Demand deposits as commitment device and the optimal debt mix of banks in a continuous-time framework

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January 2007

Abstract

Recent work in the banking theory literature stresses the role of uninsured demand deposits to discipline incumbent bank manager-owners not to extract rents from their specific abilities. In this article, we show if and how this finding can be incorporated in a continuous-time contingent claims valuation framework. Although demand deposits represent a hard claim in the sense that they introduce an exogenous default threshold through a collective withdrawal of funds (bank run), they can at the same time add firm value by acting as a commitment device. Equity holders are willing to voluntarily weaken their bargaining power by financing with deposits to increase the banks debt capacity and to exploit the additional tax shield.

Although being a stylized model, it generates testable implications. We find that the proportion of deposits in the optimal debt mix is higher if bank manager-owners have less specific abilities and if bank asset are less risky.

JEL classification: G21, G32

Keywords: Demand deposits, commitment device, bank capital structure, contingent claims valuation.

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1 Introduction

The question, whether there exists an optimal capital structure and if so, which factors determine it, has a long tradition in corporate finance research, at least with respect to non-financial firms. The question, whether there exists an optimal *bank* capital structure has only been addressed more recently. The reason may be that banks are different from non-financial firms in at least two important ways. As intermediaries, the crucial function banks perform are transformation activities, in particular risk and liquidity transformation. And secondly, as opposed to industrial firms, the liabilities of banks not only fund the business and transfer ownership but are itself a part of the business of the firm. Furthermore, financial firms are more heavily regulated.

However, with these differences in mind, it is legitimate to try to apply research methods from classical corporate finance to address the issue of optimal capital structure of commercial banks. In particular, the present article wants to put up a framework within which one can analyze the optimal decision with respect to the mix of equity, bonds and deposits for a commercial bank. Thereby, the focus is on the optimal debt mix, i.e. the question why banks want to issue demand deposits and if so how much of the overall debt capacity should consist of deposits.

We address these issues within a continuous-time model, whereby the relevant state variable is given by a flow variable, which may be interpreted as the earnings before interest and taxes (EBIT). This is in contrast to the more frequently used discretetime (two periods, three dates) models, but has the advantage to provide a more realistic picture and enables us to derive quantitatively more meaningful results.

Thus, from the methodical point of view, our model is closely related to the strand of literature that dates back to the work of Black and Cox (1976), Fischer et al. (1989) and Leland (1994). While these models have the asset value as relevant state variable, more recent contributions including Goldstein et al. (2001), Dangl and Zechner (2004) or Christensen et al. (2005) consider the EBIT as state variable.¹

While there is a huge amount of contributions dealing with capital structure issues of non-financial firms, only a limited number of papers deal with bank capital structure in a continuous-time setting. Notable exceptions are Merton (1977, 1978), Gorton and Santomero (1990), Bhattacharya et al. (2002) or Decamps et al. (2004). These articles however, are mainly concerned with the fair pricing of deposit insurance, valuation of subordinated debt or bank regulation.

With respect to content, our contribution is most closely related to Diamond and Rajan (2000, 2001), Gorton and Winton (2000) and Kashyap et al. (2002).

Diamond and Rajan (2000) stress the special role of banks as liquidity providers. They assume that a bank as relationship lender has specific abilities to generate more cash flows out of existing assets, which are assumed to be loans to entrepreneurs, than anyone else. Its specific abilities enables the bank to extract rents because she

¹ One of the reasons to consider a flow variable as state variable is to avoid a model-inherent inconsistency in that one does not need to assume the unlevered and the optimally levered firm value as a traded asset at the same time.

can threaten to withhold their abilities, or at least, the bank cannot credibly commit to not withhold them. This implies smaller pledgeable assets and in this sense, they call the loan illiquid. However, if the bank can finance its assets with demand deposits, the pledgeable income and therefore the liquidity can be increased. In a nutshell, the reason is that depositors suffer a collective action problem and will run on the bank when they believe the bank cannot redeem their claim in whole. Therefore, the bank cannot enter into negotiations and is not able to extract rents from her specific abilities. Thus, demand deposits act as a *commitment device*. The optimal bank capital structure in Diamond and Rajan (2000) is therefore the result of the trade-off between liquidity creation and costs of bank distress. A high proportion of deposits will provide more liquidity but will also increase the fragility of the bank in that the probability of bank runs increases. The model in Diamond and Rajan (2000) is formulated in a discrete-time framework, i.e. a two periods, three dates model, which provides more qualitative than quantitative results and is difficult to compare to structural models as those cited above.

