Should executives hedge their stock options and, if so, how?*

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Abstract

Executives are not permitted to hedge their options directly but can increase the certainty-equivalent value of their ESOs by hedging with an imperfectly correlated asset, even in the presence of transaction costs. The size of the gain depends on the correlation of the hedging asset's and firm's share price returns, the size of the option grant, the executive's risk aversion and the time to vesting. We characterise the optimal hedging strategy and show that sub-optimal hedging can have little benefit. Optimal private hedging increases both executive value and shareholder cost; the difference (net cost of option-based compensation) generally decreases.

JEL: G13, G30, G32, J33, M12

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1 Introduction

In recent years stock options have increasingly been used as a means of executive compensation, of which they now represent a significant component[1]. In some cases these options may also represent a sizeable proportion of the executive's wealth[2]. The value placed by executives on the options they own may, however, be affected by the strategy they adopt to manage the risk of their overall portfolio (including the option grant). This is the main issue we address in this paper: whether an executive should hedge his options, and, if so, how. We characterise the optimal hedging strategy by the executive in the presence of realistic trading frictions and restrictions and obtain numerically the resulting valuation placed by the executive on his option grant. We then compare option values if the executive does not hedge, hedges optimally and hedges sub-optimally, considering the viewpoints of both the executive and the non-executive shareholders.

The question of finding the optimal hedging and exercise strategies for the holder of the options (the executive) is complicated by features which distinguish Executive Stock Options (ESOs) from standard traded financial options[3]. ESO grants are typically *non-transferable* with long maturities and are exercisable only after an initial *vesting period*. In addition the executive faces restrictions on his ability to trade (and specifically to sell short) stock in his firm, making the as-

[1]For the U.S., Hall & Murphy (2000, 2002) report that in fiscal 1999 94% of S&P500 companies granted options to their top executives (compared with 82% in 1992), with grant date value on average 47% of total pay (using Black-Scholes estimates of option values) (21% in 1992) and that 45% of a broad sample of U.S. companies awarded options to exempt salaried employees in 1998 (12% to non-exempt and 10% to hourly paid respectively). Again using Black-Scholes estimates of option values, Conyon & Murphy (2000) find that, on average, options granted represent 10% of executive compensation for the largest (by market capitalisation) U.K. companies in fiscal 1997.

[2]Details of executives' outside wealth are difficult to obtain, however Hall & Murphy (2000) report that, in 1999, 56% of CEO pay for S&P Industrials arose from stock options (using Black-Scholes estimates of option values).

[3]See Rubinstein (1995) for a more comprehensive list.

set underlying the option grant essentially *non-tradeable*[4]. The effect of these features is that perfect market valuation methods (*e.g.* Black-Scholes) cannot be used to value ESOs; the executive's valuation will take account of the risk imposed by the option position.

Academic interest in the valuation of ESOs has increased considerably over recent years. Early models used a perfect markets setting ('Black-Scholes world') to value ESOs incorporating features such as dilution effects, delayed vesting, and non-standard options contracts. More recent valuation models[5] have recognised that the inability to trade (either the option or the underlying asset) results in the executive placing a private value on the option grant, which is lower than the perfect markets price determined in a Black-Scholes world. The difference between private (certainty equivalent) and perfect-market values is generally found to increase with the executive's risk aversion and the level of risk (volatility of the firm's share price returns). However, whilst these models use a certainty equivalent formulation to obtain the value of the option to the executive, they generally either do not consider the investment strategy for the executive's outside wealth or they specify it exogenously (*e.g.* on early exercise of the option) and generally do not alter (and do not consider optimising) the investment strategy to take account of the additional non-tradeable and non-transferable option position[6].

[4]Executives in the US, as insiders, are prohibited by Section 16-c of the Securities Exchange Act from selling short the shares of their firm (Carpenter (1998)). Black Scholes delta hedging using the shares of the firm would involve holding a negative number of shares to hedge the option grant. As noted by amongst others Core & Guay (2003), if the executive owns sufficient saleable stock prior to the option grant to be able to hedge the option grant whilst maintaining a non-negative number of shares, then the option value (in the absence of transaction costs) equals the Black-Scholes perfect market value. We do not consider this possibility and instead assume the executive initially holds no shares in the company. Holding a positive number of shares and not adjusting dynamically for the option grant would affect option values; we do not however quantify this effect.

[5]For example Lambert, Larcker & Verrecchia (1991), Huddart (1994), Kulatilaka & Marcus (1994), Carpenter (1998), Hall & Murphy (2000, 2001)

[6]Huddart (1994) and Marcus & Kulatilaka (1994) assume non-option wealth is invested

One exception to this is Henderson (2005), who values European style ESOs under CARA utility, assuming the executive can trade costlessly to maintain an optimal investment strategy in the riskless asset and the market portfolio (which is imperfectly correlated with the firm's stock returns), taking into account the position in the options. In her model, the executive thus partially hedges the (market) risk associated with the option grant. Henderson investigates the relationship between different types of risk (market and firm-specific) and finds that executive valuations increase as the proportion of market to firm-specific risk increases. However, she does not address the effects of hedging *per se* and maintains other perfect market assumptions such as the ability to trade costlessly in the partially-correlated hedging asset. In practice, there are transaction costs involved in adjusting asset holdings which should be taken into account in determining both the hedging strategy and the valuation placed on option positions by executives: this we do.

The incorporation of transaction costs into the valuation of financial options and the associated hedging strategy has been extensively studied and even low levels of transaction costs have been found to have significant effects on both hedging strategies and option values[7].

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in the riskless asset. Carpenter (1998) assumes non-option wealth (and proceeds of early exercise) are invested optimally in the stock of the firm and the riskless asset but does not adjust the investment strategy to take account of the option holding. Hall & Murphy (2000, 2001) assume non-option wealth and the proceeds of early exercise are invested in the riskless asset but that the executive also holds a constant number of shares in the firm. Again, no adjustment is made due to the option holding. Ingersoll (2002) finds the *marginal* value of an option grant assuming the executive is constrained to hold a non-zero constant fraction of his wealth in the firm's stock. Since the model is valid for option grants which are marginal and hence small relative to overall wealth, the adjustment would also be marginal. Our model is valid for a range of sizes of option grant; indeed we find the size of the option grant has a significant effect on the executive's (and non-executive shareholders') per-option values (in Ingersoll's terminology, the subjective and objective per-option values).

[7]Leland (1985) first recognised that hedging options using the Black-Scholes delta-hedging strategy in the presence of non-zero transaction costs led to potentially unbounded

Optimal hedging strategies and certainty equivalent option values incorporating transaction costs in trading the underlying asset in a utility-based framework have been considered by in a number of papers[8], which find the optimal hedging strategy is to trade only when the option's Delta moves outside a given band. Recently Whalley (2005) has derived formulae approximating the optimal hedging strategy in the presence of transaction costs and equations for the value of portfolios of European options on a non-traded asset using a partially correlated hedging asset. However all of the above transaction cost models consider only European style financial options[9]. Incorporating early exercise in an optimising utility-based model necessitates careful modelling of the optimal investment strategy after exercise; however this feature is important for ESOs as they generally vest at some time before maturity. Our model allows for optimal early exercise in a time-consistent framework, (*i.e.* assuming optimal investment by the executive both before *and after* exercise).

We follow the formulation in Whalley (2005) for hedging using a partially correlated asset with proportional transaction costs, adding conditions for optimal early exercise after the option has vested. This gives a set of partial differential equations and, importantly, associated boundary conditions, for the certainty equivalent option value to the executive, which we solve numerically.

We find that hedging activity by the executive alters his valuation of the option grant. Executives hedge to reduce the risk associated

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costs. He proposed a discrete hedging strategy of transacting at fixed points in time and derived the value of plain-vanilla European options under this hedging strategy in closed form. This was extended to value portfolios of options by Hoggard, Whalley & Wilmott (1992) other models incorporating exogenous hedging strategies include Henrotte (1993), Whalley & Wilmott (1993), Grannan & Swindle (1996), and Avellaneda & Paras (1993), and, using a binomial framework, Boyle & Vorst (1992), Bensaid, Lesne, Pages & Schienkman (1992) and Edirisinghe, Naik & Uppal (1993).

[8] Hodges & Neuberger (1989), Hodges & Clewlow (1997), Davis, Panas & Zariphopoulou (1993), and Whalley & Wilmott (1997, 1999).

[9] Zakamouline considers numerical solutions to the problem of optimal valuation of American style options under transaction costs.

with the option grant. Optimal hedging increases the option value to them by more, the greater the potential for risk reduction, *i.e.* the greater the proportion of risk which can be hedged, measured by the correlation between the returns of the firm's stock and the hedging asset, and the greater the effect of risk reduction, and hence the greater the executive's risk aversion and the larger the size of the option grant.

Transaction costs introduce a tradeoff between the benefits of risk reduction and costs of hedging. In the presence of transaction costs, the executive's valuation of the option grant decreases and the optimal hedging strategy for the executive changes from holding a single optimal amount[10] in the hedging asset at any time (and thus trading continuously) to a strategy of allowing the amount held in the hedging asset to vary within a band. Transactions occur only if the amount actually invested in the hedging asset lies outside the band; the effect of any transaction is to bring the amount invested in the hedging asset back within the no-transaction band. The width of the band reflects the tradeoff so executives hedge less with larger transaction costs, lower risk aversion, and greater share price volatility. The width of the band affects the certainty equivalent option value directly, increasing with the (absolute) correlation between the returns of the hedged and hedging assets, but decreasing on a per-option basis with increases in both the size of the option grant and the executive's risk aversion.

Overall, we find hedging (optimally) is increasingly beneficial to executives, the larger their option grants, and, for small transaction costs, the more risk averse the executive and the greater the absolute correlation between the returns on the hedged and hedging assets.

Next, we investigate the effects of suboptimal hedging by deriving the equations satisfied by the certainty equivalent option value for an executive following a sub-optimal hedging strategy and solving these numerically. We find that using a sub-optimal 'naive' hedging strategy

[10]This amount varies with the firm's share price and the Delta of the option grant.

reduces certainty equivalent option values, potentially to less than the executive's valuation of the option if unhedged, consistent with the increased risk of the executive's overall portfolio[11] .

