ALLOCATION OF ECONOMIC CAPITAL IN BANKING – A SIMULATION BASED APPROACH

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Hans-Peter Burghof

University of Hohenheim
Chair of Banking
Schloss Hohenheim, Osthof
70599 Stuttgart
Germany

Tel.: +49 711 459-22900/2901
Fax: +49 711 459-23448
burghof@uni-hohenheim.de

Jan Müller
(corresponding author)

University of Hohenheim
Chair of Banking
Schloss Hohenheim, Osthof
70599 Stuttgart
Germany

Tel.: +49 711 459-22900/2903
Fax: +49 711 459-23448
jm@uni-hohenheim.de

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ABSTRACT

An institute's economic capital allocation taking efficiency aspects into account requires the coexistence of a certain centralisation and delegation. On the one hand information pooling concerning portfolio optimization kind considerations is afforded. On the other hand exclusively possessed knowledge by decentralized entities has to be used through delegation of decision rights. Information asymmetries cause a lack of reliable performance expectations which represent an important input factor concerning an optimization like allocation of economic capital. The model describes a bank dealing in Dow Jones Industrial Average stocks over a certain time period. The model bank applies an economic capital allocation with benchmark character equating to a pure optimization problem. Information asymmetries are overcome to a certain degree through Bayesian inference. The model is meant to be used for numerical comparative analyses providing an informative basis concerning the impact of different economic capital allocation's efficiency increasing efforts.

Keywords

allocation, bayesian inference, economic capital, return on risk adjusted capital, risk capital, rorac, saa, sample average approximation, simulation, value at risk, var, var limit system, monte carlo, optimization, programming

JEL classification

C10, C60, D80

1. INTRODUCTION

Concerning the allocation of economic capital several fundamental kinds of approaches can be found in literature. Froot/Stein (1998) provides a comprehensive risk management model with the focus on interdependencies between capital structure, capital allocation and hedging transactions. As an important conclusion the flow of information between decentral entities and a central planner is identified as vital in order to maximize shareholder value. Referring to Froot/Stein (1998) the required degree of centralization and delegation is still insufficiently analysed through research. Stoughton/Zechner (2004) in a sense continues these considerations by focusing on shareholder value maximization through economic capital (EC) allocation. In this context the optimal amount of EC is also an issue as well as possibilities of how to determine hurdle rates of certain institute's divisions. As a maximizing allocation mechanism finally an internal capital market and additionally an incentive scheme to ensure truthful reporting are identified.

In contrast there are less comprehensive analytical approaches focusing on the optimization problem behind the EC allocation issue. So does Burmester/Hille/Deutsch (1999) which provides a general definition of this optimization problem. A further analytical approach with the same focus basing on Burmester/Hille/Deutsch (1999) is presented by Straßberger (2002). Despite several improvements the approach's attribute of being analytical seems to be of limitative impact concerning its conclusions. In order to enable the application of Lagrangian function only certain polynomials can be considered as EC addressees' return functions which is quite smattering.

Concerning several issues from the field of EC allocation the use of numerical methods seem to be quite adequate. Dresel/Härtl/Johanning (2001) and (2002) analyse the impact of correlations' instability resulting from traders' independent decision making while their probabilities of success are identical. A further issue are possibilities of increasing value at risk (VAR) limits' utilization ratio. Bühler (2002) also focuses on correlations' instability by analysing its influence on the EC's amount that finally can be allocated if diversification aspects are taken into account. The phenomenon of herding among traders and its impact on VAR limit systems is in the focus of Burghof/Sinha (2005) while Beek/Johanning/Rudolph (1999) analyse the conversion of annual VAR limits into limits for shorter time periods and the impact of fixed, dynamic and loss restricting limits.

The present work integrates the problematic of information asymmetries between a central planner and decentral decision makers represented through traders varying in their probabilities of success. In a sense the mentioned particular conclusions of Froot/Stein (1998) in a way define the analyses' range. In contrast to Stoughton/Zechner (2004) the modelling of an intricate incentive system ensuring truthful reporting is avoided through providing a Bayesian inference mechanism. Furthermore hurdle rate considerations as aspects concerning the optimal bank's overall amount of EC are neglected. In order to define a benchmark method of EC allocation a model bank is introduced which applies an allocation equating to a pure optimization problem. Since EC finally represents a scarce resource to be allocated in an optimal manner the optimization point of view seems to be essential for EC allocation considerations. The benchmark case includes correlations' instability resulting from the traders' independent decision making. The model[1] is meant to serve as a basis for several comparing numerical analyses through which particular efficiency increasing efforts concerning an EC allocation can be measured. Through adjustments of the benchmark configuration cases which represent banks with less sophisticated allocation methods can be derived. The modelling and coding respectively is kept flexible in order to also allow the implementation of all kinds of already analysed issues which can be found in literature. Their efficiency impacts can then be compared in an identical experimental surrounding. Since an analytical approach seems to be less demonstrative for comparative purposes a numerical is chosen. The following chapters give a description of the benchmark case.

2. THE MODEL

The bank is designed as a group of thirty traders. Its daily business volume and ability to take risks respectively is restricted by a given amount of EC which is at the same time the bank's daily value at risk limit. By assigning certain value at risk limits to each trader the EC is allocated throughout the institute. These limits define the traders' maximum exposures. The allocation of EC and value at risk limits respectively is executed by considering correlations and the adherence to the daily overall limit of the bank. The allocating entity is characterised as the management.

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[1] The model is programmed in Excel VBA. As pseudo random number generator the Rnd-function of Excel 2003 was used.

The model's movements in stock prices are given by the discrete daily returns of the thirty stocks of the Dow Jones Industrial Average (DJIA) collected for the year 2003. The DJIA was chosen to provide the model with well-known data and has no further meaning. The trading activity is modelled as follows. At the beginning of each day a trader has to decide whether to go long or short. The choice concerning which security he should trade in is neglected. Every trader is assigned to one specific DJIA-stock. The trade volumes are given by the traders' value at risk limits which are always used to full capacity. Infinite divisibility of shares is assumed. At the end of the trading day the position is closed in any case. Hence other trading strategies e.g. holding a position for several days or making no investment are excluded. Similarities concerning the bank's and the traders' design can be found in Beeck/ Johanning/Rudolph (1999), Dresel/Härtl/Johanning (2001) and (2002).

