Does Prospect Theory Explain the Disposition Effect?

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Abstract

The disposition effect is the observation that investors hold losing stocks too long and sell winning stocks too early. A standard explanation of the disposition effect refers to prospect theory and in particular to the asymmetric risk aversion according to which investors are risk averse when faced with gains and risk-seeking when faced with losses. We show that for reasonable parameter values the disposition effect can however not be explained by prospect theory. The reason is that those investors who sell winning stocks and hold losing assets would in the first place not have invested in stocks. That is to say the standard prospect theory argument is sound ex-post, assuming that the investment has taken place, but not ex-ante, requiring that the investment is made in the first place.

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JEL classification: G11

1 Introduction

The disposition effect is the observation that investors tend to sell winning stocks while they have a disposition to keep losing stocks. This observation has been made by a series of papers, including Shefrin and Statman (1985), Odean (1998), Weber and Camerer (1998), Heath, Huddart, and Lang (1999), Locke and Mann (2001), Grinblatt and Keloharju (2000), Grinblatt and Keloharju (2001) and Ranguelova (2002).

Of course, selling winners and keeping losers as such is perfectly compatible with complete rationality. A well known result is that an expected utility maximizer, with constant relative risk aversion, would rebalance a fixed-mix portfolio strategy in a setting where the investment opportunity set is constant.[1] Therefore when prices rise (fall) he would sell (buy) the security. However, as Odean (1998) has shown investors are reluctant to sell losers even when controlling for rebalancing. Hence the disposition effect is the observation that investors show a more aggressive contrarian behavior than following the fixed-mix rule. This observation is striking. All the more because Odean (1998) shows that the prices of the winner stocks investors have sold, keep on rising, whereas the prices of the loser stocks that investors have not sold, keep falling. Therefore one can exclude private information as a potential explanations for the disposition effect. Odean (1998) further rejects other possible explanations such as taxes and transaction costs.

Since the explanations based on traditional theories cannot be sustained the mentioned authors propose behavioral explanations for the disposition effect. These are either based on perception or valuation. The perception argument is that investors (erroneously) believe in mean reverting asset prices, i.e. they believe that today's losers will outperform today's winners and that the winners of today are the losers of tomorrow. Based on such beliefs investors sell winners and hold losers. Yet, Weber and Camerer (1998) reject the hypothesis that the disposition behavior is due to the belief in mean-reverting stock prices.

The valuation argument refers to two main features of prospect theory. First, under prospect theory, see Kahneman and Tversky (1979), investors evaluate outcomes relative to a reference point which in the context of stock investments is typically the purchasing price. Second, they behave as if

[1] See Samuelson (1969) and Merton (1969).

evaluating the decision consequences on an S-shaped value function, which is concave for gains and convex for losses. This reflects risk aversion in the gain region and risk-seeking in the loss region. The standard behavioral finance argument for the disposition effect is that a gain (loss) moves the investor to his risk averse (seeking) part of the value function so that he is leaned to reduce (increase) his position in the risky assets. Therefore, the disposition effect is commonly seen as an important implication of extending prospect theory to investment decisions and securities trading.

However, this standard explanation for why investors sell winners and hold losers so far has not been proved analytically. Also it is generally *assumed* that the investor has bought the risky stock and thus the issue whether the investor really will decide in this way is neglected. Hence this standard argument is in fact an ex-post argument that corresponds to a liquidation situation as analyzed by Kyle, Ou-Yang, and Xiong (2006).

In our paper we consider a model with two consecutive portfolio choices in a stylized financial market where the investor's preferences are described by prospect theory as suggested by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). We investigate the investor's risk-taking behavior following a rise, respectively a fall, in the price of the risky asset. After analyzing the standard argument, i.e. the ex-post disposition behavior, we focus on a more complete definition of the disposition behavior, where, besides requiring investors to sell winners and to hold losers, we require them explicitly to buy the stock in the first period.

Our first point of interest is the second period behavior of the investor conditional on the stock price movement in the first period. In particular, we investigate whether prospect theory can explain the behavior of an investor prone to the ex-post disposition effect. Assuming that the investor is endowed with the stock in the first period, we call him an ex-post disposition investor if he sells the stock after a gain and keeps holding it after a loss. We show how important aspects of prospect theory, in particular loss aversion and probability weighting, interact with asymmetric risk aversion. This analysis is of interest in itself but it also will lay the foundations for the inter-temporal argument. In the inter-temporal view we investigate the agent's behavior with a focus on the more complete definition of the disposition behavior. We show interactions between loss aversion, decision weighting and asymmetric risk-taking.

Our findings are that the ex-post disposition effect arises rather for lower coefficients of loss aversion whenever the agent can undo the first period loss by investing in the risky asset. In the opposite case, i.e. when he is not able to undo the first period loss, the ex-post disposition effect arises rather for more loss averse investors. The inter-temporal disposition effect arises rather for lower coefficients of loss aversion. Furthermore investors are generally prone to the ex-post disposition effect, but hardly to the true disposition effect. The reason is that those investors who sell winning stocks too early and keep losing stocks too long would in the first place not have invested in stocks.

So even when considering explicitly the asymmetric risk-taking behavior of agents, a standard explication for the disposition behavior, investors are not prone to the disposition effect. We conclude that prospect theory can indeed explain the ex-post disposition behavior, but not the more complete inter-temporal definition of the disposition behavior.

In general, prior studies link the disposition effect to the standard argument described above using an intuitive argument. To our knowledge, only two other papers formally analyze the relation between prospect theory and the disposition effect. In independent work Barberis and Xiong (2006) investigate the trading behavior of investors with prospect theory preferences. Their analysis leads them to question, as we do, whether prospect theory predicts a disposition effect. In contrast to our contribution, these authors do not consider the impact of probability weighting, nor do they make the important distinction between the ex-post and the true disposition effect. Further they do not benchmark the disposition behavior against a fixed-mix strategy and they assume dynamic optimization in a complete market. They assume dynamic optimization in a complete market setting, which, in our view, is not appropriate for a descriptive model. Gomes (2005) studies the two-period portfolio problem of an investor with preferences that are related to, but different from, prospect theory.

The rest of the paper is organized as follows. In the next section we precisely describe the framework. In section 3 we analyze the ex-post behavior of a prospect theory investor and then we consider the ex-ante point of view. In the last two sections we offer further discussion of our results and conclude.

2 The Model

We present a two period model for portfolio choice in a stylized financial market with two assets where the investor's preferences are described by prospect theory as suggested by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). After describing the financial market and the agent's preferences, we derive the investor's optimization problem and the conditions under which the (ex-post) disposition effect arises.

In our framework, there is a financial market on which two assets are traded. A riskless asset, also called the bond, and a risky asset, the stock.[2] The evolution of the stock prices is described by a binomial process, so that at the end of the following period there are two possible states. If the stock price rises, we call the corresponding state the up state; the other state is called the down state. In the up state, which realizes with probability p, the risky investment yields a gross return R_U. Note that $0 < p < 1$. In the down state, arising with probability $1 - p$, it yields R_D. The risk-free bond yields a sure gross return of R_f. We assume that the time value of money is positive, i.e. that interest rates are non-negative. Absence of arbitrage requires that $R_U > R_f > R_D$. For simplicity and without loss of generality we assume further that $R_D < 1$. To prevent negative stock prices we assume $R_D \geq 0$. These assumptions about the financial market are summarized in the following inequality: $R_U > R_f \geq 1 > R_D \geq 0$. All the parameters are assumed to be constant over time.

The preferences of the investor are based on changes in wealth and described by prospect theory. We assume that he owns an initial endowment, W_0, and that he earns no other income. Since the majority of the evidence reports a disposition behavior for individual investors[3] we want to model a small individual investor and therefore assume that no short selling is allowed. Further we assume that the investor acts myopically [4] and that the reference point relative to which he measures his gains and losses is his initial wealth.

