LGDs using the Beta distribution.

$$l_i = \Phi^{-1}(B(LGD_i^i; a, b)) \hspace{1cm} (12)$$

where $LGD_i^i$ denotes realized LGD of borrower $i$ at time $t$;

$\Phi^{-1}(\bullet)$ denotes the inverse of the cumulative standard normal distribution; and

$B(\bullet; a, b)$ denotes the cumulative Beta distribution with shape parameters $a$ and $b$.

From equation (4), the standard deviation of $l_i^i$ at each time period $t$ is always equal to $\sqrt{1 - R_{LGD}^2}$. Pair-wise LGD correlation can therefore be estimated by calculating the following pooled estimate of standard deviation of $l_i^i$.

$$\sqrt{1 - \hat{R}_{LGD}^2} = \sqrt{\frac{\sum_{t=1}^{T} (n_t - 1) \hat{\sigma}_t^2}{\sum_{t=1}^{T} n_t - T}} \hspace{1cm} (13)$$

where $n_t$ and $\hat{\sigma}_t^2$ denote the number of LGD observations and the standard deviation of $l_i^i$ in time period $t$ respectively. Using the historical LGD data reported in Table 5, $R_{LGD}^2$ is estimated to be equal to 5.88% (5.20%) by grouping data within each quarter (year). This is a fairly low level of correlation, which perhaps reflects the diversification effect as our LGD data comes from different industries and countries. We also notice the collection periods for the loans differed significantly, possibly introducing noise in the recovery value and in effect reducing correlations. We can also estimate the time-series of the market-wide systematic LGD risks ($L_i$), which is simply the mean values of $l_i^i$ within each time period divided by the estimated value of $R_{LGD}$. These estimates are plotted in Figure 6 and will be used in the next Section to estimate the correlation in systematic PD and LGD risks.
Correlation of Market-Wide Systematic PD and LGD Risks

In the previous section, we estimate pair-wise LGD correlation $R_{LGD}$ by using historical LGD data. In this section, we focus our attention on the correlation of systematic PD and LGD risk factors, which is governed by the parametric values of $\beta_{PD}$ and $\beta_{LGD}$ of our model. The calibrations of two out of four elements of correlation are therefore covered in these two sections.\(^{18}\) They provide the essential building blocks for the implementation of our model in a practical setting.

To estimate the correlation of systematic PD and LGD risk factors, we start by restating equations (1) and (2), which describing the systematic credit risks.

\[
P_t = \beta_{PD} \times X_t + \epsilon_{PD,t} \\
L_t = \beta_{LGD} \times X_t + \epsilon_{LGD,t}
\]

Market-wide systematic PD and LGD risks ($P_t$ and $L_t$ respectively) are assumed to be driven by a single systematic risk driver $X_t$, which is normally distributed with mean zero and unit variance. The coefficients $\beta_{PD}$ and $\beta_{LGD}$ therefore govern the degrees of impact of $X_t$ on $P_t$ and $L_t$. The residual changes ($\epsilon_{PD,t}$ and $\epsilon_{LGD,t}$) are assumed to be mutually independent and also independent of $X_t$. They are normally distributed with standard deviations such that both $P_t$ and $L_t$ follow standard normal distribution, again, with zero mean and unit variance. The correlation of $P_t$ and $L_t$ is therefore simply the product of the coefficients $\beta_{PD}$ and $\beta_{LGD}$. To facilitate subsequent discussions, let us denote this correlation between market-wide systematic PD and LGD risks as $R_{M}^{2}$.

Now, the practical question is: How to estimate $R_{M}^{2}$\(^{19}\) Araten, Jacobs Jr., and Varshney (2004) conduct an empirical study and examine the relation between the average annual LGD for unsecured U.S. large corporate borrowers and the average Moody’s All-Corporate default rate for the period 1985-99. The correlation obtained from a linear regression of the former on the latter is found to be equal to 25%.\(^{20}\) As expected, the correlation becomes much lower (2%) if LGDs of secured rather than unsecured loans are used. These correlation numbers however may not be appropriate for our model. Firstly, it can be shown (e.g. in Miu and Ozdemir (2005)) that the default rate of a portfolio is a biased estimator of its average PD. Secondly, their measures of PD and LGD risks (namely, average default rates and annual LGD) are not necessarily normally distributed.