Thus the ability of an executive to hedge dynamically using an imperfectly-correlated hedging asset, and his choice of dynamic hedging strategy have significant effects on the value he places on the option part of his compensation package. However, they may also affect the value placed on that option grant by its writers, the non-executive shareholders, because of the resulting change in the executive's early exercise policy. For the non-executive shareholders, the option grant is valued in a Black-Scholes framework, taking account of the early exercise policy of the option holder (executive)[12]. Since ESOs generally vest before maturity, and the residual risk due to imperfect hedging by the executive can induce optimal early exercise, even in the absence of dividends, the (absolute value of the) cost to the shareholders of the ESO is lower than the Black-Scholes value but higher than the executive's valuation, and increases with increases in the exercise threshold.

Hedging by the executive increases his valuation of the option grant and hence his optimal early exercise strategy, thus also increasing the cost of the option grant to the shareholders. However, we find that the net cost of the grant (executive's value minus shareholder's value), whilst remaining negative, is generally lower if executives hedge (except for options close to the exercise boundary). In particular, for a standard option package granted at-the-money the net cost is lower

[11] Monoyios (2004) found suboptimal hedging of European-style traded options using an imperfectly-correlated hedging asset increased the standard deviation of and decreased the median hedging error (relative to optimal hedging), with greater effect for lower absolute correlation and higher levels of risk aversion.

[12] This distinction between the value of the option grant to the executive and the non-executive shareholders has been noted in earlier models which incorporated non-tradeability and utility-based valuation, *e.g.* Muelbroek (2001), Rubinstein (1995), Carpenter (1998) and Hall & Murphy (2000, 2001). However none of the above has investigated the effects of the executive's hedging strategy on the early exercise boundary and hence the cost to the non-executive shareholders.

if the executive hedges, particularly for short vesting periods.

The structure of the rest of the paper is as follows. Section 2 describes the model and discusses the nonlinearity of the problem. Section 3 examines the implications for executives: whether to hedge, and if they do, how their choices of hedging asset and hedging strategy affect the value of the option grant to them. Section 4 considers some of the resulting implications for the non-executive shareholders who make the option grant. Section 5 concludes and considers further work.

2 The model

2.1 Setup

We consider an executive who owns a single tranche of options on the stock of the firm he manages. The option grant gives him the right to buy n shares, each with a strike price K. The options have maturity T and time to vest T_V.

The firm's stock price follows geometric Brownian motion

$$dS = (r + \xi\eta - \delta)Sdt + \eta SdW$$

where $\xi = \frac{\mu_S - r}{\eta}$ is the Sharpe ratio for the stock and δ is its dividend yield. We abstract from leverage and dilution considerations, which should not affect our main results.

The executive is prohibited from trading in the shares of the firm, but may trade in the riskless asset and also in an imperfectly correlated risky *hedging asset*[13], M, where

$$dM = (r + \lambda\sigma)Mdt + \sigma MdW_M$$

with $dWdW_M = \rho dt$.

Trades in the hedging asset incur a cost proportional to the value traded

$$kM|dy|$$

[13]This may but need not be viewed as the market

where dy is the number of hedging assets traded.

The executive values the option grant by its certainty equivalent value, assuming optimal investment of his non-firm related wealth. For tractability, we assume CARA preferences[14], so the executive has negative exponential utility with absolute risk aversion γ:

$$U(x) = -\frac{1}{\gamma}e^{-\gamma x}.$$

At each point in time the executive maximises his discounted expected utility of wealth at some date after the maturity of the option, taking into account his choices over

1. how much of the hedging asset to hold, *i.e.* when and how much to trade, and, if appropriate,
2. when to exercise the options once they have vested[15].

We solve the optimal investment problems for the executive with and without the option grant and hence find the certainty equivalent value and the optimal trading strategy when holding the option grant.

[14]CRRA or power utility results in equations which depend on the executive's wealth level, thus increasing the dimension of the problem.

[15]We assume that options are exercised as a block and stock acquired on exercise is sold immediately. In the absence of restrictions on such a sale, or additional implications for the executive's subjective utiltity, immediate sale will be optimal for the executive; indeed, the reduction in the non-hedgeable risk obtained by selling provides the main rationale for his early exercise of the option grant. Some firms may adjust the composition of the executive's compensation depending on his current exposure to the firm's share price. Different assumptions would have differing effects on the early exercise threshold. For example, Sircar & Xiong (2003) assume that on exercise, executives receive a new grant of the same number of at-the-money options. Characterisation of these differences is beyond the scope of this paper. Simultaneous exercise is not the optimal exercise policy (Jain & Subramanian (2003) consider the effects of partial early exercise), but may correspond reasonably well to exercise decisions in practice. It is further assumed that the exercise proceeds are subsequently invested optimally in the riskless and hedging assets, whenever exercise occurs. This corresponds to *time consistency* of the subjective utility function and allows for the time consistent valuation of American style options, which is an innovation in this paper for the valuation of ESOs. See the Appendix for further discussion of time consistency.

In order to simplify the solution, we utilise the fact that transaction costs are typically small ($k \approx 1-2\%$ [16]) to find asymptotic expansions for the option value and the location of the no transaction band. In principle these could be functions of S, M, y and t; however we find that all quantities of interest are *independent* of the actual value of the hedging asset, M, which simplifies their calculation significantly.

2.2 Solution

Per-option European option values when hedged with an imperfectly correlated hedging asset incurring proportional transaction costs can be approximated by[17]

$$h^E \approx h_0^E + t_b^E + t_f^E + t_i^E$$

where h_0^E is the value of the corresponding European style option in the absence of transaction costs and t_b^E, t_f^E and t_i^E represent certainty equivalent value of per-option transaction csots during the life of the option (trading to remain within the no transaction cost band), and at the final and initial dates respectively.

h_0^E, t_b^E and t_f^E satisfy respectively equations (2), (3) and (4) with final conditions (5), (6) and (7) below; t_i is given by $t_i^E = -k\frac{\rho\eta}{\sigma}S\frac{\partial h_0^E}{\partial S}$.

We must allow additionally for optimal early exercise. Repeating the analysis (see Whalley (2005) for details) with a time-consisitent utility function leads to

$$h \approx (h_0 + t_b + t_f) + t_i.$$

with

$$t_i = -k\frac{\rho\eta}{\sigma}S\frac{\partial h_0}{\partial S}$$

and the same differential equations (2), (3) and (4), and final conditions (5), (6) and (7) for h_0, t_b and t_f respectively but with an

[16] De Jong, Nijman and Röell (1995) quote bid-ask spreads in Paris of between $0.15-0.45\%$ and on SEAQ of $0.8-2.2\%$. Individuals will generally pay higher levels of transaction costs per trade including commissions.

[17] See Whalley (2005)

additional free boundary:

$$h_0(S,t) + t_b(S,t) + t_f(S,t) \quad \geq \quad \max(S-K,0) - k\frac{\rho\eta}{\sigma}SI_{\{S\geq K\}}. \quad (1)$$

where I is an indicator function, representing the fact that the certainty equivalent of the option value including future transaction costs if unexercised is always at least as great as the payoff net of transaction costs. Optimal early exercise occurs when (1) holds with equality; in addition a smooth pasing condition will also hold:

$$h_{0_S} + t_{b_S} + t_{f_S} = 1 - k\frac{\rho\eta}{\sigma}.$$

This gives an approximation to the boundary which is independent of the current amount held in the hedging asset, My[18].

In summary, we solve

$$h_{0_t} + rSh_{0_S} - rh_0 + \frac{\eta^2}{2}S^2 h_{0_{SS}} - n\hat{\gamma}(t)\frac{\eta^2}{2}(1-\rho^2)S^2 h_{0_S}^2 = 0 \qquad (2)$$

$$\begin{aligned} t_{b_t} + rSt_{b_S} - rh_2 + \frac{\eta^2}{2}S^2 t_{b_{SS}} - n\hat{\gamma}(t)\eta^2(1-\rho^2)S^2 h_{0_S} t_{b_S} \\ -n\hat{\gamma}(t)\frac{\sigma^2}{2}\left(\left(\frac{MY^+}{n}\right)^2 - \left(\frac{MY_0^+}{n}\right)^2\right) = 0 \quad (3) \end{aligned}$$

where $(MY^+)(S,t)$ and $(MY_0^+)(t)$ are the half-widths of the no-transaction bands in terms of the amount held of the hedging asset held at t with and without the option position and are defined below, and

$$\begin{aligned} t_{f_t} + rSt_{f_S} - rt_f + \frac{\eta^2}{2}S^2 t_{f_{SS}} \\ - n\hat{\gamma}(t)\eta^2(1-\rho^2)S^2 h_{0_S} t_{f_S} = 0 \qquad (4) \end{aligned}$$

[18]We approximate the actual amount held in the hedging asset on exercise by its value to leading order (the difference between the centre of the no-transaction band including the option grant and the centre of the no-transaction band once the options have been exercised). In practice the boundary will depend on the actual amount held, as in Zakamouline (2004) and thus on Y in our variables; however dependence on Y in the option value first occurs at a higher order of magnitude ($O(\epsilon)$) and the qualitative results should not be affected by this.

numerically, using finite difference methods, with final conditions

$$h_0(S,T) = \max(S-K,0) \tag{5}$$
$$t_b(S,T) = 0 \tag{6}$$

and[19]

$$t_f(S,T) = -k\frac{|\rho|\eta}{\sigma}SI_{\{S\geq K\}} \tag{7}$$

respectively, and for options which have vested, ensuring at each step that the value is at least as great as the payoff on exercise net of transaction costs, *i.e.* (1).

Note (2) is the same as that satisfied by a costlessly partially hedged option as derived in Henderson (2005) (though the option value will differ due to the early exercise condition); if trading in the underlying shares is possible for the executive (so $\rho = 1$ and $\sigma = \eta$) this reduces to the Black-Scholes equation. $t_b < 0$ represents the costs which are incurred during the life of the option by transactions made to bring the value actually held in the hedging asset within the no-transaction band. The effect per option of unwinding the hedge on exercise is captured by the t_f term which is also negative.

