Investment Timing and Endogenous Default

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Abstract

This paper examines the effect of debt and liquidity on corporate investment in a continuous-time dynamic framework. We show that due to stockholder-bondholder agency conflicts, investment thresholds are U-shaped in leverage and decreasing in liquidity. While the underinvestment problem dominates for low-liquidity firms, there is overinvestment for high-liquidity firms. In the absence of tax effects, we derive the optimal level of liquid funds that eliminates agency costs by implementing the first-best investment policy for some given capital structure. In a second step we generalize the framework by introducing a tax advantage of debt, and we show that an interior solution for liquidity and capital structure optimally trades off tax benefits and agency costs of debt.

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1 Introduction

What are the conditions leading to under- or overinvestment of levered firms? This central question regarding the interaction of corporate investment and financing policies has been discussed in the literature for several years. The underinvestment problem is first treated explicitly by Myers (1977): When debt is in place, it can be optimal from the stockholders' perspective to forgo investment opportunities that would be favorable for an all-equity firm, since the contributed investment amount is beneficial for the bondholders as well.[1] A parallel stream of research concentrated on the opposite problem of overinvestment as a consequence of debt financing, usually related to the asset substitution problem based on Jensen and Meckling (1976).[2]

However, there are only few approaches that discuss both under- and overinvestment in a unified modelling framework. Parrino and Weisbach (1999) examine the magnitude of under- and overinvestment and their sensitivity on firm characteristics within a simulation study. A recent paper by Lyandres and Zhdanov (2005) separates over- from underinvestment and focuses on the effect of the leverage level on the firm's investment timing. Childs, Mauer, and Ott (2005) attribute the contrary investment situations of asset expansion and asset substitution to under- and overinvestment, respectively. Within a rich model they compare two specific parametrizations representing either investment situation. However, the complexity of their model precludes an analytical solution, and it remains unclear which of the two firms' characteristics are most relevant for the fundamentally different investment policies in either situation. Moreover, the asset side before investment is exogenous: The firm cannot influence whether it is in a regime of asset expansion or asset substitution.

In contrast, we capture the distinction between the two situations in an analytically tractable approach by one single parameter that the firm can endogenously choose: Its

[1]Subsequently, the underinvestment issue has been examined by many other authors, see for example Mello and Parsons (1992), who model the operating policy of a levered mine in a continuous-time dynamic approach, or a recent paper by Moyen (2006) for an analysis of debt maturity and underinvestment.

[2]One recent example is the paper by Mauer and Sarkar (2005), who analyze overinvestment in the context of a conditional debt contract.

level of liquid funds. We show that this is the key factor that determines the effect of existing debt on investment timing, i.e. on the decision when to exercise a real option that consists of launching an investment project. Whenever the firm's liquidity is sufficient to allow investment using internal funds only, no additional capital is needed and pure asset substitution can take place, in the sense that a risky investment project is substituted for risk-free liquid funds. This bears incentives to overinvest as put forward by Jensen and Meckling (1976) since the stockholders are willing to invest in projects that would not be pursued from a total firm perspective. In contrast, when the firm does not have any free liquid funds, the whole investment amount has to be injected by the stockholders. This corresponds to the pure asset expansion case and induces underinvestment in the sense of Myers (1977).

In fact, already Myers (1977) suggested that a restriction on dividends, thus retaining cash for future investment, might be a way to mitigate the underinvestment problem, and he contrasted the benefits of this idea with the Jensen and Meckling (1976) problem. However, to our knowledge this comment has not led to subsequent work dealing with under- and overinvestment that would explicitly allow the firm to vary upon debt issuance a relatively easily changeable part of its existing assets, namely its level of liquid funds, in order to endogenously mitigate agency problems.

Note that the impact of liquidity on investment is not driven by financing constraints stemming from transaction costs or information asymmetry.[3] Rather we assume symmetric information, which allows issuance of fairly priced securities and excludes financing constraints. Thus, the non-existence of binding contracts on investment policy is the only friction in our model, and since we assume that the firm is managed by its sole stockholder, we can focus exclusively on the implications of the level of liquid funds for the shareholder-bondholder conflict and thus for the firm's investment timing decision.[4]

Considering first a levered firm with some given capital structure, we show that in the absence of tax effects there exists an optimal level of liquidity that eliminates agency

[3] A large strand of literature, in contrast, does deal exactly with the impact of financing constraints on investment policy. An early example is the empirical study by Fazzari, Hubbard, and Petersen (1988). Recent approaches by Boyle and Guthrie (2003), Hirth and Uhrig-Homburg (2006), and Lyandres (2005) are more related to this work since they also model investment timing and therefore can capture overinvestment as well, in their case however resulting from financing constraints.

[4] See Opler, Pinkowitz, Stulz, and Williamson (1999) for a more detailed discussion of the possible determinants of the optimal level of liquid funds mentioned in this paragraph.

costs by implementing the ex-ante optimal investment policy. Generalizing the framework by introducing tax benefits as a reason why the firm initially chooses a positive amount of debt, we show that the importance of the level of liquidity remains valid, and that there exists an interior combination of liquid funds and debt that trades off tax benefits and agency costs of debt. A testable implication is that especially firms with significant growth opportunities should have a target level of liquid funds, even if they are financially unconstrained.

This paper is organized as follows: In Section 2 we set up our modelling framework. We then solve the model in Section 3 for equity and debt values of the levered firm before investment. In Section 4 we solve for the optimal amount of liquid funds given a certain capital structure and no tax effects. Then we discuss the interaction of liquid funds and capital structure in Section 5 given that there is a tax advantage of debt financing. Section 6 concludes.

2 Model

2.1 Framework

We consider a partially debt-financed firm at time t that holds the perpetual rights worth $F^o(V)$ to a project whose value V follows the geometric Brownian motion

$$dV = (\mu - \delta)V dt + \sigma V dW \tag{1}$$

where μ, δ, and σ are constant parameters and W is a Wiener process. The superscript o denotes the levered firm before investment, in contrast to other states of the firm that will be introduced below. Besides the project rights the firm holds an amount X in liquid funds on a money market account. While the liquid funds remain constant until investment takes place, they generate a riskless income stream rX that is distributed to the stockholders, where r is the constant risk-free interest rate.[5] This may be interpreted as a debt covenant preventing dividend payments to drive the firm's cash holdings below X.

At any point in time, the stockholders have the option to exercise the project rights. The investment requires a total amount of I, and we assume $X \in [0, I]$. Therefore the stockholders have to provide the missing amount $I - X$ needed to launch the project and to

[5] At this point, we do not consider taxes on interest income from the liquid funds.

change the firm's asset side from $F^o(V)+X$ to V. After investment the project provides a cash-flow stream of δV, therefore there are opportunity costs of waiting, and the firm will invest as soon as a critical project value V_I^o, the investment threshold, is exceeded. The threshold determines the firm's investment timing policy, since ceteris paribus investment will take place earlier given that the firm decides to invest at a lower threshold.

The extreme case of $X = 0$ can be described as pure *asset expansion*: The whole investment amount has to be contributed by the stockholders, which leads to a substantial increase in total firm value. In contrast, the case of $X = I$ can be described as pure *asset substitution*: There is no capital inflow upon investment, but the riskless returns of the liquid funds are substituted by the risky project returns. For intermediate values of $X \in (0, I)$ we can model a wide variety of investment conditions.

The debt structure consists of one perpetual coupon bond with a continuous coupon stream of C. Assuming a simple tax environment where coupon payments are tax-deductible at the firm level, the after-tax debt service is $(1-\tau)C$, where τ is the corporate tax rate. Debt service is partly covered by the stockholders' income stream rX from the liquid funds. However, the remaining amount of

$$(1-\tau)C - rX, \tag{2}$$

which is constant over time as long as the firm is in state o, has to be contributed by the stockholders. The total value of equity $E^o(V)$ and debt $D^o(V)$ is

$$E^o(V) + D^o(V) = F^o(V) + X + T^o(V), \tag{3}$$

where $T^o(V)$ denotes the value of tax benefits. Note that if the income from the firm's liquid funds is sufficient to cover the required payment, i.e. if (2) is negative, then there will never be a reason for stockholders to default before investment.

