Rescheduling of Distressed Debt and Business Risk Targeting *ex ante* the Reorganization

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Abstract

This article reconsiders the rescheduling of distressed debt by including the period preceding the financial reorganization in the analysis. We develop a sequential contingent claim analysis where equity holders take advantage of their position ex ante the default. We reveal situations where the equity price is a concave function of the firm's assets volatility. The optimal strategy for equity holders consists in targeting a specific business risk level. This in turn leads either to an upward risk shifting or to some risk avoidance before default; this does not necessary imply opportunity costs for creditors. We finally appreciate the economic significance of these latter costs and address the possible crossed influence of stake holders before and at the reorganization.

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Introduction

In situation of default, debt holders have to decide whether or not they liquidate the assets of the distressed firm. If not, many different solutions exist to reorganize the financial structure of the distressed firms. Among other things, one finds debt maturity extension, forgiveness of due payment and debt-equity swaps. Empirically however, the maturity extension is a very common form of loan modification (Asquith, Gertner and Scharfstein (1994) and Mann (1997)). This approach can be considered as sufficient if the economic viability of the firm is considered. Whatever, Longstaff (1990) has recalled that avoidance of immediate and significant liquidation costs also give debt holders some incentives to grant a delay[2]. His results show that both parties benefit from the optimal maturity extension. This scheme is retained throughout the paper as emblematic of the rescheduling methods.

The paper reconsiders the way claimants act during the financial distress and the impacts of these behaviours on operational choices. In contrast with most previous works, our analysis deals with the period preceding the default event. Often studies rather explore what can happen *at* or *ex post* the reorganization and how claimants should bargain so as to influence the equilibrium resulting from renegotiation. In this respect, the scenario considered in Longstaff (1990) is rather extreme since no game is considered at all. In his set up, debt holders govern the workout with a so substantial authority that the objective function to design the reorganized debt is uniquely defined on their net gain. To be complete on this function, note that it involves the firm's assets value, the level of business risk, the default severity and the liquidation costs that prevail at the time of default.

[2] Harding and Sirmans (2002) go a step further by showing that this technique can better align the interests of parties than the discounted payoff strategy.

We argue in this paper that equity holders should profit from their position *ex ante* the default. During this period, they can prepare the predictable reorganization at best for their interest. Once the levers in their hands are identified, and we consider the business risk as a good candidate, the key question is to know whether equity holders can optimize their position… The response is positive as long as debt holders possess no covenant limiting the creativity of equity holders. Note that the firm's assets volatility belongs essentially to the set of variables used by debt holders to characterize their objective function[3]. Then, during the period preceding the reorganization, equity holders can act strategically to set this variable to an adequate value. At the reorganization date, they can impose it to creditors as an exogenous parameter by means of a threat to 'throw in the towel', to break off the reorganization they desire or, for short, to refuse the unsatisfactory work out. Equity holders are in fact likely to be rather convincing because the equity is worth nothing at default. They have a take-it or leave-it offer in their hands. This take-it-or-leave-it approach is however not necessary when the reversal actions are costly and could destroy significant value. Overall, debt holders are considered as unable to change the level of the business risk.

We develop a parsimonious model in lines of the contingent claim analysis with perfect information that consider a specific financial reorganization scheme. Both claim holders are supposed perfectly informed in the sense that both know the way the other acts. Equity holders can anticipate *before* the default that debt holders will grant a delay. They know the way creditors will design their debt and, in particular, the objective function they maximize. So the equity holders choose the level of business risk during the period preceding the default event at best for their interest. At the reorganization date, the debt holders maximize their own objective function, providing that they use the

[3] The value of the firm's assets could have been chosen since assets sales are known to be an

target business risk chosen by equity holders. Our challenge in this paper is to solve a sequential optimization problem.

Equity holders have incentives to modify the business risk before default if this increases the value of the claim they will receive from debt holders at reorganization. The key comes from the properties of the new claim created by renegotiation. The new equity is very similar to the standard call option except that it is now a concave function of the firm's assets volatility. As a consequence, there exists an optimal business risk level for equity holders. Perhaps surprisingly, we find that the debt holders do not necessary suffer from this. Rather, the creditors *can* only face opportunity costs from such a business risk targeting. The precise picture nevertheless depends on the firm's contemporaneous assets volatility and the implied shift. We retain that the upward/downward shifting of the *ex ante* business risk of the firm affects significantly the rescheduling design and that this constitutes a potential source of opportunity costs for debt holders.

To complete our analysis, we compare first of all the above opportunity costs to the agency ones. These latter are known to be important in the analysis of corporate debt. For instance, equity holders behave strategically *ex post* the reorganization at the expense of debt holders by deciding not to refund the firm after the debt issuance (as in Leland (1994)) or by proposing take-it-or-leave-it offers based on the debt service to the creditors (Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997)). Previous studies have insisted on the asset substitution problem (see Jensen-Meckling (1976), Leland (1998) as well as Ericsson (2000) and Décamps and Faure-Grimaud (2000) for detailed discussions). We compare the opportunity costs to the agency costs induced by the asset substitution. We find that the opportunity costs faced by the debt

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other response to a near default.

holders are of a similar or higher magnitude. As a second extension, we relax hypothesises that the parties have no influence on the other's decision. From now on, equity holders have no influence on the design of the rescheduled debt at reorganization and debt holders were not allowed to intervene before the default. These assumptions can appear rather strong because equity holders often participate to the reorganization and have a say on the design of the renegotiated debt and, furthermore because debt holders are used to own covenants. To account for this, we consider different levels of intervention power and modify the objective function accordingly. The way, we proceed, is very close to that exposed in Fan and Sundaresan (2000). We essentially question here whether the potential crossed impact on the existence of a target business risk.

