## Evidence of a weekend effect in a market for state

## contingent claims

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#### Abstract

This study explores the origins and strength of the weekend effect by examining a market for state contingent claims where this phenomenon has not been previously explored; the UK horserace betting market. Conditional logit models are developed for weekend and weekday markets and prices are shown to be an inferior guide to race outcome at weekends. Evidence is provided that weekend markets are populated by a larger proportion of noise traders and that their inaccurate judgements cause the effect. It is demonstrated that the effect is sufficiently pronounced to enable abnormal returns to be earned in weekend markets.


Keywords: weekend effect, market efficiency

## Evidence of a weekend effect in a market for state contingent claims

A MARKET is considered efficient if 'prices fully reflect available information' (Fama, 1970, p. 384). The degree to which information is discounted in financial markets and the manner in which it is used sheds light on the behaviour of market participants. Systematic deviations in investment patterns from that predicted by existing theory demand attention and the weekend effect is one of the most persistent of these anomalies. Cross (1973) was the first to identify the effect, whereby stock returns are persistently and significantly lower over the weekend (or non-trading days) than on other days of the week. This suggests that market participants do not discount all available information and the weekend effect therefore represents a challenge to the efficient market hypothesis. It is not surprising, therefore, that this anomaly has attracted considerable attention, being the subject of over seventy papers since 1973. The weekend effect has been shown to exist in a variety of market types (e.g., S\&P 500: Cross, 1973; French, 1980; Gibbons and Hess, 1981; Dow Jones Industrial Average: Lakonishok and Smidt, 1988; Rogalski, 1984; the over-the-counter market: Keim and Stambaugh, 1984) and is persistent across time and national boundaries (e.g., Jaffe and Westerfield, 1985; Chang et al., 1993). A useful review of this literature is given in Marquering et al. (2006).

A number of causes have been suggested for the weekend effect. Early explanations include the interest cost resulting from trading settlement lags (e.g., Gibbons and Hess, 1981; Lakonishok and Levi, 1982), measurement errors in bid-ask spreads (e.g., Keim and Stambaugh, 1984), and the tendency for firms to release adverse information after trading closes for the weekend (e.g., Penman, 1987; Damodaran, 1989). Since the late 1980s, there has been a shift from market-based to participant-based explanations. It has, for example, been shown that individual investors tend to be more active on Mondays, particularly in sell-related transactions (e.g., Lakonishok and Smidt, 1988; Lakonishok and Maberly, 1990; Abraham and Ikenberry, 1994; Brooks and Kim, 1997). Their trading focuses on the stocks of
smaller firms where the weekend effect has been shown to be more pronounced (Lakonishok et al., 1992). On the other hand, institutional investors tend to be less active on Mondays as a significant amount of time on that day is devoted to discussion of weekly investment plans (e.g., Osborne, 1962). This is reflected in lower aggregate trading volumes on Mondays (e.g., Jain and Joh, 1988; Lakonishok and Maberly, 1990). Since individual investors are generally less informed than institutional investors it is suggested that the greater activity of individual investors on the day following the weekend or a Bank holiday distorts market prices.

A number of reasons have been proposed for the heightened activity of individual investors on the day following the weekend. In particular, it is suggested that the majority of individual investors engage in their full-time business during the week, and weekends provide them with more favourable conditions for making investment decisions, including less distractions, information overload and opportunity cost (Abraham and Ikenberry, 1994). In addition, there are several factors which serve to increase selling by individual investors on Mondays, so decreasing stock returns on that day. For example, most of the information provided by financial brokers to individual investors on weekdays advises buy-related transactions, and this is not the case over weekend (e.g., Groth et al., 1979; Stanley et al., 1980; Diefenbach, 1972; Dimson and Fraletti, 1986; Dimson and Marsh, 1986). Therefore, individual investors are more likely to sell stocks on Monday for either financing other stock purchases (recommended during weekdays) or simply for liquidity purposes. In addition, there is evidence indicating that firms tend to release bad news during closed market periods so as to avoid panic selling (e.g., Patell and Wolfson, 1982; Penman, 1987; Damodaran, 1989).

In summary, three key features of the weekend effect emerge from the literature: First, it appears to be a persistent feature of a wide range of financial markets. Second, although there is debate concerning the mechanism/s which cause the anomaly, the vast majority of recent
papers attribute it, at least in large part, to the trading behaviour of individual (c.f. institutional) investors (e.g., Miller, 1988; Lakonishok and Maberly, 1990; Abraham and Ikenberry, 1994). Third, the weekend effect implies that stock returns are not constant across days of the week, and this suggests that abnormal returns may be generated by trading on the anomaly. However, in stock markets, there is no definitive end point at which the price of a stock can be determined and at which an investor's judgments can be accurately assessed. Therefore, existing studies investigate the relationship of stock returns to 'theoretical' investments made between the day after weekend/non-trading days and other days of the week. Studies have not generally modelled the impact of the weekend effect on different stocks and consequently it could be argued that the extent to which the weekend effect challenges the weak form efficiency of these markets has not been fully explored.

In order to shed further light on the weekend effect, the aims of this paper are: First, to investigate whether an analogous weekend effect exists in a market for state contingent claims which has not to date been examined: the UK bookmaker betting market. It will be shown that this market shares many features in common with wider financial markets, enabling important lessons regarding investor behaviour to be drawn. However, unlike wider financial markets, betting markets operate throughout weekends and Bank holidays when it is expected that casual bettors (similar to individual/noise traders in wider financial markets) will be more active. Weekend betting markets are therefore analogous to Mondays in financial markets. Consequently, betting markets offer the prospect of comparing the accuracy of prices formed at weekends and on weekdays (which are expected to be populated by more informed traders)

Bookmaker betting markets also offer a unique way of measuring the proportions of informed and less informed investors. This enables the second aim of the paper to be achieved: To demonstrate a link between the proportion of less informed bettors in the market and the degree to which market prices effectively predict race outcome.