If (2) is positive, the firm declares bankruptcy as soon as the stockholders decide to no longer provide the necessary payments. This is the case as soon as a critical project value of V_B^o is underrun. Then the former stockholders have no more rights or obligations in the firm, and the former bondholders receive all the firm's assets worth

$$E^e(V) = F^e(V) + X. \tag{4}$$

The superscript e is used to indicate the state of the all-*equity* firm before investment. Note that the value of the project rights $F^e(V)$ generally differs from $F^o(V)$ because the former bondholders, now stockholders of an all-equity firm, will adopt an investment timing policy V_I^e that can be different from the levered firm's policy V_I^o.

In contrast, if the critical project value V_I^o is exceeded before bankruptcy has been declared, the project is launched. After investment the firm's assets consist solely of the project V providing a cash-flow stream of δV, which the stockholders receive as a dividend after the net coupon stream, i.e.

$$\delta V - (1 - \tau)C. \tag{5}$$

In case of $\delta V - (1 - \tau)C < 0$, the stockholders have to contribute the remaining part of the coupon in order to avoid triggering default. The firm's balance in state i is

$$E^i(V) + D^i(V) = V + T^i(V), \tag{6}$$

where the superscript i denotes the state of the firm after *investment*. Once the investment project has been launched, the evolution of the asset value follows an exogenous process given by (1), and the only decision remaining to the stockholders is to declare bankruptcy by not paying the coupons anymore. This will be the case as soon as a critical project value of V_B^i is underrun. Similar to the investment policy in state e, the default policy V_B^i after investment can be different from the default policy before investment V_B^o.

When the firm after investment (state i) declares bankruptcy, the former bondholders take over the assets worth V and become new stockholders of an all-equity firm, therefore we have

$$E^{ie}(V) = V,$$

where the superscript ie denotes the state of the all-*equity* firm after *investment*. We end up in the same situation when the all-equity firm (state e) eventually launches the investment project. However, note that in the latter case the former bondholders have to provide the missing amount $I - X$ for the transition from state e to ie, while this has been done by the original stockholders at the transition from state o to i.

The transition from one state to another contingent on the evolution of the project value V is visualized in Figure 1.

[Figure 1]

2.2 Basic Claims

In order to derive solutions for debt and equity in different states, three basic cash-flow types have to be evaluated, which will form the building blocks of all the solutions in the following subsections.[6] Once their values are given, debt and equity can be viewed as a portfolio of these three basic claims, which allows convenient representations and insightful interpretations.

To define the cash flows of our basic claims, we assume that the current project value V is between a lower bound of V_B, interpreted as a bankruptcy threshold, and an upper bound of V_I, interpreted as an investment threshold, with $V_B < V_I$. A lower bound of $V_B = 0$ means that default can never occur because of the properties of the stochastic process (1). Similarly, an upper bound of $V_I = \infty$ means that we are in a world where there is no investment opportunity.

We denote by $\mathbf{B}_I(V; V_B, V_I)$ the value of a claim that pays off 1 as soon as V_I is reached and becomes worthless at V_B. It can be shown that $\mathbf{B}_I$ satisfies the ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 \mathbf{B}_I'' + (r-\delta)V\mathbf{B}_I' = r\mathbf{B}_I \quad \forall V \in [V_B, V_I]$$

with $\mathbf{B}_I'$ and $\mathbf{B}_I''$ denoting first and second derivatives with respect to V, respectively. Solving subject to the boundary conditions $\mathbf{B}_I(V_B; V_B, V_I) = 0$ and $\mathbf{B}_I(V_I; V_B, V_I) = 1$ yields the solution

$$\mathbf{B}_I(V; V_B, V_I) = \begin{cases} 0, & \text{if } V_I = \infty \\ \left(\frac{V}{V_I}\right)^{\lambda_1}, & \text{if } V_B = 0 \text{ and } V < V_I < \infty \\ \frac{(V_B)^{\lambda_2} \cdot V^{\lambda_1} - (V_B)^{\lambda_1} \cdot V^{\lambda_2}}{(V_I)^{\lambda_1}(V_B)^{\lambda_2} - (V_I)^{\lambda_2}(V_B)^{\lambda_1}}, & \text{if } V_B > 0 \text{ and } V_B < V < V_I < \infty \end{cases}$$

with

$$\lambda_1 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\frac{2r}{\sigma^2} + \left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2} \tag{7}$$

and

$$\lambda_2 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} - \sqrt{\frac{2r}{\sigma^2} + \left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2}. \tag{8}$$

[6] This can be seen as an time-homogenous extension of Ericsson and Reneby (1998), who present building blocks for bond valuation given only a lower (bankruptcy) threshold, to a situation with both a lower and an upper bound, where the latter will be the investment threshold in our model.

Similarly, $\mathbf{B}_B(V; V_B, V_I)$ denotes the value of a claim that pays off 1 upon reaching V_B, however becomes worthless when the project value hits V_I. Its value is

$$\mathbf{B}_B(V; V_B, V_I) = \begin{cases} 0, & \text{if } V_B = 0 \\ \left(\frac{V}{V_B}\right)^{\lambda_2}, & \text{if } V_I = \infty \text{ and } 0 < V_B < V \\ \frac{(V_I)^{\lambda_1} \cdot V^{\lambda_2} - (V_I)^{\lambda_2} \cdot V^{\lambda_1}}{(V_I)^{\lambda_1}(V_B)^{\lambda_2} - (V_I)^{\lambda_2}(V_B)^{\lambda_1}}, & \text{if } V_I < \infty \text{ and } 0 < V_B < V < V_I. \end{cases}$$

The third claim with value $\mathbf{B}_O(V; V_B, V_I)$ yields the risk-free rate r as long as the project value stays in between V_B and V_I. It can be shown to be worth

$$\mathbf{B}_O(V; V_B, V_I) = 1 - \mathbf{B}_I(V; V_B, V_I) - \mathbf{B}_B(V; V_B, V_I). \tag{9}$$

Note that in order to implement a strategy that yields the risk-free rate as long as the project value remains within the interval (V_B, V_I) and a payoff of 1 upon reaching either bound, a capital investment of one monetary unit is needed. Therefore the three basic claims add up to 1.

These three basic claims can now be used to solve the model backwards. While the *ie* state is self-explaining, we will first give the solutions to the e and i states, and then use them to derive the values of equity and debt of the levered firm before investment (state o).

2.3 All-Equity Firm's Investment Decision

The assets of the firm after default (state e) as given by (4) consist of the project rights $F^e(V)$ and the liquid funds X. Since the firm after default corresponds to an all-equity firm with financially unconstrained stockholders, the firm's liquid funds do not have an impact on the investment decision, and the project rights can be valued in a classical real-option framework.[7] For such an investment option we have the boundary conditions

$$F^e(0) = 0, \tag{10}$$

$$F^e(V_I^e) = V_I^e - I, \tag{11}$$

$$F_V^e(V_I^e) = 1, \tag{12}$$

meaning that an option to invest in a worthless project is equally worthless, and that upon investment of an amount I at the critical project value V_I^e, the option can be exchanged

[7] See McDonald and Siegel (1986) and Dixit and Pindyck (1994).

for the project worth V. The first-order condition (12) ensures that immediate investment is indeed optimal above the critical project value V_I^e.

Before investment, the option value can be interpreted as a portfolio consisting of $(V_I^e - I)$ units of the basic claim $\mathbf{B}_I$ yielding one monetary unit upon investment. The resulting investment option value is

$$F^e(V) = \begin{cases} (V_I^e - I) \cdot \mathbf{B}_I(V; 0, V_I^e), & \text{if } 0 \leq V < V_I^e \\ V - I, & \text{if } V \geq V_I^e \end{cases} \tag{13}$$

with the investment threshold being

$$V_I^e = \frac{\lambda_1}{\lambda_1 - 1} \cdot I. \tag{14}$$

2.4 Levered Firm after Investment

Similarly, there are solutions in the literature for the values of equity and debt of the levered firm after investment (state i), when the firm's liquid funds have been used up in order to launch the investment project, and there is no more optionality on the asset side.[8] We start with the equity value $E^i(V)$ that satisfies the boundary conditions

$$\lim_{V \to \infty} E_V^i(V) = 1, \tag{15}$$

$$E^i(V_B^i) = 0, \tag{16}$$

$$E_V^i(V_B^i) = 0, \tag{17}$$

meaning that for sufficiently high project values there is a one-to-one relation between the equity value and the project value, and that at default the stockholders receive nothing. The first-order condition (17) ensures that the bankruptcy threshold V_B^i indeed maximizes equity value. The stockholders' position after investment consists of holding the project value and paying the net coupon stream until default, therefore the solution to $E^i(V)$ is

$$E^i(V) = \begin{cases} 0, & \text{if } 0 \leq V \leq V_B^i \\ V - V_B^i \cdot \mathbf{B}_B(V; V_B^i, \infty) - \frac{(1-\tau)C}{r} \cdot \mathbf{B}_O(V; V_B^i, \infty), & \text{if } V > V_B^i \end{cases} \tag{18}$$

with the bankruptcy threshold

$$V_B^i = \frac{\lambda_2}{\lambda_2 - 1} \cdot \frac{(1-\tau)C}{r}. \tag{19}$$

[8]See Black and Cox (1976) and Leland (1994).