The rest of this paper is organized as follows. Section 1 develops the setting and discusses the new claims created by the reorganization. Section 2 studies the way the equity price depends on the business risk level and reveals the existence of an optimal business risk level. Section 3 examines the different effects of an ex ante modification of the business risk and highlights the potential opportunity costs faced by debt holders. Section 4 extends the analysis and discusses some important issues of the paper. We appreciate here the opportunity costs by comparison with the well known agency costs. Then we address issues relating to the relative influence of equity holders and creditors before and at the reorganization.

1. The Framework

Our contingent claim analysis adopts the continuous time framework of Black, Scholes and Merton (1973, 1974). Financial markets are perfect, complete and trading takes place continuously. There exists a riskless asset paying a known and constant interest rate denoted by r. There are neither taxes, neither transaction costs nor, for the moment, liquidation costs. We consider a risky levered firm with a simple capital structure. The firm is financed by equity and a single debt whose maturity is T_1 and face value F_1. The form’s asset's value at time t is denoted by V_t and its process is assumed to be correctly described, under the risk neutral measure, by:

$$dV = rVdt + \sigma_1 VdW \tag{1}$$

where W is a Brownian motion and σ_1 denotes the firm volatility. σ_1 stands for the level of business risk. Assuming now that things are going on until the maturity T_1, the payoff at T_1 for equity holders T_1 is notoriously that of a call option $(\max(V_{T_1} - F_1;0))$. For their part, in absence of liquidation costs, the debt holders receive either the promised face value if $V_{T_1} \geq F_1$ otherwise the value of the firm’s assets (for short $\min(V_{T_1};F_1)$). If there are bankruptcy costs, the value they can get is strictly lower than the assets’ value. Denoting by $\beta \in [0,1]$ the realization rate, they will receive only βV_{T_1} in case of liquidation. To alleviate these costs, debt holders may grant an additional period of τ. By doing so, they swap the known payoff of βV_{T_1} for a new contingent claim that pays F_1 at $T_1 + \tau$ if $V_{T_1+\tau} \geq F_1$ and $\beta V_{T_1+\tau}$ otherwise. When creditors govern this financial reorganization, Longstaff (1990) demonstrates that there exists an optimal extension period τ. His procedure is based on their wealth only. Under the risk neutral measure Q, the net gain function is given by:

$$H\left(V_{T_1}, \sigma_1, T_2\right) = e^{-r(T_2 - T_1)} E_{T_1}^{Q}\left[\beta V_{T_2} 1_{V_{T_2} < F_1} + F_1 1_{V_{T_2} \geq F_1}\right] - \beta V_{T_1} . \tag{2}$$

where unnecessary references to the face value and the interest rate have been omitted in ht notation of H. Straightforward computations then yields to:

$$H\left(V_{T_1},\sigma_1,T_2\right)=-\beta V_{T_1}+\beta V_{T_1}N\left[-d_{1,V_{T_1}/F_1}\left(T_2-T_1\right)\right]+F_1 e^{-r\left(T_2-T_1\right)}N\left[d_{2,V_{T_1}/F_1}\left(T_2-T_1\right)\right] \qquad (3)$$

where $d_{1,x}(t)=\frac{\ln x+\left(r+\frac{1}{2}\sigma_1^2\right)t}{\sigma_1\sqrt{t}}$, $d_{2,x}(t)=d_{1,x}(t)-\sigma_1\sqrt{t}$ and N is the standard cumulative distribution function. Examination of H shows it is a concave function of T_2, so it may serve as objective function. The optimal extension maturity is then obtained by computing:

$$T\left(V_{T_1},\sigma_1\right)=\arg\max_{t\in[T_1,\infty[} H\left(V_{T_1},\sigma_1,t-T_1\right). \qquad (4)$$

The renegotiated debt is then evaluated by[4]:

$$D_{T_1}\left(V_{T_1},\sigma_1\right)=H\left(V_{T_1},\sigma_1,T\left(V_{T_1},\sigma_1\right)\right)+\beta V_{T_1}. \qquad (5)$$

The debt rescheduling also leads to the creation of a new claim received by equity holders at time T_1. Because our set up is very similar to that of Black, Scholes and Merton, this claim resembles to a call option. Its price at time T_1 may be denoted by:

$$E_{T_1}\left(V_{T_1},\sigma_1\right)=\text{call}\left(V_{T_1},\sigma_1,T\left(V_{T_1},\sigma_1\right)-T_1\right). \qquad (6)$$

There is however a key difference with the standard claim since its expiration date is solution to the above optimization problem. The next section illustrates that this feature alters significantly a well known property of the equity. The new equity price $E_{T_1}\left(V_{T_1},\sigma_1\right)$ is not a convex function of σ_1 as "usual".

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[4] the subscript puts some emphasis on the time T_1.

We now claim that equity holders should prepare the reorganization at best for their interest and take profits from their position *ex ante* the default i.e. during the period preceding the reorganization, say $[T_1-\varepsilon, T_1]$. As discussed in the introduction, we assume that debt holders are not entitled to act before T_1[5]. In addition, they cannot change at time T_1 the business risk level because of the dissuading costs of reversibility or/and the take-it-or-leave-it attitude of equity holders. The price property of the new equity suggests that equity holders can optimize their position by setting σ_1 to an appropriate value. If some times before T_1 (say $T_1-\varepsilon$), the debt reorganization is likely, the equity is worth:

$$E_{T_1-\varepsilon}\left(V_{T_1-\varepsilon},\sigma_1\right)=\text{call}\left(V_{T_1-\varepsilon},\sigma_1,\varepsilon\right)+e^{-r\varepsilon}\int \text{call}\left(V_{T_1},\sigma_1,T\left(V_{T_1},\sigma_1\right)-T_1\right)1_{\{V_{T_1}<F_1\}}Q_{T_1-\varepsilon}\left(V_{T_1}\right)dV_{T_1} \quad (7)$$

where $Q_{T_1-\varepsilon}\left(V_{T_1}\right)$ is the risk neutral density of V_{T_1} conditional on its ($T_1-\varepsilon$)-value. The first term of the right hand side of equation (7) is the standard call of Black, Scholes and Merton whose theoretical expiration is T_1 and the associated time-to-expiration is $T_1-(T_1-\varepsilon)$. The second term is deduced from the potential reorganization at time T_1. The debt rescheduling delivers to equity holders a new claim whose price depends on the underlying firm's asset value at that date.