Concerning VAR computation the delta-normal method is applied throughout the model. Assuming jointly normal distributed returns secures VAR being a coherent[2] risk measure. Using the delta-normal method yields a drastically eased portfolio VAR computation which saves simulation time for the key aspects of the present analysis. A further simplification is gained through generally assuming means of zero. Jorion (2001: 219 et seqq., 255 et seqq.) gives a detailed insight to delta-normal method.

In a firstly considered case with benchmark character a new allocation of the EC takes place in front of every trading day. Provided that adherence to the bank's and the traders' VAR limits is given the maximized bank's expected return indicates which size the value at risk limits allocated among the traders should correspond to in specific. Influencing factors concerning each allocation's shape are on the one hand the expectations concerning the upcoming day's standard deviations and correlations[3] estimated through an exponentially weighted moving average (EWMA) with a parameter configuration accordant to JPMorgan/Reuters (1996: 75 et. Seqq.). But on the other hand in the present model also the management's estimations concerning the skills of its traders play a certain role. Those estimations base upon a Bayesian inference process which requires further modelling

The prior used by the management concerning the occurrence of skills in the model world is identical with the model's actual occurrence of skills among all traders. This fact just implies

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[2] See Artzner (1999) for coherency issues.
[3] See Alexander (1998) for issues on volatility and correlation measurement.

a very good guess by the bank managers concerning the distribution of skills and eases the modelling by avoiding further bias. The skills of a trader are represented by a specific probability of success *p*. The distribution of skills *p* is described through the transformation of the generalized beta density function according Johnson et al. (1999: 210, 211).

Formula 1:

$$f(p)=\frac{1}{B(\alpha,\beta)}\frac{(p-\underline{p})^{\alpha-1}(\overline{p}-p)^{\beta-1}}{(\overline{p}-\underline{p})^{\alpha+\beta-1}} \text{ whereas } \underline{p}\le p\le\overline{p};\alpha,\beta>0$$

In the model the function is shaped by the parameters $\alpha=1$ and $\beta=9$. It is defined on the interval $[0{,}5,1]$. Insertion in the formula above yields the following density function.

Formula 2:

$$f(p)=4608(1-p)^8 \text{ whereas } 0{,}5\le p\le 1;\alpha=1;\beta=9$$

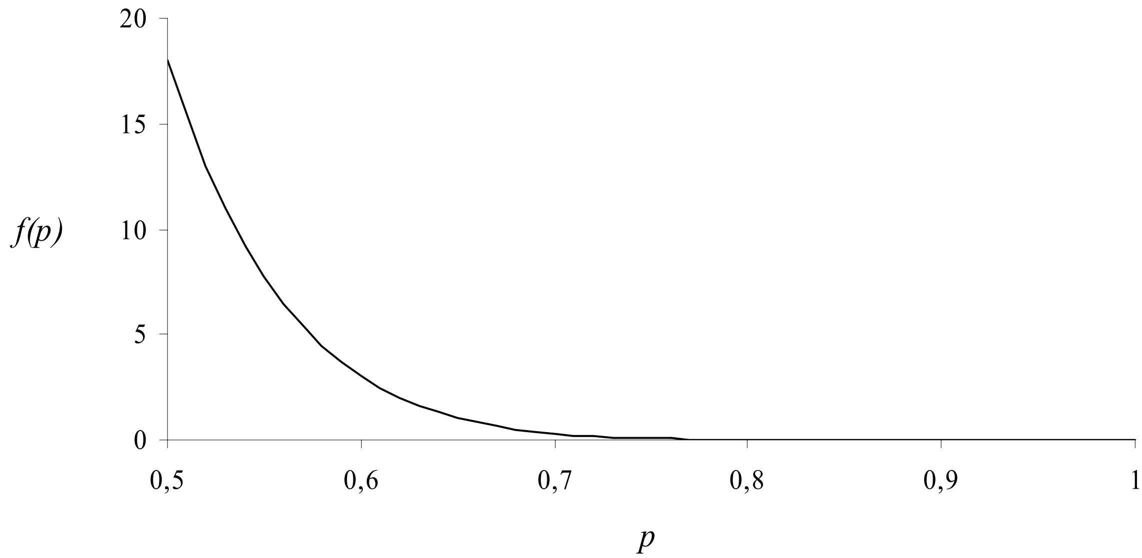

Figure 1: Actual and prior probability distribution of skills $p \sim B(1,9)$ whereas $0{,}5\le p\le 1$, $\mu=0{,}55$ and $\sigma=0{,}045$

In the model values of $p<0{,}5$ remain disregarded since such values imply a trader making wrong decisions deliberately which should not be part of the present analysis. In contrast the scenario of a trader without a clue whether to go long or short is strongly represented since the distribution's mode is 0,5. The chosen distribution meets the demand of being not entirely implausible. This is sufficient since it just has to yield a selection of traders which differ in their skills. Finally through all later comparative analyses of different cases the identical skills' distribution is used. Hence the exact distribution form is not vital concerning the future conclusions. Each trader of the model bank is determined through a draw $p_i \sim B(1,9)$ whereas ($i = 1,\dots,30$) and $0{,}5 \le p_i \le 1$. These draws' outcomes will be used in the benchmark case as through all other later analyses.

trader	1	2	3	4	5	6	7	8	9	10
$p_i \approx$	0,5465	0,5101	0,5895	0,5065	0,5606	0,5986	0,5173	0,5465	0,53	0,5147
trader	11	12	13	14	15	16	17	18	19	20
$p_i \approx$	0,6321	0,5219	0,5188	0,5549	0,5405	0,582	0,505	0,5094	0,5760	0,5969
trader	21	22	23	24	25	26	27	28	29	30
$p_i \approx$	0,5234	0,6123	0,5362	0,7113	0,513	0,5955	0,5775	0,5058	0,5849	0,5402

Table 1: The banks' employees/traders represented by the outcomes of the draws $p_i \sim B(1,9)$ whereas ($i = 1,\dots,30$), $0{,}5 \le p_i \le 1$ and $\mu_{bank} \approx 0{,}5458$

Since the specific values of p_i are unknown to the management it has to estimate those parameters as already mentioned above. This process is modelled through Bayesian updating. Thereby the estimates' accuracy develops according to the law of large numbers. In the model the Bayesian updating represents a certain learning ability of the management.

