Distressed Sales and Financial Arbitrageurs:

Front-running in Illiquid Markets*

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Abstract

We investigate the impact of an arbitrageur's activities in an illiquid market, where there is a large distressed trader and a fringe of small traders. Large traders trade strategically considering price impacts of their trades and future uncertainty on market liquidity. Prices are determined endogenously through a dynamic bargaining and trading process. We find that equilibrium strategies for large traders vary with their relative bargaining power and the level of uncertainty with respect to market liquidity. When there is no such uncertainty, the arbitrageur does not trade at all. However, when there is even a slight amount of uncertainty over future liquidity, the arbitrageur may want to sell part of her holdings before the distressed trader. Moreover, her incentive to "front-run" increases with the level of uncertainty. In most cases, the arbitrageur will front-run the distressed trader by selling quickly, and rebuild her position later at a lower price. The distressed trader's optimal response is to liquidate quickly, despite a big price decline. We note, however, that the arbitrageur does not front-run when there is little uncertainty over market liquidity, or when market liquidity improves over time. The distressed seller can then trade quickly without disturbing prices dramatically.

1. Introduction

In the past two decades, we have witnessed a number of unusual market movements caused by arbitrageurs. Some of them triggered market-wide crises that incurred government intervention. The most famous example is the 1998 LTCM crisis. Studies of the crisis provide hints that LTCM might have become a victim of predatory trading.[1]

Thus large traders, when forced to liquidate, become vulnerable when they cannot hide their trades from other market participants. Such situations can arise from: 1) margin calls from financiers; 2) binding internal or regulatory risk management requirements such as portfolio insurance and Value-at-Risk constraints triggered by unexpected price movements;[2] 3) hedging needs; 4) the need to cover a losing short position, etc. They can be turned into arbitrage opportunities under certain circumstances. Arbitrageurs' trading makes distressed traders' trading more costly or even impossible. Moreover, such strategic trading does not make an arbitrage opportunity diminish but widens it and destabilizes the market.

We study rational front-running activities in a decentralized illiquid market, where risk-neutral traders contact trading counterparties and bargain over the transaction price, mimicking an over-the-counter market. A large trader, whose market share significantly dominates a small trader's, faces an urgent need to unwind her position within a limited time. Detecting the distressed trader's urge to liquidate, another large trader (the arbitrageur) may find it profitable to front-run the distressed trader. For example, she can

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[1] For example, Cai (2003) finds evidence that market makers front-ran LTCM's trades when it faced binding margin constraints in 1998 by examining data of audit trail transactions.

[2] Researches blame some risk management strategies, e.g., portfolio insurance and dynamic hedging, for the 1987 stock market crash [see, Brady, Cotting, Kirby, Opel and Stein (1988), Grossman (1988), Grossman and Zhou (1996), Grossman and Zhou (1996)etc], and Value-at-Risk models [Danielsson and Zigrand (2002), Cuoco and Liu (2002), Morris and Shin (2003) and others].

sell ahead or at the same time as the distressed seller's trades, and rebuild her position later at a lower price. She realizes, however, this is not a free lunch. In a market with only a few scattered traders, an arbitrageur may not be able to succeed in executing her strategy. For example, she may not be able to sell and buy at the best times, and the cost of arbitrage can be too high to make a profit. On the other hand, in the presence of a potential arbitrageur, the distressed trader will be concerned about her ability to liquidate before the deadline and at an acceptable price. Therefore, the trading decisions of these two large traders - a distressed trader and an arbitrageur - constitute a trading game with limited market liquidity, in which both players make their choices simultaneously, taking into account not only the impact of her own trading but the other traders' trading as well.

We examine the above scenario in a discrete-time three-date model with two large traders, a distressed seller and an arbitrageur, each with an endowment of two units of an illiquid asset; and two small traders, who can at most buy two units of the illiquid asset when its price is driven down to a very low level. Trades occur upon the encounter of two traders, and prices are determined by bargaining between two counterparties. A trader's intrinsic type, which depends on her valuation of the illiquid asset, can be either high (i.e., a high valuation of the asset) or low (i.e., a low valuation), and may change over time with some probabilities. This gives rise to some uncertainty over future demand or supply of the asset. Such a setup implies two different indications of "illiquidity": limited trading opportunities in each period, and uncertainty about future trading opportunities. Traders are thus faced with the risk of not being able to find a trading counterparty in some future period. When choosing her trading strategy, a large trader, in particular, has to consider the other large trader's strategies, because their actions, along with the potential for type switching, jointly determine the current and future market liquidity.

We find that equilibrium strategies vary with type switching probabilities and bargaining power. When there is no uncertainty over future liquidity (i.e., no type switching), the distressed seller liquidates very quickly, whilst the arbitrageur does not trade at all. However, when there exists even a slight probability of type switching, the arbitrageur may want to front-run[3] and sell part of her holdings before the distressed seller. When uncertainty over future types gets larger, the incentive for the arbitrageur to front-run also gets stronger. For moderate levels of type switching rates and large traders' bargaining powers, the optimal strategy for the distressed seller is to liquidate her entire position in the first period; for the arbitrageur the optimal strategy is to front-run and sell two units in the first period as well. However, equilibrium strategies may be different under some extreme situations. For example, when uncertainty about future market liquidity is extraordinarily high, the arbitrageur still trades aggressively by selling two units in the first period, while the distressed trader may choose to spread the sale over two periods. The reason that the arbitrageur sells quickly under such an unfavourable market condition is due partly to the concern that she may also experience type switching, and hence be forced to liquidate. To the contrary, in a more stable market with improving market liquidity, e.g., the probability of type switching up is much larger

[3] Front-running is defined in related literature [e.g., Pritsker (2004), Brunnermeier and Pedersen (2005)] as an arbitrageur selling *before* the distressed seller. In our model, we do not distinguish between an arbitrageur selling before or at the same time as the distressed trader, but call both activities "front-running". The reason is that when the arbitrageur makes a trading decision in the first period, she does not observe the distressed trader's selling order. She is not even sure whether the distressed trader will trade in this period, because the distressed trader can choose to trade only in the second period. In the case that the distressed seller only trades in the second period but the arbitrageur sells in the first period, she front-runs the distressed trader.

than that of type switching down, the arbitrageur does not front-run at all because there is hardly any profit in doing so.

We would like to ask what causes such front-running behaviour. There are two factors: imperfect competition and liquidity uncertainty. The arbitrageur profits from front-running in two stages: first, she exhausts the limited market liquidity by trading quickly in the same direction as the distressed trader; then she trades with the distressed trader as a monopoly liquidity provider. Large traders can influence prices by trading strategically (i.e. large traders are not price takers). Their market power is further strengthened by the limited market liquidity (i.e., a limited number of traders)[4]. In our model, prices are generated endogenously through dynamic bargaining, modeled by the Nash bargaining solution, which reflects on the role of a trader's relative market power in asset pricing.

Infinitely elastic demands for assets are assumed in a frictionless economy. However, empirical evidence shows that this is not the case, even in the most liquid markets like NYSE and NASDAQ [Holthausen, Leftwich and Mayers(1987, 1990), Keim and Madhavan(1996), Chan and Lakonishok(1993), Lamont and Thaler(2003), Ofek and Richardson(2003), for a sample]. In particular, Coval and Stafford(2005) identify asset fire sales by mutual funds in equity markets due to financial distress (e.g., extreme capital flows). They present evidence that even in the most liquid markets, assets sometimes sell at fire sale prices. In addition, they find that forced trades are predictable, which creates an incentive and an opportunity for front-running. In an illiquid market, asset prices can no longer be characterized as an exogenous pricing process because impacts of trading on prices make demand curves "downward sloping". Thus when excess demand for an asset

[4] For example, Green, Hollifield and Schurhoff (2004) estimate measures of dealer bargaining power in the decentralized U.S. municipal bond markets and find that dealers' market power decreases in trading size.

suddenly increases, prices must be driven down. In a less liquid market with only limited number of traders or trading opportunities, the presence of arbitrageurs should have significant impacts on the price process.

One such recent example is Citigroup's euro-zone bond trades on August 2, 2004. On that date, Citigroup stunned the euro-zone government bond market by selling about Euros 11 billion of more than 200 debt instruments in less than two minutes on the EuroMTS trading platform. They only bought back Euros 4 billion of the bonds at lower prices around a half-hour later, booking a profit of Euros 17 million. They took advantage of information on trading intentions and prices displayed on trading platforms like MTS for market-making purpose and flooded the market. This cannot happen in a perfectly competitive security market.

Our work relates to several strands of research. First, our model relates to the literature of strategic trading and price manipulation [see, for instance, Allen and Gale(1992), Jarrow(1992), Vayanos(1999, 2001) and others]. We present a scenario that a large trader can generate arbitrage profits by trading strategically not because she is an informed trader, but because the market is imperfectly competitive and illiquid. Both of these two factors are indispensable to the arbitrageur's front-running. Imperfect competition is a major reason for price impacts of trading (or a "downward sloping" demand curve for an asset) in a security market [e.g., Kihlstrom(2000), Pritsker(2004), Subramanian and Jarrow(2001), Brunnermeier and Pedersen(2004)]. Pastor and Stambaugh(2003), who investigate effects of market-wide liquidity on asset prices, also focus on this aspect of illiquidity associated with temporary price fluctuations induced by order flows. We distinguish in our work illiquidity of an asset that is characterized by its downward sloping demand curve from market liquidity uncertainty, which arises from an uncertainty over future availability of liquidity providers. Both factors have effects on equilibrium asset prices and account for discounts in asset prices.

Our model is also related to a number of recent papers on pricing illiquid assets. In particular, we adopt the basic structure of the dynamic search and bargaining model in Duffie, Garleanu and Pedersen(2004a, b), extended by Vayanos and Wang(2003) and Weill(2003). Duffie, et al (2004a, b) present a search and matching model for assets that are traded over the counter and develop a theory of bid-ask spreads which reflects the dealer's bargaining power vs. the investors' other options in terms of trading opportunities. The purpose of this paper is not to examine how traders' behaviour is affected by the search and matching process. Rather, we use the setup of random switching in agents' expectations to motivate trades of a single claim to future consumption. Moreover, with a limited number of traders, this random switch in investors' expectations generates such liquidity uncertainty that a trader may not be able to find anybody to trade with if co-switching in expectations occurs to several traders in the market. It thus gives investors incentive to trade strategically. This is especially the case for large traders because their trades have a greater impact on either the demand or supply of the asset. We thus introduce large traders into a "thin" market and show how they choose trading strategies taking into account the strategic consequences of their trades.

Our model differs from the competitive search model of Duffie, et al (2004a, b). They assume a continuum of identically small traders, while we introduce large traders, which brings into the model imperfect competition, the possibility of strategic trading and hence price impacts of their trades. Moreover, unlike the steady state equilibrium characterized in their papers, we derive subgame perfect equilibria by backward induction.[5] With a small number of players, we can model the trading process easily and scrutinize the effect of trades initiated by different traders.

In terms of arbitrageurs' front-running behaviour, the research most related to ours includes Carlin, Lobo and Viswanathan(2005), Brunnermeier and Pedersen(2004), Attari, Mello and Ruckes(2004) and Pritsker(2004). Pritsker (2004) studies how the presence of large traders alters the equilibrium asset returns in a general equilibrium model. He finds evidence of front-running in simulations of distressed sales, in which asset prices overshoot (slightly) the competitive level. Attari, et al.(2004) illustrate how arbitrageurs can exploit a distressed trader depending upon that trader's financial constraints. Arbitrageurs may even lend to a distressed trader in order to reap even larger profits when the distressed trader begins to fail. Brunnermeier and Pedersen(2004) focus more on the price impacts of arbitrageurs' predatory trading as well as effects on market liquidity. In their model, large traders are restricted by exogenous holding limits so that they choose trading strategies such that no transaction costs are incurred in the equilibrium solution. Carlin, et al.(2005) study the strategic interactions among large traders in a continuous-time stage game with an explicitly assumed asset-pricing equation, taking account of both the permanent and transitory price effects of trades.

Studying a similar scenario of strategic trading with distressed sales, we approach the problem in a different way. In Attari, et al. (2004), Brunnermeier and Pedersen(2004) and Carlin, et al.(2005), trading volume surges because of arbitrage activities. Excess

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[5] We only consider pure strategies for each trader. Since the extensive form for this two-period game is already complicated enough, we do not consider mixed strategies for any trader.

demand from strategic traders can always be met immediately since there always are liquidity traders or long-term investors. On the other hand, we place the scenario in an illiquid market with a limited number of traders so that all trading activity has an impact on prices. In addition, all investors' trading strategies are derived from the first principle. That is, both the distressed seller and the arbitrageur choose the optimal strategy conditional on the current market liquidity and the expectation of future liquidity. Finally, we do not assume any functional form for prices. Instead, prices are determined endogenously in the process of utility maximization through dynamic bargaining and trading.

Our research also provides some insight into the design of trading mechanisms. Specifically, will the preannouncement of trading intention – so called "sunshine trading" – prevent front-running? Our dynamic model shows that the arbitrageur will front run under most circumstances when she observes the forced liquidation by the distressed trader. The key determinant underlying the "front-running" behaviour is the imperfect competition in this small-number-of-trader market. Since the arbitrageur's position is large enough to affect the demand and supply of market liquidity, she profits from first draining the market liquidity and then providing liquidity back to the market as the monopolist. Her incentive to do so quickly goes away as the number of small liquidity traders increases. This is in line with the discussion in Admati and Pfleiderer(1991). They argue that in a *competitive* market, front-running is very unlikely to occur because front-runners are actually providing a valuable market-making service rather than exploiting the trade announcer. Brunnermeier and Pedersen (2005) also point out that the profitability of predatory trading depends on how large the predators' initial position is relative to the liquidity traders'. In other words, if there are enough liquidity providers in the market, sunshine trading is a feasible strategy; otherwise, it will cause predatory trading.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 analyses agents' optimization problems and characterizes the large traders' optimal response functions. Equilibria are also characterized in cases where there is no type switching and there is a very small chance of type switching. Section 4 presents examples of subgame perfect equilibria of the game under different parameter values. Section 5 concludes. Proofs and derivations can be found in the Appendix.

2. The Model

We consider a discrete three-date model, resembling an over-the-counter market. Trades take place at date t_1 and t_2. There is no trade at the last date t_3, when investors realise returns on their investments. Investors can invest in two assets: one illiquid asset traded in the decentralized market paying a dividend $\bar{D}$ at t_3; and one risk-free asset with an infinitely elastic supply, e.g., a money market account, with a return $r < \bar{D}$ at the last date. The illiquid asset can only be traded at the meeting of two traders at a bilaterally agreed upon price. Short sales and borrowing are not allowed in this market.[6]

There are four risk-neutral agents in the market, each of whom is characterized by a triple set $\{\upsilon, \gamma, m\}$. Let $\upsilon \in \{h, l\}$ denote an agent's valuation for the illiquid asset that can be either high (h) or low (l). We regard υ as an agent's intrinsic type.[7] A high-type agent with $\upsilon = h$ values the asset at $\bar{D} > r$, while a low-type agent ($\upsilon = l$) only values the asset at $\bar{D} - \varepsilon \ll r$. Each agent owns γ units of the asset, $\gamma \in \{0,1,2\}$, and has m dollars in his money market account. Since there is no borrowing, $m \geq 0$.

At t_1, two agents are endowed with two units of the security each ($\gamma = 2$) but zero dollar (m=0). We call these two agents, "large traders". The other two agents do not own any unit of the security but have $m = M$ dollars in their money market accounts. We

[6] The borrowing constraint is justified in the following context: a large trader sometimes confronts difficulties in raising funds at a time of financial distress. It is also highly unlikely for a distressed trader to re-negotiate with a large number of claim holders in a short time. The short sale constraint is not crucial for our results in that short selling cannot be an optimal strategy for any of the four traders in this game. However, it may be a key condition in some other games where, for example, there exists an additional large high-type non-owner. Short selling could be a viable strategy for such an agent. Thus, the relaxation of the short sale constraint may change the outcome of that game for certain parameters. Although interesting, this effect is beyond the scope of this paper. We would like to focus on how a potential arbitrageur (i.e., a large high-type owner) reacts to distressed sales and how her strategies are affected by market conditions.

[7] Different intrinsic types can also be interpreted as investors' different discount rate as in Kijima and Uchida (2005).

assume that an agent with M dollars cannot afford to simultaneously purchase a unit from both large traders in any time period. In other words, they can only afford one unit at any bilaterally agreed prices. They are thus called "small traders".[8]

The bargaining and trading process is modeled as a two-round mechanism. In the first round, traders simultaneously engage in pre-trade bilateral negotiations. Before a trader contacts any potential counterparty, she only knows her own type and position. Once two traders start negotiation, they reveal their types, υ, and bargain over the transaction prices that associate with different trading sizes. We model the pre-trade bilateral bargaining price by the Nash bargaining solution to keep the model simple and tractable. Thus the price is given by

$$P(t) = q_s \Delta V_b(t) + q_b \Delta V_s(t)$$

where q_s and q_b are the bargaining power of the seller and the buyer, and ΔV_b and ΔV_s refer to the reservation values of the buyer and the seller respectively.

In the second round, orders are consummated. Traders thus choose their actions: how many units to trade and with whom to trade. We assume a finite action space A^i for trader i and payoff functions $u^i : A \to \mathrm{R}$. Let $a_t = \left(a_t^1, \ldots, a_t^4\right)$ be the actions that are played in this stage in period t. Additionally, $a_t^i \in \left\{-\gamma_t^i, \ldots, -1, 0, 1, \ldots, \gamma_t^i\right\}$, and is only observable to large players at the end of each period. Here, the minus sign indicates the "sell" action. A trader may choose to sell his entire holding, to sell part of the holding, not to trade, or to buy some units of the security if $m > 0$. Small traders, on the other

[8] However, they may be able to buy two units from one large trader as long as the price is low enough and their budget constraint is not violated. Note that, once a small trader buys two units of the illiquid asset, she becomes a large trader.

hand, only observe the actions that are played in the bargaining and trading they participate in. Thus, they cannot base their actions in period t on period-t histories.

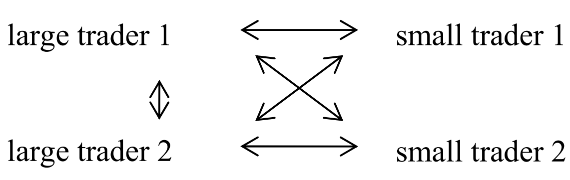
Trader i's payoff function is defined as his expected value function over the rest of the trading horizon. That is,

$$u^i = EV_t^i\left(a_t^i, a_t^{-i}\right)$$

Traders may bargain with more than one counterparty in the first round. However, engaging in bargaining does not guarantee a deal. In other words, orders in the second round may not be consistent with those in the first round. An agent will only trade at the most advantageous price. However, with this general rule, there may be a potential cycling problem that no trade will eventually take place between any two traders. Since we are interested in trading situations that are fast-paced and high pressured, we make some assumptions to eliminate the potential cycling problem and possible many-round strategic negotiations among the four players.

Assumption 1: We assume that small traders are "geographically separate". That is, small traders are unable to contact each other and can only be reached by large traders.

Therefore, negotiations and trades can only take place between two large traders and between the large traders and the small traders.



Assumption 2: Pre-trade bargaining between any two traders consists of a single round.

Once they reach an agreement on a trading size and a transaction price in a pre-trade bargain, they decide whether to trade or not with their counterparty and at this price. If they decide not to trade at this time with this counterparty, they lose the chance of trading with each other in this period. This captures the idea of a one-shot over-the-counter negotiation.

Note that changes to these two assumptions may alter the bargaining outcome and hence the general results. We realize that the bargaining/trading mechanism may not be an efficient one. However, even with four traders and two periods, the number of subgames becomes very large. To facilitate the derivation and analysis, we make these assumptions to simplify the trading outcomes.

With these two assumptions, large traders have trading advantages over small traders in that: (1) their information set in the stage of pre-trade bargaining is bigger; and (2) they have more contacts than small traders. Thus, large traders behave strategically in choosing their actions in the process of bargaining and trading in that they take account of all past actions and all contingent payoffs they may get in future periods (i.e., they maximize their expected payoff at time *t*). Small traders, on the other hand, are myopic in that they will trade with any other traders as long as their expected payoffs of trading are non-negative.

Moreover, an investor's intrinsic type (υ) evolves over the course of trading due to the exogenous probabilities of type switching. The transition function is defined as $\rho\left(\upsilon^{t+1}\middle|\upsilon^{t}\right)$. Let $\rho\left(l^{t+1}\middle|h^{t}\right)=\rho_{d}$ be the probability of switching rate from the "high-type" to the "low-type", and $\rho\left(h^{t+1}\middle|l^{t}\right)=\rho_{u}$ the opposite switching rate from the "low-type" to the "high-type". The type switching probabilities are public information. In this work, we only consider the case where all traders face the same probabilities of type switching.[9] We may think of this as a market-wide effect and thus can occur to every trader in the market. For simplicity and tractability, we assume that type migration processes are pairwise independent. Without loss of generality, we also assume that the total probability of type switching up and down is less than 1, that is, $\rho_d + \rho_u < 1$. This assumption ensures that trading between a high-type investor and a low-type investor is profitable.

