Dual-class share issues and mitigating the costs of corporate democracy

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ABSTRACT

Corporate governance is on the reform agenda all over the globe. The financial literature is scrutinizing dual-class ownership structure. This paper shows that dual-class shares, although they increase imperfections in the control market, help to mitigate another important problem-the noncontractability of the firm's investment policy. Restrictions on the issuance of nonvoting equity may cause managers, who own equity in the firm and value control, to underinvest. The costs associated with underinvestment, may, at times, outweigh the benefits of restricting managers from issuing shares with disparate voting rights.

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I. Introduction

In the movement toward shareholder democracy, equalizing the voting power and influence of corporate shares has become a touchstone for good governance. Corporate charter provisions that explicitly limit the rights of minority shareholders are completely antithetical to this movement. Thus, it is not surprising that dual-class provisions, which create a second class of common stock with reduced voting power, have come under fire. The corporate democracy movement has led policy makers, most vocal among them a high-level group of EU company law experts, to warn of the threats posed by dual-class provisions.[1] If these financial market reforms maintain their momentum, the dual-class ownership structure, which is still utilized by a substantial minority of firms around the world, may soon be seen as a relic from a past era.[2]

This paper argues for a more nuanced view of the role of dual-class shares. It shows that, although the dual-class structure does increase market imperfection in the control market when viewed in isolation, dual-class shares also mitigate another important problem-the noncontractability of the firm's investment policy. Because scale expanding investment projects lower the ability of incumbent management to resist takeovers, management with noncontractable control benefits have incentive to eschew such scale expanding investments. Dual class shares, by protecting managers from the loss of control rents, encourage value maximizing investment policies. Thus, for smaller firms facing favorable investment opportunity sets, a dual-class share ownership structure increases firm value and overall economic efficiency.

Our result complements and extends the existing literature on corporate voting, which as has analyzed nonvoting equity in the context of control contests and well documented the negative

[1] A report of the EU company law experts describes an element–the breakthrough rule–that poses a serious threat to dual class ownership structure. According to the group's recommendations, a bidder that has acquired 75% of the risk based capital of a company (i.e., the company's cash flow rights) should be able to gain control and to this end "break through"any mechanisms and structures that have been established by the company's articles or other means. Even if the company has established a dual-class structure and the bidder has acquired shares with inferior or no voting rights, the bidder will still be able to cast votes in proportion to the fraction of risk-capital that it has acquired.

[2] Ford Motor Company is the best known example of NYSE listed firm with a dual-class structure. Other well-known examples are Fox Entertainment Group (NYSE) and News Corp. (NYSE and London). Also, Anonymous (1997) reports that a number of high-tech IPOs contain dual-class shares as a defense against hostile takeovers. Becht and Roell (1999) find extraordinarily high degree of concentration of shareholder voting power in Continental Europe relative to the US and the U.K. Also, in the U.S.A. over 50% of companies have a largest shareholder who holds less than 5% of the shares, in Austria and Germany there are virtually no such companies. Voting power concentration in US, that is, percentage of companies for which largest voting power stake lies within 20 to 25% accounts for 3.5% of the NYSE listed firms and 5.5% of the NASDAQ listed firms.

shareholders' wealth effect of dual-class ownership structure (e.g., Grossman and Hart (1988), Harris and Raviv (1988), and Ruback (1988)).[3] These papers, like ours, show that holding the firm's investment policy fixed, the likelihood of a successful value increasing takeover is diminished by the creation of a limited-voting class of common stock. Our analysis shows that this reduction in takeover market efficiency can be more than compensated by improved efficiency of corporate investment policy.

Thus, this paper provides a desideratum for explaining the mixed results documented by empirical studies of the effect of dual-class recapitalization on shareholders' wealth. Partch (1987) examines 44 publicly traded firms and concludes that the creation of classified common stock does not harm current shareholders. Jarrell and Poulsen (1988) examine 94 firms that issue limited voting stock, and find significant stock price declines at the announcement of the dual-class recapitalization. They document that approximately 61 percent of the firms experience negative returns on the announcement day and 39 percent registered positive returns. In another empirical study, Lehn and Poulsen (1990) compare dual-class recapitalization to leveraged buyouts and find that firms with greater growth opportunities are likely to go for a dual-class recapitalization. Other empirical studies have examine issues such as announcement date returns, the returns on nonvoting versus voting equity, the treatment of nonvoting equity in takeovers, and the relative frequency of takeovers in firms with nonvoting equity.[4]

This paper points to differences in investment opportunity sets as a potential explanation for the considerable variation in the effect of dual class share issues both within and between firms. In this way our paper integrates analysis of the dual-class decision (heretofore viewed simply of concern to the control literature) into the rich body of research on capital structure and underinvestment, specifically, the firms/managers forgoing positive NPV investment opportunities. Underinvestment and its causes have been studied in a number of papers.

There is an interesting difference between the standard underinvestment problem and under-

[3]Other papers in the area have analyzed the optimality of the one share/one vote structure (Harris and Raviv (1989)), and the issuance of dual-class shares in an IPO (Bebchuk and Zingales (1996)).

[4]Other empirical papers in this area: Smith and Amoako-Adu (1995) find evidence that the prices of restricted voting securities take into account the fact that these shares are treated less favorably in takeovers. Maynes (1996) shows that the premium paid to superior voting shares relative to the restricted voting shares reflects the expectation of higher cashflows in takeovers for the superior voting shares. Moyer et al. (1992) find that the curtailment of shareholders' voting rights through dual-class recapitalization does not tend to entrench management, because other monitoring mechanisms substitute for traditional voting rights.

investment in a corporate control context: That is, in the control context underinvestment is not a problem when managerial ownership is very large–but neither it is a problem when ownership is very small. Because in the control context, the underinvestment results from a dilution in the manager's ownership. If the manager does not own any equity in the firm, there is no scope for dilution and, hence, no underinvestment. This is a counterintuitive result. Typically, managerial ownership of equity causes the interests of managers and shareholders to be better aligned, reducing agency costs. In this case, the underinvestment results from a dilution in the manager's ownership. If the manager does not own any equity in the firm, there is no scope for dilution and no underinvestment. Debt does not solve the underinvestment problem, because it carries the risk of bankruptcy. Nonvoting equity (a term used interchangeably with zero-vote shares) alleviates the underinvestment problem. The issuance of nonvoting stock does not result in a dilution in the manager's ownership of the firm and, therefore, does not cause an increase in the probability of the manager's losing control. As a consequence, the manager is more willing to invest in all available positive NPV projects. However, costs associated with nonvoting equity limit its effectiveness in solving the underinvestment problem.

Our results depend on the roles we assume for the manager and shareholders. The investment decision is made by the manager. If the investment decisions are made by the shareholders, and not the manager, underinvestment is no longer an issue and voting shares are optimal. If it is possible to ensure full investment through contracting, again voting shares may be optimal. There are problems with the remedies described above. A firm whose the shareholders make the investment decision is difficult to imagine because the information requirements would be stringent - the firm would have to reveal all information regarding the projects to the shareholders. Preventing competitors from gaining access to the information and stealing the firm's projects would be impossible. If less stringent information requirements are imposed, managers can ensure underinvestment by withholding projects from shareholders. Contracts are unlikely to work for the same reasons. A contractual solution would also require all investment opportunities to be known to shareholders. In addition, it would require investment opportunities to be verifiable - imposing verification costs on shareholders.

Similarly, if it is possible to ensure full investment through debt financing, again voting shares may be optimal. There are many lines of argument against full investment using debt by highly levered firms.[5] The literature on credit rationing by banks and other lending institutions may help explain the bound on corporate borrowing.[6] Likewise, managers avoid high debt ratios in an to corporate borrowing in an attempt to protect their jobs and stabilize their personal wealth.[7] Also, bankruptcy costs (the costs of liquidation or reorganization) probably discourage high debt level. Debt induced underinvestment has been considered by Myers (1977) and Berkovitch and Kim (1990). Risky debt in the firm's capital structure causes the shareholders objectives to diverge from firm value maximization, making it optimal to forgo positive NPV projects.[8]

The remainder of this article is organized as follows. The paper's formal argument is presented for a simple case in Section II. A formal model is introduced in section III. Section IV depicts the results. Extensions are discussed in section V. Conclusions are presented in section VI. All formal proofs are delegated to the appendix.

II. Basic Idea

Consider a firm that has a public value of \$2, generates a private value for the incumbent manager of \$0.2 and has 100 shares outstanding. The value, both public and private, of the existing firm is the same under the incumbent manager and his rival. The incumbent manager owns 50 shares in the firm and is wealth constrained; that is, the incumbent does not have access to additional funds that would allow him to purchase additional shares in the firm. Given that the incumbent owns half the shares in the firm, there is a zero probability of a change in control of the firm without the incumbent's consent.

The expected value of the incumbent's stake in the firm is the sum of the expected public value of the shares that he owns plus the expected private benefits of control. The expected public value of a share in the firm is the probability of the incumbent remaining in control times the public value of the firm under the incumbent, plus, the probability of the rival gaining control times the public

[5] See, for example, Thakor (1991).

[6] See, for example, Stiglitz and Weiss (1981).

[7] See, for example, Donaldson (1963).

[8] Underinvestment and its causes have been studied in a number of papers. Myers and Majluf (1984) and Cooney and Kalay (1993) obtained conditions under which firms whose existing assets are undervalued may find it optimal to forgo positive NPV projects rather than to issue under-priced equity to finance the investment. The manager is assumed to possess better information than outside shareholders and operates the firm in the best interests of the shareholders. Jensen and Meckling (1976) argued that managers who own less than 100% of the equity may find it sub-optimal to exert the effort needed to search for and analyze all positive NPV projects.

value of the firm under the rival. The expected private benefit extracted by the incumbent is the private benefit of control times the probability of remaining in control; there is zero private benefit if the manager loses control. The value of the incumbent's stake in the firm is $1.2 (=$1.0+1.0*0.2). The value of the shares owned by outside shareholders is the probability of the incumbent retaining control times the public value of the firm under the incumbent, plus, the probability of the rival gaining control times the price paid by the rival. Thus, the value of the shares owned by the existing outside shareholders is $1.0.