The final aim of the paper is to assess, to a fuller extent than that employed in previous papers, the strength of the weekend effect; in particular, to determine the degree to which the weekend effect can be exploited to earn abnormal returns and hence to measure the full extent of the weak form inefficiency which it causes.

To achieve these aims the remainder of the paper is organised as follows: The nature of the UK horserace betting market and why it offers the prospect of better understanding the strength of, and mechanism/s underlying, the weekend effect are outlined in Section I., The hypotheses to be tested are also introduced in this section. The data and methodology employed to test the hypotheses are described in Section II. The results are reported and discussed in Section III and conclusions are drawn in Section IV .

## I. Horserace Betting Markets and Hypotheses

## A. Betting Markets as Windows on Financial Market Behaviour

Throughout the last five decades, a number of researchers have aimed to shed light on investors' behaviour in financial markets by drawing lessons from horserace betting markets (e.g., Griffith, 1949; Yaari, 1965; Rosett, 1971; Dowie, 1976; Losey and Talbott, 1980; Hausch et al., 1981; Hausch and Ziemba, 1985; Asch and Quandt, 1987; Bird and McCrae, 1987; Thaler and Ziemba, 1988; Swidler and Shaw, 1995; Schnytzer and Shilony, 1995; Law and Peel, 2002; Levitt, 2004; Johnson et al., 2006). There are a number of reasons for this: First, horserace betting markets share a number of fundamental characteristics with wider financial markets. These include the complexity and interdependence of factors which influence a horse's prospects (an asset's value), easy of entry, extensive market knowledge, large numbers of participants who can be classified as casual traders (sometimes referred to as noise or less informed traders) and informed (or professional) traders (Snyder, 1978). In addition, each " $n$-horse race corresponds to a market for contingent claims with $n$ states in which the $i$ th state corresponds to the outcome in which the $i$ th horse wins the race" (Shin,

1992, p1142). In state contingent claims terms, the purchase price of a claim on horse $i$ in race $j$, which pays $£ 1$ if horse $i$ wins and nothing if it loses, is given by $1 /\left(1+O_{i j}\right)$, where $O_{i j}$ represents horse $i$ 's odds. It is argued that bettors will continue to place money on horse $i$ in race $j$ until the purchase price of a claim on this horse accurately reflects the market's best estimate of the horse's chance of winning (Figlewski, 1979).

Second, wagering markets "are especially simple financial markets, in which the pricing problem is reduced. As a result wagering markets can provide a clear view of pricing issues which are complicated elsewhere" (Sauer, 1998, p. 2021). The value of the lessons drawn from betting markets is reinforced by the fact that they, in themselves, are important markets; for example, the turnover of the UK horserace betting market in 2006 was over $£ 15,500$ million.

Third, horserace betting markets offer an important advantage over other financial markets in exploring market efficiency: they generate an unequivocal outcome (a winner) and an associated rate of return within a finite time frame (Law and Peel, 2002), and hence provide an objective benchmark against which to measure the quality of the decision to purchase (place) a particular claim (bet) or the quality of any explanatory model. The finite nature of this form of market also means that there is a large pool of markets of essentially similar type available for analysis, amounting in the UK to between 5,000 and 6,000 markets (flat races) each year. Consequently, betting markets are 'well suited for testing market efficiency' (Law and Peel, 2002, p. 327) since they possess features which enable the computation of expected value of each contingent state and they enable insights into the manner and degree to which information is used in markets to be clearly discerned.

## B. The UK Horserace Betting Market

One important feature which distinguishes the UK horserace betting market from that operating in many other parts of the world (e.g., USA, Hong Kong, Japan) is the presence of
market makers (bookmakers) at racetracks, where they operate in parallel with the pari-mutuel market. Unlike pari-mutuel market bettors (whose bets are settled at the odds prevailing at the close of the market), bookmaker market bettors can secure their return without the danger of a bandwagon effect eroding their gains, since their bets are settled at the odds available at the time the bets are struck. Consequently, the bookmaker market is likely to attract the more serious, informed bettors, including those with access to privileged information (Crafts, 1985; Sauer, 1998; Schnytzer and Shilony, 1995). In addition, the turnover of the bookmaker betting market in the UK is nearly forty times that of the pari-mutuel market. Consequently, professional bettors, who are likely to bet with larger stakes, are not likely to bet in the small pari-mutuel market, where their stakes reduce the odds at which their own bets are settled. The aim of this study is to compare the accuracy of prices formed in markets which are likely to be more/less heavily populated at different times with informed bettors and it is clear from the preceding discussion that bookmaker markets offer the best prospect of achieving this.

Odds in the pari-mutuel market are determined formulaically by the relative pattern of betting across all horses in the market but odds in the bookmaker market also reflect the views of the individual bookmakers. Independent bookmakers at the racetrack post odds at the start of the market for a given race; these represent their considered opinion of each horse's chance of success. They subsequently change their odds as a result of receiving information concerning horses' relative prospects and/or as a result of demand for bets on different horses. Bettors are free to bet with the bookmaker offering the best odds on their selection up to the start of the race. The important role played by the market makers in bookmaker markets makes these more akin to wider financial markets than pari-mutuel markets.

