A Re-examination of the Empirical Performance of the Longstaff and Schwartz Two-factor Term Structure Model Using Real Yield Data

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Abstract

This paper tests the Longstaff and Schwartz two-factor model of the term structure using real yield data from Australia, Germany, the United Kingdom and the United States. We use Hansen's generalized method of moments (GMM) framework to test the cross-sectional restrictions imposed by the LS two-factor model as well as the Cox-Ingersoll- Ross one-factor model. Further, we compare the forecasting ability from both one- and two-factor models. Our findings support the superiority of the two-factor model and, hence, indicate that both the short-term interest rate and the volatility of short-term interest rates are important factors in modeling the term structure of interest rates.

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I. Introduction

Short-term interest rates play several significant roles in modern economies. It is a key instrument for policy makers to manipulate monetary policy to influence the macro-economy. The short rate is also a key input for the valuation process of all fixed income securities and derivatives, the implementation of optimal hedging strategies, dynamic hedging and trading of bonds, and portfolio allocation decisions. As a result, over the past few decades, modeling short-term interest rates has been one of the most-studied topics in modern finance.

One area of keen interest has been the formal modelling of the term structure. Logically these models began with the simplest case based on a single-factor setup. However, in the 1980s, researchers demonstrated the inadequacy of such formulations and developed multiple-factor approaches to term structure (see, for example, Stambaugh, 1988; and Brennan and Schwartz, 1982). Most notable, for the current paper, Longstaff and Schwartz (1992) pointed out that one of the inadequacies of the short-rate model is that it implies the instantaneous returns on bonds of all maturities are perfectly correlated. Hence, they extend the Cox, Ingersoll and Ross (CIR) (1985) one-factor specification to a two-factor model of the term structure of interest rates. The LS (1992) model is theoretically appealing. It is developed to value interest-rate-sensitive contingent claims in the real economy. It belongs to the general equilibrium approach of term structure modeling. Hence, using the LS approach it is possible to endogenously determine the term structure, its dynamics, and functional form of the market price of risk.

One objection against many multiple-factor models of the term structure is that multiple-factors make the valuation problem complicated and in many cases intractable. This, however, is not a problem with the LS model. The LS provides closed-form formulae for bond pricing and its derivatives. Although the LS model is theoretically sound, their empirical tests have a serious drawback, namely, they test the term structure model of real rates using nominal rate data. The inability to reject the model using nominal data could be viewed as evidence supporting the model or alternatively it could be a reflection of the inconsistency between the theoretical underpinnings of the model and the data used.

Motivated by the above discussion, the purpose of our paper is to execute a test of the LS (1992) model using appropriately formulated real yield data in the international context. The countries we examine are the Australia; Germany; the UK and the US. This naturally leads us to our main research questions as follows:

1) How well does the LS model fit real yield data?
2) Are the results of these tests similar across different fixed-income markets?
3) How well does the LS model work in terms of forecasting yields?

Our paper is organised as follows. Section 2 briefly reviews related empirical work on term structure modelling, with particular attention given to a multifactor Cox, Ingersoll, and Ross model. Section 3 describes the data and research design adopted. Section 4 presents empirical results and Section 5 concludes.

II. Literature Review

In general, modeling the term structure of interest rates can be classified into two major approaches: the no-arbitrage and the equilibrium approaches. The arbitrage-free approach assumes that no arbitrage opportunities exist in the economy. Given the stochastic evolution of one or more interest rates, the prices of all contingent claims can be derived by imposing the no-arbitrage condition. However, the no-arbitrage approach provides no guidance about the form of the market risk premium and can sometimes lead to internal inconsistencies or arbitrage opportunities due to the arbitrary choice of the functional form adopted.[1] Examples of the no-arbitrage approach include Ho and Lee (1986), and Heath, Jarrow, and Morton (1992).

In contrast to the no-arbitrage approach, the general equilibrium approach is built upon the intertemporal capital asset pricing model of Merton (1973) and the rational expectations equilibrium model of Lucas (1978). The general equilibrium approach, developed by Cox, Ingersoll and Ross (1985), represents an equilibrium specification of the underlying economy with assumptions regarding the evolution of one or more exogenous factors or state variables in the economy and about the preferences of a representative investor. According to the CIR model, the economy is composed of identical individuals, each of whom maximize utility subject to wealth such that each individual chooses his optimal consumption, and the optimal proportion of wealth to be invested in each of the production processes, and in each of the contingent claims.

Hence, in equilibrium, the CIR model is completely consistent with stochastic production and with changing investment opportunities such that the interest rate and the expected rates of return on the contingent claims have to adjust until all wealth is invested in the physical production processes. The CIR model yields testable implications for prices of bonds whose payoffs are denominated in real terms and the closed-form expressions for the endogenously derived real prices in terms of one or more state variables. The major advantage of the equilibrium approach over the no-arbitrage approach is that the functional form of market prices of risk is obtained as part of equilibrium and both the term structure and its dynamics are endogenously determined.

[1] See Cox, Ingersoll and Ross (1985) for more discussion regarding the drawbacks of the no-arbitrage model.

Due to its theoretical appeal, the CIR model has been empirically tested and extended by many researchers since its introduction. Examples of this work include Stambaugh (1988); Brown and Dybvig (1986); Longstaff and Schwartz (1992); Gibbons and Ramaswamy (1993); Brown and Schaefer (1994); Pearson and Sun (1994); Balduzzi, Das, and Foresi (1996); Bakshi and Chen (1996); Jensen (2000); Bansal and Zhou (2002); and Chen and Scott (2003).[2] Stambaugh (1988) employs nominal Treasury-bill data to test the prediction of the CIR model that conditional expected holding period returns of discount bonds are linearly related to forward rates. He rejects a one-factor model but finds only weak evidence for more than two factors. The main limitations of the Stambuagh approach are that the data used are based on nominal yields and that neither the underlying state variables are uncovered, nor the parameters of the CIR model estimated. Brown and Dybvig (1986) use a cross-sectional method to estimate certain combinations of the parameters of a one-factor CIR model using nominal Treasury-bill data. In examining the fit of nominal Treasury-bill prices to the CIR model, Brown and Dybvig (1986) fail to make use of the theoretical distribution of an instantaneous riskless rate implied by the model and they also cannot identify the speed of adjustment to the mean and market prices of risk separately and, hence, not all four parameters of the CIR model can be estimated.

Longstaff and Schwartz (1992) developed a two-factor general equilibrium model of the term structure of real interest rates using the CIR (1985) framework. Specifically, they use the short-term interest rate and the instantaneous variance of changes in the short-term interest rate as the two factors underlying the dynamics of term structure. The advantage of this model is that its dynamics and factor risk premium are all endogenously

[2] Longstaff and Schwartz (1992); Gibbon and Ramaswamy (1993); Bakshi and Chen (1996) and Chen and Scott (2003) are most related to this paper.

determined. Longstaff and Schwartz (1992) use generalized method of moments (GMM) to test the cross-sectional restrictions imposed on the term structure by the two-factor model, using nominal Treasury data. In their GMM estimation of the two-factor model, LS use one-month T-bill rates as a proxy for the instantaneous interest rate and estimates of interest rate volatility are generated from a GARCH(1,1) model. The major drawback of the empirical application of the LS model is the inconsistency of testing a real model using nominal yield data. Further, their approach depends on the assumption that the estimates for the two factors do not contain measurement error.

Gibbons and Ramaswamy (1993) use GMM to estimate and conduct an empirical test of the one-factor real CIR model using real yield data.[3] They begin with the assumption that the price level is independent of the real economy and employ the steady-state distribution of the real interest rate to calculate the unconditional means, variances, and covariances of real yields to maturity of nominal discount bonds. The major disadvantage of GR is that they can use only Treasury bills, whose price is less sensitive to the model parameters relative to long-term coupon bonds. Further, using a steady-state density of interest rates is another disadvantage of GR, as it is the conditional density which determines the evolution of the term structure through time. Finally, their approach can be applied only to discount bonds and there are not enough coupon bonds available to construct the implied prices of discount bonds with various times to maturity without interpolation or extrapolation.