Since the default situation is assumed significant ($V_{T_1-\varepsilon}<F_1$) and ε rather small, we have:

$$\text{call}\left(V_{T_1-\varepsilon},\sigma_1,\varepsilon\right)\approx 0\,, \quad (8)$$

$$e^{-r\varepsilon}\approx 1\,, \quad (9)$$

[5] Note that this does not induce information asymmetry because this is not a question of information but a matter of contracting rights.

$$dQ\left(V_{T_1}\right) \approx \delta\left\{V_{T_1} = V_{T_1-\varepsilon}\right\} \tag{10}$$

where δ is the Dirac measure. So

$$Q\left[V_{T_1} < F_1 / V_{T_1-\varepsilon}\right] \approx 1 \tag{11}$$

and

$$E_{T_1-\varepsilon}\left(V_{T_1-\varepsilon}, \sigma_1\right) \approx \text{call}\left(V_{T_1-\varepsilon}, \sigma_1, T\left(V_{T_1-\varepsilon}, \sigma_1\right) - T_1\right). \tag{12}$$

To enhance their position at the reorganization of the capital structure, equity holders must maximize the equity price at time $T_1 - \varepsilon$ to identify the optimal business risk level. In view of the equation (12), their own objective function is almost equivalent to maximize the price of the equity received at time T_1. More formally, they need to compute:

$$\sigma_1^* = \arg\max_{\sigma_1 \in \Re^+}\left[E_{T_1-\varepsilon}\left(V_{T_1-\varepsilon}, \sigma_1\right)\right] \approx \arg\max_{\sigma_1 \in \Re^+}\left[\text{call}\left(V_{T_1-\varepsilon}, \sigma_1, T\left(V_{T_1-\varepsilon}, \sigma_1\right) - T_1\right)\right] \tag{13}$$

where:

$$T\left(V_{T_1-\varepsilon}, \sigma_1\right) = \arg\max_{t \in [T_1, \infty[} H\left(V_{T_1-\varepsilon}, \sigma_1, t - T_1\right). \tag{14}$$

Equity holders are now in position to identify the optimal business risk to the extent the solution to the above optimization procedure exists. The optimal level leads in turn to either an upward or a downward move of the existing risk profile of the firm business. Compared to σ_1 the level of business risk observed at time $T_1 - \varepsilon$, the optimal figure σ_1^* represents a more or less important modification. Various scenarios then emerge. Let's consider first that the modification is costless. The change may then be immediate or not. If not, the rise or decrease is time consuming and perhaps the period $[T_1 - \varepsilon, T_1]$ will be too short to succeed. In this case, the equity holders will have to make a take-it or leave-it offer at default time in order to end adjusting this level... Secondly, if there is some financial or transactions costs, a value C can be subtracted to the firm's assets value at time T_1 before re-running the above optimization procedure.

As a final comment, we insist that our set up implies, from now on, no game between claim holders. Our challenge is rather to solve a sequential optimization problem where the influence of the equity holders at reorganization is restricted to the fixing of the business risk. This hypothesis will be discussed in a last section.

2. On the existence of an optimal business risk for stock holders

In the precedent section, we assume there exists a level of business risk that maximize the interest of the equity holders i.e. the existence of a solution to the optimization program defined by the equations (13) and (14). This section explores in depth this issue.

A unique solution to the optimization program defined by the equations (13) and (14) exists to the extent we can prove that the derivative of $\mathrm{call}\left(V_{T_1-\varepsilon}, \sigma_1, T\left(V_{T_1-\varepsilon}, \sigma_1\right) - T_1\right)$ w.r.t. σ_1 admits a single zero. In that case, the optimal business risk level may be obviously defined as the value where the sensitivity gets to zero, i.e.:

$$\left.\frac{\partial \operatorname{call}\left(V_{T_1-\varepsilon},\sigma_1,T\left(V_{T_1-\varepsilon},\sigma_1\right)-T_1\right)}{\partial \sigma_1}\right|_{\sigma_1=\sigma_1^*}=0\,. \qquad (15)$$

As a matter of fact, there exists an analytical expression for this derivative but the dependence of the extension maturity to the volatility makes it really intricate. It does not permit any closed form solution for σ_1^* neither. So we favour a numerical approach. As base case parameters, we mainly use values retained by Longstaff (1990). The face value, due at time T_1, is worth $F_1 = 40$. The firm's asset value is either $V_{T_1-\varepsilon} = 24$ or $V_{T_1-\varepsilon} = 34$ and the actual level of the interest rate is $r = 6\%$. The associated quasi leverage ratios are approximately equal to $F_1 e^{-r\varepsilon} / V_{T_1-\varepsilon} \approx 40/24 = 1{,}666$ and $F_1 e^{-r\varepsilon} / V_{T_1-\varepsilon} \approx 40/34 = 1{,}176$. Note that, since the face value F_1 remains constant, the firm's assets value at time T_1 may be viewed as a proxy for the default severity.

Insert Figure 1.

