# Ownership Structure and the Market for Corporate Control<sup>\*</sup>

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#### Abstract

We study the impact of the ownership structure of a corporation on the characteristics and efficiency of the market for corporate control. We adopt a general mechanism design approach, in which endogenous sources of inefficiency in the market, including adverse selection, moral hazard, budget balance and voluntary trading, may preclude the possibility of efficiently restructuring control and ownership. We identify necessary and sufficient conditions for an efficient market, and describe the characteristics of efficient restructuring mechanisms, when they exist. In efficient restructuring, corporations typically increase the number of shares of the incumbent manager when he remains in control, or give him a generous golden parachute when he is deposed. Corporations are also reluctant to assign full control and full ownership to a single stockholder, unless agency costs are severe. We characterize the set of ownership structures for which efficient restructuring is possible. While the distribution of ownership among the non-controlling shareholders is irrelevant, the level of initial managerial ownership is a central determinant of this set. Typically, efficient restructuring is easier to obtain for low levels of managerial ownership.

**Keywords**: Ownership, corporate control, restructuring, mechanism design **JEL codes**: G32, G34, D82

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### 1 Introduction

The governance structure of the modern corporation is concerned with two main problems, the assignment of the right people to management and the efficient provision of incentives to managers. The former problem stems from adverse selection, while the latter stems from moral hazard. Both, however, arise from the separation of control from ownership, a defining characteristic of the modern corporation. When internal corporate governance mechanisms fail, the market for corporate control arises as one possible solution for dealing with these problems. Due to external pressure, firms frequently restructure, by firing and hiring managers and reallocating ownership, and are sometimes taken over. We seek to explain these phenomena by studying the operation of the market for corporate for corporate control and how the structure of ownership affects its efficiency.

To do so, we apply the tools of mechanism design to the problem of corporate restructuring. Thus, rather than analyzing the characteristics of specific mechanisms to transfer control, we let the market for corporate control choose the most efficient one. Under this approach, the efficiency of the market is hampered by adverse selection, moral hazard, budget balance and voluntary trading. Because of the fundamental, endogenous nature of these sources of friction, our conclusions are likely to be robust across different institutional environments.

We start by noting that either the adverse selection or the moral hazard problem, if taken in isolation, is trivially solved and the first-best allocation is obtained. When both problems are present, however, efficient restructuring of control and ownership may not be feasible. That is, there may be no ex post share rule (i.e., an allocation of ownership shares to all shareholders, conditional on the assignment of control) for which an incentive compatible, individually rational mechanism implements the efficient assignment of control and satisfies budget balance.

Indeed, our main results define necessary and sufficient conditions that characterize when a corporation can be efficiently restructured. The key to these conditions is the *optimal share rule*, so named because it implements the first-best allocation if and only if such an outcome is feasible. The properties of this share rule, which value-maximizing corporations will use, have strong positive implications for the nature of corporate restructuring in general. First, this rule treats the manager and the non-controlling shareholders very differently, typically giving the manager more shares. For example, when the manager is replaced, he receives a "golden parachute" that is set high specifically to make him willing to participate in the restructuring mechanism in the first place. Second, under this share rule managerial ownership does not converge. It never decreases when the manager retains control, but often decreases when a non-controlling shareholder assumes control. Third, the optimal share rule specifies complete dissolution, where the firm becomes fully owned and managed by a sole proprietor, if and only if agency costs are severe. Thus, our theory offers an entirely novel explanation for the persistence of the separation of control from ownership and a

prediction for when they are likely to be combined.

Perhaps more importantly, the optimal share rule also allows us to characterize the set of ex ante ownership structures for which efficient restructuring is possible. We show that, while the ex ante level of managerial ownership is a crucial determinant of this set, the ex ante distribution of ownership among the non-controlling shareholders does not matter. We also show that efficient restructuring is usually more likely to be achieved when the initial managerial ownership is low. As long as the number of shareholders is sufficiently large, efficient restructuring is possible if and only if managerial ownership is sufficiently small. For higher levels of managerial ownership, management entrenchment effects preclude efficiency.

This paper contributes also to the broader mechanism design literature. Since buyer/seller exchange is a special type of restructuring and a special case of our model, we are able to generalize the analysis of Myerson and Satterthwaite (1983) to the simultaneous exchange of control and ownership. We show that, when control and ownership can be separated, efficient bilateral exchange of control is actually possible for identical, continuous types of buyer and seller. However, this is possible only if the full separation of ownership and control introduces no agency costs.

Collecting these results, we conclude that the goals of providing incentives to managers and facilitating control transfers conflict with each other. Specifically, ex ante ownership structures that tend to mitigate agency costs also tend to exacerbate the problem of management entrenchment. Thus, too few changes of control may occur whenever the market for corporate control must deal with both moral hazard and adverse selection.

Because ownership structure affects both agency costs and the functioning of the market for corporate control, firm value depends on the ownership structure. Our analysis indicates that this relationship is quite robust. Under the ex ante structure, if managerial ownership is too low, agency costs reduce firm value. Upon restructuring, if managerial ownership is too high, the market for corporate control is inefficient, and ex post firm value is reduced.

The balance of the paper is organized as follows. In Section 2, we explain how our paper fits in the relevant literature. In Section 3, we describe the model and give some important preliminary results. In Section 4, we identify the optimal mechanism, discuss its properties and analyze the relationship between ownership structure and frictions in the market for corporate control. In Section 5, we discuss an example that illustrates our main points. Section 6 provides a discussion of the results and some concluding remarks.

### 2 Related Literature

Our mechanism design approach owes its greatest debt to Cramton, Gibbons and Klemperer (1987), the first paper to study efficient dissolution of partnerships in the presence of asymmetric information.<sup>1</sup> However, our two departures from their framework, the separation of control from ownership and the possibility of agency costs, are quite significant.<sup>2</sup> Given these changes, allocative efficiency depends both on the assignment of control and on a sufficiently high level of ex post managerial ownership. This greatly expands the set of share rules capable of potentially implementing the first-best, as efficiency no longer requires reducing the firm to single ownership. The optimal share rule is chosen from this set, essentially, to make it as easy as possible to satisfy budget balance.

In corporate finance theory, few papers directly study the effect of ownership structures on the functioning of the market for corporate control. To explain their empirical findings of a nonmonotonic relationship between management ownership and firm value, Morck, Shleifer and Vishny (1988) offer an informal theory on the trade-off between managerial incentives and entrenchment. In contrast, Stulz (1988) provides a formal theory on the trade-off between higher takeover premia and the probability of takeover, but takeover activity is not aimed at correcting sub-optimal ownership structures in his analysis, as it is in this paper.

Numerous authors have studied how ownership affects firm value, but there remains wide disagreement on the issue. In their seminal contribution, Berle and Means (1932) argue that separating control from ownership is detrimental for firm value because managers who are not owners will not be guided by profit-maximizing motives. Jensen and Meckling (1976) strengthen this vision by showing that the imperfect alignment of incentives between (controlling and owning) managers and (owning) shareholders fosters a value-reducing agency problem, which could nevertheless be mitigated if managers held stock. In contrast, Demsetz (1983) argues that a well-functioning market for corporate control tempers the agency problem. At a minimum, it renders it a short-term phenomenon. Profligate managers can be replaced, so current managers' incentives for austerity are enhanced. Generally inept managers can be replaced, too. Therefore, as long as the market for corporate control efficiently replaces such underperformers, a firm's value will not depend on its ownership structure.<sup>3</sup> If the ownership structure itself affects frictions in that market, however, it also affects the feasible ways in which firms can restructure and, consequently, firm value. Thus, our finding of a robust relationship between ownership and the efficiency of the market indicates that ownership is indeed value-relevant.

<sup>&</sup>lt;sup>1</sup>This seminal paper led to extensive work on dissolving partnerships. See for example McAfee (1992), Fieseler, Kittsteiner and Moldovanu (2001) and Moldovanu (2002).

 $<sup>^{2}</sup>$ Ornelas and Turner (2004) also separate ownership and control, but study only the problem of fully dissolving partnerships.

<sup>&</sup>lt;sup>3</sup>Empirically, several authors find that ownership structure "matters" for firm performance: see Morck, Shleifer, and Vishny (1988), McConnell and Servaes (1990, 1995), Holderness, Kroszner, and Sheehan (1999), Habib and Ljungqvist (2003), Adams and Santos (2003) and McConnell, Servaes, and Lins (2003). On the other hand, many others find no robust empirical support for the value-relevance of ownership structures: for example, Demsetz and Lehn (1985), Loderer and Martin (1997), Himmelberg, Hubbard, and Palia (1999), Demsetz and Villalonga (2001) and Coles, Lemmon, and Meschke (2003).

This paper is closely related also to the literature on possible failures of the market for corporate control, initiated with the seminal work of Grossman and Hart (1980). In their paper, an outside raider, who could acquire control and replace the incumbent manager with a superior one, will refrain from doing so if small non-controlling shareholders refuse to tender their shares in a takeover bidding game in which the tender price is set below the post-takeover share price. Because of free-riding behavior of small shareholders, "too few" changes in control occur.

Grossman and Hart's arguments suggest that the initial ownership of shares affects efficiency. For example, the size of the stakes of non-controlling shareholders affect their incentives to free-ride, while a large initial ownership stake by a single bidder also helps against the free-riding problem (Shleifer and Vishny 1986). Other related papers analyze the effects of toeholds on the ex post efficiency of private-value bidding mechanisms (Burkart 1995; Singh 1998) or common-value ones (Bulow, Huang and Klemperer 1999). Burkart et al. (1998) analyze a related but different problem: the existence of agency problems with the new controlling party after a takeover will make the post-takeover ownership structure relevant for firm value. In sum, the literature on tender offers in general implies that the ownership structure of a firm will have effects on the functioning of market mechanisms to restructure ownership and control.

In our view, the results in this literature, while insightful, are vulnerable to the critique of Demsetz (1983): if a specific takeover mechanism does not lead to an efficient outcome, why not use a different one?<sup>4</sup> For example, Grossman and Hart (1980) show that if dilution of original shareholders is possible, the free-riding problem is eliminated. Although dilution is illegal in the U.S., Müller and Panunzi (2004) show that the same outcome can be achieved when the raider finances its acquisition by issuing debt backed by the target's future cash-flows. They argue that these "bootstrap acquisitions" are legal and were also widely used in the takeover wave of the 1980s.

Our approach, by contrast, is not subject to Demsetz's critique. In appealing to the revelation principle, we permit the use of any restructuring mechanism. Thus, our finding that ownership is value-relevant is robust to *all* available mechanisms.

The meaning of the expression "market for corporate control" varies in the literature. While some reserve this term to the specific mechanism of hostile tender offers, we use the term as originally defined by Manne (1965), who views corporate control as an asset that can be bought and sold.<sup>5</sup> Thus, transactions in this market do not have to imply hostility, or the use of any specific mechanism. Consistent with this view, Manne considered friendly mergers to be the most common, and probably the most efficient, mechanism for taking over control (pp. 117-19).<sup>6</sup>

<sup>&</sup>lt;sup>4</sup>Burkart, Gromb and Panunzi (2000) and Bebchuk and Hart (2001) provide comparative analyses of two different mechanisms.

 $<sup>{}^{5}</sup>$ This definition is also broadly consistent with Jensen and Ruback's (1983) view of the market for corporate control.

<sup>&</sup>lt;sup>6</sup>Corporate restructuring activity is usually achieved by the combination of many different transactions in the

### 3 The Model

We analyze the problem of simultaneously assigning control to the most able manager and assigning him sufficiently high ownership to preclude agency costs—i.e., to prevent the manager from (inefficiently) diverting the firm's profits for private gain. After describing the setup of the model, we first show that, in the absence of direct costs of restructuring, this problem is trivial either if information about managerial abilities is public or if agency costs do not arise for any level of managerial ownership, but that when both information is private and agency costs are a potential problem, the set of efficient restructuring mechanisms is restricted and may be empty. We then provide conditions that identify necessary and sufficient conditions for when a corporation may be restructured.

#### 3.1 Setup

An all-equity firm is initially owned by n risk-neutral shareholders indexed by  $i \in N \equiv \{1, ..., n\}$ . Shareholder i owns a fraction  $r_i \in [0, 1]$  of shares, where  $\sum_{i=1}^{n} r_i = 1$ . Ownership does not imply control over the decisions taken within the firm. Instead, the firm resembles a modern corporation, in that a team of professional managers is in charge of running it. Since we wish to abstract from conflicts of interest within the management team, we model this team as a single individual with full control. We refer to the initial manager as shareholder 1. Thus,  $r_1$  is our measure of managerial ownership.

We denote the general ability of shareholder i in running the firm by  $a_i$ .<sup>7</sup> We assume that  $a_i$  is distributed according to an increasing, continuous and differentiable distribution function F with support  $[\underline{a}, \overline{a}]$ . The assumption of a common distribution is made to simplify exposition.<sup>8</sup> Managerial talent is private information. Thus, shareholder i knows his own ability  $a_i$ , but any shareholder  $j \neq i$  knows only the distribution of  $a_i$ . The expected value of  $a_i$  is denoted by  $\mu$ .