The optimal trading strategy consists of a no transaction region, within which the *number*, y, of the hedging aset hold remains constant. If the prices of the hedged (underlying share price) or hedging asset move so the value held in the hedging asset, My, is outside this band, transactions are made to bring My back within the band. The locations of the boundaries between the regions (the edges of the no-transaction band), determined endogenously, are given by .

$$My^+ = My^* + MY^+ + \ldots; \quad My^- = My^* - MY^- + \ldots,$$

before the executive has exercised and

$$My_0^+ = My_0^* + MY_0^+ + \ldots; \quad My_0^- = My^* - MY_0^- + \ldots,$$

[19] Assuming $My^*(S,T) > 0$

The leading order difference between the amounts held in the hedging asset, *i.e.* the *extra* amount held due to the option grant,

$$My^* - My_0^* \approx -\frac{\rho\eta S}{\sigma} h_{0_S},$$

is proportional to the firm's stock price multiplied by the option's delta. Note however that, since h_0 does not satisfy the Black-Scholes equation, h_{0_S} is not the Black-Scholes delta and must be calculated numerically.

The no-transaction band is symmetrical to leading order. The semi bandwidth per option, MY^+/n, depends on the level of transaction costs, the entrepreneur's risk aversion and the Delta and Gamma (first and second derivatives) of the option value and is given by the solution to

$$\begin{aligned} \left(\frac{MY^+}{n}\right)^3 &= \frac{3k\eta^2}{2\sigma^2 n\hat{\gamma}(t)} \left[\rho^2 \left(\frac{\lambda}{n\hat{\gamma}(t)\eta} + \left(\frac{\eta}{\sigma} - \rho\right) S h_{0_S} + \frac{\eta}{\sigma} S^2 h_{0_{SS}}\right)^2 \right. \\ &\quad \left. + (1-\rho^2)\left(\frac{\lambda}{n\hat{\gamma}(t)\eta} - \rho S h_{0_S}\right)^2\right] \end{aligned} \tag{8}$$

For a portfolio without exposure to the firm's stock price, there is also a bandwidth of

$$MY_0^+ = \left(\frac{3k}{2\sigma^2\hat{\gamma}(t)}\right)^{\frac{1}{3}} \left(\frac{\lambda}{\hat{\gamma}(t)}\right)^{\frac{2}{3}}$$

which is independent of n. Note if $\rho = 1$ and $\sigma = \eta$, (8) reduces to the perfect hedging bandwidth of Whalley & Wilmott (1997) which depends only on the option's Gamma, h_{SS}:

$$\frac{MY^+}{n} = \left(\frac{3k}{\sigma^2 n\hat{\gamma}(t)}\right)^{\frac{1}{3}} \left(\frac{\eta^2}{2}\left|\frac{\lambda}{n\hat{\gamma}(t)\eta} + S^2 h_{0_{SS}}\right|^2\right)^{\frac{1}{3}}$$

Imperfect correlation between the underlying and hedging asset introduces dependence also on the option's Delta, h_S in the bandwidth and hence the certainty equivalent value of transaction costs.

2.3 Discussion

The equations satisfied by the certainty equivalent per option value for the executive are nonlinear and, in particular, depend on the size of the option grant, n, and the executive's risk aversion, γ. There are two sources of deviation from the perfect markets valuation method (the Black-Scholes equation): 'cost of unhedged risk' terms, which arise in the equations for all orders of magnitude, and transaction cost terms, which occur in the equations/boundary conditions for h_2 and h_3.

The 'cost of unhedged risk' terms in the equations for the per-option value are of the form (for t_b and t_f)

$$-\left(n\hat{\gamma}(t)\eta^2(1-\rho^2)Sh_{0_S}\right)St_S$$

As the number of options in the option grant increases or the executive's risk aversion increases the magnitude of this term in the equation *per option* increases, thus reducing the value per option further relative to the Black-Scholes value. The magnitude of this effect is proportional to $n\gamma\eta^2(1-\rho^2)$ and thus increases with the size of the option grant, the executive's risk aversion and the undiversifiable risk. It is also greater for options with higher h_{0_S}, *i.e.* which are further in the money, and its cumulative effect increases with time to maturity.

For options which have not yet vested this can reduce the option value below the payoff, with greater effect the greater the remaining time until vesting. For options which have already vested, it may be optimal to exercise before maturity even if the firm pays no dividends if the option is sufficiently in-the-money (the payoff is sufficiently large). The effect of a larger grant, by increasing the magnitude of the value reducing 'cost of risk' term, is to reduce option values and hence reduce the early exercise threshold Since the value per option is then bounded below by the payoff, the reduction in value due to lack of diversification is lessened (relative to the unvested case). These effects are documented for unhedged options in Carpenter (1998) and Hall & Murphy (2000, 2001).

The leading order transaction cost term (in the equation for t_b) is

$$-n\hat{\gamma}(t)\frac{\sigma^2}{2}\left(\left(\frac{MY^+}{n}\right)^2-\left(\frac{MY_0^+}{n}\right)^2\right)$$

which is proportional to the difference in the squared semibandwidths for the executive's portfolio with and without the option position. Overall, this decreases in magnitude as $n\gamma$ increases, reflecting the economies of scale in hedging larger option grants. As $n\gamma$ increases, the overall semibandwidth increases less than proportionately with $n\gamma$ with the effect of reducing the certainty equivalent of expected transaction costs per option. The top panel in Table 1 shows how the semibandwidth per option and at-the-money total certainty equivalent value of transaction costs per option and per option value at inception vary with $\nu = n\gamma K$[20].

Insert Table 1 here.

All these terms have the effect of reducing option values overall. As $n\gamma$ increases, the economies of scale in the dynamic hedging strategy have the greatest effect so the per option certainty equivalent of expected transaction costs decreases. The exception is for low values of $n\gamma$, for which the cost of unhedged risk terms dominate and the expected transaction costs increase with $n\gamma$. For the per-option valuation overall the cost of unhedged risk terms dominate and per-option values decrease with $n\gamma$.

[20]Note the per option value depends on n and γ only via $n\gamma$. Hence we shall present results for different values of $\nu = n\gamma K$, where we have also normalised by the strike price of the option grant.

3 Implications for executives

3.1 Whether to hedge

The executive's per-option certainty equivalent valuation of an unhedged option grant[21] is shown in the Appendix to satisfy

$$f_t + rSf_S - rf + \frac{\eta^2}{2}S^2 f_{SS} - n\hat{\gamma}(t)\frac{\eta^2}{2}S^2 f_S^2 = 0 \qquad (9)$$

with final condition

$$f(S,T) = \max(S-K,0)$$

and early exercise condition $f(S,t) \geq \max(S-K,0)$ for $t > T_V$.

Similarly, as in Henderson (2005), the equation satisfied by the per-option certainty equivalent valuation of a costlessly partially hedged option grant is

$$g_t + rSg_S - rg + \frac{\eta^2}{2}S^2 g_{SS} - n\hat{\gamma}(t)\frac{\eta^2}{2}(1-\rho^2)S^2 g_S^2 = 0 \qquad (10)$$

with the same final condition

$$g(S,T) = \max(S-K,0)$$

and, as shown for the unhedged option in the Appendix, an early exercise condition for $t > T_V$ of $g(S,t) \geq \max(S-K,0)$.

Note the only difference between equations (9) and (10) is in the final term, which is proportional to the square of the options' respective Deltas. Since this term is always negative in both equations, it has the effect of unambiguously reducing option values relative to the Black-Scholes value. As it involves the square of the option's Delta,

[21] The certainty equivalent option values in Marcus & Kulatilaka (1994), Carpenter (1998) and Hall & Murphy (2000, 2001), none of which adjust the investment of the executive's wealth as a result of the option grant, are closest to the option values in this section. The different assumptions they make about the investment strategies for non-option wealth and discount rates will however affect values to some extent, in particular relative to the early exercise condition. See the Appendix for a discussion of time consistency of utility functions.

the resulting equations are nonlinear, which implies non-additivity of solutions so that the certainty equivalent value to the executive per option depends on the total number of options held.

Insert Figure 1 here

The reduction in certainty equivalent option value to the executive resulting from this extra term in both differential equations represents the effect of the additional risk the executive is forced to bear as a result of the option grant. Thus the magnitude of the term increases with the executive's risk aversion, γ, and with the size of the option grant, n. In the case of no hedging, this term is proportional to the squared *total* volatility, η^2, of the underlying shares; with partial hedging, the squared volatility in the term is instead the *undiversified*[22] volatility, $\eta^2(1-\rho^2)$. For any $|\rho| < 1$, the magnitude of the reduction in the partial hedging case is lower. This is illustrated in Figure 1, which shows the solutions to (9) and (10) in terms of values per option for the base case option grant at inception $T = 10$ and on vesting $T = T_V = 5$ for $\rho = 0.8$. The certainty equivalent option values with partial hedging are greater than the option values with no hedging in all cases; however the magnitude of the effect of undiversifiable risk on option values is attenuated if that risk can be eliminated by exercising the option (which becomes worthwhile for well in-the-money options), so the difference between the certainty equivalent values is smaller for options which have vested (the $T = 5$ lines in Figure 1). The option values at inception ($T = 10$), which have 5 years before the option vests and early exercise is possible are much lower and the difference between the partial hedging and no hedging option values is much greater, particularly for high share prices, the longer the time until the option vests.

Thus in the absence of costs it is unambiguously in the executive's best interests to hedge for any value of $\rho \neq 0$.

Insert Figure 2 here

[22]In a CAPM world, this is the share price return's idiosyncratic standard deviation.

For realistic parameter values[23], this result continues to hold in the presence of transaction costs and optimal hedging. This is shown in Figure 2 and can be seen by considering the magnitude of the terms that reduce the certainty-equivalent per-option value relative to the Black-Scholes value. The effect of hedging with an asset with correlation ρ is to reduce the magnitude of the cost of unhedged risk term in the equation from $-\eta^2\hat{\gamma}(t)S^2 f_S^2/2$ to $-\eta^2(1-\rho^2)\hat{\gamma}(t)S^2 h_{0S}^2/2$, thus increasing the value to the executive by an amount per option of approximately $O(\gamma\rho^2 n)$; however the costs associated with hedging introduce an additional term of order $\max(O(k^{\frac{2}{3}}, O((k^2\gamma n)^{\frac{1}{3}})$ which reduces the option value. If the transaction costs are sufficently small then the increase in value due to the reduction in unhedged risk dominates, as can be seen in the right panel of 1.

