The performance of a default risk model with the barrier option framework and the maximum likelihood method

(This version: December, 2006)

Heng-Chih Chou

Associate Professor
Department of Finance, Ming Chuan University,
250 Chung Shan North Road, Section 5, Taipei 111, Taiwan.
Tel.: +886 2 2882 4564 x2390; fax: +886 2 2880 9769.
E-mail address: hcchou@mcu.edu.tw.

David Wang*

Assistant Professor
Department of Finance, Chung Yuan Christian University,
200 Chung Pei Road, Chung Li 320, Taiwan
Tel.: +886 3 265 5707; fax: +886 3 265 5749.
E-mail address: dwang@cycu.edu.tw.

* Corresponding author.

The performance of a default risk model with the barrier option framework and the maximum likelihood method

Abstract

This study investigates the performance of a default risk model based on the barrier option framework with Duan's (1994, 2000) maximum likelihood method. We provide empirical validation of the model by showing that implied barriers are statistically significant for a sample of construction firms in Taiwan over the period 1994 to 2004. We then apply the model to default prediction and find that the model dominates the commonly adopted models, Merton (1974) model, the Z-Score model and the ZETA model. Moreover, we test the n-year-ahead prediction performance of the model and find evidence that the prediction accuracy of the model improves as the forecast horizon decreases. Finally, we assess the effect of estimated default risk on equity returns and find that default risk is able to explain equity returns and that default risk is a variable worth considering in asset-pricing tests, above and beyond size and book-to-market.

JEL classification: G12; G33

Keywords: Default risk model; Barrier option framework; default prediction; Maximum likelihood method

1. Introduction

Default risk modeling has gained increasing prominence over the years. Most of the interest is motivated by new regulatory requirements, such as the Basel Accord II, which provide strong incentives for financial institutions to quantify the credit risk of their portfolios. Financial institutions can either develop their own internal rating modes, or rely on third-party credit risk models to measure the default probability of their portfolios.

Basically there are two different conceptual approaches for default risk modeling, reduced form model and structural model. Reduced form models offer no economic model of default causality, and use debt prices to solve for default probabilities. Another standard theoretical paradigm for modeling credit risk is the structural model pioneered by Merton (1974). Much of the literature follows Merton's model by explicitly linking the risk of a firm's default process to the variability in the firm's asset value and viewing the market value of firm's equity as the standard call option on the market value of the firm's asset with strike price equal to the promised payment of corporate liabilities. These insights have a profound impact on financial economics, and many researches have been stimulated to verify or to criticize the approach. Black and Cox (1976) recognize one possible weakness of the approach that default only occurs at maturity of the debt. They propose to incorporate a barrier on the market value of firms' asset for triggering default prior to the maturity. As a result, the down-and-out call (DOC thereafter) option is proposed to model the firm's equity value, and the default risk can be estimated from the DOC option pricing model.

Brockman and Turtle (2003) provide empirical validation of the DOC option framework by estimating the default barriers from the market value of firm's equity and showing that implied default barriers are statistically and economically significant for a large cross-section of industrial firms. They adopt the sum of the market value of firm's equity and the book value of firm's liability as a proxy for the market value of firm's asset in their tests. Obviously the proxy is not appropriate, and Wong and Choi (2004) argue that Brockman and Turtle's (2003) empirical finding is invalid. They provide theoretical evidence to show that the proxy adopted by Brockman and Turtle (2003) for the market value of firm's asset is inappropriate for testing the validity of the DOC option framework. Besides, Duan, Gauthier and Simonato (2004) also utilize a transformed-data maximum likelihood estimation (MLE thereafter) method, based on Duan (1994, 2000), to directly estimate the market value of firm's asset along with the asset value volatility and the default barrier from the market value of firm's equity. The benefits of using MLE method are well understood in statistics and econometrics. Ericsson and Reneby (2003) and Duan *et al.* (2003) both demonstrate that the MLE method dominates over other approaches, such as Ronn and Verma (1986), in the context of structural credit risk models.

The purpose of this paper is to examine the empirical performance of the default risk model based on the barrier option framework, and the MLE method is applied to the estimation of the un-observability issue. Our sample includes 31 construction firms in Taiwan over the period 1994 to 2004. We investigate the validity of the framework by testing the statistical significance of the implied default barriers. We then apply the framework to default prediction and compare its prediction performance to the commonly adopted models, Merton model, the Z-Score model and the ZETA model. We also test the n-year-ahead prediction performance of the framework. Finally, because Vassalou and Xing (2004) show that size and book-to-market factors appear to contain no significant price information related to default risk, we also assess the effect of estimated default risks on equity returns.

The rest of the paper is organized as follows. The default risk model based on the barrier option framework is detailed in Section 2. Our sample is described in Section 3. Our empirical results are reported in Section 4. A conclusive remark is made in the final section.

2. Model

2.1 *Down-and-out call (DOC) barrier option framework*

In this section, we describe the default risk model based on the barrier option framework. The proposed framework starts from viewing the market value of firm's equity as a DOC option on the market value of firm's asset. The market value of firm's asset, denoted by V, is assumed to follow a geometric Brownian motion. Specifically, the dynamics of the firm's asset value follows a written as:

$$d \ln V_t = (\mu - \sigma^2 / 2)dt + \sigma dZ_t , \tag{1}$$

where V_t is the market value of firm's asset at time t, μ and σ are, respectively, the expected return and volatility of asset value, and Z_t is a Wiener process.

