An Application of Closed-Form GARCH Option Pricing Model on FTSE 100 Option and Volatility

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Abstract

Many empirical researches have indicated that the Black-Scholes option pricing model demonstrates systematic biases due to some unreasonable assumptions. In practice, Black-Scholes implied volatilities tend to differ across exercise prices and time to maturities. In order to solve the problem, many researchers developed closed form model (Heston and Nandi, 2000). In this study, we apply their closed form GARCH (HN GARCH) model on FTSE 100 Index option. As a benchmark, we employ the Ad Hoc Black-Scholes model of Dumas, Flemming and Whaley(1998) which use a separate implied volatility for each option to fit to the smirk/smile in implied volatilities. We find that the HN GARCH has smaller valuation errors than ad hoc BS model both in-sample and out-of-sample.

***Keywords*: closed-form GARCH option model, pricing error, implied volatility**

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1. Introduction

Black and Scholes (1973) derived a close-form option pricing model by assuming the asset price following geometric Brownian motion and log-normal distribution with constant volatility. However, Fama (1965) and Mandelbort (1966) found that stock returns exhibit both fat-tailed marginal distribution and volatility clustering. Some empirical studies had proved that Black-Scholes implied volatilities incline toward differing across exercise prices and time to maturities. The assumption of constant volatilities partly causes the system biases of Black-Scholes model. For overcoming the shortcoming, many researchers have devoted themselves to the development of option valuation models that try to grab the pattern of volatilities in the last two decades.

Early amendments include the constant-elasticity-of-variance model by Cox (1975), the jumping model by Merton (1976), the compound-option model by Geske (1979), and the displaced-diffusion model by Rubinstein (1983). Nevertheless, these models encounter the difficult that the variance rate is not observable. The latest amendments are the two types: implied volatility and stochastic volatility. The stochastic volatility includes two major kinds: continuous-time stochastic volatility models and discrete-time generalized autoregressive conditional heteroskedasticity (GARCH) models.

Continuous-time stochastic volatility models are effective for option pricing but may be difficult to implement. Although these models are assume that volatility is observable, it is very difficult to filter a continuous volatility variable from discrete observations. One alternative is the implied volatilities derived from option prices. Nevertheless, this is very time spending and computationally burdensome. Moreover, the volatility estimation techniques for continuous-time models are always nontrivial.

Furthermore, the continuous-time model can serve as the limit of a certain

GARCH model. For instance, Nelson (1990a) proved that the GARCH (1,1) model converged to a certain diffusion model. Duan (1996) argued that most of the existing bivariate diffusion models, which had been used for modeling asset returns and volatilities, could be represented as the limit of a family of GARCH models. As a particular case, nonlinear asymmetric GARCH process used for HN model was proved to contain Heston's (1993) stochastic volatility process as a continuous-time limit.

The GARCH model has an advantage over the continuous-time model because the volatility is readily observable in the historical prices of the underlying asset. Therefore, it is possible to price an option just by using the information from the observations of underlying asset. Oppositely, the continuous-time stochastic volatility model has an inherent disadvantage that it assumes that volatility is observable, but it is impossible to exactly filter volatility from spot asset prices, discrete observations, in a continuous-time stochastic volatility model. This means it is unlikely to price an option merely based on the historical prices of underlying asset.

Duan (1990) cited the econometric method, GARCH, into the discrete-time model and derive the GARCH option pricing model (1995) to improve the Black-Scholes model. The ARCH model was first introduced by Engle (1982). Bollersleve (1986) improve the ARCH model to create the GARCH model. In the GARCH process, the vital hypothesis is conditional hetreoskedasticity that the variance is determined by a series of parameters and a sequence of random variables which are white noise. To capture the negative correlation between returns and conditional volatility, Engle and Ng (1993) introduced the NGARCH model. The NGARCH model is often applied to the general theory of GARCH option pricing.

Most GARCH option pricing models are not closed-from solution. These models are typically solved by Monte Carlo simulation (Engle and Mustafa (1992), Amin and

Ng (1993), Duan (1995)), which is slow and computationally intensive for empirical analysis. More recently, Hanke (1997) has provided a network approach, Ritchken and Trevor (1999) have provided a lattice approximation to value American option and Duan, Gauthier and Simonato (1999) have provided Markov chain approach for GARCH process with single lag in the variance dynamics. Heston and Nandi (2000) invent a closed-form solution for European option pricing in a GARCH model. The model allowing not only for multiple lags in the time series dynamics of the variance process but also for correlation returns of the spot asset and variance do provide an alternative for option pricing.

We test the pricing efficiency of Heston and Nandi (2000) GARCH model (the HN GARCH) based on the data from the FTSE 100 option market. As a benchmark, we choose the Ad Hoc BS model of Dumas, Flemming and Whaley (1998, henceforth DFW), which has the flexibility of fitting to the smirk/smile of observed implied volatilities by applying a separate implied volatility for each option. We find that the HN GARCH model has smaller valuation errors than the Ad Hoc BS model in both in-the-sample and out-of-sample empirical analysis.

The rest of this article proceeds as follows. In Section 2, we introduce the methodology containing FTSE 100 option market introduction and the models we apply. Section 3 reports the empirical results including in-the-sample and out-of sample estimation. Section 4 is the conclusion.

2. Data and Methodology

The Financial Times Stock Exchange (FTSE) 100 Index was launched in 1984. It tracks the share price movement of the top 100 UK companies. It is calculated every 15 seconds throughout the trading day from 8 A.M. to 4:30 P.M.