## C. Hypotheses

It has been suggested that weekday bettors usually attend racetracks with financial ambitions and invest a considerable amount of time, effort and money in analysing the past performances of horses, jockeys and the condition of track (Kopelman and Minkin, 1991). Those individuals, who treat betting and attendance at the racetrack as a leisure pursuit, are more likely to participate at weekends and on Bank holidays because many of them are employed at other times. In general, it is argued that these individuals are likely to be less informed and skilful than weekday bettors. Support for this view is given by Saunders and Turner (1987) and McGlothlin (1956). The former refer to weekend bettors as 'uncommitted customers' who wager smaller stakes and tend to back horses on the basis of subjective preferences rather than information associated with the probability of winning. McGlothlin (1956) provides further evidence to support this view, namely that a larger average amount is bet per person on weekdays than on weekends.

It could be argued that an effect analogous to the weekend effect in wider financial markets may be present in betting markets. In particular, that greater mis-pricing occurs during periods when less informed bettors are likely to dominate the racetrack betting market. Consequently, the first hypothesis tested here is:

Odds in weekend betting markets will not be as accurate a guide to the prospects of each horse as those formed in weekday markets.

Anecdotal evidence from racetracks in the UK and more formal analysis of behaviour at US racetracks suggests that bets of casual bettors do cause substantial distortions in market prices. For example Kopelman and Minkin (1991), in a North American pari-mutuel market study found that the correlation between track odds and finishing order in weekday races was significantly higher than that in weekend races ( 0.57 c.f. 0.42 ). However, it could be argued that simply investigating the degree to which market odds correspond to the outcome of a race is not a sufficiently rigorous test of the degree to which the weekend effect distorts the market.

Horseraces result in a definite outcome and, as a result, offer the prospect of measuring more directly than through the mechanism of correlations, the strength of any weekend effect. In particular, it is possible to measure the extent to which it is possible to capitalize on any mis-pricing of horses' prospects in order to earn abnormal returns. Consequently, we are able to test a second hypothesis:

The mis-pricing of horses' prospects is sufficiently marked in weekend races for the betting market on these races to be weak form inefficient.

As indicated above, a number of previous studies examining the weekend effect in financial markets attribute the effect to the behaviour of individual (c.f. institutional) investors. However, previous studies arrive at this conclusion by inferring that individual investors are more active on certain days of the week. The current study is conducted in the UK bookmaker betting market and Shin $(1991,1992,1993)$ suggests that the actions of bookmakers provide an opportunity for measuring directly the proportions of informed and less informed (casual) bettors in a market for a given race. Consequently, in order to shed light on the origins of any mis-pricing identified, we test a third hypothesis:

Casual bettors make up a significantly greater proportion of the betting population for weekend races.

## II. Data and Methodology

## A. Data

In order to test the hypotheses, data is drawn from the UK bookmaker horserace betting market (supplied by Raceform Ltd). Closing bookmaker market prices (odds) and finishing positions are recorded for all horses which ran in flat races between 16 June 1999 and 13 Aug 2000. Races with incomplete fields (i.e. where runners were withdrawn but their odds were included in the supplied database) are not included in the analysis. As a result of this data
cleaning process, the final dataset used to test the hypotheses consisted of 60,568 runners in 5,558 races at 35 different racetracks across the UK. The mean over-round (or bookmakers' theoretical profit margin) for these races was $22 \%$. The number of horses in each race varied from 2 to 34 , with a mode of 12, and the closing market odds for horses in the sample range from $0.04 / 1$ to $100 / 1$, with a mean of $16.42 / 1$.

The dataset is divided into races run during weekdays and weekends. Weekend races are defined as races which are run over weekends and during Bank holidays, when it is expected that casual bettors will dominate the market. Weekday races are defined as those run on working days when, by definition, most bettors who are employed will be required at their place of work; these individuals are likely to be the more casual bettors. Consequently, it is expected that the serious/informed bettors, who may for example bet professionally or be involved in the racing industry, will make up a greater proportion of the betting public during weekdays.

## B. Methodology

Exploring mis-pricing in the weekend vs. weekday betting markets The approach is to compare the extent of mis-pricing in the weekend and weekday markets by assessing the degree of favourite-longshot bias in these two markets (i.e. the extent to which favourites'/longshots' odds under-/over-represent their chances of success). Two means of assessing the degree of favourite-longshot bias are adopted. The first, involves grouping horses by their odds level and assessing the extent to which expected rates of return for the groups are identical. This enables identification of odds levels where positive rates of return are obtainable. Consequently, for each odds level $i$ an expected rate of return to level stake bets $\left(R_{i}\right)$ is computed, as follows:

$$
\begin{equation*}
R_{i}=\frac{W_{i}\left(O_{i}+1\right)-N_{i}}{N_{i}} \tag{1}
\end{equation*}
$$

where $W_{i}$ is the ex-post number of winning horses at odds level $O_{i}$ and $N_{i}$ is the total number of horses in odds group $i$. In addition, cumulative rates of return to level stake wagers are calculated for bets placed on all horses with odds up to a specific level. Observing the expected returns $\left(R_{i}\right)$ or cumulative return across various odds categories enables detection of the favourite-longshot bias and whether bets in certain odds ranges offer profitable trading opportunities (e.g., Busche and Hall, 1988; Dowie, 1976; Gabriel and Marsden, 1990, 1991; Snyder, 1978).

