

**HIGH FREQUENCY ANALYSIS ON JUMPS AND LONG MEMORY  
VOLATILITY IN COMMODITY FUTURES PRICES\***

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# HIGH FREQUENCY ANALYSIS ON JUMPS AND LONG MEMORY

## VOLATILITY IN COMMODITY FUTURES PRICES\*

### Abstract

We concern the high frequency returns of 15 minute commodity futures prices. The basic FIGARCH model with the usual normality assumption is found to be inappropriate in representing the high frequency commodity futures returns and the rejection appears to be due to the jumps which are occurred in the high frequency returns. Hence, this paper relies on the generalized FIGARCH model combined with the Bernoulli distribution that allows for jumps in the high frequency commodity futures returns. This paper shows that the generalized FIGARCH-Bernoulli distribution model performs quite well and that the jumps spuriously increase the long memory persistence in the volatility process of the high frequency commodity futures returns.

*Keywords:* High frequency commodity futures, Jumps, FIGARCH, Bernoulli distribution, Long memory property.

*EMF classification code:* 420

## INTRODUCTION

We consider the high frequency 15 minute commodity futures prices of cattle, corn, hogs and gasoline.<sup>1)</sup> In particular, we focus mainly on finding an appropriate model of the high frequency commodity futures prices. Since there has been the apparent lack of any economic theory explaining the dynamics of the first two conditional moments in asset prices including commodity futures prices, many econometricians have commonly used the extended models of the traditional ARMA models for the means and the ARCH models for the variances to describe and represent the dynamic process of the asset prices. These traditional models have usually been estimated by the approximate Quasi Maximum Likelihood (QMLE) method under the assumption that the innovations are normally distributed. And, the normality assumption has been justified by Bollerslev and Wooldridge (1992).

Thus, we characterize the process of the high frequency commodity futures returns by applying some recent developments in modeling the volatility process. First, we use the relatively recent FIGARCH model of Baillie et al. (1996) with the Gaussian normality assumption in order to represent the high frequency commodity futures returns. The primary results from the Maximum Likelihood Estimation (MLE) of the basic FIGARCH model indicate that the FIGARCH model under the Gaussian normality assumption generally seems to match the dynamics of high frequency futures returns and is a satisfying starting point for studying the

underlying features of the high frequency futures returns data.

On the other hand, using the usual FIGARCH model with the normal distribution assumption leads to excess kurtosis, which may be related to conditional mean jumps in the high frequency futures returns. Franses and Ghijssels (1999) have proposed that the estimated residuals from GARCH model have excess kurtosis due to neglected additive outliers (AOs). These jumps might lead to “outliers” in the level and volatility process that cannot be taken accounted for by the simple normal distribution model (Hotta and Tsay, 1998). Accordingly, this paper analyzes jumps in the conditional mean process of the high frequency futures return series. Jumps in the conditional mean process are of significant interest, and the long memory property in the conditional variance process cannot be extracted without an appropriate specification of the conditional mean process.

The basic FIGARCH model assuming a normal distribution is unlikely to represent the process of high frequency futures returns with a mixture of distributions. For this purpose, it appears more useful to apply the jump diffusion process proposed by Press (1967) in order to properly account for the jumps. Since the statistical and economic explanations for the jumps and the long memory property are quite different, this paper employs a normal mixture distribution model, the FIGARCH model combined with Bernoulli jump process to account for the jumps in the conditional mean process and the long memory property in the conditional variance process.

Thus, in order to consider the existence of the jumps, we adopt the generalized

FIGARCH model combined with the Bernoulli distribution which allows for a jump possibility instead of the usual normal distribution assuming the jump probability is constant and is exogenously determined. We find that the FIGARCH-Bernoulli distribution model performs quite well and that a normal mixture distribution model, the FIGARCH model combined with the Bernoulli jump process, can improve estimates of the long memory parameter. Specification of the conditional mean process without considering the jumps seems to cause distorted higher estimates of the long memory parameter in the volatility process of the high frequency futures returns. This is quite understandable given that the jumps which otherwise may be spuriously associated with additional volatility are fully accounted for in the mixture distribution.

The results of this paper can provide important implications for the understanding of the intraday dynamics of the high frequency commodity prices and hence for empirical applications such as optimal hedge ratio estimation, tests for futures market efficiency, tests for the risk management, option valuation and portfolio management.