Similarly, the debt value $D^i(V)$ has to satisfy the boundary conditions

$$\begin{aligned} \lim_{V\to\infty} D^i(V) &= \frac{C}{r}, & (20) \\ D^i(V_B^i) &= V_B^i, & (21) \end{aligned}$$

meaning that for sufficiently high project values debt can be valued as if it was riskless, and that at default the bondholders receive the firm's assets. Note that there is no first-order condition – the bankruptcy threshold V_B^i is chosen by the stockholders and thus given by (19). For a project value V being above the bankruptcy threshold V_B^i, the debt value can be interpreted as a portfolio of a claim that provides the payment V_B^i at default, and a claim that pays a stream of C until default. Therefore the solution to $D^i(V)$ is

$$D^i(V) = \begin{cases} V, & \text{if } 0 \leq V \leq V_B^i \\ V_B^i \cdot \mathbf{B}_B(V; V_B^i, \infty) + \frac{C}{r} \cdot \mathbf{B}_O(V; V_B^i, \infty), & \text{if } V > V_B^i. \end{cases} \tag{22}$$

3 Levered Firm before Investment

Having presented solutions for the e and i states, and thus for the boundaries of the o state, we are now ready to derive the values of equity and debt of the levered firm before investment.

For the equity value $E^o(V)$ of the levered firm before investment, the boundary condition at V_I^o is

$$E^o(V_I^o) = E^i(V_I^o) - (I - X), \tag{23}$$

meaning that upon investment the stockholders have to provide an amount $I - X$ and afterwards hold a position worth $E^i(V_I^o)$ according to (18).

When considering the default decision, we have to distinguish two cases: If the stockholders' net payment given by (2) is negative, there is no default before investment, i.e. $V_B^o = 0$. Then debt becomes virtually riskless for very low project values, and the second boundary condition is

$$\lim_{V\to 0} E^o(V) = X - \frac{(1-\tau)C}{r} > 0.$$

However in the second case, if the stockholders' net payment given by (2) is positive, there will be an endogenous default decision characterized by the condition

$$E^o(V_B^o) = 0, \tag{24}$$

meaning that the stockholders receive nothing at default. Consequently the equity value can be interpreted as a portfolio consisting of a cash-flow stream of $rX - (1-\tau)C$ while neither investment nor default occurs, and a payment of $E^i(V_I^o) - (I - X)$ upon investment. We therefore derive the following closed-form expression for $E^o(V)$:

$$E^o(V) = \begin{cases} 0, & \text{if } 0 < V \leq V_B^o \\ \left(X - \frac{(1-\tau)C}{r}\right) \cdot \mathbf{B}_O(V; V_B^o, V_I^o) \quad + & \\ (E^i(V_I^o) - (I - X)) \cdot \mathbf{B}_I(V; V_B^o, V_I^o), & \text{if } V_B^o < V < V_I^o \\ E^i(V) - (I - X), & \text{if } V \geq V_I^o \end{cases} \tag{25}$$

In order to derive the optimal investment and bankruptcy thresholds, we have to consider the first-order conditions at the boundaries. The condition at V_I^o is

$$E_V^o(V_I^o) = E_V^i(V_I^o), \tag{26}$$

which ensures that the resulting investment threshold V_I^o indeed maximizes the value of equity. Note that we thus assume that the firm follows the *second*-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. In Section 5 we will compare this to the *first*-best investment policy, i.e. the policy that maximizes the sum of equity and debt values. If default is possible at all (i.e. if (2) is positive), the second first-order condition

$$E_V^o(V_B^o) = 0 \quad \forall V_B^o > 0 \tag{27}$$

ensures that the resulting bankruptcy threshold V_B^o also maximizes the value of equity.

On the other hand, the debt value $D^o(V)$ has to satisfy a boundary condition at V_I^o being

$$D^o(V_I^o) = D^i(V_I^o).$$

We distinguish again the two cases for V_B^0: If the stockholders' net payment given by (2) is negative, we have $V_B^o = 0$ and

$$\lim_{V \to 0} D^o(V) = \frac{C}{r}.$$

In contrast, if (2) is positive, the boundary condition at V_B^o is

$$D^o(V_B^o) = E^e(V_B^o).$$

So the debt value for $V_B^o < V < V_I^o$ corresponds to a portfolio that consists of the right to receive $E^e(V_B^o)$ upon default, a stream of C while neither default nor investment occurs, and $D^i(V_I^o)$ upon investment. For the no-default case, the first expression equals zero. The solution for $D^o(V)$ is

$$D^o(V) = \begin{cases} V, & \text{if } 0 \leq V \leq V_B^o \\ E^e(V_B^o) \cdot \mathbf{B}_B(V; V_B^o, V_I^o) + \frac{C}{r} \cdot \mathbf{B}_O(V; V_B^o, V_I^o) \quad + & \\ D^i(V_I^o) \cdot \mathbf{B}_I(V; V_B^o, V_I^o), & \text{if } V_B^o < V < V_I^o \\ D^i(V), & \text{if } V \geq V_I^o \end{cases}$$

However, there are no first-order conditions, since it is the stockholders' exclusive right to choose the investment and bankruptcy policies that are value-maximizing from their perspective. Therefore, both V_B^o and V_I^o result from the maximization of equity value (25), captured by the conditions (26) and (27).

4 Underinvestment, Overinvestment, and Liquidity

In this section, we first discuss how the effect of leverage on the firm's investment policy depends on the liquid funds. In a second step, we analyze the implications of the firm's liquid funds as such on the investment policy. We take the capital structure C_0 as given, and in order to focus on agency costs, i.e. the value effect of suboptimal investment, we abstract from tax effects ($\tau = 0$), which implies $T^i(V) = T^o(V) = 0$. In Section 5 we will then discuss the interaction of liquidity and capital structure choice given a positive tax rate ($\tau > 0$) inducing an advantage of debt financing and therefore a reason to initially issue debt.

When solving for the equity value $E^o(V)$ in (25), we have to determine at the same time the investment and bankruptcy policies of the levered firm before investment, V_I^o and V_B^o. In doing so we assume throughout this section that the firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Since we do not consider tax effects in this section, the first-best policy would be to choose the investment threshold V_I^e of the all-equity firm, and therefore we now compare the levered firm's investment policy to that of an all-equity firm. Note that the investment threshold V_I^e of the all-equity firm given by (14) has a closed-form solution that is not dependent on the firm's liquid funds X. In contrast, the investment policy of the levered firm crucially depends on X.

4.1 Effect of Leverage on Investment Policy

We first discuss the effect of leverage, measured by the debt coupon C, on the firm's investment policy. We take as a starting point that an unlevered firm ($C = 0$) follows the investment policy V_I^e regardless of its level of liquid funds. In contrast, Figure 2 illustrates that the effect of increasing leverage does depend on the firm's cash holdings. Our numerical example uses a set of parameters given in Table 1.

Table 1: Parameter values used in numerical examples.

Parameter	Value
Project investment cost	$I = 100$
Project value volatility	$\sigma = 50\%$
Project cash-flow rate	$\delta = 5\%$
Riskless interest rate	$r = 5\%$

[Figure 2]

For $X = 0$ we observe the pure asset expansion case: The stockholders are most concerned about the fact that the amount that they have to contribute to the firm also benefits the bondholders, and therefore they are reluctant to launch the investment project unless the project value is sufficiently favorable. As a consequence, the firm requires a higher project value for investment than an all-equity firm ($V_I^o > V_I^e$). It thus follows a policy that can be described as underinvestment, similar to the problem discussed by Myers (1977). This effect becomes more and more pronounced with increasing leverage, therefore the investment threshold rises more and more above the level of V_I^e for the unlevered firm.