There are two main types of FTSE 100 Index Options – calls and puts. FTSE 100

Index Options operate in much the same way as equity options. The main difference between them is that when exercised, the cash value is calculated and not the physical delivery of the underlying shares.

In the first half of 2001, the European style FTSE 100 Index option contract had an average monthly volume of 1.25 million contracts and an average monthly open interest of over 1.4 million contracts, it ranks fourth worldwide just behind the S&P 100 and S&P 500 contracts of the CBOE and the ODAX contract of the EUREX.

The standard expiration months are March, June, September and December. Additional expiration months are introduced so that contracts that expire during the three nearest calendar months are always available. The trading hours of FTSE 100 Index options are from 8 A.M. to 4:30 P.M. Traders submit orders electronically to the central limit order book. Incoming market orders are automatically matched with orders in the order book. Moreover, order can be canceled at any time. No one can submit orders directly to LIFFE Connect except exchange members. Till October 2002, there were 143 public order members. Public order members can trade their own account or on behalf of their customers. There were also 60 non-public order members. Non-public order members can only trade on their own account. There are no designated market makers with special quoting obligation or privileges in the FTSE 100 Index options.

The buyers and the sellers pay a fixed cost of ￡0.25 per trade. Order submissions and cancellations are free. However, there is a fixed fee per message for the automated price injection models.

We obtained our sample from Bloomberg. It consists of time series of reported closing prices of European style FTSE 100 index option contracts. Our sample data covers 194 trading days from 06/01/2005 to 03/03/2006. We assume the dividends to be zero and need not to subtract it from the current index level. For the risk free rate, the continuously compounded Treasury bill rates interpolated to match the maturity of the call option are used.

Two exclusionary criteria are used for filtering these data. First, only call option records in which moneyness, K/S, lies between 0.9 and 1.1 are included in the sample. This excludes some very deep out-of-the-money and deep in-the-money call options that are either infrequently traded.

Second, the call options are taken out of the sample if their prices do not satisfy the boundary condition:

$$S(t) \geq C(t,T) \geq \max(0, S(t) - PVD - Ke^{-r(T-t)}) \quad (1)$$

The first inequality must hold. If not, investor can just buy the underlying asset directly because a call option represent a right to buy one share of underlying asset so a call option never exceeds the value the underlying asset no matter what happens. The second inequality also must be satisfied since it ensures that there is no arbitrage opportunity.

The data set consist of 1302 records. 848 of the 1302 records are for in-sample empirical analysis. The rest 454 are for out-of-sample empirical analysis. In out-of-sample section, the sample of 704 is categorized into 10 sets. In terms of moneyness, the data set is divided into five categories: deep in-the-money call options with K/S < 0.96, in-the-money call options with 0.96 ≦ K/S < 0.98, at-the-money call options with 0.98 ≦ K/S < 1.02, out-of-the-money call options with 1.02 ≦ K/S < 1.05, and deep out-of-the-money call options with 1.05 ≦ K/S ≦ 1.1. The term to expiration is classified into two categories: Short-term (less than 65 days) and Long-term (between 65 and 100 days). For the difficulty of obtaining data and the data filtration by above two criteria, the bin of contract with moneyness K/S between 1.05 and 1.1 and time to maturity less than 65 days is empty.

We find the average call option prices across different moneyness and time to maturity based on the data for out-of-sample part. The important finding is that the price difference in call option prices for different moneyness and time to maturity is large.

We examine the average implied Black-Scholes volatilities from call contracts across different moneyness and time to maturity from the data for out-of-sample empirical analysis. We find that the implied volatility of each bin is not significantly different. It means the volatility smirk/smile effect of the sample is not obvious.

We find that the implied volatility curve across different strike prices and moneyness is flat. It may virtually meet the assumption of constant volatility under Black-Scholes (BS) model. We do further discussion on effect of flat implied volatility curve in the following out-of-sample empirical analysis.

Moreover, we use the daily historical closing price of FTSE 100 Index, from 1/2/2003 to 12/30/2005, to estimate the parameters in the GARCH process.

3. The Model

This part describes the option pricing process under the closed-form GARCH option valuation model (Heston & Nandi (2000)). Before, GARCH option pricing models are typically solved by simulation. It is time-spending and computationally intensive. In contrast, the HN GARCH model offers an analytical solution and would be time-saving and cost-saving for option pricing.

3.1 The HN Closed-form GARCH Option Pricing Model

At time t, a European option with strike price K at the expires at time T is worth

$$C = e^{-(T-t)}E_t^*\left[Max\left(S(T)-K,0\right)\right] = \left(\frac{1}{2}S(t)+\frac{e^{-(T-t)}}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{K^{-i\phi}f^*(i\phi+1)}{i\phi}\right]d\phi\right) - Ke^{-(T-t)}\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \mathrm{Re}\left[\frac{K^{-i\phi}f^*(i\phi)}{i\phi}\right]d\phi\right) \qquad (1)$$

where $E_t^*[\ ]$ ***denotes the expectation under risk-neutral probability measure.***

The HN closed form GARCH model enables us to calculate the expectation in the above formula once we have the characteristic function of $\log(S(T))$, instead of calculating two separate integrals. This is different from that of Heston (1993) in which we have to get two characteristic functions, respectively $f_1(\log S, v, t; \phi)$ and $f_2(\log S, v, t; \phi)$. Therefore, the model in Heston (1993) have to invert the above two characteristic functions to get the desired probabilities

$$P_j(\log S, v, T; \log k) = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \mathrm{Re}\left[\frac{K^{-i\phi} f_j(\log S, v, T; \phi)}{i\phi}\right] d\phi \quad , j = 1,2 . \qquad (12)$$

And then, substituting the two probabilities into the formula, $C(s, v, t) = SP_1 - Ke^{-r(T-t)}P_2$, generates the option value.