By following Equation (1), one can derive the closed form solution for the value of DOC options as:

$$\begin{aligned} V_E &= DOC(V, X, H) \\ &= VN(a) - Xe^{-rT} N(a - \sigma\sqrt{T}) \\ &- V(H/V)^{2\eta} N(b) + Xe^{-rT}(H/V)^{2\eta-2} N(b - \sigma\sqrt{T}) \\ &+ R(H/V)^{2\eta-1} N(c) + R(V/H) N(c - 2\eta\sigma\sqrt{T}), \end{aligned} \tag{2}$$

where V_E is the market value of firm's equity, V is the market value of firm's asset, X is the future promised payment of corporate liabilities, H is the default barrier level, σ is the asset value volatility, r is the risk-free interest rate, T is the time to maturity of corporate liabilities, R is the rebate paid to the firm's owners if the asset value reaches its barrier level, $N(\cdot)$ is the cumulative distribution function for a standard normal random variable,

$$a = \begin{cases} \dfrac{\ln(V/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \; for X \geq H, \\ \dfrac{\ln(V/H) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \; for X < H, \end{cases}$$

$$b = \begin{cases} \dfrac{\ln(H^2/VX) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \; for X \geq H, \\ \dfrac{\ln(H/V) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \; for X < H, \end{cases}$$

$$c = \frac{\ln(H/V) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } \eta = \frac{r}{\sigma^2} + \frac{1}{2}.$$

For tractability, it is assumed that in the event of default no rebate is paid to the firm's owners ($R = 0$) and the default barrier level H is proportional to the corporate liability X by a barrier-to-debt ratio α ($H = \alpha X$).

2.2 *The Maximum Likelihood Estimation method*

The estimation of the unobserved asset values, such as asset value, the volatility of asset value, and the default barrier level, can be processed cast as a transformated-data maximum likelihood estimation (MLE). This approach was proposed by Duan (1994, 2000). According Duan (2004), the advantages of the approach are that (1) the resulting estimators are known to be statistically efficient, and (2) sampling distributions are easily available for testing hypotheses.

Denoting Equation (2) by $g(V, \sigma, \alpha)$, one can apply the transformed-data maximum likelihood estimates method of Duan (1994, 2000, 2004) to obtain the log-likelihood function of discretely sampled equity values on a firm that survives the entire sample period $\{V_E^0, V_E^h, V_E^{2h}, ..., V_E^{nh}\}$ as

$$
\begin{aligned}
&L(\mu,\sigma,\alpha;V_E^0,V_E^h,V_E^{2h},...,V_E^{nh}) = -\frac{n}{2}\ln(2\pi\sigma^2 h) \\
&-\frac{1}{2}\sum_{j=1}^{n}\frac{(\ln\frac{V^{jh}}{V^{(j-1)h}}-(\mu-\frac{\sigma^2}{2})h)^2}{\sigma^2 h}-\sum_{j=1}^{n}\ln V^{jh} \\
&+\sum_{j=1}^{n}\ln(1-\exp(-\frac{2}{\sigma^2 h}\ln\frac{V^{(j-1)h}}{\alpha X}\ln\frac{V^{jh}}{\alpha X})) \qquad (3) \\
&-\ln(N(\frac{(\mu-\frac{\sigma^2}{2})nh-\ln\frac{\alpha X}{V_0}}{\sqrt{nh}\sigma})-\exp^{(\frac{2}{\sigma^2})(\mu-\frac{\sigma^2}{2})\ln\frac{\alpha X}{V_0}}N(\frac{(\mu-\frac{\sigma^2}{2})nh+\ln\frac{\alpha X}{V_0}}{\sqrt{nh}\sigma})) \\
&-\sum_{j=1}^{n}\ln\left|\frac{\partial g(V^{jh},\sigma,\alpha)}{\partial V^{jh}}\right|,
\end{aligned}
$$

where the first derivative of the equity value with respect to the asset value can be derived as

$$
\begin{aligned}
\frac{\partial g(V,\sigma,\alpha)}{\partial V} &= V\frac{1}{\sqrt{2\pi}}e^{-\frac{a^2}{2}}\frac{1}{\sigma\sqrt{T}V}+N(a)-Xe^{-\mu T}\frac{1}{\sqrt{2\pi}}e^{-\frac{(a-\sigma\sqrt{T})^2}{2}}\frac{1}{\sigma\sqrt{T}V} \\
&-V(\alpha X/V)^{2\eta}(-\frac{1}{\sqrt{2\pi}}e^{-\frac{b^2}{2}}\frac{1}{\sigma\sqrt{T}V})-V(-\alpha^{2\eta}X^{2\eta}2\eta V^{-2\eta-1})N(b) \\
&-(\alpha X/V)^{2\eta}N(b)+Xe^{-\mu T}(\alpha X/V)^{2\eta-2}(-\frac{1}{\sqrt{2\pi}}e^{-\frac{(b-\sigma\sqrt{T})^2}{2}}\frac{1}{\sigma\sqrt{T}V}) \\
&+Xe^{-\mu T}(-\alpha^{2\eta-2}X^{2\eta-2}(2\eta+2)V^{-2\eta+1})N(b-\sigma\sqrt{T}),
\end{aligned}
$$