Next we discuss the case $X = I$, which is the pure asset substitution case first treated by Jensen and Meckling (1976). The value of equity makes up only a part of the total firm value when there is debt in the firm. Therefore the stockholders relatively bear less risk than the owners of an all-equity firm when they substitute the riskless returns of the liquid funds by the risky project returns. The stockholders should then be willing to engage in risk-shifting and invest even for a lower project value than an all-equity firm ($V_I^o < V_I^e$). Thus they follow a policy that can be described as overinvestment. The more debt there is in the firm, the more favorable this is to the stockholders. Therefore worse and worse projects are accepted for increasing leverage. The investment threshold is strictly decreasing in C until it hits the bankruptcy threshold. In this degenerate case (and for any even higher C) the o state disappears completely.

To summarize, for either of these pure cases of under- and overinvestment, respectively, we observe that the distortions in investment policy resulting from stockholder-bondholder conflicts are increasing in the firm's leverage measured by the debt coupon C. This result is consistent with the simulation study by Parrino and Weisbach (1999). However, they explain under- and overinvestment in a static framework by different project characteristics, while their project is financed entirely with equity.

Both cases contain another effect that is isolated by Lyandres and Zhdanov (2005): The loss of the option value (from the stockholders' perspective) in the event of default induces early exercise in order to increase equity value, i.e. the $X = 0$ threshold is rising less steeply and the $X = I$ threshold is falling faster due to this additional effect.

The combination of the $X = 0$ and the $X = I$ cases lead to an investment threshold that is U-shaped in leverage for any interior level of liquid funds. For a small debt coupon, the advantage of shifting risk to the bondholders dominates, and therefore investment takes place at project values below V_I^e. However, for increasing coupon level default becomes more likely, and thus also the stockholders' risk to provide value to the firm that is beneficial mainly to the bondholders later on. As this motivation for underinvestment becomes more and more important, the investment threshold for intermediate X eventually reaches its minimum and starts rising again. For sufficient leverage even the all-equity threshold V_I^e is exceeded.

4.2 Effect of Liquid Funds on Investment Policy

Now we analyze the implications of the firm's liquid funds as such on the investment policy. We claim that the investment threshold V_I^o of the levered firm should be decreasing in the firm's liquid funds X: For low-liquidity firms, the properties of asset expansion and underinvestment ($V_I^o > V_I^e$), explained above in its pure form for $X = 0$, are predominant. On the other hand, for high-liquidity firms the properties of asset substitution and overinvestment ($V_I^o < V_I^e$) found in its pure form for $X = I$ are predominant. Consequently, we expect that there will be an intermediate level X_0^* of liquid funds (the subscript 0 indicates the $\tau = 0$ case discussed throughout Section 4) for which the levered firm follows the all-equity firm's investment policy ($V_I^o = V_I^e$).

[Figure 3]

Figure 3 shows that the levered firm's investment threshold indeed has the proposed shape, i.e. it is decreasing in the firm's liquid funds X. Again we use the set of parameters given in Table 1, and the debt coupon rate is set to $C_0 = 27$. The effect is also visible in Figure 2 for this fixed C_0 and selected values of X. Moreover, Figure 2 reveals that for $X = I$, the coupon level C_0 is indeed in the range where the investment threshold coincides with the bankruptcy threshold, which can also be seen at the right margin of Figure 3. There is an optimal amount of liquid funds $X_0^* = 64.51$ for which the levered firm follows the all-equity firm's investment policy ($V_I^o = V_I^e$). Otherwise, the firm chooses an investment policy that is suboptimal from a first-best perspective.

4.3 Effect of Project Risk on Investment Policy

Intuitively, we should expect that the stockholders' incentives to overinvest are increasing in project risk, since risk-shifting then becomes even more attractive. However, would this also mean that for cases in which the investment threshold is above the first-best level, the underinvestment problem is mitigated by increasing project risk?

Indeed, Figure 4 shows that the levered firm's investment threshold relative to the all-equity firm's threshold is decreasing in the project value volatility σ both in areas where there is overinvestment and underinvestment, respectively.[9] This relation shown for the case of $X = X_0^*$ and varying C in the first graph of Figure 4 holds similarly for any other level of liquid funds, including the two extreme cases of $X = 0$ and $X = I$: Even for $X = 0$ where there is underinvestment for any positive amount of debt (see Figure 2), the underinvestment problem is mitigated for increasing project risk.

Besides further confirming the negative relation of project risk and investment threshold, the second graph of Figure 4 for the case of $C = C_0$ and varying X allows the following interpretation: Given a certain leverage level measured by C, the optimal amount of liquid funds X^* needed in order to implement the first-best investment policy V_I^e (i.e. the amount of liquid funds that yields $V_I^o/V_I^e = 1$) is decreasing in project risk. While the stockholders' overinvestment incentives are increasing in project risk, their desire can be tamed by increasing the amount $I - X$ that they have to contribute to the investment.

[9] This result is in line with the findings of Parrino and Weisbach (1999).

Therefore reducing X makes the stockholders more reluctant to invest early in risky projects.

4.4 Agency Costs

Now we analyze the agency costs that the firm suffers due to a suboptimal investment policy as a result of existing debt. In this context the term 'suboptimal' refers to the total firm's perspective. We determined the firm's policy in Section 3 to be optimal from the stockholders' point of view *ex-post*, i.e. after debt is in place. However, *ex-ante*, i.e. before debt is in place, it would be in the interest of the stockholders if they could credibly commit themselves to the investment policy of an all-equity firm, since at that point in time, the debt contract will be set up according to the assumed investment policy. Therefore *ex-ante* the stockholders maximize the sum of equity and debt value, i.e. total firm value, and it would be suboptimal not to commit to the all-equity firm's investment policy. Consequently, agency costs arise since we assume that this *ex-ante* commitment is not possible.

We define agency costs AC as the difference in total firm value between an all-equity firm and a levered firm:

$$AC_{\tau=0} = E^e(V) - [E^o(V) + D^o(V)] = F^e(V) - F^o(V). \quad (28)$$

They can then be understood as the part of all-equity firm value that is lost due to the suboptimal investment policy. Using (3), (4), and $T^o(V) = 0$ for $\tau = 0$, we get the last expression, which illustrates that the agency costs for $\tau = 0$ exactly consist of the loss in option value due to the deviation from the all-equity investment policy.

As elaborated in Section 4.2, a low-liquidity levered firm chooses a higher investment threshold than an all-equity firm, resulting in agency costs of underinvestment. On the other hand, the investment threshold of a high-liquidity levered firm is lower compared to that of an all-equity firm, resulting in agency costs of overinvestment. Only for an intermediate level of liquid funds X_0^* there are no agency costs.

These costs also disappear if the current project value either exceeds both V_I^o and V_I^e, which will result in immediate investment in either case, or if it falls below V_B^o, which means that there will be immediate default and afterwards the former bondholders will follow the all-equity investment policy.

[Figure 5]

Figure 5 shows the agency costs of debt for an initial project value of $V_0 = 0.75V_I^e = 319.92$, which is also indicated by a horizontal line in Figure 3. Comparing to that figure it can be seen that indeed only for $X = X_0^* = 64.51$ there are no agency costs. For lower levels of liquid funds, i.e. $X < X_0^*$, the investment threshold V_I^o, and thus the agency costs of underinvestment, are both increasing with distance from X_0^*.

On the other hand, for $X > X_0^*$ the investment threshold V_I^o is decreasing with distance from X_0^*, and consequently the agency costs of overinvestment are increasing. However, since we assume an initial project value of $V_0 = 0.75V_I^e < V_I^e$, a new situation arises if the firm's liquid funds are high enough that the investment threshold V_I^o falls below the initial project value V_0. Going back to Figure 3, we observe that this happens for $X > 83.83$. Then immediate investment will take place right after debt issuance, while the all-equity firm would prefer to postpone the investment. In all of these states, we do no longer have possible future overinvestment, but actual overinvestment, and thus for all $X > 83.83$ the agency costs of overinvestment remain at

$$F^e(V) - (V - I),$$

which is a constant value since $F^e(V)$ is not dependent on X.