To keep the numerical example simple, we assume that the firm has to choose from three discrete investment levels - invest nothing, invest $1 or invest $2. If the firm invests nothing, there is no addition to the value of the firm, no new shares are issued to raise capital and the incumbent manager retains control since he own 50% of the firm's equity. The value of the incumbent's stake and that of the shares owned by outside investors remains the same as above.

The value of the existing firm and the additional private and public value generated under the incumbent and a rival are summarized in Table *I*. The first row of the table corresponds to the case discussed in the paragraph above, where the firm does not undertake any new investments. The second of the table correspond to the cases where the investment in the new project is 50% as large as the value of the existing firm. The third row of the table depicts a scenario where the new project is as large as the worth of the existing firm. Investment in the projects adds to the public value of the firm and to the private benefits enjoyed by the incumbent manager. The rival is assumed to be able to generate a public value that is higher than the sum of the public and private value that the incumbent can generate - the rival is thus superior to the incumbent.

For simplicity, we assume that the number of new shares that have to be issued to finance the investment can be determined by ignoring the feedback effect that positive NPV investments have on the value of the firm's existing shares. If non-voting equity is used to finance the investment, the incumbent's proportional ownership of the control rights (votes) remains at 50% and the incumbent retains the ability to prevail in all control contests. If voting equity is used to finance the investment, the incumbent's proportional ownership of the control rights drops to either 33% or 25% depending on the level of investment. We assign probabilities of 0.8 and 0.6 to the ability of the incumbent to prevail in a control contest when he owns 33% and 25% of the voting shares. Table *II* summarizes this information.

Table III shows that the incumbent's expected wealth is maximized at an investment level of \$1 when the investment is finance using voting equity and at an investment level of \$2 when the investment is financed using non-voting equity. The decision made by the existing outside shareholders is related to the type of security that the firm can issue. If the incumbent is required to finance the investment by issuing voting equity, the incumbent will invest \$1 and the expected value of the shares owned by existing outside shareholders is \$1.058. This number is derived as follows: For investment level of \$1 the expected public value is equal to the expected NPV under the incumbent plus the expected NPV under the rival or [(0.8(3.1 - 1) + 0.2(3.18 - 1)] or 2.116. The expected private benefit extracted by the incumbent is the private benefit of control times the probability of remaining in control. For investment level of 1 the expected private benefit is 0.8(0.21) or 0.168. Therefore the expected value of incumbent's stake for investment level of 1 is [0.5(2.116) + 0.168] or 1.226. The expected welfare of the outside shareholders' is the residual expected public value. For the investment level , the shareholders' expected wealth is 0.5(2.116) or 1.058. If the incumbent has a choice regarding the type of equity to issue to finance the project, the incumbent will issue non-voting equity to invest \$1 and the expected value of the shares owned by existing outside shareholders is \$1.05.

¿From the above example it can be seen that there are situations in which it is value increasing for outside shareholders to allow the incumbent to issue non-voting equity to finance investments. This increases the outside shareholders' wealth from \$1.058 to \$1.06. This is true regardless of the fact that non-voting equity is likely to entrench the incumbent and prevent better rivals from replacing him. The difference in the value of the shares owned by the existing outside shareholders when one-vote and zero-vote shares are used to finance the investment is a cost of entrenchment (for investment level \$1, the costs entrenchment is (\$1.058 -\$1.05) or \$0.08) Allowing the manager to issue non-voting shares will raise the value of the shares owned by existing outside shareholders when the loss in value due to under-investment is larger than the loss in value due to entrenchment. Examples of this situation are firms that have many growth opportunities and firms in relatively new industries. For firms that have relatively few growth opportunities the above result is unlikely to hold. In these firms under-investment is less likely to be a problem and will have a smaller negative impact on the value of the firm.

Does a contractual solution to the underinvestment problem works? Often it may be possible to make a side payment to the manager to induce him to undertake the investment. This alternative would require the outside shareholders to compensate the manager for the decrease in expected wealth associated with an investment of \$2 financed using one-vote shares. In the case presented above, contractual solution does not work. Increase in the outside shareholders expected wealth (\$1.078 -\$1.058 = \$0.02) associated with an increase in investment from \$1 to \$2 is smaller than the decrease in the incumbent manager's expected wealth (\$1.226 -\$1.205 = \$0.021). We do not explicitly model compensation contracts as a solution to the problem.

The results presented in this section are stylized. The model presented in the next section does away with most of the assumptions/simplifications used here. We assume that the incumbent manager controls some fraction strictly less than 50% of the firm's existing equity, even before making the new investment. This generalization is important. The model applies to situations where insiders' holding is relatively small, case in most matured markets like United States. Also, the relationship between managerial ownership and the probability of retaining control and investment is endogenously determined. The probability of incumbent retaining control is jointly determined by the distribution from which the rival's quality is drawn and the level of investment undertaken.

III. Model

There are four players in our model – (*i*) the incumbent management team, (*ii*) existing shareholders, (*iii*) new investors, and (*iv*) the rival management team. All participants are risk-neutral and the discount rate is zero; all securities have prices equal to their expected pay-off. The incumbent manager is the person who controls the firm and is charged with the day-to-day running of the firm. The incumbent manager is also the person who makes all investment decisions.[9]

Existing shareholders are the people who own the firm. They decide, through a simple majority rule, on issues related to changes in control and on the menu of securities that the firm can issue to raise new capital. The incumbent manager is also a shareholder in the firm. New investors are the people who buy the securities that the firm issues to finance the investment.[10] The final player is the rival manager. The rival, if he values the firm higher than the incumbent, offers to buy the

[9]This is a standard assumption in the literature. See, for example, Jensen and Meckling (1976), Myers (1977), Myers and Majluf (1984), Cooney and Kalay (1993), Zwiebel (1996).

[10]The investors purchasing the new securities may be existing shareholders. We do not require that the new investors are a different group of people from the old shareholders.

firm.

The model considers a firm that faces an investment opportunity. The project generates public value for the shareholders of the firm and a private benefit that accrues to the firm's manager. The expected public value of the investment under the best possible (highest quality) manager is given as $x + P(x)$. The realized public value of the project is given as $x + P(x) + \varepsilon_x$, where ε_x is uniformly distributed over the interval $(-\sigma_x,\ +\sigma_x)$. The public value (from here on we drop the expected for brevity) is the sum of the amount invested, x, and the NPV of the investment, $P(x)$. The expected private benefit is $p(x) = \alpha P(x)$.[11]

The NPV and the private benefit realized from the investment opportunity are affected by the quality of the manager who controls the firm. The quality of a manager is given as $(a_i,\ b_i)$, where $0 \leq a_i,\ b_i \leq 1$. The subscript $i \in \{I, R\}$ (Incumbent, Rival). The lowest quality manager is one with $a_i = b_i = 0$ and the best possible manager is one with $a_i = b_i = 1$. The public value of the firm under a manager of quality (a_i, b_i) is $x + a_i P(x)$; the private benefit generated is $b_i p(x)$. Therefore, a_i is referred to as the public quality and b$_i$ as the private quality of the manager. The functional form of the public value allows us to ensure that even the worst quality manager generates an expected cashflow equal to the investment, that is, an NPV of zero. The total value, sum of public value and private benefit, that is generated by a manager of quality (a_i, b_i) is $x + (a_i + \alpha b_i)P(x)$.[12] The quality of the incumbent is known, while the quality of the rival is drawn from a bivariate uniform distribution.

The temporal evolution of events is as follows. Shareholders decide on the types of securities that the firm can issue to finance the investment. Next, the incumbent manager decides the level of investment, x, and issues securities to finance the investment. A rival arrives, and if he can take over the firm, he bids for the firm and gains control. The firm is liquidated in the final period and the public value is paid out to the investors as a dividend. The person in control obtains the private benefit. The quality of the rival is uncertain at the beginning of the scenario, but is revealed at the

[11] Defining the private benefit in this way allows us to ensure that the public value and the private benefit are maximized at the same level of investment. If the public value and private benefit are maximized at different levels of investment, the manager has an incentive to either overinvest or underinvest. This incentive would have a confounding effect on the problem under consideration, and is therefore eliminated by choosing this form for the private benefit. The effect of the public value and the private benefit being maximized at different levels of investment is an interesting problem that can be considered separately (e.g., Zwiebel (1996)).

[12] The total value generated by a manager of the worst possible quality $(a_i = 0, b_i = 0)$ is x; the total value generated by a manager of the best possible quality $(a_i = 1, b_i = 1)$ is $x + (1 + \alpha)P(x)$. Thus, $x \leq x + (a_i + \alpha b_i)P(x) \leq x + (1 + \alpha)P(x)$.

time of his arrival. Figure 1 plots the time-line described above. The change in the value of the firm after a takeover is driven by the difference in quality between the incumbent manager and the rival.

The control contest is a critical stage of the process and is explained further. We assume that the incumbent does not tender in a control contest.[13] To gain control of the firm, the rival has to offer shareholders a greater amount for their shares than the incumbent. If the rival cannot offer a greater amount, he does not bid and the incumbent retains control. If he can offer a greater amount, he pays shareholders an amount equal to the higher of what the incumbent can offer and the public value of the firm under his control, and gains control. Shares have a value from two sources. The first source is the public value; shareholders receive is in the form of dividends when the firm is liquidated. This is referred to as the value of the dividend. The second source is the private benefit that is extracted in a control contest. This accrues only to shareholders who can vote in the takeover contest and is realized only when there is a change in control. This is referred to as the value of the vote.