The management distinguishes between $n = 1000$ types of traders. Hence the interval $[0{,}5, 1]$ of all possible p values is divided into n equal segments on the scale of $\Delta p = 0{,}0005$. The prior probabilities θ_j $(j = 1,\dots,n)$ concerning the occurrences of the specific types of traders are computed via the cumulative beta distribution function according to Johnson et al. (1999: 211) which is introduced firstly.

Formula 3:

$$F(p) = \frac{\int_{\underline{p}}^{\overline{p}} \left((p-\underline{p})^{\alpha-1} (\overline{p}-p)^{\beta-1} (\overline{p}-\underline{p})^{-(\alpha+\beta-1)} \right) dp}{B(\alpha,\ \beta)} \quad \text{whereas } \underline{p} \le p \le \overline{p}; \alpha, \beta > 0$$

Insertion of $\underline{p} = 0{,}5$, $\overline{p} = 1$, $\alpha = 1$ and $\beta = 9$ leads to the following term.

Formula 4:

$$F(p) = 1 - (2 - 2p)^9$$

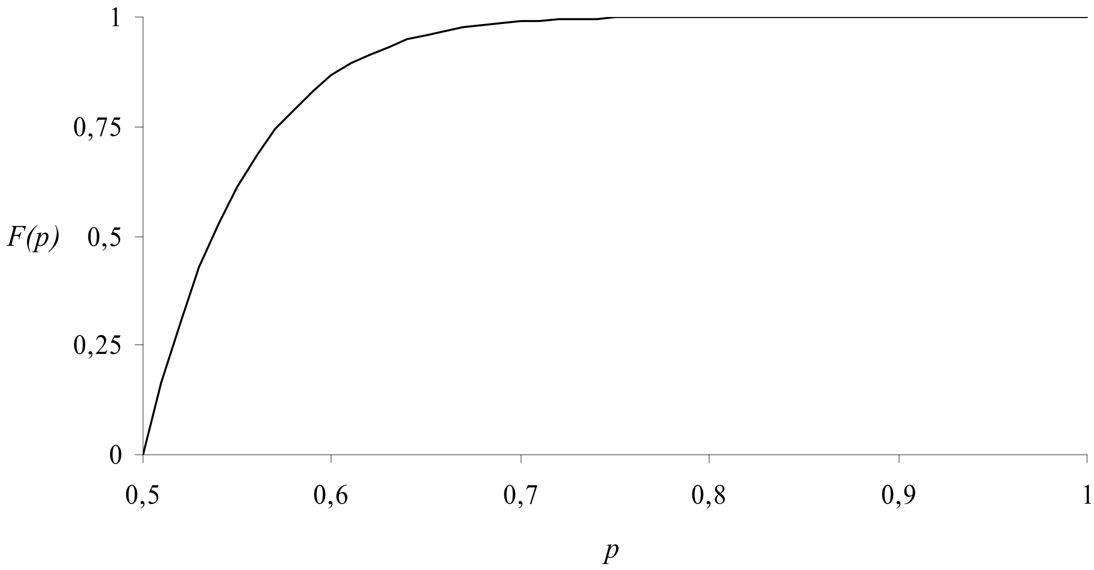

Figure 2: Cumulative distribution function of skills $p \sim B(1,9)$ whereas $0,5 \leq p \leq 1$

Through customization of the function $F(p)$ the occurrences θ_j of the traders' types can be computed.

Formula 5:

$$\overset{n}{\underset{j=1}{\theta_j}} = (1{,}001 - 0{,}001\,j)^9 - (1 - 0{,}001\,j)^9 .$$

The types' specific probabilities of success p_j are represented through the centres of the n intervals determined through Δp.

Formula 6:

$$\overset{n}{\underset{j=1}{p_j}} = \frac{1}{2} - \Delta p\left(\frac{1}{2} - j\right) \text{ whereas } n = 1000,\ \Delta p = 0{,}0005$$

The graph below illustrates the knowledge of the bank management concerning each trader at the beginning of the first trading day. Hence it knows only the occurrences θ_j of the several types of traders which serve as prior probabilities for the start of the Bayesian inference process and their corresponding probabilities of success p_j. Consider *P* and *L* denoting the events "profit" and "loss".

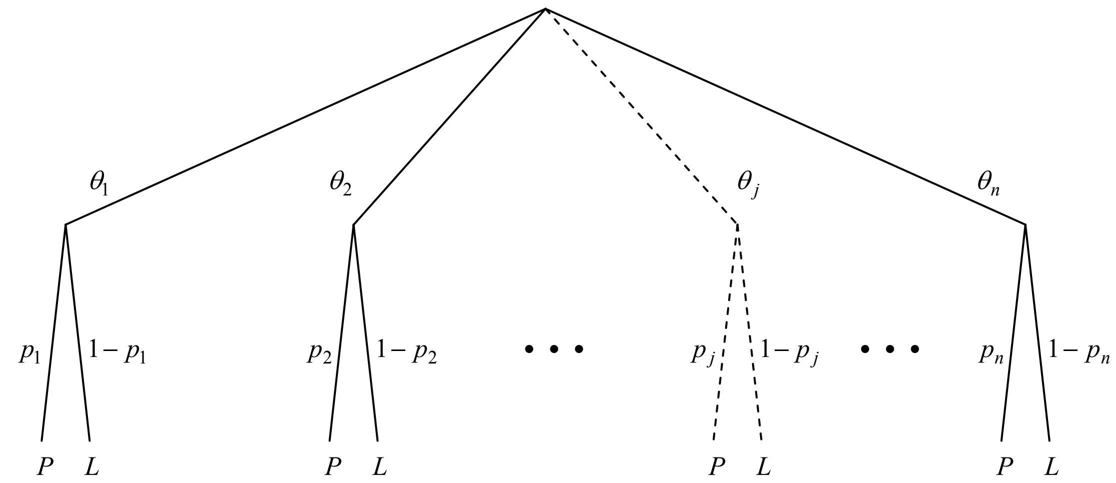

Figure 3: The probabilities' structure of each trader at the beginning of the first trading day which is known to the management

At the end of the first day there is new information regarding which trader made a profit or a loss. In a first step this information allows inferences concerning the type a trader belongs to.

In order to give a detailed example this process is shown by picking one trader who generated a profit. In this case every θ_j in the structure is adjusted by the following two terms which are based on Bayes' theorem.