A second approach is also adopted which assesses the degree of favourite-longshot bias more directly. A model is developed to estimate horses' winning probabilities using the normalized, odds implied winning probability, $p_{i j}^{s}$, which is computed as follows:

$$
\begin{equation*}
p_{i j}^{s}=\frac{\frac{1}{O_{i j}+1}}{\sum_{i=1}^{n_{j}} \frac{1}{O_{i j}+1}} \tag{2}
\end{equation*}
$$

where $O_{i j}$ is the final odds of horse $i$ running in race $j$ (with $n_{j}$ runners). In order to develop the model, a 'winningness' index $W_{i j}$ for horse $i$ in race $j$, is defined as follows:

$$
\begin{equation*}
W_{i j}=\beta \operatorname{Ln}\left(p_{i j}^{s}\right)+\varepsilon_{i j} \tag{3}
\end{equation*}
$$

where $\beta$ is a coefficient which measures the importance of $\operatorname{Ln}\left(p_{i j}^{s}\right)$ in determining the likelihood of horse $i$ winning race $j$ and $\varepsilon_{i j}$ is an independent error term. $W_{i j}$ is defined such that the horse which is observed to win a particular race has the largest winningness index of all runners in that race. The estimated probability of horse * winning race $j$ (i.e. $p_{*_{j}}^{e}$ ) is given as follows:

$$
\begin{equation*}
p_{*_{j}}^{e}=\operatorname{Prob}\left(W_{*_{j}}>W_{i j}, i=1,2, \ldots . n_{j}, i \neq *\right) \tag{4}
\end{equation*}
$$

Consequently,

$$
\begin{equation*}
p_{*_{j}}^{e}=\operatorname{Prob}\left(\beta \operatorname{Ln}\left(p_{*_{j}}^{s}\right)+\varepsilon_{*_{j}}>\beta \operatorname{Ln}\left(p_{i j}^{s}\right)+\varepsilon_{i j}, i=1,2 \ldots . n_{j}, i \neq *\right) \tag{5}
\end{equation*}
$$

The $W_{i j}$ cannot be observed directly. However, whether horse $i$ wins race $j$ can be observed and a win/lose variable $w_{i j}$ (dichotomy variable) is defined such that:

$$
\begin{align*}
& w_{i j}=1 \text { if } w_{i j}=\operatorname{Max}\left(w_{1 j}, w_{2 j}, \ldots, w_{n j j}\right) \\
& w_{i j}=0 \text { otherwise } \tag{6}
\end{align*}
$$

Consequently, the estimated probability of horse ${ }^{*}$ winning race $j$ can be represented as follows:

$$
\begin{equation*}
p_{*_{j}}^{e}=\operatorname{Prob}\left(w_{*_{j}}=1 \mid \operatorname{Ln}\left(p_{i j}^{s}\right), i=1,2 \ldots, n_{j}\right) \tag{7}
\end{equation*}
$$

McFadden (1974) demonstrates that if it is assumed that the error terms $\varepsilon_{i j}$ in equation (5) are independent and distributed according to the double exponential distribution this produces the conditional logit function, where the probability of horse $i$ winning race $j$ is given as follows (Johnson and Bruce, 2001, p.285):

$$
\begin{equation*}
p_{i j}^{e}=\frac{\exp \left[\beta \operatorname{Ln}\left(p_{i j}^{s}\right)\right]}{\sum_{i=1}^{n_{j}} \exp \left[\beta \operatorname{Ln}\left(p_{i j}^{s}\right)\right]}=\frac{\left(p_{i j}^{s}\right)^{\beta}}{\sum_{i=1}^{n_{j}}\left(p_{i j}^{s}\right)^{\beta}} \tag{8}
\end{equation*}
$$

$\beta$ is estimated by maximising the joint probability of observing the results of all $N$ races in the training sample. If $\beta=1, p_{i j}^{e}$ is equal to $p_{i j}^{s}$, suggesting that the final market odds are non-biased. However, if $\beta>1$, this suggests mis-pricing whereby $p_{i j}^{s}$ is smaller than $p_{i j}^{e}$ for favourites and $p_{i j}^{s}$ is greater than $p_{i j}^{e}$ for longshots; this is the traditional favourite-longshot bias. If $\beta<1$, this suggests that mis-pricing results in the reverse
favourite-longshot bias (Bacon-Shone, Lo and Busche, 1992). A formal $t$-test can be conducted to test the null hypothesis that $\beta=1$ (Lo, 1994) since, given the underlying assumptions regarding the error terms in equation (5) (i.e. independent and distributed according to the double exponential distribution), the estimate of $\beta$ will be a consistent estimator and approximately normally distributed with a mean $\beta$, and with a standard deviation which can be derived by the maximum likelihood procedure when the sample size is large (Johnson and Bruce, 2001). Separate estimates for $\beta$ were developed for weekend and weekday races.

Exploring the strength of the weekend effect

To test the second hypothesis, namely that the mis-pricing of horses' prospects is sufficiently marked in weekend races for the market to be weak form inefficient, the dataset is partitioned into a training (16 June 1999-15 May 2000) and a holdout sample (16 May- 13 August 2000) in the proportions $66.66 \%$ and $33.33 \%$, respectively, as suggested by a number of previous studies (e.g., Baesens, et al., 2003). The models discussed above are developed for weekend and weekday markets based on the training sample of races. The models, based solely on the closing market odds, are then employed to estimate winning probabilities $\left(p_{i j}^{e}\right)$ of horses running in the holdout races. These probabilities are used as inputs to a Kelly wagering strategy (Kelly, 1956), which identifies how much to bet on each horse running in weekday and weekend races between 16 May - 13 August 2000. Rates of return resulting from the Kelly wagering strategy for weekend and weekday races are compared. If a positive rate of return is achieved for weekend races this will suggest that the weekend market is weak form inefficient and if, simultaneously, a negative return is obtained for weekday races this will suggest that a weekend effect exists in UK horserace betting markets. The approach adopted here, of modelling winning probabilities and using these as inputs to a Kelly betting strategy, makes optimal use of the information contained in market prices. This approach is
not possible in many financial markets. Consequently, the size of any positive return identified here in relation to the weekend market will provide a valuable measure of the strength of the weekend effect.