The public value of the firm is divided over $N + n$ shares that are outstanding, where N is the number of existing shares and n is the number of new shares issued to finance the investment.[14] Therefore, the per-share public value of the firm under the incumbent is $\frac{x+P(x)a_I}{N+n}$. The per-share public value if the rival gains control is $\frac{x+P(x)a_R}{N+n}$. The incumbent's private valuation allows him to offer outside shareholders an additional $b_I\, p(x) = b_I\, \alpha P(x)$. Similarly, the rival can offer an additional $b_R\, p(x) = b_R\, \alpha P(x)$. Since we have assumed that the incumbent does not tender, the private value accrues to the voting shares that are not owned by the incumbent. Let β be the proportion of existing voting shares owned by the incumbent.[15] There are $(1-\beta)N + n$ voting shares that are owned by outside shareholders if voting shares are issued to finance the investment; and $(1-\beta)N$ voting shares that are owned by outside shareholders if nonvoting shares are issued

[13] We get similar qualitative results if the incumbent is allowed to tender in the control contest. If the manager tenders his shares in a control contest the rival's private benefit is divided over a larger number of shares, N+n. This puts the rival at a disadvantage relative to the incumbent.

[14] For simplicity we assume that the value of the firm prior to the new investments is zero.

[15] Bloch and Kremp (1999) in their recent study of French companies state Concentration of direct ownership and voting power is very high in France. Around 40% of unlisted firms have, as first shareholder, individuals owning directly more than 50% of the capital. For the CAC 40 firms, individuals are not the largest blockholder, but when they effectively are present as blockholders, they hold around 30% of the voting rights and have the control in fact. Also, a recent study by Allouche and Amann (1995) showed that, in 1992, 28.3% of the top 1,000 industrial French companies were controlled by families with shareholding ranging between 40% to 60%.

to finance the investment. The nonvoting shares cause the private benefit to be divided over a smaller number of shares, increasing the importance of the manager's private quality relative to his public quality. We assume that the incumbent manager owns fewer than half of the shares outstanding, that is, $\beta \leq 0.5$.[16]

The analysis in this paper is restricted to just two types of securities – nonvoting shares (the holders of these securities are only entitled to a share of the public value of the firm; they do not get to vote on a takeover) and voting shares (these securities entitle their holders to a share in the public value of the firm and to vote on changes in control). The effect of multiple classes of securities and the problem of optimal security design (the "best"combination of dividend and vote) are not formally addressed here. A discussion later in the paper addresses these issues.

At this stage we introduce additional notation to make the problem easier to understand. A superscript $j \in \{0, 1\}$ on variables indicates the value of the variable if the firm issues new shares with $j$votes per share ($j = 0$ corresponds to nonvoting shares, while $j = 1$ corresponds to voting shares). Let n^j = number of new shares issued to finance the investment; ϕ^j = probability of no takeover, if j-vote shares are issued to finance the investment; V_D^j =public value per share if j-vote shares are issued to finance the investment; V_{vote}^j = value of a pure vote claim if j-vote shares are issued to finance the investment.

The value per share of the i-vote shares when j-vote shares are issued to finance the investment, V_i^j, is important. If the new investment is financed using voting shares there is only one type of share outstanding and their value is given by V_1^1. If nonvoting shares are issued to finance investment, then there are two different types of share outstanding and their values are given by V_1^0, for the old voting shares, and V_0^0, for the newly issued nonvoting shares. The value of the voting shares is equal to the value of the dividend received plus the value of the vote, while the value of the nonvoting shares is equal to just the value of the dividend received. The value of the voting shares is equal to the value of the dividend received plus the value of the vote, $V_1^j = V_D^j + V_{vote}^j$, while the value of the nonvoting shares is equal to just the value of the dividend received, $V_0^0 = V_D^j$. The number of new shares that the firm has to issue to finance the investment, n^j, depends on the type of security that is issued, because $n^j = \frac{x}{V_i^j}$.

[16]If $\beta \geq 0.5$ the results are trivial - nonvoting shares can be used to ensure that the incumbent retains control; in this case the outside shareholders control fewer than 50% of the votes and cannot prevent the manager from issuing nonvoting shares.

The two decision problems can now be formally set up. Both the manager and the outside shareholders are assumed to be interested in maximizing their expected wealth. For the manager the decision variable is the level of investment, x. Given that the manager does not tender his shares to the rival, this is equivalent to

$$\max_{x} \left[N\beta V_D^j(x) + \phi b_I \, \alpha P(x) \right]. \tag{1}$$

The objective function above has two parts: The first part is related to the public value of the firm and reflects the fact that the manager is similar to any other shareholder. The second part is related to the private benefit of incumbency, and is realized only if the manager retains control of the firm. The solution to the manager's problem gives the manager's optimal response to restrictions on the type of security that the firm can issue.

Let $\hat{x}^j$ be the solution to the manager's optimization problem given that he issues $j-$vote shares to finance the investment. Outside shareholders maximize the value of their shares, picking the type of security that the manager can issue and taking the manager's optimal response as given. Thus, the decision problem of the outside shareholders is

$$\max_{j=0,1} V_1^j\left(\hat{x}^j\right). \tag{2}$$

To solve the two optimization problems, given by equations 1 and 2, we need (i) the probability that there is no takeover, (ii) the value of the dividend, and (iii) the value of the vote for the case where the firm issues voting shares and for the case where the firm issues nonvoting shares.

A change in control occurs when the rival can offer a higher per-share value to the outside shareholders than the incumbent. The probability of a takeover can be obtained by considering the case of nonvoting shares and voting shares separately. The incumbent can retain control if he can offer more for the shares than the rival. If voting shares are used to finance the investment, this is equivalent to

$$\frac{a_I \, P(x) + x}{N + n^1} + \frac{b_I \, \alpha P(x)}{(1-\beta) N + n^1} \geq \frac{a_R \, P(x) + x}{N + n^1} + \frac{b_R \, \alpha P(x)}{(1-\beta) N + n^1}. \tag{3}$$

The first term on the LHS of equation 3 is the per share public benefit that is generated with the incumbent in control. The second term on the LHS is related to the incumbent's private benefit. The denominator is smaller in this case since the private benefit is distributed only to the outside shareholders. The RHS terms are related to the public and private benefit per share generated under the rival. If nonvoting shares are issued to finance the investment, the incumbent retains control if

$$\frac{a_I\ P(x)+x}{N+n^0}+\frac{b_I\ \alpha P(x)}{(1-\beta)\,N}\geq\frac{a_R\ P(x)+x}{N+n^1}+\frac{b_R\ \alpha P(x)}{(1-\beta)\,N}. \tag{4}$$

In this case the private benefit is distributed only to those outside shareholders who own voting shares. The holders of the nonvoting shares do not share the private benefits since they cannot affect the outcome of the control contest. After simplifications, equations 3 and 4 can be expressed as

$$\underline{b}_R^j\ \leq b_I\ \frac{a_I\ -a_R}{(1+\kappa^j)\,\alpha},\text{ where } j=0 \text{ or } 1. \tag{5}$$

Note that if $j=1$, that is, if voting shares are issued to finance the investment, then $\kappa^1=\frac{\beta N}{(1-\beta)N+n^1}$; if $j=0$, then $\kappa^0=\frac{\beta N+n^0}{(1-\beta)N}$. The term $\underline{b}_R^j$ is the lowest private quality of the rival that makes takeover possible. Similarly, we simplify and rearrange equations 3 and 4, to define $\underline{a}_R^j$ and $\overline{a}_R^j$:

$$\underline{a}_R^j\ =\max\left[a_I-(1-b_I)\left(1+\kappa^j\right)\alpha,\ 0\right]\quad\text{and}\quad\overline{a}_R^j\ =\min\left[a_I+b_I\left(1+\kappa^j\right)\alpha,\ 0\right]. \tag{6}$$

Rivals with public quality higher than $\overline{a}_R^j$ can gain control of the firm irrespective of their private quality (i.e., even if $b_R\ =0$). Rivals with public quality lower than $\underline{a}_R^j$ cannot gain control of the firm, even if they have the highest possible private quality ($b_R\ =1$).

Next, consider the effects of increasing investment, x, on $\underline{a}_R^j$, $\overline{a}_R^j$ and $\underline{b}_R^j$. Since $\left(1+\kappa^j\right)$ appears in all three expressions, let us first see the effect of increasing investment on $\left(1+\kappa^j\right)$. The number of new shares needed to finance the investment increases as the size of the investment increases; that is, n^1 and n^0 both increase in x. Thus, $\left(1+\kappa^0\right)$ increases as x increases. In the expression for $\left(1+\kappa^1\right)$, the numerator and the denominator are increasing at the same rate; thus, $\left(1+\kappa^1\right)$ decreases as x increases. Thus, $\overline{a}_R^0$, $\underline{a}_R^1$, and $\underline{b}_R^1$ increase as x increases; and, $\underline{a}_R^0$, $\overline{a}_R^0$, and $\underline{b}_R^0$decrease as x increases. In figure 3 and figure 4 we plot the effects of increasing investments on $\overline{a}_R^j$, $\underline{a}_R^j$, and $\underline{b}_R^j$.

The probability of the incumbent's retaining control, that is, "no takeover," is given as

$$\phi^j = \int_0^{\underline{a}_R^j} \int_0^1 db_R da_R + \int_{\underline{a}_R^j}^{\overline{a}_R^j} \int_0^{\underline{b}_R^j} db_R da_R. \tag{7}$$

The first term is the region where the rival's public quality is very low. In this region the rival has no hope of gaining control regardless of his private quality. The second term is the region where the rival's public quality is such that the incumbent retains control if the rival's private quality is lower than $\underline{b}_R^j$; otherwise, rival gains control.