Brown and Schaefer (1994) assume that the long-term rate and the spread between the short and long rate are the two factors driving the dynamics of the term structure. They fit the two-factor CIR model using swap rates data for six countries: US, UK, GER,

[3] See Gibbons and Ramaswamy (1993) for a detailed discussion of the importance of testing term structure of real rates with matching real yield data.

YEN, ITL, and NGL. Pearson and Sun (1994) use nominal yield data and maximum likelihood estimation to estimate a two-factor CIR model where real rates and the price level are the two factors. They use the conditional density of state variables to estimate the one- and two-factor model and reject both. To obtain tractable likelihood functions, Pearson and Sun (1994) assume no measurement errors[4] by restricting the number of cross sections for the bond rates to be equal to the number of factors. They acknowledge that they should include price level data in estimating and testing the model, however, they do not do so due to the averaging problem of CPI data.[5] Balduzzi, Das, and Foresi (1996) use nominal yield data to estimate a two-factor CIR model, where the level of the short rate and a linear combination of any two rates, termed as 'central tendency', are the two factors. While this model complements the LS model, their objective is to use the two-factor model to understand the dynamics of the short rate.

Bakshi and Chen (1996) integrate asset pricing theory with models from monetary economics and provide a complete analysis of the joint determination of the price level, inflation, equity prices, and the real and nominal term structures. They show that the nominal and real term structures can have completely different properties, including fundamentally different risk structures. They suggest that real term structure is completely driven by technological shocks, whereas its nominal counterpart is driven by monetary shocks. In the situation in which monetary and technological(real) shocks are uncorrelated, the process followed by the real term structure will be independent of that followed by the nominal term structure. Their results sharply contrast with CIR (1985) and Sun (1992). Finally, they suggest that to model the real term structure, one should use one factor (real shock) as a proxy but to model the nominal term structure, one should use a two-factor model, driven by both real and inflationary shocks.

[4] Measurement errors arise when there are more bond rates than unobservable factors or state variables.
[5] See Pearson and Sun (1994) for further discussion of the averaging problem of CPI data.

Jensen (2000) uses Efficient Method of Moment (EMM) to estimate the LS model using weekly three-month nominal rates and he rejects the LS model. Further, he suggests that the inadequacy of the LS model stems from problems in accommodating the volatility process. The major drawback of Jensen is that he uses only one nominal yield to estimate the real term structure. Bansal and Zhou (2002) develop a term structure model in which the short-interest rate and the market price of risk are subject to discrete regime shifts. Using the EMM and the same data set as LS, they reject standard affine models that do not include a regime shift with up to three factors and find that the two-factor regime-switching model with a regime-dependent risk premium best fits the data. Finally, Chen and Scott (2003) estimate a multifactor CIR using nominal yield data and a Kalman filter estimation technique. They find support for a three-factor model, which is an extension of the LS model.

III. Data and Research Method

A. Data

Our dataset consists of monthly observations of bill and bond yields and consumer price index (CPI) for Australia (AUS), Germany (GER), the United Kingdom (UK), and the United States (US). The data on AUS, GER, and UK yields and CPI for all countries are collected from Datastream, while the US yields are obtained from the Center for Research in Security Prices (CRSP). For AUS, we collect 3-month, 6-month dealer bill rates, and 5-year and 10-year bond yields from February 1976 to 2006, resulting in 361 observations for the full sample. For GER, we collect 1-month, 2-year, 3-year, 5-year, 7-year, 10-year, 20-year and 30-year nominal yield from January 1996 to February 2006, resulting in 122 observations for the full sample. For UK, we collect 1-month, 3-month 2-year, 3-year, 5-year, 7-year, 10-year, 15-year, 20-year and 30-year nominal yield from

January 1996 to April 2005, resulting in 113 observations for the full sample. For the US, we collect 1-month, 3-month, 6-month and 9-month Treasury yield from the Fama file in CRSP and 1-year, 2-year, 3-year, 4-year, and 5-year from the Fama-Bliss file in CRSP from June 1964 to December 2005, resulting in 452 observations for the full sample. Finally, for all countries, with the exception of Australia, we collect monthly data on CPI from Datastream – in the case of Australia only quarterly CPI exist.[6]

To assess the forecasting performance of each model, we also partition our sample into two sub-samples, creating an in-sample and an out-of-sample analysis for each country. For AUS, the in-sample period is February 1976 to December 2002, resulting in 323 observations while the out-of-sample period is January 2003 to February 2006, resulting in 38 observations. For GER, the in-sample period is January 1996 to December 2002, resulting in 83 observations while the out-of-sample period is January 2003 to February 2006, resulting in 38 observations. For the UK, the in-sample period is January 1996 to December 2002, resulting in 83 observations while the out-of-sample period is January 2003 to April 2005, resulting in 28 observations. Finally, for the US, the in-sample period is June 1964 to December 1989, resulting in 307 observations while the out-of-sample period is January 1990 to December 2002, resulting in 145 observations.

Table 1 reports summary statistics for the consumer price index and nominal yields for all countries examined. The shapes of the yield curve are similar across all sample periods for all countries. That is, on average, the yield curve is typically upward sloping, except for the UK, which exhibits a 'hump' shape for the in-sample and full sample periods. However, the yield volatilities (standard deviations) are quite disparate across countries and sample periods. For the AUS yields, the yield volatilities decrease with the maturities

[6] For simplicity we linearly interpolate monthly CPI figures between each quarterly observation.

for all sample periods. For the GER yields, yield volatilities increase with maturity for the out-of-sample and full sample period but have a V shape with a minimum volatility at the three year term-to-maturity. For the UK yields, the yield volatilities decreases with maturity for the out-of-sample period but have a V shape with a minimum at three years and five years for the in-sample and full sample period, respectively. For the US yields, the yield volatilities display a 'hump' shape with a peak at nine months for all sample periods. Finally, for all sample periods, nominal yields for all countries exhibit a high degree of persistence with the full-sample period being the most persistent and the out-of-sample period being the least persistent.

[Table 1 About Here]

B. Method

This study is built upon the two-factor LS (1992) model.[7] The short-term interest rate *r* and the variance of changes in the short-term interest rate *V*, which are easily estimated or obtained, are chosen as two underlying factors driving the term structure of real yields. That is, the equilibrium value of $F(r,V,\tau)$, a riskless unit discount bond with τ periods until maturity derived by Longstaff and Schwartz (1992) is:

$$F(r,V,\tau) = A^{2\gamma}(\tau)B^{2\eta}(\tau)\exp(\kappa\tau + C(\tau)r + D(\tau)V) \tag{1}$$

where

$$A(\tau) = \frac{2\phi}{(\delta+\phi)\left(\exp(\phi\tau)-1\right)+2\phi},$$

$$B(\tau) = \frac{2\psi}{(\nu+\psi)\left(\exp(\psi\tau)-1\right)+2\psi},$$

[7] See Longstaff and Schwartz (1992) for a detailed derivation of the model.

$$C(\tau)=\frac{\alpha\phi\left(\exp(\psi\tau)-1\right)B(\tau)-\beta\psi\left(\exp(\phi\tau)-1\right)A(\tau)}{\phi\psi(\beta-\alpha)},$$

$$D(\tau)=\frac{\psi\left(\exp(\phi\tau)-1\right)A(\tau)-\phi\left(\exp(\psi\tau)-1\right)B(\tau)}{\phi\psi(\beta-\alpha)},$$

and

$$\nu=\xi+\lambda,$$

$$\phi=\sqrt{2\alpha+\delta^2},$$

$$\psi=\sqrt{2\beta+\nu^2},$$

$$\kappa=\gamma(\delta+\phi)+\eta(\nu+\psi).$$

Equation 1 implies that the equilibrium value of a riskless unit discount bond with τ periods until maturity is a function of variables *r, V,* and τ, and depends on six underlying parameters: $\alpha,\beta,\gamma,\delta,\eta,$ and ν. Longstaff and Schwartz (1992) further express the yield on a τ-maturity bond, Y_τ, and change in yield on a τ-maturity bond, ΔY_τ, as follows:

$$Y_\tau=-\frac{\left(\kappa\tau+2\gamma\ln A(\tau)+2\eta\ln B(\tau)+C(\tau)r+D(\tau)V\right)}{\tau} \tag{2}$$

$$\Delta Y_\tau=-\frac{C(\tau)}{\tau}\Delta r-\frac{D(\tau)}{\tau}\Delta V \tag{3}$$

The specification in equation 3 and the assumption that volatility follows a GARCH(1,1) process:[8]

$$\begin{aligned}&r_{t+1}-r_t=\alpha_0+\alpha_1 r_t+\alpha_2 V_t+\varepsilon_{t+1},\\&\varepsilon_{t+1}\sim N(0,V_t),\\&V_t=\beta_0+\beta_1 r_t+\beta_2 V_{t-1}+\beta_3\varepsilon_t^2\end{aligned} \tag{4}$$

[8] This specification allows us to empirically test the cross-sectional restrictions implied by equation 3 as a set of over-identifying restrictions on a system of moment equations using the GMM approach of Hansen (1982).

allow Longstaff and Schwartz (1992) to estimate their two-factor model.

