# VALUATION OF ELECTRICITY FORWARD CONTRACTS: THE ROLE OF DEMAND AND CAPACITY


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**WORK IN PROGRESS**

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<u>(WORK IN PROGRESS)</u>


## Abstract

We propose a two factor model for the valuation of electricity derivatives contracts. We consider as state variables demand and available generation capacity. We model the state variables as affine jump-diffusions processes. Demand is modelled as a seasonal, mean-reverting process with seasonal volatility. Capacity is modelled as a mean-reverting process with (negative) jumps. Consistent with the empirical evidence, we model spot prices as a convex function of these two state variables. With this general specification we  derive a new pricing formula for electricity forwards in closed form. As a consequence, we also can express the forward risk premium in an analytical form. We are therefore able to provide a more detailed analysis on the different components of the forward risk premium (and forward prices), extending previous work by Barlow (*Math. Fin.*, 2002), Bessembinder and Lemmon (*J. of Finance*, 2002) and Longstaff and Wang (*J. of Finance*, 2004).

Since we also obtain the characteristic function of electricity prices, option prices could also be obtained in closed form up to the inversion of the Fourier Transform.

We empirically calibrate the model with data form the PJM market.

## 1. INTRODUCTION

The goal of this paper is to introduce a general framework to analyze the effect of supply ("generation capacity") and demand variables both on spot power prices and on forward power prices. Given the characteristics of electricity as a commodity, the electricity spot price is very volatile. As a commodity, fluctuations of electricity prices are due to changes in demand and supply conditions. Among the characteristics of electricity prices, two of them are crucial to understand its behavior. On one hand, electricity is non-storable. On the other hand, and related to its non-storability, electricity is an instantaneous commodity in the sense that supply and demand must be in equilibrium at all points in time. Electricity must be generated instantaneously to satisfy consumer's demand, instead of being produced and stored until it's demanded, like in other commodities. This fact implies the introduction of a supply variable related to "effective generation capacity" of an electric system much important than in other commodities in order to understand changes in spot prices and in power derivatives prices. In order to analyse the effect of demand and supply on spot (and forward) prices, we present a model that allow us to obtain in closed form forward prices. Moreover, the model is quite flexible allowing different specifications both for the modelling of the state variables and for the relationship between state variables and electricity prices. By obtaining electricity forward prices in closed form, we have a model that clearly states the determinants of the forward risk premium.

This paper is organized as follows. Firstly, we present an overview of previous work on valuation of electricity derivatives. Secondly, we analyze the characteristics of supply and demand of electricity. In Section 4 we propose a model for electricity spot price as a function of demand and "generation capacity". In Section 5 we present a model under the risk-neutral probability measure and we derive the valuation formulas. The last section presents some conclusions and possible extensions.

## 2. MODELS FOR THE VALUATION OF ELECTRICITY DERIVATIVES

There already exist some work on the valuation of electricity derivatives, although the literature is quite recent. On one hand, there is some work following the line of research initiated by and Schwartz and Smith (2000)[1] (SS, hereafter). In this line of research there exist some specific papers that deal with the valuation of electricity derivatives. Among these we have the papers of Lucia and Schwartz (2000) and Villaplana (2003). In both papers the state variables are modelled as

[1] Schwarzt and Smith (2000) propose a model where (log-) spot price is a function of a long-term (equilibrium) and a short-term variable. They apply the model to oil data and show the analytical equivalence of this model with more typical models that rely on the concept of convenience yield (see for instance Gibson and Schwartz, and Schwartz,1997). It must be noted that the convenience yield concept is related to the existence of inventories. Therefore since electricity is non-storable, models that use the concept of convenience yield cannot be used for the valuation of electricity derivatives.

non-observable taking into account the short-term movements and the long-term trends in electricity prices. While Lucia and Schwartz (2000) apply SS model to the valuation of electricity futures contracts traded in the Nord Pool (Scandinavian market), Villaplana (2003) extends SS model introducing the possibility of jumps in the short-term process, derives the valuation formulas under the new model and realizes an empirical analysis with forward data from the Pennsylvania-New Jersey-Maryland market (PJM hereafter).

On the other hand, given the particular characteristics of electricity as a commodity, we can identify the relevant observable state variables, mainly associated to supply[2] and demand variables. This line of research has been pursued by Eydeland and Geman (1998), Pirrong and Jermakyan (1999 and 2000), Barlow (2002), Bessembinder and Lemmon (2002), Longstaff and Wang (2004), Skantze et al. (2000) and Skantze and Ilic (2001) among others.

Pirrong and Jermakyan (1999 and 2000) propose to model the equilibrium price as a function of two state variables, electricity demand and the futures price of the marginal fuel. By introducing the futures price of the marginal fuel, the authors are trying to introduce an (observable) state variable related to the supply side. The authors consider electricity price should be an increasing and convex function of demand. Moreover, they introduce a seasonal (deterministic) function to model spot prices. In Pirrong and Jermakyan (2000) the specification of the price as a function of state variables is quite flexible and the estimation is carried through semi-parametric techniques. In Pirrong and Jermakyan (1999) a parametric functional form is imposed, we will back to this issue later on. One of their main findings is that electricity forward prices will differ from expected spot prices at maturity because the existence of an endogenous demand risk premium. The authors applied the model to data from PJM market. This market is characterized by high demand levels (and volatility) in summer months. Therefore, those forward contracts that mature in summer are more expensive, not just because spot prices during summer months are higher but mainly because the existence of an endogenous demand risk premium. Bessembinder and Lemmon (2002) have adopted an equilibrium perspective and they explicitly model the economic determinants in the forward market. Their model present many realistic features of the electricity market. In particular, producers face marginal production costs that may increase steeply with output. Aggregate demand is exogenous and stochastic. They demonstrate that the forward premium is positively related to the skewness of the spot price. One of the key insights is that the risk of price spikes due to sudden positive shocks in power demand can have important effects on the size and the sign of the forward premium. In their equilibrium model, the resulting expression for spot price is given by the

[2] Since electricity is non-storable, these models depart from the traditional literature on commodity derivatives pricing, that relies on the concept of convenience yield.

following expression: $P = a\left(\frac{D}{N}\right)^{c-1}$, where $D$ is the demand level, $N$ number of (symmetric) producers (generators), $a$ and $c$ are constants, $c > 2$. Note that by considering $N$ as a constant, the authors are assuming generation capacity is not a random variable. $N$ therefore would be a proxy for the generation capacity of a system. As Bessembinder and Lemmon (2002, p.1353) point out: *"if the cost parameter c is greater than two, marginal costs increase at an increasing rate with (are convex in) output.*(…) *The rapid increases in marginal costs implied by production levels that approach capacity can be approximated by considering the effect of increasing the cost convexity parameter, c. Note also that if c is greater than two, the distribution of power prices will be positively skewed even when the distribution of power demand is symmetric"*. Therefore, the parameter $c$ captures the convexity of the cost function.

One of their main findings is the existence of a positive forward risk premium (forward price higher than expected spot price) whenever expected demand or volatility of demand is high, the reason is the right skewness of spot price distribution. The observed positive skewness of spot prices is due to the existence of a convex cost function ("supply stack"). It can be shown in case of increasing marginal costs, spot price distribution will exhibit positive skewness even in the case of symmetric electricity demand distribution. As Bessembinder and Lemmon (2002) state "*the distribution of wholesale power prices will be positively skewed if marginal production costs are convex or if the demand distribution is itself positively skewed*".

Longstaff and Wang (2004) focus on the question of how electricity forward prices are related to expected spot prices. Their goal is to analyze the theoretical predictions presented in Bessembinder and Lemmon (2002). They analyze empirically forward prices in the PJM market, finding an important risk premium that consider is the result of "*the rationality and risk aversion of economic agents participating in the market*". Longstaff and Wang (2004) point out "*total demand approaching or exceeding the physical limits of power generation*" is an important economic risk (related also to quantity risk). and "*the risk of price spikes as demand approaches system capacity is an extreme type of risk which may have important implications for the relation between spot and forward prices*". As Krapels (2000) states "*for all intents and purposes, electricity cannot be stored. What this basically means is that if demand surpasses production in a given period of time (hourly, daily or monthly market) there is no upper boundary to price levels. As seen in the various electricity markets in North America over the past few years, spot/cash price levels had no problem reaching factors of 100 or 200 normal prices*". Therefore in those situations where the demand level is near the maximum capacity of the system, the behavior of electricity prices can be quite abrupt, since electricity must be generated trough inefficient plants with a higher marginal cost (convexity of the supply function). In the empirical analysis carried out by Longstaff and Wang (2004), the authors try to establish a relation between forward premium and difference between maximum capacity and expected demand. However, there exist a problem of data availability when the analyst tries to empirically implement the capacity variable. Although the authors consider capacity as a relevant variable in order to explain the evolution of electricity forward prices and risk premium, they must assume, because lack of data, generation capacity is constant.

One of the goals of this paper is to provide a theoretical framework where the difference between capacity and demand can be related to the behavior of derivatives prices.

The papers by Barlow (2002), Skantze et al. (2000) and Skantze and Ilic (2001) must also be commented. All these papers have in common that they impose a functional form for the relationship between price and the state variables. State variables are demand and a non-specified variable related to the supply side. Barlow (2002) proposed a "non-linear Ornstein-Uhlenbeck" process for the description of observed electricity prices. Basically the author consider the demand as the relevant state variable, and model it as a mean-reverting process incorporating a non-constant mean given by a deterministic seasonal function. From empirical observation he considers electricity price is a convex function of electricity demand. In his empirical analysis shows how effectively the model is able to generate spikes, trough a non-linear filter that connects diffusive demand with electricity prices. The convexity between demand and prices is the element that generates jumps in electricity prices although demand is modelled as a diffusive process. This author does not consider the valuation of futures contracts or any other kind of derivative.

An interesting approach is the one suggested by Skantze et al. (2000) and Skantze and Ilic (2001). In their model the authors impose a exponential functional form between electricity spot price and state variables. State variables are demand and a non-observable residual variable, related to supply conditions. In particular they assume the price of electricity is given by the following equation:

$$S_h = e^{aL_h + b_h}$$

From this specification hourly electricity prices would be governed by a combination of demand $L$ and supply $b$ states. The parameters describing the load state dynamics can be estimated directly from the time history of the market demand. As, has been pointed out previously the problem in introducing a supply variable (generation capacity) is the non-observability of this type of variables. The authors propose a simple methodology that consists in extracting the state variable $b_h$. From our point of view the problem is that in their analysis supply is considered a residual variable. Under their modelling approach any change in price that is not directly related to a demand shock will be captured by this residual variable. It also must be noted the relationship between price and demand is less clear the higher the price (or the level of demand). See Figure 1 in the appendix for the PJM market. Finally it must be noted that Skantze et al. (2000, 2001) do not derive valuation formulas and do not analyze the relationship between their model assumptions and the relationship between spot and futures prices.