Under prospect theory, the overall value of a prospect is given by the sum of the subjective values of the outcomes weighted by the agent's decision

[2]The assumption that only one stock is traded on the market can be justified by mental accounting, an element in the standard argument. Mental accounting stands for the concept that individuals divide their assets into separate and non-transferable portions.

[3]E.g. Feng and Seasholes (2005) show that the more sophisticated investors are and the more trading experience they have, the less they are prone to the disposition effect.

[4]Assuming a myopic behavior for individual investors is appropriate for a descriptive model. It is consistent with the concept of narrow framing, i.e. the observation that individuals focus on the immediate future.

weights associated with the probability of the outcome. The overall value of a prospect yielding a gain x with probability p and a loss y with probability $1-p$ is given by: $V(x,p;y,1-p) = w(p)v(x) + w(1-p)v(y)$. The decision weights w measure the impact of events on the desirability of prospects. Following the authors the decision weights take the following form

$$w(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{\frac{1}{\gamma}}}, \text{ for some } 0 \leq \gamma \leq 1. \tag{1}$$

The value function v assigns to each outcome x a number $v(x)$ which reflects the subjective value of that outcome. The key features of prospect theory are the coding of outcomes into gains and losses, that a loss hurts more than an equivalent gain and asymmetric risk-taking behavior. Based on empirical evidence Tversky and Kahneman (1992) proposed a two part power function

$$v(x) = \begin{cases} (x)^{\alpha} & \text{if } x \geq 0 \\ -\beta(-x)^{\alpha} & \text{if } x < 0 \end{cases}. \tag{2}$$

The parameter β is the coefficient of loss aversion and reflects the fact that losses hurt more than equivalent gains, which is true for all $\beta > 1$. Using data from their experiments the authors estimated β to be equal to 2.25. The coefficient α measures the agent's risk aversion and takes on values between zero and one. The authors estimated α to be equal to 0.88. Observe that in the domain of gains, i.e. $x \geq 0$, the value function is concave, implying that the agent is risk averse, whereas for the domain of losses the function is convex, i.e. the investor prefers to gamble instead of facing a sure loss. We assume that all parameters are constant over time.

The investor's portfolio decision consists of allocating his wealth to the two assets traded in the financial market. In state S he maximizes his utility by allocating a fraction λ_S of his wealth in the risky asset and $1-\lambda_S$ in the riskless asset.

In $t=0$ the investor owns his initial wealth W_0. With probability p the stock price goes up and the good state realizes. In this case the investors wealth is W_U.[5] The investor's wealth position in the up state equals his initial wealth multiplied by the portfolio return. The portfolio return depends on the returns offered by the traded securities and on the investors portfolio

[5]Note that from now on we simply use the unambiguous short cut U, for the up state in $t=1$, and D for the down state in $t=1$. Similarly for $t=2$.

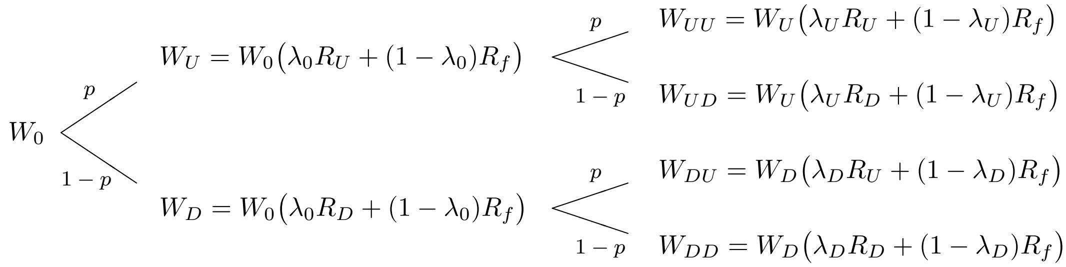

Figure 1: Evolution of the investors wealth, W_S.

decision, i.e. the fraction of wealth invested in the risky asset, λ_0. The bad state realizes with probability $1-p$ and the stock price depreciates. The agent's wealth position is W_D.

As we assume that in our model all the parameters are constant over time, the setting in the second period has the same structure as in the first period. After the investor has made his first period investment decision the state of nature in $t=1$ realizes. The market parameters, the investment decision λ_0 and the realized state of nature determine the agent's wealth in $t=1$. In the second period the investor allocates his wealth from $t=1$ to the two assets traded in the financial market. The investors wealth position in $t=2$ equals his position in $t=1$ multiplied by the return of his portfolio in the second period. The situation the investor is confronted with is depicted in Figure 1.

In each period $t=0,1$ and state $S=U,D$ the investor solves the following optimization problem

$$max_{1\geq\lambda_S\geq 0} \quad V(\lambda_S),$$

where

$$\begin{aligned} V &= w(p)v(W_{SU}-W_0) + w(1-p)v(W_{SD}-W_0), \qquad (3)\\ W_{SU} &= W_S\big(\lambda_S R_U + (1-\lambda_S)R_f\big),\\ W_{SD} &= W_S\big(\lambda_S R_D + (1-\lambda_S)R_f\big), \end{aligned}$$

$w(p)$ is defined in (1), $v(x)$ in (2) and $S=0,U,D$.[6]

The disposition effect, i.e. the observation that investors sell winners and hold losers, arises whenever $\lambda_D > \lambda_0 > \lambda_U$. We require the inequalities to

[6]Note that in the text we use the convenient short-hand notations W_U for W_{0U} and W_D for W_{0D}.

be strict to make a clear distinction between a disposition behavior and a fixed-mixed investment strategy as chosen by a expected utility maximizer with constant relative risk aversion.

The condition for the occurrence of the disposition effect is computationally not tractable.[7] To gain some insight into the solution we restrict the fraction of wealth invested in the risky asset to be either zero or one, which implies that the agent chooses to invest fully or not to invest at all in the risky asset.[8] After having understood this more tractable case in Section 3.3 we come back to the general case introduced above.

If in $S = 0$ the expected utility from holding the risky asset exceeds the utility from investing in the risk free bond the agent will invest in stocks. Otherwise, the agent prefers to invest his entire wealth in the risk-free bond. Hence he invests his entire wealth in the risky asset whenever $V(\lambda_0 = 1) > V(\lambda_0 = 0)$, or

$$w(p)(R_U - 1)^\alpha - w(1-p)\beta(1 - R_D)^\alpha > (R_f - 1)^\alpha. \tag{4}$$

Next we state the corresponding conditions for the other states. Note that in $t = 0$, when the investor chooses to invest in the stock he experiences a gain whenever the stock price rises; if it falls he experiences a loss. However, in $t = 1$ we have to distinguish different cases which imply different possible portfolio performances in terms of gains and losses, which in turn implies different valuations.

In the first case, where $R_U R_D > 1$ and $R_f R_D > 1$ [9], the agent, who invests in $S = U$ his entire wealth in the risky asset, experiences a gain in both states and he makes a sure gain, if he invests in the riskless bond. If the down state realized in the first period, the investor who buys the risky asset in $S = D$ may make a gain, if after the bad state the good state realizes, or a loss, after the realization of two consecutive down states. If he chooses to put his wealth in the risk-free alternative, he makes a sure gain.

In the second case, where $R_U R_D > 1$ and $R_f R_D < 1$, the investor who in $S = U$ invests his entire wealth in the risky asset, experiences a gain in

[7] For a detailed discussion of the solution of the above optimization problem we refer the reader to Vlcek (2005).

[8] A possible interpretation is that the risky asset is a project that absorbs all the agent's wealth. If the agent decides not to invest in the project he simply keeps his wealth in a risk-free bank account.