Comparing with Black-Scholes formula, HN GARCH model is a function of current asset price and the conditional variance. Since the conditional variance is a function of the observed path of asset prices, the option formula is a function of current and lagged asset prices. In contrast to continuous-time stochastic volatility models, volatility under HN GARCH model is a readily observable function of historical assets and is not necessary to be estimated with other procedures.

3.2 *Ad Hoc* Black-Scholes Model

A troubling aspect of the analysis is that we have not yet indicated what size of prediction error should be considered “large”. One way to gauge the prediction error is to measure them against a benchmark. Since HN GARCH model has more parameters than BS model, which owns only one parameter—volatility, it is unfair to compare these two models. Moreover, DFW (1998) had indicated that Ad Hoc BS model could price more accurately than BS model. Therefore, we choose Ad Hoc BS model as the benchmark.

DFW (1998) fitted the BS model to the reported structure of option prices each week by using the following model to describe the Black- Scholes implied volatility.

$$\sigma = \max(0.01, \min(a_0 + a_1 K + a_2 K^2 + a_3 \tau + a_4 \tau^2 + a_5 K\tau)) \tag{13}$$

σ is the annual implied volatility for an option with exercise price K and time to maturity τ. A minimum value of 0.01 for the local volatility is imposed to prevent negative values.

The empirical analysis following the methodology is presented below. First, we use Maximum Likelihood (MLE) method to estimate the parameters of the GARCH process based on the FTSE 100 index historical prices. Second, because HN GARCH model is a closed-form solution, we can estimate the parameters of HN GARCH model by Nonlinear Least Square (NLS) method based on historical prices of FTSE 100 option. Su and Fung (2004) had found that the pricing efficiency of HN GARCH model in which parameters are estimated by MLE is not virtually accurate. It may be due to that the information set of index is not necessary the same with that of index option prices since the information of daily index time series is backward-looking while index option prices are forward-looking. Finally, we compare the pricing accuracy of HN GARCH model with Ad Hoc Black-Scholes in both in-the-sample and out-of-sample empirical analysis.

4. Empirical Analysis

The empirical analysis starts with the estimation of GARCH model by Maximum Likelihood Estimation (MLE) with time series data on index returns in part A. In part B, we report the model comparisons including in-sample and out-of-sample comparisons in which we estimate the parameters of the close-form solution by

Nonlinear Least Square (NLS) method.

4.1 Estimation

The empirical analysis focuses mainly on the single lag version of GARCH model. We set $\triangle$ = 1 and use daily index return to model the volatility. In GARCH model, all parameters can be estimated directly from the history of asset prices. We do the estimation with the Maximum Likelihood Estimation (MLE) used by Bollerslev (1986). In order to illustrate the importance of skewness parameter, γ_1, we perform this estimation with an unrestricted model and a restricted model in which the skewness parameter, γ_1, equals zero. When γ_1 equal zero, the GARCH model is a symmetric model (henceforth, we called it symmetric GARCH). We do the estimation based on the data of FTSE 100 Index ranging from 1/2/2003 to 12/30/2005. Table 1 shows the maximum likelihood estimates of GARCH model for the whole sample period. Table 2 presents the MLEs for year 2003, 2004, and 2005. Figure 1 and Figure 2 show the annualized volatility of unrestricted model and restricted model for the whole sample period, respectively.

Table 1 compares the parameters of asymmetric model and symmetric model estimated based on the whole sample. We focus on the discussion of skewness parameter, γ_1. The volatility of volatility, α_1, is 7.2039E-06 from asymmetric ($\gamma_1 \neq 0$) GARCH model and 6.6393E-06 from symmetric ($\gamma_1 = 0$) GARCH model. The parameter used to measure the degree of mean reversion ($\beta_1 + \alpha_1\gamma_1^2$) is 0.884 from asymmetric GARCH model and 0.892 from symmetric GARCH model. The annualized long-run mean of volatility, $\sqrt{252(\omega+\alpha_1)/(1-\beta_1-\alpha_1\gamma_1^{2})}$, under asymmetric and symmetric GARCH is 12.50% and 12.46%, respectively. There is no significant difference between these two models. By comparing figures 1 and 2, we find that the annualized level of $\sqrt{h(t+1)}$ of the two models is similar. It is due to that the skewness parameter is not significantly different from zero by likelihood ratio test. In contrast, we find the skewness parameter of year 2004 and 2005 is significantly different from zero by using likelihood ratio test. One could plug the parameters obtained from the above MLEs (using historical FTSE 100 index levels) into the options valuation formula to get the call option prices. However, the information set of index levels is not necessarily the same with that of option prices. Option prices often embed with the expectation about the future evolution of price of the underlying asset. Su and Fung (2004) had indicated that plugging the parameters obtained from MLEs into HN GARCH pricing model to calculate option prices is not extremely accurate. HN GARCH model and Ad Hoc Black-Scholes model are all closed-form solution. Therefore, we use the Nonlinear Least Square method to estimate the parameters of HN GARCH model and Ad Hoc Black-Scholes model in the following part, model comparisons. In model comparisons, we do NLS procedure with the downhill simplex method of Nelder and Mead, which is different from Levenberg -Marquardt method used by Heston and Nandi (2000)

4.2 Model Comparisons

This part contains in-sample and out-of-sample comparisons. We compare the pricing accuracy of HN GARCH model with two option pricing models including Black-Scholes (BS) model and Ad Hoc Black-Scholes (Ad Hoc BS) Model. We get the parameter estimates and in-sample valuation errors from minimizing the root mean square error (RMSE), $\frac{\sum_{i=1}^{N_t}\sum_{t=1}^{T}e(i,t)}{TN_t}$, where T denotes the number of days in the sample, N_t is the number of options traded on specific day in the sample, and $e(i,t)$ is the difference between model value of option and market price of that option at time t.