The remaining of this section outlines an approach to estimate $R_{M}^{2}$. In the previous section, as a by-product in the estimation of pair-wise LGD correlation, we also obtain the most-likely estimations of the systematic LGD risk factor $L_t$ over time. We can therefore calculate $R_{M}^{2}$ if we also have estimations of the systematic PD risk factor $P_t$ over the same sample period.

\(^{18}\) Among the remaining two elements, the calibration of pair-wise PD correlation has already been well-documented in the literature.

\(^{19}\) The estimations of $\beta_{PD}$ and $\beta_{LGD}$ individually are more difficult than that of $R_{M}^{2}$, given that we need to first of all identify and recover the unobservable factor $X_t$.

\(^{20}\) Using the logarithmic of default rate rather than the raw default rate yields a higher correlation of 44%.
Suppose, in each period $t$, we observe $k_t$ defaults in a uniform portfolio starting with $n_t$ borrowers. It can be shown (e.g. in a way similar to that in Vasicek (1987)) that the probability of observing $k_t$ given the realization of $P_t$ is:

$$
\Omega(k_t, n_t; P_t, R_{PD}, DP) = \binom{n_t}{k_t} \times (\Phi(z(P_t, R_{PD}, DP))^k \times (1 - \Phi((z(P_t, R_{PD}, DP))))^{n_t - k_t}.
$$

(14)

where $DP$ is the constant default point; $\Phi(\cdot)$ is the cumulative normal distribution function and

$$
z(P_t, R_{PD}, DP) = \frac{1}{\sqrt{1 - R_{PD}^2}} (DP - R_{PD}P_t)
$$

The first step is to estimate $DP$ by maximizing the time-series sum of the unconditional log likelihood of observing $k_t$ and $n_t$ from $t = 1$ to $T$. Given that $P_t$ follows a standard normal distribution, the unconditional log likelihood function becomes

$$
\log (L) = \sum_{t=1}^{T} \log \int_{-\infty}^{\infty} \Omega(k_t, n_t; u_t, \hat{R}_{PD}, DP) \cdot \phi(u_t) \cdot du_t.
$$

(15)

where $\phi(\cdot)$ denotes the density function of the standard normal distribution.

As shown in Vasicek (1987), as $n_t$ becomes large, the above function can be approximated by

$$
\log (L) \approx \sum_{t=1}^{T} \left\{ \log \left( \frac{1 - \hat{R}_{PD}^2}{\hat{R}_{PD}^2} \right) + \frac{1}{2} \left( \Phi^{-1} \left( \frac{k_t}{n_t} \right) \right)^2 - \left( \frac{\sqrt{1 - \hat{R}_{PD}^2} \cdot \Phi^{-1} \left( \frac{k_t}{n_t} \right) - DP}{2\hat{R}_{PD}} \right)^2 \right\}.
$$

(16)

We conduct the estimation with the historical default rates of a uniform mid-market loan portfolio from 1991 to 2000. The default rates are reported in Table 6. By maximizing the log likelihood function in equation (16), the default point ($DP$) is estimated to be equal to -1.8981, which corresponds to a PD of 2.9%. It therefore serves as an estimate of the “long run” PD of the obligors in this uniform portfolio of mid-market loans.

---

21 Since $n_t$ is typically large in practice, it is more convenient to approximate function $\Omega(\cdot)$ with a Poisson distribution function with lambda equal to $n_t$ times $N(\hat{z})$.

22 Though we could have estimated both $R_{PD}$ and $DP$ at the same time, here we assume a $R_{PD}$ of 0.25, which is consistent with our illustrations in the previous sections. The conclusions regarding the relation between the systematic PD and LGD risks drawn subsequently are found to be robust to changes in this assumed value.

23 The obligors are typically BBB-rated. Government guaranteed loans are excluded from the sample. Using historical default rates of large corporate portfolio does not materially change the subsequent estimated correlation of systematic PD and LGD risks.