$$
a = \begin{cases} \dfrac{\ln(V/X)+(\mu+\sigma^2/2)T}{\sigma\sqrt{T}} \ for X \geq H, \\ \dfrac{\ln(V/H)+(\mu+\sigma^2/2)T}{\sigma\sqrt{T}} \ for X < H, \end{cases}
$$

$$b = \begin{cases} \dfrac{\ln(H^2/VX) + (\mu + \sigma^2/2)T}{\sigma\sqrt{T}} \ for X \geq H, \\ \dfrac{\ln(H/V) + (\mu + \sigma^2/2)T}{\sigma\sqrt{T}} \ for X < H. \end{cases}$$

The MLE is to find the values of V, σ, and α where the data set has the highest likelihood of occurrence. Given the market value of firm's equity V_E, the future promised payment of corporate liabilities X, the drift value of the physical dynamics of the firm's asset value μ, and the time to maturity of corporate liabilities T, one can obtain the maximum likelihood estimates for the barrier-to-debt ratio α, the market value of firm's asset V and the asset value volatility σ by numerically maximizing the function in (3).

According to Brockman and Turtle (2003), Equation (2) also implies a risk-neutral default probability over the interval $[0, T]$ that one can write as

$$\text{default probability} = N\left(\frac{(\ln(\alpha X) - \ln V) - (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right)$$

$$+\exp\left(\frac{2(r-\frac{\sigma^2}{2})(\ln(\alpha X)-\ln V)}{\sigma^2}\right)\left(1-N\left(\frac{-(\ln(\alpha X)-\ln V)-(r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right)\right). \qquad (4)$$

Similar probability measures can be found in Cox and Miller (1965), Ingersoll (1987) and Rich (1994). Although Equation (4) estimates only a risk-neutral probability of default, this still provides a meaningful ranking of firms according to their susceptibility to default.

3. Data

We collect data from the Market Observation Post System (M.O.P.S.) of the Taiwan Stock Exchange. Attention is given to the 31 firms in Taiwan's construction industry. The sample covers an eleven-year period of quarterly observations from 1994 to 2004. The whole data set consists of 1,364 firm-quarters which provide sufficient data to perform the maximum likelihood estimation.

Table 1 lists the 31 firms in Taiwan's construction industry in our sample. For each firm considered, we report its stock code, company name, date of listing and date of bankruptcy. A total of 5 bankrupted firms are recorded. Table 2 presents the basic summary statistics for our firm data. For each variable considered, we report the sample mean, median, standard deviation, minimum and maximum for the 1,364 available observations.

Total market value of equity is measured as the total number of shares outstanding times the stock price. Debt value is computed as the book value of assets minus the book value of equity. The reported debt proportion is given by the ratio of debt value to the book value of assets. The riskless rate is proxied by the one-year savings rate from the Bank of Taiwan. The average market value of equity for our sample is slightly larger than ten million NT dollars. However, the typical firm has a much smaller market value of equity of just over three million NT dollars. The sample mean debt load exceeds six billion NT dollars with a median of approximately four billion NT dollars. The average and median debt ratios display a similar mean and median of approximately 55%. The annualized risk-free interest rate ranges from a low of 1.32% to a high of 7.37% over the sample period. The mean and median values are approximately 5%-5.4%.

4. Empirical results

4.1. Evaluation of implied default barriers

To estimate the barrier-to-debt ratio α, the market value of firm's asset V and the asset value volatility σ, the maximum likelihood approach requires the market value of firm's equity V_E, the future promised payment of corporate liabilities X, the risk-free interest rate r and the time to maturity of corporate liabilities T. The market value of firm's equity V_E is measured as the total number of shares outstanding times the stock price. The future promised payment of corporate liabilities X is computed as the book value of assets minus the book value of equity. The risk-free interest rate r is proxied by the one-year savings rate from the Bank of Taiwan. The time to maturity of corporate liabilities T is set as 10 years which is considered in Brockman and Turtle (2003).

With a complete specification of the market value of firm's equity V_E, the future promised payment of corporate liabilities X, the risk-free interest rate r and the time to maturity of corporate liabilities T, we can proceed to estimating the barrier-to-debt ratio α, the market value of firm's asset V and the asset value volatility σ. First, the values of V_E, X, r and T are substituted into the log-likelihood function in (3). Then, the estimates of α, V and σ are obtained by maximizing the log-likelihood function via the numerical scheme of Nelder and Mead (1965).

Table 3-5 report the maximum likelihood estimates for the barrier-to-debt ratio α, the market value of firm's asset V and the asset value volatility σ, respectively. The means of

α are all statistically significant at the 5% level. This suggests the existence of implied default barriers. Furthermore, the means of α are all less than one, and this implies that the firms in our sample have a financial structure of default barrier level below its total liability.