The net increase in value to the executive is greater

- the larger the option grant
- the greater the reduction in risk due to hedging
- the more risk averse the executive
- the greater the time until maturity and the time until vesting of the option

3.2 Choice of hedging asset

An executive may have a choice of partially-correlated tradeable assets with which to hedge including *e.g.* index futures and stocks or baskets of stocks in similar industries[24]. Potential assets will differ in their volatility, correlation with the stock price being hedged and level of transaction costs incurred in making a trade.

[23] As long as $O(\rho^2) \gg \max(O(k^{\frac{2}{3}}/(\gamma n)))$

[24] We assume that the executive invests only in a single risky hedging asset and chooses his investment strategy optimally over time. Allowing for multiple risky assets in a non-CAPM framework would complicate the analysis, specifically of the strategy in the absence of the option, but would contribute little to the valuation or hedging of the option itself.

We have seen that it is only the correlation between the hedging and hedged assets which affects the benefits of hedging due to the reduction in the unhedged risk, whereas all three (correlation, the volatility of the hedging asset and the transaction cost level) affect the certainty-equivalent value of expected transaction costs associated with the hedge. The magnitude of the reduction in certainty-equivalent per option value due to unhedged risk decreases as the absolute level of correlation increases, whereas the reduction in per-option value due to expected transaction costs increases with absolute correlation and the transaction cost level.Numerical simulation shows transaction costs have a greater impact for close to the money options, whereas changes in correlation have the greatest effect for in-the-money options which have not yet vested and for large changes in $1-\rho^2$, *i.e.* for high $|\rho|$. From the equations we can also see that the magnitude of the effect of unhedged risk on per-option value increases with $n\gamma$, whilst the effect of transaction costs decreases with $n\gamma$ due to economies of scale. There is thus a trade-off between higher absolute correlation and higher transaction costs which varies with $n\gamma$.

Insert Table 2 here

Table 2 lists the per-option values assuming partial hedging for hedging assets with varying levels of correlation and transaction costs for at-the-money and in-the-money ($S/K = 2$) option grants at inception and on vesting for base case parameters (so $n\gamma K = \nu = 1$). This can be used to compare benefits of alternative hedging assets. In general higher option values are achieved by selecting hedging assets with higher absolute correlation (keeping the transaction cost level fixed) and lower transaction costs (keeping ρ fixed). For at-the-money options at inception[25] for $n\gamma K = 1$, the effects of doubling the transaction cost level are approximately equivalent to decreasing the absolute correlation by 0.1. For deeper in-the-money options (at inception, so

[25]This is the most relevant panel, since hedging asset decisions will be made at inception of the grant and the majority of options are issued at-the-money (see *e.g.* Murphy (1999)).

with 5 years until vesting), the effects of unhedged risk increase by more than the increase in expected transaction costs, so hedging assets with higher values of absolute correlation give higher option values for a greater range of transaction cost levels. For options which have vested, the possibility of early exercise decreases the magnitude of the reduction in option values due to unhedged risk; thus for at-the-money options, transaction costs are (slightly) relatively more important in the choice of hedging asset, and this effect is more pronounced for in-the-money options.

Insert Table 3 here

Table 3 lists equivalent per-option values for varying ρ and k and for different effective sizes of option grant ($\nu = n\gamma K = 0.1$ and 10) for at-the-money options at inception. Smaller option grants or less risk-averse executives ($\nu = 0.1$) reduce the effects of both unhedged risk and transaction costs on per-option values but the decrease is greater for the unhedged risk component. Hence, for at-the-money options at inception with $\nu = 0.1$, low transaction costs are relatively more important in choosing a hedging asset than absolute correlation. (The option value for hedging assets with $\rho = 0.5$ and $k = 0.005$ is greater than that for $\rho = 0.9$ and $k = 0.01$.) Conversely, for larger option grants and/or more risk-averse executives ($\nu = 10$), high absolute correlation is relatively more important. Note that for $\nu = 0.1$, as ρ decreases from 1 for non-zero transaction costs, the certainty-equivalent per-option value first decreases and then increases, thus demonstrating directly the trade-off between risk reduction and transaction cost effects. This trade-off arises because the effects of decreasing absolute correlation on the unhedged risk component of per-option values decreases as $|\rho|$ decreases from 1 (thus increasing the benefits to hedging and option values) whereas the certainty equivalent of expected transaction costs *decreases* as $|\rho|$ decreases. The effect arises for small ν because the magnitude of the unhedged risk component varies more with ν than the transaction cost component.

Thus, whilst better hedging assets have high absolute correlation and low transaction costs, the choice between alternative hedging assets depends on the size of the option grant and the executive's risk aversion. Choosing a better hedging asset will increase the executive's value by more, the more risk-averse the executive, the longer the time until maturity and until vesting, and the larger the option grant.

3.3 Hedging strategy

Both transaction costs and the imperfect correlation between hedged and hedging assets change the optimal hedging strategy from the Black-Scholes hedging strategy, which for an option with value $V^{BS}(S,t)$ involves holding a number $-\Delta_{BS} = \partial V^{BS}/\partial S$, or equivalently an amount with value

$$-S\frac{\partial V^{BS}}{\partial S}$$

in the hedging asset, which in this case would be perfectly correlated with the shares of the firm.

Optimal partial hedging with an imperfectly correlated hedging asset in the absence of transaction costs involves continuously adjusting the amount held in the hedging asset, My, to ensure the *extra* amount held in the hedging asset because of the option grant[26] is:

$$-n\frac{\rho\eta}{\sigma}S\frac{\partial g}{\partial S}$$

where $g(S,t)$ satisfies (10). This expression is similar to that for costless perfect hedging, in that it involves the share price multiplied by the option's Delta, but this is now multiplied by a factor[27] $\rho\eta/\sigma$, with magnitude[28] which may be greater than or less than 1.

[26]The total amount held with the option position is $-n(\rho\eta/\sigma)Sg_S + (\lambda/(\sigma\hat{\gamma}(t)))$; the amount held in the absence of the option position is $(\lambda/(\sigma\hat{\gamma}(t)))$.

[27]Note both the option value and the hedging strategy reduce to their Black-Scholes equivalents for perfect hedging, for which $\rho = 1$ and $\sigma = \eta$.

[28]In theory $\rho\eta/\sigma$ has the same sign as ρ, which can thus be either positive or negative, although in practice for *e.g.* the market as a hedging asset, $\rho > 0$.

In the presence of transaction costs, the optimal hedging strategy is to hold an amount My in the hedging asset such that

$$My^*(S,t) - MY^+(S,t) \le My \le My^*(S,t) + MY^+(S,t)$$

i.e. so that the additional holding due to the option grant, $My - My_0$, remains within a no-transaction band centred, to leading order, on

$$-n\frac{\rho\eta}{\sigma}S\frac{\partial h_0}{\partial S}.$$

The semibandwidth per option for the full investment problem including the option grant is given by

$$\begin{aligned} \frac{MY^+}{n} &= \left(\frac{3k}{2\sigma^2\hat{\gamma}(t)n}\right)^{\frac{1}{3}} \left|\rho^2\left(\frac{\lambda}{\hat{\gamma}(t)n} + \left(\frac{\eta}{\sigma} - \rho\right)\eta S h_{0_S} + \frac{\eta^2}{\sigma}S^2 h_{0_{SS}}\right)^2 \right. \\ &\qquad \left. + (1-\rho^2)\left(\frac{\lambda}{\hat{\gamma}(t)n} - \rho\eta S h_{0_S}\right)^2\right|^{\frac{1}{3}}. \qquad (11) \end{aligned}$$

For $k = 0$ the bandwidth is zero and the equations reduce to the costless hedging case above. As transaction costs increase, the bandwidth, and hence the level of risk incurred in the investment strategy, increases, decreasing the total number of transactions during the life of the option. Different levels of transaction costs give different solutions to the tradeoff between reducing risk and incurring additional costs of hedging.

Imperfect correlation thus changes the magnitude of the optimal amount held in the hedging asset, whereas transaction costs change the nature of the hedging strategy from continuous to bandwidth hedging.

Insert Figure 3 here

Figure 3 shows the per option delta for options with 10 years to maturity and 5 years to vesting for different correlations with the hedging asset. Note the magnitude of the optimal amount which should be held in the hedging asset per option on a partially correlated share ($= (\rho\eta/\sigma)Sh_{0_S}$) is less than the Black-Scholes magnitude and decreases as $1-\rho^2$ increases for two reasons: because of the direct effect on the multiplier $\rho\eta/\sigma$ and also because h_{0_S} decreases as $1-\rho^2$ increases.

Insert Figure 4 here

Figure 4 shows the additional amount held in the hedging asset due to the option grant for our base case. Note the perfect hedging amount can lie outside the hedging band for risk averse executives with large option grants and low correlation and transaction cost levels.

We now consider the impact on in certainty-equivalent value from using the wrong hedging strategy in the partially correlated asset. Any deviation from the optimal hedging strategy[29] will reduce value. In the Appendix we show that if an executive follows a non-optimal policy of holding an amount

$$-nD(S,t)$$

in the hedging asset (or $D(S,t)$ per option), then the per-option value $d(S,t)$ satisfies:

$$\begin{aligned} d_t + rSd_S - rd + \frac{\eta^2}{2}S^2 d_{SS} - n\hat{\gamma}(t)\frac{\eta^2}{2}(1-\rho^2)S^2 d_S^2 \\ - n\hat{\gamma}(t)\frac{\sigma^2}{2}\left(D(S,t) - \frac{\rho\eta}{\sigma}Sd_S\right)^2 = 0 \quad (12) \end{aligned}$$

with final condition

$$d(S,T) = \max(S-K, 0)$$

Note there are now two terms that reduce the option value: the first represents the effects of the level of unhedged risk under an optimal hedging strategy, as in equation (10) for costless partial hedging, whereas the second term represents the effects of the additional risk induced by sub-optimal hedging on the option value and is proportional to the square of the difference between the optimal $((-\rho\eta/\sigma)Sd_S)$ and actual $(D(S,t))$ amounts of the hedging asset held at time t. Thus positive and negative deviations have an equally negative effect on the option value.

[29] For this section we consider only the leading order term and hence ignore the effect of differences in total transaction costs between the two strategies.