In order to study the particular case of a distressed sale by a large trader, we impose some restrictions on a large trader's type switching probabilities.

Assumption 3: We assume that a *large* high-type owner, having once suffered a type switch from "high" to "low", cannot switch back to the high type until she liquidates the long position.

This assumption tries to capture situations of urgent liquidations, such as margin calls from brokers or hedging needs arising from violent market moves. With this assumption, a large distressed seller (i.e., a large low-type owner of the illiquid asset) seeks to trade aggressively to avoid the extra low liquidation value at the last date (i.e., $\bar{D} - \varepsilon \ll r$).

The market can be formalized as a stochastic dynamic game. It starts with nature first choosing the intrinsic type for each investor. Hence there are many possible scenarios.[10] We would like to study the price impact of an arbitrageur's strategic trading on a

[9] We can also consider the case where traders face heterogeneous type switching rates. For example, A does not switch type but everybody else does. This should have two effects on every trader's value function: A's reservation value of no trade increases; and the chance of trading in the second period also increases. A would front-run if and only if her profits from trading in the second period would compensate for the price discount she gives up in the first period. To avoid repetition, we do not analyze this case in this paper.

[10] Obviously, some cases do not provide trading opportunities and hence are trivial. For example, two large traders are both high-type owners and two small traders are both low-type non-owners. To avoid repetition, we will not study every single game in detail.

distressed large trader's trading behaviour and asset prices. We therefore only focus on the following scenario. At t_1, one of the two large traders is of low-type and will be referred to as *D*, the ***D***istressed large trader. The other large trader is a high-type owner, referred to as *A* (***A***rbitrageur). Each of them is endowed with two units of the illiquid asset. Two small traders (*S*), who are identical and anonymous at t_1, are both high-type non-owners with endowments of *M* dollars to buy one unit each.

Figure 1 summarizes the sequence of events. At the beginning of each trading date, each agent recognizes her own type and endowment. Agents who decide to trade in this period simultaneously contact some other agents. (Note that, in this small market, large traders always know where to find other traders so that there is no search process involved in this game.) When two agents meet, they reveal their types and demands and start pre-trade bargaining over the transaction price. An agent may contact and bargain with more than one trader but trade at the most advantageous price. If the two parties engaged in bargaining reach an agreement, a transaction occurs. If an agent does not meet any trading counterparties or reach an agreement at a trading date, she has to wait until the next trading date to resume trading. Note that a trader's intrinsic type is subject to change over time. Thus at the start of the next trading date in sequence, agents learn their new types, and trade if necessary.

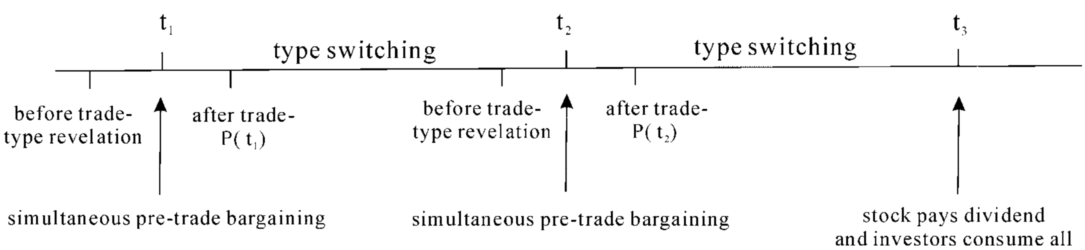


Figure 1. Timing of Type Switching and Trading

Here, we make some further assumption on players' bargaining power.

Assumption 4: We assume that a trader's relative bargaining power is constant over time but different across various parings.[11]

We denote the relative bargaining power of *A* and *D* with respect to small traders as q_A and q_D. The two small traders (*S*) are initially identical: they each have bargaining power equal to 1/2. The relative bargaining power of *A* to *D* is $q_A/(q_A+q_D)$. Since *A* is the only high type large trader with no need to trade, we assume $1>q_A>1/2$. We assume *D*'s bargaining power is less than *A*'s but can be either greater or less than a small trader's, i.e., $1>q_A>q_D>0$.

In the process of type switching and trading, information is symmetric in that 1) the type switching probabilities are public information and 2) No trader can hide his identity when he enters a negotiation. Thus, it is easy for *A* to find out *D*'s distressed situation as soon as they start a negotiation at the first trading date. Small traders, however, are not aware of *D*'s position because they are separated from each other and have limited ability to infer from previous actions. Therefore, only *A*, the arbitrageur has information and incentive to trade against the distressed trader. Small traders can thus be treated as liquidity providers in this game.

2.1 Definition of Illiquidity

[11] The notion of bargaining power describes the fraction of the joint surplus of trading that one party obtains in bargaining over the transaction price. A player's bargaining power is a measure of her control over bargaining and is determined by various factors. It reflects a player's discount rate relative to her opponent's, her position, trade size, outside options, as well as degree of investor sophistication. Thus, it partly captures the idea of "market power", which in this model reflects how many units of the security a trader owns. Although it is natural to conjecture that a player's relative bargaining power varies with time and holdings, we assume the relative bargaining power of the four traders is constant over time for simplicity and tractability.

The dynamic model in this work differs from the search model in Duffie, et al. (2004a, b) in two respects. First, they assume a continuum of traders while we only have four traders in our model. Second, agents in their model are identical except for their types, while in our model agents are heterogeneous in both their types and initial endowments. These differences give rise to two indicators of "illiquidity". Trading opportunities are restricted by the limited number of traders in the market. Thus, the market is "illiquid" because of the reduction in trading counterparties. We regard this illiquidity as exogenous. As well, the impact of a type shift by any agent is a significant change in the potential volume that can be bought or sold. Therefore, the possibility of type switching generates further uncertainty over future trading opportunities. Traders are thus reluctant to trade in early periods because they fear that their type change in the future and they may not be able to trade at all. This reduction in trades can be regarded as "endogenous" illiquidity. When choosing trading strategies, large traders explicitly take into account both "exogenous" and "endogenous" illiquidity, i.e., the current number of counterparties and the potential that counterparties can be found in future periods.

Since there is only one asset traded in the market, we use the term "market liquidity" to refer to tradability of this asset. In a market with a small number of traders, who are also likely to incur demand shocks, there is always a chance that a trader cannot find a trading counterparty. We name this phenomenon "liquidity uncertainty". There is no such uncertainty in Duffie, et al.'s model because type switching has no effect on the aggregate number of buyers and sellers as there are an infinite number of agents in the market. Also, as shown in Liang and Milne (2005), if there are n traders but very few trading periods, or t trading periods but only a few traders, (n and t are allowed to go to infinity), the market is always liquid in the sense that a trader can liquidate her position at any speed or at any time she wants. Thus, neither of the two attributes of "illiquidity" – a small number of traders and the possibility of type switching – is dispensable for the existence of "liquidity uncertainty". If, on the one hand, the type switching probabilities go to zero, there is no uncertainty over the future counterparties and thus no liquidity risk. In this case, the trading strategies and prices are determined only by the four traders' positions and types at the beginning of the game. On the other hand, as the number of traders increases (to infinity, in the limit) a purely competitive market emerges, and again there is no liquidity uncertainty.

In the next section we analyze the traders' trading problem and characterize large traders' optimal strategies.

3. Front-running and Price Manipulation

3.1 Agents' Optimization Problem

The purpose of this paper is to determine both A and D's equilibrium strategies, and the resulting price functions. We consider only pure strategies for all traders. Figure 2 demonstrates all possible trading actions that can be taken by both large traders in the first period. $a_{t_1}^i \in \{0,-1,-2\}$ for $i = A, D$. For A and D choosing strategies simultaneously, the complete action set for either of them is greatly extended. Define $\left(a_{t_1}^A, a_{t_1}^D\right)$ as a pair of actions for A and D at t_1. $\left(a_{t_1}^A, a_{t_1}^D\right) \in \{(-1,0)$, $(-1,-1)$, $(-1,-2)$, $(-2,0)$, $(-2,-1)$, $(-2,-2)$, $(0,0)$, $(0,-1)$, $(0,-2)\}$. The elements of this set correspond to type configurations I through IX as shown in Figure 2. If A does not trade in the first period, the game then degenerates to the one which is similar to the game studied in Liang and Milne(2005), in which there is only one distressed large seller.

When a large trader chooses a trading action at t_1, she must bear in mind that her choice alone cannot determine which subgame she will be in at t_1 and t_2. When she makes a trading decision at t_1, she not only has to weigh the trade-off between the price discount caused by a quick sale and the possibility of being unable to trade in a later period due to type switching, but must also consider the impact of the other large trader's trading on prices and the future liquidity. Thus, a large trader really has to consider three factors when making a trading decision: (1) the impact of her trade on the other large

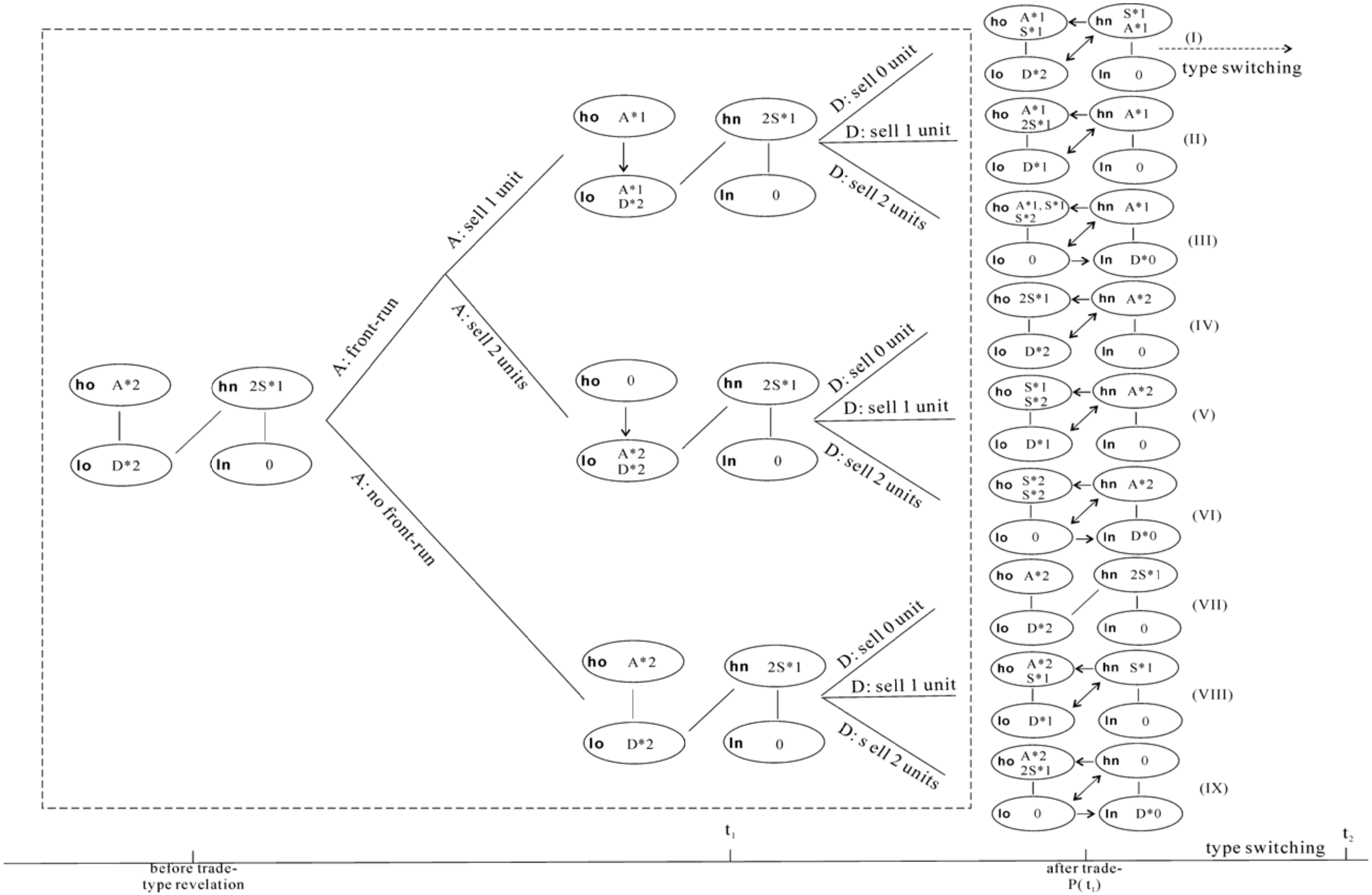


Figure 2. The dynamics of the trading and type evolution in the first period

trader, and vice versa; (2) the joint effects of their trades on prices and future market liquidity; and (3) the changes in future liquidity due to the exogenous type switching.

In this three-date game, agents invest in either an illiquid asset or a liquid asset to maximize their payoff at t_3. That is, an agent i maximizes her expected value by choosing her trading strategies at t_1 and t_2.

$$\max_{\psi_{t_1},\psi_{t_2}} \quad EV_{t_3}^{i}$$

where ψ_{t_1} and ψ_{t_2} are pure strategy profiles at t_1 and t_2, $\psi_t = \left(\psi_t^i, \psi_t^{-i}\right)$. We solve for the subgame perfect equilibrium by backward induction.

We now define the equilibrium outcome to this dynamic trading game.

Definition: *An outcome profile consists of a trading strategy profile and associated transaction prices* $\left(\psi(t), P(t)\right)$. *An equilibrium outcome profile* $\left(\psi(t)^*, P(t)^*\right)$ *is an outcome profile such that for a particular trader configuration at each time, and given split-the-difference negotiations, a large trader cannot improve her expected payoff by adopting any other strategy profile* $\left(\psi(t)', P(t)'\right)$, *and no small trader can improve her expected payoff in a pairwise negotiation with large traders.*

We define "front-running" in this paper as the *ho*-trader *A* sells ahead or at the same time as *D*'s trade. Notice first that "front-running" will not arise in models such as Duffie, et al.(2004a) and Kijima and Uchida(2005), in which a continuum of investors who can only own one unit is assumed. In equilibrium, trades can only occur in those models between low valuation asset owners and high valuation non-owners. Other types of investors, i.e., high valuation owners and low valuation non-owners will have no incentive to trade. Kijima and Uchida (2005) also consider the case where some high valuation owners sell to get instant cash. In contrast, the "front-running" activity of the high-type owner A in our model, if it occurs, is rational. Her incentive to participate in the game comes from either limited market liquidity or the possibility of price manipulation due to her dominant position and bargaining power (or both). She also needs to determine the timing and extent of front-running. The following lemma delineates the timing of front-running.

Lemma 1: *Given that A does not switch type between t_1 and t_2, if $a_{t_1}^A = 0$, then $a_{t_2}^A = 0$.*

This lemma states that if A does not front-run in the first period, she will not sell in the second period either. The game is then simplified by Lemma 1. If A does not trade at t_1, D behaves as a monopoly seller much like in Liang and Milne (2005), except that now, with a probability of ρ_d, D has to compete with A_{lo} (i.e., A incurs type switching) in the second period for selling to a small buyer.

3.2 Large Traders' Equilibrium Strategies

The large traders' optimal strategies result from comparing the expected payoffs of all possible actions. When choosing a strategy at a trading date, a large trader considers two effects of her choice: (1) what bargaining situations she may be involved in at that date, and in turn, what price she may receive for the action she chooses to play; (2) given her choice today, what bargaining situations are possible at the next date. If there is no type switching, the second effect goes away. It is natural, then, to ask how big the effect of type switching is on a trader's strategy. In particular, does the high type owner A still have an incentive to participate in trading if there is no such uncertainty over future types? The following proposition characterizes the equilibrium in a benchmark case with vanishing type switching rates.

Proposition 1: *When* $\rho_d \to 0$ *and* $\rho_u \to 0$*,*

(i) $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^* = \left(0, -2\right)$. *That is, D sells two units and A never trades in the first period. The equilibrium does not depend on A and D's bargaining power,* q_A *and* q_D*;*

(ii) D's trades have no impact on prices, i.e., she sells one unit or two units in the first period at the same price when A does not trade. That price is:

$$P = \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\left(1 - q_D\right)^2$$

When there is no risk of type switching, *D* always sells two units at t_1 no matter what *A*'s strategy is. On the other hand, *A* will sell two units if *D* sells one unit or does not trade, but switch to a "no trade" strategy if *D* liquidates her whole position in the first period. This is because, when *D* sells two units at t_1, *A* has to sell at a lower price without any chance to buy back even one unit in the second period. Thus, if there is no uncertainty over future trading opportunities, *A* never finds a forced liquidation to be a profitable arbitrage opportunity and *D* can exploit her monopoly power over small traders and sell as quickly as she wants.

We next consider the game with both type switching rates being positive, but very small. We ignore second and higher order terms that contain ρ_d or ρ_u in traders' value functions. This approximation is appropriate because for very small type switching rates (e.g., $\rho_d = \rho_u = 0.01$) the chance that more than one agent undergoes type switching is close to zero ($\rho_d^2 = 0.0001$). Thus, we may just focus on situations where only one trader experiences type switching.

Proposition 2: *(i) When both* ρ_d *and* ρ_u *are strictly positive but very small, A has an incentive to front-run D's trades. Specifically, when* $\rho_d \geq \underline{\rho}_d\left(q_A,q_D\right)$ *,* $\left(\psi_{t_1}^A,\psi_{t_1}^D\right)^* = \left(-1,-2\right)$*; when* $\rho_d < \underline{\rho}_d\left(q_A,q_D\right)$*,* $\left(\psi_{t_1}^A,\psi_{t_1}^D\right)^* = \left(0,-2\right)$*.* $\underline{\rho}_d\left(q_A,q_D\right)$ *is given by*

$$\underline{\rho}_d\left(q_A,q_D\right) = \frac{\left(1-q_D\right)^2\left(2-q_D\right)}{\left(1-q_A\right)\left(2-q_D\right)+2\left(1-q_D\right)\left(3-3q_D+q_D^2\right)}$$

(ii) A would front-run and sell one unit for a smaller type switching rate ρ_d*, ceteris paribus, if: her bargaining power increases; D's bargaining power increases; A's bargaining power decreases while D's increases; or both A and D's bargaining power increases. That is,*

$$\frac{\partial\underline{\rho}_d\left(q_A,q_D\right)}{\partial q_A} < 0,$$

$$\frac{\partial\underline{\rho}_d\left(q_A,q_D\right)}{\partial q_D} < 0,$$

$$\frac{\partial^2\underline{\rho}_d\left(q_A,q_D\right)}{\partial q_A\partial q_D} < 0.$$

When the type switching rate ρ_d is below some level, *D* sells two units and *A* does not trade in the first period. When ρ_d increases above that level, *D* sells two units but *A* sells one unit. For example, even if the chance of switching from high type to low type (i.e., ρ_d), is as small as 1%, we can still find some values for q_A and q_D (e.g., $q_A = 0.87$, $q_D = 0.86$) such that in equilibrium, *A* partly front-runs (i.e., sells one unit) while *D* liquidates quickly (sells two units) at t_1.

Ignoring higher order terms of ρ_d and ρ_u is equivalent to reducing traders' opportunity set in the second period (although most of these trading opportunities are associated with very small probabilities). Next, we analyze large traders' equilibrium strategies considering every possible bargaining and trading situation that may arise in the second period.

We first characterize each large trader's optimal response function to the other large trader's action. Take *D*'s response to *A*'s actions at t_1 for example. Given *A*'s action, *D* compares her value functions for all possible responses. The optimal response is the strategy that rewards her with the highest expected payoff. We summarize the two large traders' optimal responses at t_1 in the following lemma.

Lemma 2: *For* $1 > q_A > q_D > 0$, $\rho_u + \rho_d < 1$ *and* $\rho_{u/d} \in [0,1]$,

(i) $V_{t_1}^A\left(a_{t_1}^A = -1, a_{t_1}^D = 0\right) > V_{t_1}^A\left(a_{t_1}^A = 0, a_{t_1}^D = 0\right)$,

$V_{t_1}^A\left(a_{t_1}^A = -1, a_{t_1}^D = -1\right) > V_{t_1}^A\left(a_{t_1}^A = 0, a_{t_1}^D = -1\right)$;

(ii) $V_{t_1}^D\left(a_{t_1}^D = -1, a_{t_1}^A = 0\right) > V_{t_1}^A\left(a_{t_1}^D = 0, a_{t_1}^A = 0\right)$,

$V_{t_1}^D\left(a_{t_1}^D = -2, a_{t_1}^A = -1\right) > V_{t_1}^A\left(a_{t_1}^D = -1, a_{t_1}^A = -1\right)$,

$V_{t_1}^D\left(a_{t_1}^D = -1, a_{t_1}^A = -2\right) > V_{t_1}^A\left(a_{t_1}^D = 0, a_{t_1}^A = -2\right)$.

Under the most general conditions of the four parameters, if *D* does not sell or only sells one unit, "do not trade" is a strictly dominated strategy for *A*; if *D* sells two units, *A* may choose any of the three strategies. Similarly, if *A* does not front-run, "do not trade" is a strictly dominated strategy for *D*; if *A* front-runs and sells one unit, "sell one unit" is a strictly dominated strategy for *D*; if *A* sells two units, *D* never chooses the strategy of "no trade".