24 The value of $DP$ is found to be sensitive to the assumption of pair-wise PD correlation. If we assume a $R_{PD}^2$ of only 10%, $DP$ becomes -2.0793, which corresponds to a PD of 1.9%. Nevertheless, the subsequent analysis
The expected value of \( P_t \) in each time period \( t \) conditional on observing \( k_t \) and \( n_t \) can be shown to be equal to
\[
E(P_t | k_t, n_t; DP, R_{PD}) = \frac{DP - \Phi^{-1}(k_t/n_t) \cdot \sqrt{1 - R_{PD}^2}}{R_{PD}}
\]  

(17)

Given the estimated value of -1.8981 for \( DP \) and the assumed value of 0.25 for \( R_{PD}^2 \), we estimated the time-series of the systematic PD risks \( (P_t) \) using equation (17) with the historical default rates reported in Table 6. In Figure 6, we plot the estimated \( P_t \) along with the systematic LGD risk factor \( (L_t) \) obtained from the previous section. Systematic PD and LGD risks are found to be evolving in a synchronized fashion over the 90s. Specifically, both PD and LGD risks are relatively high in 1996, 1997, and from 2000 onwards. On the other hand, both risks are relatively low in 1994, 1995 and 1999.

Finally, \( R_{LV}^2 \) is estimated to be equal to 0.53 by calculating the correlation between \( P_t \) and \( L_t \) over the years where we have data for both variables. This is however considered to be a noisy estimate of the systematic correlation given the short time-series of historical data. Moreover, the correlation is found to be much smaller and close to zero (not reported here) when LGD information of mid-market portfolio is used instead of that of large-corporate. It is as expected given the fact that most of the mid-market loans are secured and thus less related to systematic PD risks. This finding is also considered to be consistent with the results documented in Araten, Jacobs Jr., and Varshney (2004).

Conclusions

In this paper we examine the Basel’s downturn LGD requirement. We first discuss the LGD rating philosophy with respect to the downturn LGD requirement, showing that the conservatism in LGD to account for the lack of correlations can also be accommodated in a cyclical PIT framework. We also show that the use of a-cyclical bottom-of-the-cycle LGD effectively increases the risk horizon over one year for capital calculation.

We then examine the correlations between PD and LGD in detail, showing that there are different types of correlations in question. We propose a stylized model to analyze and model the PD and LGD correlations in its entirety. Using historical default data of a loan portfolio, we estimate the correlations of LGD risk drivers among different obligors and the regarding the relation between the systematic PD and LGD risks is found to be robust to changes in the assumed value of \( R_{PD}^2 \).

25 Here, we normalize the two time series with their respective standard deviations. For visual illustration purpose, we present the negative of the values of \( P_t \) in Figure 7. In our model, the lower the value of \( P_t \), the higher is the PD risk.

26 Numerically, the correlation between \( P_t \) and \( L_t \) is actually -0.53, which suggests a positive correlation between systematic PD and LGD risks.
correlation of systematic PD and LGD risk factors. The former is found to be small whereas the latter can be substantial.

We use these estimated correlations together with our proposed model, to assess how much the mean LGD needs to be increased in order to compensate for the lack of consideration of correlations in the Basel capital formula and the commonly-used credit risk models. Admittedly, not having an accurate measure of the correlations between the idiosyncratic factors impairs our ability to more accurately determine the degree of conservatism in downturn LGD requirement. However, our analysis shows that even at a moderate level of idiosyncratic correlations (say $\theta^2 = 0.2$) and at a level of systematic correlation (say $\beta^2 = 0.5$), which is comparable with that we estimated, we need to increase the mean LGD by about 37% in order to achieve the correct economic capital under a model where correlations are ignored. The required mark-up for secured loans, which are common for mid-market borrowers, is likely to be much smaller. Our hope is that this paper provides not only a yardstick for the level of markup required, but also a framework that the banks can use themselves based on their internal data to estimate and justify their LGD choices for different portfolios.
References


Table 1: Simulated economic capitals (in $) at 99% and 99.9% for a uniform portfolio of 2,000 loans (each $1) under different scenarios of systematic and idiosyncratic sensitivity factors $\beta$ and $\theta$. Pair-wise PD correlation is set at 0.25 (i.e. $R_{PD}^2$ equals to 0.5). We use a pair-wise LGD correlation of 0.059 (i.e. $R_{LGD}^2$ equals to 0.24) from the estimation results of the subsequent section. Economic capital is defined as the difference between the mean portfolio value and the respective critical values at the loss tail at one-year risk horizon.