We have seen that the difference between the Black-Scholes amount and the optimal amount increases as $1-\rho^2$ increases, particularly when the option has not yet vested, so the effect of incorrectly hedging using the Black-Scholes delta would be more likely to destroy value for low values of ρ with long times to vesting.

Insert Figure 5 here

We illustrate the magnitude of the effects of mishedging in Figure 5 for a hedging strategy of

$$D = SN(d_1),$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right)}{\eta\sqrt{T}} + \frac{1}{2}\eta\sqrt{T}$$

i.e. the stock price multiplied by the Black-Scholes European call option delta[30].

Using a naive hedging strategy can thus significantly reduce the value of the option grant to the executive. The value of a sub-optimally hedged option can be lower than that of an unhedged option because of the extra risk resulting from the sub-optimal investment strategy[31].

3.4 Early exercise thresholds

Hedging thus increases the certainty equivalent option value for the executive, decreasing the discount from the perfect markets value. For options which have not yet vested, the certainty equivalent option

[30] In order to isolate the effects of the sub-optimal hedging, we considered the value if the option is exercised according to the exercise threshold associated with optimal hedging. Allowing for optimal exercise given the sub-optimal hedging strategy gave only a negligible difference in value. For assets which do not pay any dividends during the life of the option, the consistent choice for an executive hedging using the Black-Scholes European call option delta would be to exercise only at maturity; however this exercise strategy for a partially-hedged option would result in significant loss in value due to the sub-optimal exercise strategy.

[31] For example, comparing (12) and (9), this will be true if $D(S,t) > 2(\rho\eta/\sigma)Sd_S \ \forall S, t$.

value can fall below the payoff for options which are sufficiently in-the-money, because of the unhedged risk which the options impose on the executive's overall wealth, which increases with the value of the options. Similarly, for options which have already vested and are sufficiently in the money, it will be worthwhile exercising the options before maturity[32] even if the stocks pay no dividends over the life of the options[33].

Table 1 gives exercise thresholds on vesting for different values of ν and ρ[34] and shows that hedging increases the optimal exercise threshold and that the effect is greater for higher $|\rho|$, larger option grants and more risk averse executives.

4 Implications for shareholders

Shareholders are assumed to value the options granted to the executive at their perfect market value (value in a Black-Scholes world)[35]. However, for options which vest before maturity, the value is not equal to the equivalent Black-Scholes option value because the exercise strategy is determined by the executive.

Thus shareholders solve

$$q_t + rSq_S - rq + \frac{\eta^2}{2}S^2 q_{SS} = 0 \tag{13}$$

[32] Recall we assume all options are exercised simultaneously. Allowing for optimal partial exercise will not change the comparative statics of our results.

[33] Equations (9) and (10) can be viewed as Black-Scholes equations with effective dividend yields of $n\hat{\gamma}(t)\eta^2 Sh_{0_S}/2$ and $n\hat{\gamma}(t)\eta^2(1-\rho^2)Sh_{0_S}/2$ respectively, *i.e.* with dividend yields which increase with the stock price.

[34] The results shown are for costless hedging; incorporating transaction costs reduces early exercise thresholds, but as with the effects on option values, the magnitude of the dfference is small.

[35] The shareholders may be sufficiently well-diversified, or able to trade costlessly in assets which span the risks associated with the share price, or risk-neutral. Note that, although the shareholders act *as if* they were risk-neutral, they need not be. In the subsequent discussion we shall use the term 'effectively risk-neutral' to describe the shareholders because their behaviour is consistent with risk-neutrality; however, our results are valid for a wider range of scenarios than one in which shareholders are risk-neutral.

for the per-option value, q, of the grant, subject to

$$q(S,t) = -\max(S-K,0) \quad \text{if} \quad S \geq S^*(t) \tag{14}$$

where $S^*(t)$, $t > T_V$ is the early exercise boundary for the relevant option valuation problem for the executive

$$S^*(t) \;=\; \inf\{S_t : h(S_t) = \max(S_t - K, 0) - k\frac{\rho\eta}{\sigma}SI_{\{S \geq K\}}\}$$

Risk-averse executives exercise earlier than would be optimal for the shareholders, who behave as if they are risk-neutral. This early exercise decreases the cost of the grant (increasing total shareholder value) due to the early cash receipt of the options' strike price. This is illustrated in Figure 6 for options on a non-dividend paying share. In the bottom panel we show option values on vesting. The solid line represents the Black-Scholes option value, which would be the absolute value of the cost to the shareholders if the executive did not exercise early. The dashed line represents the certainty-equivalent value of the option to the executive, assuming he exercises it optimally (note that this value smooth-pastes to the payoff at the optimal exercise threshold, S^*). The dotted line represents the absolute value of the actual cost to the shareholders of the option, given the executive's actual exercise strategy. This has lower absolute value than the Black-Scholes value because of the forced early exercise, but higher absolute value than the executive's certainty-equivalent value, because of the unhedged risk that the risk-averse executive is forced to bear, which lowers his CE value relative to the effectively risk-neutral shareholders' value. In particular, since it is the executive and not the shareholders who determines the early exercise threshold, the shareholders' option value *does not* smooth paste at the exercise threshold.

Insert Figure 6 here

In the top panel we show option values at inception, *i.e.* with 5 years to vesting. Note how the value to the executive lies below the payoff for deeply in-the-money options whereas the cost to the non-executive shareholders is strictly greater than the payoff.

Insert Figure 7 here

In Figure 7 we show the difference between the shareholders' and executive's valuations of the same option grant. This represents the deadweight cost of forcing the risk-averse executive to bear unhedged risk through his compensation, and is the 'inefficiency cost' discussed by Hall & Murphy (2000, 2001). Note that the values coincide only once the option is exercised; since the shareholders' option value does not smooth paste at the exercise threshold, the cost is strictly negative for all share prices below this level. For options which have not yet vested, the difference increases further due to the cost to the risk-averse executive of the unhedged risk which cannot be eliminated by exercising the option. The total deadweight cost is thus larger for options with greater times to vesting, particularly for deeply in-the-money options; earlier vesting reduces this net deadweight cost.

If executives hedge their option grant with a partially-correlated hedging asset, they increase their value of the option grant but also increase the cost of the option grant to the shareholders (principals).

For options which have vested, the executive's early exercise decision is based on a trade-off between his cost of bearing unhedged risk if he does not exercise and the cost (in lost interest on the strike price) if he does exercise. Shareholders, on the other hand, bear only the cost of the executive's early exercise. Partial hedging reduces the executive's cost of unhedged risk for all stock prices, thus resulting in a new, higher, exercise threshold. Thus for in-the-money options where either hedged or unhedged options are close to being exercised[36], the increase in the cost to the shareholders if the executive hedges can be greater than the increase in value to the executive, *i.e.* the *net* cost can be greater for hedged options (see the top lines in Figure 7). However, for lower stock prices, the effect of the decrease in option values due to non-diversified risk, which is greater for unhedged options, dominates

[36]By 'close to being exercised', we mean options which either have already vested or have a short time to vesting, and for share prices close to the early exercise threshold; these options will have a low expected time until exercise.

and the net cost of the option grant is greater for unhedged options.

Note that, for European-style options, the cost to the shareholders is unchanged by partial hedging by the executive (since the non-executive shareholder value is affected only by changes in early exercise patterns), so the increase in value to the executive is always greater than the increase in cost to the shareholders. More generally, during the vesting period (when the executive is not allowed to exercise) the net difference (cost) will increase more slowly for hedged than for unhedged option grants and overall the net difference is reduced by decreasing the time until vesting of the options. Thus, *ex ante* contracts which minimise the deadweight loss involve short vesting periods.

5 Conclusions and further work

We have derived the optimal hedging strategy for risk-averse executives endowed with an option grant which they are unable to hedge using the firm's shares which they can hedge only with a partially-correlated hedging asset, incurring proportional transaction costs at every trade. We show that hedging their options is generally beneficial to executives, increasing their private valuation of the option grant, and also that hedging decreases the 'inefficiency cost' to the firm of using options to compensate risk-averse executives. The effects of optimal hedging on option values are greater, the larger the option grant, the greater the executive's risk aversion, the longer the time to vesting and to maturity, and the greater the hedgeable risk ($\rho^2\eta^2$). For relatively large option grants and/or risk-averse executives, it is worthwhile for executives to hedge as much of the risk as possible, *i.e.* using the hedging asset which has maximal correlation. For smaller option grants and/or less risk-averse executives, transaction costs become relatively more important and may also influence the choice of hedging asset. However it is important for executives to use the correct hedging strategy for the particular hedging asset: use of a sub-optimal strategy reduces the value of the option grant, potentially to lower than its value if left unhedged.

In common with many other models of executive stock options which involve risk-averse executives, essentially risk-neutral shareholders, partial hedging and exogenous distribution of future share prices, the net cost of option compensation in this model is always negative, *i.e.* the value to the executives of option-based compensation is always strictly lower than the cost to the shareholders of providing that compensation, because of the unhedged risk it forces executives to bear. This means that there is no rationale for the use of option (or more generally stock-based) compensation in such a model. The widespread use of option (or more generally stock based) compensation is consistent with a belief by firms that the benefits of stock options outweigh their costs, potentially due to the incentive effect arising from an increase in the dependence of the executive's wealth on the firm's share price, which would induce effort and so increase the share price whilst this dependence continued to hold. Modelling such incentive effects in a continuous time model, as recently started in Cadenillas *et al* (2003) Agliardi & Andergassen (2003) and Whalley (2005b), could thus be a fruitful area of future research.

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6 Appendix

6.1 Time consistent utility functions

We wish to maximise the expected utility of terminal wealth at some date on or after the maturity of the options, assuming non-option wealth is invested optimally.