Some shareholders might have no managerial skills and thus have a very low  $a_i$ . However, there might be also very good potential managers who are not shareholders of the firm. Now, because we define as shareholders any individual i owning a fraction  $r_i \in [0, 1]$  of the firm—even if  $r_i = 0$ —in principle any individual in this economy could be considered a shareholder. Thus, our approach is

market for corporate control with managerial initiatives to refocus the firm towards a more efficient allocation of corporate assets (see Jensen 1987, 1988).

<sup>&</sup>lt;sup>7</sup>We treat  $a_i$  as a measure of managerial talent, but other interpretations are also possible. For example,  $a_i$  might be considered a measure of shareholder *i*'s ability to identify the right people to actually run the business.

<sup>&</sup>lt;sup>8</sup>If shareholders have past observations of the manager's performance, for example, they could in principle use them to update their beliefs about managerial ability. A natural way to incorporate this into our model is to permit the ex ante distribution of the manager's ability to be different than the ex ante distribution of the other shareholders. As long as the support of each  $a_i$  is the same, however, our main results would be unaffected by this change.

indeed very general; any potential candidate for becoming a manager must be included in the set of n shareholders.

We consider a simple technology in which profit is a linear function of the manager's ability. Thus, under the initial control structure and in the absence of agency costs, the manager knows that profit will be  $\pi = a_1$ , whereas the non-controlling shareholders expect profit  $E[a_1] = \mu$ . If upon restructuring shareholder *i* becomes the manager, the firm's profit becomes  $\pi = a_i$ .

Managers may have, however, private incentives to divert company profits to themselves. We model the extraction of private gains similarly to Burkart et al. (1998). Specifically, the manager uses a share  $\gamma$  of the firm's profit to produce "share"  $\delta(\gamma)$  for himself, which can be understood as perquisites that the manager consumes, leaving the residual share  $1 - \gamma$  to be divided among the shareholders. Thus, under the ex ante ownership structure, the manager's payoff is  $[\delta(\gamma) + (1 - \gamma)r_1]a_1$  and shareholder *i*'s payoff is  $(1 - \gamma)r_ia_1$ . The allocation of corporate resources  $\gamma$  is a choice variable to the manager. We follow the technical assumptions of Burkart et al. (1998) that  $\delta(\cdot)$  is twice continuously differentiable, increasing and concave in [0, 1], with boundary conditions  $\delta(0) = 0$  and  $\delta'(1) = 0$ . However, we relax their other assumptions in two important ways. First, we permit the marginal gain of initial extraction,  $\delta'(0)$ , to be anywhere in [0, 1]. Second, we require  $\delta(\gamma)$  to be strictly concave only if  $\delta'(0) > 0$ . Thus, we include a wide spectrum of specifications of private gains, including the case where no private gains are available ( $\delta(\gamma) = 0$ ). Note that these assumptions guarantee inefficient extraction of private benefits, since  $\delta(\gamma) < \gamma$  for all  $\gamma > 0$ . Thus, the specification of Burkart et al.'s for the extraction of private gains corresponds to the special case of ours where  $\delta'(0) = 1.^9$ 

An incumbent manager chooses to divert profits to private gains as to maximize his payoff:

$$\max_{\gamma \in [0,1]} \left[ \delta\left(\gamma\right) + (1-\gamma)r_1 \right] a_1. \tag{1}$$

Therefore, the optimal choice of  $\gamma$  is given by

$$\gamma^* = \begin{cases} h(r_1) & \text{if } \delta'(0) > r_1 \\ 0 & \text{if } \delta'(0) \le r_1, \end{cases}$$
(2)

where  $h \equiv (\delta')^{-1}$ . Thus, for sufficiently small  $r_1$ , the manager diverts profits for his private gain. Since  $\delta(\gamma) < \gamma$ , this introduces agency costs. Notice that, given our assumptions, the (privately) optimal share of profits that the manager extracts does not depend on his ability, but is non-increasing in his ownership share. Moreover,  $\gamma^* = 0$  if  $r_1 = 1$  and, unless  $\delta(\gamma) = 0$ ,  $\gamma^* = 1$  if  $r_1 = 0.10$  Thus, agency costs are absent for all  $r_1$  only if  $\delta(\gamma) = 0$ .

<sup>&</sup>lt;sup>9</sup>See Shleifer and Wolfenzon (2002) for an alternative, but equivalent, modeling approach to inefficient extraction of private benefits.

<sup>&</sup>lt;sup>10</sup> If  $\delta(\gamma) = 0$ , the manager with  $r_1 = 0$  is indifferent between any level of private extraction.

The timing of events is as follows. There is an initial, exogenous allocation of control and of ownership,  $\mathbf{r} = \{r_1, r_2, ..., r_n\}$ .<sup>11</sup> After ownership and control are allocated, each shareholder learns his ability. They then write a multilateral contract to reallocate ownership and control among themselves. Under the rules of this contract, they implement a new allocation of shares and control rights. Finally, production takes place and the firm generates profit (gross of agency costs)  $\pi = a_j$ , where j is the index of the (potentially) new manager in charge. We refer to this sequence of events as the operation of a *restructuring mechanism*, which is a procedure to change the original structure of ownership and control. We refer to the set of available restructuring mechanisms as constituting the *market for corporate control*.

#### 3.2 The restructuring problem

Suppose that there was no scope for agency costs  $(\delta(\gamma) = 0)$  and no private information. It is then obvious that, without direct costs of restructuring, the first-best efficient allocation can always be achieved, with control being assigned to the most talented shareholder regardless of the initial ownership and control structures. This is, in fact, a simple illustration of the Coase Theorem. Unlike previous models of the market for corporate control, ours does not restrict the set of contracts available to shareholders—they are free to restructure ownership and control as they wish. Thus, it is natural that ex ante ownership will be irrelevant and efficient restructuring will always be achieved in our setup in the absence of direct transaction costs. The expected surplus from restructuring in this case is the first best,  $V^{fb} \equiv E(\tilde{a} - a_1)$ , where  $\tilde{a} \equiv \max\{a_1, ..., a_n\}$ . The surplus from restructuring under asymmetric information is thus necessarily no higher than  $V^{fb}$ .

However, even when shareholders' talent is private information, fully-efficient restructuring remains possible. We illustrate this possibility using a particularly simple mechanism.

**Definition 1** The trivial restructuring mechanism has the following characteristics. After learning his ability  $a_i$ , each shareholder simultaneously announces his type. The mechanism assigns control to the agent who reports the highest ability  $\tilde{a}$ , while the ownership structure **r** remains intact throughout.

It is immediate to see that the trivial mechanism yields a Bayesian-Nash equilibrium, where all shareholders truthfully report their abilities.<sup>12</sup> This mechanism implements the first-best allocation of control, the participation constraints of all shareholders are met and the mechanism has a balanced budget. Thus, adverse selection per se is not a problem for efficient restructuring, as long as contracts are complete.

<sup>&</sup>lt;sup>11</sup>We adopt the convention that **bold** variables represent vectors.

<sup>&</sup>lt;sup>12</sup>This actually yields a Groves equilibrium, where truthful reporting is a *dominant* strategy if each  $r_i > 0$ .

Now let the manager have the power to divert some corporate resources to his private consumption. Generally, this will happen if internal control mechanisms have failed.<sup>13</sup> Nevertheless, if information about abilities were not private, it would be simple to rearrange ownership and control to eliminate agency costs completely.

Thus, notwithstanding direct costs of restructuring, implementing an efficient allocation of ownership and control is difficult only if *both* private information (adverse selection) and agency costs (moral hazard) are present. In that case, since the manager's incentives to divert company profits are stronger, the smaller is his ownership share, the problem of value maximization requires not only assigning the right person to management, but also making sure that this person's equity stake is large enough that he does not divert profits. The trivial mechanism can then be expected to yield  $V^{fb}$  only if the initial ownership share of each shareholder is sufficiently high to preclude him from diverting profits if he happens to become the new manager. If initial ownership shares do not satisfy that requirement (and they typically do not), ownership will need to be reassigned in any efficient restructuring mechanism to guarantee a sufficiently high manager's share.

#### **3.3** Mechanisms for efficient allocation of ownership and control

Here we start analyzing how efficient restructuring can be achieved when both adverse selection and moral hazard problems are present.

A corporation  $\langle \mathbf{r}, \mathbf{F}, \delta \rangle$  is fully characterized by its ex ante ownership structure  $\mathbf{r}$ , by the distribution of managerial abilities  $\mathbf{F}$  and by how private benefits may be extracted, represented by  $\delta$ . Our problem is the efficient restructuring of the ownership and control of such corporations. Using the revelation principle, it is without loss of generality to restrict attention to direct revelation mechanisms in which shareholders simultaneously report their types  $\mathbf{a} = \{a_1, ..., a_n\}$  and the mechanism determines (1) the new control structure  $\mathbf{c}(\mathbf{a}) = \{c_1, ..., c_n\}$ ; (2) the new ownership structure  $\mathbf{s}(\mathbf{a}) = \{s_1, ..., s_n\}$ ; and (3) transfer payments to shareholders  $\mathbf{t}(\mathbf{a}) = \{t_1, ..., t_n\}$ . We assume that  $c_i \in \{0, 1\}$ , where  $c_i = 1$  implies that shareholder *i* has control (so that  $\pi = a_i$ ) and  $c_i = 0$  implies that he does not have control. We call  $\langle \mathbf{c}, \mathbf{s}, \mathbf{t} \rangle$  a restructuring mechanism.

We restrict attention to mechanisms that are budget balanced. This requires

$$\begin{cases} \sum_{\substack{i=1\\n}}^{n} c_i \left( \mathbf{a} \right) = 1 \\ \sum_{\substack{i=1\\i=1}}^{n} s_i \left( \mathbf{a} \right) = 1 \\ \sum_{\substack{i=1\\i=1}}^{n} t_i \left( \mathbf{a} \right) = -K, \end{cases}$$
(3)

<sup>&</sup>lt;sup>13</sup>For example, the manager may have succeeded in effectively capturing the board of directors.

where  $K \ge 0$  is constant and represents the exogenous direct cost of restructuring that must be borne by the shareholders. This might consist of costs arising from regulations, trading costs, raising funds to place a takeover bid, etc. In a world with no exogenous transaction costs, K = 0.

A necessary condition for a mechanism to be expost efficient is that it allocates control according to  $^{14}$ 

$$c_i = \begin{cases} 1 & \text{if } a_i = \tilde{a} \\ 0 & \text{if } a_i < \tilde{a}. \end{cases}$$

$$\tag{4}$$

However, this condition is not sufficient for the mechanism to yield efficiency. It must also preclude agency costs ex post. Specifically, the ex post manager must have a sufficiently large ownership share that he does not divert profits. Letting  $s_i^{c_i}$  be the ownership share of partner *i* conditional on his control  $c_i$ , condition (2) implies that the following necessary condition must hold as well:

$$s_i^1 \ge \delta'(0) \equiv \underline{s}.\tag{5}$$

Thus, efficiency alone does not impose any constraint on the shares  $s_i^0$  received by shareholder *i* when *i* does not assume control, but it does require that shareholder *i*'s controlling share,  $s_i^1$ , must exceed <u>s</u>.<sup>15</sup>

**Lemma 1** The expost share rule in any efficient restructuring mechanism must satisfy, for every  $i \in N$ ,

$$s_{i}^{c_{i}} = \begin{cases} s_{i}^{1} \in [\underline{s}, 1] \\ s_{i}^{0} \in [0, 1]. \end{cases}$$
(6)

We allow  $s_i^0$  to be idiosyncratic across shareholders but assume that  $s_i^0$  is independent of the identity of shareholder  $j \neq i$  who is assigned control. This assumption greatly simplifies the analysis and is without loss of generality in the current setting, where all shareholders have the same ability distribution F.

Budget balance and Lemma 1 impose the following restrictions on **s**. This and the subsequent proofs are in the Appendix unless they are very short.

Lemma 2 In any efficient restructuring mechanism, budget balance implies:

1. 
$$\sum_{i \neq 1} s_i^1 = (n-1) \left(1 - s_1^0\right) - (n-2) \left(1 - s_1^1\right) \ge (n-1)\underline{s}$$
 and  
2.  $\sum_{i \neq 1} \left(s_i^1 - s_i^0\right) = (n-1) \left(s_1^1 - s_1^0\right).$ 

<sup>&</sup>lt;sup>14</sup>Since types are continuous, the case where two shareholders tie for highest type is a zero probability event and can be ignored.

<sup>&</sup>lt;sup>15</sup>It is possible that other, exogenous forces, may require a minimum managerial ownership share as well. For instance, <u>s</u> could be affected by legal or institutional forces that govern the required minimum share necessary for acquiring control. For instance, if a corporation is *required* to dissolve, then <u>s</u> = 1.

#### **3.4** Voting structure

All of our results are derived under unanimity voting. This rule maximizes the likelihood of finding fully efficient restructuring mechanisms, because it completely eliminates the free-riding effect identified by Grossman and Hart (1980). Thus, whenever fully-efficient restructuring is possible, it must be possible under a unanimity rule. In this sense, our approach is without loss of generality. However, it is important to note that, when it is not possible to achieve the first-best, the unanimity rule is inefficient, because it may block Pareto-improving changes that lead to second-best outcomes.