The figure 1 plots prices of the equity, $E_{T_1}\left(V_{T_1},\sigma_1\right)$, as a function of the business risk for different values of β. The considered volatilities range from 5% to 25%. Values for the realization rate β are those of Longstaff (1990): they range from 60% to 90%[6]. As expected, the rescheduling is a good arrangement for equity holders since they get a strictly positive value whatever the business risk is. We observe that the equity price is lower and lower as the realization

[6] It must be pointed out however that the recent study of Bris, Welch and Zhu (2006) increases the range of admissible values for the realization rate. These authors report very significant changes in firm's assets value and very low recovery for creditors. They write on page 1264: "*Chapter 7 assets drop by at least 20% in mean and 62% in median. Assuming our overly pessimistic reported-only creditor recovery, the median chapter 7 dissipates substantially all its assets even before fee are paid.*" And precise on page 1287: "*In about half of our 30 Chapter 7 liquidations, secured creditors receive nothing.* […] *the mean recovery is 32%* ". They finally add on page 1289 "*unsecured creditors receive nothing in 95% of our Chapter 7*

rate increases. A reason for this is that the optimal extension maturity is a strictly increasing function of the liquidation costs $1-\beta$. The main point however is that the equity price is not a straightforward function of σ_1. The graph displays a concave structure with respect to the volatility of the firm's assets. The humped shape is clearly inconsistent with the usual conclusions of Black-Scholes and Merton that the equity is convex and strictly increasing function of the underlying firm's assets volatility. The new claim received is affected by the level of business risk in a different way because the volatility influences the extension maturity $T(V_{T_1};\sigma_1)$. Overall, this suggests an optimal level of business risk. The following section investigates now the impacts of such a business risk targeting.

3. Effects of targeting business risk *ex ante* the reorganization

The debt rescheduling and the design of the new claim depend on the business risk level imposed by the equity holders. The target business risk level has a direct impact on the price of the rescheduled debt. We investigate now the debt price sensitivity to the business risk level and whether the strategic behaviour of equity holders is at the expense of debt holders.

3.1. The rescheduled debt as a function of the business risk level

As for equity, the impact of the business risk level on the price of the rescheduled debt can be studied formally to the extent the derivative of $D_{T_1}(V_{T_1},\sigma_1)$ with respect to σ_1 is not too-intricate to be analyzed. Here again, there exists an analytical expression for this. However, due to the indirect impact of σ_1 on $T(V_{T_1},\sigma_1)=\arg\max_{t\in[T_1,\infty[} H(V_{T_1},\sigma_1,t-T_1)$, it is not so-straightforward and we favour a numerical approach again. The Figure 2 is in every point similar to Figure 1 except that we deal with the price of the rescheduled debt. This graph

cases". Overall, these figures imply very low values for the realization rate (corresponding mean and median are respectively 80% and 38%).

shows that this is a decreasing function of the business risk (everything else being equal). Hence, debt holders will benefit or suffer from the modification of the business risk profile *ex ante* the reorganization depending on the way the business risk is shifted towards the target level… If the initial level of business risk σ_1 is lower (*resp*. greater) than σ_1^*, the equity holders will shift upward (*resp*. downward) the current level of risk and debt holders will be worse off (*resp*. better off).

Insert Figure 2.

In view of Figure 2, we can conclude that the creditors can either face opportunity costs or benefit from opportunity gains. So the *ex ante* business risk targeting is not necessary achieved at the expense of debt holders. In some cases, both parties benefit from it. A reason for this is that, in our set up, the debt rescheduling is not a zero-sum game. The total firm's value at time T_1 is not invariant to the firm's asset volatility. Denoted by v_{T_1}, it is rather given by $v_{T_1}(V_{T_1},\sigma_1) = V_{T_1} - \mathbf{BC}(V_{T_1},\sigma_1)$ where the bankruptcy costs are $\mathbf{BC}(V_{T_1},\sigma_1) = (1-\beta)V_{T_1} N\left[-d_{1,V_{T_1}/F_1}\left(T(V_{T_1},\sigma_1) - T_1\right)\right]$. Hence, v_{T_1} depends on volatility both directly and indirectly via the optimal extension maturity. The next paragraph sheds more lights on the effects of the *ex ante* business risk targeting i.e. the volatility shifting from σ_1 to σ_1^*.

3.2. Opportunity costs or gains ?

This business risk targeting is worth for equity holders, but it may have some positive and negative effect on the debt price. We can term these latter effects the opportunity gains and opportunity costs gains respectively. To study their magnitude, we assume that equity holders shift the volatility of the firm's assets from σ_1 to σ_1^* and we assess the impacts by means of a couple of measures. To this end, let's denote by S the considered price (S is either E or

D), and let's make the extension maturity of the rescheduled debt apparent. The first measure is a prices difference defined by: $S\left(V_{T_1},\sigma_1^*,T\left(V_{T_1},\sigma_1^*\right)\right)-S\left(V_{T_1},\sigma_1,T\left(V_{T_1},\sigma_1\right)\right)$.

The second one is a relative prices difference computed by
$\left[S\left(V_{T_1},\sigma_1^*,T\left(V_{T_1},\sigma_1^*\right)\right)-S\left(V_{T_1},\sigma_1,T\left(V_{T_1},\sigma_1\right)\right)\right]/S\left(V_{T_1},\sigma_1,T\left(V_{T_1},\sigma_1\right)\right)$.

All terms are computed thanks to the equations (12), (13), (14) and (5). In both measures, the price prevailing before equity holders act strategically is the benchmark price. So, when we apply these measures to the equity price, a positive value can be directly interpreted as the magnitude of the incentives for equity holders to change the "actual" level of business risk.

Insert Figure 3.