One of the reasons behind the asymmetry in spot prices is the existence of spikes in electricity spot price series. These jumps usually appear in high demand periods, given a positive demand shock has a bigger effect on the spot price given the convexity of the supply function. Another possible reason behind spikes is a reduction in the supply system. This reduction in generation capacity may be given by a decrease in the number of generators of a system or in the case of interconnected systems a reduction in the imports capacity from other electric markets, see for instance the analysis for the New England spikes June 1999, in Krapels (2000). Birnbaum et al. (2002) have empirically shown there exist an important relationship between price level and generation capacity.

In this paper we model equilibrium electricity prices as a function of two observable sate variables: demand and generation capacity. In this way we extend the work of Pirrong and Jermakyan (1999), Barlow (2002) and Bessembinder and Lemmon (2002) by considering the (maximum) capacity of a system to generate electricity as a random variable. We consider, following empirical observation and the results from the equilibrium model of Bessembinder and Lemmon (2002), electricity prices are a convex function of these two state variables. Since we obtain an analytical form for the forward risk premium, this paper is also related to the empirical analysis presented in Longstaff and Wang (2004).

## 3. DEMAND AND EFFECTIVE GENERATION CAPACITY

### 3.1 Demand

Electricity demand is mainly determined by economic activity and weather. Electricity is a necessary commodity and its use is intimately related to the economic activity. The relationship between economic activity and electricity demand makes a load a seasonal variable. On one hand, electricity demand has intra-daily seasonality and we can identify high demand and low demand hours (for instance during night, and more generally from midnight to 6:00 am). We may also observe demand evolves during the week, higher demand during weekdays and lower demand during weekends and public holidays. On the other hand, weather also influences electricity demand. Among the variables that define weather, temperature is the one with greater influence on electricity demand. Extreme high or low temperature induce an extensive use of air-conditioners or for heat demand. Electricity demand (or load) usually depends non-linearly on temperature, see for instance Meneu et al. (2001). This non-linear relationship (U-shaped) also implies volatility demand is non-constant along the year. High demand periods are also more volatile. Therefore electricity demand may be considered as a mean-reverting process, where the mean is non-constant (seasonal) and with periods of high and low volatility. In order to take into account these features, we introduce several components in our model for the evolution[3] of load.

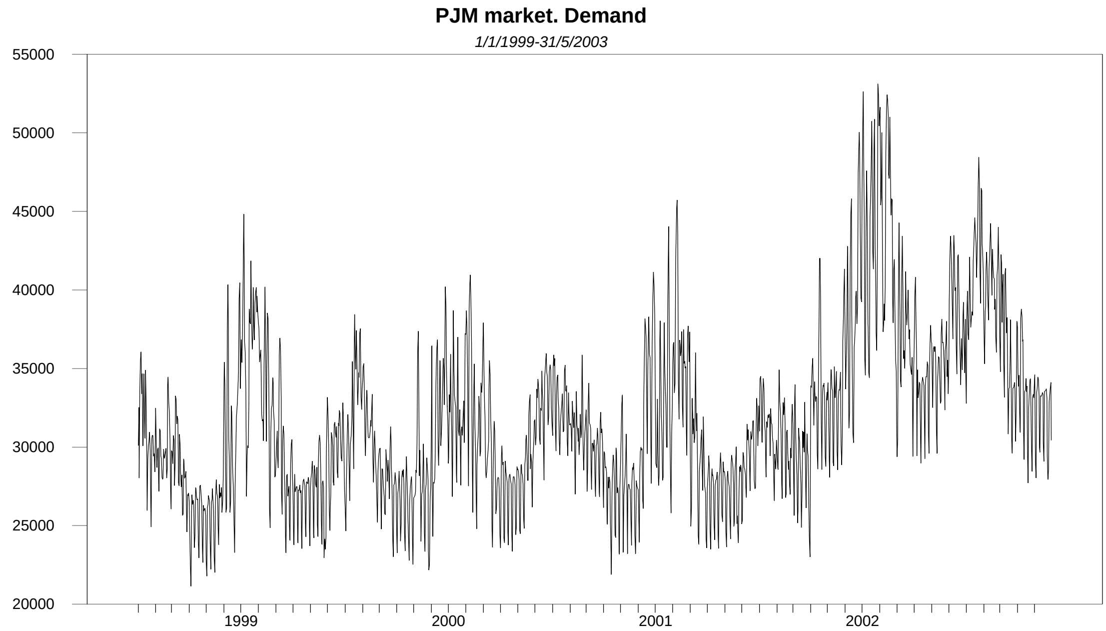
$$\begin{aligned} D_t &= g(t) + \chi_t^D \\ d\chi^D &= -k_\chi^D \chi^D dt + \sigma_\chi^D(t) dZ_x \end{aligned} \tag{1}$$

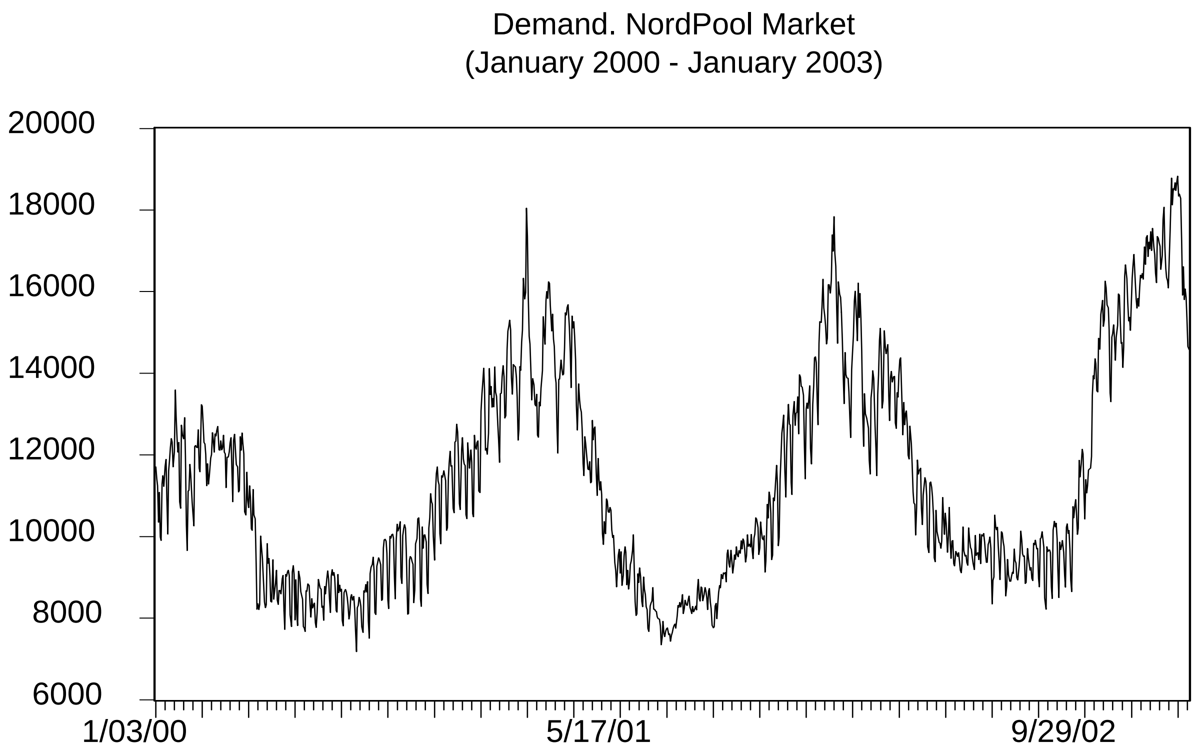
with this specification demand has a non-constant, deterministic average given by the sinusoidal function *g(t)*, the second factor ( $X$ ) is a mean-reverting process, with seasonal volatility captured by the term $\sigma_\chi^D(t)$. We may model the seasonality in the level and in the volatility either by monthly dummies or by some sinusoidal function. It also must be noted, that given this specification we may include a deterministic time trend into the deterministic function *g(t)*. If instead, we consider that the long-term demand is itself stochastic we may find useful to model demand in the following way:

$$\begin{aligned} D_t &= g(t) + \chi_t^D + \xi_t \\ d\chi^D &= -k_\chi^D \chi^D dt + \sigma_\chi^D(t) dZ_x \\ d\xi &= \mu_\xi dt + \sigma_\xi dZ_\xi \end{aligned} \tag{1b}$$

In this two factor model, the equilibrium level is uncertain and we also include a mean-reverting short-term deviation. In such a way, the long-term demand state variable is modelled trough an Arithmetic Brownian Motion.

[3] In a first step of this research we considered the possibility of modelling electricity demand as a jump-diffusion process. Given the seasonality in the volatility of electricity demand, we hypothesized the seasonality could be a result of the existence of seasonality in the jump process. Following Escribano et al. (2002) we have estimated a jump-diffusion process (with seasonal and constant intensity) to electricity demand but results did not sustain the existence of jumps in electricity demand in our sample. Therefore we consider electricity demand as a diffusion process an allow for the possibility of seasonality in both the level and the volatility process.


PJM market. Demand
1/1/1999-31/5/2003
55000
50000
45000
40000
35000
30000
25000
20000
1999
2000
2001
2002


Demand. NordPool Market
(January 2000 - January 2003)
20000
18000
16000
14000
12000
10000
8000
6000
1/03/00
5/17/01
9/29/02

From previous graphs, we observe some differences in the behavior of the series in each market. These differences are important in order to understand the behavior of spot prices, forward prices and forward risk premiums, as we will see later on. In particular, it is important to emphasize demand in the PJM market is much volatile, has a higher degree of asymmetric behavior (skewness) and higher kurtosis.