#### 3.5 Incentive compatibility and individual rationality

Let  $-i \equiv N \setminus i$ ,  $\mathbf{a}_{-i} \equiv \{a_1, ..., a_{i-1}, a_{i+1}, ..., a_n\}$  and  $E_{-i}\{.\}$  denote the expectation operator with respect to  $\mathbf{a}_{-i}$ . Under the mechanism, shareholder *i* expects to receive transfer  $T_i(a_i) \equiv E_{-i}\{t_i(\mathbf{a})\}$ . On top of the transfer, he expects to be allocated control with some probability and expects to own some shares ex post. Let  $G \equiv F^{n-1}$  be the distribution of the largest of the other shareholders' abilities,  $\tilde{a}_{-i} \equiv \max\{a_1, ..., a_{i-1}, a_{i+1}, ..., a_n\}$ , with corresponding density *g*. Thus, under an ex post efficient mechanism  $\langle \mathbf{c}, \mathbf{s}, \mathbf{t} \rangle$ , shareholder *i* has expected utility

$$U_i(a_i) = s_i^1 a_i G(a_i) + s_i^0 \int_{a_i}^{\bar{a}} u dG(u) + T_i(a_i).$$
<sup>(7)</sup>

In contrast, if no mechanism were put into place, the initial ownership and control structure would be kept intact and the shareholders would expect to receive the following dividends:

$$\begin{cases} \overline{U}_1(a_1) = \beta(r_1)a_1 \\ \overline{U}_i(a_i) = (1 - \gamma^*(r_1))r_i\mu \text{ for all } i \in \{2, ..., n\}, \end{cases}$$
(8)

where  $\beta(r_1) \equiv \delta(\gamma^*(r_1)) + (1 - \gamma^*(r_1))r_1$ . These payoffs form the basis for (interim) *individually* rational participation, which we require the mechanism to satisfy. That is, unless

$$U_i(a_i) \ge \overline{U}_i(a_i),\tag{9}$$

shareholder i prefers the original ownership and control structure to remain intact and (we assume) can effectively block any transfers of ownership and control.

Because types (abilities) are private information, a mechanism must be *incentive compatible* to yield allocative efficiency:

$$U_{i}(a_{i}) \geq U_{i}(b) = s_{i}^{1}a_{i}G(b) + s_{i}^{0}\int_{b}^{\bar{a}} u dG(u) + T_{i}(b)$$
(10)

for all  $i \in N$ ,  $a_i, b \in [\underline{a}, \overline{a}]$ . That is, conditional on all other shareholders declaring their types truthfully to the mechanism, shareholder *i* must find it optimal to do the same. The next lemma expresses this condition in a more convenient form. **Lemma 3** A restructuring mechanism  $\langle \mathbf{c}, \mathbf{s}, \mathbf{t} \rangle$  that assigns control to the shareholder with the highest announced ability is incentive compatible if and only if, for every  $i \in N$  and for all  $a_i, b \in [\underline{a}, \overline{a}]$ ,

$$T_i(a_i) - T_i(b) = -(s_i^1 - s_i^0) \int_b^{a_i} u dG(u).$$
(11)

Incentive compatibility guarantees that a mechanism that assigns control to the shareholder with the highest announced ability yields the efficient assignment of control, as in (4). To see the intuition for this result, consider  $U_i(b)$  as given in expression (10). The effect of a marginal increase in b on  $U_i(b)$  is  $dU_i(b)/db = g(b) \left[s_i^1 a_i - s_i^0 b\right] + dT_i(b)/db$ . Absent transfers, then,  $U_i(b)$ is maximized when  $b = a_i s_i^1/s_i^0$ . Thus, if the mechanism is such that shareholder *i* expects to receive more shares if he gains control than if he does not (i.e., if  $s_i^1 > s_i^0$ ), without transfers this shareholder would have an incentive to misrepresent himself as having a somewhat higher type. To counteract this incentive and induce shareholder *i* to reveal his type truthfully, the transfers under the mechanism must then be decreasing in ability, as shown in (11). Similarly, if  $s_i^0 > s_i^1$ , transfers must be increasing in ability.<sup>16</sup>

Incentive compatibility also completely pins down the *shapes* of the transfer functions, leaving undetermined only the set of fixed components in  $\{T_i(a_i)\}$ . This implies that, with the help of external subsidies to guarantee participation, one can always implement an incentive compatible mechanism. On the other hand, if there are no external subsidies, participation can only be guaranteed if the expected gains from restructuring are sufficiently large relative to the informational rents required to induce truth-telling. In reality, external subsidies are unlikely to be available. On the contrary, the implementation of a mechanism is likely to generate additional administrative costs, captured here by K.

Now, notice that an incentive compatible mechanism that yields the expost efficient assignment of control will also be individually rational if and only if the "worst-off type" of each shareholder i, denoted by  $a_i^*$ , is willing to participate in the mechanism. This type is defined so that his net utility from the mechanism is the minimum among all possible types:

$$a_i^* \in \arg\min_{a_i \in [\underline{a}, \overline{a}]} \{ U_i(a_i) - \overline{U}_i(a_i) \}.$$
(12)

Since all other possible types of shareholder i expect to gain at least as much as type  $a_i^*$  under the mechanism, it is clear that individual rationality (IR) constraints require the expected efficiency gains from transferring corporate control to the most able manager to be large enough to allow the mechanism to "bribe" the worst-off types of every shareholder. The next two lemmas use this fact to characterize the participation constraints of incentive compatible mechanisms.

<sup>&</sup>lt;sup>16</sup>Notice also the contrast with the literature on dissolving partnerships, where the assumption that  $s_i^1 = 1$  and  $s_i^0 = 0$  implies that each shareholder would always have an incentive to announce  $b = \overline{a}$  in the absence of transfers.

**Lemma 4** An incentive compatible restructuring mechanism  $\langle \mathbf{c}, \mathbf{s}, \mathbf{t} \rangle$  that assigns control to the shareholder with the highest announced ability is individually rational for shareholder 1 if and only if

$$T_1(a_1^*) \ge \max\{\left(\beta(r_1) - s_1^1\right)\bar{a}, 0\} - s_1^0 \int_{a_1^*}^{\bar{a}} u dG(u),$$
(13)

where  $a_1^* = G^{-1}\left(\min\left\{\frac{\beta(r_1)}{s_1^1}, 1\right\}\right)$  except when  $s_1^1 = \beta(r_1) = 0$ , in which case  $a_1^*$  is any element in  $[\underline{a}, \overline{a}]$ .

Lemma 4 identifies the worst-off type of manager and characterizes individual rationality for him. It has important implications. Note first that whenever  $s_1^0 > 0$ , the mechanism allows the original manager to keep shares of the firm even if he loses control. The higher is  $s_1^0$ , therefore, the "safer" the mechanism is for the manager. Accordingly, the transfer that his worst-off type requires to participate is lower, the higher is his "safeguard"  $s_1^0$ .

To reduce management entrenchment, generous severance pay packages can be optimal (Almazan and Suarez 2003). Our model has this same feature: we interpret  $s_1^0$  as a golden parachute, such as options granted to a departing manager in case of a change in control. Because granting golden parachutes is a way to let the departing manager share some of the efficiency gains of his replacement, it reduces his opposition to a change in control.

The next lemma characterizes the individual rationality constraints for the non-controlling shareholders.

**Lemma 5** An incentive compatible restructuring mechanism  $\langle \mathbf{c}, \mathbf{s}, \mathbf{t} \rangle$  that assigns control to the shareholder with the highest announced ability is individually rational for shareholder  $i \in \{2, ..., n\}$  if and only if

$$T_i(a_i^*) \ge (1 - \gamma^*(r_1))r_i\mu - s_i^0 \int_{a_i^*}^{\bar{a}} u dG(u),$$
(14)

where  $a_i^* = \underline{a}$  unless  $s_i^1 = 0$ , in which case  $a_1^*$  is any element in  $[\underline{a}, \overline{a}]$ .

**Proof.** For  $i \in \{2, ..., n\}$ , net utility  $U_i(a_i) - \overline{U}_i(a_i)$  is strictly convex with first derivative  $s_i^1 G(a_i)$ . Thus, for  $s_i^1 > 0$ , it is increasing in  $a_i$  for all  $a_i \ge \underline{a}$ . Hence  $a_i^* = \underline{a}$ . Participation is individually rational for all types of shareholder  $i \in \{2, ..., n\}$  if and only if  $U_i(\underline{a}) = s_i^1 \underline{a} G(\underline{a}) + s_i^0 \int_{\underline{a}}^{\overline{a}} u dG(u) + T_i(\underline{a}) \ge (1 - \gamma^*(r_1))r_i\mu$ , which is equivalent to condition (14).

If  $s_i^1 = 0$ , then  $U_i(a_i) - \overline{U}_i(a_i)$  has a first derivative of zero. Hence all types have the same net utility. For any  $a_i^* \in [\underline{a}, \overline{a}]$ , we must have  $T_i(a_i^*) \ge (1 - \gamma^*(r_1))r_i\mu - s_i^0 \int_{a_i^*}^{\overline{a}} u dG(u)$ .

The intuition behind this result is simple. The worst-off type of a non-controlling shareholder has the lowest possible managerial ability  $\underline{a}$ .<sup>17</sup> Such a shareholder knows that, under the mechanism,

 $<sup>^{17}</sup>$ Except where we mention them specifically, we ignore in the text the uninteresting multiplicity of worst-off types arising in the boundary cases of lemmas 4 and 5.

he will end up with a payoff of  $s_i^0 \int_{\underline{a}}^{\overline{a}} u dG(u)$ , while he expects to receive  $(1 - \gamma^*(r_1))r_i\mu$  if he does not participate. Thus, he participates only if he expects to receive a monetary transfer that is at least as large as the difference between those two values.

Lemma 5 illustrates an effect that is similar to the free-riding behavior of non-controlling shareholders analyzed by Grossman and Hart (1980): non-controlling shareholders, who do not contribute for production, will hold on to their shares unless they are paid a premium over their current value. In Grossman and Hart's specific bidding game, the price paid per share had to be at least equal to the share price after the change in control. Because we focus on the set of all implementable efficient mechanisms, we find that free-riding by non-controlling shareholders can be mitigated by means of considerably smaller price premia. Due to the unanimity rule, this premium is zero. But even when outside shareholders receive no rents from selling their shares, unless they get at least the value of their shares in the status quo case, they will block efficiency-enhancing changes of control. Thus, participation of outside shareholders remains a problem, even when they are pivotal.

### 4 Conditions for Efficient Restructuring

We begin our analysis of the feasibility of efficient restructuring with a general characterization result. We then briefly discuss a key corollary, which foreshadows one of our main results: whereas the *level* of managerial ownership matters for efficiency, the *distribution* of ownership among non-controlling shareholders is irrelevant. For ease of explanation, we start by taking the ex post ownership structure **s**, or share rule, as given. We then derive the optimal share rule later in the section.

#### 4.1 Exogenous ex post share rules

Note that conditions (4), (6), (11), (13) and (14) are necessary and sufficient to ensure that there is an efficient restructuring mechanism that is both individually rational and incentive compatible for all types of all shareholders. We use them to characterize the set of all ex ante and ex post structures for which an efficient restructuring mechanism achieves budget balance.

**Proposition 1** A corporation  $\langle \mathbf{r}, \mathbf{F}, \delta \rangle$  can be efficiently restructured with an incentive compatible, individually rational mechanism with share rule **s** if and only if **s** satisfies lemmas 1 and 2 and

$$V(\mathbf{r}, \mathbf{s}) \ge K,\tag{15}$$

where

$$V(\mathbf{r}, \mathbf{s}) = \sum_{i=1}^{n} \left[ s_{i}^{1} \int_{a_{i}^{*}}^{\bar{a}} u dG(u) - \left( s_{i}^{1} - s_{i}^{0} \right) \int_{\underline{a}}^{\bar{a}} F(u) u dG(u) \right] - \max\{ \left( \beta(r_{1}) - s_{1}^{1} \right) \bar{a}, 0 \} - (1 - \gamma^{*}(r_{1}))(1 - r_{1})\mu \quad (16)$$

and  $\{a_i^*\}$  are as defined in lemmas 4 and 5.

Condition (15) compares  $V(\mathbf{r}, \mathbf{s})$ , the expected gains from trade minus the informational rents generated by restructuring mechanism  $\langle \mathbf{c}, \mathbf{s}, \mathbf{t} \rangle$ , to its operating costs  $K \geq 0$ . Whenever condition (15) holds, any "wrong" initial allocation of control can be efficiently corrected by a mechanism with share rule  $\mathbf{s}$ . On the other hand, if it is not met, then mechanism  $\langle \mathbf{c}, \mathbf{s}, \mathbf{t} \rangle$  will either not be able to achieve ex post efficiency or will require an outside subsidy.

To analyze the consequences of Proposition 1, it proves convenient to adopt the following definition.