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Table 2 - Required Mark-up for LGD with respect to Systematic ($\beta^2$) and Idiosyncratic Correlations ($\theta^2$): Percentage increases in LGD (from 39%) to match the corresponding economic capitals reported in Table 1. Simulations of economic capitals at 99% and 99.9% for a uniform portfolio of 2,000 loans are conducted by assuming $\beta = \theta = 0$, and zero pair-wise LGD correlation $R_{LGD} = 0)$. Same as Table 1, pair-wise PD correlation is assumed to be equal to 0.25 (i.e. $R_{PD}$ equals to 0.5). In order to produce the correct economic capital numbers, these percentages therefore represent the required “mark-up” in mean LGD in a model where LGD correlations are ignored.

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Table 3: Simulated economic capitals (in $) at 99% and 99.9% for a uniform portfolio of
2,000 loans (each $1) under different scenarios of the idiosyncratic sensitivity factor $\theta$ and
the pair-wise PD correlation $R^2_{PD}$. Same as Table 1, we use a pair-wise LGD correlation of
0.059 (i.e. $R^2_{LGD}$ equals to 0.24) from the estimation results of the subsequent section. The
systematic sensitivity factor $\beta$ is set as the square root of 0.53, which corresponds to the
value estimated in the last section of this paper. Economic capital is again defined as the
difference between the mean portfolio value and the respective critical values at the loss tail
at one-year risk horizon.

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**Table 4- Required Mark-up for LGD with respect to $R_{PD}^2$ and Idiosyncratic Correlations ($\theta^2$):** Percentage increases in LGD (from 39%) to match the corresponding economic capitals reported in Table 2. Simulations of economic capitals at 99% and 99.9% for a uniform portfolio of 2,000 loans are conducted by assuming $\beta = \theta = 0$, and zero pair-wise LGD correlation (i.e. $R_{LGD} = 0$). Pair-wise PD correlation is assumed to be equal to the corresponding values of 0.0, 0.1, 0.2, 0.3, 0.4 and 0.5 respectively. In order to produce the correct economic capital numbers, these percentages therefore represent the required “mark-up” in mean LGD in a model where LGD correlations are ignored.

\[ \beta^2 = 0.53; ~ R_{LGD}^2 = 0.059 \]

<table>
<thead>
<tr>
<th>$R_{PP}^2$</th>
<th>$\theta^2$</th>
<th>0.00</th>
<th>0.20</th>
<th>0.40</th>
<th>0.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>43%</td>
<td>72%</td>
<td>100%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>32%</td>
<td>60%</td>
<td>85%</td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>32%</td>
<td>56%</td>
<td>80%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>26%</td>
<td>54%</td>
<td>77%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>26%</td>
<td>51%</td>
<td>75%</td>
<td>97%</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>27%</td>
<td>48%</td>
<td>71%</td>
<td>93%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R_{PP}^2$</th>
<th>$\theta^2$</th>
<th>0.00</th>
<th>0.20</th>
<th>0.40</th>
<th>0.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>47%</td>
<td>74%</td>
<td></td>
<td>100%</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>29%</td>
<td>51%</td>
<td>73%</td>
<td>89%</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>24%</td>
<td>38%</td>
<td>59%</td>
<td>76%</td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>18%</td>
<td>30%</td>
<td>46%</td>
<td>60%</td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>20%</td>
<td>29%</td>
<td>44%</td>
<td>59%</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>19%</td>
<td>31%</td>
<td>43%</td>
<td>54%</td>
<td></td>
</tr>
</tbody>
</table>
Table 5: Summary statistics of historical LGD from 1993 to 2002 of loans made to large
corporate borrowers by a bank (Note: data are not available in 1997)