It is difficult to fully characterize properties of equilibrium strategies for both *A* and *D* with four variable parameters: ρ_d, ρ_u, q_D and q_A. Since large traders' response functions contain higher order non-linear terms of the four parameters, we can hardly derive explicit conditions for certain equilibria. We thus provide, in the next section, numerical examples to demonstrate how the equilibrium may vary with the four parameters, and how sensitive it is to slight changes in parameter values. We also consider how to interpret trends we observe from these examples.

4. Market Conditions, Bargaining Power and Equilibrium Strategies

Large traders choose optimal strategies at t_1, trading off the impact of their trades on prices against future trading opportunities. Equilibrium strategies are thus functions of four parameters: ρ_d, ρ_u, q_A and q_D, where $\rho_d + \rho_u < 1$ and $0 < q_D < q_A < 1$. Since trading strategies for a large trader are discrete (i.e., "do not trade", "sell one" or "sell two"), a change in any one parameter may cause equilibria to switch from one to another. This is clearly demonstrated in Table 1: only one parameter is changed in each panel so that it is easy to observe trends in equilibria that vary with that parameter.

In panel 1, equilibrium strategies for *A* and *D* switch from "do not trade" and "sell two units", respectively, to "sell two units" for both agents as q_D increases from 0.1 to 0.95 and q_A, ρ_d and ρ_u are set at 0.96, 0.1 and 0.1. While *D* always trades two units in the first period, *A* switches from "do not trade" to "sell two units" as q_D gets larger. This is partly because an increase in *D*'s bargaining power makes her less likely to win any second period competition with *A* to trade with a small trader. However, even if *D* loses such competitions in some subgames, she may gain more in some others due to greater bargaining power. Thus, *A* will front run as long as her expected payoff of front-running is higher than not doing so.

Next, we fix q_D, ρ_d and ρ_u, and let q_A increase (panel 2). *A*'s equilibrium strategy again switches from "do not trade" to "sell two units", while *D* always chooses to sell two units at t_1. When q_A gets bigger, *A* may lose trading opportunities in some subgames, but she gains more from bargaining in others.

Panel 1: q_D changes

$\rho_d = 0.1$, $\rho_u = 0.1$, $q_A = 0.96$

q_D	Equilibria $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^*$
0.1	(0,-2)
0.15	(0,-2)
0.2	(0,-2)
0.25	(0,-2)
0.3	(0,-2)
0.35	(0,-2)
0.4	(0,-2)
0.45	(-2,-2)
0.5	(-2,-2)
0.55	(-2,-2)
0.6	(-2,-2)
0.65	(-2,-2)
0.7	(-2,-2)
0.75	(-2,-2)
0.8	(-2,-2)
0.85	(-2,-2)
0.9	(-2,-2)
0.95	(-2,-2)

Panel 2: q_A changes

$\rho_d = 0.1$, $\rho_u = 0.1$, $q_D = 0.45$

q_A	Equilibria $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^*$
0.51	(0,-2)
0.535	(0,-2)
0.56	(0,-2)
0.585	(0,-2)
0.61	(0,-2)
0.635	(0,-2)
0.66	(0,-2)
0.685	(0,-2)
0.71	(0,-2)
0.735	(0,-2)
0.76	(0,-2)
0.785	(0,-2)
0.81	(-2,-2)
0.835	(-2,-2)
0.86	(-2,-2)
0.885	(-2,-2)
0.91	(-2,-2)
0.935	(-2,-2)

Panel 3: ρ_d changes

$\rho_u = 0.1$, $q_D = 0.55$, $q_A = 0.6$

ρ_d	Equilibria $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^*$
0	(0,-2)
0.05	(0,-2)
0.1	(-2,-2)
0.15	(-2,-2)
0.2	(-2,-2)
0.25	(-2,-2)
0.3	(-2,-2)
0.35	(-2,-2)
0.4	(-2,-2)
0.45	(-2,-2)
0.5	(-2,-1)
0.55	(-2,-1)
0.6	(-2,-1)
0.65	(-2,-1)
0.7	(-2,-1)
0.75	(-2,-1)
0.8	(-2,-1)
0.85	(-2,-1)

Panel 4: ρ_u changes

$\rho_d = 0.1$, $q_D = 0.55$, $q_A = 0.6$

ρ_u	Equilibria $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^*$
0	(-2,-2)
0.05	(-2,-2)
0.1	(-2,-2)
0.15	(-2,-2)
0.2	(-2,-2)
0.25	(-2,-2)
0.3	(0,-2)
0.35	(0,-2)
0.4	(0,-2)
0.45	(0,-2)
0.5	(0,-2)
0.55	(0,-2)
0.6	(0,-2)
0.65	(0,-2)
0.7	(0,-2)
0.75	(0,-2)
0.8	(0,-2)
0.85	(0,-2)

Table 1. Equilibrium Strategies and Model Parameters

Panels 3 and 4 show the equilibria when type switching probabilities are altered. In panel 3, we set ρ_u, q_D, q_A at 0.1, 0.55 and 0.6 respectively, and let ρ_d vary between 0 and 0.85. (ρ_d cannot take a value over 0.9 because $\rho_u = 0.1$ and $\rho_d + \rho_u < 1$.) Equilibrium strategies for *A* and *D* change three times as ρ_d gets larger. When ρ_d is very small (e.g., $\rho_d = 0$ and 0.05), *A* does not front-run and *D* sells two units at t_1. This coincides with Propositions 1 and 2 for zero and positive but very small ρ_d and ρ_u. *A* starts to front-run while *D* keeps selling two units as ρ_d gets larger (e.g., $\rho_d = 0.1$). Interestingly, when ρ_d increases to a relatively large value (e.g., $\rho_d = 0.5$), *A* still sells two units but *D* switches to sell only one unit in the first period. *D* would rather take the chance of not being able to trade the leftover unit in the second period than compete with *A* to sell two units (at a big discount) in the first period.

Panel 4 presents equilibrium strategies for *A* and *D* as ρ_u increases ($\rho_d = 0.1$, $q_D = 0.55$, $q_A = 0.6$). We may infer that when ρ_u gets larger, *A* loses the incentive to front-run. This makes sense because for a large ρ_u and a small ρ_d, *D* is able to make a sale more easily, and at a better price, since small buyers' expected payoff from the strategy "buy-and-hold-for-two-periods" is higher. On the other hand, if *A* front-runs in the first period, she may not be able to buy back units in the second period, or if she can then only by paying a higher price due to *D*'s competition.

We note that the way in which the equilibrium changes with one parameter also depends on the value of the other three parameters. Examples in Table 2 give us some sense of this complexity. Within each panel, ρ_u is the only parameter that changes; across the three panels ρ_d changes to show how the trend in each panel varies with ρ_d.

Panel 1

ρ_d **= 0.05**, q_D = 0.55, q_A = 0.6

ρ_u	Equilibria $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^*$
0	(0,-2)
0.01	(0,-2)
0.02	(0,-2)
0.03	(0,-2)
0.04	(0,-2)
0.05	(0,-2)
0.06	(0,-2)
0.07	(0,-2)
0.08	(0,-2)
0.09	(0,-2)
0.1	(0,-2)
0.11	(0,-2)
0.12	(0,-2)
0.13	(0,-2)
0.14	(0,-2)
0.15	(0,-2)
0.16	(0,-2)
0.2	(0,-2)
0.3	(0,-2)
0.4	(0,-2)
0.5	(0,-2)
0.6	(0,-2)
0.7	(0,-2)
0.8	(0,-2)
0.9	(0,-2)

Panel 2

ρ_d **= 0.1**, q_D = 0.55, q_A = 0.6

ρ_u	Equilibria $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^*$
0	(-2,-2)
0.01	(-2,-2)
0.02	(-2,-2)
0.03	(-2,-2)
0.04	(-2,-2)
0.05	(-2,-2)
0.06	(-2,-2)
0.07	(-2,-2)
0.08	(-2,-2)
0.09	(-2,-2)
0.1	(-2,-2)
0.11	(-2,-2)
0.12	(-2,-2)
0.13	(-2,-2)
0.14	(-2,-2)
0.15	(-2,-2)
0.16	(-2,-2)
0.2	(-2,-2)
0.3	(0,-2)
0.4	(0,-2)
0.5	(0,-2)
0.6	(0,-2)
0.7	(0,-2)
0.8	(0,-2)

Panel 3

ρ_d **= 0.83**, q_D = 0.55, q_A = 0.6

ρ_u	Equilibria $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^*$
0	(-2,-2)
0.01	(-2,-2)
0.02	(-1,-2)
0.03	(-2,-1)
0.04	(-2,-1)
0.05	(-2,-1)
0.06	(-2,-1)
0.07	(-2,-1)
0.08	(-2,-1)
0.09	(-2,-1)
0.1	(-2,-1)
0.11	(-2,-1)
0.12	(-2,-1)
0.13	(-2,-1)
0.14	(-2,-1)
0.15	(-2,-1)&(0,-2)
0.16	(-2,-1)&(0,-2)

Table 2. Equilibrium Strategies with Different Type Switching Rates

In general, A is more inclined to front-run as ρ_d gets larger while ρ_u gets smaller. Consequently, trading volume increases as A starts to front-run.

All the different equilibria arising in the above numerical examples are compiled in Table 3. These examples are selected to represent different conditions for market liquidity. We choose the same moderate bargaining powers for these examples (q_D = 0.55, q_A = 0.6).

	ρ_d	ρ_u	q_D	q_A	Equilibria $\left(\psi_{t_1}^A, \psi_{t_1}^D\right)^*$
1	0.05	0.1	0.55	0.6	(0,-2)
2	0.05	0.6	0.55	0.6	(0,-2)
3	0.1	0.1	0.55	0.6	(-2,-2)
4	0.83	0.02	0.55	0.6	(-1,-2)
5	0.83	0.1	0.55	0.6	(-2,-1)
6	0.83	0.15	0.55	0.6	(-2,-1) & (0,-2)

Table 3. Examples of Different Equilibria

Example 1 represents relatively stable market liquidity because both type switching probabilities are very small. The arbitrageur does not front-run because there is almost no uncertainty over future liquidity. Example 2 demonstrates another interesting situation where the probability of type switching from low to high, ρ_u, is much greater than the opposite switching probability, ρ_d, which implies improving conditions for market liquidity. *A* does not have an incentive to front-run in this case either because, as there are more liquidity providers (i.e., high type non-owners) in the second period, it is less likely that *A* will profit from such a strategy. On the other hand, under both conditions, the distressed trader trades quickly without driving the price down too much.

When ρ_d and ρ_u are large enough that the probability of co-switching is non-negligible (example 3), *A* finds it profitable to front-run and sell two units in the first period while *D* will also liquidate her whole position in the first period.

Examples 4, 5 and 6 illustrate cases where ρ_d becomes much larger than ρ_u. These can be thought of as scenarios where market liquidity dries up quickly. Examples 4 and 5 imitate a highly volatile situation where small traders are reluctant to trade due to the high probability of type switching down. Under such conditions, the arbitrageur still trades aggressively (i.e., sells one unit in example 4 and two units in example 5) due to concern that type switching may occur to her in the second period. Example 5 shows that *D* may have to offload her position slowly because of *A*'s competition. In example 6, there are two equilibria and we cannot easily eliminate either of them. These three examples also suggest that *A*'s strategy can become very sensitive to market conditions for future liquidity when ρ_d is very large.

We also find evidence of price depression when the arbitrageur, *A*, front-runs aggressively in the first period. Comparing prices at which *D* sells two units in the first period when *A* front-runs (-2,-2) against prices when *A* does not front-run (0,-2), we find that for most parameter values, *D* receives a lower price selling two units when *A* front-runs at t_1. This is not surprising because *D* has to sell two units to one small buyer if *A* front-runs, whereas she can sell one unit to each of the small buyers otherwise.

Numerical examples suggest that the distressed large trader's strategy is affected by the existence of the arbitrageur. Some of the examples show that *D* may be forced to spread sales over two periods to avoid the harsh competition with *A* in the first period under certain market conditions (e.g., example 5 in Table 3). When selling two units in the first period, the arbitrageur's trading affects the distressed seller in two ways. First, she sells in the first period to exhaust market liquidity; second, she then becomes the monopoly buyer in the market and can further exploit the distressed seller by driving down prices in the second period. *D* cannot trade as quickly as she wants to because of *A*'s predatory trading. Clearly, this presents a serious risk to a large investor holding illiquid assets, and must be considered as part of her risk management strategy.

In summary, our model shows that time-varying market liquidity does explain arbitrageurs' trading behavior. When there is no risk of type switching, *A* would never trade. However, when there is a positive probability of type switching, even if very small, it may be in *A*'s interest to trade in the first period. Extra volume arises because there is uncertainty over future liquidity. The arbitrageur trades to take advantage of the distress of other traders, or for his own benefit (to avoid experiencing type switching himself), or possibly for both reasons. The way the arbitrageur trades, however, depends on the expectation of future liquidity. During times when liquidity dries up quickly, the arbitrageur trades aggressively, which provides evidence of a "flight to liquidity". During times of improving liquidity, the arbitrageur retreats from trading. This also implies that distressed sales during such times do not create arbitrage opportunities.

5. Conclusions

We study an arbitrageur's front-running activities in an illiquid market. The model includes two large traders who must trade strategically in a decentralized market with limited liquidity providers. The forced liquidation of a large distressed trader provides a profitable trading opportunity to the other large trader, motivating her to front-run the distressed trader's trades. Equilibrium strategies for the large traders may vary with market conditions. For a majority of market conditions, the arbitrageur front-runs by quickly selling her entire position and then later rebuilds her holdings. The distressed trader, on the other hand, also liquidates her whole position quickly, despite severe price depression. In some extreme situations, such as when the likelihood of type switching from high to low is much greater than that for the reverse, the equilibrium strategy for the arbitrageur is still to fully front-run, but the distressed trader may choose to spread sales over two periods. There can also be multiple equilibria for this game, conditional on model parameters such as type switching probabilities and bargaining power.

The large traders' strategies are jointly affected by imperfect competition and uncertainty over market liquidity, which thereby determine the endogenous price functions. For the distressed trader, these two factors result in liquidity discounts embodied in prices. For the arbitrageur, imperfect competition and liquidity uncertainty allow her to effectively drain the market of liquidity and later become the only liquidity provider. Her ability to move prices depends on both the number of small traders in the market and the uncertainty over traders' potential types. Through examples, we have shown when there is little uncertainty over market liquidity, or when market liquidity improves over time, the arbitrageur does not front-run.

Now we understand how large traders choose their strategies by working out the subgame perfect equilibrium of this two-period model. With the same framework, we can also study how a short-sale constraint affects an arbitrageur's decision on front-running, by letting the arbitrageur be a high type non-owner and able to borrow from small traders. The arbitrageur can either provide liquidity to the distressed seller directly, or become a competitor for liquidity by borrowing the asset from small high-type owners and then selling it in the market. We conjecture that borrowing costs have a crucial effect on the arbitrageur's trading strategy, since the risks of being unable to buy back the illiquid asset (which is mandatory) may require such a very high premium that front-running is no longer profitable.

Appendix A

Proof of Lemma 1:

Consider the case that A does not switch type between t_1 and t_2. If A does not sell while D sells 2 units in the first period, A has no incentive to sell in the second period, because even if she can find a buyer (i.e., a high-type non-owner) in the market, she is no better off by selling the asset than holding it until t_3. If A does not sell and D sells one unit in the first period, A then has to compete with D in the second period if she wants to sell one unit to the remaining small buyer. Since $q_A > q_D$, A's bargaining price with a small buyer, S_{hn}, is always greater than D's. A small buyer hence always buys from the distressed seller D_{lo}. Finally consider the case that both A and D do not sell at t_1, and no one undergoes type switching between t_1 and t_2. If D does not sell at t_2, A sells to a small buyer one or two units at her reservation value (note that she is a high-type owner), i.e., $\left(\bar{D} - \rho_d \varepsilon\right)/r$, which is exactly the same payoff as her expected payoff to holding the asset until t_3. If D sells at t_2, A's expected payoff to selling any unit will be no more than that of holding the asset. Therefore, A has no incentive to sell at t_2 if she does not sell at t_1. In sum, if A, still being the high type by t_2, tries to trade at in the second peroid, her payoff of selling any unit would be less than or equal to the payoff of holding it until t_3. Therefore, she will not sell in the second period unless her type switches. □

Proof of Lemma 2:

An agent i's optimization problem is to maximize her expected value by choosing her trading strategies at t_1 and t_2.

$$\max_{\psi_{t_1}, \psi_{t_2}} \quad EV_{t_3}^{i}$$

where ψ_{t_1} and ψ_{t_2} are pure strategy profiles at t_1 and t_2, $\psi_t = \left(\psi_t^i, \psi_t^{-i}\right)$. We first briefly describe how we solve for the subgame perfect equilibrium.

The dynamic game is solved by standard backward induction. We first calculate each trader's payoff at the last date t_3, $X_{t_3}^i\left(\Gamma_{t_3}\right)$. For a subgame at t_2, we calculate a trader's expected payoff to any feasible trading action $a_{t_2}^i$, i.e., $E_{t_2}\left[X_{t_3}^i\left(\Gamma_{t_3}\left|a_{t_2}^i\right.\right)\right]$. Comparing a trader's expected utility gains across her action set, we determine her optimal strategy and value function for this subgame at t_2.

$$V_{t_2}^i\left(\Gamma_{t_2}\right) = \max_{\psi_{t_2}} E\left[X_{t_3}^i\left(\Gamma_{t_3}\left|\left(\psi_{t_2}^i, \psi_{t_2}^{-i}\right)\right.\right)\right]$$

where Γ_t denotes a subgame at t.

Moving back to the first period, we repeat the above steps and determine each trader's value function for every feasible trading action at t_1, conditional on the other traders' trading activities. That is,

$$V_{t_1}^i\left(a_{t_1}^i, \Gamma_{t_1}\left|a_{t_1}^{-i}\right.\right) = E\left[V_{t_2}^i\left(\Gamma_{t_2}\left|\left(a_{t_1}^i, a_{t_1}^{-i}\right)\right.\right)\right]$$

This is trader i's expected value for actions $\left(a_{t_1}^i, a_{t_1}^{-i}\right)$ taken at t_1. An optimal strategy for trader i at t_1 is the strategy that maximizes $V_{t_1}^i\left(\psi_{t_1}^i, \Gamma_{t_1}\left|\psi_{t_1}^{-i}\right.\right)$ given other traders' strategies $\psi_{t_1}^{-i}$.

Next, we characterize a large trader' optimal response function to the other large trader's strategy.

***D*'s Optimal Responses to *A*'s Action at t_1**

(i) *A*: do no trade at t_1

Given A does not trade in the first period, D's corresponding trading strategies can be to sell one unit (VIII-(0,-1)), to sell two units (IX-(0,-2)) or no trade (VII-(0,0)). The following figure (A-1) illustrates all possible evolutions of the game if both large traders choose not to trade (i.e., VII-(0,0)).

In subgame (VII-i) at t_2, D can liquidate her long position by selling one unit to each small high type non-owner (S_{hn}) at the bilateral bargaining price $P_{t_2}^{D_{lo}-S_{hn}}$, which is determined by

$$(1-q_D)\left[P_{t_2}^{D_{lo}-S_{hn}} - \frac{\bar{D}-\varepsilon}{r}\right] = q_D\left[\frac{\bar{D}-\rho_d \Delta t \varepsilon}{r} - P_{t_2}^{D_{lo}-S_{hn}}\right] \tag{A-1}$$

$$\Rightarrow \qquad P_{t_2}^{D_{lo}-S_{hn}} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}(1-q_D+q_D\rho_d) \tag{A-2}$$

In subgames (VII-ii), D and A compete to sell to small buyers. The pre-trade bargaining prices between D_{lo} and a S_{hn} and between A_{lo} and a S_{hn} are

$$P_{t_2}^{D_{lo}-S_{hn}} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}(1-q_D+q_D\rho_d)$$

and,

$$P_{t_2}^{A_{lo}-S_{hn}} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}(1-q_A+q_A\rho_d).$$

Since $q_A > q_D$, $P_{t_2}^{D_{lo}-S_{hn}} < P_{t_2}^{A_{lo}-S_{hn}}$. A S_{hn} compares two bargaining prices with D and A and buys at a lower price. Thus in this subgame, D sells two units to two small buyers but A cannot sell any.