**Definition 2** For restructuring mechanism  $\langle \mathbf{c}, \mathbf{s}, \mathbf{t} \rangle$ , the **net surplus** of the mechanism is  $V(\mathbf{r}, \mathbf{s})$ . Alternatively, the **net friction** of the mechanism is  $V^{fb} - V(\mathbf{r}, \mathbf{s})$ .

Studying properties of the net surplus V allows us to characterize the effects of ownership  $\mathbf{r}$  on the efficiency of the market for corporate control. A larger V implies that efficient restructuring is possible for larger values of K and yields a surplus that is closer to  $V^{fb}$ . Intuitively,  $V^{fb} - V(\mathbf{r}, \mathbf{s})$  represents friction in the market for corporate control because it measures the extent to which informational rents reduce the surplus available to induce participation by the pivotal worst-off types of shareholders.

Now, note that the distribution of ownership rights among non-controlling shareholders does not enter (16).

**Corollary 1** The initial distribution of shares among non-controlling shareholders,  $(r_2, ..., r_n)$ , does not affect the net surplus,  $V(\mathbf{r}, \mathbf{s})$ .

This is a simple yet very strong result. It implies, in particular, that whether non-controlling shareholders are initially large or small is irrelevant for whether a particular share rule implements efficient restructuring.

#### 4.2 Optimal mechanisms

The characterization result of Proposition 1 tells us little about the types of mechanisms corporations will actually use to restructure. Assuming shareholders can bargain together and implement their decisions effectively, they will choose **s** to maximize ex post firm value. When efficient restructuring is possible, they will limit themselves to the set of share rules capable of achieving  $V^{fb}$ . In this subsection, we give necessary and sufficient conditions for whether this set is non-empty, that is, for whether corporation  $\langle \mathbf{r}, \mathbf{F}, \delta \rangle$  can be efficiently restructured. The key instrument is a particular share rule, defined next, which is in this set if and only if it is non-empty.

**Definition 3** The optimal share rule  $s(r_1)$  satisfies

$$\mathbf{s}(r_1) \in \arg\max_{\mathbf{s}\in B} V(r_1, \mathbf{s}),$$

where B is the set of all share rules that satisfy the balanced budget conditions in lemmas 1 and 2. An **optimal restructuring mechanism** is an incentive compatible, individually rational, ex post efficient mechanism with ex post share rule  $\mathbf{s}(r_1)$ .

Note that, in light of Corollary 1, we replace  $\mathbf{r}$  by  $r_1$  in the argument of V. The continuity of the value function,  $V(r_1, \mathbf{s}(r_1))$ , follows from the theorem of the maximum, while the existence of optimal restructuring mechanisms follows from the continuity of  $V(r_1, \mathbf{s})$  with respect to  $\mathbf{s}$  and from the fact that B is a non-empty compact set.

The next proposition describes the properties and pivotal nature of the optimal share rule.

**Proposition 2** A corporation  $\langle \mathbf{r}, \mathbf{F}, \delta \rangle$  can be efficiently restructured if and only if  $V(r_1, \mathbf{s}(r_1)) \geq K$ , where the optimal share rule  $\mathbf{s}(r_1)$  is unique and requires:

- *i.*  $s_i^0(r_1) = \frac{1-s_1^1(r_1)}{n-1}$  for all  $i \neq 1$ .
- ii.  $s_i^1(r_1) = \underline{s}$  for all  $i \neq 1$ .
- *iii.*  $s_1^0(r_1) = 1 \underline{s} \frac{n-2}{n-1} \left[ 1 s_1^1(r_1) \right].$

 $iv. \ s_1^1(r_1) = \begin{cases} 0 \quad if \quad \underline{s} = 0 \ and \ r_1 = 0 \\\\ \underline{s} \quad if \quad \underline{s} > 0 \ and \ \overline{a} - \int_{G^{-1}\left(\frac{\beta(r_1)}{\underline{s}}\right)}^{\overline{a}} G(u) du - \int_{\underline{a}}^{\overline{a}} u dF(u)^n < 0 \\\\ 1 \quad if \quad \overline{a} - \int_{G^{-1}(\beta(r_1))}^{\overline{a}} G(u) du - \int_{\underline{a}}^{\overline{a}} u dF(u)^n > 0. \end{cases}$ 

Otherwise,  $s_1^1(r_1) \in [\underline{s}, 1]$  is interior and satisfies

$$\int_{a_w}^{\bar{a}} G(u) du = \bar{a} - \int_{\underline{a}}^{\bar{a}} u dF(u)^n, \tag{17}$$

where  $a_w \equiv G^{-1}\left(\frac{\beta(r_1)}{s_1^1(r_1)}\right)$ . In any case,  $s_1^1(r_1) \ge r_1$ , with  $s_1^1(r_1) > r_1$  for  $r_1 \in (0,1)$ .

The optimal share rule treats the manager and the non-controlling shareholders quite differently.<sup>18</sup> The reason is that the identity of the worst-off type of manager depends crucially on his "winning" share  $s_1^1$ , while the identities of the worst-off types of non-controlling shareholders do not depend on their ex post shares at all.

Hypothetically, if there were no budget balance requirements, increasing any "losing" share,  $s_i^0$ , would unambiguously increase V, as such a change clearly increases the sum of expected transfers to the worst-off types (see equation (16)). Given budget balance, however, the optimal share rule increases those values in  $\{s_i^0\}$  which have a greater positive effect on V. An increase in the sum of "losing" shares for the non-controlling shareholders,  $\sum_{i\neq 1} s_i^0$ , lowers the "winning" share of the original manager,  $s_1^1$ . This implies that the worst-off type of the latter,  $a_1^* = G^{-1}\left(\frac{\beta(r_1)}{s_1^1}\right)$ , has a higher ability. As a result, the net surplus from restructuring from the perspective of the worst-off type of manager is reduced. In contrast, an increase in the manager's golden parachute,  $s_1^0$ , does not affect the identities of the worst-off types of other shareholders and, therefore, does not affect the surplus from restructuring. Accordingly, the optimal share rule specifies  $s_1^0(r_1) \ge s_i^0(r_1)$ .<sup>19</sup> Similarly, because a larger  $s_1^1$  yields a smaller  $a_1^*$  (so that this type expects greater gains from restructuring), while larger values of  $\{s_i^1\}_{i\neq 1}$  do not change  $\{a_i^*\}_{i\neq 1}$ , the optimal share rule specifies  $s_1^1(r_1) \ge s_i^1(r_1)$ .<sup>20</sup>

Note that there is, essentially, an "optimal" worst-off type of manager  $a_w$ , given by condition (17).  $V(r_1, \mathbf{s})$  is concave in  $s_1^1$ , so when it is possible to choose  $s_1^1(r_1) \in [\underline{s}, 1]$  such that (17) holds, then  $a_1^* = a_w$  and we say that  $s_1^1(r_1)$  is *interior*. When  $\underline{s} > 0$  and  $r_1$  is small, the value of  $s_1^1$  that sets  $a_1^* = a_w$  may be infeasibly low, in which case the corner solution  $s_1^1(r_1) = \underline{s}$  arises (and  $a_1^* < a_w$ ). On the other hand, when  $r_1$  is large, the value of  $s_1^1$  that sets  $a_1^* = a_w$  may be infeasibly high, in which case  $s_1^1(r_1) = 1$  is optimal (and  $a_1^* > a_w$ ).<sup>21</sup>

While clearly having normative implications, the properties of the optimal share rule also have several positive implications for corporate restructuring. Key features of  $\mathbf{s}(r_1)$  are chosen precisely to ensure the manager's participation in restructuring. The golden parachute serves as a particularly effective tool—each of the non-controlling shareholders' winning shares is kept at <u>s</u> to ensure the largest possible  $s_1^0$ . Thus, our analysis indicates that there may be strong efficiency reasons for sweetening a severance package to convince a CEO to participate in a restructuring. This may also

<sup>&</sup>lt;sup>18</sup>While the optimal sharing rule,  $\mathbf{s}(r_1)$ , is unique, there will typically be a multiplicity of optimal mechanisms that yield  $V(r_1, \mathbf{s}(r_1))$ ). This is because in general there is a multiplicity of transfer rules  $\mathbf{t}$  consistent with incentive compatibility and individual rationality. The differences between these transfer rules are trivial, however—recall that incentive compatibility pins down the shapes of  $\{T_i(a_i)\}$ , leaving open only the sizes of the shift parameters  $\{T_i(a_i^*)\}$ .

<sup>&</sup>lt;sup>19</sup>Since any non-controlling shareholder receives an expost managerial share  $\underline{s}$ , set just large enough to preclude agency costs, we have that  $s_1^1(r_1) \ge s_i^1(r_1) = \underline{s}$ .

 $s_{1}^{20}s_{1}^{0}(r_{1}) \ge s_{i}^{0}(r_{1})$  implies that  $1 - \underline{s} - \frac{n-2}{n-1}[1 - s_{1}^{1}(r_{1})] \ge \frac{1}{n-1}[1 - s_{1}^{1}(r_{1})]$ , which holds as long as  $s_{1}^{1}(r_{1}) \ge \underline{s}$ .

<sup>&</sup>lt;sup>21</sup>The case  $r_1 = 0, \underline{s} = 0$  is special because the worst-off type  $a_1^*$  can be any  $a \in [\underline{a}, \overline{a}]$  for the optimal  $s_1^1(0) = 0$ .

help to explain why deposed CEOs frequently receive stock and/or stock options as part of their severance pay.

The optimal share rule has the property that restructuring does not lead to convergence of managerial ownership. If the manager remains in control after the operation of an optimal mechanism, his ownership stake will typically increase. However, if he is deposed, the new manager's ownership is set at  $\underline{s}$ , which will often be smaller than  $r_1$ . Thus, we should expect to see CEOs with long tenures increase their ownership stakes through time, but should see managerial ownership reduced when a long-tenured CEO is replaced.

#### 4.2.1 Complete dissolution

The nature of optimal restructuring also contributes to the literature on *partnerships*. In past work, started by the seminal contribution of Cramton et al. (1987), the typical nature of payoffs to the partners is different than to our shareholders. Most notably, control is not modeled, so agency costs do not emerge and complete dissolution is the only ex post share rule that yields an efficient outcome. Here, by contrast, ex post efficiency depends on the assignment of control and on whether ex post managerial ownership precludes agency costs. In this context, complete dissolution is typically suboptimal.

**Corollary 2** The optimal share rule specifies complete dissolution  $(s_i^1 = 1 \text{ and } s_i^0 = 0 \text{ for all } i)$  if and only if agency costs are extreme,  $\delta'(0) = 1$ .

**Proof.** We show in the proof of Proposition 2 that  $V(r_1, \mathbf{s})$  is increasing in  $s_1^0$ . Therefore, unless  $\underline{s} = 1$ , the optimal  $s_1^0$  is strictly positive, and  $s_i^1 < 1$  for at least one i = 1, ..., n. We know that  $\underline{s} = 1$  if and only if  $\delta'(0) = 1$ .

Thus, complete dissolution is optimal if and only if it is the only share rule that prevents agency costs ex post. It follows directly that, when agency costs are less than extreme, complete dissolution may fail to implement  $V^{fb}$  in situations where  $\mathbf{s}(r_1)$  will do so. In such situations, partnerships are unlikely to completely dissolve. Thus, our theory complements that of Cramton et al. (1987) by offering an explanation both for the persistence of a partnership structure and for the conditions under which dissolution may emerge endogenously.

#### 4.3 Ownership structure and the possibility of efficient restructuring

It follows directly from Corollary 1 that the possibility of efficient restructuring does not depend on the initial distribution of ownership among non-controlling shareholders, since the only characteristic of  $\mathbf{r}$  that affects the optimal share rule is  $r_1$ . While that distribution does affect whether the trivial mechanism implements efficient restructuring, as discussed earlier, it does not matter for the size of  $V(r_1, \mathbf{s}(r_1))$ .

In contrast, the ex ante level of managerial ownership,  $r_1$ , is a key determinant of  $V(r_1, \mathbf{s}(r_1))$ and, consequently, of the possibility of efficient restructuring. It determines the severity of the participation constraints of non-controlling shareholders, the ex ante level of agency costs, and the worst-off type of manager, both directly and through influencing  $\mathbf{s}(r_1)$  in an optimal mechanism. Thus, it has strong positive implications for firm value.

To begin analyzing the impact of  $r_1$ , we hold **s** fixed, as in Proposition 1. It is clear from (16) that there are three channels through which managerial ownership *directly* affects efficiency. First, agency costs aside, an increase in managerial ownership  $r_1$  slacks the outside shareholders' participation constraints, increasing  $V(r_1, \mathbf{s})$  by a factor of  $\mu$  at the margin. Larger initial shares for non-controlling shareholders (and thus lower initial managerial ownership) make it more expensive to induce the participation of low-ability non-controlling shareholders. Thus, low levels of initial managerial ownership may have adverse efficiency effects, because efficient transfers of control are less likely to be feasible when the minimum compensation for the non-controlling shareholder—as represented in the last term of equation (16)—is larger. We refer to this force as the *outside shareholder participation effect*: it becomes easier to induce the participation of outside shareholders in a mechanism that reallocates control as the initial stake in the hands of insiders increases.