Following Longstaff and Schwartz (1992), we first estimate the conditional volatility of the short-term real yield using GARCH (1,1) with variance in the mean equation.[9] The changes in observed values of the real yield, the short-term real yield and its conditional volatility serves as input into Hansen's (1982) GMM estimation with the following moment conditions:[10]

$$h_t(\theta) = \begin{bmatrix} e_t \\ e_t \Delta r \\ e_t \Delta V \end{bmatrix} \tag{5}$$

where $e_t = \Delta\hat{Y}_\tau - \Delta Y_\tau$, θ is a vector of parameters to be estimated, and $\Delta\hat{Y}_\tau$ is the change in observed yield on a τ-maturity bond. The number of moment conditions depends on the number of maturities of the observed yield. Thus, if we have observed yields on n different maturities and m_2 parameters, we will have $3n$ moment conditions, resulting in $3n\text{-}m_2$ over-identifying restrictions on a system of moment equations using the GMM approach.

In addition, for comparative purposes we also estimate the LS one-factor model and obtain GMM difference statistics to test which model performs better. To obtain the specification of the one-factor CIR model, we follow LS (1992) by imposing the restrictions $\alpha = \delta = \gamma = 0$ and express the yield on a τ-maturity bond, Y_τ, and change in yield on a τ-maturity bond, ΔY_τ, as follows:

$$y_\tau = \frac{-\ln AA(\tau)}{\tau} + \frac{BB(\tau)}{\tau} r \tag{6}$$

[9] For the UK and GER short rates, we fit GARCH(1,1) model since it fits these data better.
[10] The only difference between the LS (1992) and our paper is the input to the GMM estimation. Before estimating model using GMM, we adjust the nominal yield by the inflation (CPI). Effectively, we test the model of the real yield with the real yield data.

$$\Delta Y_\tau = \frac{BB(\tau)}{\tau}\Delta r \tag{7}$$

where

$$AA(\tau) = \left[\frac{2\psi \exp((\upsilon+\psi)^{\frac{\tau}{2}})}{(\upsilon+\psi)(\exp(\psi\tau)-1)+2\psi}\right]^{2\eta},$$

$$BB(\tau) = \frac{2(\exp(\psi\tau)-1)}{(\upsilon+\psi)(\exp(\psi\tau)-1)+2\psi}$$

For the one-factor LS/CIR model, the changes in observed values of real yield, and the short-term real yield are the inputs to Hansen's (1982) GMM estimation with the following moment conditions:[11]

$$h_t(\theta) = \begin{bmatrix} e_t \\ e_t\Delta r \end{bmatrix} \tag{8}$$

where $e_t = \Delta\hat{Y}_\tau - \Delta Y_\tau$, θ is a vector of parameters to be estimated, and $\Delta\hat{Y}_\tau$ is the change in observed yield on a τ -maturity bond. The number of moment conditions depends on the number of maturities of the observed yield. Thus, if we have observed yields on n different maturities and m_1 parameters, we will have $2n$ moment conditions, resulting in $2n$-m_1 over-identifying restrictions on a system of moment equations using the GMM approach.

C. Forecasting

To assess the performance of both the one-factor CIR and two-factor LS/CIR models, we perform a forecasting exercise as follows. First, for both models, we estimate GMM parameters during the in-sample period. Given the GMM estimates, we then back out

[11] The key difference between LS (1992) and our paper is the input to the GMM estimation. Before estimating the model using GMM, we adjust the nominal yield by inflation (CPI). Effectively, we test the model of the real yield with real yield data (as opposed to nominal yield data).

two unknown parameters, $\lambda,$ and η for the two-factor model, and one unknown risk-premium parameter for the one-factor model during the in-sample periods.[12]

Next, we fit various ARIMA-(G)ARCH specifications to the CPI, the two unknown parameters $\lambda,$ and η for the two-factor model and the one unknown risk-premium parameter for the one-factor model during the in-sample period in order to identify the best time-series specifications for these parameters.[13] We then adopt the identified time series specifications to the rolling sample window to obtain out-of-sample forecasts for these series.

Given the GMM estimates and the time-series estimates for the remaining unknown parameters, we can obtain the real yield for each term structure model. We then convert the real yield to nominal yield using the forecasted CPI series and finally compute the following forecast errors for both in- and out-of-sample periods:

$$\begin{aligned}
ME &= \frac{1}{N}\sum_{i=1}^{N}\left(\hat{y}_i - y_i\right),\\
MAE &= \frac{1}{N}\sum_{i=1}^{N}\left|\hat{y}_i - y_i\right|,\\
RMSE &= \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{y}_i - y_i\right)^2},\\
U &= \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{y}_i - y_i\right)^2}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\hat{y}_i\right)^2}+\sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y_i\right)^2}},
\end{aligned} \tag{9}$$

where *ME* is the mean error; *MAE* is the mean absolute error; *RMSE* is the root mean square error; *U* is Theil's inequality; $\hat{y}_i, y_i$ are forecasted and actual yields, respectively;

[12] For the forecast exercise, we use Chan, Karolyi, Longstaff and Schwartz (1992) moment conditions to obtain GMM estimates for three unknown parameters in the one-factor CIR model.

[13] The best model is selected as that which achieves the minimum value of the Schwarz Bayesian criterion and the Akike criterion. Results of this process are suppressed to conserve space but are available upon request.

and N is the number of observations. For ME, MAE, and RMSE, a smaller value indicates a better forecast. Theil's inequality ranges from 0 to 1, and the closer its value is to zero, the better is the forecast.

IV. Empirical Results

A. GMM Estimates

Table 2 reports GMM estimates for both one-factor CIR and two-factor LS/CIR models for all four countries examined. Like LS (1992), we find that the cross-sectional restrictions imposed by the two-factor model cannot be rejected by data from all countries examined at conventional significance levels, whereas the cross-sectional restrictions imposed by the one-factor model can be rejected for Australia and the US. Results from the two-factor model are very impressive since the cross-sectional restrictions are imposed on yields with maturities up to 10 years (for AUS yields), 30 years (for GER and the UK yields), and 5 years (for US yields). This implies that the two-factor model holds for the entire yield curve (i.e. short-term, immediate and long-term maturities).[14] Like LS (1992), we find that the fit between the actual and the two-factor model standard deviations is very close across all sample periods, except for the out-of-sample period of AUS, GER and UK. For example, Table 1 shows that the actual standard deviations range from 0.0269 in the 3-month US yield to 0.0233 in the 5-year US yield for the full sample period. The standard deviation implied by the two-factor model ranges from 0.0253 in the 3-month US yield to 0.0225 in the 5-year US yield. The mean difference between the actual and two-factor model standard deviations is only 0.001 for the US and AUS yields, 0.006 for the UK yield, and 0.0275 for the GER yield

[14] This result is particularly impressive since Longstaff and Schwartz (1992) note that previous empirical studies often find the explanatory power of equilibrium term structure models drops rapidly for maturities in excess of one year and they find their 1992 model holds for only short-term and intermediate maturities.

for the full sample period.[15] Finally, the GMM difference statistics reported in Table 2 are significant at 5% level for all countries examined. This implies universal superiority of the two-factor model over the one-factor model.