The plan of the rest of this paper is as follows. The next section describes the 15 minute commodity futures returns of cattle, corn, hog and gasoline and the basic properties of the high frequency commodity futures returns data. In particular, a strong intraday periodicity and a long memory property are found to be very significant in the high frequency commodity futures returns. This is followed by the application of the long memory volatility, FIGARCH model to

represent the high frequency commodity futures returns. For the analysis of the high frequency commodity returns, we first apply the Flexible Fourier Form (FFF) proposed by Gallant (1981, 1982) to eliminate the intraday periodicity in the high frequency commodity futures returns and then uses the basic FIGARCH model of Baillie et al. (1996) with a normal distribution to estimate the long memory property in the volatility process of the high frequency filtered commodity futures returns. The FIGARCH model is found to be econometrically superior to the model of the regular stable GARCH model. But the primary results show excess kurtosis which cannot be accounted for by the normal distribution model.

And, the next section then analyzes jumps in the conditional mean process of the high frequency futures returns using a normal mixture distribution model. The FIGARCH model combined with Bernoulli process is to represent the conditional mean jumps and the long memory volatility process of the high frequency commodity futures returns. In particular, the Bernoulli jump process is found to be quite appropriate for accounting for the conditional mean jumps and in capturing the effects of the jumps on the high commodity frequency commodity futures returns data. The final section provides a brief conclusion.

## **BASIC ANALYSIS OF HIGH FREQUENCY FUTURES RETURNS**

We examine four high frequency commodity futures data; cattle, corn, hogs, and

gasoline, which are obtained from the Futures Industry Institute data center. The high frequency commodity futures prices are for real-time transaction records, which we initially convert to 15-minute price intervals by using the last price quoted before the end of every 5-minute interval over the trading day.<sup>2)</sup> Cattle and hogs are both important livestock commodities in U.S. agriculture but their different life cycles mean different inherent price dynamics, even though they are found to have a lot of similarity in the stochastic properties of prices for these two livestock commodities as presented by Baillie et al. (2007). Corn is a major annual crop that is of critical importance to U.S. agriculture since it is used heavily as animal feed, and Gasoline is included to see if results are markedly different for a natural resource based commodity.

The returns of the 15-minute commodity futures prices are defined in the conventional manner as continuously compounded rates of return and calculated as the first difference of the natural logarithm of prices. The  $n$ -th interval return during day  $t$  is defined as

$$R_{t,n} = 100 \times [\ln(P_{t,n}) - \ln(P_{t,n-1})] \quad (1)$$

where  $t = 1, \dots, T$  (trading days),  $n = 1, \dots, K$  (intraday intervals) and  $P_{t,n}$  is the futures price for the  $n$ -th intraday interval during trading day  $t$ . The details of the basic statistics and the sample periods used for the raw (unadjusted) 15-minute futures returns are provided in Table 1. For example, the sample mean of the 15-minute corn futures return is found to be -0.0041 which are very close to zero and indistinguishable at the standard significance level given the sample

deviations of 0.01 and 0.1. And, Figure 1 displays corresponding picture of the 15 minute corn future returns representing that the returns are centered on zero but there exist several jumps (large changes) and obvious volatility clustering in the high frequency futures return series.

However, the high frequency corn returns appear not to be normally distributed since the sample skewness and kurtosis are -0.2142 and 5.2148, which are all found to be statistically significant. In particular, the estimated kurtosis statistics for the high frequency corn futures returns is found to be relatively large, which implies the rejection of a Gaussian normal distribution assumption. The high excess kurtosis may be due to the occurrences of numerous jumps that have taken place in the high frequency corn futures returns as presented in Figure 1. These jumps could lead to the level and volatility outliers that the normal distribution cannot take into account (Hotta and Tsay, 1998). Actually, the high frequency corn futures returns are characterized by several large jumps or shifts followed by ostensibly random movement. The jumps in the high frequency corn futures returns may be caused by several financial and economic events in the corn futures markets. The corresponding graphs for the other commodities are not shown to reserve space but they all exhibit the quite similar pattern.

The volatility processes in the high frequency commodity futures returns are further analyzed. Figure 2 plots the sample autocorrelations of high frequency corn futures returns for lags of up to 10 trading days in 15-minute intervals displayed in the horizontal axis for the



returns, the squared returns and the absolute returns of the raw 15-minute high frequency corn futures returns series. In particular, Figure 2 shows that there generally exists a small negative but significant first-order autocorrelation in returns, which may be due to the non-synchronous trading phenomenon while higher order autocorrelations are not significant at conventional levels. The autocorrelation functions of the absolute returns exhibit a pronounced U shape suggesting substantial intraday periodicity and decay very slowly at the hyperbolic rate, which is a typical feature of a long memory property. These are in line with the findings of Cai et al. (2001) who characterized similar intraday periodicity in the 5-minute high frequency gold prices. To conserve space the corresponding graphs for the other commodities are not shown but are available upon request to the authors. However, it is observed that the similar shapes and the amplitudes of the intraday periodicity in the autocorrelations of absolute returns exist in the other commodities. This seasonal pattern seems to be closely related to the intraday trading activity in commodity futures markets as presented by Muller et al. (1990) and Cai et al. (2001).