Formula 7:

$$\overset{n}{\underset{j=1}{\theta_j}} \rightarrow \overset{n}{\underset{j=1}{prob(j|P)_j}} = \frac{\theta_j p_j}{\sum_{j=1}^{n} \theta_j p_j} \quad \text{whereas } n = 1000,\ P = \text{profit}$$

In contrast if the trader caused a loss, the term changes.

Formula 8:

$$\overset{n}{\underset{j=1}{\theta_j}} \rightarrow \overset{n}{\underset{j=1}{prob(j|L)_j}} = \frac{\theta_j (1 - p_j)}{\sum_{j=1}^{n} \theta_j (1 - p_j)} \quad \text{whereas } n = 1000,\ L = \text{loss}$$

Thus the formulas yield the probabilities concerning a trader of belonging to type j under the condition that this trader made a profit or a loss. Since in the model there is new information every day these computations can take place in an according frequency while the input value for the shown adjustment process is always its outcome of the day before. Concerning the example the input values of the second day are represented through $\overset{n}{\underset{j=1}{prob(j|P)_j}}$ and $\overset{n}{\underset{j=1}{prob(j|L)_j}}$ respectively.

The focus is again on the picked profit generating trader i. The estimate's computation concerning his probability of success p_i^e is achieved by using $\overset{n}{\underset{j=1}{prob(j|P)_j}}$. Finally the products' sum of all adjusted occurrences in the structure with the types' probabilities of success is built.

Formula 9:

$$p_i^e(P) = \sum_{j=1}^{n} p_j \, prob(j|P)_j = \sum_{j=1}^{n} p_j \frac{\theta_j p_j}{\sum_{j=1}^{n} \theta_j p_j} \text{ whereas } n = 1000, \; P = \text{profit}$$

If the trader causes a loss the term changes as shown subsequently.

Formula 10:

$$p_i^e(L) = \sum_{j=1}^{n} p_j \, prob(j|L)_j = \sum_{j=1}^{n} p_j \frac{\theta_j (1 - p_j)}{\sum_{j=1}^{n} \theta_j (1 - p_j)} \text{ whereas } n = 1000, \; L = \text{loss}$$

According to the law of large numbers this Bayesian inference process approaches probability p_j of type j which is closest to the trader's actual probability of success p_i. Hence the accuracy of the estimates rises with the number of days and inferences respectively and with the size of the variable n standing for the number of distinguished types of traders. Since the inference process begins with the start of a simulation the analysed bank always corresponds with a just founded one. This fact could be mitigated by randomly replacing traders through further draws from $B(1,\ 9)$ after also randomly chosen time periods. The following gives an impression of the Bayesian inference mechanism's functionality.

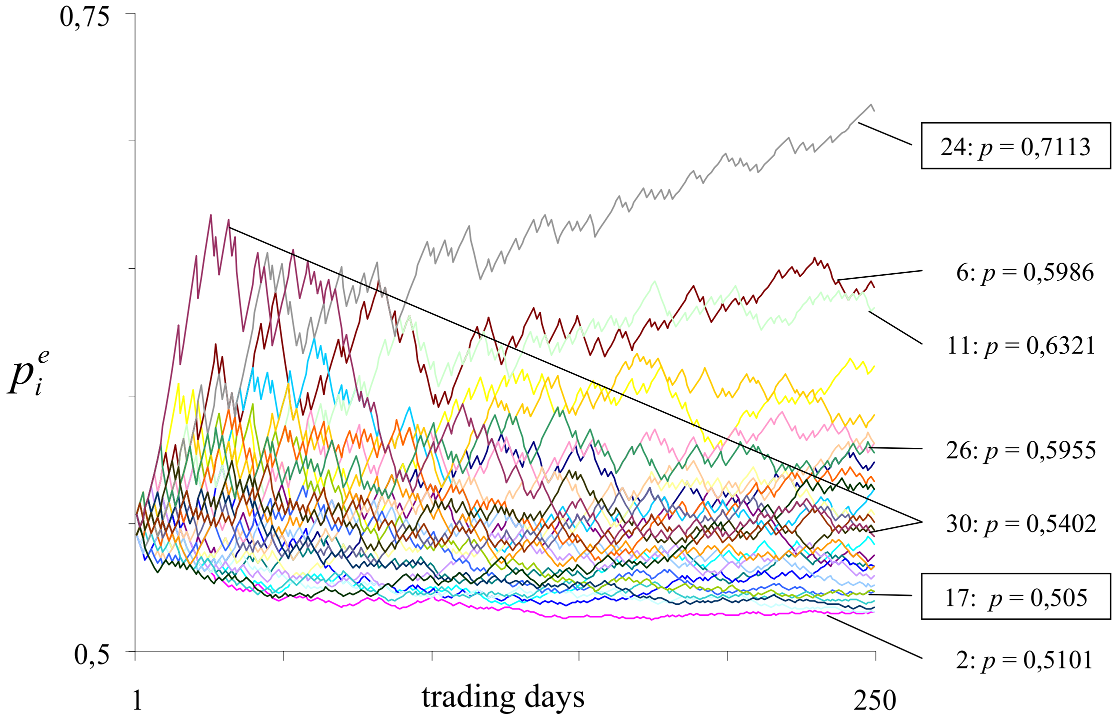

Figure 4: Exemplary results of Bayesian inference

After a certain number of days a ranking among the traders corresponding to their p_i^e values becomes apparent. Bayesian inference reproduces the traders' ranking which is based on their actual probabilities of success p_i. Are further random draws neglected which could represent a certain fluctuation among the bank's employees this ranking's accuracy depends simply on the number of trading days the simulation runs.

3. OPTIMIZATION PROBLEM: EXPECTED RETURN

Finally the present optimization problem is solved through sample average approximation[4] (SAA) also known as the sample path or the stochastic counterpart method. More generally speaking Monte Carlo sampling and the field of stochastic programming[5] are applied. The SAA is used in connection with the so called exterior method. Hence during one optimization process always the same Monte Carlo sample is accessed if random data is required. To be precise in the model such a sample is produced once a day for the simulation of the upcoming one. The basic ambition is to model a bank's every day business processes. Compared to a purely my-sigma driven Markowitz[6] kind optimization approach this requires integration of traders' position taking and moreover their skills and the according management's estimators respectively. In order to integrate all influences in an optimization yielding an adjusted VAR limit allocation SAA is an appropriate method. In a closing remark concerning SAA there has to be indicated that it represents a simplification of the true optimization problem which is necessary for computational reasons and hence more precisely SAA finally just provides a solutions estimator.