[9] From the assumptions made above it follows that $R_U R_U > 1, R_U R_f > 1$ and that $R_D R_D < 1$.

	$R_U R_D > 1$, $R_f R_D > 1$		$R_U R_D > 1$, $R_f R_D < 1$		$R_U R_D < 1$, $R_f R_D < 1$	
	U	D	U	D	U	D
$\lambda_U = 1$	gain	gain	gain	gain	gain	loss
$\lambda_U = 0$	gain	gain	gain	gain	gain	gain
$\lambda_D = 1$	gain	loss	gain	loss	loss	loss
$\lambda_D = 0$	gain	gain	loss	loss	loss	loss

Table 1: Different cases which imply different possible portfolio performances in terms of gains and losses in $t = 1$.

both states and he makes a sure gain, if he invests in the riskless bond. If the down state realizes in the first period and the investor invests in the risky asset, he experiences a gain and a loss. If the investor chooses to put all his wealth in the risk-free alternative, he makes a sure loss.

In the third case, where $R_U R_D < 1$ and $R_f R_D < 1$, the investor, who buys the risky asset in $S = U$, may make a gain, if after the up state the good state realizes, or a loss, if after the up state the down state realizes. He makes a sure gain, when investing in the risk-free bond. If the down state realizes and the agent invests in the risky asset, he experiences a loss independent of which state realizes in the second period. If the investor chooses to put all his wealth in the risk-free alternative, he makes a sure loss. We summarize the possible cases and the consequences in Table 1.

In the first two cases, i.e. when $R_U R_D > 1$ and $R_f R_D > 1$ and when $R_U R_D > 1$ and $R_f R_D < 1$, the condition that the agent invests in the risky asset after the stock price appreciated in the first period is

$$w(p)(R_U R_U - 1)^\alpha + w(1-p)(R_U R_D - 1)^\alpha > (R_U R_f - 1)^\alpha. \tag{5}$$

In the third case, where $R_U R_D < 1$ and $R_f R_D < 1$ the agent prefers the risky asset to the risk-free bond whenever

$$w(p)(R_U R_U - 1)^\alpha - w(1-p)\beta(1 - R_U R_D)^\alpha > (R_U R_f - 1)^\alpha. \tag{6}$$

Similarly, the condition that the agent invests in the risky asset after the stock price depreciated is in the case where $R_U R_D > 1$ and $R_f R_D > 1$

$$w(p)(R_U R_D - 1)^\alpha - w(1-p)\beta(1 - R_D R_D)^\alpha > (R_f R_D - 1)^\alpha, \tag{7}$$

in the case where $R_U R_D > 1$ and $R_f R_D < 1$

$$w(p)(R_U R_D - 1)^\alpha - w(1-p)\beta(1 - R_D R_D)^\alpha > -\beta(1 - R_f R_D)^\alpha, \quad (8)$$

and in the case where $R_U R_D < 1$ and $R_f R_D < 1$

$$w(p)(1 - R_U R_D)^\alpha + w(1-p)\beta(1 - R_D R_D)^\alpha < (1 - R_f R_D)^\alpha. \quad (9)$$

In the described setting the disposition effect is the situation, where the agent invests in the risky asset in $t = 0$, sells the asset after the price appreciated and keeps on holding the risky stock after its price went down. This means that we observe the disposition effect whenever $\lambda_0 = 1$, $\lambda_U = 0$ and $\lambda_D = 1$. Thus the conditions for the disposition effect to occur are[10]:

1. In the case, where $R_U R_D > 1$ and $R_f R_D > 1$:

$$\begin{aligned} w(p)(R_U - 1)^\alpha - w(1-p)\beta(1 - R_D)^\alpha &\geq (R_f - 1)^\alpha, \\ w(p)(R_U R_U - 1)^\alpha + w(1-p)(R_U R_D - 1)^\alpha &\leq (R_U R_f - 1)^\alpha \quad \text{and} \\ w(p)(R_U R_D - 1)^\alpha - w(1-p)\beta(1 - R_D R_D)^\alpha &\geq (R_f R_D - 1)^\alpha. \end{aligned} \quad (10)$$

2. In the case, where $R_U R_D > 1$ and $R_f R_D < 1$:

$$\begin{aligned} w(p)(R_U - 1)^\alpha - w(1-p)\beta(1 - R_D)^\alpha &\geq (R_f - 1)^\alpha, \\ w(p)(R_U R_U - 1)^\alpha + w(1-p)(R_U R_D - 1)^\alpha &\leq (R_U R_f - 1)^\alpha \quad \text{and} \\ w(p)(R_U R_D - 1)^\alpha - w(1-p)\beta(1 - R_D R_D)^\alpha &\geq -\beta(1 - R_f R_D)^\alpha. \end{aligned} \quad (11)$$

3. In the case, where $R_U R_D < 1$ and $R_f R_D < 1$:

$$\begin{aligned} w(p)(R_U - 1)^\alpha - w(1-p)\beta(1 - R_D)^\alpha &\geq (R_f - 1)^\alpha, \\ w(p)(R_U R_U - 1)^\alpha - w(1-p)\beta(1 - R_U R_D)^\alpha &\leq (R_U R_f - 1)^\alpha \quad \text{and} \\ w(p)(1 - R_U R_D)^\alpha + w(1-p)(1 - R_D R_D)^\alpha &\leq (1 - R_f R_D)^\alpha. \end{aligned} \quad (12)$$

[10]We assume that when the investor is indifferent between the risky and the riskless asset, he purchases the stock in $S = 0$ and $S = D$ and he invests in the bond in $S = U$.

In what follows, we investigate these conditions. First we analyze the conditions for the ex-post disposition effect, i.e. the condition that the investor prefers simultaneously to invest in $S = U$ in the risk-free bond and in $S = D$ in the stock. Then we take an ex-ante perspective and require that the agent has to prefer the stock in $S = 0$, the bond in $S = U$ and the stock in $s = D$. We complete these findings with numerical results about the occurrence of the disposition effect for the case where the agent can choose any degree of investment, i.e. where λ_S is not restricted to zero or one.

3 Results

In this section we present the results of our model. We first analyze the case, where λ_S is restricted to be zero or one. This allows us to make concrete statements about the conditions for the occurrence of the disposition effect and to provide intuition for the results. Then we provide numerical results for the general case, where we quantify the occurrence of the disposition effect.

For the restricted case, we first discuss the relationship between the (ex-post) disposition effect and loss aversion. Next, we take on the traditional view, where it is implicitly assumed that the investor already owns the risky stock and analyze his behavior given the stock price movement. We show that in fact the ex-post disposition behavior is consistent with most of the parameter combinations. Then we take on a completer view, and require for the disposition effect not only that the investor sells a winning asset and keeps a losing asset, but also that the agent decides to buy the risky stock in $S = 0$. We show that the disposition effect arises very rarely.

Analyzing the conditions for the occurrence of the (ex-post) disposition effect allows us to discuss the role of loss aversion. A first observation is that if the market parameters satisfy the condition $R_U R_D > 1$ and if the disposition effect arises for a $\beta_1 > 1$, then it arises for all β_2, where $\beta_1 > \beta_2 > 1$. The intuition is that an investor that is less loss averse more readily buys the risky stock in $S = 0$ and $S = D$. Note that since the agent does not face a loss in $S = UD$ when investing in the risky asset, the condition to sell the stock in $S = U$ is independent of loss aversion.