We have estimated Model (1) with daily data. In order to do so, we have discretized the process and have estimated the parameters by Maximum Likelihood. Since we are dealing with daily data, the discretization error is negligible (Melino, 1994). The models have been estimated by Maximum Likelihood with RATS 2.5. The estimation has been done trough the BHHH algorithm. Seasonality in the mean and in the volatility function can be modelled with monthly "dummies" or with sinusoidal functions. In particular, we have estimated the following model :

$$D_t = g(t) + X_t \tag{2a}$$

$$g(t) = B0 + B2 \cdot t + D1 \cdot workday_t + C1 \cdot \sin\left((t + C2) \cdot \frac{2\pi}{365}\right) + C3 \cdot \sin\left((t + C4) \cdot \frac{4\pi}{365}\right) \tag{2b}$$

$$X_t = B1 \cdot X_{t-1} + \sigma_x^s \cdot \varepsilon_t \tag{2c}$$

$$\sigma_x^s = QS1 + QS2 \cdot spring_t + QS3 \cdot fall_t + QS4 \cdot summer_t \tag{2d}$$

where $\varepsilon_t \sim$ *i.i.d. N(0,1)*. Equation (2b) describes the deterministic seasonal behavior of the series. We have introduced a deterministic trend in order to capture the growth in the level of electricity demand because socio-economic and demographic reasons. The variable $workday_t$ is a dummy variable that takes 1 during weekdays and zero otherwise (weekend). Lastly we incorporate a general formula through sinusoidal functions to capture the seasonal behavior (we allow for two local maxima per year). In case there is just one local maximum we would get $C3 = C4 = 0$. In equation (2c) we have incorporated the dynamics of demand. This dynamics are captured trough an autoregressive component of order 1. The parameter B1 captures the speed of mean reversion of the process and it is linked to parameter $k$ in model (1), in particular a low mean reversion, low $k$ is equivalent to $B1 \approx 1$. Equation (2d) models seasonality in volatility through a set of dummy variables: $fall_t$ takes 1 if the observation is on September, October or November and zero the rest of the months; $spring_t$ takes the value of 1 if the observation is March, April and May or zero in the rest of months and finally $summer_t$ takes value 1 if the observation is in June, July or August and zero the rest of the months.

Table 2. Estimation results, model (2a)-(2d)

| | **PJM MARKET** | | | | **NORDPOOL MARKET** | | | |
|---|---|---|---|---|---|---|---|---|
| | **Model with Constant Volatility** | | **Model with Seasonal Volatility** | | **Model with Constant Volatility** | | **Model with Seasonal Volatility** | |
| Parameter | Coeff. | t-stat. | Coeff. | t-stat. | Coeff. | t-stat. | Coeff. | t-stat. |
| B0 | 24804,74 | 91,82 | 24870,49 | 99,09 | 9670,01 | 91,04 | 10101,38 | 103,45 |
| B1 | 0,7907 | 59,64 | 0,7808 | 55,56 | 0,802 | 47,99 | 0,790 | 42,95 |
| B2 | 5,86 | 25,22 | 5,64 | 26,60 | 2,97 | 21,68 | 2,26 | 17,69 |
| D1 | 3471,32 | 21,32 | 3415,11 | 22,36 | 4,68 | 0,07 | 11,17 | 0,17 |
| C1 | -1498,66 | -11,31 | -1188,87 | -7,91 | -3073,72 | -45,32 | -3011,19 | -52,52 |
| C2 | -1393,91 | -205,63 | -1394,30 | -223,90 | 257,64 | 214,54 | 258,91 | 225,81 |
| C3 | 3805,25 | 24,07 | 3621,09 | 24,05 | - | - | - | - |
| C4 | -342,70 | 24,07 | -343,31 | -343,09 | - | - | - | - |
| STDV | 2269,85 | 64,83 | | | 797,43 | 48,65 | | |
| QS1 | | | 2214,82 | 28,17 | | | 1023,62 | 21,21 |
| QS2 | | | -473,34 | -5,26 | | | -259,82 | -4,53 |
| QS3 | | | -367,14 | -3,62 | | | -174,81 | -2,92 |
| QS4 | | | 953,85 | 6,43 | | | -527,99 | -9,80 |
| *LL* | -14711,98 | | -14625,53 | | -8999,47 | | -8944,78 | |
| SC[4] | 29490,71 | | 29339,68 | | 18062,07 | | 17973,74 | |

An important aspect is the existence of a seasonal pattern both in the "*level*" of the series and in the "*volatility*". It is also important to mention the correlation between seasonal pattern in the level of the series and the seasonal pattern in the volatility of the series (i.e. *volatility* of demand is higher in those periods where *level* of demand is also high). This fact together with the convexity of the cost function generates the high volatility and seasonality in the volatility of price series. We may also show for both markets we obtain better results with the "seasonal volatility" model than with the "constant volatility". These kind of seasonality is much more important in the PJM market than in the NordPool case. As we will see below, the seasonal pattern in volatility do translate to the price of forward contracts. Therefore, in our model, ceteris paribus, in those markets where seasonal demand volatility is much important we will observe a more important seasonal behavior in forward curves (we will also show risk premium is a function of demand volatility, and therefore seasonal behavior in demand volatility will translate in more seasonality of the forward risk premium).

[4] Schwarz criterion is defined by the formula: $-2 \cdot LL + m \cdot \ln(n)$ where *LL* is log-likelihood value, *m* is the number of estimated parameters and *n* is the number of observations.

### 3.2. Generation Capacity

In order to better understand the evolution of spot price series and forward prices, generation capacity ("effective generation capacity") of a power system should be considered a random variable. Some of the fluctuations on generation capacity may be known in advance by the market participants (for instance because the existence of seasonality and/or planned outages), but many other fluctuations in generation capacity are random. Bessembinder and Lemmon (2002), consider that "*power function varies seasonally, as producers schedule planned maintenance outages during periods of low expected power demand*". Although these reductions in available capacity are planned reductions, and may be known by market participants, they have also been the cause of some of the jumps observed jumps in price series. As an example, Krapels (2000) show how effectively the increase in prices in the New England market during 7-8 June 1999, was due to a combination of a reduction in available capacity plus an increase in demand, that in normal situations would not have affected the usual level of prices. Another example, is the Scandinavian market, NordPool. In NordPool electricity is mainly generated through hydro resources (see also Lucía and Schwartz (2002) for a description of the market). During last months of year 2002 and the beginning of 2003 the level of hydro reservoirs in this market is at historical minimum levels. As a consequence spot prices and therefore also forward prices have increased substantially. For a relationship between spot prices and hydro reservoirs you can also see Botterud et al. (2002). Kollberg et al. (1999) present another example of the relationship between generation capacity and fluctuations of forward prices in NordPool: "*... a shock that affected futures prices at Nord Pool was the decision by the Swedish government to close down one nuclear reactor at Barsebäck. At a time when the supply of electricity was already regarded as constrained in the Nordic region, this decision to cut production resources even further made the market react in a powerful way. Suddenly, there was a shift in all forward and futures contracts with maturity after the closing date*". Therefore, those observed fluctuations in prices (and the jumps) are not only caused by abrupt changes in demand conditions. Apart from the planned changes in capacity (planned outages) it is also common to observe non-planned outages. Maybe because reductions or congestion in the transmission capacity of connected areas.

We may also consider the possibility that reductions in supply have their causes in the exercise of market power by some generators. That may be the case of some of the episodes of price increases observed in California, Joskow and Kahn (2001).

In this paper, we do not plan to analyze the causes of the changes in the effective generation capacity but we only consider it must be taken as a random variable. In this way, we extend the papers by Barlow (2002), Bessembinder and Lemmon (2002) and Pirrong and Jermakyan (1999 and 2000) among others.

Given the characteristics of the variable capacity we propose the following model with mean reversion and jumps:

$$dc = k_c(\theta_c - c)dt + \sigma_c dZ_c + J_c(\eta_{J,c})d\Pi(\lambda_c) \tag{3a}$$

where $\theta_c$ is a parameter for the central tendency of the variable capacity *c*, and the speed of mean reversion is given by the parameter $k_c$. The variable capacity has two random terms, one is a diffusive process and the other one is governed by a Poisson process, that has associated a random jump $J_c$. The volatility parameter is $\sigma_c^2$ and the occurrence of a jump is governed by a Poisson process, where $\lambda_c$ captures the frequency of the process. The jump size may be constant or may be given from a probability distribution. The diffusive process is independent of the Poisson process and the jump size. The Poisson process (governing the probability of observing a jump) and the jump size are also independent, see Duffie, Pan and Singleton (2000) and Piazzesi (2002).

Although we have assumed the equilibrium level of *c* is given by the constant $\theta_c$, we may incorporate the possibility of a seasonal behavior. Seasonality in capacity would be interesting for those markets where electricity is mainly generated by hydro resources, for instance in the Scandinavian market. On the other hand, we may also incorporate the possibility of permanent changes in the equilibrium level $\theta_c$. These permanent changes may be explained by new generating capacity or interconnections. We could incorporate these changes by considering $\theta_c$ as a deterministic step function. If at some point in time new capacity is added to the system we would incorporate it trough a change in $\theta_c$, from a low equilibrium level, $\theta_c^L$, to a high equilibrium level, $\theta_c^H$. Another possibility would be to consider the existence of a long-term stochastic variable, governing the equilibrium level of capacity[5].

In fact it must be noted, that the typical expression for a mean-reverting process (i.e. equation (3a) without the jump component) may be written as the sum of a constant equilibrium level $\theta_c$, and a mean-reverting process (fluctuating around zero). That is, the following specification

$$C_t = \theta + \chi_t^c \tag{3b.1}$$

[5] As long as we retain the conditions required by Affine Jump Diffusion processes, we can extend the model, to incorporate some extra uncertainty terms.

$$d\chi_t^c = -k_c \chi^c dt + \sigma_\chi^c dZ_{\chi,c} \tag{3b.2}$$

is equivalent to

$$dC = k_c(\theta - c)dt + \sigma_c dZ_c \tag{3.c}$$

Therefore we may also extend the model for the capacity expression by introducing a long term ("equilibrium") factor, so a more general model for generation capacity would be:

$$\begin{aligned} C_t &= \theta_t + \chi_t^c \\ d\theta &= \mu_\theta dt + \sigma_\theta dZ_\theta \\ d\chi^c &= -k_c \chi^c dt + \sigma_c dZ_{\chi,c} \end{aligned} \tag{3.d}$$

We could also incorporate a jump component in the short-term mean-reverting factor like in expression (3). With respect the jump size distribution, we are interested in those jump size distributions that allow us to obtain closed form formulas, therefore we could work with a Gaussian or an Exponential jump size distribution, see Deng (1999) and Escribano et al. (2002) for the introduction of jump component directly in the spot price specification. In this paper we will assume jumps are negative and come from an Exponential distribution (the mean jump size under the empirical probability distribution will be given by the parameter $\eta_{J,c}$).

Summarizing, in model 3a, we assume capacity follows a mean-reverting process with constant volatility and we allow the possibility of sudden changes in the effective generation capacity of a market. With the jump component we may, for instance, analyse the probability assigned by the market to a decrease in the capacity of interconnection between two areas (reduction in the possibility of power imports), or the possibility of outage of a (relative) big generator. The relative importance of these possibilities in a given market is an empirical question. On the other hand, the specification (3.d) allow us to concentrate on the effect of the long-term capacity variable on forward prices with longer maturities.