In subgame (VII-iii), D sells two units to a small high-type non-owner (i.e., S_{hn}) and a low-type non-owner (S_{ln}). Trade takes place between D and the S_{ln} because it is mutually beneficial. The bargaining price between the D_{lo} and the S_{ln} is given by

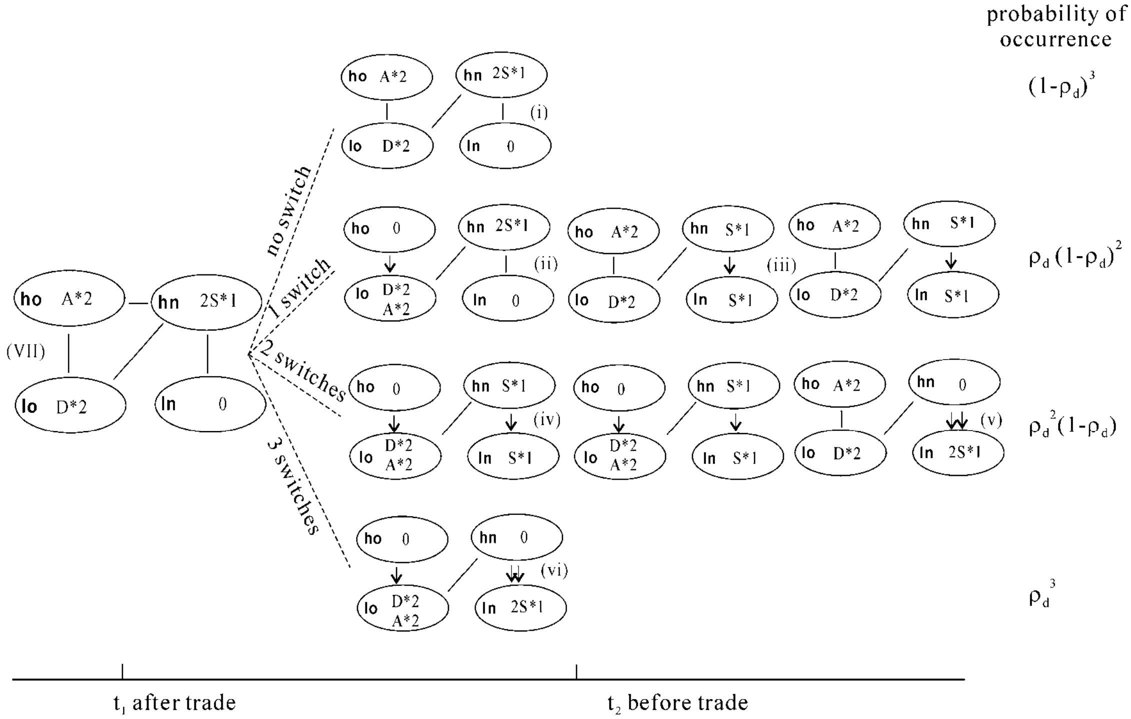


Figure A-1. Type switching and game evolution for strategy VII-(0,0)

$$\left(1-q_D\right)\left[P_{t_2}^{D_{lo}-S_{ln}}-\frac{\bar{D}-\varepsilon}{r}\right]=q_D\left[\frac{\bar{D}-\left(1-\rho_u\right)\varepsilon}{r}-P_{t_2}^{D_{lo}-S_{ln}}\right] \tag{A-3}$$

$$\Rightarrow \qquad P_{t_2}^{D_{lo}-S_{ln}}=\frac{\bar{D}}{r}-\frac{\varepsilon}{r}\left[1-q_D+q_D\left(1-\rho_u\right)\right] \tag{A-4}$$

In subgame (VII-iv), D_{lo} and A_{lo} compete to sell one unit to a S_{ln} and one unit to a S_{hn}. Since A_{lo}'s bargaining power is greater than D_{lo}'s, A_{lo} always loses the competition because the pre-trade bargaining price between A and a small trader is always larger than the bargaining price between D and a small trader. Hence in this subgame, D sells one unit to the S_{ln} and one unit to the S_{hn}.

In subgame (VII-v) and (VII-vi), D sells two units to two S_{ln}'s.

Thus D's value function of the strategy "no trade" when A does not front-run at t_1 is the expected payoff to the strategy VII-(0, 0).

$$V_{t_1}^{D_l}\left(\text{VII-}(0{,}0)\right)=\frac{1}{r}\left\{2\left(\frac{\bar{D}}{r}-\frac{\varepsilon}{r}(1-q_D+q_D\rho_d)\right)\left[(1-\rho_d)^3+\rho_d(1-\rho_d)^2\right]\right.$$

$$+\left[\frac{2\bar{D}}{r}-\frac{\varepsilon}{r}\left(2(1-q_D)+q_D\rho_d+q_D(1-\rho_u)\right)\right]$$

$$\left[2\rho_d(1-\rho_d)^2+2\rho_d^2(1-\rho_d)\right]$$

$$\left.+\left[\frac{2\bar{D}}{r}-\frac{2\varepsilon}{r}\left(1-q_D+q_D(1-\rho_u)\right)\right]\left(\rho_d^2(1-\rho_d)+\rho_d^3\right)\right\}$$

$$=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\left[1-q_D+q_D\left(2\rho_d-\rho_d^2-\rho_d\rho_u\right)\right] \tag{A.5}$$

Similarly, A's value function of the strategy VII-(0, 0) can be computed as

$$V_{t_1}^{A_h}\left(\text{VII-}(0{,}0)\right)=\frac{1}{r}\left[2\frac{\bar{D}-\rho_d\varepsilon}{r}(1-\rho_d)+2\frac{\bar{D}-\varepsilon}{r}\rho_d\right]$$

$$=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\left(2\rho_d-\rho_d^2\right) \tag{A-6}$$

Strategy VIII-(0,-1): A does not sell and D sells one share at t_1.

All possible type switching and trader distributions are demonstrated in Figure A-2. Since smaller traders are "geographically separate", only D and A are able to contact the small buyer S_{hn} in subgames VIII-ii, iii and iv. Because $q_A > q_D$ and $P_{t_2}^{D_{lo}-S_{hn}} < P_{t_2}^{A_{lo}-S_{hn}}$, D is the only seller in subgames VIII-ii, iii and iv.

Thus D's value function for taking the strategy VIII-(0,-1) is

$$V_{t_1}^{D_l}\left(\text{VIII-}(0{,}\text{-}1)\right)=P_{t_1}^{D_l-S_h}\left(1|\text{VIII}\right)+\frac{1}{r}\left\{\left(\frac{\bar{D}}{r}-\frac{\varepsilon}{r}(1-q_D+q_D\rho_d)\right)\right.$$

$$\left[(1-\rho_d)^3+2\rho_d(1-\rho_d)^2+\rho_d^2(1-\rho_d)\right]$$

$$\left.+\left[\frac{\bar{D}}{r}-\frac{\varepsilon}{r}\left(1-q_D+q_D(1-\rho_u)\right)\right]\left[\rho_d(1-\rho_d)^2+2\rho_d^2(1-\rho_d)+\rho_d^3\right]\right\}$$

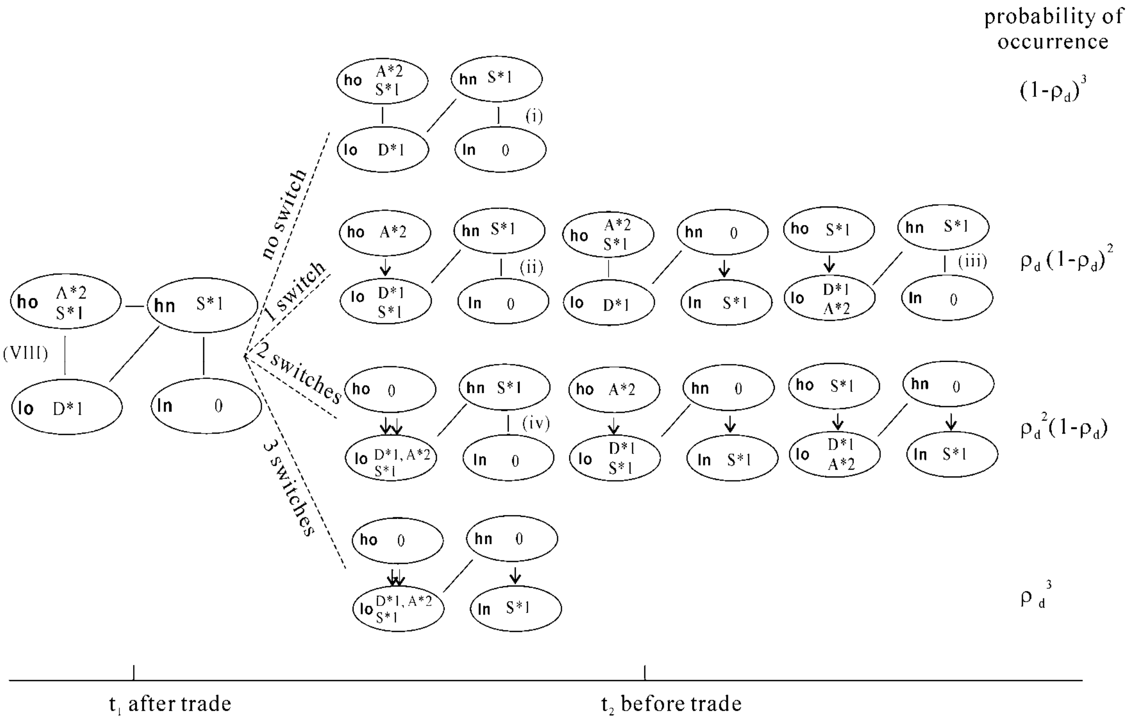


Figure A-2. Type switching and game evolution for strategy VIII-(0,-1)

$$= P_{t_1}^{D_l - S_h}\left(1\middle|\text{VIII}\right) + \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\left(1 - q_D + 2q_D\rho_d - q_D\rho_d^2 - q_D\rho_d\rho_u\right) \tag{A.7}$$

where $P_{t_1}^{D_l - S_h}\left(1\middle|\text{VIII}\right)$ is the price that D sells one unit to a S_{hn} at t_1.

The value function of A for strategy VIII-(0,-1) is exactly the same as that for strategy VII-(0,0) since she cannot make a sale if she switches to the low type.

$$V_{t_1}^{A_h}\left(\text{VIII-}(0,\text{-}1)\right) = V_{t_1}^{A_h}\left(\text{VII-}(0,0)\right) = \frac{2\bar{D}}{r^2} - \frac{2\varepsilon}{r^2}\left(2\rho_d - \rho_d^2\right) \tag{A.8}$$

Strategy IX-(0,-2): A does not sell and D sells two units to two small traders at t_1.

In subgame (IX-i), A sells two shares to D such that

$$\frac{q_D}{q_A+q_D}\left[P_{t_2}^{A_l-D_h}(\mathrm{IX-i})-\frac{1}{r}(\bar{D}-\varepsilon)\right]=\frac{q_A}{q_A+q_D}\left[\frac{1}{r}(\bar{D}-\rho_d\Delta t\varepsilon)-P_{t_2}^{A_l-D_h}(\mathrm{IX-i})\right]$$

$$P_{t_2}^{A_l-D_h}(\mathrm{IX-i})=\frac{\bar{D}}{r}-\frac{\varepsilon}{r}\left(\frac{q_D}{q_A+q_D}+\frac{q_A}{q_A+q_D}\rho_d\right) \tag{A.9}$$

In subgames IX-ii and iv, S_{lo}'s trade with D_{hn} at

$$P_{t_2}^{S_l-D_h}(\mathrm{IX-ii})=\frac{\bar{D}}{r}-\frac{\varepsilon}{r}\left[q_D(1-\rho_u)+(1-q_D)\rho_d\right] \tag{A-10}$$

In subgame IX-iii, D_{hn} contacts both A_{lo} and S_{lo} and buys one unit from each of them. In subgame IX-v, D_{hn}, the monopoly buyer, finds three sellers, A_{lo} and two S_{lo}'s. Since D_{hn} can only buy two units, whether she buys two from A_{lo} or one from each of the S_{lo}'s depends on which bargaining price is lower. She then compares two prices $P_{t_2}^{A_l-D_h}$ and $P_{t_2}^{S_l-D_h}$ given by (A.9) and (A.10). When $\frac{1-\rho_u-\rho_d}{1-\rho_d}>\frac{1}{q_A+q_D}$, $P_{t_2}^{A_l-D_h}>P_{t_2}^{S_l-D_h}$ and D_{hn} buys from the small seller S_{lo}; otherwise, when $\frac{1-\rho_u-\rho_d}{1-\rho_d}\le\frac{1}{q_A+q_D}$, $P_{t_2}^{A_l-D_h}\le P_{t_2}^{S_l-D_h}$ and D_{hn} buys from the large seller A_{lo}.

The above two inequalities also illustrate that a large trader chooses the trading counterparty by trading-off her relative bargaining power with the probability of receiving the future dividend payment. Since this condition is crucial in solving subgames in the second period, and appears repeatedly in the text and the appendix, I rewrite the condition in the following shorthand form.

$\frac{1-\rho_u-\rho_d}{1-\rho_d}>\frac{1}{q_A+q_D}$ $CTS > BP$ (i.e., Chance of Type Switching > Bargaining Power),

$\frac{1-\rho_u-\rho_d}{1-\rho_d}\le\frac{1}{q_A+q_D}$ $CTS \le BP$ (i.e., Chance of Type Switching ≤ Bargaining Power).

Thus D's value function when she chooses to sell two units when A does not sell at t_1 is

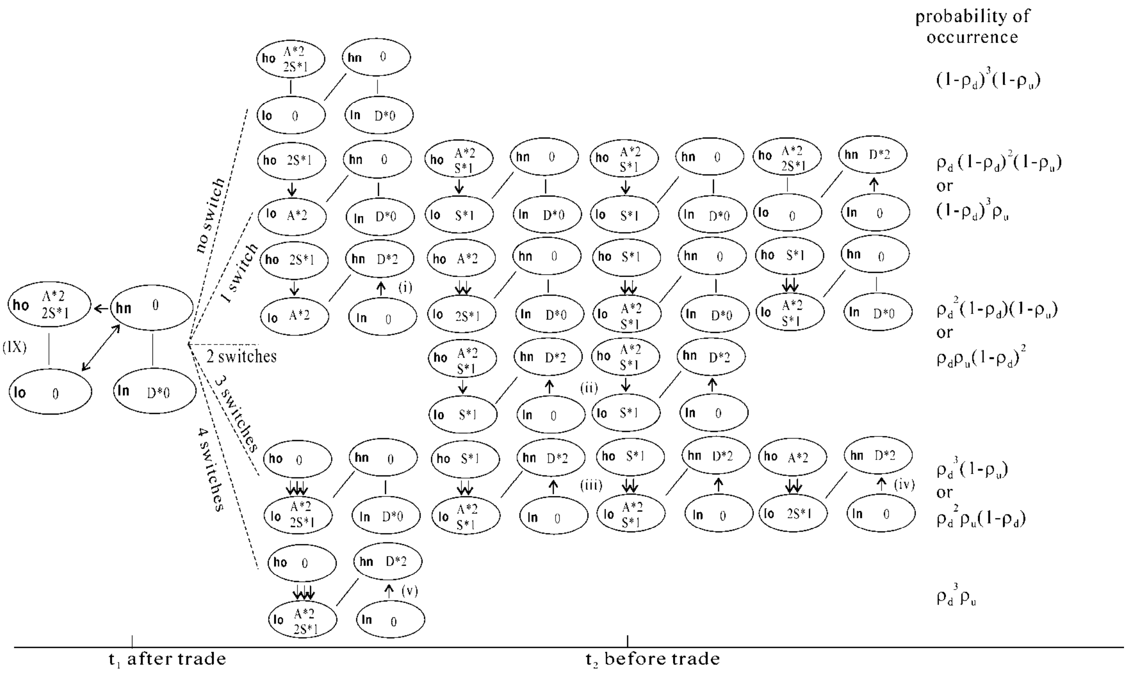


Figure A-3. Type switching and game evolution for strategy IX-(0,-2)

$$
\begin{aligned}
V_{t_1}^{D_l}\left(\text{IX-}(0,\text{-}2)\right) = {} & 2P_{t_1}^{D_l-S_h}\left(1\middle|\text{IX}\right) \\
& + \frac{2\varepsilon}{r^2}\left[\frac{q_D}{q_A+q_D}+\left(\frac{q_A}{q_A+q_D}-1\right)\rho_d\right]\left[\rho_d\rho_u\left(1-\rho_d\right)^2+\rho_d^2\rho_u\left(1-\rho_d\right)\right] \\
& + \frac{2\varepsilon}{r^2}\left[q_D\left(1-\rho_u\right)-q_D\rho_d\right]\left[\rho_d\rho_u\left(1-\rho_d\right)^2+2\rho_d^2\rho_u\left(1-\rho_d\right)\right] \\
& + \begin{cases} \dfrac{2\varepsilon}{r^2}\left[q_D\left(1-\rho_u\right)-q_D\rho_d\right]\rho_d^3\rho_u & \text{when} \quad CTS > BP \\[2ex] \dfrac{2\varepsilon}{r^2}\left[\dfrac{q_D}{q_A+q_D}+\left(\dfrac{q_A}{q_A+q_D}-1\right)\rho_d\right]\rho_d^3\rho_u & \text{when} \quad CTS \le BP \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
&= 2P_{t_1}^{D_l - S_h}\left(1 \middle| \mathrm{IX}\right) \\
&+\frac{2\varepsilon}{r^2}\begin{cases} q_D\left(1-\rho_u-\rho_d\right)\rho_d\rho_u + \\ \quad \dfrac{q_D}{q_A+q_D}\left(1-\rho_d\right)^2\rho_d\rho_u & \text{when } CTS > BP \\ q_D\left(1-\rho_u-\rho_d\right)\rho_d\rho_u\left(1-\rho_d^2\right)+ \\ \quad \dfrac{q_D}{q_A+q_D}\left(1-\rho_d\right)\rho_d\rho_u\left(1-\rho_d+\rho_d^2\right) & \text{when } CTS \le BP \end{cases}
\end{aligned}
\tag{A.11}
$$

A’s expected payoff for this subgame is

$$
\begin{aligned}
V_{t_1}^{A_h}\left(\mathrm{IX}\text{-}\left(0,\text{-}2\right)\right) = {} & 2\left(\frac{\bar{D}}{r^2}-\frac{\rho_d\varepsilon}{r^2}\right)\left(1-\rho_d\right) \\
& +2\frac{\bar{D}-\varepsilon}{r^2}\Big[\rho_d\left(1-\rho_d\right)^2\left(1-\rho_u\right) \\
& +2\rho_d^2\left(1-\rho_d\right)\left(1-\rho_u\right)+\rho_d^3\left(1-\rho_u\right)\Big] \\
& +\left[\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\left(\frac{q_D}{q_A+q_D}+\frac{q_A}{q_A+q_D}\rho_d\right)\right]\rho_d\rho_u\left(1-\rho_d\right)^2 \\
& +\left[\frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left(1+\frac{q_D}{q_A+q_D}+\frac{q_A}{q_A+q_D}\rho_d\right)\right]2\rho_d^2\rho_u\left(1-\rho_d\right) \\
& +\rho_d^3\rho_u\begin{cases} 2\dfrac{\bar{D}-\varepsilon}{r^2} & \text{when } CTS > BP \\ \dfrac{2\bar{D}}{r^2}-\dfrac{2\varepsilon}{r^2}\left(\dfrac{q_D}{q_A+q_D}+\dfrac{q_A}{q_A+q_D}\rho_d\right) & \text{when } CTS \le BP \end{cases}
\end{aligned}
$$

$$
=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\begin{cases} \rho_d\left(1-\rho_u\right)+\rho_d^2\rho_u+\rho_d\left(1-\rho_d\right) \\ \quad +\dfrac{q_D}{q_A+q_D}\left(1-\rho_d\right)^2\rho_d{\rho_u}^2 & \text{when } CTS > BP \\ \rho_d\left(1-\rho_u\right)+\rho_d\left(1-\rho_d\right)+2\rho_d^2\rho_u\left(1-\rho_d\right)+\rho_d^4\rho_u \\ \quad +\dfrac{q_D}{q_A+q_D}\left(1-\rho_d\right)\left[\rho_d\rho_u\left(1-\rho_d\right)+\rho_d^3\rho_u\right] \\ & \text{when } CTS \le BP \end{cases}
\tag{A.12}
$$

Next we consider D's optimal response to A's strategy of "no trade" in the first period by comparing her value functions of three strategies VII, VIII and IX. In the bargaining with two small buyers, D would choose to sell one unit as long as the portion of the marginal profit from selling this unit given up to the buyer is compensated by the profit she claims from the buyer.

$$\left(1-q_D\right)\left(V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right)-V_{t_1}^{D_l}\left(\text{VII-}(0,0)\right)\right)=q_D\left(V_{t_1}^{S_h}\left(1\middle|\text{VIII}\right)-P_{t_1}^{D_l-S_h}\left(1\middle|\text{VIII}\right)\right) \quad \text{(A.13)}$$

where $V_{t_1}^{S_h}\left(1\middle|\text{VIII}\right)$ is the value function of the small high type non-owner who buys from D.

$$\begin{aligned} V_{t_1}^{S_h}\left(1\middle|\text{VIII}\right) &= \left(\frac{\bar{D}}{r^2}-\frac{\rho_d\varepsilon}{r^2}\right)\left(1-\rho_d\right)+\left(\frac{\bar{D}}{r^2}-\frac{\left(1-\rho_u\right)\varepsilon}{r^2}\right)\rho_d \\ &= \frac{\bar{D}}{r^2}-\frac{\rho_d\varepsilon}{r^2}\left(2-\rho_d-\rho_u\right) \end{aligned} \quad \text{(A.14)}$$