If $R_U R_D < 1$ then the agent makes a loss in $S = DU$ and $S = DD$, independently of his investment decision, so that the investment decision in $S = D$ is independent of loss aversion. On the other hand, in $S = U$ the investor faces a potential loss in $S = UD$, when holding the risky asset and therefore the more he is loss averse, the more he prefers the risk-free asset. Note that the effects of an increase in loss aversion go in opposite directions for the conditions in $S = 0$ and $S = U$. In absolute terms the effect is stronger in $S = 0$, so that if the disposition effect arises for a $\beta_1 > 1$, then it arises for all β_2, where $\beta_1 > \beta_2 > 1$. Again, a lower loss aversion implies that the investor more readily invests in the risky asset in the first period.

From these statements it follows that the ex-post disposition effect arises more often for lower coefficients of loss aversion if $R_U R_D < 1$ and for higher coefficients of loss aversion if $R_U R_D > 1$.

3.1 The Ex-post Disposition Effect

In this section we assume that the investor is endowed with the risky asset and analyze his portfolio decision given a stock price movement.

The investment decision as described above depends on the parameters of the agent's preferences, α, β and γ, as well as the parameters of the financial market, i.e. the possible returns and the probabilities of the possible states. Since many different parameters are involved, we look first at different special cases in order to isolate the different effects of the parameters. As we have seen above, a lower loss aversion coefficient β favors the occurrence of the ex-post disposition effect whenever $R_U R_D > 1$ and it lowers it in the opposite case. In this section we focus on the impacts of the parameter of the decision weighting function γ and the coefficient of risk aversion α. We assume that the investor is loss averse, i.e. $\beta > 1$.

To get more insights, we vary the two parameters in the following way: the parameter of the decision weighting function γ is either fixed at 1, so that the investor weights the outcomes with the objective probabilities or it is assumed to lie between 0 and 1. When the coefficient of risk aversion α is fixed, it is kept constant either at 0 or at 1. When $\alpha = 0$ the value function is flat, in both the gain and loss domain. This implies, that after a first gain an additional gain does not yield an additional utility; similarly, after a first loss, an additional loss does not hurt more. In this sense we can say that if $\alpha = 0$ the investor is quite risk-averse in the domain of gains and quite risk-seeking in the domain of losses. The other case, where $\alpha = 1$, implies a piece wise linear value function and that the investor is risk neutral in the gain and loss domain. Note that whenever a gain and a loss can occur, the value function over the whole domain is concave. This follows from the kink at the origin, i.e. from loss aversion. Hence the investor is risk averse. Otherwise α is assumed to lie between 0 and 1.

This yields six possible situations. The more restrictions we impose on the preference parameters, the more tractable the inequalities describing the agents choices become. Allowing for more general parameter ranges often has the drawback that no analytical statements can be made, so that we have to provide numerical solutions.

Proposition 1 summarizes the results for the cases, where analytical statements can be made. The detailed proofs can be found in the appendix.

Proposition 1. *The ex-post disposition effect*

1. *An investor who weights outcomes with their objective probabilities and is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e.* $\gamma = 1$ *and* $\alpha = 0$*, is prone to the ex-post disposition effect whenever* $R_f R_D < 1$*.*

2. *An investor who is risk neutral in the gain and loss domain and weights outcomes with their objective probabilities, i.e.* $\gamma = 1$ *and* $\alpha = 1$*, is prone to the ex-post disposition effect whenever* $R_U R_D < 1$ *and* $\phi_4 \geq p \geq \phi_1$*, where* $\phi_4 = \frac{R_U R_f - 1 + \beta(1 - R_U R_D)}{R_U R_U - 1 + \beta(1 - R_U R_D)}$ *and* $\phi_1 = \frac{R_f - R_D}{R_U - R_D}$*.*

3. *An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and who is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e.* $0 < \gamma < 1$ *and* $\alpha = 0$*, is prone to the ex-post disposition effect whenever* $R_f R_D < 1$*.*

An investor who weighs outcomes with the objective probability and is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $\gamma = 1$ and $\alpha = 0$, is prone to the ex-post disposition effect whenever $R_f R_D < 1$. The reason is that in $S = U$ the agent is in the gain zone and hence quite risk averse so that he never prefers the risky stock. In $S = D$, if $R_f R_D > 1$ the investor has the opportunity to realize a sure gain by investing in the risk free bond. Therefore he prefers the risk free bond. However, if $R_f R_D < 1$, the investor being in the loss zone is quite risk-seeking and invests therefore in the risky asset. Note that these statements apply even when the investor is not loss averse.

An investor who is risk neutral in the gain and loss domain and weights outcomes with their objective probabilities, i.e. $\gamma = 1$ and $\alpha = 1$, is prone to the ex-post disposition effect whenever after a first period loss, the agent cannot undo this loss, i.e. $R_U R_D < 1$, and the probability of the occurrence of the good state is bounded by ϕ_4 from above and by ϕ_1, the martingale probability for the stock price to rise, from below. This is the situation where the stock has a very high downside risk. We emphasize that even for an investor who has no asymmetric risk taking behavior, i.e. who is risk neutral in the gain and loss domain ex-post disposition effect arises. However only for restricted parameter values.

An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and who is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $0 < \gamma < 1$ and $\alpha = 0$, is prone to the ex-post disposition effect whenever $R_f R_D < 1$. The same reasoning as in the situation where $\gamma = 1$ applies. Note that for an investor with $\alpha = 0$ probability weighting has no impact on the occurrence of the ex-post disposition effect.

For the other combinations of α and γ no unambiguous conclusions can be drawn. Therefore we provide a numerical analysis.

To illustrate the situation where $\gamma = 1$ and $0 < \alpha < 1$ we present Figure 2. It shows the parameter combinations for which the ex-post disposition effect arises for different returns of the risky asset, R_D and R_U. In the following figures the value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up state, p is fixed at 0.5. The values of R_D vary between 0 and 1 and R_U is varied between 1.1 and 2.1. For other values of p and R_f similar results are obtained.[11] The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. These values correspond to the empirical findings of Tversky and Kahneman (1992).[12] The parameter of the decision weights γ is fixed at 1. The parameter combinations, where the ex-post disposition effect occurs, are marked with black color, whereas the domains, where the conditions for the ex-post disposition effect are violated, are marked with grey color. In Figure 2 we see that the ex-post disposition effect occurs rarely, in about 12% of the cases. We observe it for moderate and low returns in the down state and high returns in the up state. We can conclude that the ex-post

[11]One of these is the case where $R_f = 1$. It implies that for the investor the alternative to buying the stock is holding cash. Note that in this case $R_f R_D < 1, \forall R_D$ and hence the first case, where $R_U R_D > 1$ and $R_f R_D > 1$ never arises. Therefore we analyze in our numerical result the general case where $R_f > 1$.

[12]For other parameter values similar results are obtained.

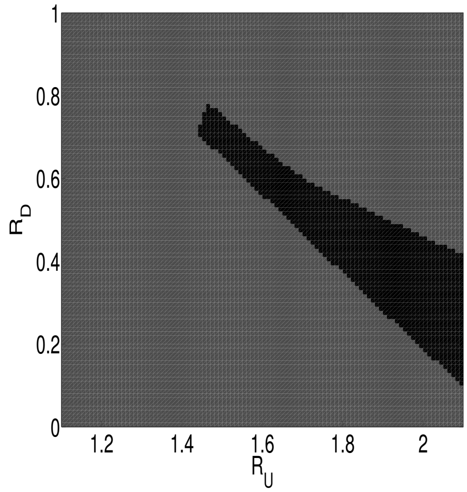

Figure 2: Return combinations for which the ex-post disposition effect arises. The values of R_D vary between 0 and 1 and R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 1. The parameter combinations, where the ex-post disposition effect occurs are marked with black color. The ex-post disposition effect occurs in about 12% of the cases.

disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and Kahneman (1992) and $\gamma = 1$ is a special case and does not occur in general.