However, when agency costs are present, an increase in  $r_1$  decreases perquisites  $\gamma^*(r_1)$ . This significantly alters the outside shareholder participation effect, because perquisite-taking shrinks the size of the aggregate participation constraint,  $(1 - \gamma^*(r_1))(1 - r_1)\mu$ , and makes it non-monotonic as a function of  $r_1$  (it is 0 for both  $r_1 = 0$  and  $r_1 = 1$ ). Indeed, when agency costs are accounted for, the outside shareholder participation effect is positive if and only if  $(1 - \gamma^*(r_1))(1 - r_1)\mu$  is decreasing in  $r_1$ .

Finally,  $r_1$  has a negative effect on  $V(r_1, \mathbf{s})$  because a higher  $r_1$  implies a higher worst-off type  $a_1^*$  for the original manager, which in turn reduces the expected gains from restructuring available to bribe that type under the mechanism. A larger  $a_1^*$  makes it harder to satisfy the IR constraint for the worst-off type of the incumbent manager. Intuitively, the worst-off type of the incumbent manager knows that he will get the least informational rent. Thus, his main incentive to participate is his expectation of sharing some of the efficiency gains through his expost ownership of shares  $s_1^1$  or  $s_1^0$ . But if his ability is high, these expected efficiency gains are small. Thus, as managerial ownership increases, so does management resistance to changes. In line with previous literature, we call this the management entrenchment effect.

Since this effect is unambiguously negative, it is clear that when the outside shareholder participation effect is also negative, V decreases with  $r_1$ . When the latter is positive, there is some ambiguity about whether it dominates the management entrenchment effect. These effects are easily summarized using the derivative of V with respect to  $r_1$ :

$$\frac{dV(r_1, \mathbf{s})}{dr_1} = (1 - r_1)\frac{d\gamma^*}{dr_1}\mu + (1 - \gamma^*(r_1))\left(\mu - a_1^*\right).$$
(18)

When no agency costs are present,  $\gamma^* = 0$  and this expression reduces to  $\mu - a_1^*$ . In this case, V is increasing in  $r_1$  if and only if the outside shareholder participation effect,  $\mu$ , dominates the management entrenchment effect,  $-a_1^*$ . When agency costs are present, these effects change because agency costs change with  $r_1$ . The first term in (18) is unambiguously negative, while the second term is negative whenever  $a_1^* > \mu$ . Thus, a sufficient condition for V being everywhere decreasing in  $r_1$  is that  $a_1^* > \mu$ .

Now consider how managerial ownership affects the possibility of efficient restructuring through its effects on the optimal share rule. We show that there is a level of managerial ownership  $\hat{r}_1 < 1$ such that, if ownership is greater than  $\hat{r}_1$ , the management entrenchment effect always dominates.

**Proposition 3** There is a  $\hat{r}_1 < 1$  such that  $V(r_1, \mathbf{s}(r_1))$  is strictly decreasing in  $r_1$  for  $r_1 > \hat{r}_1$ . Furthermore, if  $\delta(1) \ge \delta'(0)G(\mu)$ ,  $\hat{r}_1 = 0$ .

At an intuitive level, the sufficient condition for V to be decreasing everywhere is that the extraction of all private benefits is not "too inefficient" (relative to the extraction of small amounts), so that  $\frac{\delta(1)}{\delta'(0)}$  is sufficiently large. Interestingly, this always includes the case of no agency costs but, depending on  $G(\mu)$ , may not include the case of extreme agency costs.

Next, we define the set of ownership structures for which efficient restructuring is possible and prove several results related to the characterization of this set.

**Definition 4** Let  $\Phi = \{r_1 | V(r_1, \mathbf{s}(r_1)) \ge K\}$  be the set of all  $r_1$  for which efficient restructuring is possible.

#### **Proposition 4** If $\delta'(0) = 0$ and K = 0, then $\Phi = [0, 1]$ .

Thus, if there are no agency costs and no direct restructuring costs, efficient restructuring is always possible, because the trivial mechanism will always work. This result yields a subtle, yet important contribution to the broader mechanism design literature. To see this, note that bilateral exchange is a special type of restructuring that emerges in our model when ex ante managerial ownership is extreme ( $r_1 = 1$ ). Myerson and Satterthwaite (1983) show that efficient exchange under budget balance is impossible for symmetric continuous types of the buyer and seller. However, they do not explicitly model the exchange of control. We show that, when this is considered, and the asset in question is divisible, the constraint imposed (on efficient exchange) by extreme ownership changes significantly. The following corollary recasts the Myerson-Satterthwaite impossibility result. **Corollary 3** Let  $r_1 = 1$ . In this case, the corporation can be efficiently restructured if and only if  $\delta(\gamma) = 0$  and K = 0, and using the share rule  $s_1^1 = s_1^0 = 1$  and  $s_i^1 = s_i^0 = 0$  for  $i \neq 1$ .

**Proof.** We know from Proposition 2 that the optimal mechanism assigns  $s_1^1(1) = 1$ . Thus,  $V(r_1, \mathbf{s}(1))$  collapses to

$$V(1, \mathbf{s}(1)) = -\underline{s}(n-1) \int_{\underline{a}}^{\overline{a}} [1 - F(u)] G(u) du$$

which is strictly negative unless  $\underline{s} = 0$ . If  $\underline{s} = 0$ ,  $V(1, \mathbf{s}(1)) = 0$  as well, and efficient restructuring is impossible unless K = 0.

Hence, it is actually possible to efficiently restructure when  $r_1 = 1$ , but if and only if the trivial mechanism works (that is, if and only if the seller can retain full ownership when keeping or surrendering control) and the complete separation of control from ownership introduces no agency costs and entails no direct costs of restructuring. When this holds,  $\Phi = [0, 1]$  from Proposition 4. Note that, for  $\underline{s} = 1$  and n = 2,  $V(1, \mathbf{s}(1))$  collapses to the minimum outside subsidy required to implement efficient bilateral exchange in the Myerson-Satterthwaite setting.<sup>22</sup> Indeed, their setting can be interpreted as a nested, special case of our model when  $r_1 = 1$  and  $\delta'(0) = 1$ . Since  $V(r_1, \mathbf{s})$  is increasing in  $s_1^0$ , it follows directly that, when  $\delta'(0) < 1$  and both control and ownership are tradeable, efficient bilateral exchange requires a smaller outside subsidy than Myerson and Satterthwaite identified, so long as the "seller" (manager) retains some ownership shares when the "buyer" (a non-controlling shareholder) assumes control ex post.

The next two results (which are corollaries of Proposition 3) show that efficient restructuring is typically more difficult to achieve when managerial ownership is high.

**Corollary 4** If  $\delta'(0) > 0$ , then  $\Phi = [0, r'_1]$ , with  $r'_1 < 1$ , if  $\delta(1) \ge \delta'(0)G(\mu)$  and  $\Phi$  is non-empty.

**Proof.** When  $r_1 = 1$  we have

$$V(1, \mathbf{s}(1)) = -\delta'(0) (n-1) \int_{\underline{a}}^{\overline{a}} [1 - F(u)] G(u) du,$$

which is negative (and thus lower than K) if and only if  $\delta'(0) > 0$ . Since  $\hat{r}_1 = 0$  when  $\delta'(0) > 0$ ,  $\delta(1) \ge \delta'(0)G(\mu)$ , and  $\Phi$  is non-empty, continuity and monotonicity of V yield the existence of  $r'_1 \in [0, 1)$ .

If there are agency costs, efficient restructuring may not be possible for any  $r_1$ . This corollary shows, however, that if there are some levels of ownership for which efficient restructuring is possible and extraction of all private benefits is not too inefficient, then we can be quite precise with our

 $<sup>^{22}</sup>$ See Myerson and Satterthwaite (1983, p. 272, equation (7)).

characterization of  $\Phi$ : efficient restructuring is possible if and only if  $r_1 \leq r'_1 < 1$ . Thus, sufficiently high levels of managerial ownership preclude efficient restructuring.

The following corollary strengthens the sufficient condition for monotonicity of V in Proposition 3 by showing that the management entrenchment effect always dominates when the number of shareholders is "large."

**Corollary 5** For sufficiently large n,  $\hat{r}_1 = 0$  and  $\Phi = [0, r'_1]$  if it is non-empty, where  $r'_1 < 1$ .

**Proof.** Clearly,  $\hat{r}_1 = 0$  for all n if  $\delta'(0) = 0$ . If  $\delta'(0) > 0$ , then  $V(r_1, \mathbf{s}(r_1))$  is strictly decreasing in

 $r_1$  if  $\frac{\delta(1)}{\delta'(0)} \ge G(\mu) = F(\mu)^{n-1}$ . Since  $\mu < \bar{a}$ , the latter expression decreases with n and has  $\lim_{n \to \infty} F(\mu)^{n-1} = 0 < \frac{\delta(1)}{\delta'(0)}$ , completing the proof.

Notice that this result holds for any private extraction function  $\delta(\gamma)$ . Thus, the characterization result for  $\Phi$  is quite general.

This set of results puts the conflict between the moral hazard problem and the adverse selection problem in striking relief. A low level of ex ante managerial ownership is bad for incentives but good for the operation of the market for corporate control, so it aggravates the moral hazard problem but mitigates the adverse selection problem. For sufficiently high  $r_1$ , when the market for control fails to yield efficiency, second-best alternatives dictate (sometimes) keeping a less-qualified manager who will not divert profits or recruiting a more qualified manager who will extract some private benefits. As we have shown, while each problem in isolation can be overcome through the market for corporate control, private benefits and asymmetric information *jointly* create endogenous frictions which, in some cases, cannot be overcome. Most importantly, they make managerial ownership the crucial determinant of the efficiency of this market and, hence, a key determinant of firm value.

#### 5 An Example

To illustrate our results, consider an example where shareholder abilities are distributed uniformly, K = 0 and

$$\delta(\gamma) = \alpha \left(\gamma - \frac{\gamma^2}{2}\right). \tag{19}$$

Note that  $\alpha$  serves as an index of the severity of agency costs. We have

$$\gamma^*(r_1) = \begin{cases} 1 - \frac{r_1}{\alpha} & \text{if } r_1 < \alpha \\ 0 & \text{if } r_1 \ge \alpha, \end{cases}$$

so that  $\underline{s} = \alpha$ . Substituting into (16) for the uniform distribution (so that  $G(a) = a^{n-1}$ ) and using Proposition 2, we can solve for the optimal share rule:<sup>23</sup>

<sup>&</sup>lt;sup>23</sup>We omit the case where  $r_1 = \underline{s} = 0$ .

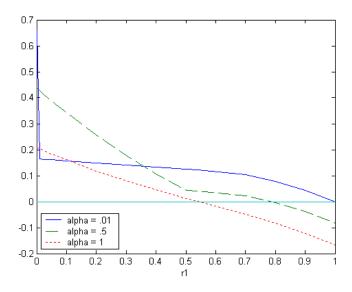


Figure 1: Net Surplus, n = 2, different alphas

$$\begin{aligned} s_i^0(r_1) &= \frac{1 - s_1^1(r_1)}{n - 1}, \\ s_i^1(r_1) &= \alpha, \\ s_1^0(r_1) &= 1 - \alpha - \frac{n - 2}{n - 1} \left[ 1 - s_1^1(r_1) \right], \\ s_1^1(r_1) &= \begin{cases} \max\left\{ \min\left\{ \frac{1}{2} \left(\alpha + \frac{r_1^2}{\alpha}\right) (n + 1)^{\frac{n - 1}{n}}, 1\right\}, \alpha \right\} & \text{if } r_1 < \alpha \\ \min\left\{ r_1(n + 1)^{\frac{n - 1}{n}}, 1 \right\} & \text{if } r_1 \ge \alpha \end{cases} \end{aligned}$$

The optimal  $s_1^1$  also identifies the worst-off type of manager:

$$a_{1}^{*} = \begin{cases} \left[ \frac{\frac{1}{2} \left( \alpha + \frac{r_{1}^{2}}{\alpha} \right)}{\alpha} \right]^{\frac{1}{n-1}} & \text{if } s_{1}^{1}(r_{1}) = \alpha \\ \left( \frac{1}{n+1} \right)^{\frac{1}{n}} & \text{if } s_{1}^{1}(r_{1}) \in (\alpha, 1) \\ \left[ \frac{1}{2} \left( \alpha + \frac{r_{1}^{2}}{\alpha} \right) \right]^{\frac{1}{n-1}} & \text{if } s_{1}^{1}(r_{1}) = 1. \end{cases}$$

Figure 1 shows  $V(r_1, \mathbf{s}(r_1))$  as a function of  $r_1$  for  $\alpha \in \{.01, .5, 1\}$  and n = 2. Note that V is everywhere decreasing in  $r_1$ ; this holds because  $\frac{\delta(1)}{\delta'(0)} = \frac{1}{2} = G(\frac{1}{2})$ . V is not monotone in  $\alpha$ , however. When  $\alpha = .01$ , the (negative) slope of V is most severe when  $r_1 < .01$ . This is due to the fact that the participation constraints for the non-controlling shareholders increase rapidly (as perquisite-taking falls) with  $r_1$ —that is, outside shareholder participation effect, net of agency costs, is strongly negative for very small  $r_1$ . When  $\alpha = .5$ , this phenomenon holds as well, but is less marked. When  $\alpha = 1$ , the participation constraints rise until  $r_1 = .5$ , then fall. This is why

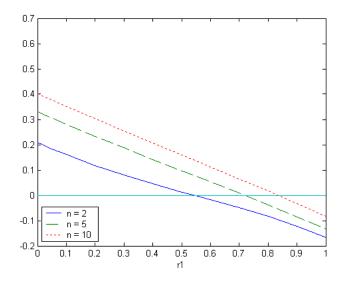


Figure 2: Net Surplus, alpha = 1, different n

V is convex for small  $r_1$  but concave for large  $r_1$ . In all three cases,  $\Phi$  is non-empty, so efficient restructuring is possible for a closed interval of  $r_1$ . This interval is largest for  $\alpha = .01$  ([0, .996]) and smallest for  $\alpha = 1$  ([0, .56]).