## 4. THE MODEL AND THE RELATIONSHIP BETWEEN SPOT PRICE AND STATE VARIABLES.

Both from the empirical evidence and from theoretical results from the equilibrium model of Bessembinder and Lemmon (2002) (see also the Appendix), we know spot price should be a convex function of the two state variables, see also Pirrong and Jermakyan (1999 and 2000). Therefore, we may specify a generic function $\varphi(\cdot)$ such that $P_t = \varphi(D_t, C_t)$ and where given the characteristics of electric sector and the convexity of the sypply function ("supply stack"), these conditions should be satisfied $\varphi_D > 0$, $\varphi_C < 0$, $\varphi_{DD} > 0$ and $\varphi_{CC} > 0$. Pirrong and Jermakyan (1999) considered the following specification for the relationship between price and state variables:

$$P(q_t, g_t, t) = l_t^{\gamma} e^{\alpha q_t^2 + s(t)}$$

where $l_t$ is the price of the marginal fuel, $q_t$ is the level of demand ("load") and $s(t)$ is a deterministic seasonal function.

It seems reasonable to consider spot price as an increasing and convex function of demand, see Pirrong and Jermakyan (2000), Barlow (2002) and Bessembinder and Lemmon (2002). On the other hand, it seems also natural to consider that a similar relationship must hold between price and capacity. When available capacity decreases, spot price should increase (for a given level of demand), and this relationship should also be convex given the characteristics of the supply stack.

Birnbaum et al. (2002) showed the existence of a convex relationship between price and "capacity utilization rate" in the PJM market. There exist a lot of candidate functional forms for modelling the relationship between demand and capacity. Given the main goal of this paper is to obtain valuation formulas in analytic form, we should also take into account such a restriction. As we said above we want to be able to exploit the results by Duffie, Pan and Singleton (2000) and Cheng and Scaillet (2004). Therefore the additional constraint we face is the existence of a price transformation (in particular logarithmic transformation) such that log-price is a linear function of state variables.

Given these constraints we propose the following model:

$$P_t = C_t^{\gamma} \cdot \beta \cdot e^{\alpha \cdot D_t} \tag{4a}$$

$$\begin{aligned} D_t &= g(t) + \chi_t^D \\ d\chi^D &= -k_\chi^D \chi^D dt + \sigma_\chi^D(t) dZ_x \end{aligned} \tag{4b}$$

$$dc = k_c(\theta_c - c)dt + \sigma_c dZ_c + J_c(\eta_{J,c})d\Pi(\lambda_c) \tag{4c}$$

$$dZ_x dZ_c = \rho dt \tag{4d}$$

where[6] $c \equiv \ln C$, $\alpha > 0$ y $\gamma < 0$.

Applying logs to (4a) we have the expression for log-price as a linear function of state variables[7]. That is we have this alternative expression for the log-price:

$$\begin{aligned}\ln P_t &= \gamma \ln C_t + \ln \beta + \alpha \cdot D_t = \\ &= \gamma \cdot c_t + \ln \beta + \alpha \cdot \left(g(t) + \chi_t\right) = \\ &= \left(\ln \beta + \alpha \cdot g(t)\right) + \left(\gamma \cdot c_t + \alpha \cdot \chi_t\right)\end{aligned}$$

The first part of this equation captures deterministic elements, and if we define $f(t) \equiv \ln \beta + \alpha \cdot g(t)$, we obtain the last expression for log-price:

$$\ln P_t = f(t) + \gamma \cdot c_t + \alpha \cdot \chi_t \tag{5}$$

Since we are modelling log-price as a linear[8] function of state variables, and given these variables do follow affine jump-diffusion processes, we can use the results by Duffie, Pan and Singleton (2000).

Before moving to present the model under the risk-neutral probability measure and derive the valuation formula we present some preliminary evidence about the adequacy of the proposed specification.

**Relationship between price and demand**

Next we do provide a more detailed analysis of the relationship between price and demand. If we impose the restriction $\gamma = 0$, we may obtain a more restricted version of the general model proposed in the equation (4a). With such restriction the spot price is given by $P_t = \beta \cdot e^{\alpha \cdot D_t}$. The relationship between price and demand is given by graph 1 and 2. From this graphs we can check the degree of convexity in the relationship between spot price and demand in PJM and NordPool

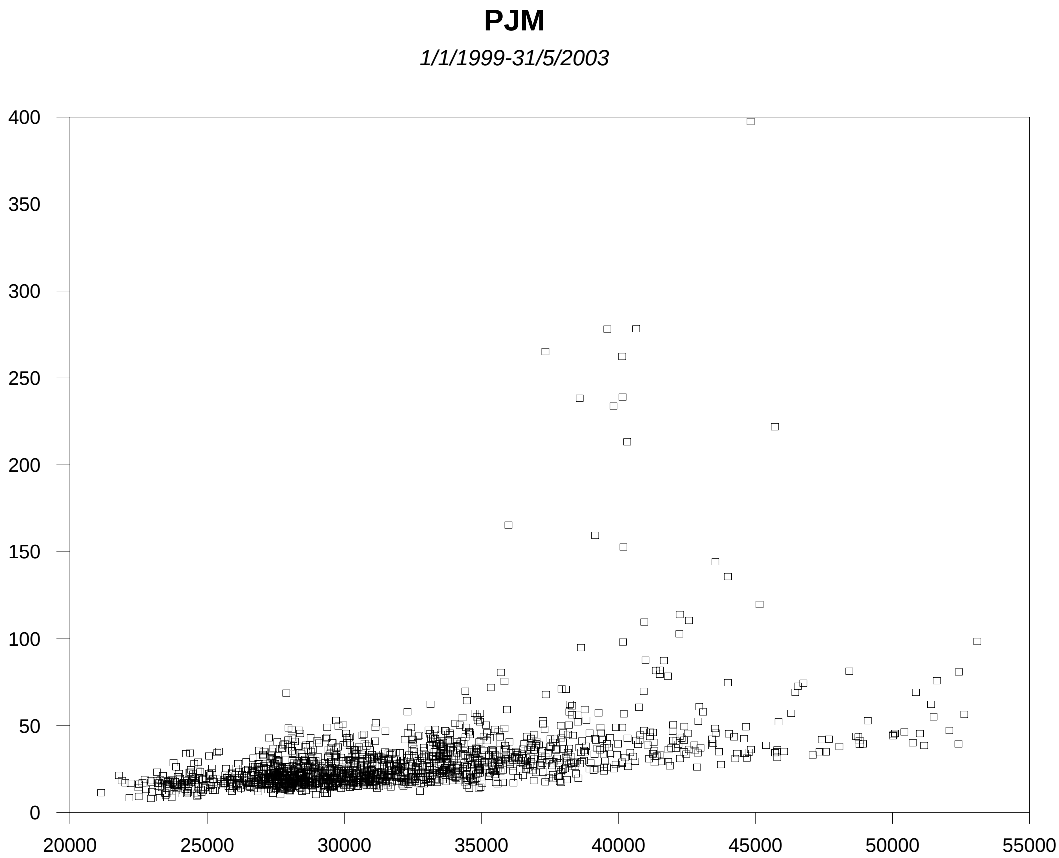
[6] It should be clear $c$ is a random variable related to the capacity of the system, and it is not the "convexity parameter" of the cost function in Bessembinder and Lemmon (2002).

[7] We have estimated equation (4a)with weekly data from NordPool, where hydro reservoirs has been used as a proxy for the variable "generation capacity". Estimation results are presented in Table A.1 in the appendix. Estimates coefficients are signficatively different from zero and have the correct sign.

[8] An alternative interesting line is that proposed in Leippold and Wu (2002) for the modelling of interest rates. The authors propose to model interest rates as a quadratic function of state variables. The authors show under this specification the tractability of the model is maintained, in the sense analytic or quasi-analytic formulas may be obtained for interest rate derivative models. However, the authors model the state variables trough diffusion processes without incorporating the possibility of jumps. A possible interesting line of research is that found in Cheng and Scaillet (2004).

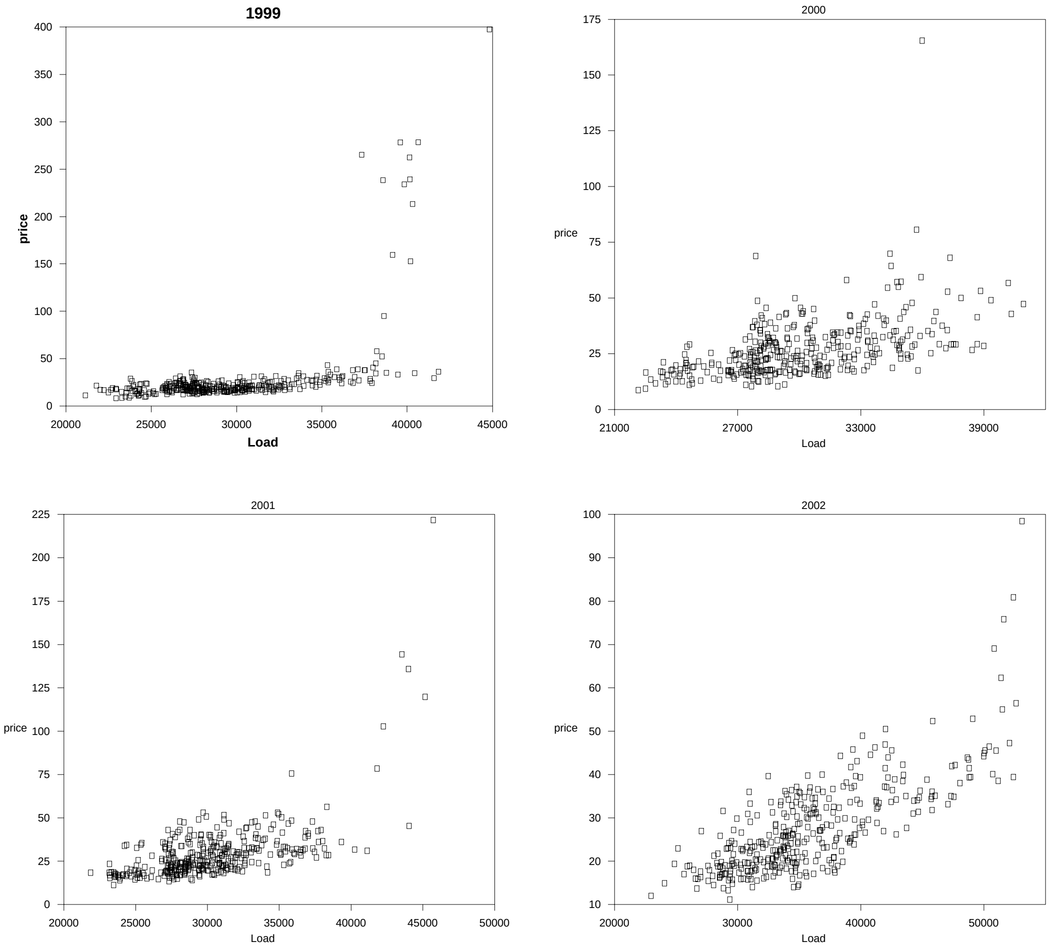
market. We may also check the existence of a threshold level, from that level on changes in demand do have a great impact on price level. The convexity of the function will depend on the cost structure of the market, and reflects the aggregate supply of the market. From the graphs we can see in the case of the PJM market, the curve presents a very clear  “inverted L” form. This form is one of the reasons behind the observed spikes in PJM price series. From a certain demand level, small positive demand shocks above this level could generate very important changes in price series. As we have also seen volatility demand is much higher during summer months (i.e. during spike periods), and is at these months were more spikes are observed in spot prices.