To illustrate the situation where $0 < \gamma < 1$ and $\alpha = 1$ we present Figure 3. It shows the parameter combinations for which the ex-post disposition effect arises for different returns of the risky asset, R_D and R_U. Except for α and γ the same parameter values as above are used. The parameter combinations, where the ex-post disposition effect occurs are marked with black color. In Figure 3 we see that the ex-post disposition effect occurs often, in about 50% of the cases. We observe it predominantly for moderate and low returns in the down state. We can conclude that the ex-post disposition behavior for an agent that is described with parameters consistent with empirical findings

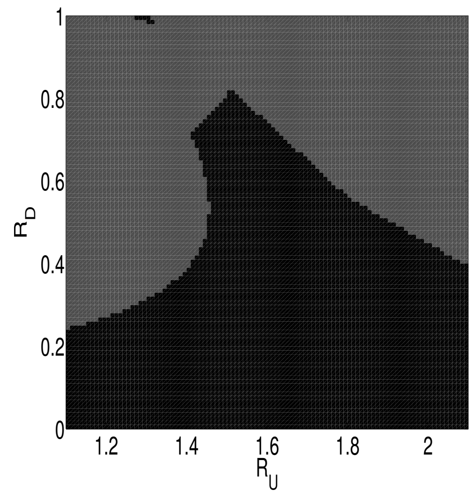

Figure 3: Return combinations for which the ex-post disposition effect arises. The values of R_D vary between 0 and 1 and R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient of risk aversion α equals 1. The parameter of the decision weights γ is fixed at 0.65. The parameter combinations, where the ex-post disposition effect occurs are marked with black color. The ex-post disposition effect occurs in about 50% of the cases.

of Tversky and Kahneman (1992) and $\alpha = 1$ does occur in general for risky assets with a high downside risk.

To illustrate the most general case, i.e. the situation where $0 < \gamma < 1$ and $0 < \alpha < 1$, we present Figure 4. Except for α and γ the same parameter values as above are used. The coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 0.65. These values correspond to the empirical findings of Tversky and Kahneman (1992).[13] The parameter combinations, where the ex-post disposition effect occurs are marked with black color. In Figure 4 we see that the ex-post disposition effect occurs

[13]Tversky and Kahneman have estimated the value of γ to be 0.61 if gains are involved and 0.69 when losses are involved. For simplicity we take the same value for gains and losses and set $\gamma = 0.65$. Again, for other parameter values similar results are obtained.

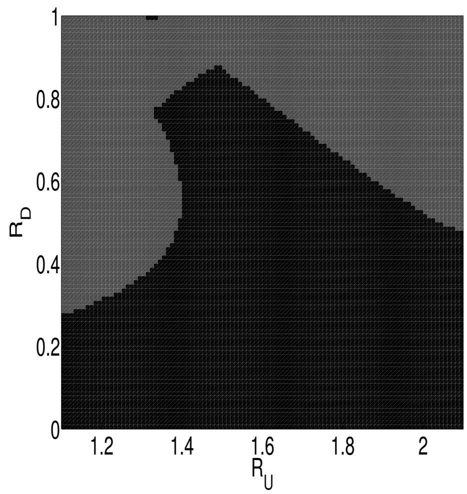

Figure 4: Return combinations for which the ex-post disposition effect arises. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 0.65. The parameter combinations, where the ex-post disposition effect occurs are marked with black color. The ex-post disposition effect occurs in about 59% of the cases.

often, in about 59% of the cases. We observe it for moderate and low returns in the down state. We can conclude that the ex-post disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and Kahneman (1992) does occur in general for risky assets with a high downside risk.

3.2 The True Disposition Effect

In this section we make one step backward in time and impose the additional condition that besides selling a winning stock and keeping a losing stock the investor has to buy the stock in the first place. So that the disposition effect arises whenever the requirements to simultaneously prefer the stock in $S = 0$

and $S = D$ and to prefer the bond in $S = D$ are satisfied. This makes the definition of the disposition effect more consistent. Since the conditions for the disposition effect in $t = 1$ stay the same as for the ex-post disposition effect we focus in this section on the ex-ante condition.

As above, we first look at different special cases in order to isolate the different effects of the parameters. We focus on the impacts of the parameter of the decision weighting function γ and the coefficient of risk aversion α and assume that the investor is loss averse.

We vary the two parameters as in the previous section: the parameter of the decision weighting function γ is either fixed at 1, so that the investor weights the outcomes with the objective probabilities or it is assumed to be between 0 and 1. When the coefficient of risk aversion α is fixed, it is kept constant either at 0 or 1. Otherwise it is assumed to be between 0 and 1. This yields six possible situations. The more restriction we impose on the preference parameters, the more tractable the inequalities describing the agents choices become. Allowing for more general parameter ranges often has the drawback that no analytical statements can be made, so that we have to provide numerical solutions.

Proposition 2 summarizes the results for the cases where analytical statements can be made. The detailed proofs can be found in the appendix.

Proposition 2. *The true disposition effect*

1. *An investor who weights outcomes with their objective probabilities and is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $\gamma = 1$ and $\alpha = 0$, never is prone to the disposition effect.*

2. *An investor who is risk neutral in the gain and loss domain and weights outcomes with their objective probabilities, i.e. $\gamma = 1$ and $\alpha = 1$, never is prone to the disposition effect.*

3. *An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and who is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, i.e. $0 < \gamma < 1$ and $\alpha = 0$, never is prone to the disposition effect.*

An investor who weights outcomes with their objective probabilities and who is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, as long as he is loss averse never invests in the risky asset in $S = 0$ This implies that he cannot be prone to the disposition effect.

An investor who is risk neutral in the gain and loss domain and weights outcomes with their objective probabilities, never is prone to the disposition effect. The reason is that he either does not purchase the stock in $S = 0$ or, that if he does so, he never sells it after a gain.

An investor who weights outcomes with the decision weights as proposed by Tversky and Kahneman (1992) and who is quite risk averse in the domain of gains and quite risk-seeking in the domain of losses, never invests in the risky asset in $S = 0$ implying that he is not prone to the disposition effect.

For the other combinations of α and γ no unambiguous conclusions can be drawn. Therefore we provide a numerical analysis.

To illustrate the situation where $\gamma = 1$ and $0 < \alpha < 1$ we present Figure 5. It shows the parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. In the following graphics the value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up state, p is fixed at 0.5. The values of R_D vary between 0 and 1 and R_U is varied between 1.1 and 2.1. For other values of p and R_f similar results are obtained. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 1. These values correspond to the empirical findings of Tversky and Kahneman (1992). [14] The parameter combinations, where the disposition effect occurs are marked with black color, whereas the domains, where the conditions for the disposition effect are violated are marked with grey color. In Figure 5 we see that the disposition effect almost never occurs, in fact overall it occurs in less than 0.5% of the cases.

To illustrate the situation where $0 < \gamma < 1$ and $\alpha = 1$ we present Figure 6. Except for α and γ the same parameter values as above are used. The parameter combinations, where the disposition effect occurs are marked with black color. In Figure 6 we see that the disposition effect occurs very rarely, in less than 0.5% of the cases. We observe it for very high returns in the down state and returns in the up state of the order 1.3. We can conclude that the disposition behavior for an agent that is described with parameters consistent with the empirical findings of Tversky and Kahneman (1992) and $\alpha = 1$ is a very special case and does not occur in general.

[14] Again, for other parameter values similar results are obtained.