4.2. Evaluation of default prediction performances

To evaluate the ability of the barrier option framework to predict default, we compare its prediction performance to the commonly adopted models, Merton model, the Z-Score model and the ZETA model. We first compare the barrier option framework to Merton model. Merton (1974) proposes a model where a firm's equity is a call option on the assets of the firm. The risk-neutral probability, *P*, that the firm will default by time *T* is the probability that shareholders will not exercise their call options to buy the assets of the firm for its debt at time *T*. It is given by

$$P = N\left(-\frac{\ln\frac{V}{X} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} + \sigma\sqrt{T}\right), \qquad (5)$$

where V is the market value of firm's assets, X is the future promised payment of corporate liabilities, r is the risk-free interest rate, σ is the asset value volatility, T is the time to maturity of corporate liabilities and $N(\cdot)$ is the cumulative distribution function for a standard normal random variable.

Following Kealhofer and Kurbat (2001), we perform a standard power test to compare default predictive power of the barrier option framework and Merton model. The power results are shown in Figure 1. The barrier option framework power curve is denoted by the solid line. The Merton model power curve is denoted by the dotted line. A power curve shows the tradeoff between type I and type II error for all possible values of the measure. The type I error is the probability of failing to identify a default in advance, and is given by the vertical distance from the chosen point to the top of the figure. The type II error is the probability of incorrectly identifying a firm as a default candidate, and is given by the horizontal distance to the chosen point from the origin. The barrier option framework power curve lies above the Merton model power curve at all points, meaning that it has less type I error for any given common level of type II error, or equivalently, less type II error for any given common level of type I error. Thus, the barrier option framework is uniformly more powerful than Merton model in predicting default.

We then compare the barrier option framework to the Z-Score model and the ZETA model. According to Altman (2000), a convenient specification of the Z-Score model can be written as

$$Z = 1.2X_1 + 1.4X_2 + 3.3X_3 + 0.6X_4 + 1.0X_5, \tag{6}$$

where Z is the value of Z-Score, X_1 is the working capital/total assets ratio, X_2 is the retained earnings/total assets ratio, X_3 is the earnings before interest and taxes/total assets ratio, X_4 is the market value of equity/book value of total liabilities ratio and X_5 is the sales/total assets ratio. Altman (2000) shows that, on a univariate level, all of the ratios indicate higher values for the non-bankrupt firms. Therefore, the greater a firm's distress potential, the lower its Z-Score. We use a cutoff value of 1.81 for Z-Score as advocated by Altman (2000).

The ZETA model is constructed as a second generation model with several enhancements to the original Z-Score approach. Because the ZETA model is a proprietary effort, there is no full disclosure of the parameters of the model. We adopt a 7-variable model as suggested by Altman (2000), and perform a discriminant analysis on our firm data to derive the model coefficients. Our own version of the ZETA model can be written as

$$\begin{aligned} ZETA &= -2.056 + 6.610X_1 - 8.852X_2 + 0.010X_3 \\ &+ 3.902X_4 - 0.029X_5 - 0.028X_6 + 0.152X_7, \end{aligned} \tag{7}$$

where *ZETA* is the value of ZETA score, X_1 is the earnings before interest and taxes/total assets ratio, X_2 is the normalized measure of the standard error of the estimate around a five-to-ten-year trend on X_1, X_3 is the logarithm of the earnings before interest and taxes/total interest payments ratio, X_4 is the retained earnings/total assets ratio, X_5 is the current assets/current liabilities ratio, X_6 is the market value of common equity/total capital ratio and X_7 is the logarithm of total assets. The discriminant analysis shows that the value of -3.57 is an adequate cutoff point for ZETA model.

In Table 6, we consider the ability of the barrier option framework, the Z-Score model, and the ZETA model to predict default within the next one, two and three years. For each panel of the table, we report output for four logistic regressions that can be shown as

$$\log(\frac{P}{1-P}) = \beta_0 + \beta_1 X_1 + \varepsilon,$$

$$\log(\frac{P}{1-P}) = \beta_0 + \beta_2 X_2 + \varepsilon, \tag{8}$$

$$\log(\frac{P}{1-P}) = \beta_0 + \beta_3 X_3 + \varepsilon,$$

$$\log(\frac{P}{1-P}) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon,$$

where P is the probability that a firm will go bankrupt, X_1 is the default probability implied by the barrier option framework, X_2 is the Z-Score value and X_3 is the ZETA value. All regressions include an intercept that we omit for brevity. Our results show that default probability increases as the likelihood of bankruptcy increases. The estimated coefficients are statistically significant at the 5% level. Z-Score and ZETA are also shown to be valuable predictors of future bankruptcy. Given the inverse scale of these two variables, we find significantly negative coefficients as expected. Our results further show that the inclusion of default probability and Z-Score increases the log-likelihood ratio relative to other models in isolation, and show that both variables are statistically significant at the 5% level. Our results demonstrate markedly better predictive ability for the barrier option framework versus the Z-Score and ZETA models. For instance, in all three panels, the log-likelihood ratio for the barrier option framework is the highest in comparison to the Z-Score and ZETA models. Thus, we find evidence that the barrier option framework dominates the Z-Score and ZETA models in its ability to predict bankruptcy over both short and longer horizons.

We also evaluate the ability of the barrier option framework to predict bankruptcy by testing its 0.25-, 0.5-, 0.75-, and 1-year-ahead prediction performances. Our results are reported in Table 7. Our results show that, for each of the 5 defaulted firms in our sample, the default probabilities increase as it approaches to the date of default. For instance, in the case of Pacific Construction Corporation, the default probabilities increase from 46.21% one-year-ahead to 73.86% one-quarter-ahead of the actual default. Thus, we find evidence that the prediction accuracy of the barrier option framework improves as the forecast horizon decreases.