Graph 1: Relationship between price and demand, PJM market.



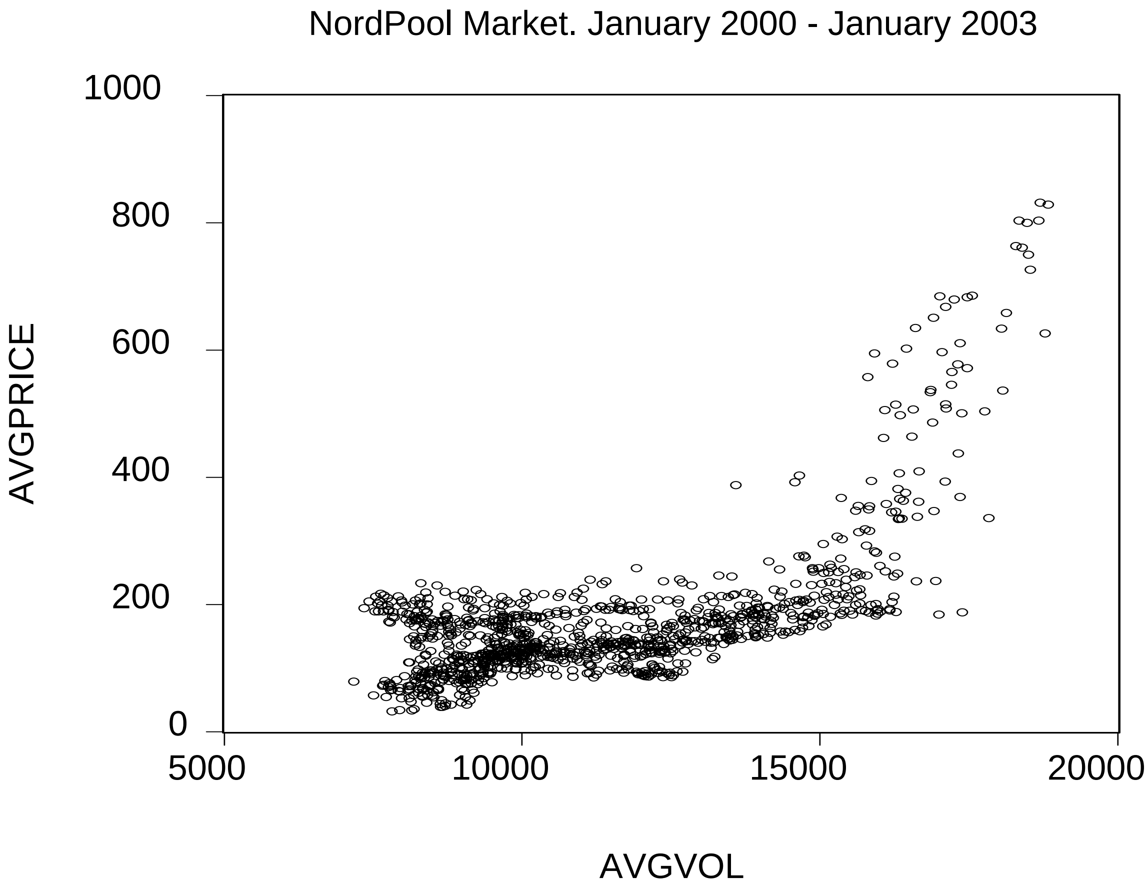
# PJM. Relationship between Price & Demand

*1/1/1999-31/5/2003*



Graph 2: Relationship between price and demand, NordPool.

Daily data (January 2000 – January 2003)



**Relationship between price and capacity**

In a similar way we may analyse in a more direct way the relationship between price and capacity if we set $\alpha = 0$. In this way we obtain a more restricted version of model (3a-3c). It may be shown in this restricted model the expected price (under the empirical probability measure) at $T$ is given by the following expression:

$$E^P(P_T|F_t) = \exp\left(\gamma e^{-k_c\tau}c + \theta_c\gamma\left(1-e^{-k_c\tau}\right) + \frac{\sigma_c^2\gamma^2}{4k_c}\left(1-e^{-k_c\tau}\right) + \frac{\lambda_c}{k_c}\ln\left(\frac{\gamma\eta_{J,c}e^{-k_c\tau}-1}{\gamma\eta_{J,c}-1}\right)\right) \tag{6}$$

where $F_t$ corresponds to the filtration at $t$ (i.e. contains information until $t$).

We can re-express this equation in the following way:

$$E^P(P_T|F_t) = \exp\left(\theta_c\gamma + \gamma(c-\theta_c)e^{-k_c\tau} + \frac{\sigma_c^2\gamma^2}{4k_c}\left(1-e^{-k_c\tau}\right) + \frac{\lambda_c}{k_c}\ln\left(\frac{\gamma\eta_{J,c}e^{-k_c\tau}-1}{\gamma\eta_{J,c}-1}\right)\right) \tag{7}$$

In this way it we may see in a clear way the relationship between capacity and price. It must be noted, the parameter $\gamma$ captures the degree of convexity in the relationship between price and capacity, and $\gamma < 0$, so if capacity goes down (up) prices go up (down). On the other hand, we can also check the effect of permanent changes in capacity on the expected price. The higher the equilibrium level $\theta$ the lower the price. It can be shown, "ceteris paribus", if $\theta_c^B < \theta_c^A$ then $E^P\left(P_T \mid F_t, \theta_c^B\right) > E^P\left(P_T \mid F_t, \theta_c^A\right)$.

It may also be analysed the effect of deviations in real capacity from the equilibrium level at a given point in time. If $c_t > \theta_c \Rightarrow \gamma(c-\theta_c)e^{-k_c\tau} < 0$, while if $c_t < \theta_c \Rightarrow \gamma(c-\theta_c)e^{-k_c\tau} > 0$. Therefore capacity increments implies a lower expected price. We may also check $\gamma$ captures the sensitivity of expected price to changes in the capacity variable.

As an illustration we reproduce in the Appendix a graph presented originally in Birnbaum et al. (2002), in which is shown the relationship between the utilization rate and the price level. This plot shows how increments in prices may occur although the utilization capacity rate is not "extremely" high[9]. As an illustration we provide in the Appendix some graphs with the relationship between

[9] As the authors point out "*wholesale prices "fly up" the moment demand surges, though there may be still significant spare capacity in the system*", the authors also consider the increase in process given an increase in the capacity utilization rate is specially pronounced in the electricity sector compared with other sectors "*electricity prices tend to start rising at lower levels of capacity utilization than do prices for other commodities*"

price series and hydro reservoirs[10] graph A.7 and A.8. We may effectively see the existence of a negative relationship between prices and hydro reservoirs.

## 5. VALUATION OF FUTURES CONTRACTS

In this section we present the specification of the model under the risk neutral probability and we present the valuation formula. As we have commented previously, the specification we propose allow us to use the results provided y Duffie, Pan and Singleton (2002).

The specification under the objective probability measure is given by equations (4a)-(4c). For the valuation of derivative contracts we have to express the model under the risk-neutral probability measure, Cox and Ross (1976) and Harrison and Kreps (1979), see also Duffie (1992). In our model this implies we have to incorporate the price of risk for the different sources of uncertainty. In our case, the sources of uncertainty are the demand and the effective generation capacity, and therefore we have to introduce two additional parameters $(\phi_x, \phi_c)$. We are going to assume constant price of risk, although a more general specification could be introduced. In particular, the market price per unit of demand and capacity risk are $\phi_x$ and $\phi_c$ respectively. Given the volatility of demand is seasonal we are incorporating seasonality in the market price of demand risk. Under the risk neutral probability measure the specification of the model is given by:

$$\begin{aligned}
&\ln P_t = f(t) + \gamma \cdot c_t + \alpha \cdot X_t \\
&dX = k_x\left(\theta_x^* - X\right)dt + \sigma_x^{seas} dZ_x^* \\
&dc = k_c\left(\theta_c^* - c\right)dt + \sigma_c dZ_c^* + J_c^*\left(\eta_{J,c}^*\right)d\Pi\left(\lambda_c\right) \\
&dZ_x^* dZ_c^* = \rho dt
\end{aligned} \qquad \textbf{(8)}$$

where $\theta_x^* = -\dfrac{\phi_x \cdot \sigma_x^{seas}}{k_x}$ and $\theta_c^* = \theta_c - \dfrac{\phi_c \cdot \sigma_c}{k_c}$.

In the case of the adjustment of the jump component, we have to make a simplifying assumption. Given empirically is not possible to separate the effect of the component of the jump size and the probability of jump, we make the simplifying assumption $\lambda_c^* = \lambda_c$, in this way the jump component is captured trough the jump size risk.

[10] The hydro reservoirs level in Nord Pool act as a “proxy” for the generation capacity in the Scandinavian market.