## **FIGARCH MODEL WITH A NORMAL DISTRIBUTION**

As with many analyses of high frequency asset price returns like stocks, bonds and exchange rates (Andersen et al., 2005), it is found that the high frequency commodity futures returns display considerable intraday periodicity, which is usually attributed to institutional

trading features. This periodicity is removed using the FFF filtering method of Gallant (1981, 1982), which is explained in detail in Baillie et al. (2007). Thus, the filtered high frequency 15-minute commodity futures returns is defined as,  $y_{t,n} = R_{t,n} / s_{t,n}$  where  $s_{t,n}$  is the intraday periodicity estimated from FFF. Figure 3 represents the correlograms of the filtered high frequency corn futures returns while the correlograms of the other commodities are not included in this paper to reserve space but they are available upon the request to the authors. It shows that the FFF filter seems to remove much of intraday periodicity presented in the raw absolute returns successfully as in Baillie et al. (2007). The filtered high frequency corn futures returns ( $y_{t,n}$ ) are virtually found to be stationary with small autocorrelations at the first few lags. On the other hand, the volatility processes of the filtered high frequency corn futures returns are found to be very persistently autocorrelated with long memory hyperbolic decay. The long memory patterns in the volatility process of the high frequency corn futures returns are almost same as the pattern in the high frequency gold futures returns in Cai et al. (2001).<sup>3)</sup> The correlograms of the other high frequency futures returns are also found to exhibit the similar patterns.

A model that is consistent with these stylized facts is the MA( $n$ )-FIGARCH( $p, d, q$ ) process,

$$y_{t,n} = R_{t,n} / s_{t,n} = \mu + \theta(L)\varepsilon_{t,n} \quad (2)$$

$$\varepsilon_{t,n}^2 = z_{t,n}\sigma_{t,n} \quad (3)$$

$$[1 - \beta(L)]\sigma_{t,n}^2 = \omega + [1 - \beta(L) - \phi(L)(1 - L)^d] \varepsilon_{t,n}^2 \quad (4)$$

where  $s_{t,n}$  is the intraday periodicity estimated from FFF, and  $z_{t,n} \sim i.i.d.(0, 1)$ ,  $P_t$  is the asset price,  $\mu$  and  $\omega$  are scalar parameters, and  $\beta(L)$  and  $\phi(L)$  are polynomials in the lag operator to be defined later. The polynomial in the lag operator associated with the moving average process is  $\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_n L^n$ , and  $d$  continues to represent the long memory parameter.

The FIGARCH model in equation (4) is motivated by noting that the standard GARCH ( $p, q$ ) model of Bollerslev (1986) can be expressed as  $\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$ ,

where the polynomials are  $\alpha(L) \equiv \alpha_1 L + \alpha_2 L^2 + \dots + \alpha_q L^q$ ,  $\beta(L) \equiv \beta_1 L + \beta_2 L^2 + \dots + \beta_p L^p$ .

The GARCH( $p, q$ ) process can also be expressed as the ARMA[ $\max(p, q), p$ ] process in squared innovations  $[1 - \alpha(L) - \beta(L)]\varepsilon_t^2 = \omega + [1 - \beta(L)]\nu_t$  where  $\nu_t \equiv \varepsilon_t^2 - \sigma_t^2$ , and is a zero mean, serially uncorrelated process which has the interpretation of being the innovations in the conditional variance. Similarly, the FIGARCH( $p, d, q$ ) process can be written naturally as

$$\phi(L)(1 - L)^d \varepsilon_t^2 = \omega + [1 - \beta(L)]\nu_t, \quad (5)$$

where  $\phi(L) = [1 - \alpha(L) - \beta(L)]$  is a polynomial in the lag operator of order  $\max(p, q)$ .

Equation (5) can be easily shown to transform to equation (4), which is the standard representation for the conditional variance in the FIGARCH( $p, d, q$ ) process. Further details concerning the FIGARCH process can be found in Baillie et al. (1996). The parameter  $d$  characterizes the long memory property of hyperbolic decay in volatility because it allows for

autocorrelations decaying at a slow hyperbolic rate.