Firstly recall the Merton problem for optimising investment in a riskless and single risky asset, M, where

$$dM = (r + \lambda\sigma)M dt + \sigma M dz,$$

investing an amount θ in the risky asset and the remainder in the riskless asset so wealth, W evolves as

$$dW = (rW + \lambda\sigma\theta)dt + \sigma\theta dz.$$

Optimising the expected utility of terminal wealth,

$$J_0(W,t;T) = \sup_{\theta_s, t\le s\le T} E_t[U(W_T)],$$

J_0 satisfies

$$0 = J_{0_t} + rWJ_{0_W} + \sup_\theta\{\lambda\sigma\theta J_{0_W} + \frac{1}{2}\sigma^2\theta^2 J_{0_{WW}}\}$$

Optimising over the amount invested in the risky asset, we obtain the optimal investment strategy,

$$\theta_0^* = -\frac{\lambda J_{0_W}}{\sigma J_{0_{WW}}},$$

and the equation satisfied by $J_0(W,t;T)$

$$0 = J_{0_t} + rWJ_{0_W} - \frac{\lambda^2}{2}\frac{J_{0_W}^2}{J_{0_{WW}}}$$

which must be solved subject to $J_0(W,T) = U(W_T)$.

To obtain a *time consistent* solution to the early exercise problem for the option grant, we must solve

$$J(W,S,t;T) = \sup_{\tau:T_V\leq\tau\leq T}\ \sup_{\theta_s,t\leq s\leq\tau} E_t[J_0(W_\tau + \Lambda(S_\tau),\tau;T)], \quad (15)$$

which is not necessarily equal to $\sup_\tau \sup_\theta E_t[U(W_\tau + \Lambda(S_\tau)]$. This effectively assumes the option payoff is invested optimally (in the riskless and market assets) immediately on exercise. Henderson (2005) [21] calls J_0 the *time consistent* utility function for cashflows at T.

For exponential utility, $U(W_T) = -\frac{A}{\gamma}e^{-\gamma W_T}$, the time consistent utility is

$$J_0(W,t;T) = -\frac{A}{\gamma}e^{-\frac{\lambda^2}{2}(T-t)}e^{-\gamma e^{r(T-t)}W}$$

J_0 is time consistent because utilities at different times before the terminal date are valued consistently (with wealth being invested optimally up to the terminal date). Henderson demonstrates this consistency directly for exponential utility functions to obtain (15) for the exponential utility function.

6.2 Optimal costless hedging

We derive equations satisfied by time consistent utility based valuation of option grants given an arbitrary investment/hedging strategy in the hedging asset. The certainty equivalent value of unhedged and suboptimally hedged options arise naturally as special cases. We then derive the optimal holdings for costless hedging and the resulting equations satisfied by the certainty equivalent option value.

As above, we maximise discounted expected time consistent utility of wealth over the early exercise and investment strategies

$$J(W,S,t;T) \quad = \quad \sup_{\tau:T_V\leq\tau\leq T}\ \sup_{\theta_s,t\leq s\leq\tau} E_t[J_0(W_\tau+\Lambda(S_\tau),\tau;T)]$$

where $\Lambda(S)$ is the payoff to the option, which depends on the firm's stock price S.

S evolves as

$$dS=(r+\xi\eta)Sdt+\eta SdZ_S$$

with $dZdZ_S=\rho dt$.

In the early exercise region $J(W,S,t)=J_0(W+\Lambda(S),t)$. In the continuation region for general investment strategy of holding θ in the hedging asset, J satisfies

$$\begin{aligned}0 \quad = \quad & J_t+rWJ_W+(r+\xi\eta)SJ_S+\frac{1}{2}\eta^2S^2J_{SS}\\ & +\lambda\sigma\theta J_W+\rho\sigma\eta\theta SJ_{SW}+\frac{1}{2}\sigma^2\theta^2J_{0_{WW}}\end{aligned}$$

or, specialising to the case of exponential utility so

$$J(S,W,t;T)=-\frac{A}{\gamma}e^{-\frac{\lambda^2}{2}(T-t)}e^{-\gamma e^{r(T-t)}(W+X(S,t))},$$

$X(S,t)$, the certainty equivalent value of the option grant, satisfies

$$\begin{aligned}0 \quad = \quad & -\frac{\lambda^2}{2\gamma e^{r(T-t)}}-rX+X_t+(r+\xi\eta)SX_S+\frac{1}{2}\eta^2S^2(X_{SS}-\gamma e^{r(T-t)}X_S^2)\\ & +\theta(\lambda\sigma-\rho\sigma\eta\gamma e^{r(T-t)}SX_S)-\frac{1}{2}\sigma^2\gamma e^{r(T-t)}\theta^2\end{aligned}$$

subject to $X(S,T)=\Lambda(S)$, $X(S,t)\geq\Lambda(S)$ for $t\geq T_V$.

In the case of no hedging (no adjustment for the option position), so $\theta=\theta_0^*=\frac{\lambda}{\sigma\hat{\gamma}(t)}$ with $\hat{\gamma}(t)=\gamma e^{r(T-t)}$,

$$\sigma^2\hat{\gamma}(t)\theta\left(\frac{\lambda}{\sigma\hat{\gamma}(t)}-\frac{\rho\eta}{\sigma}SX_S\right)-\frac{\theta^2}{2}=\frac{\lambda^2}{2\hat{\gamma}(t)}-\rho\eta\lambda SX_S$$

and X satisfies

$$0 = -rX + X_t + (r + \eta(\xi - \rho\lambda))SX_S + \frac{1}{2}\eta^2 S^2 (X_{SS} - \hat{\gamma}(t)X_S^2)$$

subject to $X(S,T) = \Lambda(S)$, $X(S,t) \geq \Lambda(S)$ for $t \geq T_V$. Writing $X = nf$ we obtain (9)

More generally, if the extra amount held in the hedging asset as a result of the option grant is $-nD(S,t)$ so $\theta = \theta_0^* - nD(S,t)$ then X satisfies

$$\begin{aligned} 0 &= -rX + X_t + (r + \eta(\xi - \rho\lambda))SX_S + \frac{1}{2}\eta^2 S^2 (X_{SS} - \hat{\gamma}(t)X_S^2) \\ &\quad + \rho\sigma\eta\hat{\gamma}(t)S(nD)X_S - \frac{1}{2}\sigma^2\hat{\gamma}(t)S^2(nD)^2 \end{aligned}$$

subject to $X(S,T) = \Lambda(S)$, $X(S,t) \geq \Lambda(S)$ for $t \geq T_V$. Setting $X = nd$ and rearranging, we obtain (12)

Finally, the optimal hedging strategy is given by

$$\begin{aligned} \theta^* &= \text{argmax}\{\theta(\lambda\sigma - \rho\sigma\eta\gamma e^{r(T-t)}SX_S) - \frac{1}{2}\sigma^2\gamma e^{r(T-t)}\theta^2\} \\ &= \frac{\lambda}{\sigma\hat{\gamma}(t)} - \frac{\rho\eta}{\sigma}SX_S \end{aligned}$$

with optimised certainty equivalent option value given by

$$0 = -rX + X_t + (r + \eta(\xi - \rho\lambda))SX_S + \frac{1}{2}\eta^2 S^2 (X_{SS} - \hat{\gamma}(t)(1-\rho^2)X_S^2)$$

subject to $X(S,T) = \Lambda(S)$, $X(S,t) \geq \Lambda(S)$ for $t \geq T_V$. Writing $X = ng$ we obtain (10)

6.3 Optimal partial hedging with transaction costs

The executive solves

$$J(S,M,B,t,y) = \sup_{\tau, t \leq \tau \leq T} \sup_{dy_s, t \leq s \leq \tau} E_t\left[J_0(M_\tau, B_\tau + \Lambda(S_\tau), \tau, y_\tau)\right] \tag{16}$$

where $J_0(M,B,\tau,y)$ is the time consistent utility function, which in this case is the solution of the optimal investment problem (in the riskless and market assets only) *i.e,* without exposure to the firm's stock price:

$$J_0(M, B, \tau, y) = \sup_{dy} E_\tau \left[U(W_T)\right],$$

$\Lambda(S)$ represents the net cash payoff to the option,

$$\Lambda(S) = n \max(S - K, 0)$$

and terminal wealth, W_T, is also assumed to be net of transaction costs

$$W_T = B_T + M_T y_T - k(M_T, y_T).$$

The state variables are assumed to evolve as:

$$dB = rB - Mdy - k(M, dy)$$

$$dM = (r + \lambda\sigma)Mdt + \sigma M dZ$$

$$dS = (r + \xi\eta)Sdt + \eta S dZ_S$$

with $\lambda = \frac{\mu_M - r}{\sigma}$ and $\xi = \frac{\mu_S - r}{\eta}$ the Sharpe ratios of the market and the firm's share price respectively.

Note S and M are assumed to be imperfectly correlated: $dZdZ_S = \rho dt$ with $|\rho| \leq 1$ and if CAPM holds we have

$$\xi\eta = \mu_S - r = \beta_S(\mu_M - r) = \beta_S\lambda\sigma = \frac{\rho\eta\sigma}{\sigma^2}\lambda\sigma = \rho\eta\lambda$$

so $\xi = \rho\lambda$.

Expanding (16) and writing

$$J(S, M, B, t, y) = -\frac{1}{\gamma} e^{-\frac{\lambda^2}{2}(T-t)} e^{-\gamma e^{r(T-t)}(B + h^w(S,M,t,y))}$$

$$J_0(M, B, t, y) = -\frac{1}{\gamma} e^{-\frac{\lambda^2}{2}(T-t)} e^{-\gamma e^{r(T-t)}(B + h^0(M,t,y))}$$

we have that either it is optimal to exercise and

$$h_w(S, M, t, y) = \Lambda(S) + h_0(M, t, y),$$

or, in the continuation region, there are three subregions:

- a **no transaction** region, within which y, the number of units of the market held, remains constant. Thus $dy = 0$ and h_w satisfies:

$$
\begin{aligned}
0 \quad = \quad & -\frac{\lambda^2}{2\hat{\gamma}(t)} + h^w_t - rh^w + (r + \xi\eta)Sh^w_S + (r + \lambda\sigma)Mh^w_M \\
& + \frac{\eta^2}{2}S^2(h^w_{SS} - \hat{\gamma}(t)h^{w2}_S) + \rho\eta\sigma SM(h^w_{SM} - \hat{\gamma}(t)h^w_S h^w_M) \\
& + \frac{\sigma^2}{2}M^2(h^w_{MM} - \hat{\gamma}(t){h^w_M}^2) \qquad (17)
\end{aligned}
$$

where $\hat{\gamma}(t) = \gamma e^{r(T-t)}$ and subject to:

$$h^w(S, M, t, y) \geq \Lambda(S, M) + h_0(M, t, y)$$

In this region the decrease in utility from holding a suboptimal number of units of the market is lower than the marginal utility loss arising from the transaction costs of adjusting the position.

$$M(1 - k) \leq h^w_y \leq M(1 + k)$$

Thus y is close to its "optimal" value, $y^*(S, M, t)$, *i.e.* the number of units of the market which would be held in the absence of transaction costs.