We may redefine, the price of the forward contracts as a function of state variables in the following way:

$$F\left(D,C,t,T\right)=E^{Q}\left(e^{\ln P_T}\left|F_t\right.\right)=e^{f(T)}\cdot E^{Q}\left(e^{\gamma\cdot c_T+\alpha\cdot X_T}\left|F_t\right.\right) \tag{9}$$

If we define the vectors $u=\left(u_1,u_2\right)=\left(\gamma,\alpha\right)$ and the vector of state variables $Y=\left(c,X\right)$ we may check we are in the framework of affine jump-diffusions, and following Duffie, Pan and Singleton (2000) we may derive an analytic formula for the characteristic function and for the price of forwards contracts since we have to calculate $E^{Q}\left(e^{u\cdot Y_T}\left|F_t\right.\right)$. If we follow Duffie, Pan and Singleton (2003) we may prove under this model the valuation formula for forward contracts with maturity at *T*, is given by the following expression:

$$F\left(x,c,t,T\right)=\exp\left\{f(T)+\gamma\cdot c_t\cdot e^{-k_c(T-t)}+\alpha\cdot x_t\cdot e^{-k_x(T-t)}+A(T-t)\right\} \tag{10}$$

where

$$A(T-t)=\theta_c^{*}\gamma\left(1-e^{-k_c(T-t)}\right)-\frac{\phi_x\alpha}{k_x}\left(1-e^{-k_x(T-t)}\right)+\frac{\left(\alpha\cdot\sigma_x^{seas}\right)^2}{4k_x}\left(1-e^{-2k_x(T-t)}\right)$$
$$+\frac{\sigma_c^2\gamma^2}{4k_c}\left(1-e^{-2k_c(T-t)}\right)+\frac{\rho\sigma_x\sigma_c\gamma\alpha}{k_x+k_c}\left(1-e^{-(k_x+k_c)(T-t)}\right)+\frac{\lambda_c}{k_c}\ln\left(\frac{\gamma\eta_{J,c}^{*}e^{-k_c(T-t)}-1}{\gamma\eta_{J,c}^{*}-1}\right)$$

From this expression we may see that effectively and increase in the level of demand and a smaller capacity level imply higher futures prices. On the other hand, higher volatility  of demand and supply will translate in higher prices of futures contracts. It is also important to note that the effect of the price of risk of demand and supply $\left(\phi_x,\phi_c\right)$, and the volatility of state variables $\left(\sigma_c,\sigma_x^{seas}\right)$ on the price of the forward contract will depend the parameters $\alpha$ and $\gamma$. That is, the characteristics of each individual market, captured trough the convexity in the relationship between price and demand and between price and supply, are very important  in order to understand the behavior of the price of forward contracts.

By obtaining an analytic formula for the price of the forward contract, we may also calculate explicitly the expression for the forward risk premium.

$$RP_t \equiv \ln F(t,T,P) - \ln E_t^P(P_T) = -\gamma \frac{\phi_c \sigma_c}{k_c}\left(1-e^{-k_c\tau}\right) - \alpha \frac{\phi_x \sigma_x^{seas}}{k_x}\left(1-e^{-k_x\tau}\right) + \frac{\lambda_c}{k_c} J^{RP} \tag{11}$$

where $J^{RP} \equiv \ln\left(\frac{\gamma\eta_{J,c}^{*} e^{-k_c\tau} - 1}{\gamma\eta_{J,c}^{*} - 1}\right) - \ln\left(\frac{\gamma\eta_{J,c} e^{-k_c\tau} - 1}{\gamma\eta_{J,c} - 1}\right)$

We are mainly interested on the first two terms in equation (11). We can show how effectively the size of the forward risk premium depends on the sensitivity of spot prices to changes in state variables, trough the parameters $(\alpha,\gamma)$. The higher the convexity in the relationship between price and demand (higher $\alpha$) the higher the risk premium[11]. On the other hand, the same situation occurs when we focus on the sensitivity of spot prices to respect the changes in capacity. It can be shown how effectively the higher the volatility of state variables $\left(\sigma_c, \sigma_x^{seas}\right)$ the higher the risk premium. Another important factor is how seasonality in the volatility of demand translates into the risk premium. In this way, if volatility of demand is seasonal, as we have seen in the results presented in table 2, the risk premium will also be seasonal. In fact, for the PJM market we have shown in previous sections the existence of an important seasonal factor in the volatility of demand. The papers by Bessembinder and Lemmon (2002), Pirrong and Jermakyan (1999 and 2000) and Villaplana (2003) present evidence and discuss the existence of a seasonal pattern in the forward risk premium. With the model proposed in this paper we are able to provide another argument to explain this seasonal pattern in the risk premium. Our model states that risk premium will be higher in those contracts that mature in periods of high volatility of demand. As it has been shown in Table 2, the PJM market is characterized by a high volatility of demand in June, July and August[12,13]. As it has been shown for instance in Pirrong and Jermakyan (2000) or in Villaplana (2003), the price of forward contracts with maturity in those months is extremely high, in part because the existence of an important risk premium.

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[11] It must be noted that if there exist a forward risk premium, empirically we should obtain $\phi_\chi < 0$ and $\phi_c > 0$.

[12] For the PJM market we have used another specification for the modelling the volatility, trough sinusoidal functions. Results are quite similar an in particular with the "dummy" specifications results are slightly better.

[13] We also have estimated a specification in which the volatility may be different in each of these months, The results show volatility is a bit higher in July although the differences are not statistically significant, so we have not presented these results.

Therefore, from equation (11) we may show how different parameters affect the behavior of the risk premium at different points in time in a specific market (for instance trough changes in the volatility of demand) and allows us to compare the reasons behind the difference in the size of risk premium in different markets. In those markets in which price is more sensible to changes in state variables (higher convexity in the relationship between price, demand and generation capacity) we will observe a higher risk premium.

From the results obtained from our model, it is clear a more exhaustive empirical analysis on the existence of forward risk premium in the PJM and NordPool market is needed. However, the empirical preliminary evidence shows our model is able to present some arguments that explain, at least partially, the differences between the observed risk premium in different markets (Lucía and Schwartz, 2002; Pirrong and Jermakyan, 2000; Bessembinder and Lemmon, 2002 and Villaplana, 2003).

## 6. CONCLUSIONS

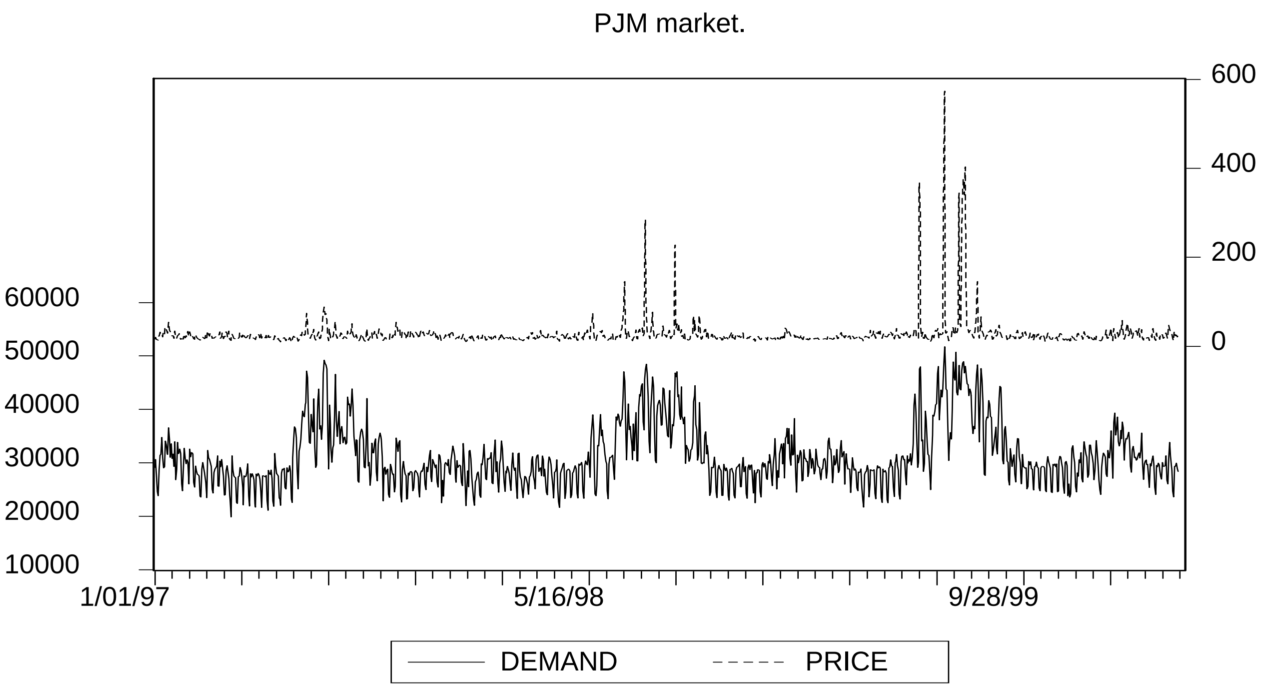
In this paper we have presented a framework for the valuation of derivatives contracts that incorporate the effect of demand and capacity (supply) variables. The goal of this paper was to present a framework to analyze the effect of demand and supply have on the prices of electricity futures contracts. We have made a brief overview on the literature and we have shown how the supply variable, “generation capacity” is considered in almost all papers a relevant variable in order to understand spot and forward prices. However we have shown most of the papers although considering it as an important variable do not introduce it in their analysis. We have shown that effectively the difficulty in implementing this variable maybe one of the reasons behind the fact that there is no model that incorporate it explicitly. However, we believe the framework we have introduced may be quite useful as a first step in the understanding of forward risk premium and on obtaining closed form valuation formulas. The framework we propose consists in modelling the state variables as affine diffusion processes (with the possibility of jumps) and we have derived the valuation formula for futures contracts. Our model extends among others the work by Barlow (2002).

We have presented an analytic expression for the risk premium and we have shown risk premium depends positively on the degree of convexity between spot price and the state variables, on the volatility of the state variables and on the price of risk. We have shown the observed seasonality in the forward risk premium in the PJM market can be explained trough the important seasonality observed in the volatility of demand, and through the relationship between price and demand in this market, characterised by an “inverted L” form.

## APPENDIX

Graph A.1: Price and Quantity time series. PJM market. Daily data.