The value of a pure dividend claim is equal to the expected dividend that the holder of the claim gets and is given by

$$V_D^j = \frac{\phi^j \left(a_I \; P\left(x\right) + x\right)}{N + n^j} + \int_0^{\underline{a}_R^j} \int_0^1 \frac{\left(a_R \; P\left(x\right) + x\right)}{N + n^j} db_R da_R + \int_{\underline{a}_R^j}^{\overline{a}_R^j} \int_0^{\underline{b}_R^j} \frac{\left(a_R \; P\left(x\right) + x\right)}{N + n^j} db_R da_R. \tag{8}$$

The first term is the probability that the incumbent retains control times the per-share public value of the firm under the incumbent. The second and third terms give the expected dividend under the rival. The third term is generated by rivals of very high public quality, who can take over the firm regardless of their private quality. The numerator gives the value of the expected total dividend received by the shareholders. This is scaled by the number of shares outstanding to get a per-share value.

The value of a pure vote claim is related to the extraction of private benefit from the rival in a takeover. To obtain an expression for the value of the vote, we classify the rival into one of three groups. The first group is that of rivals who cannot gain control of the firm (low public quality – regions considered in the probability of no takeover). If a rival from this group is drawn, no private benefit is extracted and the value of the vote is zero. Next consider the rivals who can gain control of the firm without having to pay out any of their private benefit (high public quality – the region related to the third term in the value of the dividend). There is again no extraction of the rival's private benefit. Private benefit is extracted only in the case of a rival of intermediate public quality. The payoff to the vote claim when the firm issues voting shares can be written as

$$\begin{cases} \frac{(a_I \; P(x) - a_R \; P(x))}{N + n^1} + \frac{b_I \; \alpha P(x)}{(1-\beta)N + n^1} & \text{if } \underline{a}_R^1 \le a_R \le \overline{a}_R^1 \\ 0 & \text{otherwise} \end{cases}. \tag{9}$$

Similarly, the payoff to the vote claim when the firm issues nonvoting shares is

$$\begin{cases} \frac{(a_I\ P(x) - a_R\ P(x))}{N+n^0} + \frac{b_I\ \alpha P(x)}{(1-\beta)N} & \text{if } \underline{a}_R^0 \le a_R \le \overline{a}_R^0 \\ 0 & \text{otherwise} \end{cases} . \tag{10}$$

The value of the vote is simply the expectation of these values,

$$V_{vote}^1 = \int_{\underline{a}_R^1}^{\overline{a}_R^1} \int_{\underline{b}_R^1}^{1} \frac{(a_I\ P(x) - a_R\ P(x))}{N+n^1} + \frac{b_I\ \alpha P(x)}{(1-\beta)N + n^1} db_R da_R \tag{11}$$

and

$$V_{vote}^0 = \int_{\underline{a}_R^0}^{\overline{a}_R^0} \int_{\underline{b}_R^0}^{1} \frac{(a_I\ P(x) - a_R\ P(x))}{N+n^0} + \frac{b_I\ \alpha P(x)}{(1-\beta)N} db_R da_R. \tag{12}$$

A. Simplifications

A factor that makes the model as described, mathematically involved is that the expressions for $\overline{a}_R$ and $\underline{a}_R$ have min and max functions in them. To simplify things, we break the set of possible incumbent quality into a number of subsets that we consider separately. In particular we consider the subset of a_I and b_I values that satisfy the following two conditions:

$$a_I + \alpha b_I \frac{\bar{n}+1}{1-\beta} \le 1 \tag{13}$$

and

$$a_I - \alpha(1-b_I)\frac{\bar{n}+1}{1-\beta} \ge 0, \tag{14}$$

where $\bar{n} = \frac{n}{N}$is an upper bound on the number of shares needed to finance all available positive NPV projects. The subset of a_I and b_I values conforming to these restrictions has the property that a_I and b_I are negatively correlated. This would describe a world where the total value of the firm is fixed and a_I and b_I determine how it is divided between public value and private benefit. We do not place any restrictions on the values that $\bar{n}$ can take. It would be difficult to imagine situations in which $\bar{n}$ is greater than 0.5, since $\bar{n} = 0.5$ would mean that the firm increases 50% in size as a result of the new projects (or that the firm issues half as many new shares as there are shares outstanding). The subsets of values of a_I and b_I that we chose are those values for which the min and max functions can be dropped from the expression for $\underline{a}_R$ and $\overline{a}_R$ for both the one-vote and the zero-vote cases. For small α and β this subset contains almost all possible values of a_I and b_I. To see the impact of conditions 13 and 14, consider the case where $\bar{n} = 0.5$, $\alpha = 0.1$ and $\beta = 0.1$. These values of α and β imply that the maximum private benefit that can be reaped by the manager is 10% as large as the maximum NPV obtained by investing in the project and that the manager has a 10% stake in the firm before the issue of new shares. A value of $\bar{n} = 0.5$ implies that the number of new shares issued by the firm are less than or equal to one-half the number already existing. In this case, 83.34% of all possible a_I and b_I combinations are considered. If we consider $\bar{n} = 0.5$, $\alpha = 0.1$, and $\beta = 0.1$, then 50% of all possible a_I and b_I combinations are considered.

IV. Results

The initial result depicts the type of manager who will underinvest if they are forced to finance investment using voting equity. The next two results consider the interests of existing shareholders and the manager. The types of managers under whom outside shareholders will be willing to allow the firm to issue nonvoting equity is obtained. Next, we check the manager's willingness to finance the investment using nonvoting equity. Finally, we provide a result regarding the social cost of nonvoting equity.

A. Investment financed by voting equity, and nonvoting equity

The manager chooses the investment level to maximize his expected wealth. If the investment is financed using equity, the probability of retaining control is the probability that there is no takeover. Following three terms in the manager's objective function are dependent on the level of investment - the value of the dividend, the probability of retaining control, and the private benefit of control. The value of the dividend increases with investment as positive NPV projects are undertaken. The private benefits of control also increase with investment. We consider the dependence of the probability of retaining control on the level of investment below.

Let us first consider the case in which the manager does not own equity in the firm and investments are financed by issuing voting equity. Since the manager does not own any equity in the firm, there is no possibility of dilution in ownership and the probability of the manager's retaining control is unaffected by the level of investment. Thus, the manager's objective function is increasing in investment (since the private benefit of control increases in investment) and the manager invests in all available positive NPV projects.

When the manager owns equity in the firm, the probability of his retaining control depends on the level of investment. As the firm issues voting equity the proportional holding of the manager decreases, making it easier for a rival to win a control contest. The first order condition of the manager's objective function has a negative term. If the private benefits of control are large, this negative term leads to an interior optimum making it optimal for the manager to forgo some positive NPV projects. The proposition below formalizes this result.

Proposition 1. When investments are financed by issuing voting equity, (a) if the manager does not own any equity in the firm, he invests in all available positive NPV projects; (b) if the manager owns equity in the firm, he forgoes some positive NPV projects if

$$b_I \geq \frac{1-3\beta}{4(1-2\beta)} + \sqrt{\left(\frac{1-3\beta}{4(1-2\beta)}\right)^2 + \frac{1+\beta}{6(1-2\beta)}}.$$

The condition provided in the proposition above is a sufficient condition. The manager will always forgo some projects if his private quality meets the condition in the proposition above. The manager may forgo some positive NPV projects for smaller values of b_I.

A part of the cost of underinvestment is borne by the manager since he owns equity in the firm. The larger his ownership, the larger is the cost to him of underinvestment. Also, the larger his ownership, the greater is the impact of the dilution in control. Due to dual impact, the condition given in the proposition depends on β. If the manager owns 5% of the equity in the firm, the manager will have an incentive to forgo positive NPV projects if $b_I > 0.74$. This implies that if we collect a sample of firms with 5% managerial equity ownership, we should expect to find some underinvestment in 26% of the firms.

Next, we consider the firm that finances investment by issuing nonvoting equity. The situation is slightly more complicated. The private quality of the manager determines whether the probability of retaining control increases or decreases with investment. Nonvoting equity increases the importance of the manager's private quality, relative to his public quality, in determining the outcome of a control contest. For managers with high private quality, nonvoting equity causes the probability of retaining control to increase with investment.

Proposition 2. If investment is financed using nonvoting equity, for $\frac{1}{2} \leq b_I \leq 1$ the probability of the manager's retaining control increases with investment.

The increasing probability of retaining control causes the value of the manager's objective function to increase with investment, making it optimal for the manager to invest in all available positive NPV projects. The proposition below formalizes this result.

Proposition 3. Let $\overline{n} \geq \frac{n}{N}$ be the upper bound on the number of shares needed to finance all available positive NPV projects. Then, for all $b_I \geq \frac{1-\beta(2\overline{n}+3)}{4(1-\beta(\overline{n}+2))} + \sqrt{\left(\frac{1-\beta(2\overline{n}+3)}{4(1-\beta(\overline{n}+2))}\right)^2 + \frac{\beta(1+\overline{n})}{3(1-\beta(2+\overline{n}))}}$, the incumbent manager invests in all available positive NPV projects if investment is financed using nonvoting shares.

For all values of b_I that satisfy the above condition, the manager will invest in all available positive NPV projects. There exist lower values of b_I for which nonvoting equity will ensure full investment. The cost of issuing nonvoting equity increases with the size of the investment opportunity available. For high levels of managerial ownership, high β, a large fraction of this cost is borne by the manager. It is optimal for the manager to invest in all available projects by issuing nonvoting equity only if the private benefit is large enough to cover the costs involved. Therefore, we find that the condition given in the proposition depends on β and $\overline{n}$. If the manager owns 5% of the equity in the firm and the investment opportunity available is half as large as the existing firm, $\overline{n} = 0.5$, the manager invests in all available projects if b_I is greater than 0.53.