The difficulties provided by the problem's nonlinearity are endurable since a found optimal solution in the present setting is always at the same time a global optimum. This stems from the underlying correlation matrices being positive definite which describes the case of common portfolio optimization if derivatives are excluded. Hence the VAR constraint is convex and also the objective function is of simplifying concaveness and does not provide local maxima.[7] Since the used objective function's outcomes are expected profit a maximization approach is applied.

The following remarks can be interpreted as the description of a benchmark case and hence the description of a bank behaving optimal from an entirely theoretical point of view. An example for the theoretical orientation is the assumption of a day to day EC allocation which is

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4 See Shapiro (2003: 353 et seqq.) for a general description of the sample average approximation (SAA)-method.

5 For stochastic programming in general see Birge/Louveaux (1997), Marti (2005) and Ruszczynski/Shapiro (2003).

6 See Markowitz (1959) for considerations on portfolio optimization.

7 See Fylstra (2005) for an overview concerning convexity aspects' impact on actual optimization research. Rockafellar (1993) originally emphasized the importance for optimization issues of whether convexity holds or not.

less realistic. The objective concerning each allocation is the maximisation of the bank's expected profit concerning the upcoming trading day.

Formula 11:

$$\underset{\arg\max\ \mathbf{vl_{t+1}}}{E\left[R_{t+1}\left(\mathbf{vl_{t+1}}\right)\right]} \rightarrow \max = E\left[R_{t+1}\left(\mathbf{vl^{*}_{t+1}}\right)\right]$$

s.t.

$$\sqrt{E\left[\mathbf{var^{T}_{t+1}}\left(\mathbf{vl_{t+1}}\right)\right]\cdot \mathbf{C_{t+1}} \cdot E\left[\mathbf{var_{t+1}}\left(\mathbf{vl_{t+1}}\right)\right]} = E\left[VAR_{t+1}\left(\mathbf{vl_{t+1}}\right)\right] \leq VL_{t+1} = EC_{t+1}$$

$$\mathbf{vl_{t+1}} = \mathrm{vl_i}(\mathrm{i}=1,\ldots,\mathrm{n}) \;\middle|\; \mathrm{vl_i} \in \mathbb{R}^{n},$$

$$E\left[\mathbf{var_{t+1}}\left(\mathbf{vl_{t+1}}\right)\right] = E\left[\mathrm{var_i}\left(\mathrm{vl_i}\right)\right](\mathrm{i}=1,\ldots,\mathrm{n}) \;\middle|\; E\left[\mathrm{var_i}\left(\mathrm{vl_i}\right)\right] \leq \mathrm{vl_i}$$

$$\text{whereas } \mathrm{n}=30,\ \mathrm{t}=0,\ldots,252$$

Note that vectors and matrices are not italicized. Bold face indicates vectors and matrices as a whole. Otherwise their single elements are meant. Small letters denote vectors while capital letters indicate matrices. The bank's expected return $E\left[R_{t+1}\left(\mathbf{vl_{t+1}}\right)\right]$ depends on vector $\mathbf{vl_{t+1}}$ which contains the VAR-limits $\mathrm{vl_i}\ (\mathrm{i}=1,\ldots,\mathrm{n})$ of the traders. $\mathbf{vl^{*}_{t+1}}$ denotes the limit allocation maximizing the banks expected return. The amount of EC determines the bank's daily overall VAR-limit VL and hence the constraint to the bank's expected value at risk which is just allowed to be caused by any allocation $\mathbf{vl_{t+1}}$. In doing so EC provides the constraint regarding the optimization problem. The term $E\left[\mathbf{var_{t+1}}\left(\mathbf{vl_{t+1}}\right)\right]$ denotes the vector of expectations regarding the traders' caused value at risks which depend on the allocated limits $\mathbf{vl_{t+1}}$ as well. Hence finally the expectations concerning the bank's return R_{t+1} refer to the return the traders are able to gain with a certain VAR-limit allocation. The following gives information about how the traders' market anticipation is modeled and simulated. To give a better insight the vector notation is abandoned.

Formula 12:

$$E\left[R_{t+1}\left(\mathbf{vl_{t+1}}\right)\right] = \frac{1}{m}\cdot\sum_{h=1}^{m}\sum_{i=1}^{n} R_{h,i,t+1}\left(vl_{h,i,t+1}\right) = \frac{1}{m}\cdot\sum_{h=1}^{m}\sum_{i=1}^{n} r_{h,i,t+1}\cdot V_{h,i,t+1} = \frac{1}{m}\cdot\sum_{h=1}^{m}\sum_{i=1}^{n}\frac{-vl_{i,t+1}\cdot\varepsilon_{h,i,t+1}\cdot\sqrt{\Delta t}}{z_{h,i,t+1}(\alpha,S)}$$

$$\text{whereas}\quad r_{h,i,t+1} = \sigma_{i,t+1}\cdot\varepsilon_{h,i,t+1}\cdot\sqrt{\Delta t},\quad V_{h,i,t+1} = \frac{-vl_{i,t+1}}{\sigma_{i,t+1}\cdot z_{h,i,t+1}(\alpha,S)},$$

$$\varepsilon_{h,i,t+1} \sim N(\mu,\ \Sigma),\quad \Delta t = \frac{1}{252},\quad (\mathbf{\Omega}, \mathbf{A}, P)^{m,\,n}$$

Note that control variable $h = 1, ..., m$ stands for the *m* simulations of the forthcoming trading day where $i = 1, ..., n$ denotes the *n* traders and traded stocks respectively. Take R_{t+1} for the bank's overall return and $R_{h,i,t+1}$ for the return of trader *i* during simulation iteration *h*. In contrast $r_{h,i,t+1}$ denotes each stock's rate of return. It stems from a discrete geometric Brownian motion hence the multiplication of the particular stock's actual standard deviation $\sigma_{i,t+1}$, a multivariate standard normal distributed random number $\varepsilon_{h,i,t+1}$ and the square root of the corresponding time increment. As mentioned the model's assumption of normally distributed rates of return enables the application of the Delta-Normal-Method for VAR-computation. Hence a transformation of the VAR computing term yields the market value $V_{h,i,t+1}$ referring to the maximum trading position each trader is allowed to hold. It is determined by the VAR-limit in the numerator and the standard deviation as the quantile $z_{h,i,t+1}(\alpha,S)$ in the denominator. The quantile's absolute value $|z_{h,i,t+1}(\alpha, S)|$ depends on the underlying confidence level $1-\alpha$. For the present analyses a confidence level of 99 percent is applied leading to a quantile's absolute value of $|z_{h,i,t+1}(\alpha, S)| \approx 2{,}33$. In contrast the quantile's sign changes in dependence of different states *S* which finally depend on several random numbers' interaction described subsequently.