[Table 2 About Here]

In addition, we examine the ability of the one- and two-factor models to fit actual yields at different points in time along the yield curve. Figure 1 compares the historical yields to maturity of actual nominal yields with those implied by GMM estimates of the one-factor CIR and two-factor LS models. Panel A presents the yields in December 2001 (for AUS), May 1999 (for GER), December 2001 (for the UK), and January 1972 (for the US), illustrative of when interest rates were low. Panel B presents the yields in June 1982 (for AUS), June 1996(for GER), March 1996 (for the UK), and August 1981 (for the US), illustrative of when interest rates were high. Panel C presents the yields in December 1978 (for AUS), March 1998 (for GER), April 1999 (for the UK), and December 1986 (for the US), illustrative of when interest rates were moderate. Finally, Panel D presents the average yields over the in-sample period.

As shown in Figure 1, for all countries, except for the AUS yields (during the moderate and low yield periods), the yields implied by the two-factor LS model track the actual yield curve better than those by the one-factor model for all shapes of yield curves (i.e. regardless of whether rates are high, moderate or low).

[Figure 1 About Here]

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[15] As noted by Longstaff and Schwartz (1992), the moment restrictions adopted do not impose the restriction that these moments (mean, standard deviation, or higher moments) match.

B. Forecasting Results

Table 3 reports various measures of forecast performance for both in- and out-of-sample periods for Australia. For the AUS yields, the ME, MAE, RMSE and U indicate that two-factor model forecasts the longer-term yields (5 and 10 years) better than does the one-factor model – both in-sample and out-of-sample. However, the one-factor model gives a better forecast for the short-term (6 month) yield than does the two-factor model. The overall forecast errors from the one-factor model are smaller than those from the two-factor model in both in- and out-of-sample period for the Australian real yields even though the two-factor model performs better at the longer end (5 and 10 years). This is due to the large forecast errors when using the two-factor model to forecast short-term yields.

[Table 3 About Here]

Tables 4, 5 and 6 report similar forecast error performance for Germany, the UK and the US, respectively. The key messages coming from these tables can be summarized as follows. First, for the remaining three countries, the results are fairly similar to each other. That is, we find that the two-factor model is generally superior to the one-factor model. Second, for the German sample the one-factor model does seem to do better in out-of-sample forecasting. Third, the UK and US samples provide the strongest forecasting support for the LS two-factor model – in all situations this specification dominates its simpler one-factor counterpart for these two countries.

[Tables 4, 5 & 6 About Here]

V. Conclusion

This paper adopts Hansen's generalized method of moments (GMM) framework to test the cross-sectional restrictions imposed by the Longstaff and Schwartz (1992) [LS] two-factor model as well as the Cox-Ingersoll- Ross one-factor model using real yields in four different countries: Australia, Germany, the UK and the US. Specifically, we test the term structure models, developed for the real economy using real yields. We find that we can reject the one-factor model in favour of the LS two-factor model using the GMM difference statistics for all four countries examined. In addition, we also measure the forecasting ability from both the one- and two-factor models. We find that the in-sample forecasts are superior to the out-of-sample forecasts in all cases. For all countries except for Australia, we find forecasts from the LS two-factor model are superior to those from the one-factor CIR model both for in- and out-of-sample periods. That is, our findings generally support the two-factor model and, hence, indicate that both the short-term interest rate and the volatility of changes in the short-term interest rates are important factors in modelling the term structure of interest rates.

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Table 1

Summary Statistics for Consumer Price Index and Nominal Yields

This table reports mean, standard deviation, and the first three lagged autocorrelations for the consumer price index (CPI), change in CPI, nominal yields and change in nominal yields for Australia, Germany, the UK and the US. The in-sample period is 03/76 to 12/02 for AUS (322 observations), 01/96 to 12/02 for GER, and the UK, (83 Observations) and 06/64 to 12/89 for the US (306 observations). The out-of-sample period is 01/03 to 02/06 for AUS (38 Observations), 01/03 to 02/06 for GER (38 observations), 01/03 to and 04/05 for UK, (28 observations) and 01/90 to 01/02 for the US (145 observations). NiM and NiY are *i*-month nominal yield, and *i*-year nominal yield, respectively.

Panel A: Full Sample Period										
	μ	σ	ρ1	ρ2	ρ3	μ	σ	ρ1	ρ2	ρ3
			AUS					***Change in AUS***		
CPI	0.0575	0.0385	0.9790	0.9610	0.9430	-0.000322	0.0058	-0.0030	-0.0030	0.2090
N3M	0.0957	0.0434	0.9800	0.9610	0.9400	-0.000069	0.0078	-0.0340	0.1020	-0.0740
N6M	0.0963	0.0429	0.9840	0.9670	0.9500	-0.000074	0.0068	0.0280	0.0130	0.0390
N5Y	0.0953	0.0339	0.9800	0.9680	0.9550	-0.000116	0.0054	-0.2950	0.0460	0.1850
N10Y	0.0974	0.0328	0.9860	0.9710	0.9560	-0.000131	0.0037	0.0600	-0.0110	0.1290
			GER					***Change in GER***		
CPI	0.0147	0.0359	-0.2490	0.0520	-0.1450	0.000153	0.0572	-0.6390	0.2262	-0.0521
N1M	0.0313	0.0081	0.9777	0.9499	0.9173	-0.000106	0.0015	0.2170	0.1460	0.1690
N2Y	0.0344	0.0081	0.9661	0.9190	0.8707	-0.000058	0.0021	0.1870	0.0140	0.0210
N3Y	0.0371	0.0082	0.9570	0.9000	0.8440	-0.000075	0.0023	0.1450	-0.0200	-0.0280
N5Y	0.0412	0.0082	0.9410	0.8640	0.7830	-0.000131	0.0023	0.1290	-0.1070	-0.0150
N7Y	0.0446	0.0086	0.9400	0.8660	0.7870	-0.000173	0.0022	0.1000	-0.1390	0.0730
N10Y	0.0472	0.0088	0.9370	0.8670	0.7940	-0.000199	0.0020	0.0540	-0.1480	0.1110
N20Y	0.0521	0.0087	0.9270	0.8490	0.7700	-0.000234	0.0019	-0.0080	-0.1280	0.0410
N30Y	0.0534	0.0088	0.9260	0.8480	0.7700	-0.000240	0.0018	-0.0120	-0.1610	0.0720
			UK					***Change in UK***		
CPI	0.0253	0.0390	0.0710	-0.1270	-0.3000	0.000608	0.0530	-0.3792	-0.0149	-0.1908
N1M	0.0521	0.0112	0.9820	0.9580	0.9270	-0.000146	0.0016	0.3460	0.4080	0.2890
N3M	0.0523	0.0112	0.9790	0.9530	0.9180	-0.000139	0.0018	0.2340	0.3890	0.1850
N2Y	0.0531	0.0109	0.9630	0.9130	0.8550	-0.000168	0.0024	0.2270	0.0400	-0.0800
N3Y	0.0539	0.0108	0.9590	0.9060	0.8450	-0.000192	0.0024	0.1850	0.0170	-0.0110
N5Y	0.0542	0.0103	0.9550	0.9010	0.8450	-0.000207	0.0023	0.0980	-0.0810	0.0830
N7Y	0.0545	0.0107	0.9510	0.8970	0.8410	-0.000242	0.0023	0.0570	-0.1360	0.0900
N10Y	0.0541	0.0111	0.9530	0.9030	0.8540	-0.000263	0.0022	-0.0140	-0.1620	0.1550
N15Y	0.0542	0.0117	0.9560	0.9090	0.8640	-0.000287	0.0021	-0.0610	-0.1680	0.1660
N20Y	0.0536	0.0120	0.9590	0.9160	0.8740	-0.000295	0.0020	-0.0820	-0.1530	0.2100
N30Y	0.0524	0.0125	0.9600	0.9200	0.8780	-0.000308	0.0019	-0.0770	-0.1160	0.2000
			US					***Change in US***		
CPI	0.0464	0.0363	0.6460	0.5840	0.5430	0.000014	0.0305	-0.4170	-0.0290	-0.0120
N1M	0.0601	0.0258	0.9480	0.9070	0.8700	-0.00004	0.0080	-0.1450	-0.0390	0.0250
N3M	0.0631	0.0269	0.9570	0.9170	0.8940	-0.000077	0.0072	-0.0390	-0.1850	0.2050
N6M	0.0650	0.0274	0.9530	0.9130	0.8920	-0.000080	0.0077	-0.0800	-0.2530	0.2660
N9M	0.0657	0.0282	0.9390	0.9010	0.8980	-0.000082	0.0093	-0.2130	-0.3330	0.4660
N1Y	0.0680	0.0253	0.9700	0.9340	0.9030	-0.000033	0.0056	0.1250	-0.0890	-0.0750
N2Y	0.0701	0.0246	0.9760	0.9450	0.9180	-0.000014	0.0048	0.1680	-0.0840	-0.0960
N3Y	0.0716	0.0238	0.9780	0.9520	0.9280	-0.000002	0.0045	0.1310	-0.0860	-0.1020
N4Y	0.0728	0.0235	0.9790	0.9550	0.9350	0.000006	0.0043	0.0760	-0.1000	-0.0250
N5Y	0.0735	0.0233	0.9810	0.9600	0.9400	0.000012	0.0040	0.0900	-0.0690	-0.0580