4.2.1 In-sample Comparison

We estimate from the NLS estimation in-sample period (ranging from 6/1/2005 to 10/27/2005) and the average in-sample valuation error (not reported). The risk neutral skewness is positive, which indicates that variance tends to rise when index falls, and vice versa.

The average call option price of the sample period is £118.12. The root mean square error (RMSE) of the simple BS model is £14.41. The root mean square error for the non-updated GARCH model is £10.68. Comparing these two models, the non-updated GARCH model can get the more accurate price than simple BS model. However, comparing non-updated GARCH model with the Ad Hoc BS model, we find that the Ad Hoc BS model with RMSE of 9.91 provides a better in-sample fit than non-updated GARCH model does. It is because the Ad Hoc BS model is designed to fit both the volatility smile in strike prices and the term structure of implied volatilities. Moreover, it is updated every week. Therefore, the above comparison is not extremely fair. In order to do a fair comparison, we estimated an "updated" GARCH model by minimizing the RMSE between model prices and market prices by allowing the parameters to change every week. Table 3 reports the comparison between the updated GARCH model and the Ad Hoc BS model. We find that after updating the GARCH model, the RMSE, 9.87, is smaller than that in Ad Hoc BS model. Thus, the flexibility of updating appears to make a difference in terms of its ability to fit option prices in-sample.

Table 4 describes the mean and standard deviation of the updated GARCH coefficients from the estimation. All parameters seem to be unstable in the sample period. It may result from the insufficient data or the character FTSE 100 Index. Moreover, the average skewness parameter of the updated GARCH model is negative.

There is significantly different from the skewness parameter of the non-updated GARCH model. It may be caused by the loss function, RMSE, which is used to estimate the parameters. Kristofferson and Jacobs (2001) indicated that the relatively wide range of option prices across moneyness and maturity raises the problem of heteroskedasticity for RMSE-based parameter estimation. We find the average standard deviation of market call option prices in updated periods is 92.94.

DFW (1998) have shown that a more flexible model may dominate within in-sample period but have much less predictive power for out-of-sample option prices. It occurs when a misspecified model achieves a good in-sample result by overfitting the data. We examine this issue in the following section by comparing all the models within out-of-sample period.

4.2.2 Out-of-sample Comparison

After estimating the parameters within in-sample period, we turn to verify the out-of-sample valuation performance of these option-pricing models which are estimated within in-sample period. We test the prediction performance of each option valuation model based on the call option prices ranging from 10/28/2005 to 3/3/2006. In computing the out-of-sample call option prices by the non-updated GARCH model, we estimate the parameters of this model by Nonlinear Least Squares method based on call option prices from 6/1/2005 to 10/27/2005 (106 days) and plug these parameters into this model to calculate the call option prices from 10/28/2005 to 3/3/2006. For computing the updated models including the updated GARCH, the Ad Hoc BS and the simple BS model, we use the data from 106 days to 1 day before the first day of each predicted period (one week, around 5 days) to estimate the parameters and apply these parameters to computing the call option prices in that predicted period.

For getting objective and fair comparisons, we quote four loss functions to judge which model is more efficient and accurate. There are root mean squared errors (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and (root mean implied volatility errors (RMIVE). Each loss function owns its disadvantage. For example, the disadvantage of RMSE is that it implicitly assigns a lot of weight to options with high valuations. MAPE has the disadvantage that short time to maturity and out-of-the money options with price valuations close to zero will implicitly get assigned a lot of weight. Therefore, we try to use the advantages of four loss functions to compensate the disadvantages of each loss function.

We list these loss functions in the following:

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(C_{\text{mod}\,el} - C_{market})^2} \tag{13}$$

$$MAE = \frac{1}{N}\sum_{n=1}^{N}\left|C_{\text{mod}\,el} - C_{market}\right| \tag{14}$$

$$MAPE = \frac{1}{N}\frac{\left|C_{\text{mod}\,el} - C_{market}\right|}{C_{market}} \tag{15}$$

$$RMIVE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(\sigma(C_{\text{mod}\,el}) - \sigma(C_{market}))^2} \tag{16}$$

where,

C_{model} : the call option price from models

C_{market}: the market call option price

$\sigma(C_{\text{model}})$: the BS implied volatility from model call prices

$\sigma(C_{\text{market}})$: the BS implied volatility from market call prices

N : the number of contracts

Table 5 reports the out-of-sample aggregate valuation errors across various models. The aggregate out-of-sample root of mean squared errors are 9.27, 6.77, 6.94, and 5.90, respectively for the BS, the Ad Hoc BS, the non-updated GARCH, and the updated GARCH, respectively. By comparing the valuation errors, we find the updated GARCH model outperform the other models in each loss function comparison. The out-of-sample result is similar with the in-sample result. The BS is still the worst performer on each loss function except RMIVE out-of-sample. We also find that the Ad Hoc BS performs worst on RMIVE. It may be due to the flat implied volatility cure from the data in this sample period. However, based on the other three loss functions, the Ad Hoc BS is the second best. It dominates the non-updated GARCH model. In a nutshell, the updated GARCH still dominates the other three option valuation models out-of-sample.