Formula 13:

$$z_{h,i,t+1}(\alpha,S) < 0 \quad \forall \quad S \;\Bigg|\; \overbrace{\varepsilon_{h,i,t+1} > 0 \wedge \zeta_{h,i,t+1} < p^{e}_{i,t+1}(\psi_{i,t})}^{\text{profit}} \vee \overbrace{\varepsilon_{h,i,t+1} < 0 \wedge \zeta_{h,i,t+1} > p^{e}_{i,t+1}(\psi_{i,t})}^{\text{loss}} \;,$$

$$z_{h,i,t+1}(\alpha,S) > 0 \quad \forall \quad S \;\Bigg|\; \overbrace{\varepsilon_{h,i,t+1} < 0 \wedge \zeta_{h,i,t+1} < p^{e}_{i,t+1}(\psi_{i,t})}^{\text{profit}} \vee \overbrace{\varepsilon_{h,i,t+1} > 0 \wedge \zeta_{h,i,t+1} > p^{e}_{i,t+1}(\psi_{i,t})}^{\text{loss}} \;,$$

$$\text{whereas } \zeta_{h,i,t+1} \text{ and } \psi_{i,t} \sim U(0,\ 1)$$

$\zeta_{h,i,t+1}$ as $\psi_{i,t}$ denote uniform distributed random numbers. While the first mentioned determines whether a certain trader generates a profit or loss throughout the *m* simulations the second one decides whether the trader generates a profit or loss during the models "real" trading action. Since the real trading's earnings drive the skills' estimators values $p^{e}_{i,t+1}$ these depend at the same time on $\psi_{i,t}$. The operator $\vee$ separates two constellations. Both lead to the same quantile sign but once the result is a profit and once a loss.

The generation of the random data which impacts SAA can be interpreted as spanning up a probability space $(\mathbf{\Omega}, \mathbf{A}, P)$. See e.g. Birge/Louveaux (1997: 49 et seqq.) for probability space issues. This random data's elementary origins are denoted through $\omega | \omega \in \mathbf{\Omega}$ while $\mathbf{\Omega} = \{\omega\}$. Hence SAA's specifically distributed data $\varepsilon(\omega)$ as well as $\zeta(\omega)$ depends on those origins. Note that ψ and hence $\mathrm{p^e}(\psi)$ only represents random data if the whole model is considered. To a particular SAA in the model the corresponding estimators' values are deterministic data. $\mathbf{A}$ describes a collection of random events which in turn are subsets of $\mathbf{\Omega}$. In the present work's context single events' examples are $\mathrm{A} = \varepsilon > 0$ or $\mathrm{A} = \zeta < \mathrm{p^e}(\psi)$ whereas a collection's example is $\mathbf{A} = \varepsilon > 0 \wedge \zeta < \mathrm{p^e}(\psi) = \text{profit}$. Hence according to Formula 9 the present probability space implicates four event collections. Two result in a profit and two in a loss. To each event $\mathrm{A} \in \mathbf{A}$ a probability $P(\mathrm{A})$ can be associated while $0 \le P(\mathrm{A}) \le 1$, $P(\varnothing) = 0$, $P(\mathbf{\Omega}) = 1$, $P(\mathrm{A}_1 \cup \mathrm{A}_2) = P(\mathrm{A}_1) + P(\mathrm{A}_2)$ if $\mathrm{A}_1 \cap \mathrm{A}_2 = \varnothing$.

Characteristically SAA implicates the discretization of the spanned up $(\mathbf{\Omega}, \mathbf{A}, P)$ for countability reasons which causes the mentioned fact concerning the present problem being just approximatively. As determined by Formula 12 the described optimization problem in the present work requires *m* vectors of the length *n* concerning the particularly distributed random data $\varepsilon(\omega)$ and $\zeta(\omega)$ as random sample per SAA. Hence the corresponding discrete probability space can be denoted through $(\mathbf{\Omega}, \mathbf{A}, P)^{m,\,n}$. Since the focus of the present analyses is not on the difference between the solutions estimator and the true problem's solution the value of *m* is primarily derived from computational time issues. For $m \to \infty$ the approximation would turn into the true problem's solution. Detailed SAA's accuracy and stopping criteria remarks provides Shapiro (2003: 357 et seqq.). Since furthermore the sample is generated exteriorly the optimization problem finally shrinks to a deterministic one. Hence for its solution a deterministic algorithm can be applied which will be described during the subsequent section.

4. SUBPROBLEM: EXPECTED RORAC

It is assumed that $\mathbf{vl}^*_{\mathbf{t+1}}$ maximizes the relation of expected bank's return per expected bank's VAR. This relation just describes the expectation concerning the widely spread ratio return on risk adjusted capital (RORAC). Referring to $\mathbf{vl}^{*}_{\mathbf{t+1}}$ the ratio's maximization has a sufficient condition's character. Hence in order to find $\mathbf{vl}^{*}_{\mathbf{t+1}}$ the following subproblem has to be solved.

Formula 14:

$$\underset{\arg\ \max\ \mathbf{vl_{t+1}}}{E\left[RORAC_{t+1}\left(\mathbf{vl_{t+1}}\right)\right]} \rightarrow \max = E\left[RORAC_{t+1}\left(\mathbf{vl^{**}_{t+1}}\right)\right]$$

s.t.

$$\mathbf{vl_{t+1}} = \mathrm{vl_i}(\mathrm{i} = 1, ..., \mathrm{n}) \;\middle|\; \mathrm{vl_i} \in \mathbb{R}^n$$

For technical reasons the constraints of the superior optimization problem are firstly ignored. To give a better insight vector notation is again abandoned in the following.