4.3. Evaluation of estimated default risks

Fama and French (1993) propose a three-factor model that includes beta, size and book-to-market to explain equity returns. Fama and French (1996) argue that the size and book-to-market factors proxy for default risk. Vassalou and Xing (2004), however, show that although size and book-to-market contain some default-related information, this is not the reason that the Fama-French three-factor model is able to explain the cross section of equity returns. They find that size and book-to-market appear to contain other significant price information that is unrelated to default risk, and conclude that default risk is a variable worth considering in asset-pricing tests, above and beyond size and book-to-market.

Following Vassalou and Xing (2004), we evaluate the ability of our estimated default risks to explain equity returns by extending the Fama-French three-factor model to include a forth factor, default probability. In Table 8, we report output for six regressions that can be shown as

$$R = \beta_0 + \beta_1 X_1 + \varepsilon,$$

$$R = \beta_0 + \beta_2 X_2 + \varepsilon,$$

$$R = \beta_0 + \beta_3 X_3 + \varepsilon, \tag{9}$$

$$R = \beta_0 + \beta_4 X_4 + \varepsilon,$$

$$R = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon,$$

$$R = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon,$$

where R is the equity return, X_1 is the beta value, X_2 is the firm size, X_3 is the book-to-market ratio and X_4 is the default probability implied by the barrier option framework. All regressions include an intercept that we omit for brevity.

Our results show that beta, size, book-to-market and default probability are all valuable explanatory variables of equity returns. The estimated coefficients are statistically significant at the 5% level. The signs of the estimated coefficients show that high-beta firms earn higher returns than low-beta firms, small firms earn higher returns than big firms, growth stocks (low book-to-market) earn higher returns than value stocks (high book-to-market), and low-default-risk firms earn higher returns than high-default-risk firms.

Our results further show that the inclusion of default probability in the Fama-French model increases the adjusted R^2 relative to other models in isolation, and show that default probability is statistically significant at the 5% level. Thus, we find evidence that default risk *is able to* explain equity returns, and that default risk is a variable worth considering in asset-pricing tests, above and beyond size and book-to-market. Our evidence is consistent with the finding from Vassalou and Xing (2004).

5. Conclusions

This paper investigates the performance of a default risk model based on the barrier option framework. We provide empirical validation of the model by showing that implied barriers are statistically significant for a sample of construction firms in Taiwan over the period 1994 to 2004. We then apply the model to default prediction and find that the model dominates the commonly adopted models, Merton model, the Z-Score model and the ZETA model. Moreover, we test the n-year-ahead prediction performance of the model and find evidence that the prediction accuracy of the model improves as the forecast horizon decreases. Finally, we assess the effect of estimated default risk on equity returns and find that default risk is able to explain equity returns and that default risk is a variable worth considering in asset-pricing tests, above and beyond size and book-to-market. This paper provides empirical validation of a default risk model based on the barrier option framework, applies the model to default prediction, and assesses the effect of estimated default risk on equity returns. We believe that there exists considerable potential for additional applications of the barrier option framework to future financial research.

References

Altman, E., 2000. Predicting financial distress of companies: revisiting the Z-Score and ZETA models. Working Paper, New York University.

Black, F., Cox, J., 1976. Valuing corporate securities: some effects of bond indenture provisions. Journal of Finance 31, 351-367.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. Journal of Political Economy 81, 637-654.

Brockman, P., Turtle, H., 2003. A barrier option framework for corporate security valuation. Journal of Financial Economics 67, 511-529.

Cox, D., Miller, H., 1965. The Theory of Stochastic Processes. John Wiley, New York.

Duan, J., 1994. Maximum likelihood estimation using price data of the derivative contract. Mathematical Finance 4, 155-167.

Duan, J., 2000. Correction: Maximum likelihood estimation using price data of the derivative contract. Mathematical Finance 10, 461-462.

Duan, J., Gauthier, G., Simonato, J., 2004. On the equivalence of the KMV and maximum likelihood methods for structural credit risk models. Working Paper, University of Toronto.

Fama, E., French, K., 1993. Common risk factors in the returns on bonds and stocks. Journal of Financial Economics 33, 3-56.

Fama, E., French, K., 1996. Multifactor explanations of asset pricing anomalies. Journal of Finance 51, 55-84.

Ingersoll, J., 1987. Theory of Financial Decision Making. Rowan and Littlefield, Savage, MD.

Kealhofer, S., Kurbat, M., 2001. The default prediction power of the Merton approach, relative to debt ratings and accounting variables. Research Document, KMV LLC.

Merton, R., 1974. On the pricing of corporate debts: the risk structure of interest rates. Journal of Finance 29, 449-470.

Nelder, J., and Mead, R., 1965. A simplex method for function minimization. Computer Journal 7, 308-313.

Rich, D., 1994. The mathematical foundations of barrier options. Advances in Futures and Options Research 7, 267-311.

Ronn, E. I., and Verma, A.K., 1986. Pricing risk-adjusted deposit insurance: An option-based model. Journal of Finance 41, 875-895.