The valuation errors by different option moneyness and maturity categories are presented in Table 6. In Panel A, the result in the column of options with time to maturity more than 65 days is similar to that of aggregate out-of-sample. The updated GARCH model significantly outperforms other three option valuation models, especially in valuing deep out-of-the-money (K/S>1.05) contracts and in-the-money (0.96≦K/S<0.98) contracts. In Figure 3, we can see the trend of % Error of each model from data with time to maturity more than 65 days, and we find that the biggest % Error happen at deep out-of-the-money call options and obviously, updated GARCH values these call options more accurately than the other three models do. Panel B reports the valuation errors of these four models from option contracts with time to maturity less than 65 days. We find that the BS model dominates the other three models. In Figure 4, we can find the % Error of BS is smaller than the other three models. It may due to that the implied volatility curve of the FTSE 100 call options with time to maturity less than 65 days is flat. Therefore, the assumption of constant volatility of the BS model meets the condition. Nevertheless, after valuing the call option contracts with time to maturity more than 65 days, we find that the BS model is significantly dominated by the other three models. It mainly caused by the assumption of constant volatility because the implied volatility curve derived from these call contracts with time to maturity more than 65 days across different strikes is steeper. Moreover, we find that the performance of Ad Hoc BS on pricing the call options with time to maturity less than 65 days is the worst. Even though the updated GARCH model does not outperform all other models, it is still the second best model among the four models to value the call options with time to maturity less than 65 days.

5. Conclusion

This article presents the valuation performance of GARCH model on FTSE 100 Index option market and compares it with other option pricing models, including the Ad Hoc Black-Scholes model, the benchmark. In the empirical analysis, we find that the GARCH model without parameter updating every week is dominated by the Ad Hoc Black-Scholes model in both in-sample and out-of-sample empirical analysis. However, the comparison is unfair because the Ad Hoc Black-Scholes Model is updated every week, but the non-updated GARCH model is not. Therefore, we update parameters of the GARCH model every week instead of holding them constant and find the updated GARCH model outperforms the Ad Hoc Black-Scholes both in-sample and out-of-sample. It means the updated GARCH model can both fit better in-sample and predict option prices more accurately out-of-sample than the Ad Hoc Black-Scholes, which uses a separate implied volatility for each option (specific to its strike and time to maturity) extracted from market prices and is designed to produce a very close fit to the shape of the implied volatility across strike prices and maturities.

Although we prove the updated GARCH model significantly outperform the Ad Hoc Black-Scholes model, we find that in out-of-sample empirical analysis, the simple Black-Scholes model can value the call options with time to maturity less than 65 days more accurately than the other option pricing models, including the updated GARCH model. It is because the BS implied volatility curve is flat that partly meets the assumption of constant volatility under Black-Scholes model. However, the aggregate valuation errors of the simple Black-Scholes model are still the biggest.

In the in-sample empirical analysis, we find the parameters of updated GARCH model are not stable. It may be due to the loss function, root of mean squared error (RMSE), which is used to estimate the parameters. Because the call option prices we collected are wide-ranged, it causes the RMSE-based parameters unstable. Therefore, selecting a loss function for NLS estimation that fits the data characters is vital and critical for the parameters estimation. We leave the selection of loss function for NLS estimation for future research.

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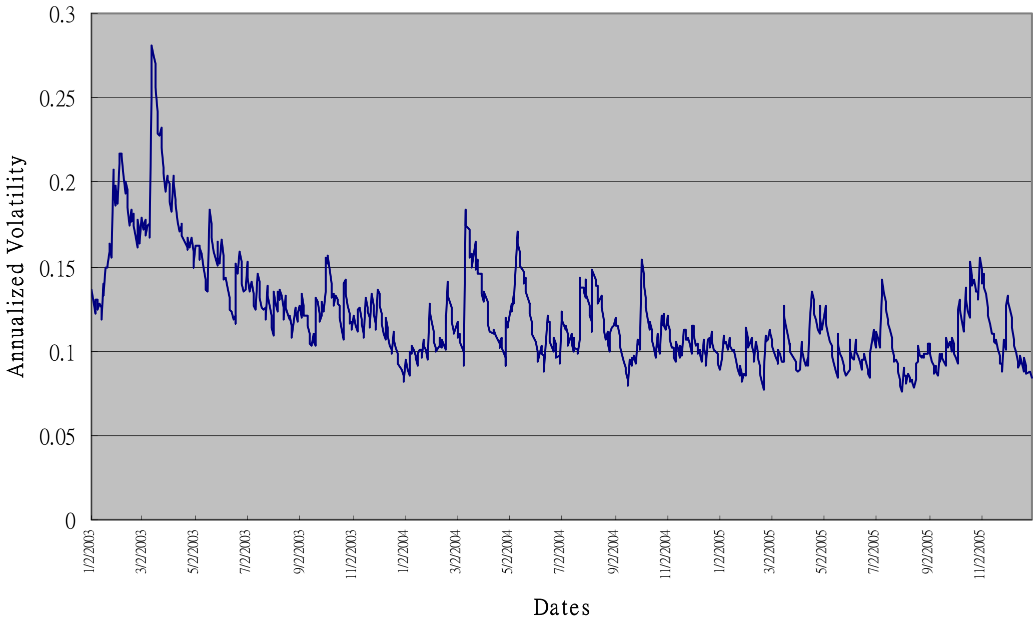

Figure 1.