Formula 15:

$$E\left[RORAC_{t+1}\left(\mathbf{vl_{t+1}}\right)\right] = \frac{E\left[R_{t+1}(\mathbf{vl_{t+1}})\right]}{E\left[VAR_{t+1}(\mathbf{vl_{t+1}})\right]} = \frac{\sum_{h=1}^{m}\sum_{i=1}^{n} -vl_{i,t+1} \cdot \varepsilon_{h,i,t+1} \cdot \sqrt{\Delta t} \cdot z^{-1}_{h,i,t+1}(\alpha, S)}{\sum_{h=1}^{m}\left|z(\alpha)\right| \cdot \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} V_{h,i,t+1} \cdot V_{h,j,t+1} \cdot \rho_{ij,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{j,t+1}}}$$

whereas $V_{h,i,t+1} = \dfrac{-vl_{i,t+1}}{\sigma_{i,t+1} \cdot z_{h,i,t+1}(\alpha, S)}$, $V_{h,j,t+1} = \dfrac{-vl_{j,t+1}}{\sigma_{j,t+1} \cdot z_{h,j,t+1}(\alpha, S)}$ and $(\mathbf{\Omega}, \mathbf{A}, P)^{m,\,n}$

Through firstly factoring out constraints a certain discrete algorithm can be used to extract the subproblem's solution. In order to provide a neat explanation the algorithm is graphed subsequently.

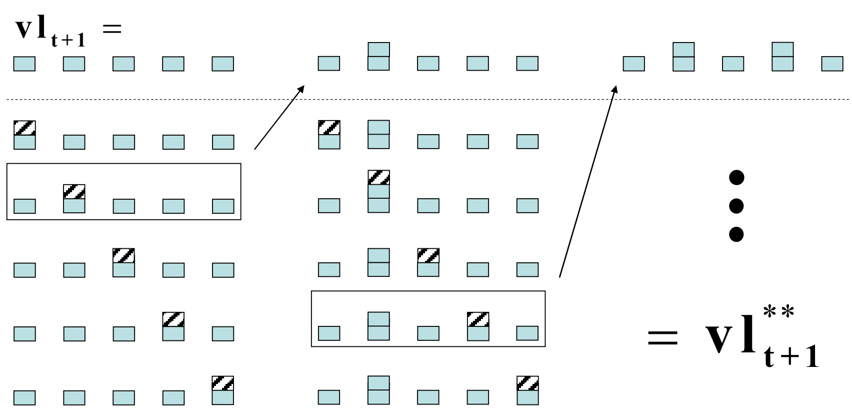

Figure 5: The functionality of the subproblem's discrete solving algorithm

Every square element denotes a trader's VAR limit. Compared to the model the description's number of traders is reduced to five. The allocation above the dotted line in the left upper corner represents the starting point. Through adding the striped limit extensions in the described manner five new allocations are generated. Subsequently each resulting allocation is tested in connection with $(\mathbf{\Omega}, \mathbf{A}, P)^{m,n}$ concerning which of them yields the highest expected RORAC. In the example the framed allocations include this attribute and hence serve as input data for the subsequent optimization iteration while every column denotes one iteration. If the firstly used limit extension of size $\Delta \mathrm{vl}$ (here $\Delta \mathrm{vl} = 250.000\ \mathrm{USD}$) does not result in at least one allocation outperforming the inputted one $\Delta \mathrm{vl}$ is adjusted through simple bisection. The bisection is continued until a certain stopping criteria is reached (here $\Delta \mathrm{vl} < 1.000\ \mathrm{USD}$). Note that in order to approach the true problem's solution additionally to the above mentioned requirements $\Delta \mathrm{vl} \to 0$ would be necessary. As stopping criteria concerning the optimization algorithm itself an expected RORAC's improvement rate smaller then a thousandth part is used. Note that the several values base on experiences and are not gathered from optimization processes themselves. Finally they result from accuracy-time-trade off considerations. In the model's benchmark case this optimization procedure is executed once in front of every trading day. The graph below represents the optimization results concerning six days which together needed 191 optimization iterations.

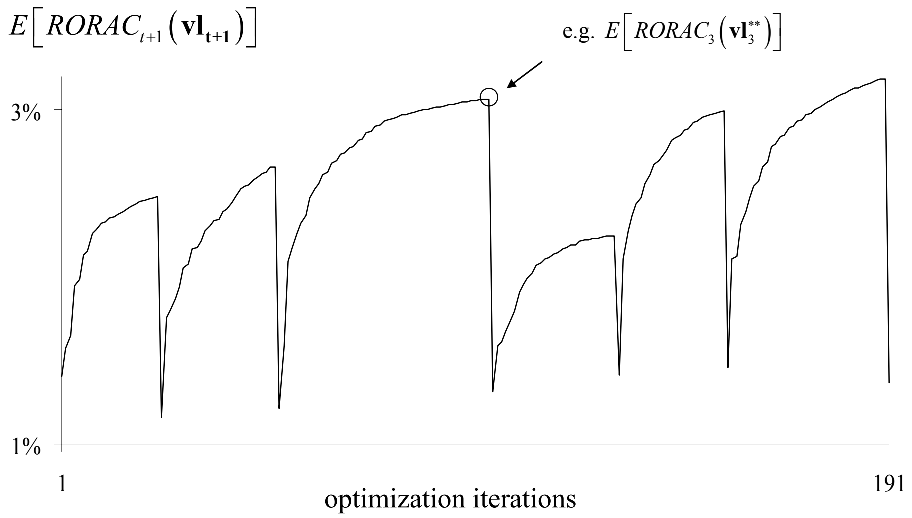

Figure 6: Exemplary results of the subproblem's discrete solving algorithm

Furthermore to identify $\mathbf{vl}^{*}_{\mathbf{t+1}}$ a particular scalar $\lambda^{\circ}_{t+1} \left| \lambda^{\circ}_{t+1} \, \mathbf{vl}^{**}_{\mathbf{t+1}} = \mathbf{vl}^{*}_{\mathbf{t+1}} \right.$ has to be identified. To track down λ°_{t+1} binary chop in connection with the interval (0, 2) is applied. Finally a search problem of the following kind exists.