Now consider the trade-off of raising  $r_1$ . When  $\alpha = .01$ , there are no agency costs ex ante for  $r_1 \in [.01, 1]$  and efficient restructuring is possible for  $r_1 \in [0, .996]$ , so expected firm value under both the status quo and under restructuring is maximized for  $r_1 \in [.01, .996]$ . For  $\alpha = .5$ , the region where both values are maximized shrinks to [.5, .78]. No such range exists for  $\alpha = 1$ . Thus, sufficiently extreme agency costs entails either an status quo loss of value or an ex post loss of value.

Figure 2 shows  $V(r_1, \mathbf{s}(r_1))$  as a function of  $r_1$  for  $n \in \{2, 5, 10\}$  and  $\alpha = 1$ . Again, V is everywhere decreasing in  $r_1$ . Clearly, V is increasing in n as well. If n = 2, efficient restructuring is possible only if  $r_1 \leq .54$ , while if n = 10, it is possible for  $r_1 \leq .83$ .

Corollary 4 and this example suggest that the negative impact of agency costs on firm value will be mitigated when managerial ownership is small. Generally, since agency problems increase the surplus from restructuring, they make it more likely that optimal ex post firm value is possible under restructuring. Consistent with many other works (e.g. Jensen and Meckling 1976), in our model agency costs are higher when managerial ownership is small. This supports the contention that agency costs make takeovers or other restructurings more likely in situations when managerial ownership stake is small.

### 6 Discussion

#### 6.1 Understanding the main results

Agency problems due to the existence of non-verifiable actions by managers can be mitigated by giving managers large ownership stakes. However, we show that this goal conflicts with the ex post ownership structure that facilitates the operation of the market for corporate control. The provision of golden parachutes to departing managers is required to alleviate their resistance to restructurings; however, if more shares are given to departing managers, fewer are available for the new manager. Thus, the optimal ownership structure targeted by a restructuring mechanism must represent a compromise between two conflicting goals: providing incentives to new managers to maximize firm value and reducing incumbent management resistance to change.

The original ownership structure also affects this trade-off. If initial managerial ownership is large, the management entrenchment problem is severe and the role of golden parachutes as a facilitator of change becomes more important. Ownership structure is therefore not neutral: a large managerial ownership means that incentives for taking value-reducing actions by current management are small, but in that case the incumbent manager is more likely to offer resistance to takeover attempts. In fact, high levels of management ownership can preclude efficient transfers of control, even when contracts for transferring control are complete. In our model, asymmetric information about managerial talent coupled with inefficient extraction of private benefits are sufficient to generate management resistance to control changes. In such a case, as the managerial block increases, the overall surplus generated by a mechanism for transferring control decreases, reducing the rents available to induce managers to participate in such a mechanism.

#### 6.2 Interpretation of the model

The model we present in this paper is very general but also very abstract. Thus, it is important to clarify what it can and what it cannot explain.

The model is not designed to explain the details of existing mechanisms of transferring control, such as proxy fights, tender offers, or mergers. Thus, it would also be misleading to use the results of our analysis to predict the outcomes of such mechanisms. There is a large literature that focuses on modeling and assessing the efficiency properties of specific mechanisms—for example, Grossman and Hart (1980), Shleifer and Vishny (1986), Burkart (1995), Singh (1998), Bulow et al. (1999), Burkart et al. (2000), and Müller and Panunzi (2004). Our mechanism design approach tells us, by contrast, what all specific mechanisms *cannot* achieve. Therefore, our approach teaches us something about the bounds and limits of the mechanisms that form the market for corporate control. We have shown that there are ownership structures that cannot be efficiently restructured by any incentive-compatible mechanism. We see this result as a fundamental property of the market

for corporate control. A direct implication is that one will never be able to fully eliminate the joint problems of management entrenchment and the lack of managerial incentives to maximize profits as long as information asymmetries remain in place.

A feature of our model that might seem too restrictive is the requirement that all shareholders must choose to participate for any change in control to occur. Thus, any shareholder, however small, can alone block a deal that could increase firm value substantially. This assumption is not as strong as it seems, however. The reason is two-fold. First, although a small shareholder cannot alone block a control transaction, the similar non-cooperative behavior of many dispersed shareholders can indeed block control changes, as Grossman and Hart (1980) point out. Thus, ignoring the participation constraints of small shareholders is generally not appropriate to achieve efficiency. The task of meeting the participation constraints of dispersed shareholders is in fact often the main difficulty in implementing a successful takeover bid. Secondly, and most importantly, our approach is not aimed at mimicking actual institutions, but rather at showing their limitations. When there are no mechanisms such that all participation constraints hold simultaneously, trade could still occur, as some shareholders can simply hold on to their shares or be forced to sell them (as in squeeze-out rules). However, no information can be gathered from them, since no one can be forced to reveal his private information. Because the mechanism would have to allocate control without full information, efficiency would not be achieved with certainty. We impose the condition of voluntary participation under unanimity precisely because we want to characterize the set of all ex ante efficient mechanisms of reallocating ownership and control.

#### 6.3 Final remarks

Our model provides an integrative framework under which the interplay between management entrenchment and agency costs can be studied under very general conditions. The market for corporate control is modeled in a way that allows it to achieve the efficient outcome whenever possible. Nevertheless, we show that information asymmetries and hidden actions create sometimes inescapable difficulties to the functioning of this market.

It should be emphasized that our theoretical framework constitutes both a departure (as it distinguishes between ownership and control) and a generalization (as it allows for agency costs) of Cramton et al. (1987). As such, it can be applied to various other settings as well. In particular, our analysis raises several issues worth of future consideration. For example, in situations where no mechanism yields ex post efficient restructuring, it is to be expected that corporations will seek "second best" mechanisms that achieve restructuring with some loss in allocative efficiency. In considering this explicitly, it would become possible to form a stronger link between the market for corporate control and firm value. We leave this extension for future research.

## A Appendix

Proof of Lemma 2. Since the assigned shares always have to satisfy budget balance, we have

$$s_i^1 = 1 - \sum_{j \neq i} s_j^0.$$
 (20)

This implies that

$$\sum_{i \neq 1} s_i^1 = \left(1 - s_1^0 - s_3^0 - s_4^0 - \ldots\right) + \left(1 - s_1^0 - s_2^0 - s_4^0 - \ldots\right) + \left(1 - s_1^0 - s_2^0 - s_3^0 - \ldots\right) + \ldots$$
$$= (n-1)\left(1 - s_1^0\right) - (n-2)\sum_{j \neq 1} s_j^0 = (n-1)\left(1 - s_1^0\right) - (n-2)\left(1 - s_1^1\right). \quad (21)$$

Since  $s_i^1 \ge \underline{s}$  for all *i*, it follows that

$$\sum_{i \neq 1} s_i^1 = (n-1) \left( 1 - s_1^0 \right) - (n-2) \left( 1 - s_1^1 \right) \ge (n-1)\underline{s}$$

From condition (20), we have also that

$$s_i^1 - s_i^0 = 1 - \sum_{j \neq i} s_j^0 - s_i^0 = 1 - \sum_j s_j^0$$

Aggregating over all  $i \neq 1$  and using condition (20) once more, we then obtain

$$\sum_{i \neq 1} \left( s_i^1 - s_i^0 \right) = (n-1)(1 - \sum_j s_j^0) = (n-1) \left( s_1^1 - s_1^0 \right), \tag{22}$$

completing the proof.  $\blacksquare$ 

**Proof of Lemma 3.** Only if. If the mechanism assigns control to the highest announced type, then if all types announce truthfully,  $U_i(a_i) = s_i^1 a_i G(a_i) + s_i^0 \int_{a_i}^{\bar{a}} u dG(u) + T_i(a_i)$ . For convenience, define  $S_i(a_i) \equiv s_i^1 G(a_i)$  and  $P_i(a_i) \equiv s_i^0 \int_{a_i}^{\bar{a}} u dG(u)$ . Then, given condition

For convenience, define  $S_i(a_i) \equiv s_i^1 G(a_i)$  and  $P_i(a_i) \equiv s_i^0 \int_{a_i}^a u dG(u)$ . Then, given condition (7), we have  $U_i(a_i) = a_i S_i(a_i) + P_i(a_i) + T_i(a_i)$ . Incentive compatibility (condition (10)) requires  $U_i(a_i) \geq a_i S_i(b) + P_i(b) + T_i(b)$  for all  $i, a_i, b \in [\underline{a}, \overline{a}]$ . We have

$$U_{i}(a_{i}) = a_{i}S_{i}(a_{i}) + P_{i}(a_{i}) + T_{i}(a_{i})$$
  

$$\geq a_{i}S_{i}(b) + P_{i}(b) + T_{i}(b)$$
  

$$= U_{i}(b) + a_{i}S_{i}(b) - bS_{i}(b).$$

Thus, if  $a_i > b$ ,

$$\frac{U_i(a_i) - U_i(b)}{a_i - b} \ge S_i(b).$$

We can similarly construct the condition

$$S_i(a_i) \ge \frac{U_i(a_i) - U_i(b)}{a_i - b}.$$

Since the last two inequalities are reversed if  $a_i < b$ , taking the limit as  $b \rightarrow a_i$ , we obtain

$$\frac{dU_i(a_i)}{da_i} = S_i(a_i). \tag{23}$$

Hence,

$$U_{i}(a_{i}) - U_{i}(b) = \int_{b}^{a_{i}} S_{i}(u) du$$
  
=  $a_{i}S_{i}(a_{i}) - bS_{i}(b) - \int_{b}^{a_{i}} u dS_{i}(u),$  (24)

where the second line uses a simple integration by parts. Using condition (7) for  $U_i(a_i)$  and  $U_i(b)$ and simplifying, equation (24) becomes

$$T_i(a_i) = T_i(b) - \int_b^{a_i} u dS_i(u) + P_i(b) - P_i(a_i).$$

Substituting back in for the definitions of  $S_i$  and  $P_i$ , we then obtain

$$T_{i}(a_{i}) = T_{i}(b) - s_{i}^{1} \int_{b}^{a_{i}} u dG(u) + s_{i}^{0} \int_{b}^{a_{i}} u dG(u)$$
  
=  $T_{i}(b) - (s_{i}^{1} - s_{i}^{0}) \int_{b}^{a_{i}} u dG(u).$ 

If. Note that

$$a_i s_i^1 [G(a_i) - G(b)] = s_i^1 \int_b^{a_i} a_i dG(u)$$

and

$$P_i(a_i) - P_i(b) = -s_i^0 \int_b^{a_i} u dG(u).$$

Adding these terms to (11), we have

$$a_i s_i^1 \left[ G(a_i) - G(b) \right] + P_i(a_i) - P_i(b) + T_i(a_i) - T_i(b) = s_i^1 \int_b^{a_i} (a_i - u) \, dG(u) \ge 0.$$

Considering the terms on the left-hand side, the inequality implies that

$$a_i s_i^1 G(a_i) + P_i(a_i) + T_i(a_i) \ge a_i s_i^1 G(b) + P_i(b) + T_i(b),$$

which is the incentive compatibility condition given by (10). Note also that this implies that utility is given by  $U_i(a_i) = s_i^1 a_i G(a_i) + s_i^0 \int_{a_i}^{\bar{a}} u dG(u) + T_i(a_i)$ , so that the mechanism assigns control to the highest announced type. **Proof of Lemma 4.** From condition (23), we have that the difference  $U_1(a_1) - \overline{U}_1(a_1)$  is convex and has first derivative  $s_1^1 G(a_1) - \delta(\gamma^*(r_1)) - (1 - \gamma^*(r_1))r_1$ . Thus, if  $s_1^1 \ge \beta(r_1)$  and  $s_1^1 > 0$ ,  $a_1^*$  is identified by the first-order condition

$$G(a_1^*) = \frac{\beta(r_1)}{s_1^1}.$$

If  $s_1^1 < \beta(r_1)$ , then  $s_1^1 G(a_1) - \delta(\gamma^*(r_1)) - (1 - \gamma^*(r_1))r_1 < 0$  for all  $a_1 \in [\underline{a}, \overline{a}]$  and  $U_1(a_1) - \overline{U}_1(a_1)$  is minimized at  $a_1^* = \overline{a}$ . Finally, if  $s_1^1 = \beta(r_1) = 0$ , all  $a_1 \in [\underline{a}, \overline{a}]$  expect to gain the same under the mechanism, so any type will do the role of the worst-off type.