A similar result arises for an initial project value $V_0 > V_I^e$, for which the all-equity firm invests immediately: We will examine that below a certain level of liquid funds X the investment threshold V_I^o will exceed V_0 and underinvestment takes place. Then the agency costs of underinvestment are

$$V - I - F^o(V),$$

and they are increasing for decreasing liquid funds X, since $F^o(V)$ becomes less valuable with the distance of X and X_0^*. In contrast, for sufficiently high X we have $V_0 \geq V_I^o$, and both the levered firm and the all-equity firm invest immediately. In that case there are no agency costs.

5 Optimal Liquidity and Capital Structure Choice

Up to now, we have taken the firm's capital structure as given without discussing how the optimal debt contract should look like. In fact, our analysis in Section 4 completely abstracts from tax effects, with the consequence that there is no model-endogenous reason at all to initially issue debt.

Therefore we now generalize this analysis: We show that when providing a reason to issue debt by introducing tax benefits ($\tau > 0$), the relationship of underinvestment, overinvestment, and liquidity elaborated in the previous section is preserved also under this more general setup. Moreover, we can now study the interaction of liquidity and capital structure choice, as well as the trade-off between the value of tax benefits and the resulting agency costs upon debt issuance.

5.1 Debt Issuance

We first present the sequence of the debt issuance process. Consider a financially unconstrained entrepreneur who owns the investment option worth $F^e(V)$ (following the all-equity exercise policy). Preparing for debt issuance, the entrepreneur first provides some amount of liquid funds X. Then the unlevered firm's balance is given by (4).

In the next step, debt is issued with a coupon C. The prospective bondholders provide a payment of $D^o(V)$ to the entrepreneur, where they correctly anticipate the value of their position in the firm after debt issuance. Upon debt issuance, the levered firm's balance changes to (3). The entrepreneur's optimization problem is to choose the liquid funds X and the capital structure C that maximize his gain upon debt issuance:

$$\max_{(X,C)} \quad E^o(V) + D^o(V) - E^e(V) = F^o(V) - F^e(V) + T^o(V). \tag{29}$$

The right-hand side allows the interpretation that the owner of the firm trades off the loss in investment option value $F^o(V) - F^e(V)$ due to a suboptimal exercise policy against the tax benefits of debt $T^o(V)$.

5.2 Agency Costs

At first it might seem reasonable to define the agency costs of debt similar to Section 4 by comparing the firm value after debt issuance to that of an all-equity firm. This would correspond to the gain upon debt issuance given by (29). However, now an all-equity firm can be made better off by issuing debt due to tax benefits. In particular, a firm that can credibly commit to the first-best investment policy will in general increase its value by issuing debt, although its value was not affected by the choice of (X, C) for $\tau = 0$. Therefore it is no longer sufficient to compare investment option values before and after debt issuance, as was reasonable in (28). Rather, we have to take into account on one hand under- and overinvestment costs driving the option value below that of an all-equity firm, and on the other hand costs due to foregone tax benefits. Our new definition of agency costs for $\tau \geq 0$ is

$$AC = [F^{o1}_{(X_1,C_1)}(V) - F^{o2}_{(X_2,C_2)}(V)] + [T^{o1}_{(X_1,C_1)}(V) - T^{o2}_{(X_2,C_2)}(V)] \tag{30}$$

The new superscript $o1$ denotes values given that the first-best investment policy is followed after debt issuance, which means that when deriving the first-best investment threshold V_I^{o1} and bankruptcy threshold V_B^{o1}, the new first-order condition

$$E_V^{o1}(V_I^{o1}) + D_V^{o1}(V_I^{o1}) = E_V^{i}(V_I^{o1}) + D_V^{i}(V_I^{o1})$$

has to be used, while condition (26) is used for the second-best investment policy (superscript $o2$, corresponds to o in the $\tau = 0$ case).[10] The subscripts in (30) point out that if the entrepreneur can credibly commit to the first-best investment policy before debt issuance, he does not only choose different investment and bankruptcy thresholds, but he chooses a combination of liquid funds and capital structure (X_1, C_1) that will in general be different from the combination (X_2, C_2) that he chooses for the second-best investment policy. Note that for $\tau = 0$, the second bracket in (30) disappears and we have $F^{o1}_{(X_1,C_1)}(V) = F^e(V)$, so we end up again at definition (28).

5.3 Numerical Example and Discussion

In a numerical example, we show the (X, C) combinations actually chosen in the first-best and second-best cases, as well as the value effect of debt issuance and the resulting agency costs of debt. Since option values and tax benefits are connected to firm values by (3) and (4), we can rewrite (30) as the difference in gains upon debt issuance:

$$\begin{aligned} AC &= [E^{o1}_{(X_1,C_1)}(V) + D^{o1}_{(X_1,C_1)}(V) - E^{e}_{(X_1)}(V)] \\ &- [E^{o2}_{(X_2,C_2)}(V) + D^{o2}_{(X_2,C_2)}(V) - E^{e}_{(X_2)}(V)] \end{aligned} \tag{31}$$

[10] The bankruptcy thresholds will in general also be different ($V_B^{o1} \neq V_B^{o2}$), although they are both chosen in stockholders' interest according to (27), or 0, respectively, if the stockholders' net payment given by (2) is negative.

Each of the square brackets represents the gain upon debt issuance relative to the all-equity firm for the first-best and second-best cases, respectively. The value of the first bracket (first-best case) as a function of the (X_1, C_1) combination chosen by the firm is shown in the first graph of Figure 6, while in the second graph the value of the second bracket (second-best case) is shown as a function of (X_2, C_2). Parameter values are again given in Table 1. In contrast to Section 4, we now use a positive tax rate of $\tau = 10\%$, and the firm chooses the optimal levels of both liquidity and debt endogenously.

It can be seen in the first graph of Figure 6 that for a firm that follows the first-best investment policy, the maximum gain by debt issuance is reached for the maximum level of liquid funds, $X_1^* = I = 100$. While the liquid funds are not needed in order to implement a certain investment policy, they work as a collateral and reduce the probability of default, and therefore the tax benefits can be enjoyed over a longer expected period of time. Since collateral causes no direct costs in our model, it is indeed optimal to use the maximum level of liquid funds. An interior level of debt (coupon $C_1^* = 30.02$) is chosen in order to achieve the maximum value of tax benefits, which is a well-known result from dynamic trade-off models.[11]

The second graph of Figure 6 shows the change in firm value upon debt issuance given that the firm follows the second-best investment policy. Still, an interior level of debt (coupon $C_2^* = 31.02$) close to that of the first-best case is chosen. However, as a result of the trade-off between tax benefits and agency costs of debt, the maximum gain by debt issuance is now reached for an interior level of liquid funds $X_2^* = 72 < I$.

As we showed in Figure 5, the firm needs a certain level of liquid funds in order to implement a second-best investment policy that mitigates conflicts of interest between stockholders and bondholders and thus agency costs of under- and overinvestment. Therefore the level of liquid funds X_2^* has to be below the maximum level of liquid funds, $X_1^* = I = 100$, which is chosen by a firm that can credibly commit to the first-best investment policy. However, while minimizing the loss $F^e(V) - F^o(V)$ was the only effect in Section 4, now the value of tax benefits, $T^o(V)$, has to be considered as well. Therefore the resulting X_2^* is on the other hand above the level of liquid funds X_0^* that the firm chooses for a similar leverage when there are no tax benefits.

The resulting maximum gains upon debt issuance are 14.23 in the first-best case, and 13.49 in the second-best case, respectively. We can then use (31) and calculate the actual

[11]See e.g. Leland (1994).

agency costs of debt as the difference of the maximum gains upon debt issuance in the first- and second-best cases. In our numerical example this corresponds to a value of $14.23 - 13.49 = 0.74$.

In order to ensure the stability of our results, we carried out a comparative statics analysis. We could confirm that the basic qualitative structure remains unchanged for a wide range of parameter values: While it is optimal to choose the maximum level of liquid funds in the first-best case, there is a significantly lower level of liquid funds in the second-best case. On the one hand, cash holdings help to increase tax benefits. However, there would be a tremendous overinvestment problem if the firm sticked to the first-best policy of holding the maximum level of liquid funds although it could not commit to the first-best investment policy.