- a **buy** region in which

$$h^w_y - M(1 + k) > 0$$

holds and thus utility is maximised by choosing dy as large as possible (as long as (S_t, M_t, t) remains in this region) Hence if (S_t, M_t, t) moves into this region, a transaction is made (y is increased) until y lies within the no transaction region again.

- a **sell** region in which

$$h^w_y - M(1 - k) > 0$$

holds and thus utility is maximised by choosing dy as negative as possible (as long as (S_t, M_t, t) remains in this region) Again, if (S_t, M_t, t) moves into this region, a transaction is made (y is decreased) until y lies within the no transaction region again.

h^0 similarly satisfies:

$$0 = -\frac{\lambda^2}{2\hat{\gamma}(t)} + h^0_t - rh^0 + (r + \lambda\sigma)Mh^0_M + \frac{\sigma^2}{2}M^2(h^0_{MM} - \hat{\gamma}(t){h^0_M}^2) \tag{18}$$

subject to:

$$M(1-k) \leq h^0_y \leq M(1+k)$$

$$h^0(M,T,y) = yM - kM|y|$$

The certainty equivalent value of the option is then $h^w(S,M,t,y) - h^0(M,t,y)$.

These non-linear three or four dimensional problems must be solved numerically. We simplify them, exploiting the fact that transaction costs, $k \ll 1$ are generally very small, by expanding in powers of k using an asymptotic expansion in order to obtain the leading order behaviour. This simplifies the problem considerably, reducing the dimension by one, and producing straightforward trading rules for executives who wish to hedge their option exposure using tradeable assets. For details of the asymptotic expansion see Whalley (2005). The leading order behaviour gives the equation for h_0, and successive non-zero terms in the expansion give t_b and t_f respectively, which are $O(k^{1/2})$ and $O(k^{3/4})$ respectively.

	MY^+/n	CETC	h	$V^*(T_V)$
$\nu = 0.1$	1.75	0.0276	0.438	≥ 4
$\nu = 0.5$	0.43	0.0271	0.378	3.06
$\nu = 1.0$	0.25	0.0259	0.330	2.40
$\nu = 2.0$	0.15	0.0234	0.272	1.95
$\nu = 10.0$	0.04	0.0141	0.122	1.34
	MY^+/n	$CETC$	h	$V^*(T_V)$
$\rho = 1.0$	0.33	0.0458	0.448	$\cdot^*$
$\rho = 0.9$	0.28	0.0326	0.367	2.95
$\rho = 0.8$	0.25	0.0259	0.330	2.40
$\rho = 0.7$	0.23	0.0207	0.307	2.18
$\rho = 0.6$	0.21	0.0165	0.291	2.05

Table 1: Hedging semibandwidth per option at inception, per-option certainty equivalent values of transaction costs and per-option values at inception and early exercise threshold on vesting for different sizes of option grants (ν) and hedging assets (ρ). $\cdot^*$ = no optimal early exercise. Parameter values where not stated: $T = 10$, $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\gamma = 1 \times 10^{-6}$, $\rho = 0.8$, $nK = 1 \times 10^6$, $\sigma = 0.2$.

*

Per-option values at inception

	$S/K=1$			$S/K=2$		
	$k=0.005$	$k=0.01$	$k=0.02$	$k=0.005$	$k=0.01$	$k=0.02$
$\rho=0.9$	0.382	0.367	0.339	1.052	1.024	0.972
$\rho=0.8$	0.342	0.330	0.309	0.921	0.899	0.859
$\rho=0.7$	0.316	0.307	0.290	0.836	0.820	0.789
$\rho=0.6$	0.298	0.291	0.278	0.778	0.765	0.741
$\rho=0.5$	0.285	0.280	0.270	0.737	0.727	0.709

Per-option values on vesting

	$S/K=1$			$S/K=2$		
	$k=0.005$	$k=0.01$	$k=0.02$	$k=0.005$	$k=0.01$	$k=0.02$
$\rho=0.9$	0.290	0.280	0.263	1.059	1.042	1.010
$\rho=0.8$	0.271	0.264	0.250	1.017	1.003	0.976
$\rho=0.7$	0.259	0.253	0.242	0.999	0.988	0.965
$\rho=0.6$	0.251	0.246	0.237	0.993	0.983	0.965
$\rho=0.5$	0.245	0.241	0.234	0.993^*	0.985^*	0.970^*

Table 2: Per-option values for costly hedging with different hedging assets (varying ρ and k). Parameter values where not stated: $T=10$, $T_V=5$, $\eta=0.3$, $r=0.04$, $\gamma=1\times10^{-6}$, $\nu=nK=1\times10^{6}$, $\sigma=0.2$, $\cdot^*$ option exercised; value = payoff net of costs.

At-the-money per-option values at inception

	$\nu = nK\gamma = 0.1$			$\nu = nK\gamma = 10$	
	$k = 0.005$	$k = 0.01$	$k = 0.02$	$k = 0.005$	$k = 0.01$
$\rho = 0.9$	0.458	0.442	0.411	0.183	0.173
$\rho = 0.8$	0.450	0.437	0.412	0.128	0.122
$\rho = 0.7$	0.444	0.434	0.414	0.102	0.098
$\rho = 0.6$	0.441	0.433	0.417	0.088	0.085
$\rho = 0.5$	0.439	0.432	0.421	0.079	0.077

Table 3: Per-option at-the-money option values with costly hedging at inception. Parameter values where not stated: $T = 10$, $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\sigma = 0.2$.

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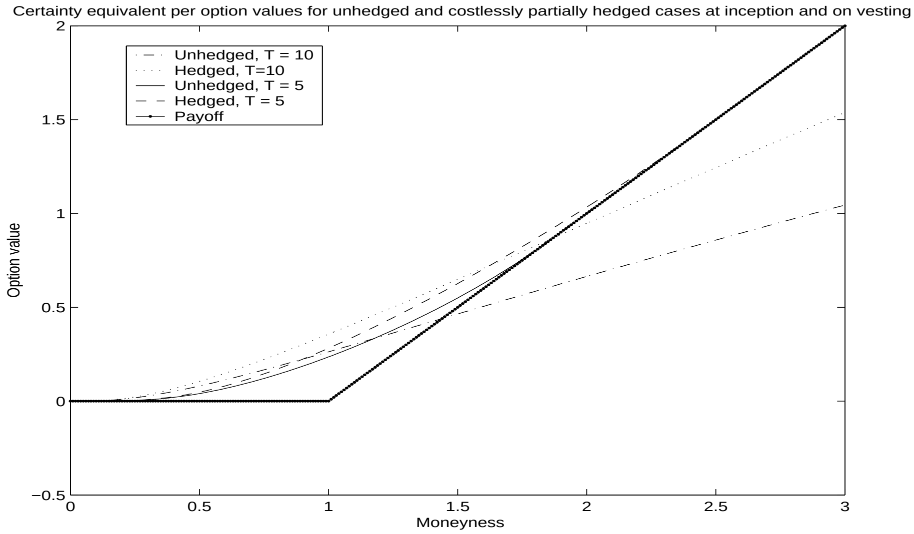

Figure 1: Value per option as a function of moneyness, S/K, for unhedged and costless hedging cases. Parameter values where not stated: $T = 10$, $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\gamma = 1 \times 10^{-6}$, $nK = 1 \times 10^6$, $\sigma = 0.2$ and $\rho = 0.8$.

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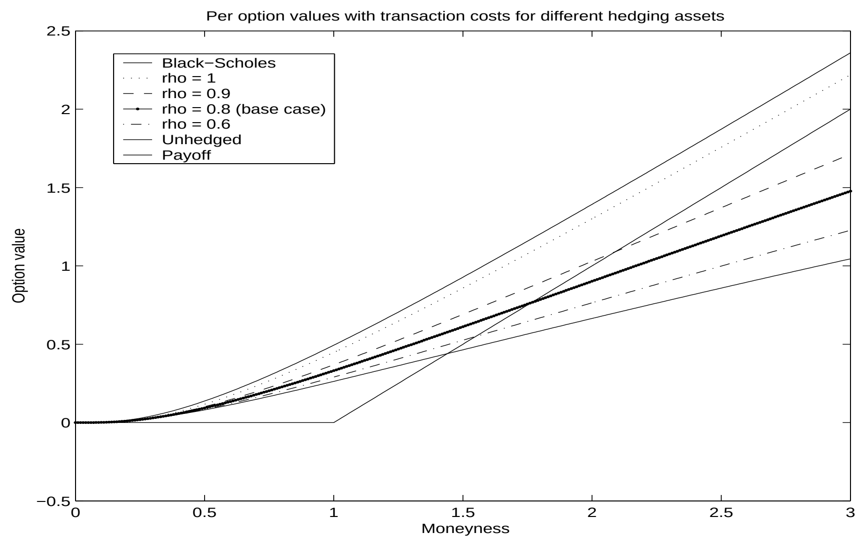

Figure 2: Certainty equivalent value per option as a function of moneyness, S/K, under optimal hedging with transaction costs for different choices of hedging asset (ρ) and no hedging $\rho = 0$. Parameter values where not stated: $T = 10$, $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\gamma = 1 \times 10^{-6}$, $nK = 1 \times 10^6$, $\sigma = 0.2$ and $k = 0.01$.