Substituting $V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right)$, $V_{t_1}^{D_l}\left(\text{VII-}(0,0)\right)$ and $V_{t_1}^{S_h}\left(1\middle|\text{VIII}\right)$ into (A.13), we have

$$\begin{aligned} P_{t_1}^{D_l-S_h}\left(1\middle|\text{VIII}\right) = \frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\Big[&\left(1-q_D\right)^2+2q_D\left(2-q_D\right)\rho_d-q_D\left(2-q_D\right)\rho_d^2 \\ &-q_D\left(2-q_D\right)\rho_d\rho_u\Big] \end{aligned} \quad \text{(A.15)}$$

$$\begin{aligned} V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right) = \frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\Big[&\left(1-q_D\right)\left(2-q_D\right)+2q_D\left(3-q_D\right)\rho_d-q_D\left(3-q_D\right)\rho_d^2 \\ &-q_D\left(3-q_D\right)\rho_d\rho_u\Big] \end{aligned}$$

(A.16)

D would sell two units instead of one unit as long as

$$\left(1-q_D\right)\left(V_{t_1}^{D_l}\left(\text{IX-}(0,\text{-}2)\right)-V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right)\right)=q_D\left(V_{t_1}^{S_h}\left(1\middle|\text{IX}\right)-P_{t_1}^{D_l-S_h}\left(1\middle|\text{IX}\right)\right) \quad \text{(A.17)}$$

where the marginal small buyer's value function in subgame (IX) is

$$
\begin{aligned}
V_{t_1}^{S_h}\left(1\middle|\mathrm{IX}\right) &= \left(\frac{\bar{D}}{r^2}-\frac{\rho_d\varepsilon}{r^2}\right)(1-\rho_d)+\left(\frac{\bar{D}}{r^2}-\frac{(1-\rho_u)\varepsilon}{r^2}\right)\rho_d(1-\rho_u) \\
&+\left[\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left(q_D(1-\rho_u)+(1-q_D)\rho_d\right)\right]\left[\rho_d\rho_u(1-\rho_d)^2+2\rho_d^2\rho_u(1-\rho_d)\right] \\
&+\rho_d^3\rho_u\begin{cases}\dfrac{\bar{D}}{r^2}-\dfrac{\varepsilon}{r^2}\left[q_D(1-\rho_u)+(1-q_D)\rho_d\right] & \text{when } CTS > BP \\ \dfrac{\bar{D}}{r^2}-\dfrac{(1-\rho_u)\varepsilon}{r^2} & \text{when } CTS \leq BP\end{cases} \\
&= \frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\begin{cases}\rho_d(1-\rho_d)\rho_d(1-\rho_u)^2+q_D\rho_d\rho_u{}^2(1-\rho_u)+ & \\ \quad+(1-q_D)\rho_d^2\rho_u & \text{when } CTS > BP \\ \rho_d(1-\rho_d)+\rho_d(1-\rho_u)^2+\rho_d^3\rho_u(1-\rho_u) & \\ \quad+q_D\rho_d\rho_u(1-\rho_u)\left(1-\rho_d^2\right)+(1-q_D)\rho_d^2\rho_u\left(1-\rho_d^2\right) & \\ & \text{when } CTS \leq BP\end{cases}
\end{aligned}
$$

(A.18)

Therefore the price at which D sells two units when A does not sell is

$$P_{t_1}^{D_l-S_h}\left(1\middle|\mathrm{IX}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{\left(2-q_D\right)r^2}$$

$$\begin{cases}\left(1-q_D\right)^2\left(2-q_D\right)+q_D\left(2-q_D\right)^2\rho_d-q_D\rho_d^2+q_D\rho_d\left(1-\rho_u\right)^2\\ \quad+q_D^2\rho_d\rho_u\left(1-\rho_u\right)++q_D\left(1-q_D\right)\rho_d^2\rho_u\\ \quad+q_D\left(1-q_D\right)\rho_d\left(1-\rho_u-\rho_d\right)\left(3-q_D+2\rho_u\right)\\ \quad+\dfrac{2q_D\left(1-q_D\right)}{q_A+q_D}\rho_d\rho_u\left(1-\rho_d\right)^2 \quad \text{when } CTS>BP\\ \left(1-q_D\right)^2\left(2-q_D\right)+q_D\left(2-q_D\right)^2\rho_d-q_D\rho_d^2+q_D\rho_d\left(1-\rho_u\right)^2\\ \quad+q_D\rho_d^3\rho_u\left(1-\rho_u\right)+q_D^2\rho_d\rho_u\left(1-\rho_u\Delta t\right)\left(1-\rho_d^2\Delta t^2\right)\\ \quad+q_D\left(1-q_D\right)\rho_d^2\rho_u\left(1-\rho_d^2\right)\\ \quad+q_D\left(1-q_D\right)\rho_d\left(1-\rho_u-\rho_d\right)\left(3-q_D+2\rho_u-2\rho_d^2\rho_u\right)\\ \quad+\dfrac{2q_D\left(1-q_D\right)}{q_A+q_D}\rho_d\rho_u\left(1-\rho_d\right)\left(1-\rho_d+\rho_d^2\right)\\ \qquad\qquad\qquad\qquad\qquad \text{when } CTS\le BP\end{cases}$$

(A.19)

$$V_{t_1}^{D_l}\left(\mathrm{IX\text{-}}\left(0,\text{-}2\right)\right)=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{\left(2-q_D\right)r^2}$$

$$\begin{cases}\left(1-q_D\right)^2\left(2-q_D\right)+q_D\left(2-q_D\right)^2\rho_d-q_D\rho_d^2+q_D\rho_d\left(1-\rho_u\right)^2\\ \quad+q_D^2\rho_d\rho_u\left(1-\rho_u\right)+q_D\left(1-q_D\right)\rho_d^2\rho_u^3\\ \quad+q_D\left(1-q_D\right)\rho_d\left(1-\rho_u-\rho_d\right)\left(3-q_D\right)-q_D^2\rho_d\rho_u\left(1-\rho_u-\rho_d\right)\\ \quad+\dfrac{2q_D\left(1-q_D\right)}{q_A+q_D}\rho_d\rho_u\left(1-\rho_d\right)^2 \quad \text{when } CTS>BP\\ \left(1-q_D\right)^2\left(2-q_D\right)+q_D\left(2-q_D\right)^2\rho_d-q_D\rho_d^2+q_D\rho_d\left(1-\rho_u\right)^2\\ \quad+q_D\rho_d^3\rho_u\left(1-\rho_u\right)+q_D^2\rho_d\rho_u\left(1-\rho_u\right)\left(1-\rho_d^2\right)\\ \quad+q_D\left(1-q_D\right)\rho_d^2\rho_u\left(1-\rho_d^2\right)-q_D^2\rho_d\rho_u\left(1-\rho_u-\rho_d\right)\left(1-\rho_d^2\right)\\ \quad+q_D\left(1-q_D\right)\rho_d\left(1-\rho_u-\rho_d\right)\left(3-q_D\right)\\ \quad-\dfrac{q_D^2}{q_A+q_D}\rho_d\rho_u\left(1-\rho_d\right)\left(1-\rho_d+\rho_d^2\right)\\ \qquad\qquad\qquad \text{when } CTS\le BP\end{cases} \tag{A.20}$$

We compare the value functions of D's three strategies and determine her optimal response to A's strategy of "no trade" in the first period.

$$
\begin{aligned}
&V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right)-V_{t_1}^{D_l}\left(\text{VII-}(0,0)\right) \\
&=\frac{\varepsilon}{r^2}\left[q_D\left(1-q_D\right)\left(1-2\rho_d+\rho_d^2\right)+q_D\left(1-q_D\right)\rho_d\rho_u\right]>0
\end{aligned}
$$

Thus "no trade" is strictly dominated by "sell one" for D when A does not front-run.

Comparison between $V_{t_1}^{D_l}\left(\text{IX-}(0,\text{-}2)\right)$ and $V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right)$ is not that straightforward. Which strategy is better depends on the relationship between the type switching probabilities ρ_d, ρ_u, and the relative bargaining powers q_D and q_A. For example, let $\rho_d=0.2$, $\rho_u=0.2$, $q_D = 0.55$, $q_A = 0.8$, such that the condition $CTS > BP$ is satisfied. Comparing D's value functions of "sell one unit" and "sell two units" given the above parameter values, we find $V_{t_1}^{D_l}\left(\text{IX-}(0,\text{-}2)\right)>V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right)$. We then decrease q_A to 0.6, other parameters being equal, to satisfied the opposite condition $CTS \le BP$. We Comparing value functions under this condition, we find that $V_{t_1}^{D_l}\left(\text{IX-}(0,\text{-}2)\right)$ is still greater than $V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right)$. For most of parameter values, we find that

$$
V_{t_1}^{D_l}\left(\text{IX-}(0,\text{-}2)\right)> V_{t_1}^{D_l}\left(\text{VIII-}(0,\text{-}1)\right) > V_{t_1}^{D_l}\left(\text{VII-}(0,0)\right)
$$

Therefore, when A does not front-run in the first period, D never chooses to "wait", and she almost always chooses to sell two units.

(ii) A: front-run and sell one unit at t_1.

When A front-runs and sells one unit at t_1, D can respond by selling one unit (strategy II-(-1,-1)) or two units (strategy III-(-1,-2)) or not selling in this period ((strategy I-(-1,0)). We analyze both A and D's expected payoffs to the three strategies.

Strategy I-(-1,0): D does not sell while A sells one share.

Subgame (I-i) is the monopoly seller case, in which D_{lo} bargains with S_{hn} and A_h simultaneously and sells to each of them one unit at $P_{t_2}^{D_l-S_h}$ and $P_{t_2}^{D_l-A_h}$ given below.

$$P_{t_2}^{D_l-S_h} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}\left(1 - q_D + q_D \rho_d\right) \tag{A.21}$$

$$P_{t_2}^{D_l-A_h} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}\left(1 - \frac{q_D}{q_A + q_D} + \frac{q_D}{q_A + q_D}\rho_d\right) \tag{A.22}$$

In subgame (I-ii) and (I-v), A occurs type switching to the low type so that she competes with D_{lo} to sell to the only small high type non-owner. Since the pre-trade bargaining price between D_{lo} and S_{hn} is lower than the bargaining price between A_{lo} and S_{hn}, the small buyer buys from D at $P_{t_2}^{D_l-S_h}$. In subgame (I-iv), D sells one unit to A_h and

one unit to S_{ln} at $P_{t_2}^{D_{lo}-S_{ln}} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}\left[1 - q_D + q_D\left(1 - \rho_u\right)\right]$ (A.23)

In subgame (I-vi) and (I-viii), D_{lo} and A_{lo} compete to sell one unit to S_{ln} and it is D_{lo} who makes the transaction with S_{ln}.

Subgame (I-iii) is a little bit tricky in that there are multiple heterogeneous buyers, i.e., D_{lo} and S_{lo}, and sellers, A_{ho} and S_{hn}. Since small traders cannot contact each other, D and A can contact all potential trading counterparts while small traders can only contact the large traders. In this case, D_{lo} contacts both A_{ho} and S_{hn} and tries to sell one unit to each of them. A_{ho} receives two ask prices from D_{lo} and S_{lo} and only buys one unit at the lower of the two prices. Since the small buyer, S_{hn}, is only contacted by D_{lo}, she thus trade with D_{lo} at the bilateral bargaining price immediately. A compares two pre-trade bargaining prices with D_{lo} and S_{lo}. $P_{t_2}^{D_l-A_h}$ is given by (A.22) and

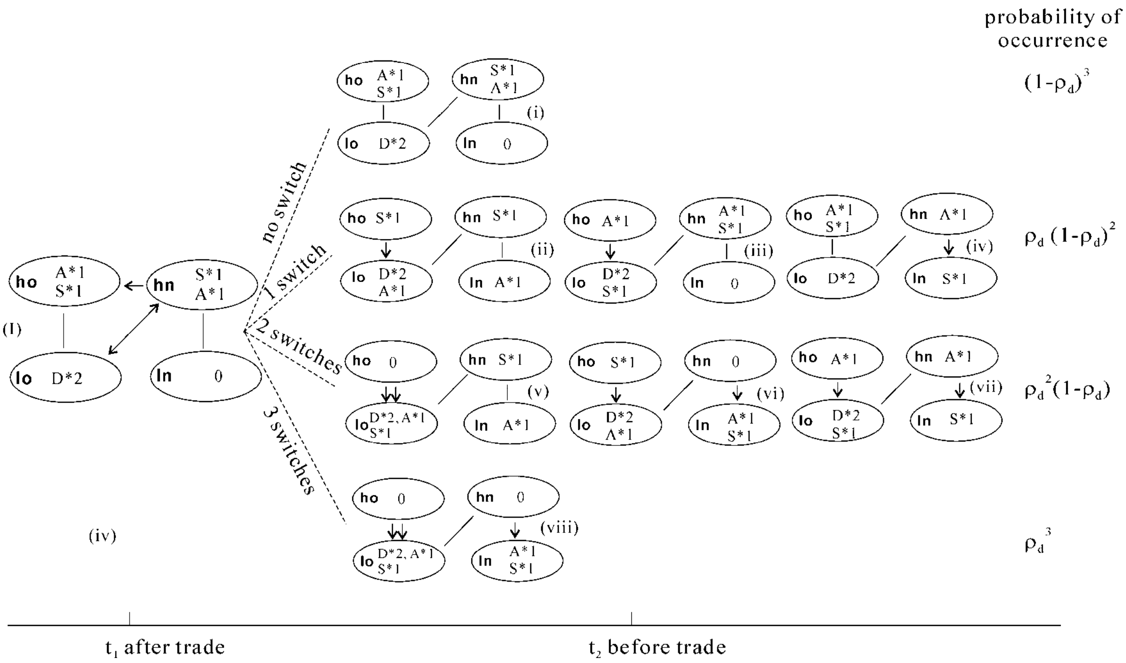


Figure A-4. Type switching and game evolution for strategy I-(-1,0)

$$P_{t_2}^{S_l-A_h} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}\left[q_A(1-\rho_u)+(1-q_A)\rho_d\right] \tag{A.24}$$

She trades with the S_{lo} if $P_{t_2}^{D_l-A_h} > P_{t_2}^{S_l-A_h}$ (i.e., $CTS > BP$), and trades with D_{lo} otherwise.

Subgame (I-vii) is similar to (I-iii). D_{lo} sells one unit to A_h and one to S_{ln} when $CTS > BP$, and sells only one unit to S_{ln} otherwise.

Now we calculate D's value function for strategy I-(-1,0) at t_1.

$$V_{t_1}^{D_l}\left(\text{I-}(\text{-1,0})\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\begin{cases} -\dfrac{q_D}{q_A+q_D}(1-\rho_d)^3 + 2 - q_D(1-\rho_d)^2 - q_D\rho_d\rho_u \\ \qquad\qquad\qquad \text{when } CTS > BP \\ -\dfrac{q_D}{q_A+q_D}(1-\rho_d)^2 + 2 - q_D(1-\rho_d)^2 - q_D\rho_d\rho_u \\ \qquad\qquad\qquad \text{when } CTS \le BP \end{cases}$$

(A.25)

A's expected payoff to strategy I-(-1,0) is

$$V_{t_1}^{A_h}\left(\text{I-}(\text{-}1{,}0)\right)=P_{t_1}^{A_h-S_h}\left(1|\text{I}\right)+\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}$$
$$\begin{cases}-\dfrac{q_A}{q_A+q_D}(1-\rho_d)^3+\rho_d(2-\rho_d)-q_A\rho_d(1-\rho_d)^2 \\ \quad +q_A\rho_d\rho_u(1-\rho_d) & \text{when } CTS>BP \\ -\dfrac{q_A}{q_A+q_D}(1-\rho_d)^2+\rho_d(2-\rho_d) & \text{when } CTS\le BP\end{cases}$$

(A.26)

Strategy II-(-1,-1): Both D and A sell one unit at t_1.

If no switch occurs during t_1 and t_2, traders are then in subgame (II-i) at the next trading date. D_l trades with A_h at the bilateral Nash bargaining price $P_{t_2}^{D_l-A_h}$. In subgame (II-ii), A_h, the monopoly buyer, buys from a small seller, S_{lo}, if $CTS > BP$ and from D_{lo} if $CTS \le BP$.

Subgame (II-iii) is a little different from (II-ii) in that when $CTS > BP$, A_h buys one unit from any one of the two S_{lo}'s randomly with probability ½.

Therefore the expected payoff for D_l and A_h for strategy II-(1,1) are

$$V_{t_1}^{D_l}\left(\text{II-}(-1,-1)\right)=P_{t_1}^{D_l-S_h}\left(1|\text{II}\right)+\left[\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\left(\frac{q_A}{q_A+q_D}-\frac{q_D}{q_A+q_D}\rho_d\right)\right](1-\rho_d)^3$$
$$+\left(\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\right)\left[\rho_d(1-\rho_d)^2+2\rho_d^2(1-\rho_d\Delta t)\rho_d^3\right]$$
$$+\left(\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\right)\begin{cases}\left[2\rho_d(1-\rho_d)^2+\rho_d^2(1-\rho_d)\right] \\ \qquad\qquad \text{when } CTS>BP \\ \left(\dfrac{q_A}{q_A+q_D}-\dfrac{q_D}{q_A+q_D}\rho_d\right)\left[2\rho_d(1-\rho_d)^2+\rho_d^2(1-\rho_d)\right] \\ \qquad\qquad \text{when } CTS\le BP\end{cases}$$

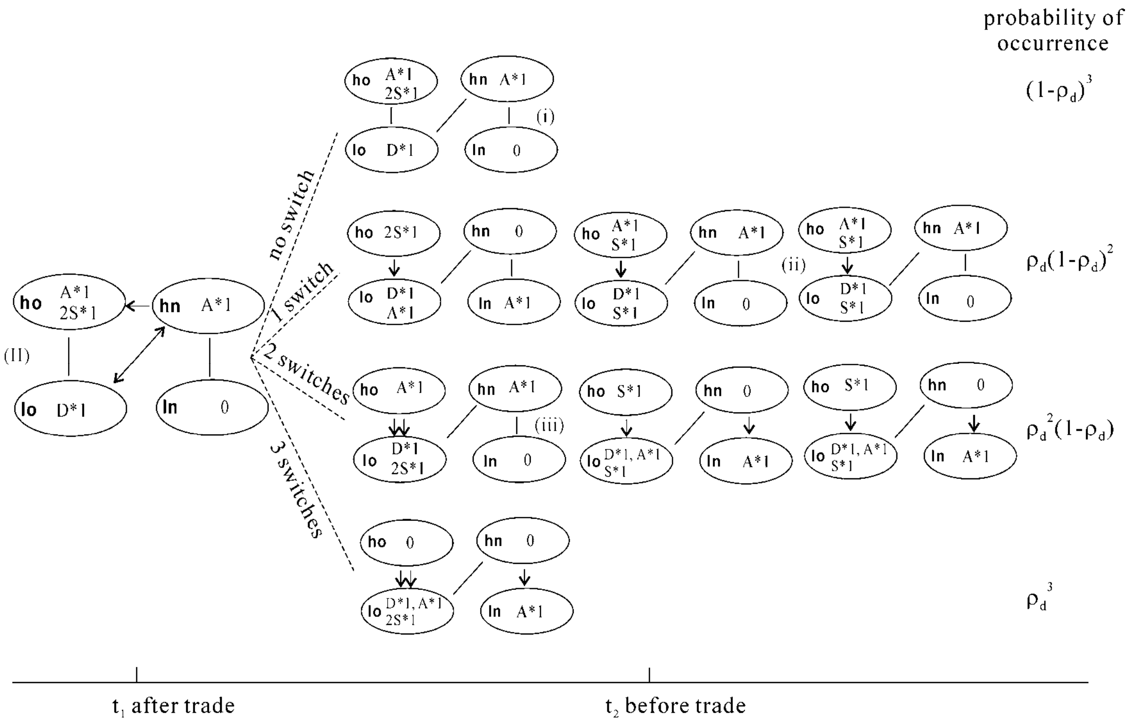


Figure A-5. Type switching and game evolution for strategy II-(-1,-1)

$$= P_{t_1}^{D_l - S_h}\left(1\middle|\mathrm{II}\right) + \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\begin{cases} 1 - \dfrac{q_D}{q_A + q_D}\left(1-\rho_d\right)^4 & \text{when } CTS > BP \\ 1 - \dfrac{q_D}{q_A + q_D}\left(1-\rho_d\right)^2 & \text{when } CTS \le BP \end{cases} \tag{A.27}$$

$$\begin{aligned} V_{t_1}^{A_h}\left(\text{II-}(\text{-1,-1})\right) &= P_{t_1}^{A_h - S_h}\left(1\middle|\mathrm{II}\right) + \left(\frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\right)\rho_d \\ &+ \left[\frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\left(-\frac{q_A}{q_A+q_D} + \left(1 + \frac{q_A}{q_A+q_D}\right)\rho_d\right)\right]\left(1-\rho_d\right)^3 \\ &+ \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\begin{cases} \left[-q_A + q_A\rho_u + \left(1+q_A\right)\rho_d\right]\left[2\rho_d\left(1-\rho_d\right)^2 + \rho_d^2\left(1-\rho_d\right)\right] \\ \qquad \text{when } CTS > BP \\ \left(-\dfrac{q_A}{q_A+q_D} + \left(1+\dfrac{q_A}{q_A+q_D}\right)\rho_d\right)\left[2\rho_d\left(1-\rho_d\right)^2 + \rho_d^2\left(1-\rho_d\right)\right] \\ \qquad \text{when } CTS \le BP \end{cases} \end{aligned}$$

$$= P_{t_1}^{A_h - S_h}\left(1|\text{II}\right)+\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\begin{cases}\rho_d\left(2-\rho_d\right)-q_A\rho_d\left(1-\rho_d-\rho_u\right)\left(1-\rho_d\right)\left(2-\rho_d\right) & \\ \quad -\frac{q_A}{q_A+q_D}\left(1-\rho_d\right)^4 & \text{when } CTS > BP \\ \rho_d\left(2-\rho_d\right)-\frac{q_A}{q_A+q_D}\left(1-\rho_d\right)^2 & \text{when } CTS \le BP\end{cases}$$

(A.28)

The expected payoff to a small high-type owner who bought one unit at t_1 is

$$V_{t_1}^{S_{ho}}\left(1|\text{II}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\begin{cases}\rho_d+\frac{1}{2}\rho_d^2\left(1-\rho_d\right)+\left[q_A\left(1-\rho_d-\rho_u\right)+\rho_d\right]\rho_d\left(1-\rho_d\right)\left(1-\frac{1}{2}\rho_d\right) & \\ & \text{when } CTS > BP \\ \rho_d\left(2-\rho_d\right) & \text{when } CTS \le BP\end{cases}$$

(A.29)

Strategy III-(-1,-2): D sells two units and A sells one unit at t_1.