Figure 3 gathers together two different graphs (each focusing on a specific measure). On each graph, we represent the two different claims for direct comparison. The left graph plots the prices difference in numeraire (say, euros) whereas the right one considers the relative price differences in percentage. The abscissa displays different magnitudes for the risk shifting. The shift is defined with the help of a variable u ranging from $-\frac{2}{3}$ to $+\frac{2}{3}$ (in percentage in the graphs.). We set $\sigma_1 \equiv \sigma_1^*(1+u)$ in order to a/ center the abscissa on the target risk level and to b/ be able to appreciate the different values each other. A negative (*resp*. positive) value for u, say 10%, means that the value is actually 10%-lower (*resp*. 10%-greater) than the optimal level. This also implies that equity holders will be prompted to shift upward (*resp*. downward) the business risk of the firm.

Figure 3 offers different insights. We can first check the obvious result that when the initial firm's assets volatility σ_1 is equal to the optimal σ_1^* (i.e. $u=0$), equity holders have no incentive to make any changes in the firm's business. In all the other cases, the equity holders benefit from changing the business risk level. When the initial volatility is lower ($\sigma_1 < \sigma^* \Leftrightarrow u < 0$), the equity holders are better off to increase the business risk level. This is especially perceptible on the right graph that indicates a raise of more than 40% of the initial equity price. This benefit decreases as the modification tends to be moderate, i.e. as the initial σ_1 is initially closer to σ^*. Almost symmetrically, debt holders suffer from such a risk shifting. The negative values observed on both graphs are measures of the opportunity costs faced by the creditors. When the initial volatility is 66% smaller than the optimal one, the absolute costs are about 4 times higher than the equity holders' gain. In relative values, however, the risk shifting appears less detrimental for debt holders than it is beneficial for equity holders. When the initial volatility is greater than the optimal level of business risk ($\sigma_1 > \sigma^* \Leftrightarrow u > 0$), the situation is rather different since both parties benefit from the business risk targeting. Compared to the equity holders, debt holders are even better off in absolute values! In relative values, however, the gain is comparable. To conclude, we retain that the business risk targeting can induce opportunity costs but it may also align interests of both parties.

4. Discussion and extension

Our previous analysis can be extended in different ways. A natural question arising now is the economic significance of the opportunity costs. Another issue concerns the assumptions underlying our analysis because it can be argued that both parties can participate before and at reorganization time more actively.

4.1. Do opportunity costs matter?

To answer the first question, we propose to compare the opportunity costs with costs resulting from the asset substitution problem *ex post* the reorganization. From standard analysis of corporate finance, the post-default asset substitution leads to some agency costs for lenders. The risk shifting is beneficial for stockholders at the expense of the debtholders. Of course, agency costs can be avoided *ex ante* by an appropriate covenant and e.g. Bhanot and Mello (2006) have recently suggested a trigger clause to monitor the rating of the debt. They serve however our purpose by providing benchmarks.

In view of the previous sections, σ_1^* is the level of the business risk used by the debt holders to compute the extension maturity i.e. to design the rescheduled debt. This level may be or not equal to the pre-default level of firm's assets volatility σ_1. When $\sigma_1 < \sigma_1^*$, debt holders face some opportunity costs that can be measured with the above relative measure of prices difference. After the debt is rescheduled, the equity holders can act (once again) strategically to increase the volatility of the firm's asset. Assuming that they act immediately after to the reorganization, the price of the new claims may be rewritten $E\left(V_{T_1}, \sigma_2, T\left(V_{T_1}; \sigma_1^*\right)\right)$ and $D\left(V_{T_1}, \sigma_2, T\left(V_{T_1}; \sigma_1^*\right)\right)$ to explicit the different volatilities. The agency costs implied by the upward shift of the business risk can be appreciated by a relative difference of debt prices. We compute $\left(D\left(V_{T_1}, \sigma_2, T\left(V_{T_1}, \sigma_1^*\right)\right) - D\left(V_{T_1}, \sigma_1^*, T\left(V_{T_1}, \sigma_1^*\right)\right)\right) / D\left(V_{T_1}, \sigma_1^*, T\left(V_{T_1}, \sigma_1^*\right)\right)$.

Table 1 provides and compares opportunity costs versus agency costs. The opportunity costs are implied by the *ex ante* business risk targeting while the agency costs are caused by the classical ex post asset substitution problem. Both consider comparable shifts in the business risk. The shifts are computed with $\left(\sigma_1^* - \sigma_1\right)/\sigma_1^*$ and $\left(\sigma_2 - \sigma_1^*\right)/\sigma_1^*$ as percentage of the optimal business risk. Other parameters are $V_{T_1} = 34$. $F_1 = 40$ and $r = 6\%$.

Insert Table 1.

Table 1 reports the magnitudes of the respective costs. Obviously, we find negative values for every relative prices difference we compute. We can observe in both cases, that the larger the risk shifting, the more significant the costs are and that the same is verified for the liquidation costs. The main point is however that the opportunity costs appear larger than the agency costs. In view of this, we can conclude that the opportunity costs are economically significant … but we must warn that these costs are not exclusive one another and that nothing prevents creditors to suffer from both. We now discuss some underlying assumptions.

4.2. Crossed influences of claim holders before and at reorganization

Because our set up is parsimonious, it has neglected the crossed influences of parties on the other's decision. We have assumed that equity holders have no say on the design of the extended debt. This may be regarded as strong because financial reorganization may lead to a real agreement between parties. Second, we have assumed that debt holders have no influence on the choice of the business risk level. This could be in conflict with some existing debt covenant. So we relax now each assumption separately. We then concatenate them in a single extended set up.