Table 1 (Continued)

Summary Statistics for Consumer Price Index and Nominal Yields

Panel B: In-Sample Period										
	μ	σ	ρ1	ρ2	ρ3	μ	σ	ρ1	ρ2	ρ3
			AUS					***Change in AUS***		
CPI	0.0611	0.0391	0.9800	0.9620	0.9450	-0.0004	0.0060	-0.0030	-0.0030	0.2170
N3M	0.1006	0.0433	0.9790	0.9600	0.9370	-0.0001	0.0083	-0.0340	0.1020	-0.0740
N6M	0.1013	0.0427	0.9830	0.9660	0.9480	-0.0001	0.0072	0.0270	0.0130	0.0390
N5Y	0.1003	0.0323	0.9810	0.9710	0.9590	-0.0001	0.0057	-0.3020	0.0550	0.1890
N10Y	0.1024	0.0310	0.9880	0.9760	0.9640	-0.0001	0.0039	0.0580	0.0020	0.1420
			GER					***Change in GER***		
CPI	0.0140	0.0337	-0.0830	-0.0170	-0.1490	0.0011	0.0485	-0.4820	0.0870	-0.0110
N1M	0.0355	0.0061	0.9550	0.9000	0.8360	-0.0001	0.0017	0.2200	0.1400	0.1450
N2Y	0.0387	0.0056	0.9110	0.8020	0.6880	-0.0001	0.0022	0.1530	0.0930	0.0780
N3Y	0.0415	0.0055	0.8860	0.7570	0.6210	-0.0001	0.0024	0.1230	0.0750	0.0250
N5Y	0.0455	0.0057	0.8920	0.7640	0.6450	-0.0001	0.0024	0.1120	-0.0510	0.0260
N7Y	0.0490	0.0063	0.9220	0.8290	0.7370	-0.0002	0.0022	0.1040	-0.0260	0.1100
N10Y	0.0513	0.0071	0.9410	0.8750	0.8110	-0.0002	0.0021	0.0660	-0.0950	0.1770
N20Y	0.0561	0.0070	0.9400	0.8750	0.8120	-0.0002	0.0020	0.0010	-0.1080	0.0650
N30Y	0.0574	0.0070	0.9420	0.8790	0.8210	-0.0002	0.0019	-0.0060	-0.1570	0.1050
			UK					***Change in UK***		
CPI	0.0242	0.0426	0.0940	-0.1390	-0.2900	0.0006	0.0573	-0.3540	-0.0460	-0.1850
N1M	0.0562	0.0105	0.9640	0.9200	0.8640	-0.0003	0.0018	0.3500	0.4130	0.2920
N3M	0.0564	0.0106	0.9630	0.9190	0.8600	-0.0003	0.0020	0.2430	0.4020	0.2070
N2Y	0.0578	0.0093	0.9360	0.8580	0.7700	-0.0003	0.0025	0.1730	0.0710	-0.0510
N3Y	0.0585	0.0093	0.9400	0.8690	0.7920	-0.0003	0.0024	0.1420	0.0500	0.0100
N5Y	0.0584	0.0094	0.9460	0.8870	0.8250	-0.0003	0.0023	0.0650	-0.0590	0.1060
N7Y	0.0586	0.0102	0.9480	0.8920	0.8350	-0.0003	0.0024	0.0300	-0.1280	0.1050
N10Y	0.0577	0.0113	0.9570	0.9120	0.8670	-0.0003	0.0024	-0.0370	-0.1510	0.1700
N15Y	0.0577	0.0123	0.9620	0.9220	0.8820	-0.0004	0.0023	-0.0800	-0.1540	0.1890
N20Y	0.0569	0.0129	0.9670	0.9300	0.8920	-0.0004	0.0021	-0.0840	-0.1350	0.2300
N30Y	0.0554	0.0137	0.9690	0.9340	0.8950	-0.0004	0.0021	-0.0710	-0.0960	0.2110
			US					***Change in US***		
CPI	0.0549	0.0387	0.6500	0.5920	0.5120	0.0001	0.0323	-0.4210	0.0350	-0.0750
N1M	0.0671	0.0267	0.9500	0.9080	0.8590	0.00007	0.0082	-0.0820	0.0750	-0.1220
N3M	0.0707	0.0274	0.9650	0.9230	0.8860	0.00014	0.0069	0.1060	-0.0860	-0.0550
N6M	0.0733	0.0272	0.9670	0.9250	0.8890	0.00014	0.0066	0.1520	-0.0880	-0.1050
N9M	0.0748	0.0269	0.9670	0.9250	0.8880	0.00013	0.0066	0.1590	-0.0980	-0.1260
N1Y	0.0754	0.0262	0.9650	0.9240	0.8900	0.00013	0.0066	0.1050	-0.1090	-0.0910
N2Y	0.0769	0.0260	0.9740	0.9400	0.9120	0.00013	0.0055	0.1430	-0.1070	-0.1120
N3Y	0.0781	0.0255	0.9770	0.9490	0.9260	0.00013	0.0050	0.1030	-0.1070	-0.1170
N4Y	0.0789	0.0254	0.9780	0.9540	0.9340	0.00013	0.0049	0.0460	-0.1190	-0.0300
N5Y	0.0794	0.0252	0.9800	0.9590	0.9400	0.00012	0.0045	0.0580	-0.0880	-0.0710

Table 1 (Continued)