6 Conclusion

In this work we have shown that existing debt can induce both underinvestment and overinvestment. This basic finding does not directly depend on capital market frictions, as we have shown that no tax benefits, bankruptcy costs, or other direct advantages or disadvantages of debt financing are needed to explain distortions in the investment policy.

Instead, we have introduced the firm's level of internal liquid funds as the key factor that determines the effect of existing debt on investment. We have shown that for low-liquidity firms the investment situation is mainly determined by the characteristics of the asset expansion problem and underinvestment takes place, while overinvestment due to the asset substitution problem is dominating the investment situation for high-liquidity firms.

Consequently, we find that investment thresholds are decreasing in liquidity for levered firms. Note that we do not assume that the firm is financially constrained. In our case, underinvestment is not due to the stockholders' difficulties to raise the investment amount on external capital markets. Rather, they decide willingly that they do not want to provide the amount since it also benefits the bondholders. In contrast, for a sufficient level of liquidity, the benefits to the stockholders that arise from asset substitution outweigh the cost from asset expansion, and overinvestment takes place.

There is an optimal level of liquidity for which there is a trade-off between these two effects, and the ex-ante optimal investment policy of an all-equity firm can be implemented. Even if the latter is not directly enforceable, we can therefore eliminate the agency costs of debt. For any other firm liquidity, the overall level of agency costs is determined by the degree of deviation from the optimal level of liquidity.

We showed that the distortions in investment policy resulting from stockholder-bondholder conflicts are increasing in the firm's leverage. This holds for the extreme cases of a firm holding no liquid funds, resulting in underinvestment, and enough liquid funds to finance the investment project, inducing overinvestment, respectively. However, the two effects interact for any interior level of liquid funds, inducing an investment threshold that is U-shaped in leverage.

Increasing project risk leads to earlier investment. This holds both in cases where there is overinvestment and underinvestment, respectively. Consequently, while underinvestment can be mitigated for increasing project risk, the overinvestment problem becomes more severe when the shareholders have more incentives for risk-shifting. Again, the ex-ante optimal investment policy can still be implemented by choosing an appropriate, i.e. a lower level of liquidity for increasing project risk.

In a second step of our analysis, we have examined the interaction of liquidity and capital structure choice given that a tax advantage of debt makes leverage favorable. In that case, a firm that can credibly commit to a certain investment policy has a motivation to issue debt, and it will use the maximum level of liquid funds as a collateral for a higher value of tax benefits. In contrast, if the investment policy cannot be bound by contracts, the firm chooses an interior level of liquid funds in order to implement an investment policy that both mitigates conflicts of interest between stockholders and bondholders on investment policy and helps to increase the value of tax benefits.

Overall, we have shed light on the key factor that determines the actual effect of existing debt, namely the firm's level of liquidity, and thus we have provided some answers to the introducing question which conditions lead to under- or overinvestment of levered firms.

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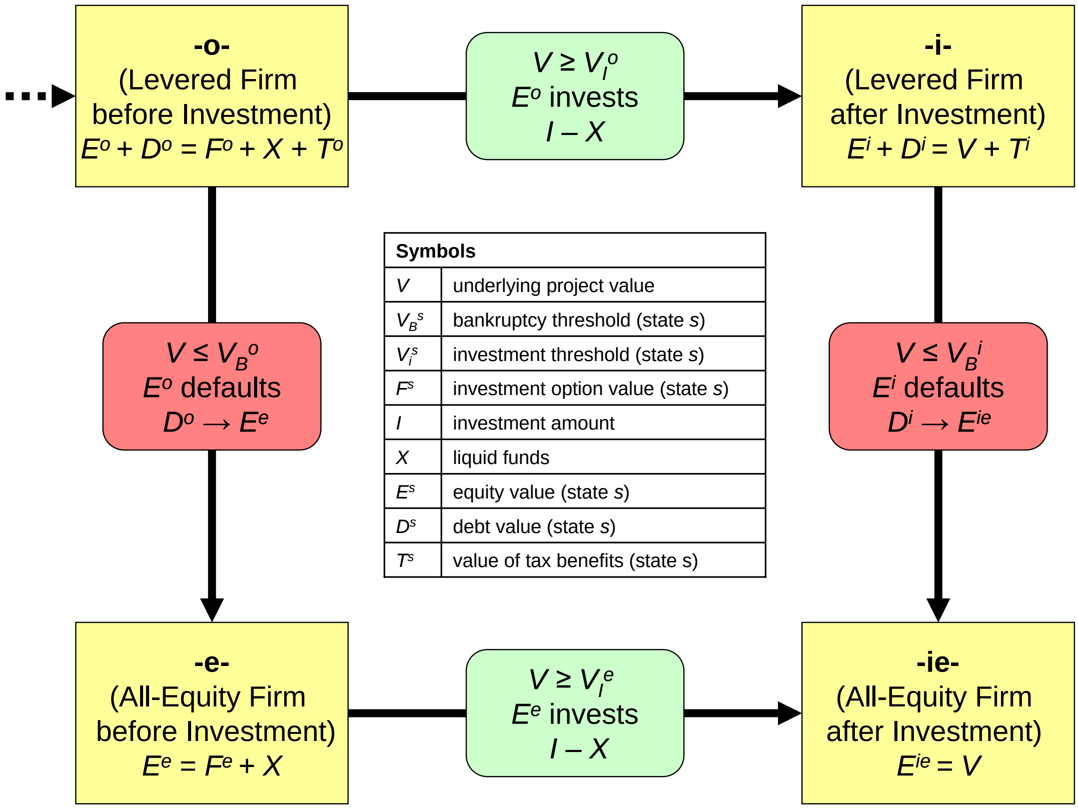

Figure 1: Transition of States.

This figure shows the possible state transitions of the firm contingent on the evolution of the project value V. Starting in state o, the firm will invest as soon as V_I^o is exceeded. Then the stockholders have to provide the amount $I - X$. In contrast, if the project value first falls below V_B^o, the firm will default. After default (state e), the former bondholders will be the owners of an all-equity firm and themselves decide to invest by providing the amount $I - X$ as soon as V_I^e is exceeded. On the other hand after investment (state i), default is still possible, and the firm will default if the project value falls below V_B^i. In either of the latter states, the last transition leads the firm to state ie in which both investment and default have already taken place. The dotted arrow into state o represents the preceding debt issuance decision of an all-equity firm.