First of all, let's assume that equity holders can participate to the financial reorganization at time T_1. They will influence the characterization of the objective function to maximize. If their relative bargaining power is denoted by η, one has $0 \leq \eta \leq 1$ and $1-\eta$ represents the debt holders "remaining" bargaining power. The objective function to consider at reorganization is:

$$H_{total}\left(V_{T_1},\sigma_1,T_2;\eta\right)=H_{debt}^{1-\eta}\left(V_{T_1},\sigma_1,T_2\right)H_{eq}^{\eta}\left(V_{T_1},\sigma_1,T_2\right). \quad (16)$$

where $H_{debt}(V_{T_1},\sigma_1,T_2)=H(V_{T_1},\sigma_1,T_2)$ and $H_{eq}(V_{T_1},\sigma_1,T_2)=\text{call}(V_{T_1},\sigma_1,T_2)$ are the respective net gain functions[7]. The optimal extension maturity is then obtained by computing:

$$T(V_{T_1},\sigma_1;\eta)=\arg\max_{t\in[T_1,\infty[} H_{total}(V_{T_1},\sigma_1,t-T_1;\eta) \tag{17}$$

subject to the constraints that H_{debt} and H_{eq} remain positive. Obviously, H_{eq} is, by definition, positive for all parameters values. H_{debt} should be positive, otherwise this means that debtors could swindle creditors. Note that, of course, if $\eta=0$, the creditors have all the bargaining power and the solution drops to $T(V_{T_1},\sigma_1;0)=T(V_{T_1},\sigma_1)$.

Let's assume now that debt holders can intervene before the default and influence the choice of risk profile *ex ante* T_1. Their intervention then modifies the objective function. Assuming that the covenant gives them an intervention power of γ, $0<\gamma<1$ (γ is lower than one otherwise this means that they control the firm). The complementary value $1-\gamma$ is associated to the equity holders' position. It is worth to introduce formally $G(V_{T_1-\varepsilon},\sigma_1;\gamma)$ the *ex ante* objective function. This function underlies our previous analysis since we have essentially assumed that $\gamma=0$ and posed:

$$G(V_{T_1-\varepsilon},\sigma_1;0)\equiv G(V_{T_1-\varepsilon},\sigma_1)=E_{T_1-\varepsilon}(V_{T_1-\varepsilon},\sigma_1)\approx \text{call}(V_{T_1-\varepsilon},\sigma_1,T(V_{T_1-\varepsilon},\sigma_1)-T_1). \tag{18}$$

If debt holders are able to influence the decisions on the operations *ex ante* the reorganization, the objective function has to be extended to:

[7] This forms an asymmetric generalized Nash bargaining problem.

$$G\left(V_{T_1-\varepsilon},\sigma_1;\gamma\right)=D^{\gamma}_{T_1-\varepsilon}\left(V_{T_1-\varepsilon},\sigma_1\right)E^{1-\gamma}_{T_1-\varepsilon}\left(V_{T_1-\varepsilon},\sigma_1\right)^{1-\gamma}\ . \tag{19}$$

where

$$D_{T_1-\varepsilon}\left(V_{T_1-\varepsilon},\sigma_1\right)=d\left(V_{T_1-\varepsilon},\sigma_1,\varepsilon\right)+e^{-r\varepsilon}\int d\left(V_{T_1},\sigma_1,T\left(V_{T_1},\sigma_1\right)-T_1\right)1_{\{V_{T_1}<F_1\}}Q_{T_1-\varepsilon}\left(V_{T_1}\right)dV_{T_1}$$

with d the $t-$time price of the corporate debt with no option to reschedule but facing potential bankruptcy costs. As we did for the equity in the equations (8) to (12), this expression can be approximated and we find: $D_{T_1-\varepsilon}\left(V_{T_1-\varepsilon},\sigma_1\right)\approx d\left(V_{T_1-\varepsilon},\sigma_1,T\left(V_{T_1-\varepsilon},\sigma_1\right)-T_1\right)$. The optimal business risk level could be found by maximizing $G\left(V_{T_1-\varepsilon},\sigma_1;\gamma\right)$ but the complete picture is obtained when both influences are considered in the same framework, as what follows.

When both parties can intervene on the other's choice, *ex ante* the reorganization i.e. at time $T_1-\varepsilon$, claim holders have to consider:

$$\begin{aligned}G\left(V_{T_1-\varepsilon},\sigma_1;\eta,\gamma\right)&=D^{\gamma}_{T_1-\varepsilon}\left(V_{T_1-\varepsilon},\sigma_1,T\left(V_{T_1-\varepsilon},\sigma_1;\eta\right)\right)E^{1-\gamma}_{T_1-\varepsilon}\left(V_{T_1-\varepsilon},\sigma_1,T\left(V_{T_1-\varepsilon},\sigma_1;\eta\right)\right)\\&\approx d^{\gamma}\left(V_{T_1-\varepsilon},\sigma_1,T\left(V_{T_1-\varepsilon},\sigma_1;\eta\right)\right)\mathrm{call}^{1-\gamma}\left(V_{T_1-\varepsilon},\sigma_1,T\left(V_{T_1-\varepsilon},\sigma_1;\eta\right)\right)\end{aligned}$$

where $T\left(V_{T_1-\varepsilon},\sigma_1;\eta\right)$ satisfies

$$T\left(V_{T_1},\sigma_1;\eta\right)=\arg\max_{t\in[T_1,\infty[}H_{total}\left(V_{T_1},\sigma_1,t-T_1;\eta\right)\ . \tag{20}$$

with $H_{total}\left(V_{T_1},\sigma_1,T_2;\eta\right)=H^{1-\eta}_{debt}\left(V_{T_1},\sigma_1,T_2\right)H^{\eta}_{eq}\left(V_{T_1},\sigma_1,T_2\right)$. The analysis provided in the previous sections is then valid insofar as the above objective $G\left(V_{T_1-\varepsilon},\sigma_1;\eta,\gamma\right)$ admits a maximum value with respect to σ_1. This is what we question in Figure 4. In such a case, the target business risk level is computed with:

$$\sigma_1^* = \arg\max_{\sigma_1 \in \Re^+} G\left(V_{T_1-\varepsilon}, \sigma_1; \eta, \gamma\right) \qquad (21)$$

Insert Figure 4.