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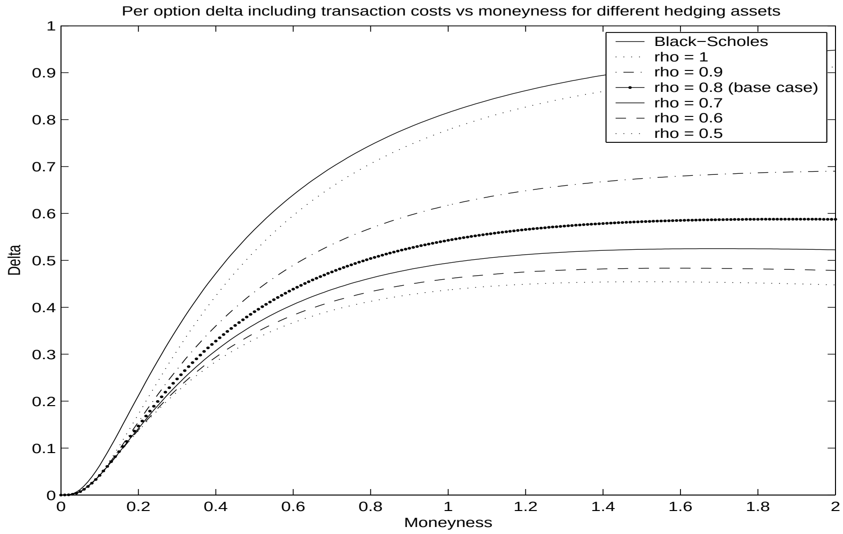

Figure 3: Per option delta as a function of moneyness for costless partial hedging with different hedging assets (varying ρ). Parameter values where not stated: $T = 10$, $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\gamma = 1 \times 10^{-6}$, $nK = 1 \times 10^{6}$, and $\sigma = 0.2$.

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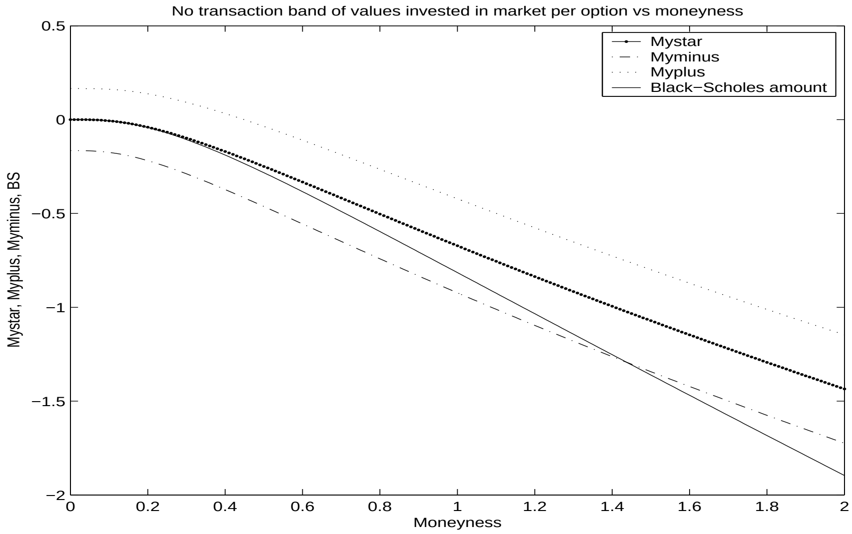

Figure 4: Centre and edges of no-transaction band per option as a function of moneyness, S/K. Parameter values: $T = 10$, $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\gamma = 1 \times 10^{-6}$, $nK = 1 \times 10^{6}$, $\sigma = 0.2$, $\rho = 0.8$ and $k = 0.01$.

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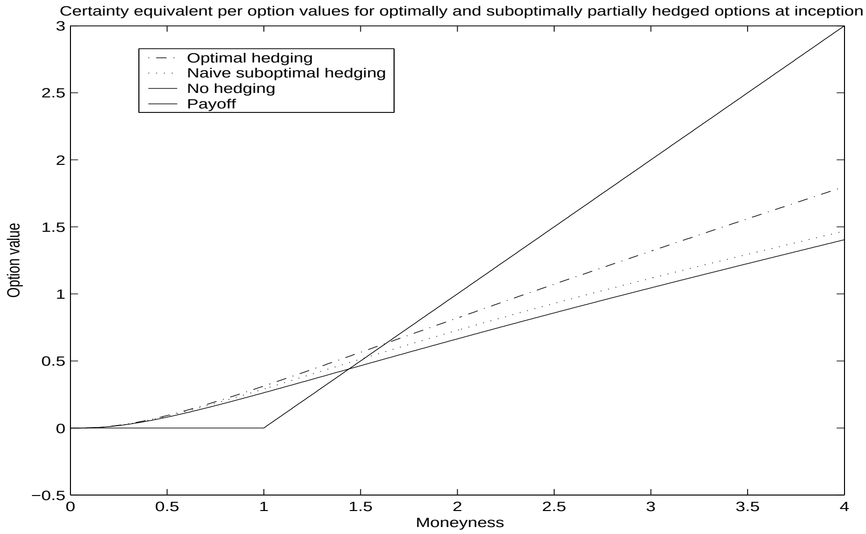

Figure 5: Certainty equivalent value per option as a function of moneyness, S/K, under optimal and suboptimal hedging for different volatilities of hedging asset. Parameter values where not stated: $T = 10$, $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\gamma = 1 \times 10^{-6}$, $nK = 1 \times 10^6$, $\rho = 0.5$, and $\sigma = 0.2$.

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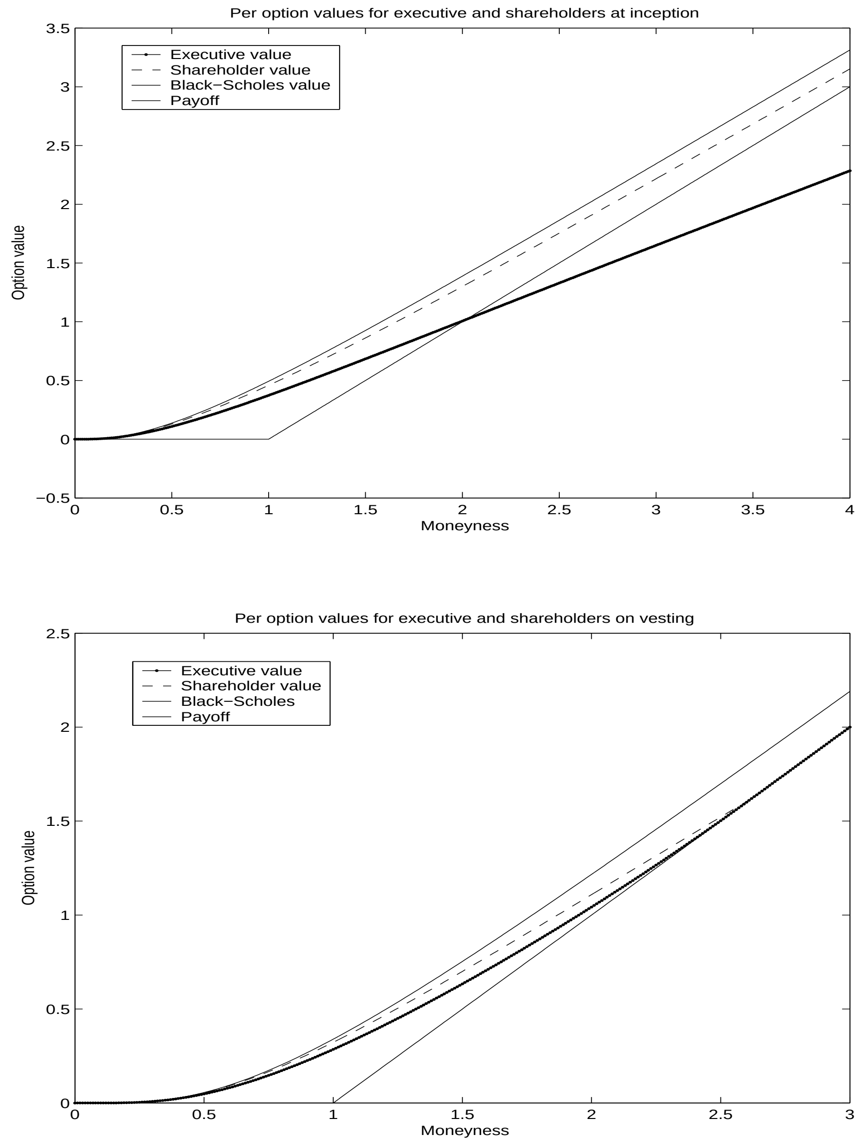

Figure 6: Values of option grant to shareholders and executive at inception (upper panel) and on vesting (lower panel) Parameter values where not stated: $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\gamma = 1 \times 10^{-6}$, $nK = 1 \times 10^6$, $\sigma = 0.2$, $\rho = 0.8$ and $k = 0.01$.

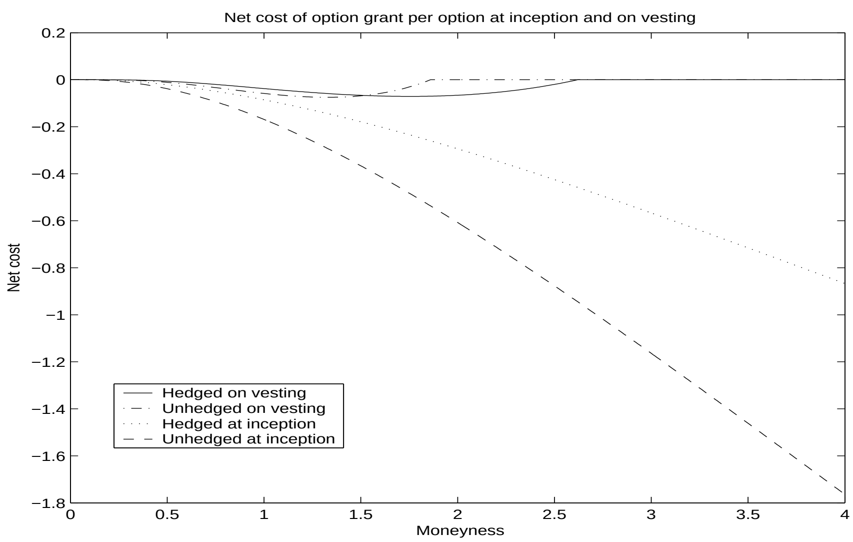

Figure 7: Difference between option valuations by shareholders and executives at inception and on vesting. Parameter values where not stated: $T_V = 5$, $\eta = 0.3$, $r = 0.04$, $\gamma = 1 \times 10^{-6}$, $nK = 1 \times 10^{6}$, $\sigma = 0.2$, $\rho = 0.8$ and $k = 0.01$.