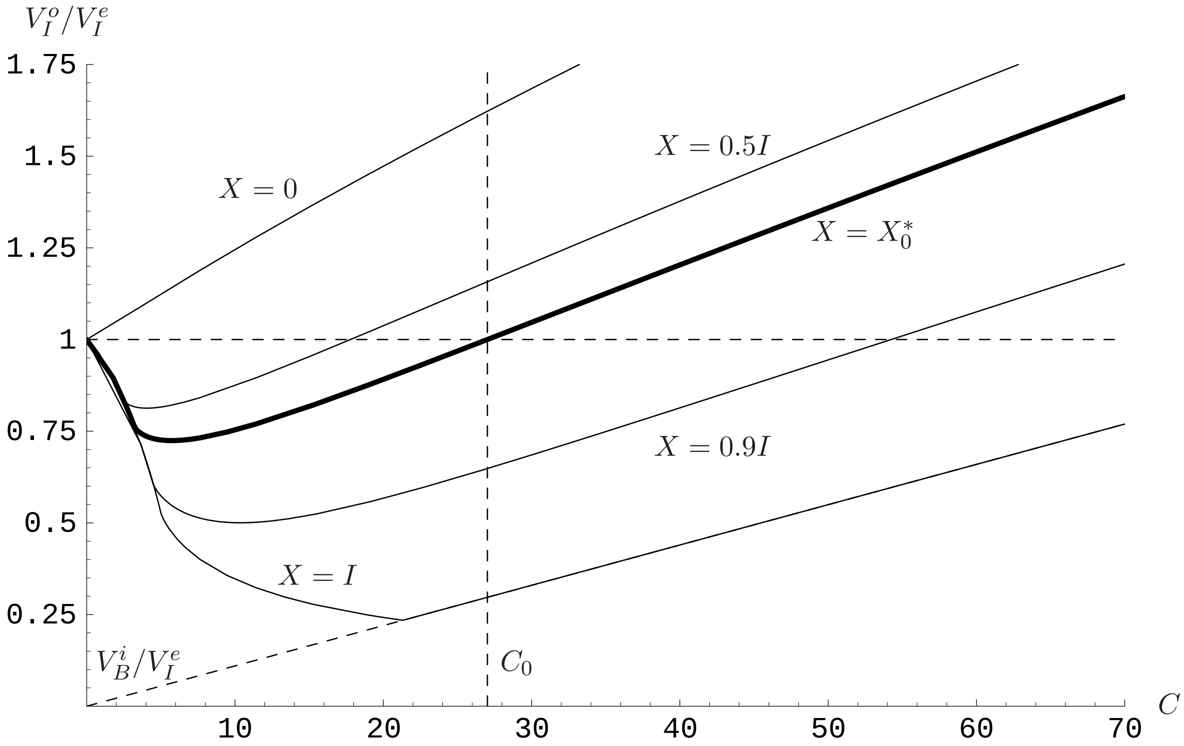

Figure 2: Levered Firm's Investment Threshold as a Function of Leverage.
The levered firm's investment threshold V_I^o, normalized by the investment threshold of the unlevered firm V_I^e, is shown for different levels of liquid funds (solid lines) as a function of the firm's leverage measured by the debt coupon C. The firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Also shown is the investment threshold of the unlevered firm, which is normalized to 1 and independent of C, and the bankruptcy threshold after investment V_B^i/V_I^e. The coupon level C_0 will be fixed in Figure 3. Parameter values are given in Table 1, taxes are not considered ($\tau = 0$).

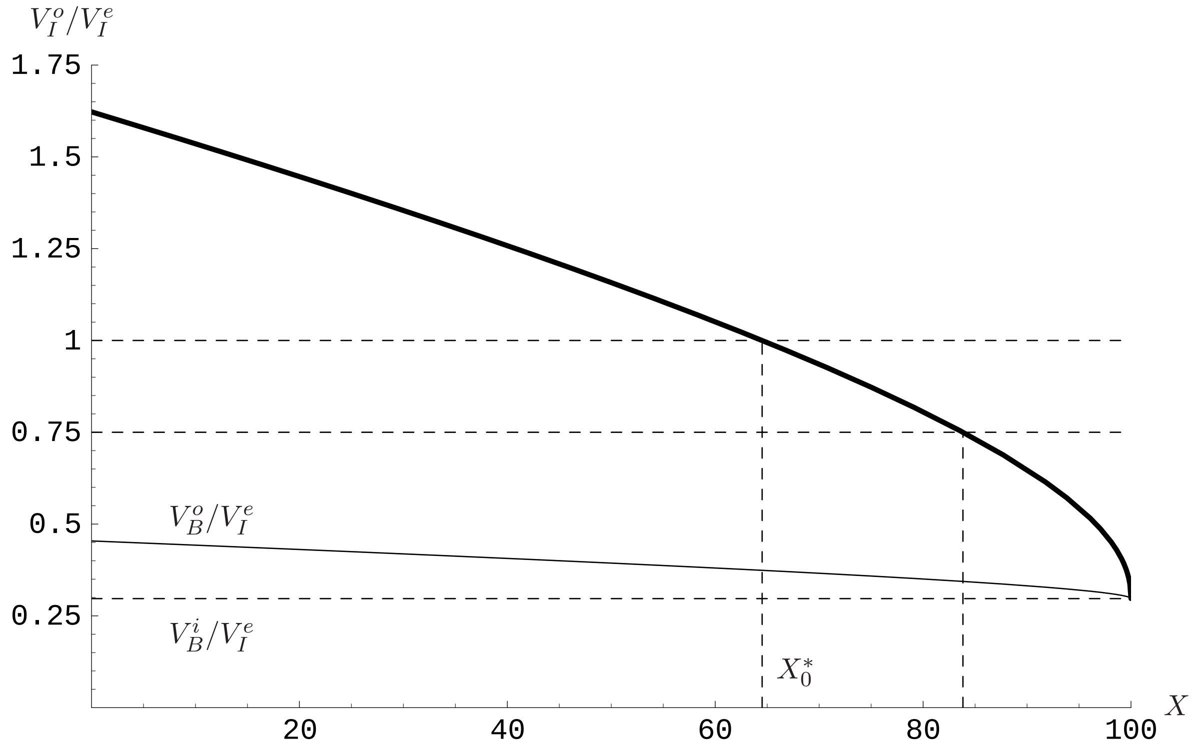

Figure 3: Levered Firm's Investment Threshold as a Function of Liquid Funds.
The levered firm's investment threshold V_I^o (solid line), normalized by the investment threshold of the unlevered firm V_I^e, is shown as a function of the firm's liquid funds X (for $C = C_0$), given that the firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Also shown are the investment threshold of the unlevered firm which is normalized to 1 and the $V_0/V_I^e = 0.75$ line indicating the level later used as the initial project value, as well as the bankruptcy thresholds before and after investment, V_B^o and V_B^i, respectively. Parameter values are given in Table 1, taxes are not considered ($\tau = 0$).