("Cantidad": quantity; "Precio": price)



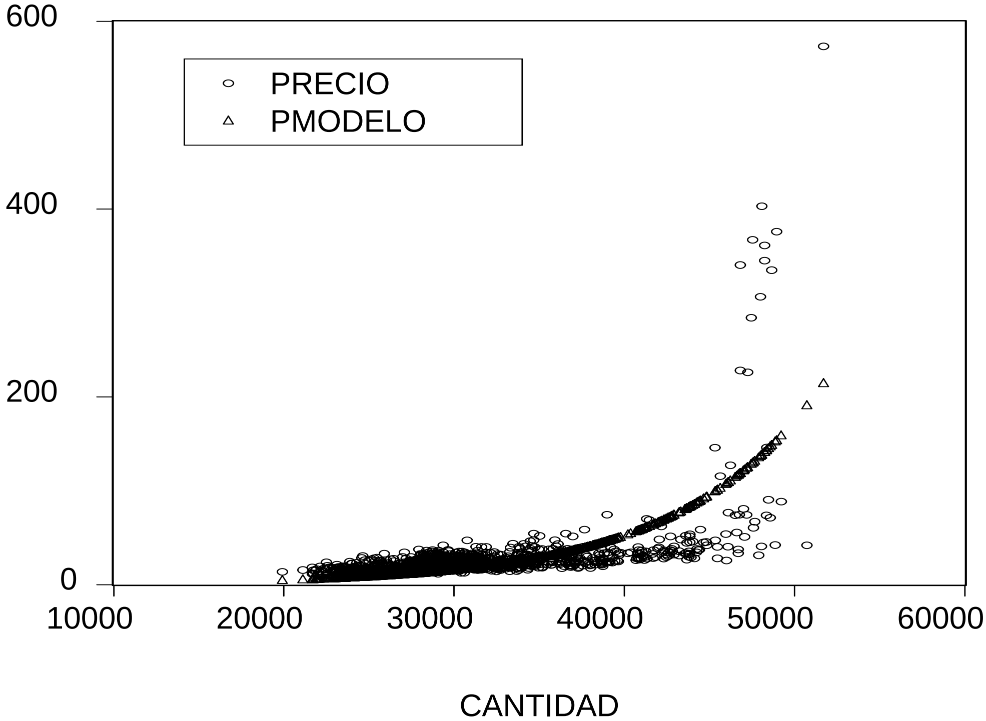
Graph A.2: Relationship between Price and Demand in the PJM market.

"PMODELO" is defined from the following specification: $P_t = \beta \cdot e^{\alpha \cdot D_t}$

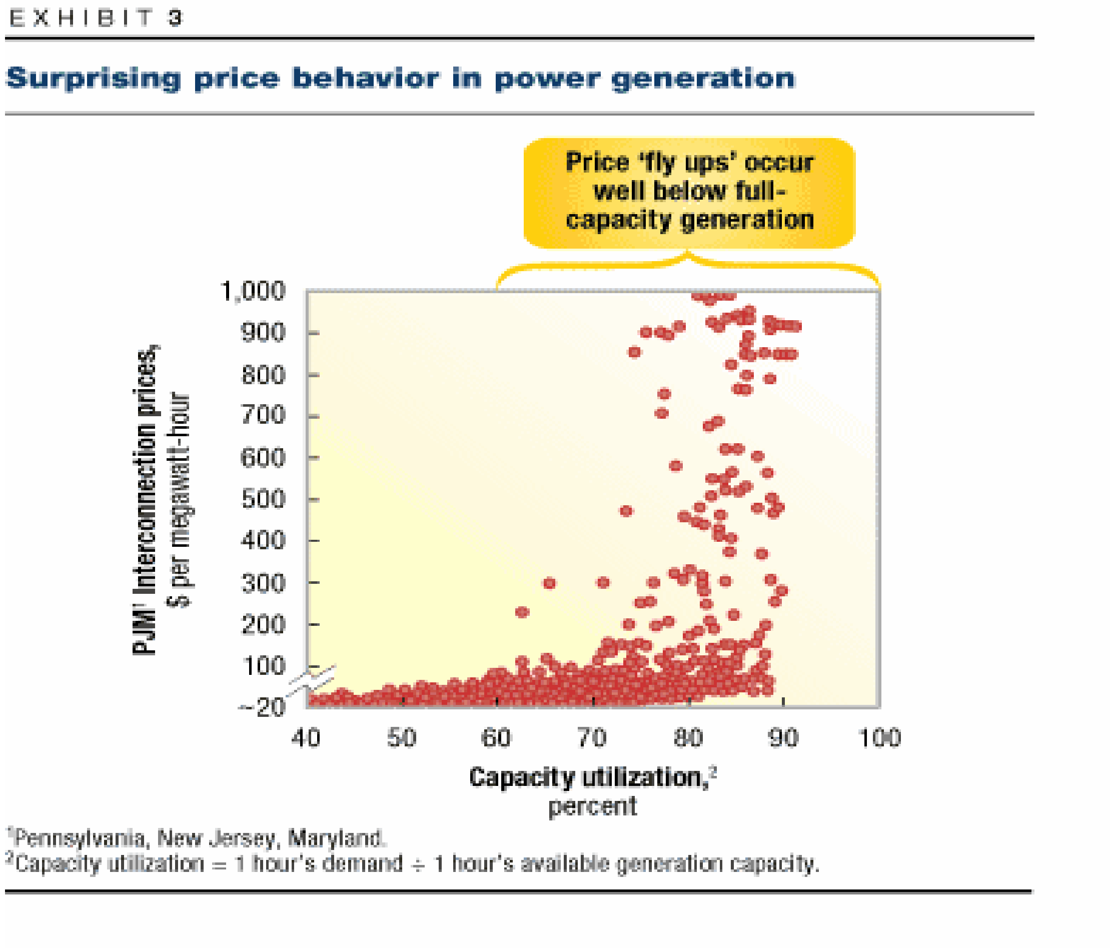
"PRECIO": is the observed price

"CANTIDAD": is the observed demand

The estimation of this simple model is just for illustrative purposes, in order to show the kind of relationship between price and the level of demand.



Graph A.3: Relationship between price and capacity utilization, PJM market.

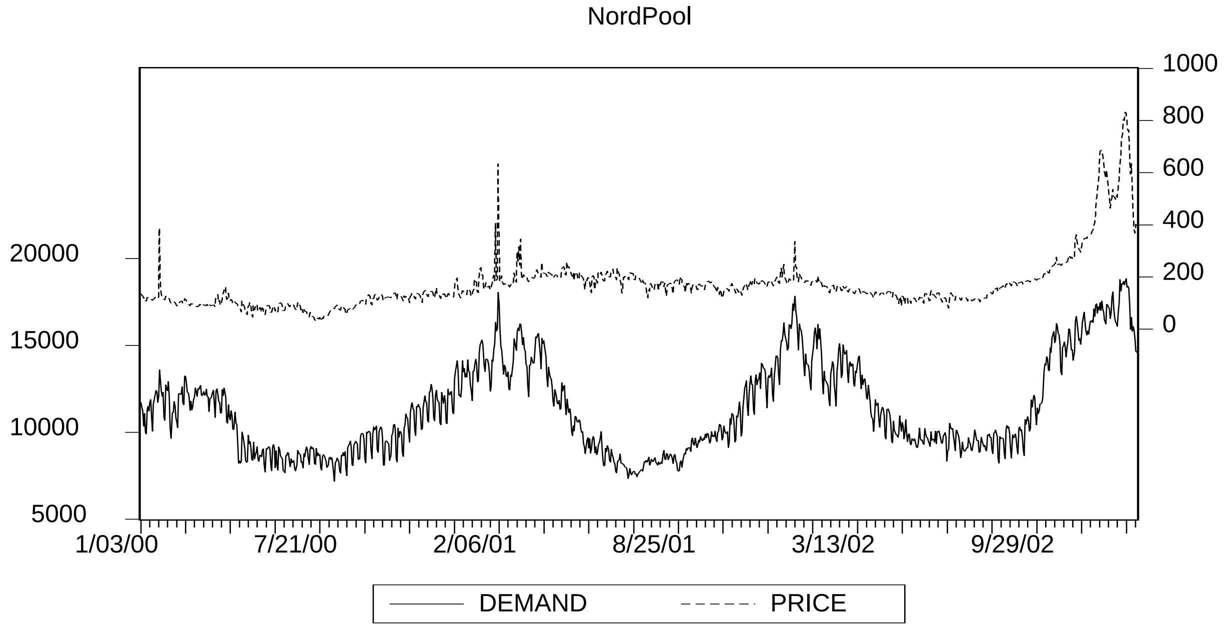



Source: Birnbaum et al. (2002). McKinsey Quarterly

Graph A.4: Price and demand series. Nord Pool. Daily data.

“Cantidad”: demand

“Precio”: price



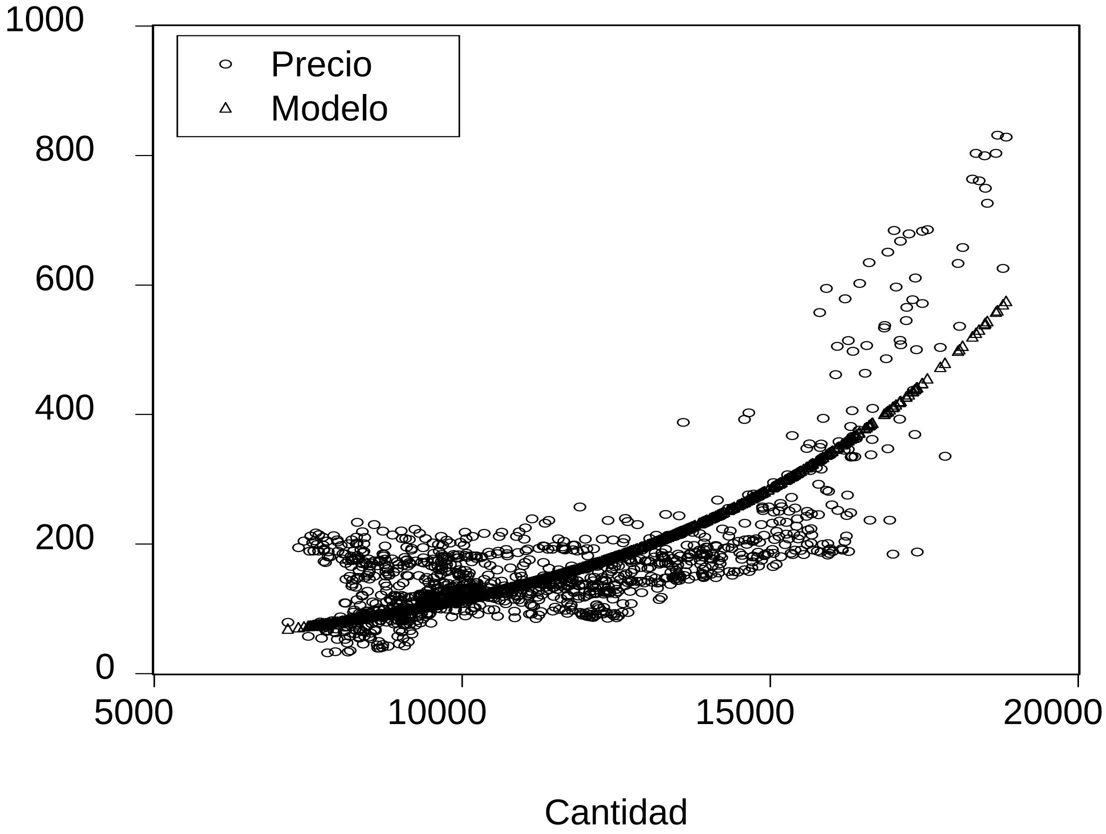
Graph A.5: Relationship between price and demand in the NordPool.