When both large traders sell and one large trader sells more than one unit, she has to sell two units to one small buyer because a small buyer is assumed not being able to buy from two different sellers at the same time due to her budget constraint. But she may be able to buy two units from one trader as long as the price is low enough. Hence if a large seller wants to sell two units when there is another seller on the market, i.e., the supply of the illiquid asset is greater than the demand for this asset, she has to sell two units together as if she only sells one unit. Also note that if a small trader, S_{hn}, buys two units in the first period, she becomes a large owner and then suffers the “large trader’s curse” that if she occurs type switching to the low type, i.e., she becomes a low-type owner, she cannot switch back to the high type again unless she liquidates her entire position.

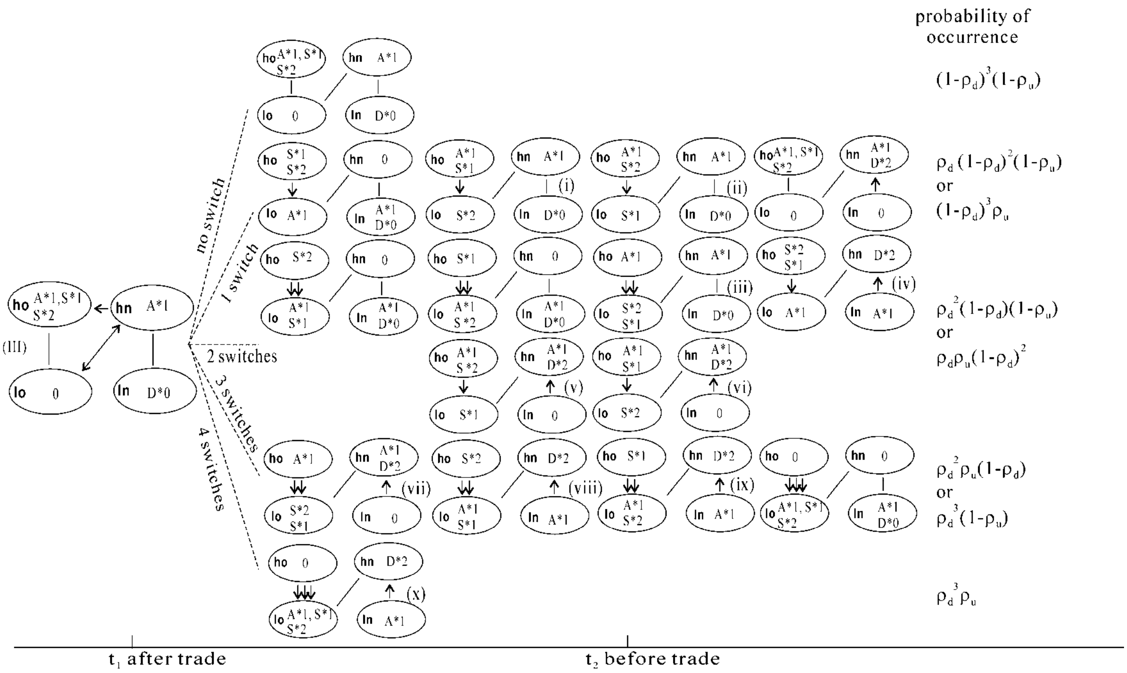


Figure A-6. Type switching and game evolution for strategy III-(-1,-2)

In subgames (III-i, ii, iii), A is the only buyer who buys one unit from a small low type seller. In subgame (III-iii), A buys from the small low type owner who bought two units at t_1 at $\left[\bar{D}-\varepsilon\left(q_A+\left(1-q_A\right)\rho_d\right)\right]/r$. In subgame (III-iv), D_{hn} buys one unit from A_l at

$$\frac{\bar{D}}{r}-\frac{\varepsilon}{r}\left(\frac{q_D}{q_A+q_D}+\frac{q_A}{q_A+q_D}\rho_d\right).$$

In subgame (III-v) and (III-vi), D_{hn} and A_h compete to buy from a low-type owner. The low-type owner, S_{lo}, compares two bargaining prices with D_{hn} and A_h and sells to D_{hn} since $P_{t_2}^{S_l-D_h}$ is higher than $P_{t_2}^{S_l-A_h}$.

In subgames (III-viii, ix, x), D_{hn} is the monopoly buyer who trades at the lowest bargaining price. In subgame (III-viii), D_{hn} buys one unit from A_l and one from S_l. In subgames (III-ix) and (III-x), D_{hn} buys two units from the small trader who has two units to sell.

In subgame (III-vii), there are two buyers, D_{hn} and A_h, and two sellers, S_{lo}'s. They contact each other and bargain. For the large S_{lo} who has two units to sell in this period, she prefers to sell to D_{hn} since $P_{t_2}^{S_{lo}^{x2}-D_h}$ is greater than $P_{t_2}^{S_{lo}^{x2}-A_h}$.

$$P_{t_2}^{S_{lo}^{x2}-D_h} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}\left[q_D + (1-q_D)\rho_d\right] \tag{A.30}$$

$$P_{t_2}^{S_{lo}^{x2}-A_h} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}\left[q_A + (1-q_A)\rho_d\right] \tag{A.31}$$

Similarly, the small S_{lo} who bought only one unit in the last period wants to sell to D_{hn} as well. That is,

$$P_{t_2}^{S_{lo}^{x1}-D_h} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}\left[q_D(1-\rho_u) + (1-q_D)\rho_d\right] \tag{A.32}$$

$$P_{t_2}^{S_{lo}^{x1}-A_h} = \frac{\bar{D}}{r} - \frac{\varepsilon}{r}\left[q_A(1-\rho_u) + (1-q_A)\rho_d\right] \tag{A.33}$$

D_{hn} compares two prices $P_{t_2}^{S_{lo}^{x2}-D_h}$ and $P_{t_2}^{S_{lo}^{x1}-D_h}$ and buys from the large S_{lo} since the price $P_{t_2}^{S_{lo}^{x2}-D_h}$ is the lower of the two. Thus, D_{hn} trades with the large S_{lo} while A_h trades with the small S_{lo}, and all of them are better off.

Hence we can calculate the expected payoffs to all four traders at t_1.

$$\begin{aligned} V_{t_1}^{D_{ln}}\left(\text{III-}(-1,-2)\right) = 2P_{t_1}^{D_l-S_h}\left(2|\text{III}\right) + \frac{\varepsilon}{r^2}\Bigg[&\frac{q_D}{q_A+q_D}\rho_d\rho_u(1-\rho_d)^2 \\ &+ q_D\rho_d\rho_u(3-\rho_d)(1-\rho_d) - q_D\rho_d\rho_u^2(1-\rho_d)\Bigg] \end{aligned} \tag{A.34}$$

$$\begin{aligned} V_{t_1}^{A_h}\left(\text{III-}(-1,-2)\right) = P_{t_1}^{A_h-S_h}\left(1|\text{III}\right) + \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\Bigg[&-\frac{q_A}{q_A+q_D}\rho_d\rho_u(1-\rho_d)^2 + 2\rho_d - \rho_d^2 \\ &+ q_A\rho_d(1-\rho_d)^2(\rho_d-2) + q_A\rho_d\rho_u(1-\rho_d)(1+\rho_d)(2\rho_d+3) \\ &+ q_A\rho_d\rho_u^2(1-\rho_d)(2\rho_d-1)\Bigg] \end{aligned} \tag{A.35}$$

$$
\begin{aligned}
V_{t_1}^{S_{ho}^{x1}}\left(1|\mathrm{III}\right) = \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\Big[& \rho_d\left(1-\rho_d\right) + \rho_d^2\left(1-\rho_d\right)^2\left(1-\rho_u\right) \\
& + \rho_d^3\rho_u\left(1-\rho_d\right) + 2\rho_d^2\left(1-\rho_u\right)^2\left(1-\rho_d\right) + \rho_d^3\left(1-\rho_u\right) \\
& + \rho_u\rho_d^2\left(1-\rho_d\right) + \left(1-\rho_d-\rho_u\right)q_A\rho_d\left(1-\rho_d\right) \\
& \left(1-\rho_d-\rho_u+2\rho_d\rho_u\right) + \left(1-\rho_d-\rho_u\right)q_D\rho_d\rho_u\left(1-\rho_d\right)\Big]
\end{aligned} \tag{A.36}
$$

$$
\begin{aligned}
V_{t_1}^{S_{ho}^{x2}}\left(2|\mathrm{III}\right) = \frac{2\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\Big[& 2\rho_d\left(1-\rho_d\right) + 2\rho_d^2 + \rho_d\left(1-\rho_d\right)\left(1-\rho_u\right)\left(1+\rho_d\right) \\
& + q_A\rho_d\left(1-\rho_d\right)^2\left(1-\rho_u\right) + 2q_D\rho_d\rho_u\left(1-\rho_d\right)\Big]
\end{aligned} \tag{A.37}
$$

Obtained all four traders' expected payoffs to strategies I, II and III, we next consider D's optimal response to A's strategy of "front-run and sell one unit" in the first period.

According to equation (1) in the text, D chooses to sell one unit over not to trade if

$$
\left(1-q_D\right)\left[V_{t_1}^{D_l}\left(\mathrm{II}-(-1,-1)\right) - V_{t_1}^{D_l}\left(\mathrm{I}-(-1,0)\right)\right] = q_D\left[V_{t_1}^{S_h}\left(1|\mathrm{II}\right) - P_{t_1}^{D_l-S_h}\left(1|\mathrm{II}\right)\right] \tag{A.38}
$$

Substituting $V_{t_1}^{D_l}\left(\mathrm{II}-(-1,-1)\right)$, $V_{t_1}^{D_l}\left(\mathrm{I}-(-1,0)\right)$ and $V_{t_1}^{S_h}\left(1|\mathrm{II}\right)$ given by (A.27), (A.25) and (A.29), we have the price $P_{t_1}^{D_l-S_h}\left(1|\mathrm{II}\right)$.

$$
P_{t_1}^{D_l-S_h}\left(1|\mathrm{II}\right) = \frac{\bar{D}}{r^2}
- \frac{\varepsilon}{r^2}
\begin{cases}
\begin{aligned}
& -\frac{q_D\left(1-q_D\right)}{q_A+q_D}\rho_d\left(1-\rho_d\right)^3 + 1 - q_D + q_D\rho_d - q_D\left(1-q_D\right)\left(1-\rho_d\right)^2 \\
& -q_D\left(1-q_D\right)\rho_d\rho_u + \tfrac{1}{2}q_D\rho_d^2\left(1-\rho_d\right) \\
& +\left[q_Aq_D\left(1-\rho_d-\rho_u\right) + q_D\rho_d\right]\rho_d\left(1-\rho_d\right)\left(1-\tfrac{1}{2}\rho_d\right)
\end{aligned} & \text{when } CTS > BP \\
\begin{aligned}
& 1 - q_D + 2q_D\rho_d - q_D\rho_d^2 - q_D\left(1-q_D\right)\left(1-\rho_d\right)^2 \\
& -q_D\left(1-q_D\right)\rho_d\rho_u
\end{aligned} & \text{when } CTS \le BP
\end{cases} \tag{A.39}
$$

$$V_{t_1}^{D_l}\left(\mathrm{II}-(-1,-1)\right)=\frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2}$$

$$\begin{cases} -\frac{q_D}{q_A+q_D}(1-\rho_d)^3(1-q_D\rho_d)+2-q_D+q_D\rho_d-q_D(1-q_D)(1-\rho_d)^2 \\ \quad -q_D(1-q_D)\rho_d\rho_u+\frac{1}{2}q_D\rho_d^2(1-\rho_d) \\ \quad +\left[q_Aq_D(1-\rho_d-\rho_u)+q_D\rho_d\right]\rho_d(1-\rho_d)(1-\frac{1}{2}\rho_d) \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{when } CTS > BP \\ -\frac{q_D}{q_A+q_D}(1-\rho_d)^2+2-q_D+2q_D\rho_d-q_D\rho_d^2-q_D(1-q_D)(1-\rho_d)^2 \\ \quad -q_D(1-q_D)\rho_d\rho_u \qquad\qquad\qquad\qquad \text{when } CTS \le BP \end{cases}$$

(A.40)

D chooses to sell two units as long as

$$(1-q_D)\left[V_{t_1}^{D_l}\left(\mathrm{III}-(-1,-2)\right)-V_{t_1}^{D_l}\left(\mathrm{I}-(-1,0)\right)\right]=q_D\left[V_{t_1}^{S_h}\left(2|\mathrm{III}\right)-2P_{t_1}^{D_l-S_h}\left(2|\mathrm{III}\right)\right] \quad (A.41)$$

$$P_{t_1}^{D_l-S_h}\left(2|\mathrm{III}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{2r^2}$$

$$\begin{cases} \frac{q_D(1-q_D)}{q_A+q_D}(1-\rho_d)^2(\rho_d+\rho_d\rho_u-1)+\rho_d\rho_u(1-\rho_d) \\ \quad \left[q_D(1-q_D)(3-\rho_d-\rho_u)+2q_D^2\right]+2(1-q_D)+2q_D\rho_d \\ \quad -q_D(1-q_D)(1-\rho_d)^2-q_D(1-q_D)\rho_d\rho_u+q_D\rho_d(1-\rho_d^2)(1-\rho_u) \\ \quad +q_Aq_D\rho_d(1-\rho_d)^2(1-\rho_u) \qquad \text{when } CTS > BP \\ \frac{q_D(1-q_D)}{q_A+q_D}(1-\rho_d)^2(\rho_d\rho_u-1)+\rho_d\rho_u(1-\rho_d) \\ \quad \left[q_D(1-q_D)(3-\rho_d-\rho_u)+2q_D^2\right]+2(1-q_D)+2q_D\rho_d \\ \quad -q_D(1-q_D)(1-\rho_d)^2-q_D(1-q_D)\rho_d\rho_u+q_D\rho_d(1-\rho_d^2)(1-\rho_u) \\ \quad +q_Aq_D\rho_d(1-\rho_d)^2(1-\rho_u) \qquad \text{when } CTS \le BP \end{cases} \quad (A.42)$$

D's expected utility of selling two units is

$$V_{t_1}^{D_l}\left(\text{III-}(-1,-2)\right)=\frac{2\overline{D}}{r^2}-\frac{\varepsilon}{r^2}$$

$$\begin{cases} -\frac{q_D^2}{q_A+q_D}\rho_d\rho_u\left(1-\rho_d\right)^2-\frac{q_D(1-q_D)}{q_A+q_D}\left(1-\rho_d\right)^3 \\ \quad -q_D^2\rho_d\rho_u\left(1-\rho_d\right)\left(1-\rho_d-\rho_u\right)+2\left(1-q_D\right)+2q_D\rho_d \\ \quad -q_D\left(1-q_D\right)\left(1-\rho_d\right)^2-q_D\left(1-q_D\right)\rho_d\rho_u+q_D\rho_d\left(1-\rho_d^2\right)\left(1-\rho_u\right) \\ \quad +q_Aq_D\rho_d\left(1-\rho_d\right)^2\left(1-\rho_u\right) \qquad \text{when } CTS>BP \\ -\frac{q_D^2}{q_A+q_D}\rho_d\rho_u\left(1-\rho_d\right)^2-\frac{q_D(1-q_D)}{q_A+q_D}\left(1-\rho_d\right)^2 \\ \quad -q_D^2\rho_d\rho_u\left(1-\rho_d\right)\left(1-\rho_d-\rho_u\right)+2\left(1-q_D\right)+2q_D\rho_d \\ \quad -q_D\left(1-q_D\right)\left(1-\rho_d\right)^2-q_D\left(1-q_D\right)\rho_d\rho_u+q_D\rho_d\left(1-\rho_d^2\right)\left(1-\rho_u\right) \\ \quad +q_Aq_D\rho_d\left(1-\rho_d\right)^2\left(1-\rho_u\right) \qquad \text{when } CTS\le BP \end{cases}$$

(A.43)

We now compare $V_{t_1}^{D_l}\left(\text{II-}(-1,-1)\right)$ and $V_{t_1}^{D_l}\left(\text{III-}(-1,-2)\right)$. It is easy to check that $V_{t_1}^{D_l}\left(\text{III-}(-1,-2)\right)$ is greater than $V_{t_1}^{D_l}\left(\text{II-}(-1,-1)\right)$ for both $CTS > BP$ and $CTS \le BP$. Thus "sell one unit" is a strictly dominated strategy for *D* when *A* chooses to sell one unit in the first period.

(iii) *A*: front-run and sell two units at t_1

(iv) *D*: do not trade at t_1

Strategy IV-(-2,0): *D* does not sell while *A* sells two units at t_1.

Since *D* does not sell in the first period, *A* sells two units to two small traders at the bilateral bargaining price. If no switch occurs, *D* can only find one buyer, the arbitrageur, at t_2 on the market (subgame IV-i). *A* is then able to rebuild her position by buying two units back from *D*. If one or both small traders, S_{ho}, occur type switching, e.g., subgame (IV-ii and iii), A_{hn} buys from S_{ho} if $CTS > BP$ and from *D* if $CTS \le BP$. Thus the expected payoff to *D* is

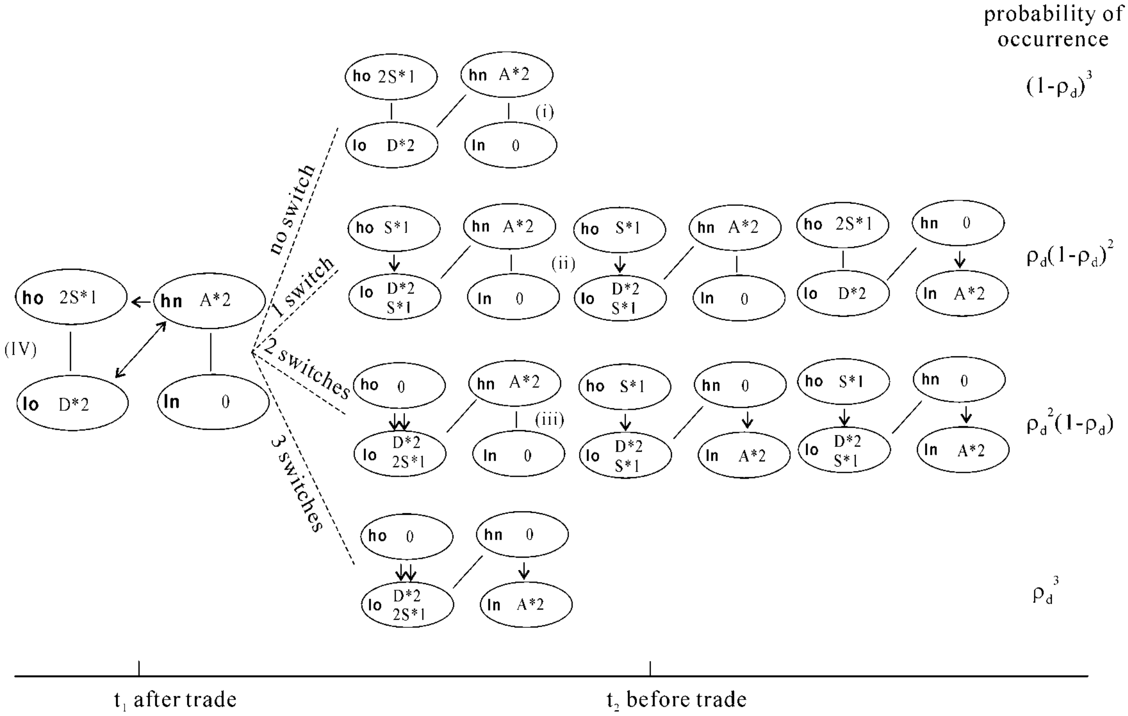


Figure A-7. Type switching and game evolution for strategy IV-(-2,0)

$$V_{t_1}^{D_l}\left(\text{IV-}(\text{-}2{,}0)\right)=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\begin{cases}\left[1-\frac{q_D}{q_A+q_D}\left(1-\rho_d\right)^3\right] & \text{when } CTS>BP\\ \left[1-\frac{q_D}{q_A+q_D}\left(1-\rho_d\right)^2\right] & \text{when } CTS\le BP\end{cases} \tag{A.44}$$

The expected payoff to A is then

$$\begin{aligned}V_{t_1}^{A_{hn}}\left(\text{IV-}(\text{-}2{,}0)\right)&=2P_{t_1}^{A_h-S_h}\left(1\middle|\text{IV}\right)\\&+\frac{2\varepsilon}{r^2}\begin{cases}\frac{q_A}{q_A+q_D}\left(1-\rho_d\right)^3+q_A\left[\rho_d\left(1-\rho_d\right)^2-\rho_d\rho_u\left(1-\rho_d\right)\right]\\ \qquad\qquad\qquad\qquad\qquad \text{when } CTS>BP\\ \frac{q_A}{q_A+q_D}\left(1-\rho_d\right)^2 \qquad\qquad \text{when } CTS\le BP\end{cases}\end{aligned} \tag{A.45}$$

$$\begin{aligned}V_{t_1}^{S_{ho}}\left(1\middle|\text{IV}\right)&=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\\&\begin{cases}\rho_d\left(1-\rho_d^2\right)+\rho_d^2\left(1-\rho_u\right)+q_A\rho_d\left(1-\rho_d\right)\left(1-\rho_d-\rho_u\right)\\ \qquad\qquad\qquad\qquad\qquad \text{when } CTS>BP\\ \rho_d\left(2-\rho_d-\rho_u\right) \qquad\qquad \text{when } CTS\le BP\end{cases}\end{aligned} \tag{A.46}$$

Strategy V-(-2,-1): D sells one unit while A sells two units at t_1.