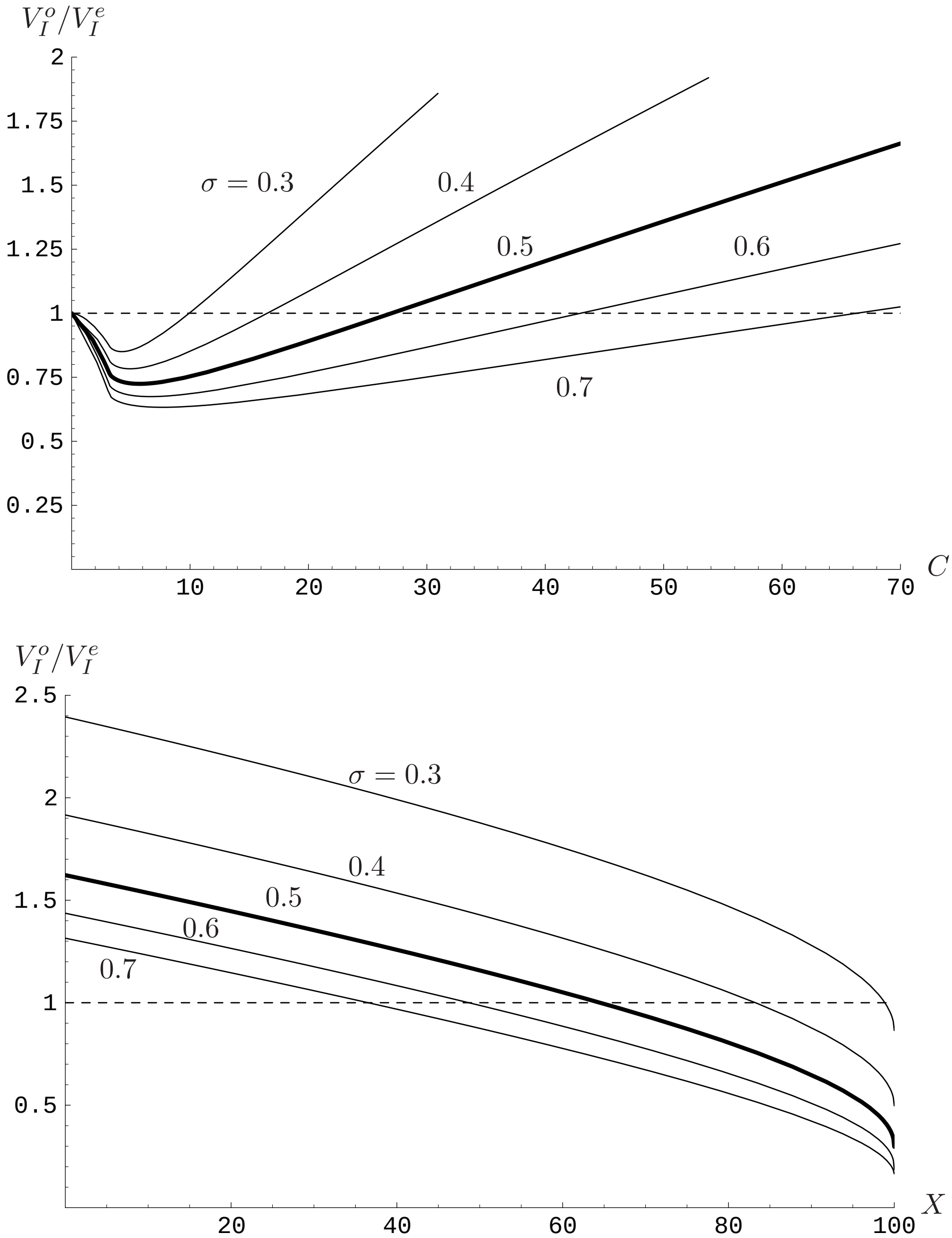

Figure 4: Levered Firm's Investment Threshold for Varying Project Value Volatility. The levered firm's investment threshold V_I^o (solid lines), normalized by the investment threshold of the unlevered firm V_I^e, is shown as a function of the firm's leverage measured by the debt coupon C (for $X = X_0^*$), and the firm's liquid funds X (for $C = C_0$), respectively, given that the firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Also shown is the investment threshold of the unlevered firm which is normalized to 1. Parameter values are given in Table 1, taxes are not considered ($\tau = 0$).

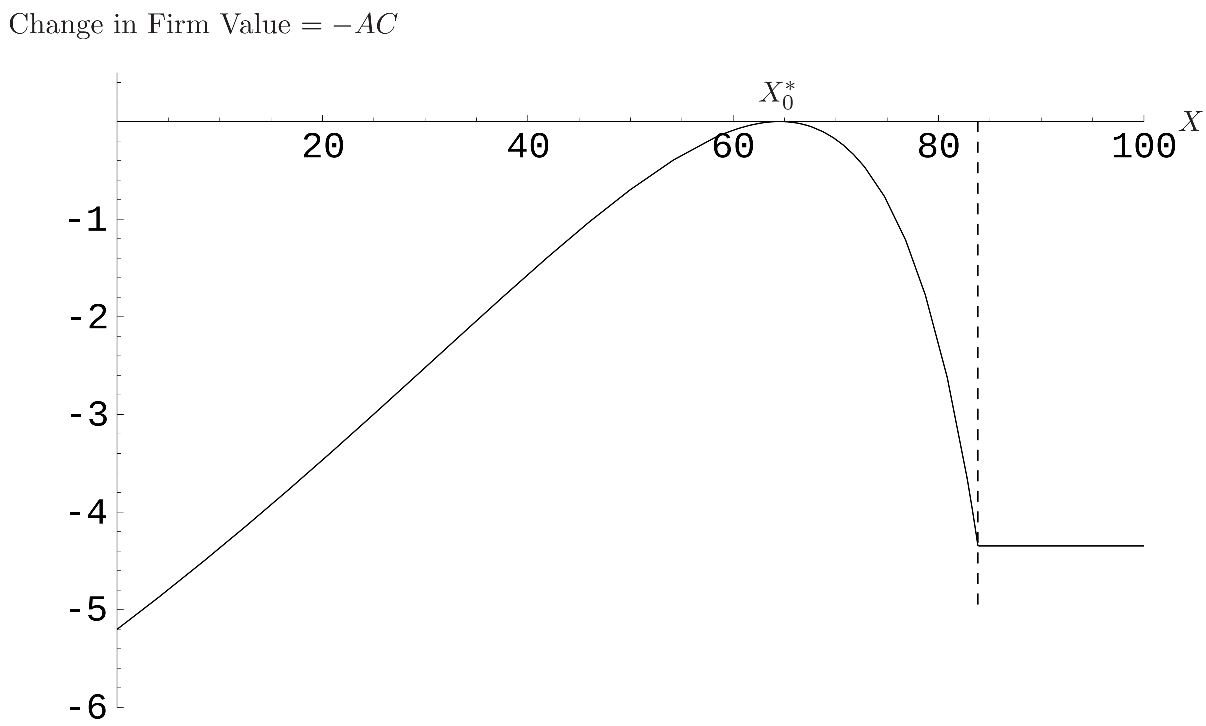

Figure 5: Agency Costs of Debt.
The agency costs of debt, measured by the (negative) change in firm value upon debt issuance, are shown as a function of the firm's liquid funds X, given that the firm follows the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance. Parameter values are given in Table 1, the initial project value considered is $V_0 = 0.75V_I^e = 319.92$, and taxes are not considered ($\tau = 0$).

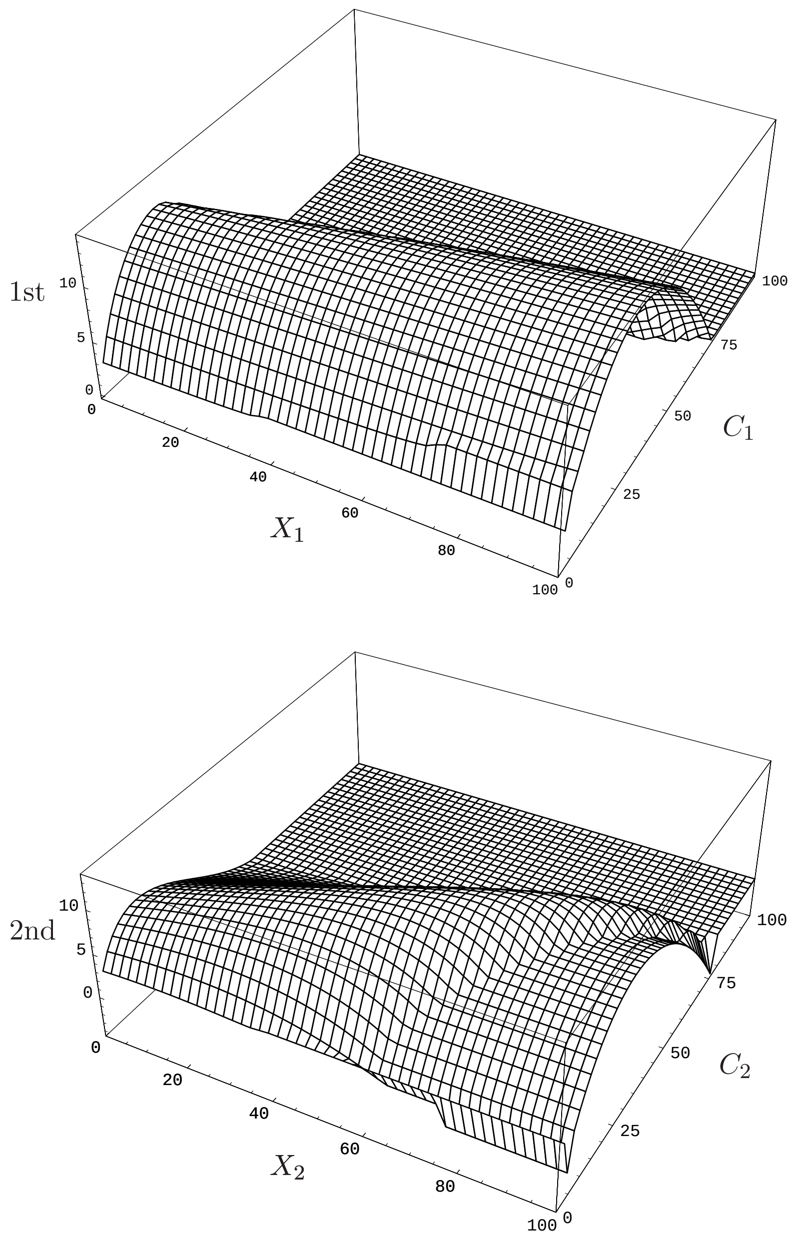

Figure 6: Optimal Liquidity and Capital Structure.
The change in firm value upon debt issuance is shown as a function of the firm's liquid funds $X_{1/2}$ and debt coupon rate $C_{1/2}$, given that the firm follows the first-best investment policy, i.e. the policy that is value-maximizing before debt issuance (first graph), or the second-best investment policy, i.e. the policy that is equity value-maximizing after debt issuance (second graph), respectively. Parameter values are given in Table 1, the initial project value considered is $V_0 = 0.75V_I^e = 319.92$, and the tax rate is $\tau = 10\%$.