<table>
<thead>
<tr>
<th></th>
<th>no. of observations</th>
<th>mean</th>
<th>Median</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993</td>
<td>48</td>
<td>35%</td>
<td>26%</td>
<td>32%</td>
</tr>
<tr>
<td>1994</td>
<td>29</td>
<td>30%</td>
<td>24%</td>
<td>27%</td>
</tr>
<tr>
<td>1995</td>
<td>18</td>
<td>29%</td>
<td>30%</td>
<td>17%</td>
</tr>
<tr>
<td>1996</td>
<td>4</td>
<td>54%</td>
<td>56%</td>
<td>33%</td>
</tr>
<tr>
<td>1997</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>1998</td>
<td>7</td>
<td>38%</td>
<td>33%</td>
<td>31%</td>
</tr>
<tr>
<td>1999</td>
<td>7</td>
<td>26%</td>
<td>25%</td>
<td>16%</td>
</tr>
<tr>
<td>2000</td>
<td>5</td>
<td>35%</td>
<td>36%</td>
<td>31%</td>
</tr>
<tr>
<td>2001</td>
<td>16</td>
<td>55%</td>
<td>64%</td>
<td>34%</td>
</tr>
<tr>
<td>2002</td>
<td>31</td>
<td>53%</td>
<td>47%</td>
<td>30%</td>
</tr>
<tr>
<td>All</td>
<td>165</td>
<td>39%</td>
<td>31%</td>
<td>30%</td>
</tr>
</tbody>
</table>
Table 6: Historical default rates (from 1991 to 2000) of a uniform portfolio of mid-market borrowers

<table>
<thead>
<tr>
<th>Year</th>
<th>Starting number of borrowers ($n_t$)</th>
<th>Number of defaulted borrowers ($k_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>9,349</td>
<td>183</td>
</tr>
<tr>
<td>1992</td>
<td>10,539</td>
<td>135</td>
</tr>
<tr>
<td>1993</td>
<td>13,306</td>
<td>218</td>
</tr>
<tr>
<td>1994</td>
<td>15,049</td>
<td>196</td>
</tr>
<tr>
<td>1995</td>
<td>14,203</td>
<td>138</td>
</tr>
<tr>
<td>1996</td>
<td>13,887</td>
<td>228</td>
</tr>
<tr>
<td>1997</td>
<td>13,809</td>
<td>205</td>
</tr>
<tr>
<td>1998</td>
<td>14,082</td>
<td>198</td>
</tr>
<tr>
<td>1999</td>
<td>16,680</td>
<td>186</td>
</tr>
<tr>
<td>2000</td>
<td>15,510</td>
<td>253</td>
</tr>
</tbody>
</table>
Figure 1:

![Business Cycle Graph]

- PIT Cyclic
- TTC / A-Cyclical Average

- LGD or Capital

- Recessions: Expansion, Recession, Business Cycle (t)
Figure 2

The diagram illustrates the business cycle with phases such as recession and expansion. It highlights the bottom of the cycle and the TTC/A-Cyclical Average. The PIT Cyclical is also indicated on the graph.
Figure 3

- Conservative Cyclic
- PIT / Cyclic
- Conservative A-Cyclic
- A-Cyclic Average

LGD or Capital

Business Cycle (t)
Figure 4

Additional capital buffer

"A"

1 year

Full turn, 3 to 4 yrs

Conservative A-Cyclical

PIT Cyclical

Business Cycle (t)

Capital
Figure 5

\[ PD \]
- Systemic Risk Driver
  - Specific Risk Driver
    - Correlated Credit Risk Driver
      - Correlated \( R^2_{PD} \) Credit Draws

\[ LGD \]
- Systemic Risk Driver
  - Specific Risk Driver
    - Correlated LGD Driver
      - Correlated \( R^2_{LGD} \) LGD Draws

\[ R_G \]
\[ R_S \]
Figure 6: Time-series plot of estimated systematic PD and LGD risks. They are the normalized time-series of $-P_t$ and $L_t$ estimated by MLE. (Note: LGD data are not available in 1997)