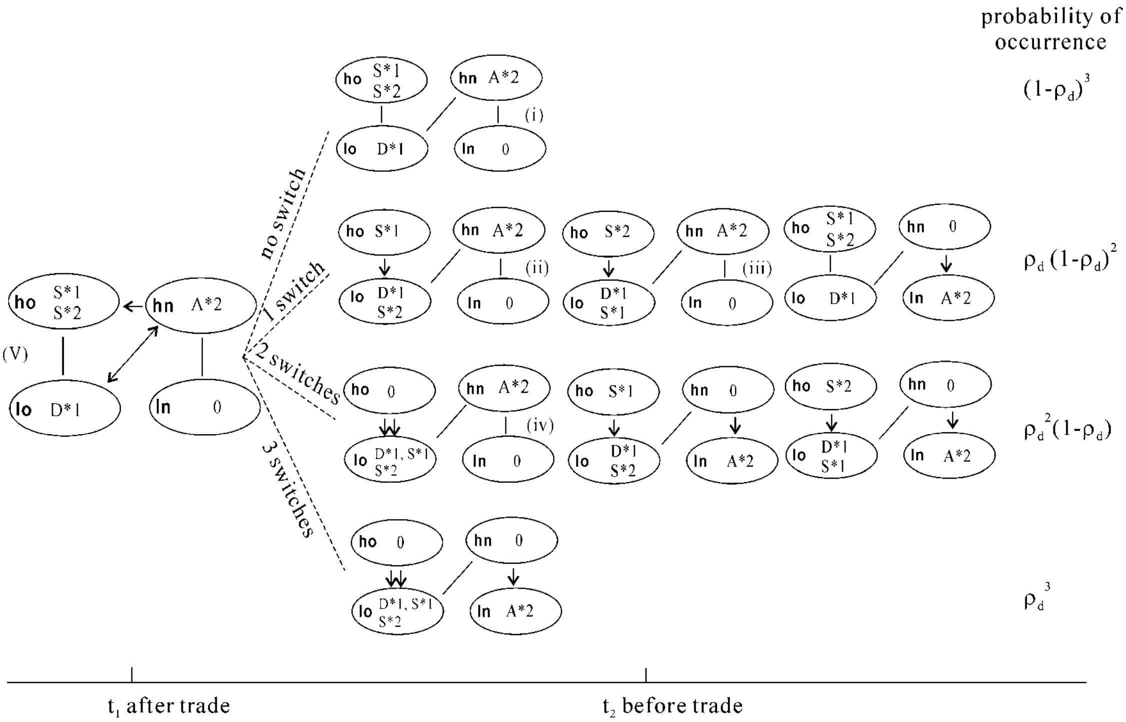


Figure A-8. Type switching and game evolution for strategy V-(-2,-1)

In subgame (V-ii, iii and iv), D competes with two other traders, S_{lo}'s, to sell to the monopoly buyer A. In subgame (V-ii and iv), A buys two units from the S_{lo} who buys two units in the first period and occurs type switching. In subgame (V-iii), A buys one unit each from D and S_{lo}. We compute D and A's value functions as follows.

$$
\begin{aligned}
V_{t_1}^{D_l}\left(\text{V-}(-2,-1)\right) = P_{t_1}^{D_l-S_h}\left(1|\text{V}\right) &+ \left[\frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\left(\frac{q_A}{q_A+q_D} + \frac{q_D}{q_A+q_D}\rho_d\right)\right] \\
&\left[(1-\rho_d)^3 + \rho_d(1-\rho_d)^2\right] \\
&+ \frac{\bar{D}-\varepsilon}{r^2}\left[2\rho_d(1-\rho_d)^2 + 3\rho_d^2(1-\rho_d) + \rho_d^3\right] \\
= P_{t_1}^{D_l-S_h}\left(1|\text{V}\right) &+ \frac{\bar{D}}{r^2} - \frac{\varepsilon}{r^2}\left[1 - \frac{q_D}{q_A+q_D}(1-\rho_d)^3\right] \qquad \text{(A.47)}
\end{aligned}
$$

$$V_{t_1}^{A_{hn}}\left(\text{V-}(\text{-2,-1})\right)=2P_{t_1}^{A_h-S_h}\left(2|\text{V}\right)+\frac{\varepsilon}{r^2}\frac{q_A}{q_A+q_D}(1-\rho_d)(1-\rho_d)^3$$
$$+\frac{2\varepsilon}{r^2}q_A(1-\rho_d)\left[\rho_d(1-\rho_d)^2+\rho_d^2(1-\rho_d)\right]$$
$$+\frac{\varepsilon}{r^2}\left[\frac{q_A}{q_A+q_D}(1-\rho_d)q_A(1-\rho_u-\rho_d)\right]\rho_d(1-\rho_d)^2$$

$$=2P_{t_1}^{A_h-S_h}\left(2|\text{V}\right)+\frac{\varepsilon}{r^2}\left[\frac{q_A}{q_A+q_D}(1-\rho_d)^3+q_A\rho_d(1-\rho_d)^2(3-\rho_d-\rho_u)\right]$$

(A.48)

$$V_{t_1}^{S_{ho}^{x2}}\left(2|\text{V}\right)=\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\left[\rho_d+\rho_d^2(1-\rho_d)+q_A\rho_d(1-\rho_d)^2\right] \quad \text{(A.49)}$$

$$V_{t_1}^{S_{ho}^{x1}}\left(1|\text{V}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\Big[\rho_d+\rho_d^2(1-\rho_d)(2-\rho_d)-\rho_u\rho_d^2(1-\rho_d)$$
$$-\rho_u\rho_d^2+q_A\rho_d(1-\rho_d)^2(1-\rho_u-\rho_d)\Big] \quad \text{(A.50)}$$

Strategy VI-(-2,-2): Both D and A sell two units at t_1.

In subgame (VI-i), A buys back two units from S_{lo}. In subgame (VI-ii), A randomly trades with one of the two S_{lo}'s at the bilateral bargaining price $P_{t_2}^{A_{hn}-S_{lo}^{x2}}$ given by (A.31). In subgame (VI-iii), D_{hn} buys two units from S_{lo}. In subgame (VI-iv), both D_{hn} and A_{hn} buy two units back from two S_{lo}'s. Subgame (VI-vi) is similar to (VI-ii), in which D_{hn} randomly buys two units from the two S_{lo}'s at $P_{t_2}^{D_{hn}-S_{lo}^{x2}}$.

Thus D and A's expected payoffs to strategy VI-(-2,-2) are as follows.

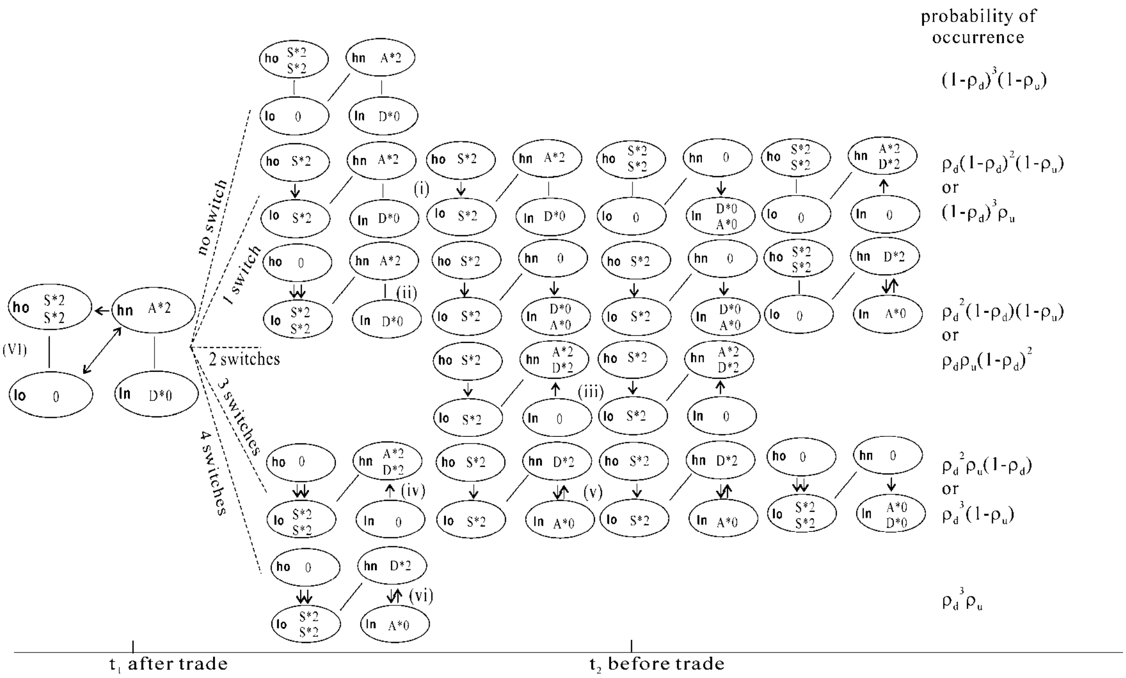


Figure A-9. Type switching and game evolution for strategy VI-(-2,-2)

$$V_{t_1}^{D_l}\left(\text{VI-}(\text{-2,-2})\right)=2P_{t_1}^{D_l-S_h}\left(2\middle|\text{VI}\right)+\frac{2\varepsilon}{r^2}q_D\left(1-\rho_d\right)\left[2\rho_d\rho_u\left(1-\rho_d\right)^2+3\rho_d^2\rho_u\left(1-\rho_d\right)+\rho_d^3\rho_u\right]$$
$$=2P_{t_1}^{D_l-S_h}\left(2\middle|\text{VI}\right)+\frac{2\varepsilon}{r^2}q_D\rho_d\rho_u\left(1-\rho_d\right)\left(2-\rho_d\right) \tag{A.51}$$

$$\begin{aligned}V_{t_1}^{A_h}\left(\text{VI-}(\text{-2,-2})\right)=2P_{t_1}^{A_h-S_h}\left(2\middle|\text{VI}\right)+\frac{2\varepsilon}{r^2}q_A\Big[&\rho_d\left(1-\rho_d\right)^2\left(1-\rho_u\right)\\&+\rho_d\left(1-\rho_d\right)^3\left(1-\rho_u\right)+\rho_d^2\rho_u\left(1-\rho_d\right)^2\Big]\end{aligned} \tag{A.52}$$

$$\begin{aligned}V_{t_1}^{S_{ho}^{\text{x2}}}\left(2\middle|\text{VI}\right)=\frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\Big[&2\rho_d+3\rho_d^2-4\rho_d^3+\rho_d^4-3\rho_u\rho_d^2+5\rho_u\rho_d^3\\&-2\rho_u\rho_d^4+q_A\left(1-\rho_d\right)\left[2\rho_d\left(1-\rho_d\right)^2\left(1-\rho_u\right)+\rho_d^2\left(1-\rho_d\right)\right]\\&+q_D\left(1-\rho_d\right)\left[2\rho_d\rho_u\left(1-\rho_d\right)+\rho_u\rho_d^2\right]\Big]\end{aligned} \tag{A.53}$$

We next determine D's optimal trading strategy when A chooses to sell two units at t_1. D compares her expected payoffs of "sell one unit" and "no trade".

$$\left(1-q_D\right)\left[V_{t_1}^{D_l}\left(\mathrm{V}-(-2,-1)\right)-V_{t_1}^{D_l}\left(\mathrm{IV}-(-2,0)\right)\right]= \\ q_D\left[\left(V_{t_1}^{S_h}\left(1\middle|\mathrm{V}\right)-P_{t_1}^{D_l-S_h}\left(1\middle|\mathrm{V}\right)\right)-\left(V_{t_1}^{S_h}\left(1\middle|\mathrm{IV}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{IV}\right)\right)\right] \tag{A.54}$$

To solve for $P_{t_1}^{D_l-S_h}\left(1\middle|\mathrm{V}\right)$, we need to know $P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{IV}\right)$ first. Thus we look at *A*'s optimal strategy when *D* does not sell in the first period. *A* compares her expected payoffs to strategies VII-(0, 0), I-(-1, 0) and IV-(-2, 0). *A* would choose "sell one unit" over "no trade" if

$$\left(1-q_A\right)\left[V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)-V_{t_1}^{A_h}\left(\mathrm{VII}-(0,0)\right)\right]=q_A\left[V_{t_1}^{S_h}\left(1\middle|\mathrm{I}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{I}\right)\right] \tag{A.55}$$

$$P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{I}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\begin{cases}\frac{q_A(1-q_A)}{q_A+q_D}+\left(2-\frac{3q_A(1-q_A)}{q_A+q_D}\right)\rho_d+\left(\frac{3q_A(1-q_A)}{q_A+q_D}-1\right)\rho_d^2-\frac{q_A(1-q_A)}{q_A+q_D}\rho_d^3 \\ -q_A\rho_d\rho_u \qquad \text{when } CTS>BP \\ \frac{q_A(1-q_A)}{q_A+q_D}+\left(2q_A-\frac{2q_A(1-q_A)}{q_A+q_D}\right)\rho_d+\left(\frac{q_A(1-q_A)}{q_A+q_D}-1\right)\rho_d^2 \\ -q_A\rho_d\rho_u \qquad \text{when } CTS\le BP\end{cases} \tag{A.56}$$

$$V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)=\frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2} \\ \begin{cases}-\frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^3+4\rho_d-2\rho_d^2-q_A\rho_d\left(1-\rho_d\right)^2-q_A\rho_d^2\rho_u \\ \qquad \text{when } CTS>BP \\ -\frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^2+\left(2+2q_A\right)\rho_d-2\rho_d^2-q_A\rho_d\rho_u \\ \qquad \text{when } CTS\le BP\end{cases} \tag{A.57}$$

A would choose to sell two units instead of one if

$$\left(1-q_A\right)\left[V_{t_1}^{A_h}\left(\mathrm{IV}-(-2,0)\right)-V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)\right]=q_A\left[V_{t_1}^{S_h}\left(1\middle|\mathrm{IV}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{IV}\right)\right] \tag{A.58}$$

$$P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{IV}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\begin{cases}\frac{q_A(1-q_A)}{q_A+q_D}\left(1-\rho_d\right)^3+2\rho_d-\rho_d^2-q_A\rho_d\rho_u \\ \qquad \text{when } CTS>BP \\ \frac{q_A(1-q_A)}{q_A+q_D}\left(1-\rho_d\right)^2+\frac{2\left(1+q_A-q_A^2\right)}{2-q_A}\rho_d-\rho_d^2-q_A\rho_d\rho_u \\ \qquad \text{when } CTS\le BP\end{cases}$$

(A.59)

$$V_{t_1}^{A_h}\left(\mathrm{IV}-(-2,0)\right)=$$

$$\frac{2\bar{D}}{r^2}-\frac{2\varepsilon}{r^2}\begin{cases}-\frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^3+2\rho_d-\rho_d^2-q_A\rho_d\left(1-\rho_d\right)^2-q_A\rho_d^2\rho_u \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{when } CTS>BP \\ -\frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^2+\frac{2\left(1+q_A-q_A^2\right)}{2-q_A}\rho_d-\rho_d^2-q_A\rho_d\rho_u \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{when } CTS\le BP\end{cases}$$

(A.60)

Compare A's value functions $V_{t_1}^{A_h}\left(\mathrm{VII}-(0,0)\right)$, $V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)$ and $V_{t_1}^{A_h}\left(\mathrm{IV}-(-2,0)\right)$.

$$V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)-V_{t_1}^{A_h}\left(\mathrm{VII}-(0,0)\right)$$
$$=\frac{\varepsilon}{r^2}\begin{cases}\frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^3+q_A\rho_d\left(1-\rho_d\right)^2+q_A\rho_d^2\rho_u & \text{when } CTS>BP \\ \frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^2+2\left(1-q_A\right)\rho_d+q_A\rho_d\rho_u & \text{when } CTS\le BP\end{cases}$$

Obviously $V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)>V_{t_1}^{A_h}\left(\mathrm{VII}-(0,0)\right)$.

$$V_{t_1}^{A_h}\left(\mathrm{IV}-(-2,0)\right)-V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)$$
$$=\frac{\varepsilon}{r^2}\begin{cases}\frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^3+q_A\rho_d\left(1-\rho_d\right)^2+q_A\rho_d^2\rho_u & \text{when } CTS>BP \\ \frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^2+2\frac{q_A^2-q_A}{2-q_A}\rho_d+q_A\rho_d\rho_u & \text{when } CTS\le BP\end{cases}$$

When $CTS>BP$, $V_{t_1}^{A_h}\left(\mathrm{IV}-(-2,0)\right)>V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)$; when $CTS\le BP$, $V_{t_1}^{A_h}\left(\mathrm{IV}-(-2,0)\right)$ can be either greater or less than $V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)$.

Therefore, given D does not trade in the first period, A's optimal strategy is to sell two units when $CTS>BP$ and to sell one unit or two units when $CTS\le BP$.