Summary Statistics for Consumer Price Index and Nominal Yields

Panel C: Out-of-Sample Period										
	μ	σ	ρ1	ρ2	ρ3	μ	σ	ρ1	ρ2	ρ3
			AUS					***Change in AUS***		
CPI	0.0262	0.0037	0.8080	0.6170	0.3910	-0.0001	0.0021	-0.0010	-0.0010	-0.3470
N3M	0.0536	0.0036	0.9200	0.8140	0.7070	0.0002	0.0010	0.1530	0.0030	0.0860
N6M	0.0540	0.0040	0.9080	0.7790	0.6680	0.0002	0.0013	0.1410	-0.1420	0.0670
N5Y	0.0527	0.0033	0.7100	0.3770	0.2640	0.00007	0.0024	0.0410	-0.3740	-0.0570
N10Y	0.0543	0.0030	0.6770	0.2920	0.1340	0.0000	0.0024	0.0840	-0.3400	-0.1900
			GER					***Change in GER***		
CPI	0.0163	0.0408	-0.5140	0.1950	-0.0780	-0.0019	0.0734	-0.7050	0.3050	-0.0510
N1M	0.0220	0.0022	0.7760	0.5660	0.3850	0.0355	0.0061	0.0930	0.1830	0.2350
N2Y	0.0247	0.0024	0.6390	0.1870	-0.1200	0.0387	0.0056	0.2990	-0.2670	-0.1850
N3Y	0.0272	0.0028	0.6700	0.2890	0.0830	0.0415	0.0055	0.1940	-0.3030	-0.1780
N5Y	0.0316	0.0033	0.7700	0.4960	0.3430	0.0455	0.0057	0.1680	-0.3070	-0.0990
N7Y	0.0350	0.0036	0.8050	0.6010	0.5520	0.0490	0.0063	0.0830	-0.4540	0.0150
N10Y	0.0381	0.0040	0.8520	0.7070	0.6470	0.0513	0.0071	0.0100	-0.3460	-0.0070
N20Y	0.0433	0.0049	0.9070	0.8140	0.7570	0.0561	0.0070	-0.0760	-0.2760	0.0190
N30Y	0.0444	0.0051	0.9160	0.8280	0.7690	0.0574	0.0070	-0.0810	-0.2630	0.0280
			UK					***Change in UK***		
CPI	0.0304	0.0321	-0.1060	-0.0510	-0.3270	0.0019	0.0470	-0.5200	0.1570	-0.1730
N1M	0.0411	0.0054	0.9390	0.8710	0.7850	0.0004	0.0012	0.1950	0.2780	0.1640
N3M	0.0419	0.0059	0.9370	0.8710	0.7830	0.0004	0.0015	0.1290	0.3010	0.0370
N2Y	0.0422	0.0056	0.9040	0.7100	0.5410	0.0002	0.0022	0.4290	-0.1290	-0.2350
N3Y	0.0435	0.0051	0.8830	0.6760	0.5080	0.0002	0.0023	0.3090	-0.1700	-0.1210
N5Y	0.0450	0.0042	0.8430	0.6110	0.4450	0.0001	0.0023	0.1830	-0.2090	-0.0070
N7Y	0.0457	0.0037	0.8270	0.5910	0.4320	0.0001	0.0022	0.1410	-0.2200	0.0130
N10Y	0.0464	0.0031	0.7950	0.5500	0.3990	0.0000	0.0019	0.0750	-0.2420	0.0770
N15Y	0.0468	0.0023	0.7160	0.4140	0.2530	0.0000	0.0017	0.0230	-0.2590	0.0340
N20Y	0.0466	0.0019	0.6540	0.3600	0.2280	0.0000	0.0016	-0.0770	-0.2550	0.0890
N30Y	0.0459	0.0016	0.6080	0.3060	0.1700	0.0000	0.0014	-0.1220	-0.2280	0.0810
			US					***Change in US***		
CPI	0.0283	0.0215	0.2320	0.0660	0.2500	-0.0001	0.0264	-0.3950	-0.2320	0.1590
N1M	0.0452	0.0155	0.8620	0.7680	0.7470	-0.0003	0.0075	-0.2550	-0.3110	0.3820
N3M	0.0470	0.0167	0.8550	0.7720	0.7830	-0.0005	0.0076	-0.3000	-0.3650	0.6600
N6M	0.0474	0.0180	0.8240	0.7370	0.7680	-0.0005	0.0095	-0.3200	-0.4250	0.6440
N9M	0.0463	0.0200	0.7480	0.6770	0.8080	-0.0005	0.0133	-0.4070	-0.4550	0.7690
N1Y	0.0523	0.0132	0.9470	0.8770	0.7960	-0.0004	0.0026	0.3700	0.1680	0.1050
N2Y	0.0555	0.0122	0.9450	0.8690	0.7820	-0.0003	0.0028	0.3560	0.0920	0.0070
N3Y	0.0580	0.0113	0.9410	0.8620	0.7720	-0.0003	0.0029	0.2910	0.0470	-0.0280
N4Y	0.0600	0.0110	0.9430	0.8660	0.7830	-0.0002	0.0029	0.2440	0.0070	-0.0080
N5Y	0.0610	0.0108	0.9450	0.8720	0.7910	-0.0002	0.0028	0.2420	0.0220	-0.0010

Table 2

GMM Tests of the Cross-sectional Restrictions on Monthly Changes in Real Yield Implied by the One- and Two-Factor LS/CIR Models

This table reports the GMM tests of the cross-sectional restrictions on the change in real yield implied by the one- and two-factor CIR models for the following countries: Australia (AUS), Germany (GER), United Kingdom (UK), and the United States (US). For the two-factor model, the four parameters are estimated from a system of 9 moment conditions for AUS, 21 for GER, 27 for the UK, and 24 for the US, resulting in 5, 17, 23 and 20 over-identifying restrictions, respectively. For the one-factor model, the two parameters are estimated from a system of 9 moment conditions for AUS, 21 for GER, 27 for the UK, and 24 for the US, resulting in 7, 19, 25 and 22 over-identifying restrictions, respectively. The data are monthly and are expressed in annualised form. The estimation sample period is 03/76 to 12/02 for AUS (322 Observations), 01/96 to 12/02 for GER and the UK, (83 Observations) and 06/64 to 12/89 for the US (306 observations).

	AUS		GER		UK		US	
	estimate	t-statistic	estimate	t-statistic	estimate	t-statistic	estimate	t-statistic
Panel A: LS Two-factor Model								
α	0.00238805	1.73	0.00021143	2.82	0.0000611	1.99	0.01570509	2.83
β	0.13436835	3.11	0.01000275	1.58	0.08080306	3.62	1.3377721	4.14
δ	0.11378453	3.27	0.00042386	0.55	0.00168989	2.38	0.02684566	3.09
υ	-6.2609993	-2.02	0.96888511	41.53	0.20547776	4.24	13.8249038	2.28
χ^2 Test (p-value)	1.284744	0.94	5.343945	1.00	14.993125	0.89	19.89715	0.46
Degrees of freedom	5		17		23		20	
Panel B: CIR One-factor Model								
β	0.07272244	12.18	0.01189804	6.50	0.26307852	9.04	-0.2385942	-3.72
υ	-2.9191919	-4.66	0.1759616	43.12	1.00091047	110.46	0.94015877	42.88
χ^2 Test (p-value)	22.17311	0.00	21.26354	0.32	24.197491	0.51	46.975479	0.00
Degrees of freedom	7		19		25		22	
GMM difference statistic (p-value)	20.888366	0.00	15.919595	0.00	9.204366	0.01	27.078329	0.00

Table 3

Forecast Error Metrics from the One- and Two-Factor LS/CIR models: Australian Data

This table reports for Australian Data: mean error (ME), mean absolute error (MAE), root mean square error (RMSE), and Theil's inequality for the Longstaff and Schwartz (1992) two-factor model (LS2) and the Cox, Ingersoll and Ross (1985) one-factor model (CIR1). The in-sample period is 03/76 to 12/02 (322 Observations) and the out-of-sample period is 01/03 to 02/06 (38 Observations).

	ME		MAE		RMSE		Theil's Inequality	
Term to Maturity	LS2	CIR1	LS2	CIR1	LS2	CIR1	LS2	CIR1
Panel A: In-sample forecasts								
6m	-0.01406	-0.00056	0.01406	0.00264	0.01574	0.00410	0.07679	0.01868
5y	-0.00269	0.00091	0.00274	0.00451	0.00372	0.00613	0.01792	0.02885
10y	0.00230	-0.00068	0.00262	0.00230	0.00383	0.00321	0.01766	0.01508
Overall	-0.00482	-0.00011	0.00647	0.00315	0.00960	0.00464	0.04569	0.02160
Panel B: Out-of-sample forecasts								
6m	-0.00800	-0.00055	0.00818	0.00180	0.00847	0.00279	0.08437	0.02590
5y	-0.00114	0.00104	0.00378	0.00265	0.00574	0.00354	0.05477	0.03315
10y	0.00028	-0.00051	0.00420	0.00285	0.00642	0.00374	0.05874	0.03459
Overall	-0.00295	-0.000005	0.00539	0.00243	0.00697	0.00338	0.06644	0.03144

Table 4

Forecast Error Metrics from the One- and Two-Factor LS/CIR models: German Data

This table reports for German Data: mean error (ME), mean absolute error (MAE), root mean square error (RMSE), and Theil's inequality for the Longstaff and Schwartz (1992) two-factor model (LS2) and the Cox, Ingersoll and Ross (1985) one-factor model (CIR1). The in-sample period is 01/96 to 12/02 (83 Observations) and the out-of-sample period is 01/03 to 02/06 (38 Observations).