The above model (2), (3), and (4) is estimated for futures returns on our four commodities of interest by maximizing the Gaussian log likelihood function,

$$\ln(L; \Theta) = -\left(\frac{T}{2}\right) \ln(2\pi) - \left(\frac{1}{2}\right) \sum_{t=1}^T \sum_{n=1}^k [\ln(\sigma_{t,n}^2 + \varepsilon_{t,n}^2 \sigma_{t,n}^2)] \quad (6)$$

where  $\Theta$  is a vector containing the unknown parameters to be estimated. However, it has long been recognized that most asset returns are not well represented by assuming  $z_t$  in equation (2) is normally distributed; for example see McFarland et al.(1982). Consequently, inference is usually based on the QMLE of Bollerslev and Wooldridge (1992), which is valid when  $z_t$  is non-Gaussian. Denoting the vector of parameter estimates obtained from maximizing (6) using a sample of T observations on equations (2), (3) and (4) with  $z_t$  being non-normal by  $\hat{\Theta}_T$ , then the limiting distribution of  $\hat{\Theta}_T$  is

$$T^{1/2}(\hat{\Theta}_T - \Theta_0) \rightarrow N[0, A(\Theta_0)^{-1} B(\Theta_0) A(\Theta_0)^{-1}], \quad (7)$$

where  $A(\cdot)$  and  $B(\cdot)$  represent the Hessian and outer product gradient respectively, and  $\Theta_0$  denotes the vector of true parameter values. Equation (7) is used to calculate the robust standard errors that are reported in the subsequent results in this paper, with the Hessian and outer product gradient matrices being evaluated at the point  $\hat{\Theta}_T$  for practical implementation.

Considerable previous work in the literature has examined high frequency returns in stock, equity and foreign exchange markets, but to date very little analysis has been done on

high frequency commodity returns (Cai et al., 2001; Martens and Zein, 2004; Baillie et al. 2007).

This section of the paper represents an extensive analysis of the volatility properties of high frequency commodity futures returns using the FIGARCH model with the normal distribution of Baillie et al. (1996). The orders of the MA and GARCH polynomials in the lag operator are chosen to be as parsimonious as possible but still provide an adequate representation of the autocorrelation structure of the high frequency data. The exact parametric specification of the model that best represents the degree of autocorrelation in the conditional mean and conditional variance of high frequency commodity returns are found to be the MA(1)-FIGARCH(1,  $d$ , 1) model for cattle, hogs and corn and the MA(1)-FIGARCH(0,  $d$ , 1) model for gasoline.<sup>4)</sup>

Table 2 presents results of applying the above model to high frequency commodity futures returns for the four commodities discussed earlier. All the models have small but significant MA(1) parameter estimates, which is usually attributed to the non-synchronous trading phenomenon. The estimated long memory volatility parameters,  $d$ , are in the range between 0.20 and 0.35 for most of the commodities considered and are generally statistically significant indicating the significant long memory characteristics in the volatility of the high frequency returns. Thus, the hypotheses that  $d = 0$  (stationary GARCH) and also  $d = 1$  (integrated GARCH) are consistently rejected for all commodities using standard significance levels.

Table 2 also reports the robust Wald test statistics, denoted by  $W$ , for testing the null hypothesis of GARCH versus a FIGARCH data generating process. Under the null,  $W$  will have an asymptotic  $\chi_1^2$  distribution and, from Table 2, the GARCH model is rejected for every commodity at standard significance levels. This robust Wald test supports the conclusion obtained both here and in Jin and Frechette (2004) that FIGARCH is superior to GARCH for modeling the conditional variances of the high frequency commodity futures returns. Evidently, the long memory property is the characteristic feature of high frequency commodity futures returns, and FIGARCH represents a significant improvement over GARCH. Thus, the estimated MA-FIGARCH models appear to describe the futures return data quite well so that it may be a satisfying starting point to analyze the nature of the underlying distributions in the high frequency commodity returns.

On the other hands, the focus of this paper is primarily directed at the assumption of the Gaussian normal distribution. Under the normality assumption, the estimated excess kurtosis are found to be 4.7, 6.6, 6.1 and 4.7 for the high frequency futures returns of the cattle, corn, hog and gasoline respectively, which are enough to reject the normal distribution. The normal distribution assumption seems to lead to excess kurtosis, and the excess kurtosis may be resulted from the jump (large changes) in the high frequency commodity futures prices caused by several financial and economic events as presented in Figure 1 for the corn futures returns. These events

concerning expected future flows can result in price changes well above normal and might be better captured by jumps rather than normal innovations. These jumps might lead to the level and volatility outliers which can not be taken into account for by the simple normal distribution as Hotta and Tsay (1998) presented. Thus, the assumption of the normal distribution seems to be inappropriate to represent the high frequency commodity returns series properly due to the jumps.