“PMODELO” is defined from the following specification: $P_t = \beta \cdot e^{\alpha \cdot D_t}$

“PRECIO”: is the observed price

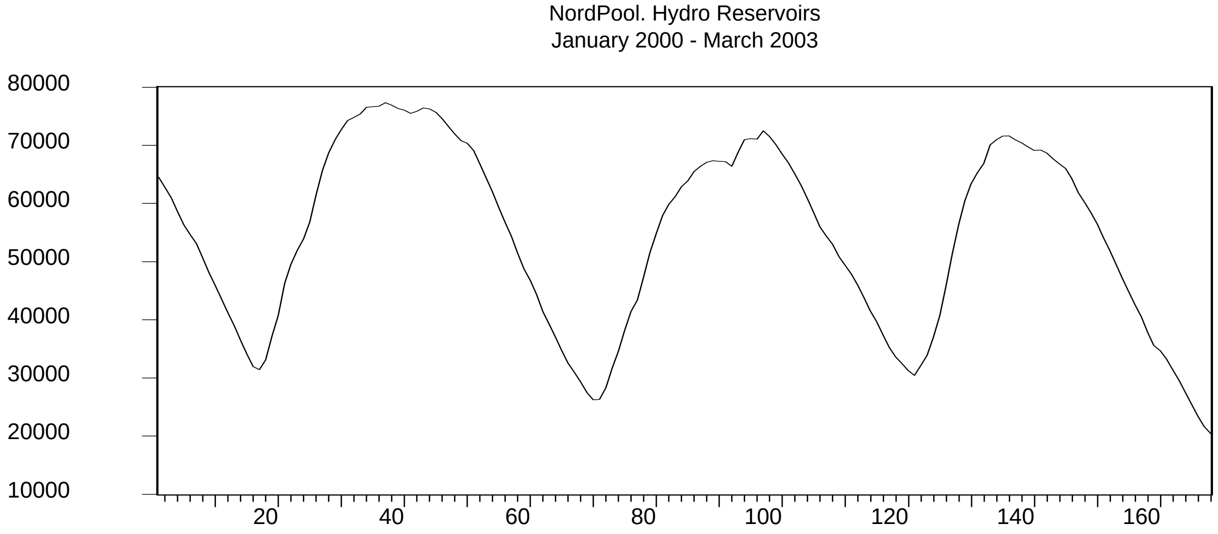
“CANTIDAD”: is the observed demand

The estimation of this simple model is just for illustrative purposes, in order to show the kind of relationship between price and the level of demand.



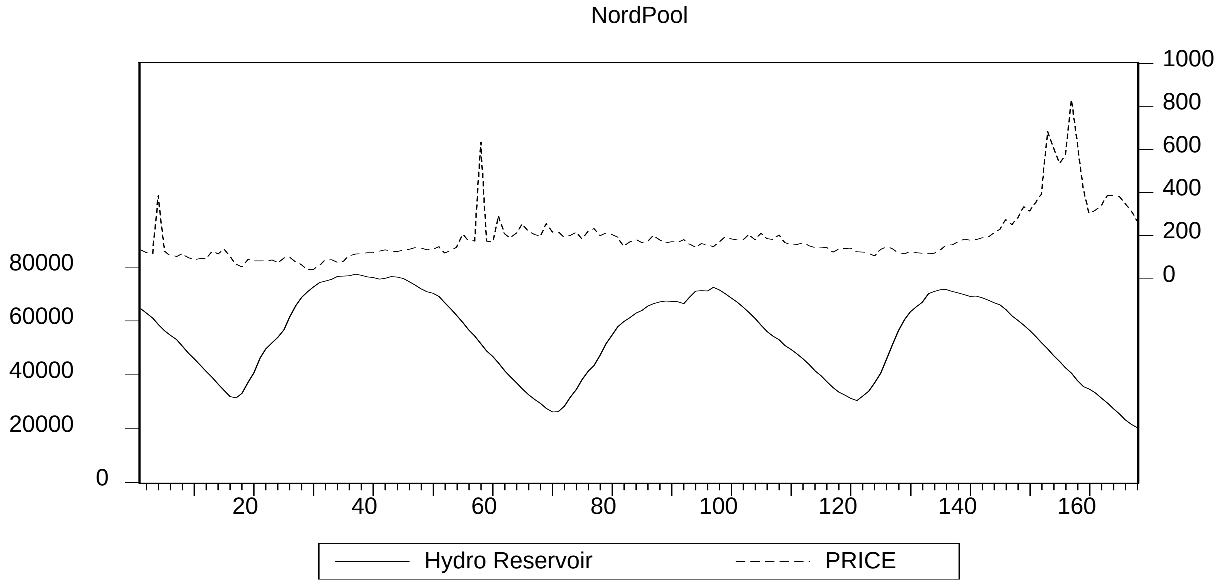
Graph A.6: NordPool market. Hydro reservoirs (GWh). Weekly data.

January 2000 – March 2003 (168 obs.)




Graph A.7: Price and Hydro Reservoir. NordPool.

Weewkly data: January 2000 – March 2003 (168 obs.)



Graph A.8: Relationship between price and hydro reservoirs. NordPool market. Weekly data. January 2000 – March 2003 (168 obs.)

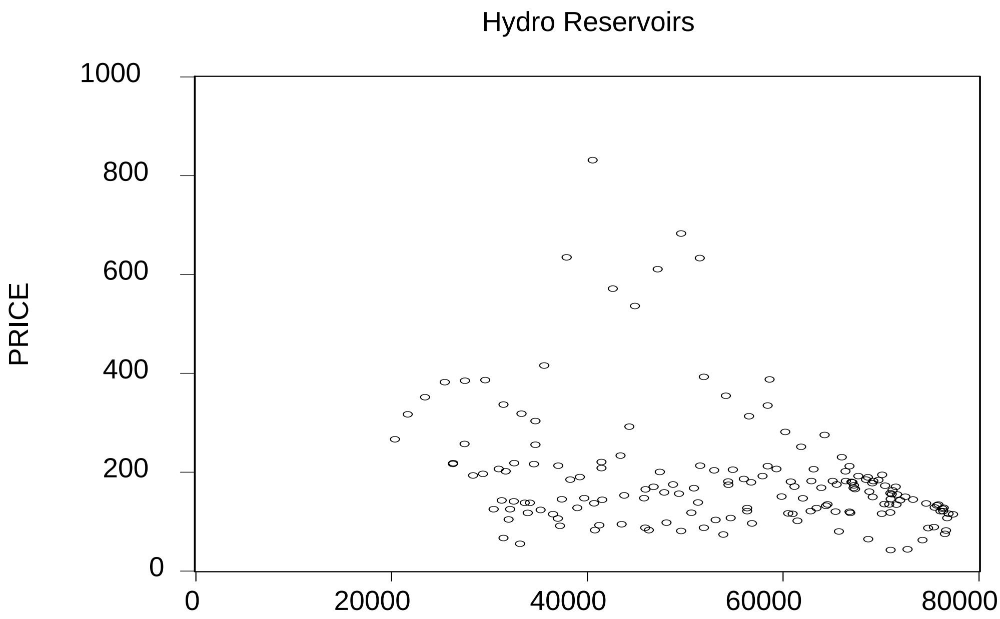


Table A.1: Estimation results, equation (4a). Nord Pool. Weekly data.

In order to run the regresión we have log-linearized equation (4a) in such a way we have:

$$P_t = C_t^{\gamma} \cdot \beta \cdot e^{\alpha \cdot D_t} \qquad \Rightarrow \qquad \ln P_t = \ln(\beta) + \gamma \cdot \ln(C_t) + \alpha \cdot D_t$$

Redefining parameters and variables this is the regression equation and the results:

**Table A.1:Estimation Results, Equation (4a). Nord Pool.**
Dependent Variable: LPRICE
Method: Least Squares
Sample: 1 168
Included observations: 168
LPRICE =C(1)+C(2)*LWATER+C(3)*VOL

| | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| C(1) | 5.968774 | 0.976608 | 6.111738 | 0.0000 |
| C(2) | -0.219197 | 0.086434 | -2.536012 | 0.0121 |
| C(3) | 0.000128 | 1.05E-05 | 12.15228 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.522616 | Mean dependent var | 5.103357 |
| Adjusted R-squared | 0.516829 | S.D. dependent var | 0.508770 |
| S.E. of regression | 0.353648 | Akaike info criterion | 0.776668 |
| Sum squared resid | 20.63607 | Schwarz criterion | 0.832453 |
| Log likelihood | -62.24008 | F-statistic | 90.31669 |
| Durbin-Watson stat | 0.296281 | Prob(F-statistic) | 0.000000 |

where:

C(1) = $\ln(\beta)$, C(2) = $\gamma$, C(3) = $\alpha$.

LPRICE: log-price

LWATER: log-hydro reservoirs, “proxy” for capacity variable ($C_t$)

VOL: demand ($D_t$)

APPENDIX: Relationship between spot price specification and producers cost function.

Bessembinder and Lemmon (2002).

Basic setup:

- one period model
- $N_P$ power producers
- Power Production Cost function: $TC_i = F + \frac{a}{c}\left(Q_{P_i}\right)^c$
- Profit (power producers can sell either in the spot or in the forward market):

$$\Pi_{P_i} = P^W Q_{P_i}^W + P^F Q_{P_i}^F - F - \frac{a}{c}\left(Q_{P_i}\right)^c$$

- Clearing Condition: $N_P \cdot Q_{P_i}^W = Q^D$
- Equilibrium price: $P^W = a\left(\frac{Q^D}{N_P}\right)^{c-1}$

Therefore for a given generic cost function :

$$TC_i = F + f\left(Q_{P_i}\right) \Rightarrow \Pi_{P_i} = P^W \cdot Q_{P_i}^W - F - f\left(Q_{P_i}\right)$$

$$\max_{Q_P^W} \Pi_{P_i} \Rightarrow P^W - f'\left(Q_{P_i}^W\right) = 0 \Rightarrow Q_{P_i}^W = f'\left(P^W\right)^{-1}$$

and the resulting clearing condition and equilibrium price are:

$$N_P \cdot Q_{P_i}^W = Q^D \Rightarrow N_P \cdot f'\left(P^W\right)^{-1} = Q^D$$

$$\Rightarrow P^W = f'\left(\frac{Q^D}{N_P}\right)$$

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