	ME		MAE		RMSE		Theil's Inequality	
Term to Maturity	LS2	CIR1	LS2	CIR1	LS2	CIR1	LS2	CIR1
Panel A: In-sample forecasts								
2y	0.00331	0.00052	0.00341	0.00836	0.00404	0.01117	0.04955	0.13944
3y	0.00210	-0.00138	0.00218	0.00927	0.00273	0.01357	0.03175	0.16081
5y	0.00018	-0.00424	0.00044	0.01005	0.00058	0.01687	0.00632	0.18685
7y	-0.00180	-0.00701	0.00181	0.00979	0.00214	0.01913	0.02207	0.20158
10y	-0.00232	-0.00874	0.00239	0.01050	0.00308	0.02099	0.03052	0.21374
20y	-0.00188	-0.01284	0.00193	0.01394	0.00221	0.02411	0.01993	0.23132
30y	0.00206	-0.01395	0.00207	0.01499	0.00239	0.02498	0.02033	0.23568
Overall	0.00023	-0.00680	0.00203	0.01098	0.00264	0.01929	0.02674	0.20413
Panel B: Out-of-sample forecasts								
2y	0.00940	0.00070	0.03967	0.02761	0.05034	0.03737	0.59576	0.53414
3y	0.00683	-0.00266	0.04356	0.02481	0.05598	0.03399	0.60702	0.49101
5y	0.00293	-0.00785	0.04685	0.02242	0.06180	0.03146	0.60362	0.44635
7y	0.00019	-0.01171	0.04819	0.02222	0.06471	0.03134	0.59386	0.43162
10y	-0.00162	-0.01512	0.04934	0.02293	0.06702	0.03185	0.58352	0.42619
20y	-0.00177	-0.02073	0.05129	0.02513	0.06962	0.03400	0.55623	0.43065
30y	0.00298	-0.02188	0.05353	0.02593	0.07030	0.03446	0.54120	0.43279
Overall	0.00271	-0.01132	0.04749	0.02443	0.06319	0.03355	0.57732	0.45030

Table 5

Forecast Error Metrics from the One- and Two-Factor LS/CIR models: UK Data

This table reports for UK Data: mean error (ME), mean absolute error (MAE), root mean square error (RMSE), and Theil's inequality for the Longstaff and Schwartz (1992) two-factor model (LS2) and the Cox, Ingersoll and Ross (1985) one-factor model (CIR1). The in-sample period is 01/96 to 12/02 (83 Observations) and the out-of-sample period is 01/03 to 04/05 (28 Observations).

	ME		MAE		RMSE		Theil's Inequality	
Term to Maturity	LS2	CIR1	LS2	CIR1	LS2	CIR1	LS2	CIR1
Panel A: In-sample forecasts								
3m	0.00028	0.00015	0.00088	0.00227	0.00110	0.00355	0.00957	0.03094
2y	0.00056	-0.00022	0.00269	0.00784	0.00320	0.01614	0.02725	0.13674
3y	0.00012	-0.00071	0.00238	0.00879	0.00297	0.01956	0.02508	0.16326
5y	0.00026	-0.00032	0.00174	0.00939	0.00212	0.02299	0.01788	0.19012
7y	-0.00018	-0.00039	0.00127	0.00969	0.00157	0.02482	0.01329	0.20363
10y	0.00031	0.00064	0.00079	0.00954	0.00112	0.02647	0.00950	0.21750
15y	-0.00042	0.00066	0.00123	0.01074	0.00162	0.02769	0.01387	0.22629
20y	-0.00024	0.00149	0.00137	0.01142	0.00169	0.02837	0.01459	0.23273
30y	0.00010	0.00305	0.00202	0.01259	0.00240	0.02932	0.02116	0.24262
Overall	0.00009	0.00048	0.00160	0.00914	0.00210	0.02341	0.01798	0.19464
Panel B: Out-of-sample forecasts								
3m	0.01020	-0.00223	0.02522	0.02763	0.03131	0.03351	0.30796	0.35612
2y	0.01218	-0.01600	0.02598	0.02728	0.03212	0.03279	0.30879	0.40626
3y	0.01170	-0.01939	0.02579	0.02891	0.03188	0.03449	0.30066	0.42845
5y	0.01120	-0.02268	0.02554	0.03075	0.03158	0.03643	0.29118	0.45007
7y	0.01107	-0.02413	0.02553	0.03152	0.03155	0.03741	0.28774	0.46049
10y	0.01070	-0.02546	0.02535	0.03214	0.03135	0.03826	0.28305	0.46809
15y	0.01046	-0.02631	0.02514	0.03257	0.03125	0.03884	0.28075	0.47409
20y	0.01057	-0.02632	0.02511	0.03253	0.03128	0.03884	0.28174	0.47586
30y	0.01073	-0.02584	0.02508	0.03220	0.03136	0.03854	0.28548	0.47680
Overall	0.01098	-0.02093	0.02542	0.03061	0.03152	0.03664	0.29150	0.44139

Table 6

Forecast Error Metrics from the One- and Two-Factor LS/CIR models: US Data

This table reports for US Data: mean error (ME), mean absolute error (MAE), root mean square error (RMSE), and Theil's inequality for the Longstaff and Schwartz (1992) two-factor model (LS2) and the Cox, Ingersoll and Ross (1985) one-factor model (CIR1). The in-sample period is 06/64 to 12/89 (306 Observations) and the out-of-sample period is 01/90 to 01/02 (145 Observations).