Vassalou, M., Xing, Y., 2004. Default risk in equity returns. Journal of Finance 56, 831-868.

Wong, H., Choi, T., 2004. The impact of default barrier on the market value of firm's asset. Working Paper, Chinese University of Hong Kong.

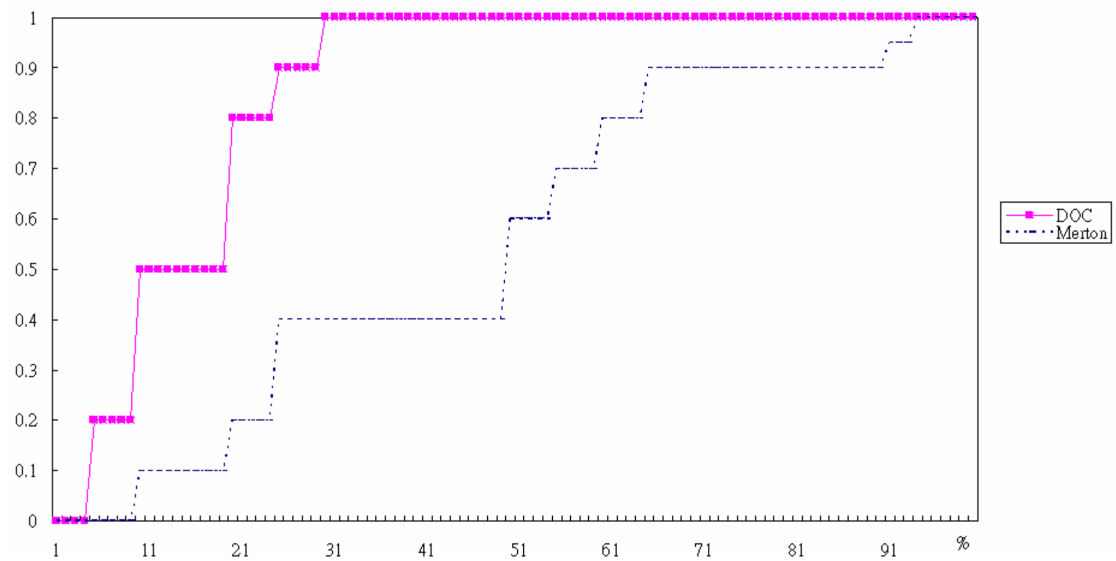

Figure 1. Power curves comparing bankruptcy predictive power of the barrier option framework and Merton model. *Notes:* We show a standard power test result to compare bankruptcy predictive power of the barrier option framework and Merton model. The barrier option framework power curve is denoted by the solid line. The Merton model power curve is denoted by the dotted line.

Table 1
The list of the 31 firms in Taiwan's construction industry in our sample

No	Stock Code	Company Name	Date of Listing	Date of Bankruptcy
1	1436	Fui Industrial Corporation	04/11/1988	
2	2501	Cathay Real Estate Development Corporation	10/28/1967	
3	2504	Goldsun Development & Construction Corporation	03/14/1978	
4	2505	Kuo Yang Construction Corporation	11/14/1979	
5	2506	Pacific Construction Corporation	02/02/1980	05/08/2003
6	2509	Chainqui Development Corporation	05/20/1988	
7	2511	Prince Housing & Development Corporation	04/24/1991	
8	2514	Long Bon Development Corporation	09/26/1992	
9	2515	Bes Engineering Corporation	03/02/1993	
10	2516	New Asia Construction & Development Corporation	05/25/1993	
11	2520	Kindom Construction Corporation	10/27/1993	
12	2523	Der Pao Construction Corporation	08/12/1994	
13	2524	King's Town Construction Corporation	10/18/1994	
14	2525	Pao Hsiang Construction Corporation	06/13/1997	09/10/2002
15	2526	Continental Engineering Corporation	11/01/1994	
16	2527	Hung Ching Development & Construction Corporation	03/06/1995	
17	2528	Crowell Development Corporation	03/10/1995	
18	2530	Delpha Construction Corporation	10/12/1995	
19	2533	Yuh Chen United Technologies Corporation	12/30/1995	
20	2534	Hung Sheng Construction Corporation	02/12/1996	
21	2535	Da Cin Construction Corporation	03/11/1996	
22	2537	Ezplace Corporation	09/06/1996	05/07/2002
23	2538	Kee Tai Properties Corporation	11/01/1996	
24	2539	Sakura Development Corporation	07/16/1997	05/07/2002
25	2540	Jin Shang Chang Development Corporation	12/26/1989	04/22/2003
26	2542	Highwealth Construction Corporation	05/03/1999	
27	2543	Hwang Chang General Contractor Corporation	10/15/1999	
28	2545	Huang Hsiang Construction Corporation	09/11/2000	
29	2546	Kedge Construction Corporation	09/11/2000	
30	2547	Radium Life Tech Corporation	12/22/2000	
31	2548	Huaku Construction Corporation	08/26/2002	

Notes: We list the 31 firms in Taiwan's construction industry in our sample. For each firm considered, we report its stock code, company name, date of listing and date of bankruptcy.