#### **FIGARCH-BERNOULLI DISTRIBUTION MODEL WITH JUMPS**

Since the presence of the jumps is primarily responsible for the rejection of the usual normality assumption, it seems to call for the use of another model. One model to be considered is to introduce jumps through the use of a normal mixture distribution. We employ a normal mixture distribution model, the jump diffusion model proposed by Press (1967), in order to account for the conditional jumps in the high frequency futures returns. Initially, Press (1967) proposed a jump diffusion model for stock prices under the assumption that the logarithm of the stock price follows a Brownian motion process on which i.i.d. normal distributed jumps are superimposed. In particular, we analyze the impact of jumps in the conditional mean process on the long memory property in the conditional variance process of the high frequency commodity futures returns series by using a normal mixture distribution model. Efficient estimation of the parameters of

continuous time processes is generally challenging. In order to give an alternative perspective on the continuous time formulation, it is considered interesting to fit a normal mixture model in discrete time, taking advantage of the relatively simple formulation.

Hence, in order to model the jumps occurred in the high frequency commodity futures returns appropriately, we rely on a jump-diffusion FIGARCH model that assumes the high frequency commodity returns are drawn from a mixture of normal distribution and jump process. In particular, we consider this model in the context of a Bernoulli distribution. The Bernoulli distribution models the stochastic jumps in the 15-minute high frequency commodity futures returns series. The main characteristic of the Bernoulli process is that over a fixed time period, one relevant information arrives in foreign exchange markets and a jump occurs in the high frequency commodity futures returns with probability ( $\lambda$ ) which is drawn from a Bernoulli distribution and is forced in the (0,1) interval. The jump size is given by the random variable  $v$ , which is assumed to be  $NID(v, \delta^2)$ .

The combined MA(1)-FIGARCH (1,d,1) model with Bernoulli distribution is,

$$y_{t,n} = \mu + \lambda v + \varepsilon_{t,n} + b\varepsilon_{t,n} \quad (8)$$

$$\varepsilon_{t,n} \sim (1 - \lambda)N(\lambda v, \delta^2) + \lambda N(v - \lambda v, \sigma_{t,n}^2 + \delta^2) \quad (9)$$

$$\sigma_{t,n}^2 = \omega + \beta\sigma_{t,n-1}^2 + [1 - \beta L - (1 - \phi L)(1 - L)^d] \varepsilon_{t,n}^2 \quad (10)$$

The high frequency commodity futures returns are still specified as following the MA(1)



process, with a jump probability ( $\lambda$ ) which is constant and is drawn from a Bernoulli distribution ( $0 < \lambda < 1$ ) and  $\nu$  is the mean of the jump distribution while  $\delta^2$  captures the variance of the jump distribution implying the additional volatility related to the jumps. The volatility process is the FIGARCH(1,d,1) model as developed earlier. The log likelihood function for the combined model has the following form,

$$\ln(\xi) = -\left(\frac{T}{2}\right) \ln(2\pi) - \sum_{t=1}^T \sum_{n=1}^k \left\{ \left[ \frac{(1-\lambda)}{h_{t,n}} \right] \times \exp\left[ -\frac{(\varepsilon_{t,n} + \lambda\nu)^2}{2h_{t,n}^2} \right] + \left[ \frac{\lambda}{(h_{t,n}^2 + \delta^2)^{\frac{1}{2}}} \right] \times \exp\left[ -\frac{(\varepsilon_{t,n} - (1-\lambda)\nu)^2}{2(h_{t,n}^2 + \delta^2)} \right] \right\} \quad (11)$$

The form of the likelihood function for the Bernoulli-normal mixture distribution is basically similar to that proposed by Vlaar and Palm (1993) which studied foreign exchange rates in the EMS (European Monetary System) using a GARCH framework. And, the normalized residuals are used for the statistical inference instead of the usual standardized residuals since the standardization may not lead to i.i.d. residuals in the mixture distribution model with time dependent variance as suggested by Vlaar and Palm (1997) and Beine and Laurent (2003).

Hence, this paper investigates the high frequency futures returns by combining the FIGARCH model with the Bernoulli jump diffusion models to consider jumps in the conditional mean and capture the long memory property in the conditional variance. Jump process is included in an attempt to reduce the influence of the conditional mean jumps on the MA-FIGARCH specification. The estimated parameters for the high frequency futures returns series over

different commodities are reported in Table 3.

The estimated parameters ( $\beta$ ) for the probability of a jump are all significant at the conventional level of significance across different commodities, implying that the jumps are quite significant in the conditional mean process for the high frequency commodity futures returns. The jumps intensities ( $\lambda$ ) calculated from the estimated ( $\beta$ ) are 0.18, 0.13, 0.15 and 0.11 for the high frequency commodity futures returns of cattle, corn, hog and gasoline respectively, which indicate the probability of jumps in the high frequency commodity futures returns occurring during the sample period. One interesting issue concerns the interpretation of the jumps and whether or not they correspond to economic and financial events in the commodity futures markets. Without more detailed information, it is difficult to distinguish these effects. We would leave this issue for a future study.