To summarize, we find that managers with high levels of private quality, underinvests if he is forced to finance the investments using voting equity. For these same high levels of private quality the manager is willing to invest fully if he is allowed to finance the investment using nonvoting equity.

B. Welfare of existing shareholders and the incumbent manager

So far we have obtained conditions under which there is increased investment if nonvoting equity is issued to raise funds. But increased investment financed by nonvoting equity is not always in the best interests of both the outside shareholders and the manager. There are costs to issuing nonvoting equity. These costs are considered here.

The value of voting equity is made up of two parts - the value of the dividend received and the value of the private benefits extracted in a takeover. Since investors in nonvoting equity are not entitled to vote in control contests, they do not receive the extracted private benefit. Therefore, investors are willing to pay a lower amount for nonvoting equity than for voting equity. This means that a larger number of nonvoting shares than voting shares have to be issued to finance a given level of investment, reducing the per-share dividend that is available to owners of the existing voting shares.

The second potential cost arises in cases where the issuance of nonvoting shares increases the entrenchment of the incumbent. The value of the vote depends on two factors – the private benefit extracted in a takeover and the probability of a takeover. Holding the private benefit extracted constant, a reduction in the probability of a takeover reduces the value of the voting rights. There is an offsetting gain. The voting rights remain concentrated in the hands of the old shareholders. This leads to the sharing of the extracted private benefit among a smaller number of investors, increasing the per-share benefit.

To summarize, the issuance of nonvoting shares impacts existing shareholders and the manager in three ways – (*i*) lower per-share dividends, (*ii*) a lower probability of a change in control, and (*iii*) a higher per-share takeover premium conditional on a takeover. Let us consider the value of the existing voting shares. The lower per share dividends result in a lower public value for these shares. Similarly, the lower probability of a change in control results in a lower value for the vote, while a higher per-share takeover premium causes a higher value for the vote. Our first result obtains conditions under which the value of existing voting shares is higher if nonvoting shares are used to finance the investment. This gives conditions under which existing shareholders will agree to finance new investment with nonvoting shares.

Proposition 4. Assume that b_I satisfies the conditions given in propositions 1 and 3 and that the manager invests x if voting equity is used to finance the investment. Outside shareholders prefer investment financed by nonvoting equity if $1 - \frac{P(x)}{P(\bar{x})} \geq \frac{\alpha^2(1+\bar{n})^2}{(1-\beta)(1+a_I)^2}\left[\frac{1}{3} - b_I\left(1 - b_I\right)\right]$.

The result above gives the level of underinvestment that is needed before shareholders are willing allow the manager to raise funds by issuing nonvoting shares. The LHS of the inequality above is a measure of the loss to shareholders because of underinvestment, while the RHS is a measure of the costs related to the issuance of nonvoting shares. It is optimal for outside shareholders to allow the manager to issue nonvoting shares when the gains realized from reduced underinvestment outweigh the costs. The level of underinvestment needed for outside shareholders to prefer nonvoting shares is quite small. If $\bar{n} = 0.5$, $\alpha = 0.1$, and $\beta = 0.1$, outside shareholders will find the issuance of nonvoting shares to finance an investment optimal, even if there is just 1% underinvestment. If $\bar{n} = 0.5$, $\alpha = 0.3$, and $\beta = 0.1$, outside shareholders will find nonvoting shares optimal if there is 8.33% underinvestment.

Now, consider the manager's expected wealth. The lower per-share dividend affects the manager's wealth negatively. The lower probability of takeover increases the incumbent manager's expected wealth since it increases the probability that the incumbent will obtain the private benefit of control. The increase in the takeover premium does not affect the manager since we have assumed that he does not tender in a takeover. The propositions below provide results on the types of managers who are better off if the firm issues nonvoting stock.

PROPOSITION 5. Assume that b_I satisfies the conditions given in propositions 1 and 3 and that the manager invests x if voting equity is used to finance the investment. Outside shareholders prefer investment financed by nonvoting equity if $b_I \geq \frac{1-\beta(\bar{n}+3)}{2(2-\beta(\bar{n}+4))} + \frac{1}{2}\sqrt{\left(\frac{1-\beta(\bar{n}+3)}{2(2-\beta(\bar{n}+4))}\right)^2 + \frac{4}{3}\frac{1+\beta(1+\bar{n})}{(2-\beta(4+\bar{n}))}}$.

The lowest b_I for which the incumbent is better off if the firm finances investment using nonvoting stock depends on the incumbent's ownership in the firm. As the incumbent's ownership increases, the cost to him of the dilution in dividend increases, making it expensive to issue nonvoting shares. The private benefit of control needs to be large enough to offset this higher cost.

Comparing the results of Proposition 2 and Proposition 5 provides some interesting insights. From Proposition 2 we see that nonvoting shares cause the probability of the incumbent's retaining control to be higher if $\frac{1}{2} \leq b_I$. From Proposition 5 we notice that the incumbent manager is better off, if the firm issues nonvoting shares and $b_I \geq \frac{1-\beta(\bar{n}+3)}{2(2-\beta(\bar{n}+4))} + \frac{1}{2}\sqrt{\left(\frac{1-\beta(\bar{n}+3)}{2(2-\beta(\bar{n}+4))}\right)^2 + \frac{4}{3}\frac{1+\beta(1+\bar{n})}{(2-\beta(4+\bar{n}))}} \geq \frac{1}{2}$. The minimum value of b_I is higher in proposition 5 than in proposition 2. The divergence exists because of the cost related to dividend dilution. If $\beta = 0.1$ and $\bar{n} = 0.5$, the manager prefers nonvoting shares if b_I is greater than 0.74.

The final question that we answer in this subsection addresses the instances when we will observe firms issuing nonvoting shares. The answer depends on the balance of power between the manager and the shareholders. If shareholders have the upper hand and can force the manager to issue a particular type of security, the condition given in proposition 4 will determine when the firm will issue nonvoting shares. On the other hand, if shareholders can only specify a menu of securities, the conditions in Proposition 4 and 5 will both have to be satisfied before the firm issues nonvoting shares.

C. Low public quality and the control contest

We finally turn to the issue of economic efficiency. Other studies have found that dual-class shares allow control of the firm to remain in/pass to the hands of inferior managers, lowering economic efficiency. We show that it is true that dual-class shares do allow inferior managers to win control contests. A statement on economic efficiency, though, requires a trade-off between the costs of underinvestment and the cost of inefficiently managed firms. This requires assumptions regarding the ability of other firms to undertake projects that the firm under consideration has forgone. We leave this aspect of the problem to future research.

Grossman and Hart (1988) show that voting shares are optimal because they ensure that the firm ends up in the hands of the manager who is of higher public quality. Our result is similar to their result. The difference between the voting shares and the nonvoting shares is that nonvoting shares cause the private quality of managers to have a larger impact on the outcome of the control contest. Consider a rival with private quality higher than the incumbent's private quality, that is, $b_R > b_I$. Nonvoting shares favor this rival in a control contest, making it easier for him to gain control of the firm; that is, he can gain control for lower values of a_R, values for which he would lose the control contest if the firm had financed its investment using voting shares. Similarly, if $b_R < b_I$, that is, if the incumbent has a higher private quality, nonvoting shares would favor the incumbent in a control contest, making it easier for him to retain control of the firm. That is, an incumbent can keep control of the firm for lower levels of a_I, values for which he would lose control of the firm if the investment had been financed using voting shares. The proposition below formalizes this result.

Proposition 6. The minimum public quality required to gain/retain control of the firm is lower in firms financed with dual classes equity.

The fact that a manager of lower public quality may gain control of firms will be an important issue for market regulators. If other mechanisms can be used to discipline managers, the cost of this problem will be small. Moyer et al. (1992) find that alternative monitoring mechanisms emerge in firms after the issuance of dual-class shares. This is an additional issue that will make the costs and benefits of dual-class shares difficult to evaluate. The next section discusses extensions to our model. It also looks at the effect of relaxing some of our assumptions.

V. Extensions

In this section we consider two related issues. The first is the issuance of shares with less than one vote per share. The reason that these may be useful is that they would have lower costs related to dividend dilution than zero-vote shares. Firms in Japan are allowed to issue multiple classes of shares. Second, we discuss the costs/ benefits of multiple classes of shares.

A. Optimal vote-dividend combination

This issue can be thought of in two different ways. The first is to consider shares that have 1 unit of dividend and θ votes and to find the optimal value of θ. The second is to allow the firm to simultaneously issue both voting and nonvoting shares. We first consider θ vote shares.

The optimality of θ-vote shares, with $0 < \theta < 1$ will depend on the size of the investment opportunity available to the firm. For a class of shares, the vote will have value only if a sufficient mass of votes of that class exist so that these shares can be used by the manager to block a takeover. This means that managers will issue θ-vote shares only if the investment opportunity is large enough that $\beta N + n\theta \geq \frac{1}{2}(N + n\theta)$.

The reasoning that obtains the above inequality is as follows: Consider a firm with two classes of shares, one-vote and θ-vote, outstanding. Suppose $n\theta$ is small so that the above inequality is not fulfilled. In this case the manager has no incentive to bid for this, θ-vote, class of shares; blocking the rival requires the manager to bid for the voting shares. The rival has no incentive to bid for these shares either. The outcome of the control contest is determined solely by the owners of the voting shares. This causes the vote to have zero value in the θ-vote shares, giving the manager no incentive to issue θ-vote shares.

If $\bar{x}$ is small, $\theta = 0$ is likely to be optimal. This is because the number of shares that are issued is going to be small for small $\bar{x}$, and the total number of votes held by shareholders in that class will be insufficient to meet the above condition. Our model has assumed that shareholders are homogeneous. Heterogeneity among shareholders may result in cases where θ-vote shares may become optimal even when $\bar{x}$ is small.