The effects of the crossed influence of equity holders and debt equity holders on the *ex ante* targeting are illustrated in Figure 4 which gathers six different graphs. The left graphs consider three arbitrary levels of debt holders influence on the *ex ante* business risk choice while the right graphs consider equity holders' influence on the reorganization at time T_1. In the upper graphs either γ or η is null so the influence is one-sided. These graphs mainly show that the concavity of the *ex ante* objective function with respect to the business risk level is still observed and this is true whatever the magnitudes of the respective influence are. These graphs then lead to some new questions but the additional insights and the related issues are left for further research. We rather discuss as a concluding remark a comment on another assumption. Our set up has essentially considered only one debt rescheduling. Because our set up is sequential, it allows at least theoretically a solution for the multiple rescheduling case. From a practical viewpoint however the picture is different because of the computational costs this represents.

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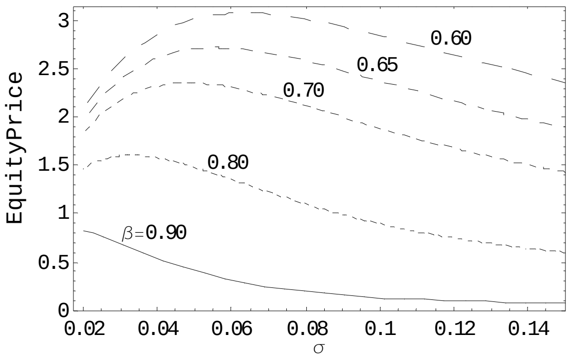
Table 1: *Ex ante* risk targeting versus *ex post* asset substitution: a comparison of relative costs

	Opportunity Costs (*ex ante* reorganization)					Agency Costs (*ex post* reorganization)				
Risk Shifting	10 %	20 %	30 %	40 %	50 %	10 %	20 %	30 %	40 %	50 %
β = 60	-1.499	-3.137	-4.919	-6.845	-8.913	-1.366	-2.617	-3.777	-4.861	-5.883
β = 65	-1.302	-2.727	-4.281	-5.966	-7.782	-1.186	-2.275	-3.286	-4.234	-5.128
β = 70	-1.108	-2.324	-3.653	-5.098	-6.662	-1.010	-1.940	-2.805	-3.617	-4.385
β = 80	-0.732	-1.538	-2.423	-3.392	-4.448	-0.668	-1.286	-1.864	-2.409	-2.926
β = 90	-0.366	-0.770	-1.215	-1.704	-2.243	-0.335	-0.647	-0.939	-1.217	-1.482

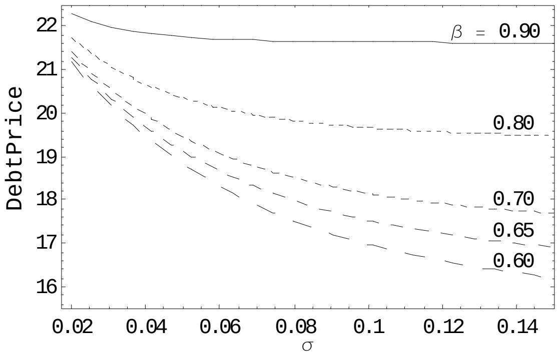
This table provides and compares the costs associated to a shift in the business risk *ex ante* versus *ex post* the reorganization. The opportunity costs come from *ex ante* business risk targeting while the agency costs are implied by the classical *ex post* assets substitution problem. The shifts are expressed in percentage of the optimal business risk. They are respectively computed by $\left(\sigma_1^* - \sigma_1\right)/\sigma_1^*$ and $\left(\sigma_2 - \sigma_1^*\right)/\sigma_1^*$. Other parameters are $V_{T_1} = 34$. $F_1 = 40$ and $r = 6\%$.

Figure 1 The price of the new equity when the maturity is optimally chosen by debt holders



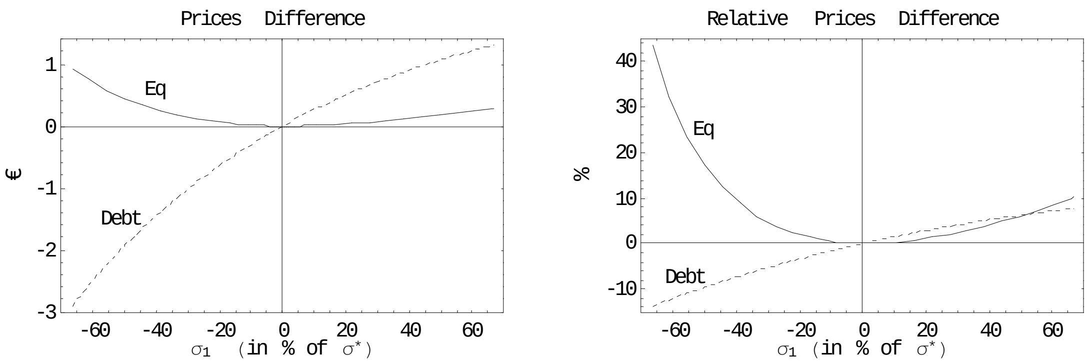
The graph plots the price of the claim received by the equity holders at reorganization as a function of the business risk and for different value of β. The extension maturity is set optimally by debt holders. β ranges from 60% to 90%. Parameters are $V_{T_1} = 24$, $F_1 = 40$, $\beta = 60\%$ and $r = 6\%$.