The estimated parameters ( $\nu$ ) which represent the impacts of the jumps on the mean process are found to be insignificant for the high frequency commodity futures returns. This may be due to a general pattern of very quick and effective exchange rate conditional mean adjustment after the jumps. However, the effects of the jumps on the volatility process ( $\delta^2$ ) of the high frequency futures returns are estimated to be very significant and much greater than those on the mean process. The effects of jumps on volatility process appear to be more important and more significant than the effects on the mean process. These results are generally similar to the

case of the high frequency gold futures returns in Cai et al. (2001)

In particular, the estimated long memory parameters of the high frequency returns are 0.1733, 0.1134, 0.1129 and 0.1437 for the high frequency commodity futures returns of cattle, corn, hog and gasoline respectively, and they are all very significant. The long memory parameters are found to be much lower than those estimated from the basic MA-FIGARCH model without considering the jumps. This suggests that the long memory property of the high frequency commodity futures returns may be significantly affected by jumps in the conditional mean process and higher values of the long memory parameters can be induced when jumps in the conditional mean process are not accounted for. This result is quite understandable given that the jumps which otherwise may be spuriously associated with additional volatility are fully accounted for in the mixture distribution model.<sup>5)</sup> And, the estimation results show that the kurtosis statistics are reduced significantly for the various commodity futures returns after the jumps are accounted for.

Thus, the greater long memory property and the excess kurtosis seem to be related to asymmetric adjustments to conditional variance in response to the jumps, which is much more gradual and persistent than the conditional mean adjustments. In particular, the jumps appear to be the driving force behind the long memory property in the volatility process of the high frequency futures returns. This confirms that the mixture distribution generally outperforms the

simple normal distribution and that hat accounting for non-uniform flows of information can significantly improve the fit of the model.

## **CONCLUSION**

We examine the properties of high commodity frequency returns of 15-minute cattle, corn, hog and gasoline futures prices. The strong intraday periodicity and the long memory property are found to have a significant impact on the volatility process of the high frequency futures returns. First, after eliminating the intraday periodicity by FFF method, we apply the FIGARCH model with a usual normality assumption and find that the FIGARCH model provides a better representation of the long memory property in the volatility process of the high frequency commodity futures return series than the usual GARCH model. The general appropriateness and robustness of the FIGARCH model persist for different high frequency commodity futures returns. But, there still exists high excess kurtosis in the high frequency commodity futures returns implying the rejection of the normality assumption, which may be caused by jumps in the conditional means of the high frequency commodity futures returns. Jumps in the conditional mean process of the high frequency returns data may be caused by economic and financial events in commodity futures markets.

These features can be better modeled by using the FIGARCH model combined with the

Bernoulli jump process. Such model is constructed to investigate the effects of conditional mean jumps on the long memory property for different high frequency commodity futures returns data. The combined FIGARCH-Bernoulli model appears to be quite appropriate for describing jumps in the conditional mean process and the long memory property in the volatility process of the high frequency futures returns series. In particular, the long memory parameters and the values of kurtosis estimated from the combined models are found to be much lower than those from the basic FIGARCH model without considering the conditional mean jumps. This constitutes strong evidence that the specification of the conditional mean process without considering the jumps may spuriously distort the estimates of the long memory parameters.

It is hoped that these results may be helpful in deepening our understanding the dynamics of commodity futures prices and in developing empirical applications such as optimal hedge ratio estimation, tests for futures market efficiency, tests for the announcement effect of market news, option valuation, risk management and portfolio management.

## Endnotes

1. Cai et al. (2001) have used 5-minute gold futures prices to find the effects of US news on the high frequency gold futures returns. And Martens and Zein (2004) have used high frequency oil prices data to investigate the realized daily volatility measures.
2. The original commodity futures prices data in this paper is the same as in Baillie et al. (2007). For more information and data availability see <http://www.theifm.org>.
3. The long memory pattern is found in the volatility process of daily commodity futures prices. See Baillie et al. (2007), Jin and Frechette (2004), Martens and Zein (2004), Brunetti and Gilbert (2000) and Crato and Ray (2000).
4. The Box-Pierce portmanteau statistics show that the models specified for each commodity do a good job of capturing the autocorrelations in the mean and volatility of the commodity return series. In each case there is no evidence of additional autocorrelation in the standardized residuals or squared standardized residuals, indicating that the chosen model specification provides an adequate fit. It is interesting to note that the autocorrelation in the mean tends to persist more for cattle and hogs than for the other commodities (i.e. more MA terms in the mean required for an adequate fit). Furthermore, these commodities also seem to require more flexible models to capture their autocorrelation in volatility as well (i.e. more GARCH terms required for an adequate fit).

5. Several recent papers present that the decrease of the persistence of shocks in foreign exchange rates when accounting for jumps appropriately. See Diebold and Inoue (1999), Granger and Hyung (1999) and Beine and Laurent (2003).