Allowing firms to simultaneously issue both nonvoting and voting shares will increase the set of firms that find it optimal to issue dual-class shares. This assertion is based on the following line of reasoning. Existing one-vote shareholders prefer nonvoting shares when the level of underinvestment is higher than $\frac{\alpha^2(1+\bar{n})^2}{(1-\beta)(1+a_I)^2}\left[\frac{1}{3} - b_I\left(1 - b_I\right)\right]$. As $\bar{n}$ decreases, the outside shareholders will find it optimal to allow the manager to finance investments using nonvoting shares even for low levels of underinvestment. If the investments are partly financed using voting shares and this analysis were carried out over the remaining projects, the relevant $\bar{n}$ would have a smaller value. This means that existing shareholders, the owners of the voting shares, would be more willing to allow managers to issue nonvoting shares. In this case the existing shareholders could allow the manager the choice of issuing voting shares or a mix of θ-voting shares per nonvoting share issued.

B. Multiple classes of shares

We considered a firm that issues only two classes of shares - voting and nonvoting shares. One logical extension to this model is to consider multiple classes of shares. Is it optimal either for the manager or for existing shareholders to issue multiple classes of shares? Shares that give their owners fractional voting rights can be considered. Thus, the firm could simultaneously issue shares with θ_0, θ_1, θ_2, and θ_3 votes (an example can be $\theta_0 = 0, \theta_1 = 0.33, \theta_2 = 0.5$, and $\theta_3 = 1$). In the above framework, the shares with fractional votes will be issued if the fractional votes have value. The fractional votes will have value if there is a sufficient mass of each of these classes of shares outstanding so that the rival is forced to buy them to take control of the firm.

The manager can raise the cost of a takeover for the rival by issuing multiple classes of shares. This does not mean that it is optimal for the manager to issue multiple classes of shares. The manager bears a cost when he issues multiple classes of shares. This cost is in the form of lower dividends. The existing shareholders are likely to find multiple classes of shares detrimental to their interest. As the number of classes of shares increases the probability of a change in control is likely to decrease very quickly. The compensating factor, investment, is unlikely to go up fast enough to increase the value of the shares held by outside shareholders. Thus, multiple classes of shares are unlikely to be optimal for old shareholders.

VI. Conclusions

A firm with a set of positive NPV projects available is considered. It is shown that if the firm requires outside equity financing to undertake the projects it will find separation of the vote and dividend claim optimal in some cases. Raising equity capital has two effects – (*i*) the value of the firm increases since positive NPV projects are being undertaken and (*ii*) the proportion of the firm's shares owned by the manager decreases, increasing the likelihood that the manager will lose control of the firm. A manager who values control will find it optimal to forgo some positive NPV projects. A pure dividend claim makes it possible for the manager to finance the investment without increasing the chances of losing control of the firm, increasing the manager's willingness to undertake all positive NPV projects.

This paper provides a theoretical justification for the easing of regulations on issuance of dual-class shares that has been occurring in developing countries. It also allows us to explain the results of the empirical studies that have found a positive abnormal return to an announcement by a firm that it will be issuing a second class of shares. In our model, the announcement of dual-class recapitalization would indicate to shareholders that the severity of underinvestment in the firm would be reduced and thus would increase the value of their shares.

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Appendix

A. Basic Results

Before we present the proofs of the propositions, here are some basic results that we will use repeatedly. By definition we have $1+k^0(x)=\frac{N+n^0(x)}{(1-\beta)N}$, $1+k^1(x)=\frac{N+n^1(x)}{(1-\beta)N+n^1(x)}$, where $n^j(x)=0$ at $x=0$ and $n^j(x)>0$ for all $\bar{x}\geq x>0$. Thus, $1+k^0(0)=1+k^1(0)=\frac{1}{(1-\beta)}$. For $x>0$ we get

$$1+k^0(x)=\frac{1}{(1-\beta)}+\frac{n^0(x)}{(1-\beta)N}>\frac{1}{(1-\beta)} \tag{BR-1}$$

and

$$1+k^1(x)=\frac{1}{(1-\beta)}-\frac{\beta n^1(x)}{(1-\beta)N\left[(1-\beta)N+n^1\right]}<\frac{1}{(1-\beta)}. \tag{BR-2}$$

Thus $1+k^0(x)\geq 1+k^1(x)$ for all $\bar{x}\geq x>0$. Integrating the expression for the value of dividend in section II and simplifying, we obtain

$$\left(N+n^j\right)V_D^j=x+\frac{P\left(x\right)}{2}\left[1+a_I^2-\left(b_I^2-b_I+\frac{1}{3}\right)\left(1+k^j\right)^2\alpha^2\right]. \tag{BR-3}$$

Integrating and simplifying the expression for payoff to the vote-claim given in section II, we get

$$\left(N+n^j\right)V_{vote}^j=\frac{P\left(x\right)\left(1+k^j\right)^2\alpha^2}{6}. \tag{BR-4}$$

Using equations BR-3 and BR-4 we derive the value of existing voting shares:

$$\left(N+n^j\right)V_1^j=x+\frac{P\left(x\right)}{2}\left[1+a_I^2+b_I(1-b_I)\left(1+k^j\right)^2\alpha^2\right]. \tag{BR-5}$$

If the firm issues voting stock to finance the investment, $n^1V_1^1=x$, giving

$$NV_1^1=\frac{P\left(x\right)}{2}\left[1+a_I^2+b_I(1-b_I)\left(1+k^1\right)^2\alpha^2\right]. \tag{BR-6}$$

If nonvoting shares are used to finance the investment, $n^0V_0^0=x$, giving

$$NV_0^0=\frac{P\left(x\right)}{2}\left[1+a_I^2+\left[b_I(1-b_I)-\frac{1}{3}\right]\left(1+k^0\right)^2\alpha^2\right] \tag{BR-7}$$

and

$$NV_1^0=\frac{P\left(x\right)}{2}\left[1+a_I^2+b_I(1-b_I)\left(1+k^0\right)^2\alpha^2-\frac{1}{3}\left(1+k^0\right)\alpha^2\left[\left(1+k^0\right)-\frac{1}{1-\beta}\right]\right]. \tag{BR-8}$$

B. Proof of Proposition 1

The manager chooses the investment level to maximize his objective function. We show that the first derivative of the manager's objective function evaluated at $\bar{x}$ is negative. This means that the manager stops investing at some level below $\bar{x}$. when investments financed using voting equity.
(a) Zero managerial ownership

$$MO^1 = P(x)\alpha b_I \left[a_I + \alpha(1+k)(b_I - \frac{1}{2})\right]. \tag{P1-1}$$

Since manager does not own any shares in the firm, $(1+k^1) = 1$. Substituting for $(1+k^1)$ and differentiating with respect to x gives

$$\frac{dMO^1}{dx} = \alpha b_I \left[a_I + \alpha(b_I - \frac{1}{2})\right] \frac{dP(x)}{dx} \geq 0 \text{ for all } x \leq \bar{x}. \tag{P1-2}$$

Therefore, the manager invests in all available positive NPV projects.
(b) Nonzero managerial ownership

$$MO^1 = \beta NV_D^1 + \Pr()\alpha b_I P(x). \tag{P1-3}$$

Also, we know

$$NV_1^1 = \frac{P(x)}{2}\left[1 + a_I^2 + b_I(1-b_I)\left(1+k^1\right)^2 \alpha^2\right], \tag{P1-4}$$

$$NV_{vote}^1 = \frac{P(x)\,N\left(1+k^1\right)^2\alpha^2}{6(N+n^1)}, \tag{P1-5}$$

which gives

$$NV_D^1 = \frac{P(x)}{2}\left[1 + a_I^2 + b_I(1-b_I)\left(1+k^1\right)^2\alpha^2\right] - \frac{P(x)\,\alpha^2\left(1+k^1\right)k^1}{6\beta}, \tag{P1-6}$$

given $V_D^1 = V_1^1 - V_{vote}^1$. Substituting into the expression for the manager's objective function, equation P1-3 gives

$$MO^1 = P(x)\,A(k^1), \tag{P1-7}$$

where $A(k^1)$ is new notation and is given by

$$\begin{aligned} A(k^1) = \frac{1}{2}&\left[\beta(1 + a_I^2 + \alpha^2 b_I(1-b_I)\left(1+k^1\right)^2) - \frac{1}{3}\alpha^2\left(1+k^1\right)k^1\right] \\ &+ \alpha b_I\left[a_I + \alpha\left(1+k^1\right)\left(b_I - \frac{1}{2}\right)\right]. \end{aligned} \tag{P1-8}$$

Differentiating with respect to x and rearranging terms, we have

$$\frac{dMO^1}{dx} = A(k^1)\frac{dP\left(x\right)}{dx} + P\left(x\right)\frac{dA(k^1)}{dk^1}\frac{dk^1}{dx}. \tag{P1-9}$$

To sign $\frac{dMO^1}{dx}$ we need to sign $\frac{dA(k^1)}{dk^1}$. Differentiating $\frac{dA(k^1)}{dk^1}$ with respect to k^1,

$$\frac{dA(k^1)}{dk^1} = \beta\alpha^2 b_I(1-b_I)\left(1+k^1\right) - \frac{1}{6}\alpha^2\left(1+2k^1\right) + \alpha^2 b_I\left(b_I - \frac{1}{2}\right). \tag{P1-10}$$

Cancelling common terms and simplifying we find that in order for $\frac{dA(k^1)}{dk^1} > 0$, the following condition needs to hold:

$$\beta b_I\left(1-b_I\right)\left(1+k^1\right) - \frac{1}{6}\left(1+2k^1\right) + b_I\left(b_I - \frac{1}{2}\right) > 0 \tag{P1-11}$$

or

$$b_I\left(b_I - \frac{1}{2}\right) + \beta b_I\left(1-b_I\right) - \frac{1}{6} > \left[\frac{1}{3} - \beta b_I\left(1-b_I\right)\right]k^1. \tag{P1-12}$$

The RHS is greater than zero, but we do not know the value of k^1.We replace k^1 by its maximum value $\frac{\beta}{1-\beta}$, and get

$$b_I\left(b_I - \frac{1}{2}\right) + \beta b_I\left(1-b_I\right) - \frac{1}{6} > \left[\frac{1}{3} - \beta b_I\left(1-b_I\right)\right]\frac{\beta}{1-\beta}. \tag{P1-13}$$

On further simplifying we get

$$b_I \geq \frac{1}{4}\left(\frac{1-3\beta}{1-2\beta}\right) + \frac{1}{4}\sqrt{\left(\frac{1-3\beta}{1-2\beta}\right)^2 + \frac{8\left(1+\beta\right)}{3\left(1-2\beta\right)}}. \tag{P1-14}$$

Hence, the proof.