Let's go back to D's decision when A chooses to sell two units at t_1. Substituting (A.58) into equation (A.54), (A.54) can be rewritten as

$$\left(1-q_D\right)\left[V_{t_1}^{D_l}\left(\mathrm{V}-(-2,-1)\right)-V_{t_1}^{D_l}\left(\mathrm{IV}-(-2,0)\right)\right]=$$
$$q_D\left[\left(V_{t_1}^{S_h}\left(1|\mathrm{V}\right)-P_{t_1}^{D_l-S_h}\left(1|\mathrm{V}\right)\right)\right]-\frac{\left(1-q_A\right)q_D}{q_A}\left[V_{t_1}^{A_h}\left(\mathrm{IV}-(-2,0)\right)-V_{t_1}^{A_h}\left(\mathrm{I}-(-1,0)\right)\right]$$

(A.61)

$$P_{t_1}^{D_l-S_h}\left(1|\mathrm{V}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\begin{cases}1-q_D\left(1-\rho_d\right)+\frac{q_D\left(q_A+q_D-1-q_A^2\right)}{q_A+q_D}\left(1-\rho_d\right)^3-q_Aq_D\rho_d^2\rho_u \\ \quad +q_D\rho_d^2\left(1-\rho_d\right)\left(2-\rho_d-\rho_u\right) \\ \quad +q_D\rho_d\left(1-\rho_d\right)^2\left[1-q_A\left(\rho_d+\rho_u\right)\right] \quad \text{when } CTS>BP \\ 1-q_D+\frac{q_D\left(3q_A-4q_A^2\right)}{2-q_A}\rho_d+\frac{q_D}{q_A+q_D}\left(1-\rho_d\right)^2\left[q_A\left(1-q_A\right)-\left(1-q_D\right)\left(1+\rho_d\right)\right] \\ \quad +q_D\left(1-q_A\right)\rho_d\rho_u-q_D\rho_d^2\rho_u+q_D\rho_d^2\left(1-\rho_d\right)\left(2-\rho_d-\rho_u\right) \\ \quad +q_Aq_D\rho_d\left(1-\rho_d\right)^2\left(1-\rho_d-\rho_u\right) \quad \text{when } CTS\le BP\end{cases} \tag{A.62}$$

$$V_{t_1}^{D_l}\left(\mathrm{V}-(-2,-1)\right)=\frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\begin{cases}2-q_D\left(1-\rho_d\right)+\frac{q_D\left(q_A+q_D-2-q_A^2\right)}{q_A+q_D}\left(1-\rho_d\right)^3-q_Aq_D\rho_d^2\rho_u \\ \quad +q_D\rho_d^2\left(1-\rho_d\right)\left(2-\rho_d-\rho_u\right) \\ \quad +q_D\rho_d\left(1-\rho_d\right)^2\left[1-q_A\left(\rho_d+\rho_u\right)\right] \quad \text{when } CTS>BP \\ 2-q_D+\frac{q_D\left(3q_A-4q_A^2\right)}{2-q_A}\rho_d+\frac{q_D}{q_A+q_D}\left(1-\rho_d\right)^2\left[q_A\left(1-q_A\right)-2+q_D+q_D\rho_d\right] \\ \quad +q_D\left(1-q_A\right)\rho_d\rho_u-q_D\rho_d^2\rho_u+q_D\rho_d^2\left(1-\rho_d\right)\left(2-\rho_d-\rho_u\right) \\ \quad +q_Aq_D\rho_d\left(1-\rho_d\right)^2\left(1-\rho_d-\rho_u\right) \quad \text{when } CTS\le BP\end{cases}$$

(A.63)

D chooses to sell two units if

$$\left(1-q_D\right)\left[V_{t_1}^{D_l}\left(\mathrm{VI}-(-2,-2)\right)-V_{t_1}^{D_l}\left(\mathrm{IV}-(-2,0)\right)\right]=$$
$$q_D\left[\left(V_{t_1}^{S_h}\left(2|\mathrm{VI}\right)-2P_{t_1}^{D_l-S_h}\left(2|\mathrm{VI}\right)\right)-\left(V_{t_1}^{S_h}\left(1|\mathrm{IV}\right)-P_{t_1}^{A_h-S_h}\left(1|\mathrm{IV}\right)\right)\right] \tag{A.64}$$

which can be rewritten as

$$(1-q_D)\left[V_{t_1}^{D_l}(\mathrm{VI}-(-2,-2))-V_{t_1}^{D_l}(\mathrm{IV}-(-2,0))\right]=$$

$$q_D\left[\left(V_{t_1}^{S_h}(2|\mathrm{VI})-2P_{t_1}^{D_l-S_h}(2|\mathrm{VI})\right)\right]-\frac{(1-q_A)q_D}{q_A}\left[V_{t_1}^{A_h}(\mathrm{IV}-(-2,0))-V_{t_1}^{A_h}(\mathrm{I}-(-1,0))\right]$$

$$P_{t_1}^{D_l-S_h}(2|\mathrm{VI})=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{2r^2}\begin{cases}\frac{q_D}{q_A+q_D}(1-\rho_d)^3\left[q_A(1-q_A)-2(1-q_D)\right]+q_D(2-q_D)\rho_d\rho_u(1-\rho_d)(2-\rho_d)\\ \quad +2(1-q_D)+2q_D\rho_d+q_D\rho_d^2(1-\rho_d)(3-\rho_d)\\ \quad -q_D\rho_d^2\rho_u(1-\rho_d)(3-2\rho_d)+2q_Aq_D\rho_d(1-\rho_d)^3(1-\rho_u)\\ \quad +q_Aq_D\rho_d^2(1-\rho_d)^2+q_D(1-q_A)\rho_d(1-\rho_d)^2+q_D(1-q_A)\rho_d^2\rho_u\\ \qquad\qquad\qquad\qquad\qquad\qquad\text{when } CTS>BP\\ \frac{q_D}{q_A+q_D}(1-\rho_d)^2\left[q_A(1-q_A)-2(1-q_D)\right]+q_D(2-q_D)\rho_d\rho_u(1-\rho_d)(2-\rho_d)\\ \quad +2(1-q_D)+2q_D\rho_d+q_D\rho_d^2(1-\rho_d)(3-\rho_d)\\ \quad -q_D\rho_d^2\rho_u(1-\rho_d)(3-2\rho_d)+2q_Aq_D\rho_d(1-\rho_d)^3(1-\rho_u)\\ \quad +q_Aq_D\rho_d^2(1-\rho_d)^2-\frac{q_D(1-q_A)^2}{2-q_A}\rho_d+q_D(1-q_A)\rho_d\rho_u\\ \qquad\qquad\qquad\qquad\qquad\qquad\text{when } CTS\le BP\end{cases}$$

(A.65)

$V_{t_1}^{D_l}(\mathrm{VI}-(-2,-2))$ is given by (A.51). We rewrite it here,

$$V_{t_1}^{D_l}(\mathrm{VI}-(-2,-2))=2P_{t_1}^{D_l-S_h}(2|\mathrm{VI})+\frac{2\varepsilon}{r^2}q_D\rho_d\rho_u(1-\rho_d)(2-\rho_d)\qquad\text{(A.66)}$$

To characterize *D*'s optimal strategy when *A* sells two units in the first period, we examine numerical examples by assigning real values to parameters ρ_d, ρ_u, q_A and q_D.

Firstly, we cannot find such values for ρ_d, ρ_u, q_A and q_D that $V_{t_1}^{D_l}(\mathrm{IV}-(-2,0))$ is greater than $V_{t_1}^{D_l}(\mathrm{V}-(-2,-1))$. In other words, *D* almost never chooses to do nothing when *A* sells two units in the first period.

Secondly, we can find examples in which D either sells one unit or two units. For example, when $\rho_d = \rho_u = 0.2$, $q_D = 0.55$, $q_A = 0.8$ (or 0.6) (i.e., $CTS > BP$ or $CTS \le BP$ correspondingly), $V_{t_1}^{D_l}\left(\mathrm{VI}-(-2,-2)\right)$ is greater than $V_{t_1}^{D_l}\left(\mathrm{V}-(-2,-1)\right)$. But when ρ_d = 0.81, ρ_u = 0.09, q_D = 0.93, q_A = 0.98 (or 0.95), $V_{t_1}^{D_l}\left(\mathrm{VI}-(2,2)\right)$ is less than $V_{t_1}^{D_l}\left(\mathrm{V}-(-2,-1)\right)$ in both cases. Hence, for most parameter values, D would choose to sell two units when A sells two units. For some extreme parameter values, D may choose to sell only one unit.

(v) D: sell one unit at t_1.

When D sells one unit at t_1, A's strategy set is {VIII-(0,-1), II-(-1,-1), V-(-2,-1)}. A would choose to sell one unit such that

$$\left(1-q_A\right)\left[V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)-V_{t_1}^{A_h}\left(\mathrm{VIII}-(0,-1)\right)\right]=q_A\left[V_{t_1}^{S_h}\left(1\middle|\mathrm{II}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{II}\right)\right] \quad \text{(A.67)}$$

where $V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)$, $V_{t_1}^{A_h}\left(\mathrm{VIII}-(0,-1)\right)$ and $V_{t_1}^{S_h}\left(1\middle|\mathrm{II}\right)$ are given by (A.28), (A.8) and (A.29) respectively.

$$P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{II}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\begin{cases}\frac{q_A(1-q_A)}{q_A+q_D}\left(1-\rho_d\right)^4+\left(2-q_A\right)\rho_d+\left(\frac{5}{2}q_A-1\right)\rho_d^2-2q_A\rho_d^3 \\ +\frac{1}{2}q_A\rho_d^4+q_A\left(2-q_A\right)\rho_d\left(1-\rho_d\right)\left(1-\rho_d-\rho_u\right)\left(1-\frac{1}{2}\rho_d\right) \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{when } CTS > BT \\ \frac{q_A(1-q_A)}{q_A+q_D}\left(1-\rho_d\right)^2+2\rho_d-\rho_d^2 \qquad \text{when } CTS \le BP\end{cases} \quad \text{(A.68)}$$

$$V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)=\frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2}$$

$$\begin{cases} -\frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^4+\left(4-q_A\right)\rho_d+\left(\frac{5}{2}q_A-2\right)\rho_d^2-2q_A\rho_d^3 \\ +\frac{1}{2}q_A\rho_d^4-q_A^2\rho_d\left(1-\rho_d\right)\left(1-\rho_d-\rho_u\right)\left(1-\frac{1}{2}\rho_d\right) \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{when } CTS>BT \\ -\frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^2+4\rho_d-2\rho_d^2 \qquad \text{when } CTS\le BP \end{cases}$$

(A.69)

Comparing $V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)$ and $V_{t_1}^{A_h}\left(\mathrm{VIII}-(0,-1)\right)$, we have

$$\begin{aligned} &V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)-V_{t_1}^{A_h}\left(\mathrm{VIII}-(0,-1)\right) \\ &=\frac{\varepsilon}{r^2}\begin{cases} \frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^4+q_A\rho_d+q_A^2\rho_d\left(1-\rho_d\right)\left(1-\rho_d-\rho_u\right)\left(1-\frac{1}{2}\rho_d\right) \\ \quad -\frac{5}{2}q_A\rho_d^2+2q_A\rho_d^3-\frac{1}{2}q_A\rho_d^4 \qquad \text{when } CTS>BP \\ \frac{q_A^2}{q_A+q_D}\left(1-\rho_d\right)^2 \qquad\qquad\qquad\qquad \text{when } CTS\le BP \end{cases} \end{aligned}$$

It is easy to prove that $V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)-V_{t_1}^{A_h}\left(\mathrm{VIII}-(0,-1)\right)>0$. Hence, “no trade” is strictly dominated by “sell one unit”.

A chooses to sell two units over one unit according to the following equation.

$$\begin{aligned} &\left(1-q_A\right)\left[V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)-V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)\right]= \\ &\qquad q_A\left[\left(V_{t_1}^{S_h}\left(2\middle|\mathrm{V}\right)-2P_{t_1}^{A_h-S_h}\left(2\middle|\mathrm{V}\right)\right)-\left(V_{t_1}^{S_h}\left(1\middle|\mathrm{II}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{II}\right)\right)\right] \end{aligned} \tag{A.70}$$

Replacing $q_A\left[V_{t_1}^{S_h}\left(1\middle|\mathrm{II}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\mathrm{II}\right)\right]$ by (A.67), the above equation can be rewritten as

$$\left(1-q_A\right)\left[V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)-V_{t_1}^{A_h}\left(\mathrm{VIII}-(0,-1)\right)\right]=q_A\left(V_{t_1}^{S_h}\left(2\middle|\mathrm{V}\right)-2P_{t_1}^{A_h-S_h}\left(2\middle|\mathrm{V}\right)\right)$$

(A.71)

$$\begin{aligned} P_{t_1}^{A_h-S_h}\left(2\middle|\mathrm{V}\right)=\frac{\bar{D}}{r^2}-\frac{\varepsilon}{2r^2}\Big[&\tfrac{q_A\left(1-q_A\right)}{q_A+q_D}\left(1-\rho_d\right)^3+4\rho_d-2\rho_d^2 \\ &+2q_A\left(1-q_A\right)\rho_d\left(1-\rho_d\right)^2\left(2-\rho_d-\rho_u\right)\Big] \end{aligned} \tag{A.72}$$

$$V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)=\frac{2\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\Big[-\tfrac{q_A^2}{q_A+q_D}(1-\rho_d)^3+4\rho_d-2\rho_d^2$$
$$-q_A\rho_d(1-\rho_d)^2(1-\rho_d-\rho_u) \tag{A.73}$$
$$-2q_A^2\rho_d(1-\rho_d)^2(2-\rho_d-\rho_u)\Big]$$

Comparing $V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)$ and $V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)$, we have

$$V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)-V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)$$
$$=\frac{\varepsilon}{r^2}\begin{cases}\tfrac{q_A^2}{q_A+q_D}(1-\rho_d)^3\rho_d-q_A\rho_d\left(\rho_u+\tfrac{1}{2}\rho_d\right)(1-\rho_d)^2+2q_A^2\rho_d(1-\rho_d)^2 & \\ \quad +q_A^2\rho_d(1-\rho_d)(1-\rho_d-\rho_u)\left(1-\tfrac{3}{2}\rho_d\right) & \text{when } CTS>BP \\ -\tfrac{q_A^2}{q_A+q_D}(1-\rho_d)^2\rho_d+2q_A\rho_d(1-\rho_d)^2 & \\ \quad +q_A\rho_d(1-\rho_d)^2(1-\rho_d-\rho_u)+2q_A^2\rho_d(1-\rho_d)^2(2-\rho_d-\rho_u) & \\ & \text{when } CTS\le BP\end{cases}$$

It is easy to see that when $CTS \le BP$,

$$V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)-V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)>0.$$

That is, *A* always sells two units. When $CTS > BP$, it is not straightforward to prove

$$V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)-V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)>0$$

Examples show that for most values of ρ_d, ρ_u, q_A and q_D, $V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)$ is greater than $V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)$. There also exists extreme values that $V_{t_1}^{A_h}\left(\mathrm{V}-(-2,-1)\right)$ is less than $V_{t_1}^{A_h}\left(\mathrm{II}-(-1,-1)\right)$. Therefore, for the condition $CTS > BP$, *A* may choose to sell one unit or sell two units depending on parameter values.

(vi): *D*: sell two units at t_1.

We consider *A*'s optimal response to *D*'s strategy of "sell two units" in the first period. *A* chooses to sell one unit over not to trade if

$$\left(1-q_A\right)\left[V_{t_1}^{A_h}\left(\text{III}-(-1,-2)\right)-V_{t_1}^{A_h}\left(\text{IX}-(0,-2)\right)\right]=$$
$$q_A\left[\left(V_{t_1}^{S_h}\left(1\middle|\text{III}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\text{III}\right)\right)-\left(V_{t_1}^{S_h}\left(1\middle|\text{IX}\right)-P_{t_1}^{D_l-S_h}\left(1\middle|\text{IX}\right)\right)\right] \tag{A.74}$$

where $V_{t_1}^{A_h}\left(\text{III}-(-1,-2)\right)$, $V_{t_1}^{A_h}\left(\text{IX}-(0,-2)\right)$ and $V_{t_1}^{S_h}\left(1\middle|\text{III}\right)$ are given by (A.35), (A.12) and (A.36). Replacing $V_{t_1}^{S_h}\left(1\middle|\text{IX}\right)-P_{t_1}^{D_l-S_h}\left(1\middle|\text{IX}\right)$ by (A.17), the above equation can be rewritten as

$$\left(1-q_A\right)\left[V_{t_1}^{A_h}\left(\text{III}-(-1,-2)\right)-V_{t_1}^{A_h}\left(\text{IX}-(0,-2)\right)\right]=$$
$$q_A\left[\left(V_{t_1}^{S_h}\left(1\middle|\text{III}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\text{III}\right)\right)\right]-\frac{\left(1-q_D\right)q_A}{q_D}\left[V_{t_1}^{D_l}\left(\text{IX}-(0,-2)\right)-V_{t_1}^{D_l}\left(\text{VIII}-(0,-1)\right)\right] \tag{A.75}$$

We can derive $P_{t_1}^{A_h-S_h}\left(1\middle|\text{III}\right)$ from (A.75). This equation also shows that the bargaining price between the arbitrageur and a small buyer is affected by the distressed trader *D*'s strategic consideration.

The value function of *A* selling one unit $V_{t_1}^{A_h}\left(\text{III}-(-1,-2)\right)$ is given by (A.35), that is,

$$\begin{aligned}
V_{t_1}^{A_h}\left(\text{III-}(\text{-}1,\text{-}2)\right)=P_{t_1}^{A_h-S_h}\left(1\middle|\text{III}\right)+\frac{\bar{D}}{r^2}-\frac{\varepsilon}{r^2}\Bigg[&-\frac{q_A}{q_A+q_D}\rho_d\rho_u\left(1-\rho_d\right)^2+2\rho_d-\rho_d^2\\
&+q_A\rho_d\left(1-\rho_d\right)^2\left(\rho_d-2\right)+q_A\rho_d\rho_u\left(1-\rho_d\right)\left(1+\rho_d\right)\left(2\rho_d+3\right)\\
&+q_A\rho_d\rho_u^2\left(1-\rho_d\right)\left(2\rho_d-1\right)\Bigg]
\end{aligned}$$

Similarly, *A* chooses the strategy of "sell two units" over "sell one unit" according to

$$\left(1-q_A\right)\left[V_{t_1}^{A_h}\left(\text{VI}-(-2,-2)\right)-V_{t_1}^{A_h}\left(\text{III}-(-1,-2)\right)\right]=$$
$$q_A\left[\left(V_{t_1}^{S_h}\left(2\middle|\text{VI}\right)-2P_{t_1}^{A_h-S_h}\left(2\middle|\text{VI}\right)\right)-\left(V_{t_1}^{S_h}\left(1\middle|\text{III}\right)-P_{t_1}^{A_h-S_h}\left(1\middle|\text{III}\right)\right)\right]$$

which can be written as

$$(1-q_A)\left[V_{t_1}^{A_h}(\mathrm{VI}-(-2,-2))-V_{t_1}^{A_h}(\mathrm{IX}-(0,-2))\right]=$$

$$q_A\left[\left(V_{t_1}^{S_h}(2|\mathrm{VI})-2P_{t_1}^{A_h-S_h}(2|\mathrm{VI})\right)\right]-\frac{(1-q_D)q_A}{q_D}\left[V_{t_1}^{D_l}(\mathrm{IX}-(0,-2))-V_{t_1}^{D_l}(\mathrm{VIII}-(0,-1))\right] \tag{A.76}$$

Substituting $V_{t_1}^{A_h}(\mathrm{VI}-(-2,-2))$, $V_{t_1}^{A_h}(\mathrm{IX}-(0,-2))$, $V_{t_1}^{S_h}(2|\mathrm{VI})$ back into this equation, $P_{t_1}^{A_h-S_h}(2|\mathrm{VI})$ can be derived. $V_{t_1}^{A_h}(\mathrm{VI}-(-2,-2))$ can also be calculated by (A.52).

We compare *A*'s three value functions of strategies IX-(0,-2), III-(-1,-2) and VI-(-2,-2) by examining numerical examples. We find that the arbitrageur may choose any of the three strategies for both conditions $CTS > BP$ and $CTS \leq BP$.

For example, when $CTS > BP$, for ρ_d = 0.05, ρ_u = 0.3, q_A = 0.92 and q_D = 0.55, $V_{t_1}^{A_h}(\mathrm{IX}-(0,-2))$ is the largest. Hence *A* chooses to do nothing. For ρ_d = 0.83, ρ_u = 0.05, q_A = 0.92 and q_D = 0.55, *A*'s optimal response is to sell one unit. But for most moderate values such as ρ_d = 0.1, ρ_u = 0.1, q_A = 0.6 and q_D = 0.55, *A*'s optimal strategy is to sell two units.

Similarly, when $CTS \leq BP$, for ρ_d = 0.05, ρ_u = 0.3, q_A = 0.6 and q_D = 0.55, *A* does not front-run. For ρ_d = 0.83, ρ_u = 0.02, q_A = 0.6 and q_D = 0.55, *A*'s optimal response is to sell one unit. For ρ_d = 0.1, ρ_u = 0.1, q_A = 0.56 and q_D = 0.55, *A* chooses to sell two units given *D* sells two units in the first period as well. □

Proof of Proposition 1:

This result follows from substituting ρ_d and ρ_u with zero in *A* and *D*'s value functions at t_1. □

Proof of Proposition 2:

When both ρ_d and ρ_u are strictly positive but very small, for instance, 0.01, we ignore all higher order terms containing second and higher orders of ρ_d and ρ_u in A and D's value functions at t_1. It is then easy to determine a large trader's optimal response to the other large trader's strategy at t_1. We find that D always chooses to sell two units no matter what strategy A chooses in the first period, but A's response to D's strategy may change with different parameter values. When D does not trade or only sells one unit in the first period, A always front-runs and sells two units. However, "sell two units" is a strictly dominated strategy for A when D sells two units. A may choose not to trade or only sell one unit when D liquidates very quickly in the first period. In particular, A choose "sell one unit" over "no trade" only if $\rho_d \geq \underline{\rho}_d\left(q_A, q_D\right)$, where

$$\underline{\rho}_d\left(q_A, q_D\right)=\frac{\left(1-q_D\right)^2\left(2-q_D\right)}{\left(1-q_A\right)\left(2-q_D\right)+2\left(1-q_D\right)\left(3-3q_D+q_D^2\right)} \tag{A.77}$$

Differentiating (A.77) with respect to q_A and q_D, one finds that

$$\frac{\partial\underline{\rho}_d\left(q_A, q_D\right)}{\partial q_A}<0,$$

$$\frac{\partial\underline{\rho}_d\left(q_A, q_D\right)}{\partial q_D}<0,$$

$$\frac{\partial^2\underline{\rho}_d\left(q_A, q_D\right)}{\partial q_A \partial q_D}<0.$$

□

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