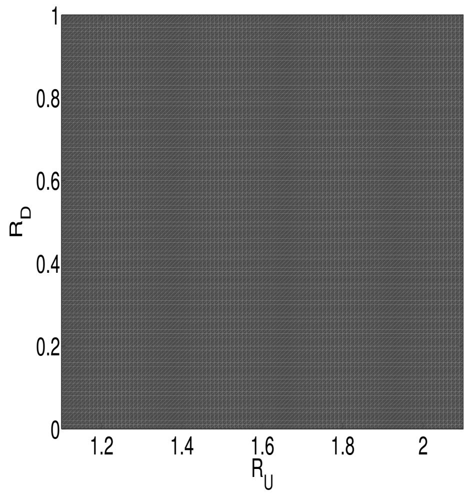

Figure 5: Parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 1. The parameter combinations, where the disposition effect occurs are marked with black color. The disposition effect occurs in less than 0.5% of the cases.

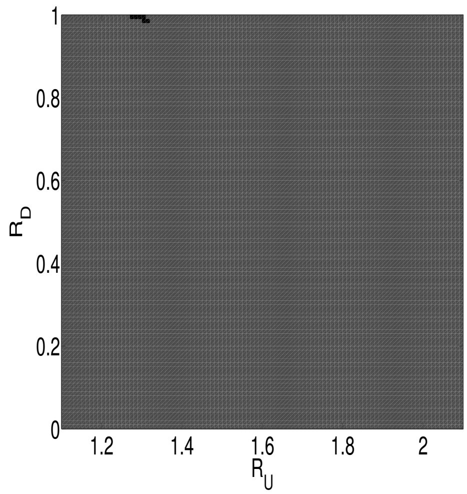

Figure 6: Parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 1. The parameter of the decision weights γ is fixed at 0.65. The parameter combinations, where the disposition effect occurs are marked with black color. The disposition effect occurs in less than 0.5% of the cases.

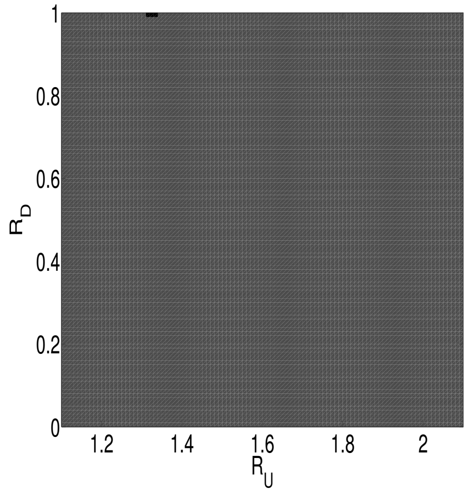

Figure 7: Parameter combinations for which the disposition effect arises for different returns of the risky asset, R_D and R_U. The values of R_D vary between 0 and 1 and the R_U is varied between 1.1 and 2.1. The value of the gross risk free rate, R_f, is kept constant at 1.1 and the probability of the occurrence of the up state, p is fixed at 0.5. The loss aversion coefficient β is kept constant at 2.25 and the coefficient for risk aversion α equals 0.88. The parameter of the decision weights γ is fixed at 0.65. The parameter combinations, where the disposition effect occurs are marked with black color. The disposition effect occurs in less than 0.5% of the cases.

To illustrate the general case, i.e the situation where $0 < \gamma < 1$ and $0 < \alpha < 1$, we present Figure 7. We see that the disposition effect occurs very rarely, in less than 0.5% of the cases. We observe it for very high returns in the down state and returns in the up state of the order 1.3. We can conclude that the disposition behavior for an agent that is described with parameters consistent with empirical findings of Tversky and Kahneman (1992) is a very special case and does not occur in general.

To gain more insight on the different drivers of the disposition effect we present Figure 8, where we take a preference oriented view. We present the parameter combinations where the disposition effect occurs in the general case in dependence of risk aversion α and loss aversion β; α ranges from 0 to 1 and β from 1 to 5. The market parameters are fixed for one of the cases

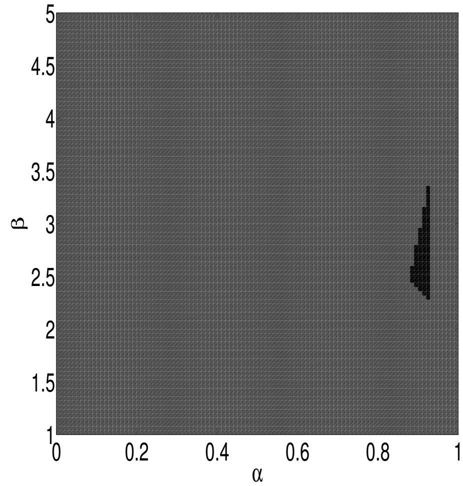

Figure 8: Parameter combinations for which the disposition effect arises in dependence of risk aversion α and loss aversion β; α ranges from 0 to 1 and β from 1 to 5. The market parameters are fixed for the case where we observed the disposition effect, i.e. $p = 0.5$, $R_U = 1.32$, $R_f = 1.1$, $R_D = 0.99$ and $\gamma = 0.65$.

where we observed the disposition effect, i.e. $p = 0.5$, $R_U = 1.32$, $R_f = 1.1$, $R_D = 0.99$ and $\gamma = 0.65$. Again we observe that the disposition effect occurs only for a very small part of the possible parameter combinations and cannot be considered a systematic phenomenon.

3.3 Any Degree of Investment

In order to generalize our results, we relax the restriction that the investor has to invest either fully or not at all. Since in this case, the conditions for the occurrence of the disposition effect are computationally not tractable we provide numerical results.

While we allow the investor to chooses any degree of investment, we maintain all other assumptions; in particular the one that short selling is not allowed. The disposition effect, i.e. the observation that investors sell winners and hold losers, arises whenever $\lambda_D > \lambda_0 > \lambda_U$. Whereas in the setting where λ_S is restricted to be either zero or one, the definition of an

		Disposition Effect
$\gamma = 1,$	$\alpha = 0$	$< 0.5\%$
$\gamma = 1,$	$\alpha = 1$	$< 0.5\%$
$\gamma = 1,$	$0 < \alpha < 1$	$< 0.5\%$
$0 < \gamma < 1,$	$\alpha = 0$	$< 0.5\%$
$0 < \gamma < 1,$	$\alpha = 1$	$< 0.5\%$
$0 < \gamma < 1,$	$0 < \alpha < 1$	$< 0.5\%$

Table 2: Any Degree of Investment. We quantify the occurrence of the disposition effect for the following parameter values: $p = 0.5$, $R_U \in [1.1, 2.1]$ $R_f = 1.1$ and $R_D \in [0, 1]$. If no other parameter values are assumed, then $\alpha = 0.88$, $\beta = 2.25$, and $\gamma = 0.65$.

ex-post disposition behavior is evident, it is not in the case where the investor can choose any degree of investment. Particularly, the assumption about the first period endowment is ambiguous. Since the quantity of the first period endowment influences significantly the statements about the occurrence of the ex-post disposition effect, we will not provide such results.

As above, we quantify the occurrence of the disposition effect for different values of α, γ, R_U and R_D. The results, in Table 2, were calculated for the following parameter values: $p = 0.5$, $R_U \in [1.1, 2.1]$ $R_f = 1.1$ and $R_D \in [0, 1]$. If no other parameter values are assumed, then $\alpha = 0.88$, $\beta = 2.25$, and $\gamma = 0.65$.

The results show that even when allowing for any degree of investment, the disposition effect does practically not occur, supporting the results from above. In the next section, we will discuss our results.

4 Discussion

In this section we discuss the results. Since the conditions for the occurrence of the disposition effect are more tractable in the case where λ_S is restricted we mainly refer to this case.

We first discuss the role of loss aversion on the conditions under which the (ex-post) disposition effect occurs. A general observation is that an investor who is less loss averse more readily invests in the risky alternative. The less a potential loss hurts, the more he invests in the risky alternative. Concerning the conditions for the disposition effect the consequences of an increase in loss aversion go in opposite directions for $S = 0$ and $S = U$. In absolute terms the effect is stronger in $S = 0$, so that the disposition effect occurs more often for low coefficients of risk aversion.