C. Proof of Proposition 2

ϕ = Probability of no takeover when the firm issues j-vote shares to finance the investment.

$$\phi = \frac{U_a^j + L_a^j}{2} = a_I + \left(1+k^j\right)\left(b_I - \frac{1}{2}\right)\alpha. \tag{P2-1}$$

We need to show that $\frac{d\phi}{dx} > 0$. Taking the derivative of equation P2-1 with respect to x,

$$\frac{d\phi}{dx} = \alpha\left(b_I - \frac{1}{2}\right)\frac{dk^0}{dx}. \tag{P2-2}$$

Since $\frac{dk^0}{dx} > 0$, $\frac{d\phi}{dx} > 0$ if $b_I > \frac{1}{2}$. Hence, the proof.

D. Proof of Proposition 3

MO^0 =Manager's objective function when the firm issues j-vote shares to finance the investment. We need to show that $\left.\frac{dMO^0}{dx}\right|_{\bar{x}} > 0$. We know from section II that

$$\begin{aligned} MO^0 &= \frac{\beta P(x)}{2}\left[1 + a_I^2 - \frac{\alpha^2\left(3b_I^2 - 3b_I + 1\right)\left(1 + k^0\right)^2}{3}\right] \\ &+ \alpha b_I P(x)\left[a_I + \left(1 + k^0\right)\left(b_I - \frac{1}{2}\right)\alpha\right]. \end{aligned} \tag{P3-1}$$

We can rewrite equation P3-1 in similar fashion to equation P1-8:

$$MO^0 = P(x)\, A\left(k^0\right), \tag{P3-2}$$

where

$$\begin{aligned} A\left(k^0\right) &= \frac{1}{2}\left[\beta\left(1 + a_I^2 + \alpha^2 b_I\left(\left(1 - b_I\right)\left(1 + k^0\right)^2\right)\right) - \frac{1}{3}\alpha^2\left(1 + k^0\right)^2\beta\right] \\ &+ \alpha b_I\left[a_I + \alpha\left(1 + k^0\right)\left(b_I - \frac{1}{2}\right)\right]. \end{aligned} \tag{P3-3}$$

Differentiating with respect to x and rearranging terms

$$\frac{dMO^0}{dx} = A\left(k^0\right)\frac{dP(x)}{dx} + P(x)\frac{dA\left(k^0\right)}{dk^0}\frac{dk^0}{dx}. \tag{P3-4}$$

We have $A(k^0), P(x)$, $\frac{dP(x)}{dx}$ and $\frac{dk^0}{dx} \geq 0$. To show that $\left.\frac{dMO^0}{dx}\right|_{\bar{x}} > 0$, we need to show that $\left.\frac{dA(k^0)}{dk^0}\right|_{\bar{x}} > 0$. Differentiating $A(k^0$ with respect to k^0 we have

$$\frac{dA(k^0)}{dk^0} = \beta\alpha^2 b_I(1 - b_I)\left(1 + k^0\right) - \frac{1}{3}\alpha^2\left(1 + k^0\right)\beta + \alpha^2 b_I\left(b_I - \frac{1}{2}\right). \tag{P3-5}$$

Cancelling the common terms and simplifying, we find that in order for $\frac{dA(k^0)}{dk^0} > 0$, the following condition needs to hold:

$$b_I\left(b_I - \frac{1}{2}\right) + \beta b_I\left(1 - b_I\right) - \frac{1}{3}\beta > \left[\frac{1}{3} - \beta b_I\left(1 - b_I\right)\right]k^0. \tag{P3-6}$$

The RHS is greater than zero, but we do not know the value of k^0. We replace k^0 by its maximum value. This gives

$$b_I \geq \frac{1}{4}\left(\frac{1 - \beta(\bar{n} + 3)}{1 - \beta(\bar{n} + 2)}\right) + \sqrt{\frac{1}{16}\left(\frac{1 - \beta(\bar{n} + 3)}{1 - \beta(\bar{n} + 2)}\right)^2 + \frac{\beta(\bar{n} + 1)}{3\left(1 - \beta(\bar{n} + 2)\right)}}. \tag{P3-7}$$

Hence, the result.

E. Proof of Proposition 4

Consider the values of b_I for which the manager invests in all available positive NPV projects if nonvoting equity is used to finance the investment. Assume that the manager invests some x if voting equity is used to finance the investment. We have to obtain conditions such that $V_1^0(\bar{x}) \geq V_1^1(x)$. Substituting for $V_1^0(\bar{x})$ and $V_1^1(x)$ from equations BR-6 and BR-8, we get

$$\frac{1+a_I^2}{2}\left[P(\bar{x})-P(x)\right]+\frac{\alpha^2 b_I(1-b_I)}{2}\left[P(\bar{x})\left(1+k^0\right)^2-P(x)\left(1+k^1\right)^2\right] \tag{P4-1}$$

$$\geq \frac{P(\bar{x})\alpha^2\left(1+k^0\right)}{6}\left[\left(1+k^0\right)-\frac{1}{1-\beta}\right]. \tag{P4-2}$$

Simplifying,

$$\left(1+a_I^2\right)\left[1-\frac{P(x)}{P(\bar{x})}\right]+\alpha^2 b_I(1-b_I)\left[\left(1+k^0\right)^2-\frac{P(x)}{P(\bar{x})}\left(1+k^1\right)^2\right] \tag{P4-3}$$

$$\geq \frac{\alpha^2\left(1+k^0\right)}{3}\left[\left(1+k^0\right)-\frac{1}{1-\beta}\right]. \tag{P4-4}$$

Let us denote $u=\frac{P(x)}{P(\bar{x})}$, an indicator of the proportion of projects undertaken if voting shares are issued to finance the investment (a measure of underinvestment). Simplifying further we get

$$\left(1+a_I^2\right)(1-u) \tag{P4-5}$$

$$\geq \frac{\alpha^2\left(1+k^0\right)}{3}\left[\left(1+k^0\right)-\frac{1}{1-\beta}\right]-\alpha^2 b_I(1-b_I)\left[\left(1+k^0\right)^2-u\left(1+k^1\right)^2\right]. \tag{P4-6}$$

Further rearranging gives

$$\left(1+a_I^2\right)(1-u) \tag{P4-7}$$

$$\geq \alpha^2\left(1+k^0\right)^2\left[\frac{1}{3}-b_I(1-b_I)\right]-\alpha^2\left[\frac{\left(1+k^0\right)}{3(1-\beta)}-u\left(1+k^1\right)^2 b_I(1-b_I)\right]. \tag{P4-8}$$

Dropping the second term on the RHS which is positive gives

$$1-u \geq \frac{\alpha^2(\bar{n}+1)}{\left(1+a_I^2\right)(1-\beta)^2}\left[\frac{1}{3}-b_I(1-b_I)\right]. \tag{P4-9}$$

Hence, the result.

F. Proof of Proposition 5: Incumbent manager prefers nonvoting shares

We prove this proposition in two parts. We first hold the investment level fixed and obtain conditions under which the manager is better off if the investment is financed using nonvoting shares; then, we show that the manager remains better off if the investment level is increased. ¿From Propositions 2 and 4 we know that $MO^j = A(k^j)P(x)$, where j stands for the type of shares issued. To show that $MO^0 \geq MO^1$, we need to obtain conditions under which $A(k^0) \geq A(k^1)$. Substituting for $A(k^0)$ and $A(k^1)$ from equations P3-3 and P1-8 and simplifying we get

$$\begin{aligned} b_I\left(1-b_I\right) \quad \geq \quad & \frac{1}{6}\left(\frac{\beta\left(1+k^0\right)^2-k^1\left(1+k^1\right)}{\left(1+k^0\right)-\left(1+k^1\right)}\right) \\ & -\frac{\beta b_I\left(1-b_I\right)}{2}\left(\frac{\left(1+k^0\right)^2-\left(1+k^1\right)^2}{\left(1+k^0\right)-\left(1+k^1\right)}\right). \end{aligned} \tag{P5-1}$$

The fact that $\beta\left(1+k^0\right)=\frac{\beta\left(N+n^0\right)}{(1-\beta) N} \leq \frac{\beta N+n^0}{(1-\beta) N}=k^0$ gives

$$\begin{aligned} & b_I\left(1-b_I\right) \\ \geq \quad & \frac{\left(1+k^0\right) \beta}{6}+\frac{\left(1+k^1\right)}{6}-\frac{\beta b_I\left(1-b_I\right)}{2}\left(\left(1+k^0\right)+\left(1+k^1\right)\right). \end{aligned} \tag{P5-2}$$

Substituting the maximum value of k^0 and k^1 and simplifying further and solving for b_I gives

$$b_I \geq \frac{1}{2}\left(\frac{1-\beta(\bar{n}+3)}{2-\beta(\bar{n}+4)}\right)+\frac{1}{2} \sqrt{\left(\frac{1-\beta(\bar{n}+3)}{2-\beta(\bar{n}+4)}\right)^2+\frac{4(1+\beta(\bar{n}+1))}{3(2-\beta(\bar{n}+2))}}. \tag{P5-3}$$

Equation P5-3 ensures that the manager is better off if the level of investment remains the same. The second part of the proof requires us to show that the manager is better off if the level of investment increases. This requires that $MO^0(x_2) > MO^0(x_1)$, where $x_2 > x_1$. Proposition 3 gives conditions under which $\frac{dMO^0}{dx} > 0$. Thus, if b_I satisfies both the conditions in Proposition 3 and the condition given by equation P5-3, the manager is better off if investment is financed using nonvoting equity. If b_I satisfies the condition obtained in Proposition 3, equation P5-3 is also satisfied. Hence, the result.