Table 2
Descriptive statistics for our firm data

	Mean	Median	Standard Deviation	Minimum	Maximum
Market value of equity*	10,021	3,462	21,722	95	192,066
Debt value*	6,438,650	3,913,803	7,137,323	14,792	35,050,850
Debt proportion	0.5412	0.5656	0.1714	0.0250	0.9998
Riskless rate	0.0543	0.0490	0.0223	0.0132	0.0737

Notes: We present the basic summary statistics for our firm data. For each variable considered, we report the sample mean, median, standard deviation, minimum and maximum for the 1,364 available observations. Total market value of equity is measured as the total number of shares outstanding times the stock price. Debt value is computed as the book value of assets minus the book value of equity. The reported debt proportion is given by the ratio of debt value to the book value of assets. The riskless rate is proxied by the one-year savings rate from the Bank of Taiwan. * denotes thousands of New Taiwan dollars (NT$1,000).

Table 3
Maximum likelihood estimates for the barrier-to-debt ratio α

No	Stock Code	Mean*	Standard Deviation	Minimum	Maximum
1	1436	0.32	0.14	0.16	0.60
2	2501	0.19	0.01	0.18	0.20
3	2504	0.19	0.02	0.18	0.21
4	2505	0.31	0.03	0.29	0.34
5	2506	0.65	0.07	0.59	0.72
6	2509	0.24	0.03	0.22	0.27
7	2511	0.23	0.02	0.21	0.25
8	2514	0.35	0.02	0.34	0.37
9	2515	0.26	0.02	0.24	0.28
10	2516	0.31	0.06	0.25	0.37
11	2520	0.21	0.07	0.14	0.27
12	2523	0.12	0.03	0.10	0.15
13	2524	0.68	0.03	0.66	0.71
14	2525	0.58	0.21	0.34	0.73
15	2526	0.22	0.01	0.22	0.23
16	2527	0.17	0.01	0.16	0.17
17	2528	0.79	0.03	0.77	0.82
18	2530	0.61	0.06	0.55	0.67
19	2533	0.41	0.06	0.36	0.48
20	2534	0.32	0.01	0.31	0.33
21	2535	0.21	0.03	0.17	0.23
22	2548	0.58	0.18	0.40	0.75
23	2537	0.87	0.05	0.81	0.90
24	2538	0.68	0.16	0.50	0.80
25	2539	0.35	0.05	0.30	0.39
26	2540	0.19	0.01	0.19	0.19
27	2542	0.63	0.04	0.60	0.68
28	2543	0.25	0.04	0.22	0.30
29	2545	0.30	0.02	0.28	0.32
30	2546	0.48	0.02	0.45	0.50
31	2547	0.42	0.02	0.40	0.44

Notes: We report the maximum likelihood estimates for the barrier-to-debt ratio α. For each firm in our sample, we report the mean, standard deviation, minimum and maximum of the maximum likelihood estimates. * denotes significance at the 5% level.

Table 4
Maximum likelihood estimates for the market value of firm's asset *V*

No	Stock Code	Mean*	Standard Deviation*	Minimum*	Maximum*
1	1436	5,136,917	912,939	4,120,946	5,888,437
2	2501	20,643,582	1,937,885	19,162,415	25,459,135
3	2504	22,209,507	963,242	21,449,542	23,292,821
4	2505	8,049,311	2,367,964	5,736,148	10,468,514
5	2506	33,828,776	9,456,599	24,841,037	43,693,184
6	2509	3,846,590	1,161,769	2,894,512	5,141,084
7	2511	14,966,235	11,413,608	1,901,248	22,998,741
8	2514	9,798,480	539,262	9,480,214	10,421,114
9	2515	42,287,414	1,031,582	41,184,652	43,228,794
10	2516	8,227,797	607,503	7,750,113	8,911,524
11	2520	12,600,510	1,979,276	10,911,547	14,778,441
12	2523	10,305,196	202,833	10,104,483	10,510,084
13	2524	4,528,204	1,448,418	3,274,582	6,113,785
14	2525	7,704,374	1,668,520	5,937,946	9,253,739
15	2526	31,301,966	321,307	31,041,751	31,661,103
16	2527	8,346,006	860,857	7,658,992	9,311,674
17	2528	4,477,052	1,442,873	3,185,421	6,034,267
18	2530	15,286,459	2,200,674	12,776,591	16,885,469
19	2533	2,647,525	540,618	2,240,909	3,261,034
20	2534	20,165,778	2,117,678	18,776,925	22,603,154
21	2535	5,896,557	495,218	5,340,201	6,289,157
22	2548	3,503,404	806,078	2,764,183	4,362,837
23	2537	2,223,884	154,845	2,045,786	2,326,637
24	2538	9,672,178	755,105	8,800,315	10,116,770
25	2539	7,047,529	3,556,563	4,127,648	11,008,442
26	2540	2,052,796	305,413	1,851,847	2,404,252
27	2542	5,591,517	666,505	5,007,346	6,317,524
28	2543	3,480,242	124,614	3,350,762	3,599,341
29	2545	3,461,990	943,103	2,914,638	4,550,987
30	2546	1,972,713	242,783	1,700,325	2,166,330
31	2547	6,670,999	975,351	6,020,764	7,792,486

Notes: We report the maximum likelihood estimates for the market value of firm's asset V. For each firm in our sample, we report the mean, standard deviation, minimum and maximum of the maximum likelihood estimates. * denotes thousands of New Taiwan dollars (NT$1,000).