When the investor cannot lose his first period gain when holding the risky stock in $S = U$, i.e. when $R_U R_D > 1$, his decision in that node is independent of loss aversion. Therefore a lower coefficient of loss aversion favors the occurrence of the ex-post disposition effect. In the opposite case, i.e. when $R_U R_D > 1$ and the investor cannot undo a first period loss, a higher coefficient of loss aversion favors the ex-post disposition effect.

We summarize the impact of risk aversion, α, and probability weighting, γ, in Table 3. In some cases we are able to make analytical statements. In others we rely on numerical computations. When we quantify the occurrence of the (ex-post) disposition effect, we use the following parameter values: $p = 0.5$, $R_U \in [1.1, 2.1]$ $R_f = 1.1$ and $R_D \in [0, 1]$. If no other parameter values are assumed explicitly, then $\alpha = 0.88$, $\beta = 2.25$, and $\gamma = 0.65$.[15]

A first observation is that the ex-post disposition effect occurs quite often. It occurs particularly often in the case where $\alpha = 0$, i.e. where after an initial gain (loss) an additional gain (loss) does not yield any additional utility (pain). In this case the investor sells winners and holds losers, unless he can undo a first period loss by investing in the risk-free alternative, i.e. $R_f R_D > 1$.

In the cases where we can make analytical statements our results show clearly that the ex-post disposition effect occurs when the risky asset has a high downside risk, i.e. for low values of R_D. In the other cases, we observe from Figures 2 to 4 that the ex-post disposition effect arises rather for low values of R_D; again, when the risky asset has a high downside risk.

An investor who owns a risky asset that has a high downside risk, after a first period loss, i.e. in $S = D$, is deep in the loss zone. Therefore he is risk seeking. This implies that he will prefer the risky alternative to the risk free one. He holds the losing stock. On the other hand, after a gain, the investor

[15] Note that if the investor did decide his investment decision with a fair coin then we would observe the ex-post disposition effect in 25% of the cases, while the true disposition effect would occur in 12.5% of the cases.

		Ex-Post Disposition Effect	Disposition Effect
$\gamma = 1$,	$\alpha = 0$	If $R_f R_D < 1$, (90%)	Never
$\gamma = 1$,	$\alpha = 1$	If $R_U R_D < 1$, (6%)	Never
$\gamma = 1$,	$0 < \alpha < 1$	13%	$< 0.5\%$
$0 < \gamma < 1$,	$\alpha = 0$	If $R_f R_D < 1$, (90%)	Never
$0 < \gamma < 1$,	$\alpha = 1$	50%	$< 0.5\%$
$0 < \gamma < 1$,	$0 < \alpha < 1$	59%	$< 0.5\%$

Table 3: Summary of Results. We quantify the occurrence of the (ex-post) disposition effect for the following parameter values: $p = 0.5$, $R_U \in [1.1, 2.1]$ $R_f = 1.1$ and $R_D \in [0, 1]$. If no other parameter values are assumed, then $\alpha = 0.88$, $\beta = 2.25$, and $\gamma = 0.65$.

is in the gain zone and hence risk averse. Therefore, he prefers the safe investment to the risky stock. This preference is amplified for stocks with a high downside risk because of loss aversion, since the investor probably faces a loss even after the stock price went up in the first period. Therefore he sells winners.

Regarding probability weighting, we observe that the ex-post disposition effect occurs for lower values of γ. Under prospect theory investors overweight small probabilities and underweight moderate and high probabilities. In the case, where both states are equally likely, the investors underweight both outcomes. The more they underweight, the less attractive the risky alternative becomes. Therefore they sell winners. Moreover under the condition that $R_U R_D < 1$ and $R_f R_D < 1$ even in $S = D$ decision weighting works in favor of the ex-post disposition effect. The reason is that it lowers the disutility from investing in the stock.

The conditions for the occurrence of the true disposition effect are practically never satisfied. Even in the cases, where the ex-post disposition effect occurs for most of the parameter values. In the cases where we make analytical statements, we can prove that the true disposition effect never occurs. In the cases where we make numerical calculations, the occurrence of the disposition effect shrinks drastically. This shows, that investors that behave like ex-post disposition investors, would not have invested in the risky asset in the first place.

The intuition behind this result is the following. As we have seen above, the ex-post conditions are satisfied for stocks with a high down-side risk. However, the investor does not want to invest in such an asset in $S = 0$. On the other hand, if the stock is attractive enough and the investor chooses to buy it in $S = 0$ then he will not sell it in $S = U$.

Other numerical analysis for different parameter values, not shown here, confirm that the ex-post conditions are satisfied more often than conditions for the disposition effect and that the differences can be quite substantial.

Similar results are obtained for other forms of value functions, as e.g. the piece-wise exponential function. For preference parameter values that approximate best the empirical evidence found by Tversky and Kahneman[16] and market parameters as used above, we found that the ex-post disposition effect occurs in 59% of the cases, whereas the true disposition effect occurs in less than 0.5%.

Moreover, introducing editing rules of prospect theory, as e.g. segregation, does not change the results substantially. For the parameter values used above, we found that the ex-post disposition effect occurs in 65% of the cases and the true disposition effect in less than 0.5%. Finally, requiring dynamic instead of myopic optimization makes the risky asset more attractive in the first period because one anticipates to optimally react to the future course of events. However, whenever the agent prefers to invest in the risky asset in the first period, he prefers to keep it after its price appreciated. In this case the true disposition effect also occurs in less than 0.5% of the parameter combinations.

Hence we have shown that various approaches, incorporating different types of value functions and editing rules, have difficulties to model the disposition effect. Moreover allowing for any degree of investment does not explain the disposition effect. This suggests that in order to explain the disposition effect one must depart from the traditional forward looking optimization paradigm in a more radical way than replacing the von Neumann-Morgenstern utility function in the expected utility paradigm by the value function of prospect theory.

A possible alternative explanation could be to model the disposition effect as a consequence of a backward looking optimization. Given the past

[16]For a discussion and the concrete parameter values we refer the reader to DeGiorgi, Hens, and Levy (2005).

investment decision, the agent transforms the outcome such that he gets the highest utility: if the investment decision is successful, the agent realizes his gain, i.e. he transforms the outcome to a realized gain. If he incurs a loss, he keeps the outcome as a paper loss, i.e. he keeps holding the asset. One could model such a behavior using two mental accounts, one for realized gains and losses and the other for paper gains and losses. Clearly in such a model the positions in the paper account have less weight than the ones in the realized account: paper losses hurt less than realized losses and realized gains give more utility than paper gains. Hence behavior consistent with the disposition effect makes the best out of a given investment decision. Note that for this argument neither loss aversion nor asymmetric risk aversion is needed since it is sufficient to assume that the utility of a gain is positive while that of a loss is negative. However this behavior is not forward looking because the resulting asset allocation may not be optimal in the future. This explanation corresponds to the story told by Gross (1982), page 150: *Investors who accept losses can no longer prattle to their loved ones, "Honey, it's only a paper loss."*

5 Conclusions

In the literature the disposition effect is explained by two main features of prospect theory, namely that decision-makers frame their choices in terms of potential gains and losses and that they maximize an S-shaped value function, which is concave for gains and convex for losses. The argument is often made without considering loss aversion. As we have shown, the assumption of no loss aversion favors the occurrence of the disposition effect. However, even for investors that are not loss averse, the disposition behavior is rather a rare result. Further, in the standard argument, it is generally *assumed* that the investor has bought the risky stock in the first place. Therefore, the issue whether the investor really will decide in this way is neglected. This implies that the standard argument is in fact an ex-post argument. Our model shows that the inter-temporal disposition behavior occurs only for very restricted parameter values. In general, the model predicts that those investors who sell winning stocks too early and keep losing stocks too long would in the first place not have invested in stocks. We conclude that prospect theory can indeed explain the ex-post disposition behavior, but not the more complete and inter-temporal definition of the disposition behavior. Possible alternative explanations for the disposition effect could include mental accounting combined with backward looking optimization.