Participation is individually rational for all types of shareholder 1 if and only if  $U_1(a_1^*) \equiv s_1^1 a_1^* G(a_1^*) + s_1^0 \int_{a_1^*}^{\bar{a}} u dG(u) + T_1(a_1^*) \geq \beta(r_1) a_1^*$ . Thus, if  $s_1^1 \geq r_1$  and  $s_1^1 > 0$ , we have

$$s_1^1 a_1^* \left(\frac{\beta(r_1)}{s_1^1}\right) + T_1 \left(a_1^*\right) \ge \beta(r_1) a_1^* - s_1^0 \int_{a_1^*}^{\bar{a}} u dG(u),$$

which simplifies to

$$T_1(a_1^*) \ge -s_1^0 \int_{a_1^*}^{\bar{a}} u dG(u).$$

(It is straightforward to see that this condition also holds for  $s_1^1 = \beta(r_1) = 0$ , only that in such a case  $a_1^*$  is defined as any element of the set  $[\underline{a}, \overline{a}]$ ). This yields condition (13), since  $\max\{(\beta(r_1) - s_1^1) \overline{a}, 0\} = 0$  when  $s_1^1 \ge \beta(r_1)$ . If  $s_1^1 < \beta(r_1)$ , participation is individually rational for the original manager if and only if

$$s_1^1 \bar{a} G(\bar{a}) + T_1(\bar{a}) \ge \beta(r_1) \bar{a}_2$$

which simplifies to

$$T_1(\bar{a}) \ge (\beta(r_1) - s_1^1)\bar{a}.$$

This yields condition (13), since  $s_1^0 \int_{a_1^*}^{\bar{a}} u dG(u) = 0$  when  $a_1^* = \bar{a}$ .

For all subsequent proofs, let  $\{a_i^*\}$  be defined as in lemmas 4 and 5.

**Proof of Proposition 1.** Only if. From condition (11), incentive compatibility of a mechanism that assigns control to the shareholder with the highest announced ability implies that  $T_i(a_i) =$ 

 $T_{i}\left(a_{i}^{*}\right)-\left(s_{i}^{1}-s_{i}^{0}\right)\int_{a_{i}^{*}}^{a_{i}}udG\left(u\right)$ . Taking ex ante expectations, we obtain

$$E_{i}\{T_{i}(a_{i})\} = E_{i}\{T_{i}(a_{i}^{*})\} - (s_{i}^{1} - s_{i}^{0}) \int_{a_{i}=\underline{a}}^{\overline{a}} \int_{u=a_{i}^{*}}^{a_{i}} udG(u)dF(a_{i})$$

$$= T_{i}(a_{i}^{*}) - (s_{i}^{1} - s_{i}^{0}) \left( \int_{u=a_{i}^{*}}^{\overline{a}} \int_{a_{i}=u}^{\overline{a}} dF(a_{i}) udG(u) - \int_{u=\underline{a}}^{a_{i}^{*}} \int_{a_{i}=\underline{a}}^{u} dF(a_{i}) udG(u) \right)$$

$$= T_{i}(a_{i}^{*}) - (s_{i}^{1} - s_{i}^{0}) \left( \int_{a_{i}^{*}}^{\overline{a}} [1 - F(u)] udG(u) - \int_{\underline{a}}^{a_{i}^{*}} F(u) udG(u) \right)$$

$$= T_{i}(a_{i}^{*}) - (s_{i}^{1} - s_{i}^{0}) \left( \int_{a_{i}^{*}}^{\overline{a}} udG(u) - \int_{\underline{a}}^{\overline{a}} F(u) udG(u) \right)$$

where the second line is obtained by changing the order of integration. Because of budget balance,  $\sum_{i=1}^{n} t_i = -K$ , it must be true that the sum of expected transfers is -K as well :  $\sum_{i=1}^{n} E_i\{T_i(a_i)\} = -K$ . Thus,

$$\sum_{i=1}^{n} T_i(a_i^*) = \sum_{i=1}^{n} \left[ \left( s_i^1 - s_i^0 \right) \left( \int_{a_i^*}^{\bar{a}} u dG(u) - \int_{\underline{a}}^{\bar{a}} F(u) u dG(u) \right) \right] - K.$$
(25)

From the individual rationality constraints (13) and (14), it is also true that

$$\sum_{i=1}^{n} T_i(a_i^*) \ge \left[ \max\{ (\beta(r_1) - s_1^1)\bar{a}, 0\} + (1 - \gamma^*(r_1))(1 - r_1)\mu - \sum_{i=1}^{n} s_i^0 \int_{a_i^*}^{\bar{a}} u dG(u) \right],$$
(26)

since  $\sum_{i \neq 1} r_i = 1 - r_1$ . Using the expression for  $\sum_{i=1}^n T_i(a_i^*)$  in (25), inequality (26) can be rewritten as

$$\sum_{i=1}^{n} \left[ s_{i}^{1} \int_{a_{i}^{*}}^{\bar{a}} u dG(u) - \left( s_{i}^{1} - s_{i}^{0} \right) \int_{\underline{a}}^{\bar{a}} F(u) u dG(u) \right] - \max\{ (\beta(r_{1}) - s_{1}^{1})\bar{a}, 0\} - (1 - \gamma^{*}(r_{1}))(1 - r_{1})\mu \ge K.$$

If. Following Cramton et al. (1987 p. 628), consider a transfer of the form

$$t_i(\mathbf{a}) = c_i - (s_i^1 - s_i^0) \int_{\underline{a}}^{a_i} u dG(u) + \frac{1}{n-1} \sum_{j \neq i} (s_j^1 - s_j^0) \int_{\underline{a}}^{a_j} u dG(u),$$

where  $\sum_{i=1}^{n} t_i(\mathbf{a}) = -K$  implies  $\sum_{i=1}^{n} c_i = -K$ . After changing the order of integration, we obtain

$$T_i(a_i) = c_i - (s_1^1 - s_1^0) \int_{\underline{a}}^{a_i} u dG(u) du + \frac{1}{n-1} \sum_{j \neq i} (s_j^1 - s_j^0) \int_{\underline{a}}^{\overline{a}} u[1 - F(u)] u dG(u).$$

This guarantees that the mechanism is incentive compatible and that it assigns control to the shareholder with the highest announced ability. Thus, by showing that it is also individually rational, we show that it is efficient.

Individual rationality requires

$$\begin{cases} i = 1: T_1(a_1^*) \ge \max\{\left(\beta(r_1) - s_1^1\right)\bar{a}, 0\} - s_1^0 \int_{a_1^*}^{\bar{a}} u dG(u) \\ i > 1: T_i(a_i^*) \ge (1 - \gamma^*(r_1))r_i\mu - s_i^0 \int_{a_i^*}^{\bar{a}} u dG(u). \end{cases}$$

Since equation (15) asserts that

$$V(\mathbf{r}, \mathbf{s}) - K \ge 0,$$

we can choose

$$c_{1} = \max\{\left(\beta(r_{1}) - s_{1}^{1}\right)\bar{a}, 0\} - s_{1}^{0}\int_{a_{1}^{*}}^{\bar{a}}udG(u) + \frac{1}{n}\left[V(\mathbf{r}, \mathbf{s}) - K\right] \\ + \left(s_{1}^{1} - s_{1}^{0}\right)\int_{\underline{a}}^{a_{1}^{*}}udG(u)du - \frac{1}{n-1}\sum_{j\neq i}\left(s_{j}^{1} - s_{j}^{0}\right)\int_{\underline{a}}^{\bar{a}}u[1 - F(u)]udG(u)du$$

and, for i > 2,

$$c_{i} = (1 - \gamma^{*}(r_{1}))r_{i}\mu - s_{i}^{0}\int_{a_{i}^{*}}^{\bar{a}}udG(u) + \frac{1}{n}\left[V(\mathbf{r}, \mathbf{s}) - K\right] \\ + (s_{i}^{1} - s_{i}^{0})\int_{\underline{a}}^{a_{i}^{*}}udG(u)du - \frac{1}{n-1}\sum_{j\neq i}(s_{j}^{1} - s_{j}^{0})\int_{\underline{a}}^{\bar{a}}u[1 - F(u)]udG(u).$$

With some simple algebra using the definition of  $V(\mathbf{r}, \mathbf{s})$ , it can be shown that  $\sum_{i=1}^{n} c_i = -K$ . We have

$$T_{1}(a_{1}^{*}) = \max\{\left(\beta(r_{1}) - s_{1}^{1}\right)\bar{a}, 0\} - s_{1}^{0}\int_{a_{1}^{*}}^{\bar{a}}udG(u) + \frac{1}{n}\left[V(\mathbf{r}, \mathbf{s}) - K\right]$$
  

$$\geq \max\{\left(\beta(r_{1}) - s_{1}^{1}\right)\bar{a}, 0\} - s_{1}^{0}\int_{a_{1}^{*}}^{\bar{a}}udG(u)$$

and, for i > 1,

$$T_i(a_i^*) = (1 - \gamma^*(r_1))r_i\mu - s_i^0 \int_{a_i^*}^{\bar{a}} u dG(u) + \frac{1}{n} \left[ V(\mathbf{r}, \mathbf{s}) - K \right] \ge (1 - \gamma^*(r_1))r_i\mu - s_i^0 \int_{a_i^*}^{\bar{a}} u dG(u),$$

completing the proof.  $\blacksquare$ 

**Proof of Proposition 2.** We first identify  $\mathbf{s}(r_1)$ , then prove the "if and only if" statement. In identifying  $\mathbf{s}(r_1)$ , it is easiest to first prove part *iii*, followed by parts *ii*, *i* and *iv*.

(Part *iii*) Consider first the manager's golden parachute,  $s_1^0$ . The budget balance conditions of Lemma 2 yield substitutions of  $\sum_{i \neq 1} s_i^1$  and  $\sum_{i \neq 1} (s_i^1 - s_i^0)$  out of expression (16), so  $V(r_1, \mathbf{s})$  may be rewritten as a function of  $s_1^1$  and  $s_1^0$  only:

$$V(r_1, \mathbf{s}) = s_1^1 \int_{a_1^*}^{\bar{a}} u dG(u) + \left[ (n-1)(1-s_1^0) - (n-2)(1-s_1^1) \right] \int_{\underline{a}}^{\bar{a}} u dG(u) - n \left( s_1^1 - s_1^0 \right) \int_{\underline{a}}^{\bar{a}} F(u) u dG(u) - (1-r_1)\mu - \max\{ (r_1 - s_1^1)\bar{a}, 0 \}.$$
 (27)

We then have

$$\frac{dV(r_1, \mathbf{s})}{ds_1^0} = \int_{\underline{a}}^{\overline{a}} F(u) u dG(u) - (n-1) \int_{\underline{a}}^{\overline{a}} [1 - F(u)] u dG(u)$$
$$= (n-1) \int_{\underline{a}}^{\overline{a}} [1 - F(u)] G(u) du > 0, \qquad (28)$$

where the second line uses the fact that  $G(u) = F(u)^{n-1}$ . Clearly, then, net surplus is maximized for the largest possible  $s_1^0$  that is compatible with  $s_1^1(r_1)$  and with the budget balance condition from Lemma 2:

$$\sum_{i \neq 1} s_i^1 = (n-1) \left( 1 - s_1^0 \right) - (n-2) \left( 1 - s_1^1 \right) \ge (n-1)\underline{s}.$$
(29)

The left-hand side of this inequality is positive when  $s_1^0 = 0$  and decreases in  $s_1^0$ . Thus,  $V(r_1, \mathbf{s})$  is maximized when the constraint holds with equality, implying that the optimal golden parachute is

$$s_1^0(r_1) = 1 - \underline{s} - \frac{n-2}{n-1} [1 - s_1^1(r_1)], \qquad (30)$$

where  $s_1^1(r_1)$  is also chosen optimally.

(Part *ii*) To find  $\{s_i^1(r_1)\}_{i \neq 1}$ , note that part *iii* also implies that  $\sum_{i \neq 1} s_i^1 = (n-1)\underline{s}$ . Since each  $s_i^1$  must be at least  $\underline{s}$ , we have that  $s_i^1(r_1) = \underline{s}$  for  $i \neq 1$ .

(Part *i*) To find  $\{s_i^0(r_1)\}_{i\neq 1}$ , rewrite (20) isolating the summation on the left-hand side for each  $i \neq 1$  that may gain control. This gives us the following n-1 conditions:

$$\begin{cases} s_1^0(r_1) + s_3^0 + s_4^0 + \dots + s_n^0 = 1 - \underline{s} \\ s_1^0(r_1) + s_2^0 + s_4^0 + \dots + s_n^0 = 1 - \underline{s} \\ \dots \\ s_1^0(r_1) + s_2^0 + s_3^0 + \dots + s_{n-1}^0 = 1 - \underline{s} \end{cases}$$

where we have used the fact that  $s_i^1(r_1) = \underline{s}$  for all  $i \neq 1$  in the optimal mechanism. It is easy to see that these conditions are only satisfied for

$$s_2^0 = s_3^0 = \dots = s_n^0.$$

Using this condition, and substituting (30) into one of the n-1 conditions above, we find that

$$s_i^0(r_1) = \frac{1 - s_1^1(r_1)}{n - 1}$$

for all  $i \neq 1$ .