A Kelly strategy requires that a fraction $f_{j}(i)$ of current wealth is bet on horse $i$ in race $j$. Let $f_{j}=\left\lfloor f_{j}(1), \ldots, f_{j}\left(n_{j}\right)\right\rfloor$ be the total fraction of wealth bet on race $j$. Given a bankroll with a starting value of one unit, if horse* with odds $O_{*_{j}}$ wins race $j$, this will lead to current wealth increasing by a factor of:

$$
\begin{equation*}
1-\sum_{i=1}^{n_{j}} f_{j}(i)+f_{j}(*)\left(O_{*_{j}}+1\right) \tag{9}
\end{equation*}
$$

Consequently, the Kelly strategy consists of choosing $f_{j}$ to maximise the expected $\log$ payoff, $F\left(f_{j}\right)$ where

$$
\begin{equation*}
F\left(f_{j}\right)=\sum_{*=1}^{n_{j}} p_{*_{j}} \log \left(f_{j}(*)\left(O_{*_{j}}+1\right)+1-\sum_{i=1}^{n_{j}} f_{j}(i)\right) \tag{10}
\end{equation*}
$$

This betting strategy identifies which races and which horses to bet on, as well as how much to bet on each of the identified horses. This strategy has been shown to be asymptotically optimal by Breiman (1961), in the sense that it maximises the asymptotic rate of growth for wealth, with 0 probability of ruin if arbitrarily small bets are permitted. Using the Kelly criterion, the total wealth grows at an exponential rate, though the standard deviation remains proportional to total wealth and thus also grows exponentially. In applying a Kelly wagering strategy, the absolute amount of money bet on a race is, to a large extent, influenced by the current wealth level, which is dependent on the results of previous bets. In order to remove potential bias resulting from the order of races in the test sample, reinvestment of winnings is not applied here.

In order to demonstrate the power of the approach adopted to measure the true strength of the weekend effect, the returns obtainable from weekend markets based on a number of more naive betting strategies are calculated. These approaches do not rely on a forecasting system or an optimal wagering strategy. The first of these involves betting $£ 1$ on each horse in the holdout sample. The result of this strategy is essentially equivalent to the cumulative rate of return of the market. The second approach bets enough money on each horse to win $£ 1$. This strategy helps to remove the favourite/longshot bias by betting more on favourites and less on longshots. This is a crude way of accounting for the favourite/longshot bias but it does not account for the actual performances of horses at each odds level. The third naive approach involves betting $£ 1$ on the favourite (i.e. the horse with the shortest final market odds) in each race. Clearly a Kelly betting strategy should make greater use of information contained in market prices since it not only uses the estimated model probabilities but also suggests an optimal fraction of bankroll to bet on horses.

If positive returns are identified for weekend races using the Kelly betting strategy it might be argued that this could have arisen as a result of a fortunate selection of the holdout sample. To overcome this problem a Jackknife (or leave-one-out) procedure, first proposed by Quenouille (1949), is employed. It is a re-sampling technique with the aim of providing a means of testing the level of confidence of an estimator. The procedure involves repeatedly setting one (different) race aside; the remaining races are used for developing a conditional logit model. This model is used to assess the winning probabilities of each horse in the set aside race and an ordinary Kelly betting strategy is employed to bet on this race. The resulting winnings are recalculated during each re-sampling and re-modelling process. This enables a test of the null hypothesis that the resulting winnings are equal to or less than zero. Let $W_{j}(j$ $=1, \ldots, n$, where $n$ is the number of races in the sample) be the observed winnings obtained by using the fitted model and the normal Kelly criterion to bet on race $j$. Let $\mu^{J}$ be the mean of
$W_{j}$. The $\mu^{J}$ can be viewed as a continuous non-linear function of the estimated model parameter $\beta$, as denoted in equation (8), hence it cannot be assumed that $\mu=\mu^{J}$, where $\mu$ is the asymptotic winnings (per race). However, maximum likelihood estimation produces consistent estimators, so the estimate of $\beta$ will converge to some limit as the size of the re-sampling set increases. Thus, it is reasonable to assume that $\mu \approx \mu^{J}$. It could be argued that since the leave-one-out process produces highly similar models in most of the in-sample races, the resulting $W_{j}$ are highly dependent. However, most of the variance in the distribution of $W_{j}$ comes from each race $j$ on which the model is tested. Thus the $W_{j}$ will be approximately independent, which allows a confidence interval for $\mu$ to be determined.