Table 5
Maximum likelihood estimates for the asset value volatility σ

No	Stock Code	Mean	Standard Deviation	Minimum	Maximum
1	1436	0.10	0.01	0.09	0.11
2	2501	0.23	0.02	0.02	0.25
3	2504	0.15	0.03	0.13	0.18
4	2505	0.12	0.03	0.10	0.15
5	2506	0.39	0.02	0.37	0.40
6	2509	0.18	0.04	0.14	0.21
7	2511	0.28	0.02	0.26	0.29
8	2514	0.02	0.00	0.02	0.02
9	2515	0.05	0.00	0.05	0.06
10	2516	0.17	0.00	0.17	0.18
11	2520	0.16	0.02	0.14	0.18
12	2523	0.02	0.00	0.02	0.03
13	2524	0.09	0.00	0.09	0.09
14	2525	0.26	0.01	0.25	0.27
15	2526	0.10	0.00	0.10	0.10
16	2527	0.12	0.00	0.11	0.12
17	2528	0.36	0.01	0.35	0.36
18	2530	0.07	0.01	0.06	0.07
19	2533	0.16	0.00	0.16	0.16
20	2534	0.20	0.02	0.18	0.21
21	2535	0.14	0.00	0.14	0.15
22	2548	0.15	0.01	0.14	0.16
23	2537	0.14	0.04	0.11	0.18
24	2538	0.13	0.01	0.12	0.14
25	2539	0.13	0.02	0.11	0.14
26	2540	0.36	0.03	0.33	0.38
27	2542	0.14	0.03	0.12	0.17
28	2543	0.12	0.01	0.11	0.13
29	2545	0.15	0.00	0.15	0.16
30	2546	0.04	0.00	0.04	0.04
31	2547	0.15	0.03	0.12	0.18

Notes: We report the maximum likelihood estimates for the asset value volatility σ. For each firm in our sample, we report the mean, standard deviation, minimum and maximum of the maximum likelihood estimates.

Table 6
Logistic regression results comparing the barrier option framework to the Z-Score model and the ZETA model

	Default Probability	Z-Score	ZETA	LLR
Panel A: Bankrupted within the next one year				
	3.894			20.452
	(0.011)*			
		-0.177		14.837
		(0.031)*		
			-25.210	16.156
			(0.040)*	
	2.862	-0.464		27.351
	(0.037)*	(0.035)*		
Panel B: Bankrupted within the next two years				
	3.800			52.778
	(0.009)**			
		-0.190		45.270
		(0.036)*		
			-28.644	48.125
			(0.040)*	
	2.618	-0.433		90.148
	(0.008)**	(0.022)*		
Panel C: Bankrupted within the next three years				
	3.696			57.249
	(0.001)**			
		-0.195		37.506
		(0.042)*		
			-28.033	41.070
			(0.037)*	
	2.527	-0.459		89.802
	(0.000)**	(0.031)*		

Notes: We report output for various logistic regression models comparing the ability of the barrier option framework (Default Probability), the Z-Score model (Z-Score) and the ZETA model (ZETA) to predict bankruptcy within the next one, two and three years. For each logistic regression considered, we report the estimated coefficients along with the associated *p*-values in the parentheses and the log-likelihood ratio (LLR). * denotes significance at the 5% level. ** denotes significance at the 1% level.

Table 7
N-year-ahead bankruptcy predictions for the 5 bankrupted firms in our sample

Company Name	Date of Bankruptcy	0.25-Year-Ahead Default Probability	0.5-Year-Ahead Default Probability	0.75-Year-Ahead Default Probability	1-Year-Ahead Default Probability
Ezplace Corporation	05/07/2002	30.70%	30.09%	20.99%	21.66%
Sakura Development Corporation	05/07/2002	76.00%	64.00%	62.00%	55.00%
Pao Hsiang Construction Corporation	09/10/2002	40.10%	37.38%	35.27%	34.05%
Jin Shang Chang Development Corporation	04/22/2003	88.00%	77.00%	68.71%	71.54%
Pacific Construction Corporation	05/08/2003	73.86%	72.16%	54.16%	46.21%

Notes: We report n-year-ahead bankruptcy predictions for the 5 bankrupted firms in our sample. For each firm considered, we report its company name, date of bankruptcy, 0.25-, 0.5-, 0.75-, and 1-year-ahead default probabilities.

Table 8
Regression results on the relative importance of beta, size, book-to-market (BM), and default probability (DP) characteristics for equity returns

Beta	Size	BM	DP	Adjusted R^2
1.7628				0.1150
(0.0000)**				
	-0.4730			0.1090
	(0.0200)*			
		-0.0002		0.1120
		(0.0002)**		
			-0.8842	0.1040
			(0.0190)*	
1.5543	-0.3061	-0.0001		0.3370
(0.0000)**	(0.0146)*	(0.0070)**		
1.4886	-0.3392	-0.0001	-0.4725	0.4401
(0.0000)**	(0.0097)**	(0.0012)**	(0.0200)*	

Notes: We report regression results on the relative importance of beta, size, BM, and DP characteristics for equity returns. For each regression considered, we report the estimated coefficients along with the associated *p*-values in the parentheses and the adjusted R^2. * denotes significance at the 5% level. ** denotes significance at the 1% level.