G. Proof of Proposition 6

Minimum public quality required is $a_I+\alpha b_I\left(1+k^j\right) \geqslant a_R+\alpha b_R\left(1+k^j\right)$. Rearranging terms and noting that the minimum a_I that allows the manager to win the control contest is obtained by replacing the inequality with an equality $\underline{a}_I^j=a_R+\alpha\left(b_R-b_I\right)\left(1+k^j\right)$ where $\underline{a}_I^j$ is the minimum public quality that is required for the manager to win the control contest, given that j-vote shares are issued to finance the investment. We need to show that $\underline{a}_I^0 \leqslant \underline{a}_I^1$. Thus, we obtain

$$\begin{aligned} \underline{a}_I^0-\underline{a}_I^1 &= \alpha\left(b_R-b_I\right)\left(1+k^0\right)-\alpha\left(b_R-b_I\right)\left(1+k^1\right) \\ &= \alpha\left(b_R-b_I\right)\left(\left(1+k^0\right)-\left(1+k^1\right)\right) \end{aligned} \tag{P6-1}$$

¿From equations BR-1 and BR-2 we know that $\left(1+k^0\right)-\left(1+k^1\right)>0$. Thus, if $b_I \geq b_R$ then $\underline{a}_I^0<\underline{a}_I^1$.

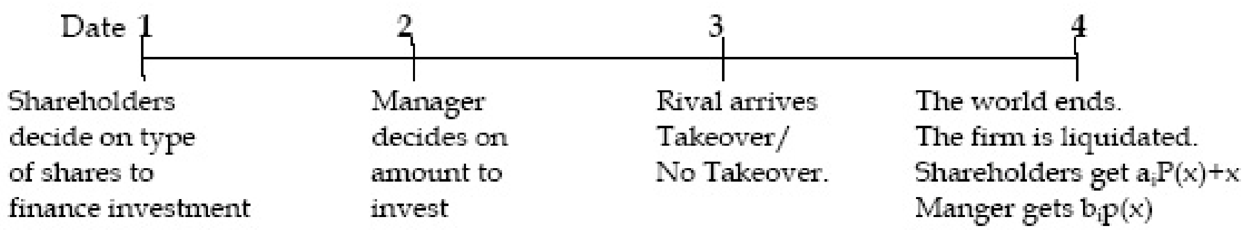

Figure 1: Temporal evolution of events in the model. The firm has a set of positive NPV projects that it can invest in. The shareholders first decide on the menu of securities that they allow the manager to use to raise the necessary funds. Their decision is based on the incumbent manager's quality and the form of the NPV function, $P(x)$. The manager then decides how much to invest, raises the required funds by issuing new shares, and invests these funds. A control contest occurs next and the winner gets the firm. At date 4 the world ends, the firm is liquidated, shareholders receive the public value of the firm, and the manager in control receives the private benefit.

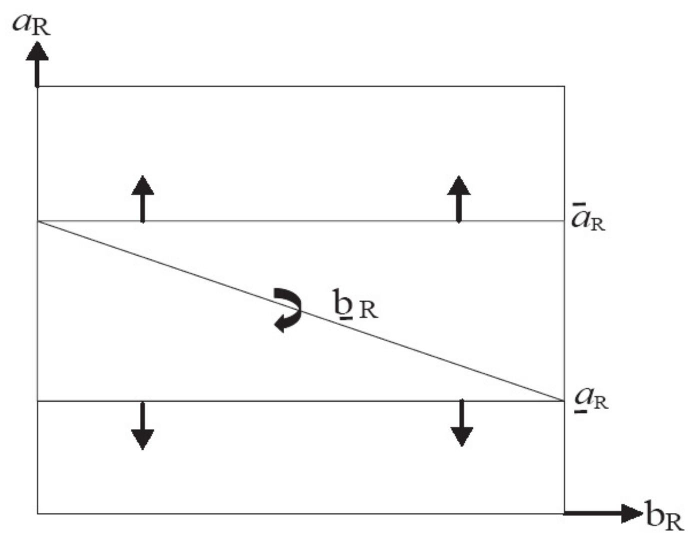

Figure 2: Effect of level of investment on the takeover region. If the firm issues nonvoting shares to fund the investment project, the position and the direction of movement of $\bar{a}^0_R$, $\underline{a}^0_R$, and $\underline{b}^0_R$ as x increases are depicted in the figure.

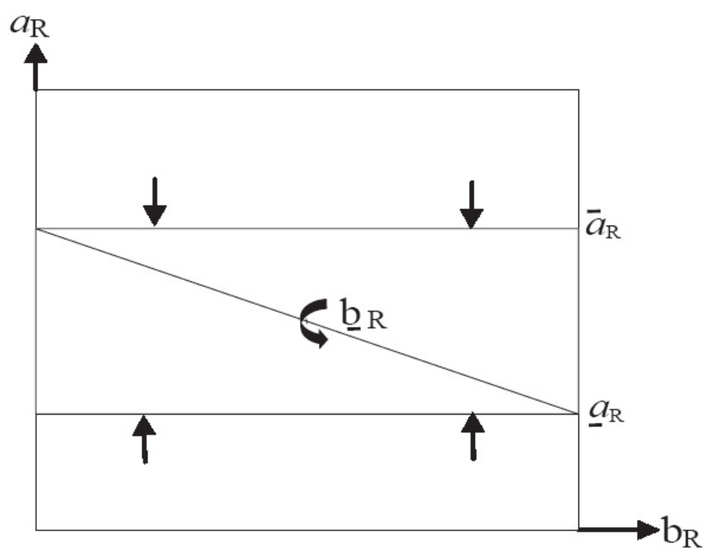

Figure 3: Effect of level of investment on the takeover region. If the firm issues voting shares to fund the investment project, the position and the direction of movement of $\bar{a}^1_R$, $\underline{a}^1_R$, and $\underline{b}^1_R$ as x increases are depicted in the figure.

Table I

Details of the Projects in the hand of the Incumbent Management

Initial number of shares outstanding: 100
Managerial ownership: 50 shares

Existing Firm		**Investment Opportunity**					**Number of new shares issued**
Public Value	Private benefit	Investment	Manager		Rival		
			Addition to public value	Addition to private benefit	Addition to public value	Addition to private benefit	
2	0.2	0	0	0	0	0	0
2	0.2	1	1.1	0.01	1.18	0	50
2	0.2	2	2.12	0.011	2.21	0	100

Figure 4: Table I summarizes the value of the existing firm and the additional private and public value generated due to the new investment opportunity under the incumbent and a rival. The first row corresponds to the situation where the incumbent management does not undertake any new investments. The second and third rows correspond to the cases where the investment in the new project is 50% and 100% as large as the value of the existing firm. Investment in the projects adds to the public value of the firm and to the private benefits enjoyed by the incumbent manager.

Table II

Ownership and the Probability of Retaining Control under Different Types of Equity Financing and Different Investment Levels

Voting Shares Issued to Finance Investments		**Non-Voting Shares Issued to Finance Investments**	
Managerial Ownership of Voting Rights	Probability of Retaining Control	Managerial Ownership of Voting Rights	Probability of Retaining Control
50%	1	50%	1
33.3%	0.8	50%	1
25%	0.6	50%	1

Figure 5: The first half of Table II shows that if voting equity is used to finance the investment, the incumbents proportional ownership of the control rights drops. We assume that it drops to 33% if the new investment opportunity is 50% of the existing value of the firm or it drops to 25% if the of the new investment opportunity is as large as the existing firm. We assign probabilities of 0.8 and 0.6 to the ability of the incumbent to prevail in a control contest when he owns 33% and 25% of the voting shares. The second half of Table II shows that if non-voting equity is used to finance the investment. The incumbents proportional ownership of the control rights (votes) remains at 50% and the incumbent retains the ability to prevail in all control contests.

Table III

Expected Wealth of the Incumbent Management and the Rival under Different Types of Equity Financing and Different Investment Levels

	Voting Shares Issued to Finance Investments		**Non-Voting Shares Issued to Finance Investments**	
Investments	Manager	Shareholders	Manager	Shareholders
0	1.200	1.000	1.200	1.000
1	1.226	1.058	1.260	1.050
2	1.205	1.078	1.271	1.060

Figure 6: The expected value of the incumbent's stake in the firm is the sum of the expected public value of the shares that he owns plus the expected private benefits of control. The expected public value of a share in the firm is the probability of the incumbent remaining in control times the public value of the firm under the incumbent, plus, the probability of the rival gaining control times the public value of the firm under the rival. For investment level of 2 the expected public value is equal to the expected value under the incumbent plus the expected value under the rival or [(0.6(4.12 - 2) + 0.4(4.21 - 2)] or 2.156. The expected private benefit extracted by the incumbent is the private benefit of control times the probability of remaining in control. For investment level of 1 the expected private benefit is 0.6(0.211) or 0.127. Therefore the expected value of incumbent's stake for investment level of 2 is [0.5(2.156) + 0.127] or 1.205. The expected welfare of the outside shareholders' is the residual expected public value. For the investment level , the shareholders' expected wealth is 1.078.