Formula 16:

$$E\left[VAR_{t+1}(\lambda_{t+1}\mathbf{vl}^{**}_{\mathbf{t+1}})\right] \underset{\lambda_{t+1}\,\arg}{\rightarrow} E\left[VAR_{t+1}(\lambda^{\circ}_{t+1}\mathbf{vl}^{**}_{\mathbf{t+1}})\right]$$

s.t.

$$\lambda^{\circ}_{t+1}\mathbf{vl}^{**}_{\mathbf{t+1}} = \mathbf{vl}^{*}_{\mathbf{t+1}}, \; \alpha - 1 = 0{,}99, \; \lambda_{t+1} \in \mathbb{R}$$

During the search the interim solutions λ_{t+1} are tested through simulation.

Formula 17:

$$E\left[VAR_{t+1}(\lambda_{t+1}\mathbf{vl}^{**}_{\mathbf{t+1}})\right] = \sum_{h=1}^{l} \left|z(\alpha)\right| \cdot \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} V_{h,i,t+1} \cdot V_{h,j,t+1} \cdot \rho_{ij,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{j,t+1}}$$

$$\text{whereas } V_{h,i,t+1} = \frac{-vl^{**}_{i,t+1}}{\sigma_{i,t+1} \cdot z_{h,i,t+1}(\alpha, S)}, \; V_{h,j,t+1} = \frac{-vl^{**}_{j,t+1}}{\sigma_{j,t+1} \cdot z_{h,j,t+1}(\alpha, S)},$$

$$(\mathbf{\Omega}, \mathbf{A}, P)^{l,\, n}$$

To give an impression concerning the model's mechanisms' functionality the following diagram presents some results of the repeated EC assignments to the traders.

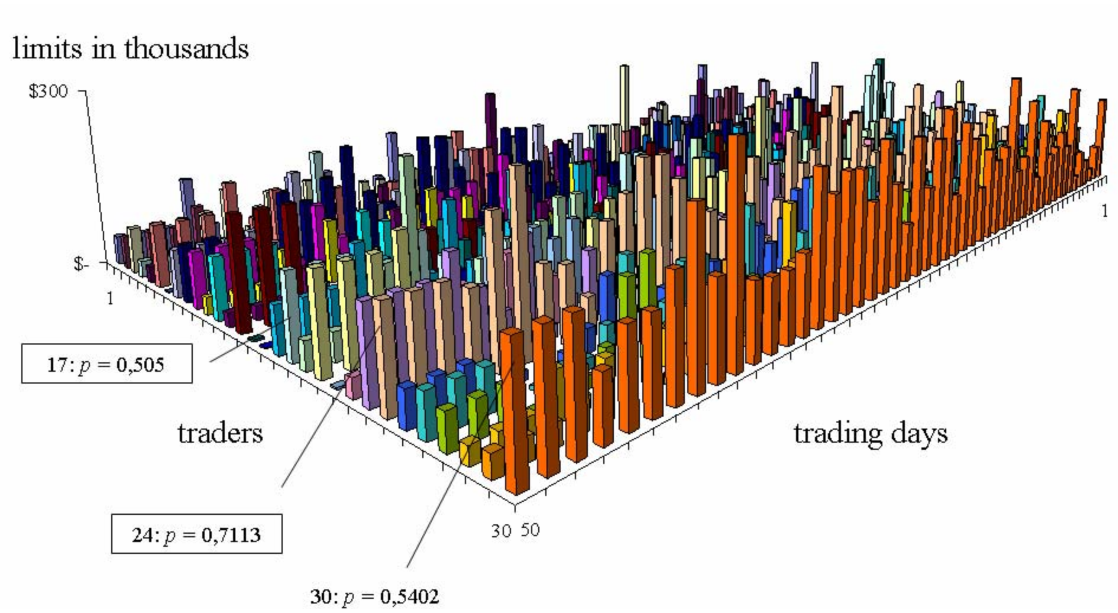

Figure 7: Exemplary illustration concerning the limit allocation during simulation

The diagram shows relatively high VAR limits during the first fifty days in context with trader number thirty which is surprising since he is just provided with an average probability of success. His limits' size stems from the fact that he was quite successful during the first fifty days which resulted in relatively high estimators p_{30}^{e} as could be seen in figure four. Hence in his case the optimization process yielded high limits while top trader number twenty four did not yet entirely reach his skills' corresponding rank in the allocation process.

5. CONCLUSIONS

Through adjustments concerning the configuration of the above described benchmark model several cases can be derived which represent banks with less sophisticated allocation methods concerning EC. In order to prevent avoidable bias which impairs the comparability of the numerical analyses' results the analyses base on an identical row of pseudo random numbers and probability space $(\mathbf{\Omega}, \mathbf{A}, P)^{model}$ respectively. Furthermore every bank deals on DJIA's stock price movements of the year 2003.

At least five major fields concerning potential analyses can be identified. Firstly tests will be undertaken in order to measure the impact of the Bayesian inference on allocations' efficiency. Is ex ante information in the form of expected performances as vital as it seems at first sight? How important is this kind of information compared with other input factors of the allocation optimization process? Do homogeneity and heterogeneity aspects concerning the EC's addressees play a crucial role in this context?

Another field is represented through the forms of correlations' integration. Completely neglecting them appears to be out of the question. However to small institutes this might be the preferred strategy for convenience reasons. How bad is this solution compared to others? And how strong are efficiency gains of additionally considering correlations' instability resulting from addressees' decision making through the optimization process?

The allocations' and optimizations' frequency is a further range of potential research. A daily execution surely is far from practice. From a practical point of view an annual allocation of limits appears possible since this surely represents the state of the art regarding most of the bigger institutes. How high are efficiency decreases to be accepted if the frequency is distinctly lowered compared to the benchmark case?

Further questions rank around the allocation of profits and losses. Should they immediately increase and decrease the corresponding VAR limit? How strong is the impact on efficiency compared to configuration adjustments from other mentioned fields?

In order to close this collection of ideas heuristic allocation methods can be named which could be used instead. Can a uniform allocation of the model bank's EC among its traders compete with more sophisticated solutions? For example a RORAC or earnings based ranking of the traders could be used as an indication which trader should get the biggest and lowest VAR limit respectively. How does a random allocation perform and should underperforming traders be excluded from trading?

Structuring and executing these kinds of thoughts should lead to a data collection which allows drawing some relevant conclusions concerning what could and what should be considered by institutes in order to reach a certain efficiency degree regarding their use of the scarce resource EC.

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