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**Table 1: Basic Statistics for Raw High Frequency Commodity Futures Returns**

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	Cattle	Corn	Hog	Gasoline
First Obs.	99/05/03 9:30	99/05/03 13:15	99/05/03 9:30	99/0503 10:15
Last Obs.	00/12/28 13:00	00/12/28 13:15	00/12/28 13:00	00/12/28 15:00
Sample Size	6075	6594	6000	8020
Mean	0.0036	-0.0041	0.0037	0.0069
Variance	0.0183	0.0755	0.0384	0.1590
Skewnes	-0.2142	-0.0704	-0.1634	-0.2142
Kurtosis	5.2148	9.9384	13.0998	5.4554

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**Table 2: Estimated MA-FIGARCH model for Filtered High Frequency Commodity Futures Returns**

	Cattle	Corn	Hogs	Gasoline
$\mu$	0.0031 (0.0015)	-0.0060 (0.0027)	0.0091 (0.0033)	0.0113 (0.0039)
$\theta$	-0.0525 (0.0144)	-0.0955 (0.0143)	-0.0490 (0.0158)	-0.0272 (0.0128)
$d$	0.2097 (0.0366)	0.2263 (0.0539)	0.3503 (0.0620)	0.2113 (0.0251)
$\omega$	0.0024 (0.0019)	0.0027 (0.0015)	0.0030 (0.0012)	0.0325 (0.0049)
$\beta$	0.4234 (0.3876)	0.8519 (0.0812)	0.7242 (0.0816)	0.0963 (0.0283)
$\phi$	0.3450 (0.3802)	0.8137 (0.0968)	0.5485 (0.0964)	-----
$m_3$	-0.111	0.233	-0.199	-0.294
$m_4$	4.728	6.648	6.138	4.695
Q(50)	51.138	62.609	50.327	32.422
Q <sup>2</sup> (50)	37.736	48.162	53.041	45.245
$W$	32.777	17.648	31.943	71.131

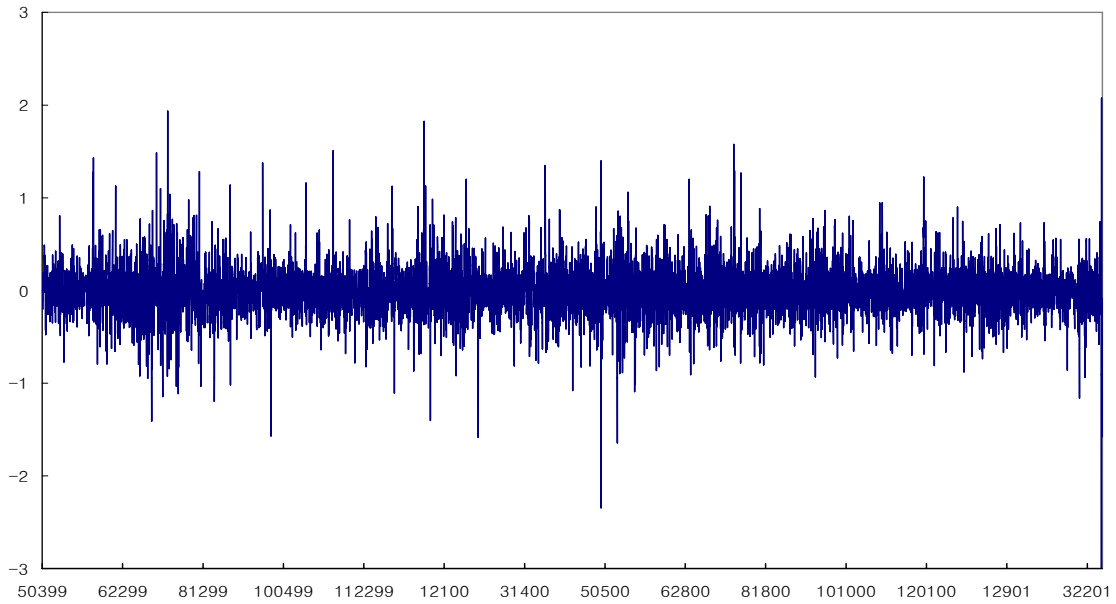
Notes: Robust standard errors based on QMLE are in parentheses below the corresponding parameter estimates. The diagnostic statistics Q(50) and Q<sup>2</sup>(50) are portmanteau statistics based on the first 50 autocorrelations of the standardized residuals and the autocorrelations of the squared standardized residuals respectively. The statistics  $m_3$  and  $m_4$  are the sample skewness and kurtosis respectively of the standardized residuals.  $W$  is the robust Wald statistic for testing the GARCH specification against FIGARCH.