Exploring the origin of the weekend effect

In order to test the third hypothesis, that the origin of the weekend effect is the greater proportion of casual bettors operating in the weekend market (c.f. the weekday market) use is made of Shin's $(1991,1992,1993)$ argument that market makers (bookmakers) react to the presence of informed traders by deliberately shortening the prices on longshots. This arises since bookmakers seek to protect themselves from the actions of informed traders who are aware, beyond that which could be gleaned from publicly available information, of the true prospects of success of particular horses. Fearing that they face such bettors, bookmakers increase the bid-ask spread in order to reduce the risk of losing a large amount of money to informed traders (particularly on longshots), but ensuring the prices are attractive to bettors (c.f. other bookmakers). Consequently, bookmakers reduce the odds on longshots, as this has the effect of reducing the potentially substantial losses they might face from informed traders' bets on long odds runners (Shin, 1991, 1992, 1993; Vaughan Williams and Paton, 1997). Support for a larger favourite-longshot bias in markets more heavily populated by informed traders is given by Bruce and Johnson (2003). Shin (1993) models the betting market as an
extensive form game, using the proposed link between bid-ask spread and the prevalence of informed traders to develop a means of estimating the extent of betting by these privileged agents. The result is Shin's $z$ value, which measures the proportion of informed trader activity within a market. Based on a sample of 136 UK races Shin (1993) estimates $z$ to be about $2 \%$; this is confirmed by Vaughan Williams and Paton (1997) and Law and Peel (2002). Cain et al. (2001) confirm the value of Shin's $z$ as a useful measure by demonstrating a strong positive relationship between Shin's $z$ and another well established indicator of informed trader activity (i.e. a significant reduction in a horse's odds).

In order to clarify whether there is a significant difference in the proportion of informed traders in weekend and weekday betting markets the extent to which Shin's $z$ values differ between these two sets of races is tested. There are several statistical tests which could be used to test the hypothesis but the Mann-Whitney $U$ test is employed because it does not require any assumptions regarding the distribution of the test variable (Wackerly et al. 2002). It simply requires that all observations are independent and are randomly selected from the population. Consequently, the Mann-Whitney test is employed to compare the Shin's $z$ value $z_{i}^{h}$ of each race $i$ run at weekends (i.e. holiday periods: $h$ ) with the Shin's $z$ value $z_{j}^{d}$ of each race $j$ run on weekdays $(d)$. If the weekend and weekday periods have the same median Shin's $z$ value then each $z_{i}^{h}$ will have a probability 0.5 of being greater or smaller than each $z_{j}^{d}$ after mixing and ranking all the observations by magnitude. Consequently, the hypothesis is constructed as follows: $H_{o}: P\left(z_{i}^{h}>z_{j}^{d}\right)=0.5 ; H_{1}: P\left(z_{i}^{h}<z_{j}^{d}\right) \neq 0.5 . U_{h}$ and $U_{d}$ are the number of times a $z_{i}^{h}$ from period $h$ is greater/smaller than a $z_{j}^{d}$ from period $d$, respectively. If the periods have the same median, $U_{h}$ and $U_{d}$ are expected to be approximately equal. Consequently, the statistical test is derived from taking the smaller of
$U_{h}$ and $U_{d}$ (Shier, 2006). This test is performed two-tailed using SPSS software (version 13) and a p-value $<0.05$ is considered statistically significant.

## III. Results and Discussion

## A. Mis-pricing Evidenced by Expected Rates of Return

The cumulative returns to level stake bets placed on each horse with odds less than or equal to a particular final market price are calculated for weekend and weekday races separately (without splitting them into training and test samples). The results are plotted in Figure 1. It appears that the favourite-longshot bias exists in both markets. However, positive rates of return can only be found up to odds levels of $0.2 / 1$ for weekday races but up to odds levels of 1.1/1 for weekend races. In addition, the highest expected rate of return for bets on horses up to a particular odds level was higher for races run at weekends (16\%) than for races run on weekdays (5\%). This suggests that horses with very short odds are more seriously undervalued in the weekend betting market. Consequently, although the average rates of return for bets placed on all horses in both the weekend and weekday market are very similar ( $-31.48 \%$ and $-31.90 \%$ respectively), positive returns are more likely to be made in the weekend market (c.f. the weekday market) if bets are restricted to short-odds horses. These results provide a hint of greater mis-pricing in the weekend market and the extent to which this can be exploited to develop a profitable betting strategy is explored below.
[Figure 1 about here]

## B. Mis-pricing Evidenced by Model Parameters

Conditional logit models are developed for weekend and weekday races with $\log$ of normalized final market odds probabilities as explanatory variables $\left(\ln \left(p_{i j}^{s h}\right)\right.$ and $\ln \left(p_{i j}^{s d}\right)$,
respectively). These models are represented by equations (11) and (12) and the coefficients $\beta_{h}$ and $\beta_{d}$ are estimated from races run between 16 June 1999 and 15 May 2000:

$$
\begin{align*}
& p_{i j}^{e h}=\frac{\exp \left(\beta_{h} \ln \left(p_{i j}^{s h}\right)\right.}{\sum_{i=1}^{n_{j}} \exp \left(\beta_{h} \ln \left(p_{i j}^{s h}\right)\right)}=\frac{\left(p_{i j}^{s h}\right)^{\beta h}}{\sum_{i=1}^{n_{j}}\left(p_{i j}^{s h}\right)^{\beta h}}  \tag{11}\\
& p_{i j}^{e d}=\frac{\exp \left(\beta_{d} \ln \left(p_{i j}^{s d}\right)\right.}{\sum_{i=1}^{n_{j}} \exp \left(\beta_{d} \ln \left(p_{i j}^{s d}\right)\right)}=\frac{\left(p_{i j}^{s d}\right)^{\beta d}}{\sum_{i=1}^{n_{j}}\left(p_{i j}^{s d}\right)^{\beta d}}
\end{align*}
$$