(Part *iv*) Next consider the optimal  $s_1^1$ . Using the results from parts *i* to *iii* of the proposition, we can substitute into (27) to yield the function  $V'(r_1, s_1^1)$ :

$$V'(r_1, s_1^1) = s_1^1 \int_{a_1^*}^{\bar{a}} u dG(u) + \left[1 - s_1^1 - (n-1)\underline{s}\right] \int_{\underline{a}}^{\bar{a}} u dF(u)^n + (n-1)\underline{s} \int_{\underline{a}}^{\bar{a}} u dG(u) - (1 - \gamma^*(r_1))(1 - r_1)\mu - \max\{(\beta(r_1) - s_1^1)\bar{a}, 0\}.$$
 (31)

The  $s_1^1$  that maximizes this expression defines the optimal  $s_1^1(r_1)$ .

Assume, for the moment, that  $\underline{s} = 0$ . If  $r_1 = 0$  and  $s_1^1 > 0$ , then  $a_1^* = \underline{a}$  and

$$\frac{\partial V'(r_1, s_1^1)}{\partial s_1^1} = \int_{\underline{a}}^{\overline{a}} u dG(u) - \int_{\underline{a}}^{\overline{a}} u dF(u)^n < 0,$$

so  $s_1^1(r_1) = 0$  if  $r_1 = 0$  and  $\underline{s} = 0$  (this follows from the continuity of V').

For positive  $r_1$  if  $\underline{s} = 0$ , or for all  $r_1$  if  $\underline{s} > 0$ , we will show that it is optimal to have  $s_1^1 > \beta(r_1) \ge r_1$  in this case. Note first that, since  $s_1^1$  is defined on  $[\underline{s}, 1]$ , a compact set, if  $V'(r_1, s_1^1)$  is bounded then it must achieve a maximum on this set. To show that the optimal  $s_1^1$  is unique and greater than  $r_1$ , it suffices to show that  $V'(r_1, s_1^1)$  is strictly increasing in  $s_1^1$  when  $s_1^1 \le \beta(r_1)$ , strictly concave for  $s_1^1 > \beta(r_1)$ , and has a continuous first derivative and is finite when  $s_1^1 = 1$ .

First, when  $s_1^1 \leq \beta(r_1)$ , we have

$$\frac{\partial V'(r_1, s_1^1)}{\partial s_1^1} = \bar{a} - \int_{\underline{a}}^{\bar{a}} u dF(u)^n > 0, \qquad (32)$$

so  $V'(r_1, s_1^1)$  is increasing and linear in  $s_1^1$  in this range. Next, when  $s_1^1 > \beta(r_1)$ , we have

$$\frac{\partial V'(r_1, s_1^1)}{\partial s_1^1} = a_1^* G(a_1^*) + \int_{a_1^*}^{\bar{a}} u dG(u) - \int_{\underline{a}}^{\bar{a}} u dF(u)^n = \bar{a} - \int_{a_1^*}^{\bar{a}} G(u) du - \int_{\underline{a}}^{\bar{a}} u dF(u)^n,$$

where the second line follows from a simple integration by parts. Since  $a_1^* = \bar{a}$  at  $s_1^1 = \beta(r_1) > 0$ , this expression collapses to (32) at that point, so the derivative is continuous at  $s_1^1 = \beta(r_1)$ . It is straightforward to show that  $V'(r_1, s_1^1)$  is strictly concave for  $s_1^1 > \beta(r_1)$ :

$$\frac{\partial^2 V'(r_1, s_1^1)}{\partial \left(s_1^1\right)^2} = G(a_1^*) \frac{da_1^*}{ds_1^1} < 0.$$

Finally, since  $V'(r_1, s_1^1)$  is finite when  $s_1^1 = 1$ , if there exists an  $s_1^1 \in [\underline{s}, 1]$  such that  $\frac{\partial V'(r_1, s_1^1)}{\partial s_1^1} = 0$ , then that  $s_1^1$  is the unique maximizer of  $V'(r_1, s_1^1)$ . In that case, the optimal  $s_1^1$  is defined implicitly by the first-order condition (17). Otherwise, either

$$\bar{a} - \int_{G^{-1}\left(\frac{\beta(r_1)}{\underline{s}}\right)}^{\bar{a}} G(u) du - \int_{\underline{a}}^{\bar{a}} u dF(u)^n < 0,$$

in which case  $s_1^1 = \underline{s}$  is the unique maximizer, or

$$\bar{a} - \int_{G^{-1}(\beta(r_1))}^{\bar{a}} G(u) du - \int_{\underline{a}}^{\bar{a}} u dF(u)^n > 0,$$

in which case  $s_1^1 = 1$  is the unique maximizer. This completes our identification of  $\mathbf{s}(r_1)$ .

To show that a corporation with initial managerial ownership  $r_1$  can be efficiently restructured if and only if  $V(r_1, \mathbf{s}(r_1)) \ge K$ , note that if  $V(r_1, \mathbf{s}(r_1)) \ge K$ , the corporation can be efficiently restructured using  $\mathbf{s}(r_1)$ , because  $\mathbf{s}(r_1)$  satisfies the budget balance conditions in lemmas 1 and 2, so there are no agency costs ex post, and  $V(r_1, \mathbf{s}(r_1)) \ge K$  implies that  $\mathbf{s}(r_1)$  yields an efficient transfer of control by Proposition 1. On the other hand, if the corporation can be efficiently restructured, then  $V(r_1, \mathbf{s}) \ge K$  for some  $\mathbf{s}$  that satisfies the budget balance conditions in (6). Since  $V(r_1, \mathbf{s}(r_1)) \ge$  $V(r_1, \mathbf{s})$  by the definition of  $\mathbf{s}(r_1)$ , we have that  $V(r_1, \mathbf{s}(r_1)) \ge K$ . Since  $\mathbf{s}(r_1)$  satisfies budget balance as well, the corporation can be efficiently restructured using share rule  $\mathbf{s}(r_1)$ .

**Proof of Proposition 3.** To prove our results, we begin by showing that  $a_w > \mu$ .

Consider the term  $\int_{\underline{a}}^{\overline{a}} u dF(u)^n$ . Consider a random variable X that follows F and a random variable Y that follows G. Define the random variable Z as

$$Z \equiv \max\left\{X, Y\right\}.$$

Thus,

$$E\left[Z
ight] = \int_{\underline{a}}^{\overline{a}} ud\left[G\left(u
ight)F(u)
ight].$$

The expected Z conditional on already knowing a is

$$h(a) \equiv E[Z \mid a] = aG(a) + \int_{a}^{\overline{a}} u dG(u).$$

By the law of iterated expectations, we then have

$$E[Z] = E_a[E[Z \mid a]] = E_a[h(a)].$$

Thus, we can conclude that

$$E_{a}\left[h\left(a\right)\right] = \int_{\underline{a}}^{\overline{a}} ud\left[G\left(u\right)F(u)\right].$$

Now let us go back to the first-order condition for an optimal  $s_1^1$ . Suppose we are in an interior optimum. The problem is to find  $a_w$  such that

$$a_w G(a_w) + \int_{a_w}^{\overline{a}} u dG(u) - \int_{\underline{a}}^{\overline{a}} u d(G(u) F(u)) = 0,$$

or equivalently,

$$h\left(a_{w}\right) - E_{a}\left[h\left(a\right)\right] = 0.$$

Notice that h(a) is increasing and convex. Thus, Jensen's inequality implies that

$$h\left(\mu\right) - E_{a}\left[h\left(a\right)\right] < 0,$$

where  $\mu = E[a]$ , thus the value  $a_w$  that solves

$$h\left(a_{w}\right) - E_{a}\left[h\left(a\right)\right] = 0$$

is such that  $a_w > \mu$ .

Now, differentiating  $V(r_1, \mathbf{s}(r_1))$  with respect to  $r_1$ , we obtain

$$\frac{dV(r_1, \mathbf{s}(r_1))}{dr_1} = \frac{\partial V}{\partial r_1} + \frac{\partial V}{\partial s_1^1} \frac{\partial s_1^1(r_1)}{\partial r_1} \\
= -[1 - \gamma^*(r_1)] a_1^* - \left[\gamma^*(r_1) - 1 - (1 - r_1) \frac{d\gamma^*}{dr_1}\right] \mu \\
= (1 - r_1) \frac{d\gamma^*}{dr_1} \mu + (1 - \gamma^*(r_1)) (\mu - a_1^*),$$

where the envelope theorem zeroes out the last term in the first line of algebra above.

Consider initially the case where the solution is interior. Since  $\frac{d\gamma^*}{dr_1} = \frac{1}{\delta''(\gamma)} < 0$  by the concavity of  $\delta(\cdot)$  and we have just seen that  $\mu < a_w$ , we have

$$\frac{dV(r_1, \mathbf{s}(r_1))}{dr_1} = (1 - r_1)\frac{d\gamma^*}{dr_1}\mu + (1 - \gamma^*(r_1))(\mu - a_w) < 0,$$

where  $a_w$  is defined in (17), for all  $r_1$  such that it holds for a feasible  $s_1^1(r_1)$ . Now, if  $s_1^1(r_1) = 1$ , then  $a_1^* \ge a_w > \mu$ . Thus, if  $s_1^1(r_1) > \underline{s}$ , then V is clearly decreasing in  $r_1$ .

Next, suppose that  $r_1$  is small enough so that  $s_1^1 = \underline{s} > 0$  is optimal. Differentiating expression (31) when  $s_1^1 = \underline{s} = \delta'(0) > 0$ , we obtain

$$\frac{dV(r_1, s_1^1 = \underline{s})}{dr_1} = (1 - r_1)\frac{d\gamma^*}{dr_1}\mu + (1 - \gamma^*(r_1))\left[\mu - G^{-1}\left(\frac{\beta(r_1)}{\delta'(0)}\right)\right].$$

Since  $\beta(r_1)$  is increasing in  $r_1$ , the term in square brackets is decreasing in  $r_1$ . Thus, if this term is negative for any  $\hat{r}_1$ , it is negative for all  $r_1 \geq \hat{r}_1$ . Moreover, if  $\mu - G^{-1}\left(\frac{\delta(\gamma^*(0))}{\delta'(0)}\right) = \mu - G^{-1}\left(\frac{\delta(1)}{\delta'(0)}\right) \leq 0$ , then  $\hat{r}_1 = 0$ . Clearly, this holds whenever  $\delta(1) \geq G(\mu)\delta'(0)$ .

Finally, consider the boundary case of  $\delta'(0) = 0$ , so that  $\underline{s} = 0$  and, for  $r_1 > 0$ ,

$$\frac{dV(r_1, \mathbf{s}(r_1))}{dr_1} = \mu - a_1^*.$$

The preceding analysis holds, as  $a_1^* > \mu$  for all r, implying that V is decreasing everywhere when  $\delta'(0) = 0$ , which implies the condition  $\delta(1) \ge G(\mu)\delta'(0)$ . Note that the above derivative holds as  $r_1$  approaches 0, but is undefined at  $r_1 = 0$ .

**Proof of Proposition 4.** Suppose  $\delta'(0) = 0$  and K = 0. It suffices to show that the trivial mechanism, where  $s_i^1 = s_i^0 = r_i$  for all *i*, achieves expost efficiency for all  $r_1$ . Since  $\delta'(0) = 0$ , we

have  $\underline{s} = 0$  and the trivial mechanism's share rule precludes agency costs ex post. Since  $s_i^1 = s_i^0$  for all *i*, we have, using equation (25) from the proof of Proposition 1,

$$\sum_{i=1}^{n} T_i(a_i^*) = \sum_{i=1}^{n} \left[ \left( s_i^1 - s_i^0 \right) \left( \int_{a_i^*}^{\bar{a}} u dG(u) - \int_{\underline{a}}^{\bar{a}} F(u) u dG(u) \right) \right] = 0.$$

Individual rationality requires that equation (26) hold:

$$\sum_{i=1}^{n} T_i(a_i^*) = 0 \ge \left[ \max\{(r_1 - s_1^1)\bar{a}, 0\} + (1 - r_1)\mu - \sum_{i=1}^{n} s_i^0 \int_{a_i^*}^{\bar{a}} u dG(u) \right].$$

Given the trivial mechanism's share rule,  $a_1^* = \overline{a}$  and the right-hand side of the above expression becomes

$$(1-r_1)\left[\mu - \int_{\underline{a}}^{\overline{a}} u dG(u)\right] \le 0,$$

so that  $V(r_1, \mathbf{s}^t) \ge 0$  for any  $r_1$ , where  $\mathbf{s}^t$  is the share rule for the trivial mechanism. Since  $V(r_1, \mathbf{s}(r_1)) \ge V(r_1, \mathbf{s}^t)$ , the proof is complete.

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