**Table 3: Estimated MA-FIGARCH-Bernoulli jump model with for Filtered High Frequency Commodity Futures Returns**

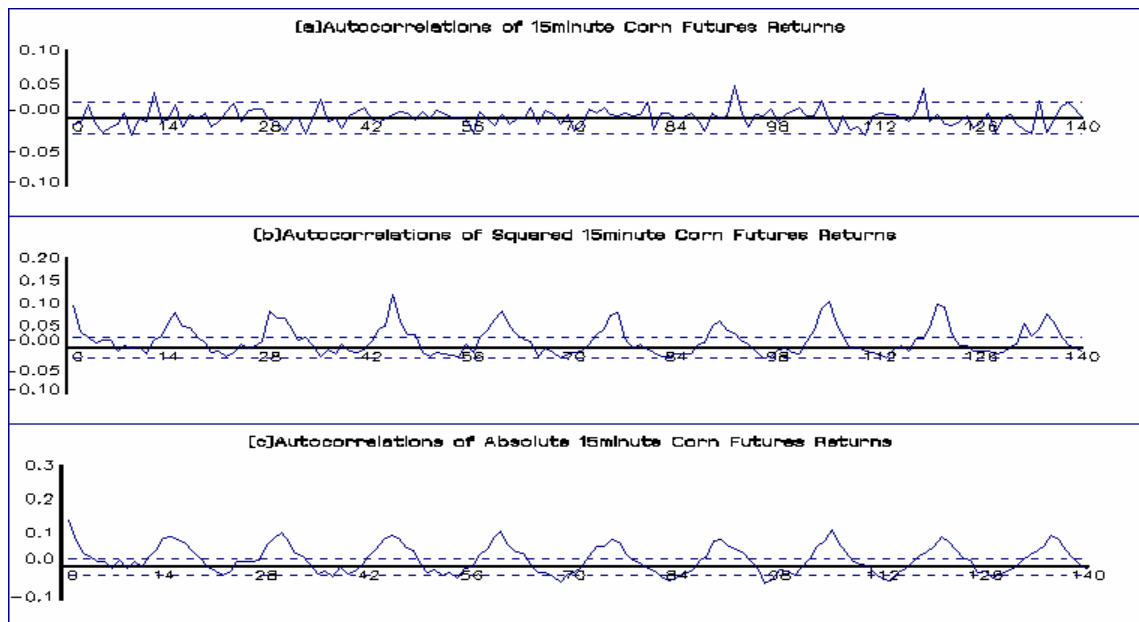
	Cattle	Corn	Hogs	Gasoline
$\mu$	0.0051 (0.0019)	-0.0091 (0.0028)	0.0089 (0.0034)	0.0221 (0.0044)
$j$	1.4889 (0.4475)	1.9376 (0.1871)	1.7171 (0.4255)	2.1077 (0.3941)
$\lambda$	[0.184]	[0.126]	[0.152]	[0.108]
$\nu$	-0.0122 (0.0101)	0.0383 (0.0226)	-0.0092 (0.0240)	-0.1320 (0.0520)
$\delta^2$	0.0294 (0.0073)	0.2258 (0.0352)	0.2161 (0.0761)	0.3122 (0.0735)
$\theta$	-0.0668 (0.0140)	-0.1203 (0.0126)	-0.0655 (0.0142)	-0.0437 (0.0128)
$d$	0.1733 (0.0303)	0.1134 (0.0233)	0.1129 (0.0202)	0.1437 (0.0170)
$\omega$	0.0001 (0.0003)	0.0009 (0.0007)	0.0000 (0.0001)	0.0184 (0.0067)
$\beta$	0.6978 (0.2908)	0.8514 (0.0926)	0.9651 (0.0285)	0.0364 (0.0208)
$\phi$	0.6382 (0.3035)	0.8342 (0.0997)	0.9736 (0.0242)	- -
$m_3$	-0.067	0.143	-0.021	-0.193
$m_4$	1.625	3.619	3.732	1.687
Q(50)	20.625	28.838	26.117	23.652
Q <sup>2</sup> (50)	12.723	10.704	17.793	7.290

Notes: the same as Table 2 except that a jump intensity of  $\lambda$ , where  $\lambda = [1 + \exp(j)]^{-1}$ ,  $0 < \lambda < 1$ , and is specified to be generated by the Bernoulli distribution. The jump size is given by the random variable  $v_t$  which is assumed to be NID ( $\nu$ ,  $\delta^2$ ).

**Figure 1: 15 minute Corn Futures Returns**



**Figure2: Correlograms of Raw 15 minute Corn Futures Returns for 10 trading days**



**Figure3: Correlograms of Filtered 15 minute Corn Futures Returns for 10 trading days**