This figure shows the daily annualized spot volatility from the restricted/symmetric GARCH model from 1/2/2003 to 12/30/2005 using daily FTSE 100 Index levels.

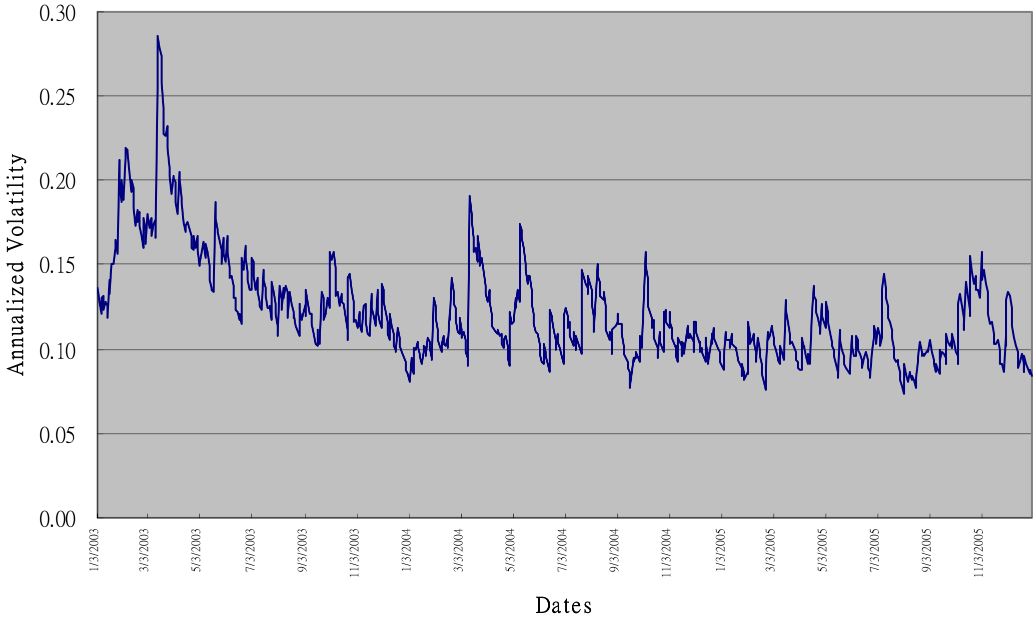

Figure 2.

This figure shows the daily annualized spot volatility from the unrestricted/asymmetric GARCH model from 1/2/2003 to 12/30/2005 using daily FTSE 100 Index levels.

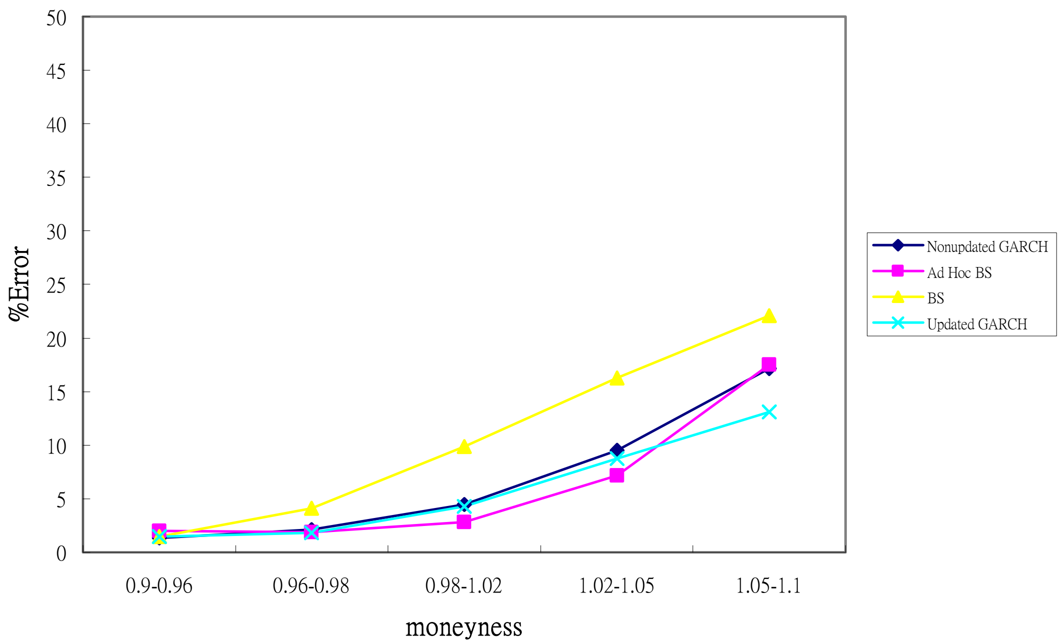

Figure 3.

This figure shows the percentage out-of-sample valuation errors (i.e. 100 $\times$ RMSE $\div$ Average option prices) for call options (more than 65 days) by various models.