The results of estimating the conditional logit models are reported in Table 1. The coefficients associated with the log of normalized odds probability are significantly different from zero at the $5 \%$ level in both the weekend and weekday models. This implies that market prices contain a considerable amount of information regarding the relative competitiveness among horses in a race. In addition, the coefficients are significantly greater than one for both models, indicating that the favourite-longshot bias exists in races run on weekdays and weekends. Furthermore, the weekend market has a higher degree of favourite-longshot bias ( $\beta_{h}=1.2421$ ) than the weekday market $\left(\beta_{d}=1.2362\right)$, although the difference in these coefficients is not significant at the $5 \%$ level $(t=0.1)$.
[Table 1 about here]
Taken together, the results of comparing both expected returns at different odds levels and the degree of favourite-longshot bias for weekend and weekday races support hypothesis 1, namely that odds formed by the wagering decisions of bettors during weekend periods are not as accurate a guide to horses' prospects as those formed in weekday periods. The strength of this case, in terms of the degree of favourite-longshot bias on weekdays and weekends, is stronger than the raw results suggest. This arises because Shin $(1991,1992,1993)$ provides a strong theoretical case for the favourite-longshot bias being greater in bookmaker betting
markets when a greater proportion of informed traders are present. It is likely therefore that in weekday markets (where, it is shown below, a greater proportion of informed traders are present) a large part of the bias arises from the actions of the market makers (bookmakers) restricting the odds on longshots to protect themselves from the bets of informed traders. Empirical support for this view is given by Vaughan Williams and Paton (1997) and Bruce and Johnson (2003). The results here demonstrate that the favourite-longshot bias in weekend markets is at least as large as that for weekday races, and yet bookmakers' actions are far less likely to be the cause of this bias when the proportion of informed traders is smaller. These results therefore provide evidence that the wagering decisions of bettors cause the mis-pricing observed during weekends; and it is demonstrated below that this mis-pricing enables abnormal returns to be earned.

## C. The Strength of the Weekend Effect

The weekend and weekday models estimated in the previous section are employed to predict winning probabilities for horses running in the weekend and weekday holdout races, respectively. A standard Kelly wagering strategy is employed (without re-investment of winnings), using these predicted probabilities together with final market odds, to determine the optimal proportion of one's bankroll to bet on a given horse (assuming an initial wealth $£ 1000$ ). The ratio of winnings to stakes over the whole holdout sample period (16 May - 13 August 2000) resulting from this strategy is presented in Table 2, together with equivalent ratios for alternative, more naive, betting strategies. The results demonstrate that for races run during both weekend and weekday periods the wealth derived from employing the model probabilities (using the Kelly criterion) always exceeds that obtained from naive betting strategies. More importantly, positive returns are not achievable over the holdout period from races held on weekdays, even when the best betting strategy is adopted (i.e. Kelly criterion: $-4.55 \%)$. However, positive returns are produced using the Kelly strategy during this period
from betting on weekend races (19.21\%). In addition, a Kelly wagering strategy, with reinvestment of winnings, applied to the 528 holdout races run at weekends, produces a significant increase in wealth (223\%). A similar strategy applied to the 1,382 holdout races run on weekdays results in a substantial decrease in wealth (see Figure 2).
[Table 2 and Figure 2 about here]

Analysis of the weekday and weekend races indicates that the contrasting returns obtained from these events do not derive from differences in the mean bookmakers' over-round or theoretical profit (1.22 and 1.21, respectively, $\mathrm{t}_{\text {difference }}=0.84, \mathrm{p}=0.20$ ) or the mean number of runners per race ( 10.41 and 10.74 , respectively, $\mathrm{t}_{\text {difference }}=1.36, \mathrm{p}=0.09$ ). In addition, to confirm that the above results do not derive from the particular selection of the holdout sample, separate Jackknife tests were conducted for races run during weekends and for those run on weekdays, as outlined in the methodology section. The Jackknife procedure for races run during weekends results in a mean return per race $(\mu)$ of 0.00101 (standard error $=$ 0.00066 ). A test of the hypothesis $\mu=0$ against the alternative $\mu>0$ is significant at the $6.4 \%$ level. On the contrary, the Jackknife results for races run on weekdays (mean return per race $=-0.00042$ and standard error $=0.00037$ ) suggest that the return per race is not significantly different from zero.

Taken together, the results suggest that a betting strategy based simply on the final market prices in weekend races can be used to generate positive returns, but this is not possible for weekday races. To date no study in financial markets has been able to demonstrate so clearly the strength of the weekend effect. This arises from the opportunity which betting markets afford of modelling winning probabilities (based on actual winning frequencies) and making full use of the information contained in market odds through the use of the Kelly wagering strategy. In summary, the results reported here support hypothesis 2 ,
namely that mis-pricing of horses in races run during weekends leads to a betting market which is weak-form inefficient.

## D. Investigating the Origin of the Weekend Effect

In formulating the hypotheses it was speculated that greater mis-pricing of horses may occur during the weekend because of the greater involvement of casual bettors. It was argued that weekday markets are likely to be populated with a greater proportion of more serious bettors and that their informed betting decisions should result in prices more in line with actual winning probabilities. The results of testing hypotheses 1 and 2 confirm this view.

To test the hypothesis that less informed bettors make up a greater proportion of weekend race markets the difference between Shin's $z$ values for weekend and weekday races run between 16 June 1999 and 13 August 2000 is explored. The median Shin's $z$ value for the 4,037 races run on weekdays was 0.0243 (mean $=0.0252$, standard error $=0.0001$ ) and the median Shin's $z$ value for the 1,521 races run during weekends was 0.0232 (mean $=0.0243$, standard error $=0.0002$ ). The Mann-Whitney $U$ test for difference between these two population values produced a $p$ value of 0.0003 . This result suggests, in line with hypothesis 3 , that the proportions of informed bettors (c.f. casual bettors) are greater in weekday markets. It appears, therefore, that an important factor in causing the weekend effect in betting markets is the wagering decisions of casual bettors. This is in line with much of the speculation concerning the origin of the weekend effect in wider financial markets (e.g., Miller, 1988; Lakonishok and Maberly, 1990; Abraham and Ikenberry, 1994).