Figure 2 The price of the renegotiated debt



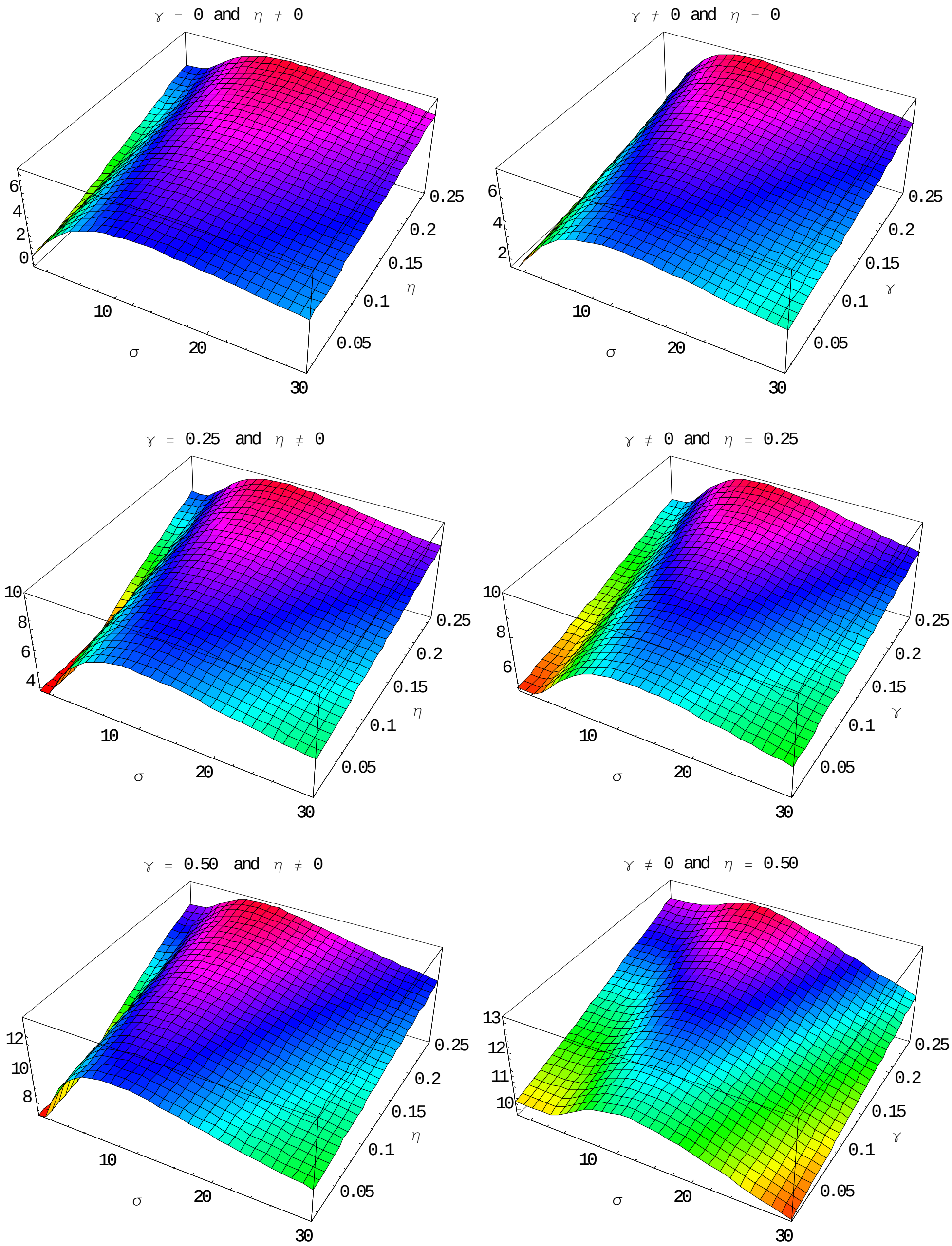
The graph plots the price of the renegotiated debt $D_{T_1}\left(V_{T_1},\sigma,T\left(V_{T_1},\sigma\right)\right)$ as a function of the business risk and for different value of β. The extension maturity is set optimally by debt holders. β ranges from 60% to 90%. Other parameters are $V_{T_1}=24$, $F_1=40$ and $r=6\%$.

Figure 3 Impact of the changes of the business risk & the potential opportunity costs.



The graphs plot the prices difference and the relative prices difference for equity and debt as a function of the initial volatility expressed as a percentage of the target business risk. The initial volatility ranges from $\frac{1}{3}\sigma^*$ to $\frac{5}{3}\sigma^*$. For the claim S, the price difference is computed as $S(\sigma^*) - S(\sigma)$ and the relative price difference by $(S(\sigma^*) - S(\sigma))/S(\sigma)$ (this latter being expressed in percentage). Parameters are $V_{T_1} = 24$, $F_1 = 40$, $\beta = 60\%$ and $r = 6\%$, the extension maturity being set optimally by debt holders.

Figure 4 Crossed influences and the *ex ante* G -function



This figure deals with the *ex ante* objective function G. The left graphs plot G as a function of σ_1 the business risk and the influence of creditors η. Three different levels are considered for γ: 0, 0.25, 0.50. The right graphs plot G as a function of σ_1 the business risk and the influence of equity holders γ. Three different levels are considered for η: 0, 0.25, 0.50. Parameters are $V_{T_1} = 34$, $F_1 = 40$, $\beta = 40\%$ and $r = 10\%$. The extension maturity is set optimally by debt holders thanks to the extended H -function.