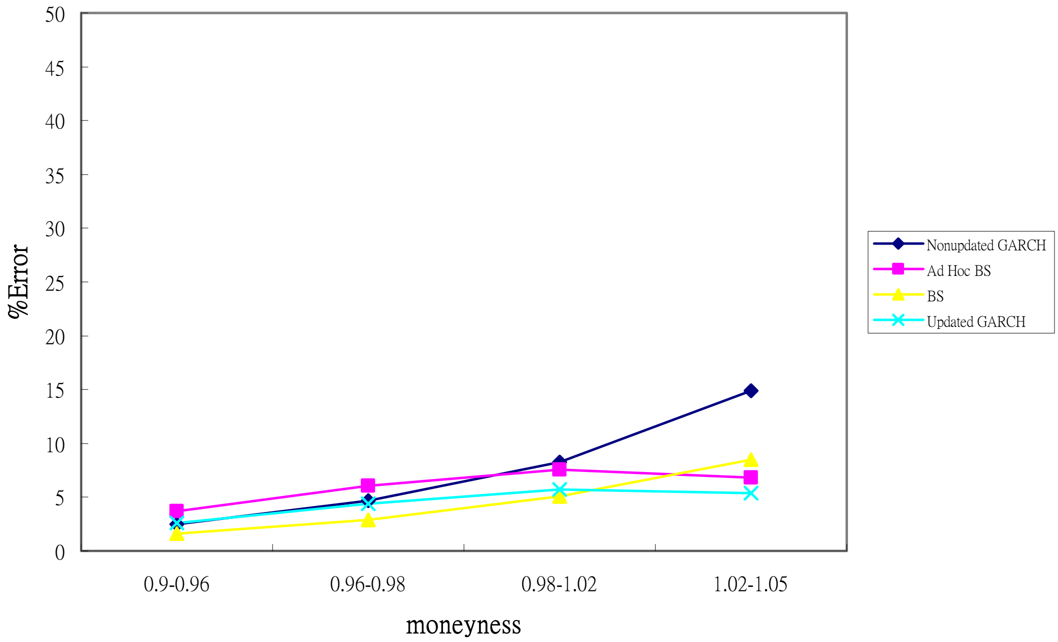

Figure 4.

This figure shows the percentage out-of-sample valuation errors (i.e. 100 × RMSE ÷ Average option prices) for call options (less than 65 days) by various models.

Table 1. Maximum Likelihood Estimation

	α_1	β_1	γ_1	ω	λ	θ	$\beta_1+\alpha_1\gamma_1^2$	Log-Likelihood
GARCH (Spot)	7.2039E-06	0.884	-0.146	0	0.0475	12.50%	0.884	3330.856
	(2.81E-05)	(7.88E-01)	(4.31E+00)	(2.27E-05)	(5.92E-01)			
GARCH, $\gamma_1=0$,(Spot)	6.6393E-06	0.892		0	-0.0928	12.46%	0.892	3331.783
	(7.06E-06)	(2.07E-01)		(6.42E-06)	(4.37E-01)			

Maximum Likelihood Estimates of the GARCH with p = q = 1, and △ = 1 (day) using the spot FTSE 100 levels for the unrestricted ($\gamma_1 \neq 0$)and restricted ($\gamma_1 = 0$) model.

$$\log(S(t)) = \log(S(t-\Delta)) + r + \lambda h(t) + \sqrt{h(t)}z(t).$$
$$h(t) = \omega + \beta_1 h(t-\Delta) + \alpha_1 (z(t-\Delta) - \gamma_1 \sqrt{h(t-\Delta)})^2$$

The log-likelihood function is $\sum_{t=1}^{T} -0.5(\log(h(t)) + z(t)^2)$, where T is the number of days in the sample. The daily index levels from 1/2/2003 to 12/30/2005 are used. Number of Observations = 758. Asymptotic standard errors appear in parentheses. θ defined to be equal to $\sqrt{252(\omega+\alpha_1)/(1-\beta_1-\alpha_1\gamma_1{}^2)}$ is annualized (252days) long-run volatility (standard deviation) implied by the parameters estimates. $\beta_1+\alpha_1\gamma_1^2=1$ measures the degree of mean reversion in that $\beta_1+\alpha_1\gamma_1^2=1$ implied that the variance process is integrated.

Table 2. Maximum Likelihood Estimation

	α_1	β_1	γ_1	ω	λ	θ	$\beta_1+\alpha_1\gamma_1^2$	Log-Likelihood	#Observation
2005									
GARCH (Spot)	2.05E-06	0.7342	217.9788	2.91E-06	32.24	**0.08624**	**0.832**	**1185.646**	252
	1.80E-06	0.1761	137.6015	2.85E-06	90.86				
GARCH, $\gamma_1=0$ **,(Spot)**	2.11E-06	0.8705		1.71E-06	74.10	**0.08613**	**0.871**	1181.398	252
	1.51E-06	0.1214		2.81E-06	88.18				
2004									
GARCH (Spot)	1.27E-06	0.546	476.8648	5.88E-06	24.10	**0.10462**	**0.835**	**1148.355**	254
	2.07E-07	0.3009	189.5706	3.83E-06	**28.20**				
GARCH, $\gamma_1=0$ **,(Spot)**	1.80E-06	0.8464		4.76E-06	**74.01**	**0.10372**	**0.846**	1144.397	254
	1.30E-06	0.0978		4.42E-06	**134.13**				
2003									
GARCH (Spot)	1.54E-05	0.8805	0.090976	0.00E+00	0.11	**0.18015**	**0.88**	**1010.749**	252
	0.00000726	0.0991	0.019645	7.60E-06	0.79				
GARCH, $\gamma_1=0$ **,(Spot)**	1.56E-05	0.8788		0	0.07	**0.18026**	**0.879**	1010.683	252
	9.33E-06	0.1336		9.80E-06	0.09				

Table 3. In-sample comparison of the Ad Hoc BS model and the updated GARCH model

	RMSE	Average price	Observation
***Ad Hoc* BS**	9.91	118.12	848
HN GARCH model*-updated*	9.87	118.12	848

In-sample valuation errors (in ￡) from weekly estimation using call option prices in the period from 6/1/2005 to 10/27/2005. Average price is the average call option price in the sample period.