## IV. Conclusion

Betting markets and wider financial markets are similar in a number of respects and one of the conclusions to emerge from this study is that a phenomenon relating to weekends exists in both markets. Taken together the results offer strong support for the three hypotheses developed earlier. Mis-pricing is shown to occur in races run during weekends and during
weekdays. However, mis-pricing on weekdays is not sufficient to enable a betting strategy to be constructed, based solely on final market odds, which results in abnormal returns. In contrast, a Kelly betting strategy based on final market odds can be used to earn substantial profits for races run at weekends. The finding is remarkable given that the average over-round or theoretical bookmaker profit on these races is $21 \%$ and this suggests that the weekend effect is an important phenomenon in betting markets.

A secondary but important conclusion to emerge from this study is the value of using betting markets to shed light on behaviour in wider financial markets. The advantage of exploring the weekend effect in betting markets is that it allows direct measurement of the full strength of the effect. In particular, because races produce a definitive result, it is possible to model the relationship between market odds and winning probabilities. Consequently, estimates of winning probabilities can be derived and these can be used in a Kelly wagering strategy to make full use of the biased odds available in weekend markets. The advantage of exploring market efficiency in this manner was clearly demonstrated by the fact that more naive betting strategies did not suggest mis-pricing in weekend markets, whereas the modelling/Kelly approach clearly demonstrated a significant inefficiency. In addition, as explained above, exploring the weekend effect in betting markets offers a means of attributing its cause; in fact the evidence presented leads to the conclusion that it is the biased decisions of more casual investors on weekend races which are an important factor in causing this phenomenon.

Overall the results indicate that the weekend effect is a widespread phenomenon which has an important impact on market efficiency. Since the results indicate that casual investors play an important part in creating this inefficiency, future work examining in more detail the behaviour of these individuals is needed.

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Figure 1: Rates of return from bets at given final market odds: Weekend and non-weekend races
$\rightarrow$ Weekend $\rightarrow$ - Weekday


Midpoints of odds categories

Figure 2: Log of cumulative wealth resulting from applying a Kelly wagering strategy (with re-investment of winnings) for races run at (a) weekends and on (b) weekdays during the holdout period (16 May- 13 August 2000).


Table 1: Coefficients and statistical tests for the weekend and weekday conditional logit models based on the log of normalized final market odds probabilities estimated for races run between 16 June 1999 and 15 May 2000

| Model | Weekend races | Weekday races |
| :--- | :---: | :---: |
| Coefficient $(\beta)$ | $1.2421^{\mathrm{a}}$ | $1.2362^{\mathrm{b}}$ |
| Standard Error | 0.0515 | 0.0299 |
| t-ratio | 24.0964 | 41,2899 |
| $L(\theta=0)$ | $-3,472.03$ | $-9,201.53$ |
| $L(\theta=\overparen{\theta})$ | $-1,910.59$ | $-5,122.24$ |
| $L L$ ratio statistic | $3,122.88$ | $8,158.58$ |
| Adj $\dddot{R}^{2}$ | 0.4497 | 0.4433 |
| No. of races | 993 | 2,655 |
| No. of horses | 10,793 | 29,721 |

[^0]Table 2: A comparison of returns from different wagering strategies on races run at (a) weekends and on (b) weekdays in the out-of-sample period (16 May - 13 August 2000)

| Wagering strategies | Weekend races |  |  |  |  |  | Weekday races |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | No. of bets | No. of races bet | No. of races won | Amt. bet (£) | Profits <br> (£) | Rate of return (\%) | No. of bets | No. of races bet | No. of races won | Amt. <br> bet <br> (£) | Profits <br> (f) | Rate of return (\%) |
| Kelly strategy ${ }^{\text {a }}$ | 187 | 162 | 76 | 4,720 | 910 | 19.21 | 424 | 370 | 165 | 11,520 | -520 | -4.55 |
| $£ 1$ bet on each horse | 5,669 | 528 | 85 | 5,669 | -1,834 | -32.35 | 14,385 | 1,382 | 228 | 14,385 | -4,449 | -30.92 |
| Return $£ 1$ on each horse if the horse wins | 5,669 | 528 | 0 |  | -113 | -17.65 | 14,385 | 1,382 | 0 | 1,685 | -303 | -17.97 |
| $£ 1$ bet on favourite of each race ${ }^{\text {b }}$ | 575 | 528 | 189 | 575 | -6 | -1.13 | 1,506 | 1,382 | 484 | 1,506 | -91 | -6.02 |

${ }^{\text {a }}$ A Kelly strategy can involve betting on more than one horse in a race.
${ }^{\mathrm{b}}$ Where a race has more than one favourite, $£ 1$ is bet on each favourite.


[^0]:    ${ }^{\mathrm{a}}$ The coefficient of the weekend model is significantly different from 1 ( $t$-value 4.7, $\mathrm{p}<0.01, \mathrm{n}=993$ )
    ${ }^{\mathrm{b}}$ The coefficient of the weekday model is significantly different from 1 ( $t$-value $7.9, \mathrm{p}<0.01$, $\mathrm{n}=2,655$ )