A Appendix

A.1 Proof of Proposition 1

1. We analyze the two conditions for $t = 1$ for the parameter combination $\gamma = 1$ and $\alpha = 0$. In the first case, where $R_f R_D > 1$ and the second case, where $R_U R_D > 1$ and $R_f R_D < 1$, the condition to sell the asset after a gain yields

$$p + (1 - p) \leq 1, \tag{13}$$

which is satisfied for all $0 < p < 1$. The condition for the investor to prefer the risky asset in $S = D$ in the first case yields

$$-(1 - p)\beta \geq 1 - p, \tag{14}$$

which yields a contradiction for all $\beta \geq 1$ and $0 < p < 1$, so that no ex-post disposition effect occurs. In the second case the condition yields

$$p \geq -p\beta, \tag{15}$$

which is satisfied for all $\beta \geq 1$ and $0 < p < 1$, so that the ex-post disposition effect does arise. In the third case, where $R_U R_D < 1$, the condition to sell the winning stock yields

$$-(1 - p)\beta \leq 1 - p, \tag{16}$$

which is satisfied for all $\beta \geq 1$ and $0 < p < 1$, and to hold a losing stock yields

$$p + (1 - p) \leq 1, \tag{17}$$

which is satisfied for all $0 < p < 1$, so that the ex-post disposition effect does arise.

Note that in the above inequalities the assumption about the investor's behavior when being indifferent is crucial. In case where the inequalities were strict, the ex-post disposition effect would not occur.

2. For the first case, where $R_f R_D > 1$, the ex-post condition is satisfied whenever

$$\begin{aligned}
&\phi_1 \geq p \geq \phi_2 \\
&\qquad \text{where} \quad \phi_1 = \frac{R_f - R_D}{R_U - R_D}, \\
&\qquad\qquad\quad\;\; \phi_2 = \frac{R_f R_D - 1 + \beta(1 - R_D R_D)}{R_U R_D - 1 + \beta(1 - R_D R_D)}.
\end{aligned} \tag{18}$$

In absence of arbitrage and for all $\beta > 1$ it follows that $\phi_2 > \phi_1$, so that this conditions is never satisfied.[17] For the case, where $R_U R_D > 1$ and $R_f R_D < 1$, the ex-post disposition effect arises whenever

$$\begin{aligned}
&\phi_1 \geq p \geq \phi_3 \\
&\qquad \text{where} \quad \phi_3 = \frac{\beta R_D (R_f - R_D)}{R_U R_D - 1 + \beta(1 - R_D R_D)}.
\end{aligned} \tag{19}$$

Note that in absence of arbitrage and for all $\beta > 1$ it follows that $\phi_3 > \phi_1$, so that this condition is never satisfied. For the case, where $R_U R_D < 1$, the ex-post disposition effect arises whenever

$$\begin{aligned}
&\phi_4 \geq p \geq \phi_1 \\
&\qquad \text{where} \quad \phi_4 = \frac{R_U R_f - 1 + \beta(1 - R_U R_D)}{R_U R_U - 1 + \beta(1 - R_U R_D)}.
\end{aligned} \tag{20}$$

Note that in absence of arbitrage and for all $\beta > 1$ $\phi_3 > \phi_1$.

3. In the first case, where $R_f R_D > 1$ the agent prefers to invest his wealth in $S = U$ in the risk free asset if

$$w(p) + w(1-p) \leq 1, \tag{21}$$

[17] Note that for an investor that is not loss avers, i.e. $\beta = 1$, $\phi_2 = \phi_1$ for all parameters, so that the investor is prone to the ex-post disposition effect in the special case where $p = \phi_2 = \phi_1$.

which is true for all $0 < \gamma < 1$ and $0 < p < 1$. The condition to prefer to invest in the risky asset in $S = D$ yields

$$-w(1-p)\beta \geq 1 - w(p), \tag{22}$$

which yields a contradiction for all $\beta \geq 1$ and $0 < w(x) < 1$. So that no ex-post disposition effect occurs. In the second case, where $R_U R_D > 1$ and $R_f R_D < 1$, the agent prefers to invest his wealth in $S = U$ in the risk free asset if

$$w(p) + w(1-p) \leq 1, \tag{23}$$

which is true for all $0 < \gamma < 1$ and $0 < p < 1$. The condition to prefer to invest in the risky asset in $S = D$ yields

$$w(p) \geq \big(w(1-p) - 1\big)\beta, \tag{24}$$

which is satisfied for all $\beta \geq 1$and $0 < w(x) < 1$. So that the ex-post disposition effect occurs in this case. In the third case, where $R_U R_D < 1$, the agent prefers to invest his wealth in $S = U$ in the risk free asset if

$$-w(1-p)\beta \leq 1 - w(p), \tag{25}$$

which is true for all $\beta \geq 1$ and $0 < w(x) < 1$. The condition to prefer to invest in the risky asset in $S = D$ yields

$$w(p) + w(1-p) \leq 1, \tag{26}$$

which is true for all $0 < \gamma < 1$ and $0 < p < 1$. So that the investor behaves from an ex-post perspective as a disposition investor whenever the investor makes a sure loss investing in the risk free asset in $S = D$.

□

A.2 Proof of Proposition 2

1. For the parameter combination $\gamma = 1$ and $\alpha = 0$ the condition to invest in the risky asset $t = 0$ writes:

$$-(1-p)\beta \geq 1 - p, \tag{27}$$

which is a contradiction for all $0 < p < 1$ and $\beta \geq 1$, since the left hand side is negative. Therefore the quite risk averse investor who weights outcomes with their objective probability never invests in the risky asset in $t = 0$ implying that he cannot be prone to the disposition effect.

2. For the parameter combination $\gamma = 1$ and $\alpha = 1$ in the first case, where $R_f R_D > 1$, the condition that the investor buys the stock in the first period and sells it after a gain yields

$$\begin{aligned} p(R_U - 1) - (1-p)\beta(1 - R_D) - R_f + 1 &\geq 0, \\ p(R_U R_U - 1) + (1-p)(R_U R_D - 1) - R_U R_f + 1 &\leq 0. \end{aligned} \tag{28}$$

These conditions cannot be satisfied simultaneously since combining them yields $(1-p)(\beta - 1)(R_D - 1) \geq 0$ which is a contradiction for all $0 < p < 1, \beta > 1$ and $R_D < 1$[18]. For the case, where $R_U R_D > 1$ and $R_f R_D < 1$ the conditions for the investor to buy the risky asset in $t = 0$ and to sell it after a gain, are the same as in the case, where $R_U R_D > 1$ and $R_f R_D > 1$.

For the case, where $R_U R_D < 1$ and $R_f R_D < 1$, the condition that the investor buys the stock in the first period and sells it after a gain yields $(1-p)(\beta - 1) \leq 0$ which is a contradiction for all $0 < p < 1$ and $\beta > 1$.

3. For the parameter combination $0 < \gamma < 1$ and $\alpha = 0$ the condition for $t = 0$ writes:

$$-w(1-p)\beta \geq 1 - w(p), \tag{29}$$

which is a contradiction for all $0 < w(p) < 1$ and $\beta \geq 1$. So that the quite risk averse investor never invests in the risky asset in $t = 0$ implying that he is not prone to the disposition effect.

[18] Note that an investor who is not loss averse, i.e. $\beta = 1$, would buy the stock in the first period and sell it after a gain.

□

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