Table 4. Mean estimates from the updated GARCH model using Nonlinear Least Squares

Parameter	Mean	Standard Deviation
α_1	3.59478E-06	6.37E-06
β_1	0.218902879	0.308171
γ_1^*	-1150.685404	1155.317
ω	4.17739E-07	9.4E-07

Table 5. Out-of-sample valuation error

	Loss Function					
Models	RMSE	MAPE	MAE	RMIVE	Average prices	# Obs
BS	9.27	0.075	7.20	0.0111	151.19	454
***Ad Hoc* BS**	6.77	0.056	5.25	0.0126	151.19	454
***HN GARCH*-non-updated**	6.94	0.066	5.61	0.0111	151.19	454
***HN GARCH*-updated**	5.90	0.045	4.40	0.0105	151.19	454

Table reports the aggregate out-of-sample valuation errors (in ₤) for call options by various models. The data range from 10/28/2005 to 3/3/2006. BS is the Black-Scholes model in which volatility is assumed being constant. Ad Hoc BS is an Ad Hoc version of Black-Scholes model with strike and maturity specified implied volatility; both the BS and Ad Hoc BS are parameter-updated each period

Table 6. Out-of-sample valuation error for call options

Panel A

		Time to Maturity			
		>=65			
Model	Moneyness	RMSE	%Error	MAE	RMIVE
BS					
	0.9-0.96	5.79	1.46	4.59	0.0146
	0.96-0.98	11.62	4.12	10.31	0.0137
	0.98-1.02	15.18	9.85	14.28	0.0130
	1.02-1.05	10.40	16.32	8.75	0.0089
	1.05-1.1	5.80	22.11	4.65	0.0071
***Ad Hoc* BS**					
	0.9-0.96	7.94	2.01	6.85	0.0230
	0.96-0.98	5.24	1.86	4.28	0.0068
	0.98-1.02	4.32	2.80	3.20	0.0036
	1.02-1.05	4.55	7.15	3.18	0.0038
	1.05-1.1	4.60	17.52	4.06	0.0061
***HN GARCH*-non-updated**					
	0.9-0.96	5.29	1.34	4.31	0.0164
	0.96-0.98	5.92	2.10	4.90	0.0069
	0.98-1.02	6.87	4.46	4.98	0.0058
	1.02-1.05	6.02	9.44	4.12	0.0050
	1.05-1.1	4.51	17.18	3.79	0.0054
***HN GARCH*-updated**					
	0.9-0.96	5.73	1.45	4.87	0.0185
	0.96-0.98	5.22	1.85	4.13	0.0061
	0.98-1.02	6.65	4.32	4.65	0.0056
	1.02-1.05	5.52	8.67	3.40	0.0045
	1.05-1.1	3.44	13.12	2.54	0.0042

Panel B

		Time to Maturity			
		<65			
Model	Moneyness	RMSE	%Error	MAE	RMIVE
BS					
	0.9-0.96	5.37	1.59	4.98	0.0188
	0.96-0.98	6.14	2.89	4.92	0.0157
	0.98-1.02	5.07	5.12	3.91	0.0073
	1.02-1.05	3.26	8.46	2.49	0.0042

***Ad Hoc* BS**					
	0.9-0.96	12.41	3.67	11.60	0.0348
	0.96-0.98	12.86	6.05	12.30	0.0262
	0.98-1.02	7.51	7.59	6.38	0.0107
	1.02-1.05	2.62	6.81	2.07	0.0032
***HN GARCH*-non-updated**					
	0.9-0.96	8.34	2.47	7.40	0.0260
	0.96-0.98	9.92	4.67	9.14	0.0221
	0.98-1.02	8.21	8.30	7.33	0.0115
	1.02-1.05	5.73	14.89	5.14	0.0064
***HN GARCH*-updated**					
	0.9-0.96	8.82	2.61	7.97	0.0278
	0.96-0.98	9.34	4.40	8.55	0.0215
	0.98-1.02	5.71	5.77	4.63	0.0089
	1.02-1.05	2.06	5.34	1.70	0.0023

Panel C

Model	#Obs	Time to Maturity	RMSE	MAPE	MAE	RMIVE
BS						
	276	>=65	11.16	0.096	9.31	0.0118
	178	<65	5.09	0.043	3.92	0.0099
***Ad Hoc* BS**						
	276	>=65	5.24	0.053	4.07	0.0098
	178	<65	8.61	0.061	7.09	0.0159
***HN GARCH*-non-updated**						
	276	>=65	5.94	0.056	4.49	0.0082
	178	<65	8.27	0.081	7.34	0.0144
***HN GARCH*-updated**						
	276	>=65	5.56	0.043	3.95	0.0085
	178	<65	6.39	0.047	5.08	0.0130

Reported out-of-sample valuation errors by moneyness and maturity for call options. Moneyness is defined to be K/S where K is the strike price and S is the spot price. %Error is the ratio of the RMSE to the average call option prices for that option category. Panel A reports the valuation errors of contracts, which are categorized by moneyness, with time-to-maturity more than or equaling to 65 days. Panel B reports the valuation errors of contracts, which are categorized by moneyness, with time-to-maturity less than 65 days. Panel C reports the out-of-sample valuation errors of each model. The call option contracts are separated into ≧65 days and <65 days. #Obs means the number of observations.