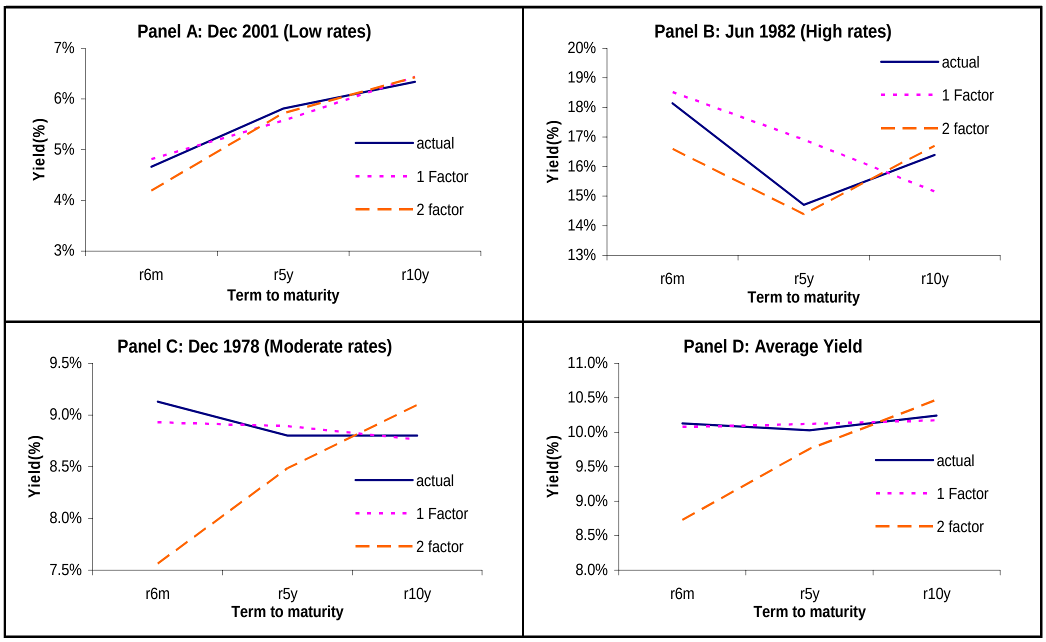
	ME		MAE		RMSE		Theil's Inequality	
Term to Maturity	LS2	CIR1	LS2	CIR1	LS2	CIR1	LS2	CIR1
Panel A: In-sample forecasts								
3m	0.00115	-0.00259	0.00191	0.00303	0.00258	0.00465	0.01688	0.03117
6m	0.00005	-0.00409	0.00088	0.00445	0.00123	0.00603	0.00785	0.03959
9m	-0.00066	-0.00469	0.00094	0.00505	0.00131	0.00660	0.00826	0.04275
1y	-0.00057	-0.00438	0.00119	0.00475	0.00159	0.00621	0.00994	0.03987
2y	-0.00001	-0.00247	0.00154	0.00307	0.00203	0.00410	0.01249	0.02563
3y	0.00019	-0.00067	0.00114	0.00134	0.00161	0.00185	0.00980	0.01129
4y	0.00012	0.00103	0.00052	0.00131	0.00074	0.00169	0.00448	0.01011
5y	-0.00005	0.00277	0.00129	0.00324	0.00178	0.00413	0.01065	0.02437
Overall	0.00003	-0.00189	0.00118	0.00328	0.00169	0.00475	0.01049	0.02982
Panel B: Out-of-sample forecasts								
3m	0.00507	-0.00022	0.00746	0.00986	0.01082	0.01351	0.10411	0.13414
6m	0.00586	0.00039	0.00822	0.01045	0.01251	0.01499	0.11809	0.14655
9m	0.00803	0.00252	0.01016	0.01147	0.01641	0.01821	0.15396	0.17733
1y	0.00307	-0.00245	0.00626	0.00974	0.00851	0.01292	0.07668	0.12087
2y	0.00328	-0.00223	0.00651	0.00977	0.00865	0.01247	0.07393	0.11088
3y	0.00375	-0.00178	0.00672	0.00982	0.00883	0.01210	0.07240	0.10339
4y	0.00427	-0.00128	0.00687	0.00998	0.00914	0.01205	0.07238	0.09966
5y	0.00536	-0.00018	0.00752	0.01005	0.01004	0.01216	0.07764	0.09829
Overall	0.00484	-0.00065	0.00747	0.01014	0.01091	0.01370	0.09437	0.12321

Figure 1

Actual Nominal Yields versus those Implied by GMM Estimates

This figure compares actual historical yields to maturity with those implied by GMM estimates of the one-factor CIR and two-factor LS models. Panel A presents the yields in December 2001 (for AUS); May 1999 (for GER); December 2001 (for the UK) and January 1972 (for the US) when interest rates were low. Panel B presents the yields in June 1982 (for AUS); June 1996(for GER); March 1996 (for the UK) and August 1981 (for the US) when interest rates were high. Panel C presents the yields in December 1978 (for AUS); March 1998 (for GER); April 1999 (for the UK) and December 1986 (for the US) when interest rates were moderate. Panel D presents the average yields over the in-sample period. The sample period is 03/76 to 12/02 for AUS (322 Observations); 01/96 to 12/02 for GER and the UK (83 Observations) and 06/64 to 12/89 for the US (306 observations).

AUS



GER

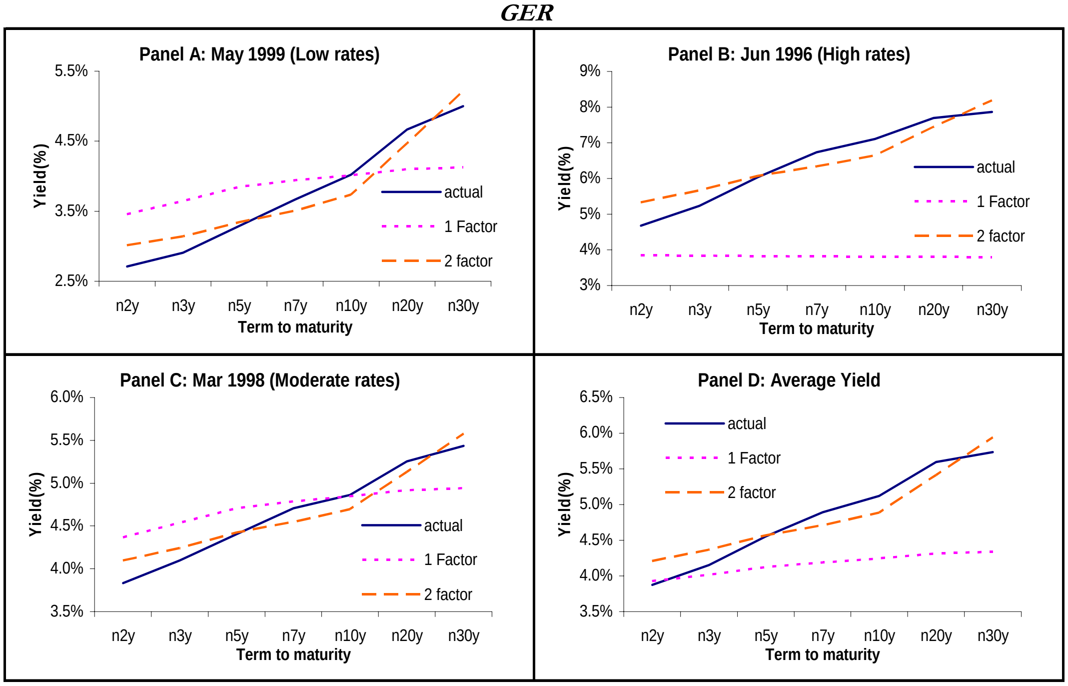
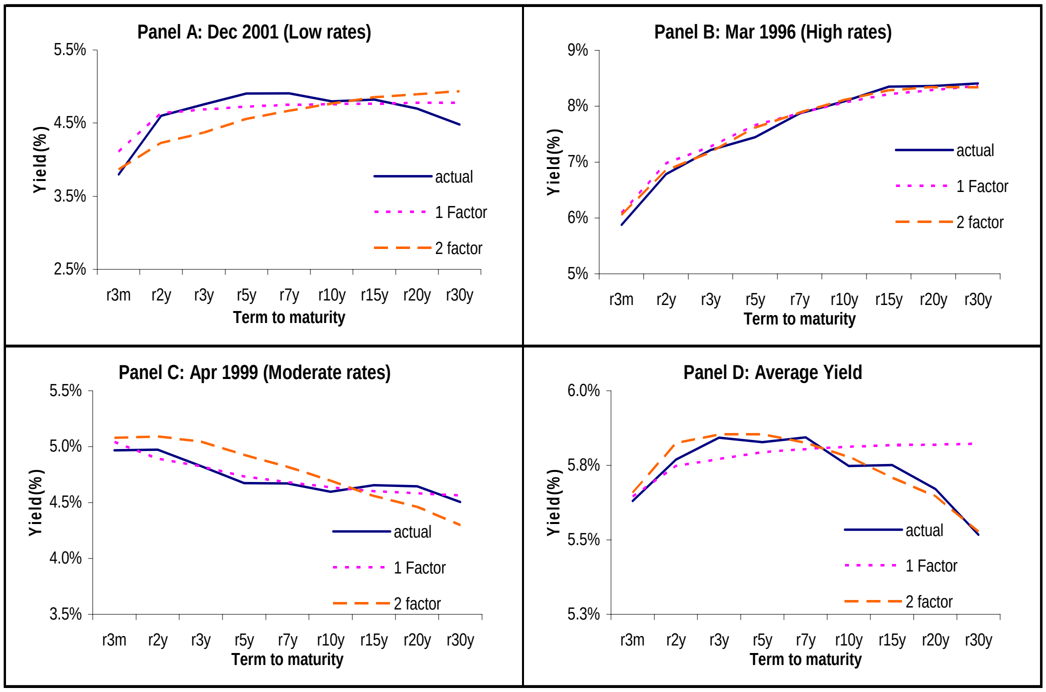


Figure 1 (Continued)

Actual nominal yields versus those implied by GMM estimates

UK



US