The fact that demand deposits provide a way for the bank managers to commit themselves not to use their specific abilities to extract rents depends on the assumption that deposits are not insured. As soon as deposit insurance enters the model, bank runs will not occur and thus the crucial disciplining mechanism disappears.

With respect to models that include deposit insurance, the focus is on the fair pricing of the premium banks should pay for the deposit insurance,² and on the mandatory issuance of subordinated debt as a regulatory mechanism that intends to curtail banks risk-taking through market discipline. This so-called subordinated debt proposal has been put forward by f.ex. Gorton and Santomero (1990), Evanoff and Wall (2000) or Levonian (2001).

Another strand of literature focuses primarily on the valuation of deposits. In particular, Jarrow and van Deventer (1998) propose an arbitrage-free pricing framework for demand deposits. Thereby, they assume that the interest paid on a demand deposit is lower than the risk-free rate, i.e. that banks can earn a rent. To ensure arbitrage-free pricing they must invoke a market segmentation hypothesis. Focusing on interest-rate risk, they derive closed-form valuation formulas. However, their model neither incorporates default risk or taxes, nor addresses the problem of optimal debt mix and capital structure. Similar approaches, that assume a premium on deposit interest rates, are due to Hutchison and Pennacchi (1996), O'Brien (2000), or Kalkbrener and Wiling (2004). Besides deriving valuation formulas for the rents generated by deposits, they provide corresponding empirical tests. However, as mentioned above, the problem of why deposits may be issued in the first place is not addressed. On the one hand, the premium on demand deposits always favors financing with deposits over issuing bonds. So, the question why a bank may want to issue demand deposits is irrelevant in these models, since it is already answered by definition. On the other hand, as long as demand deposits provide a rent and

 $^{^{2}}$ As referenced above, early contributions in this respect are Merton (1977, 1978).

absent default risks, the bank should be financed exclusively by deposits. Therefore, the determination of an optimal debt mix is not possible. This, however, are questions we want to address in this article.

The implication of the bank capital theory of Diamond and Rajan (2000) is that banks with a high capital ratio will create less liquidity than those with low capital ratios. This view is not uncontested, and an alternative line of reasoning comes to the opposite conclusion. According to f.ex. Bhattacharya and Thakor (1993), Repullo (2004), and von Thadden (2004),³ bank capital serves as a cushion against risks that are associated with liquidity creation. Thus, banks with high capital ratios can absorb more risks and can therefore create more liquidity than those banks with low capital ratios. A recent empirical study by Berger and Bouwman (2005) attempts to test which of the two competing hypotheses can be confirmed by available data on liquidity creation.⁴ They find that for large banks,⁵ the data confirms the risk absorption hypothesis, since large banks with higher capital ratios created more liquidity. However, for small banks they find support for the hypothesis in line with Diamond and Rajan (2000), in that small banks with high capital ratios created less liquidity than those with low capital ratios.

So, at least for the sub-sample of small banks, there is evidence that demand deposits may indeed act as commitment device that enables bank to increase liquidity creation.

This article can be viewed as showing why a bank may want to issue demand deposits in the first place, and how deposits can act as commitment device in a continuous-time structural model of bank capital structure. The proposed framework has the advantage to provide quantitative and potentially testable implications for the capital structure, in particular for the mix of bonds and deposits.

As mentioned above, in Diamond and Rajan (2000) the threat of a bank run drives the banker's rents to zero, since depositors face a collective action problem which rules out the possibility to negotiate over the surplus that could be generated by the specific abilities of the banker. However, while on the one hand Diamond and Rajan (2000) stress the important role of the collective action problem, their result that the banker can credibly commit not to extract rents and pass on the full value of the loan depends on the other hand crucially on the assumption that after a run has occurred the depositors, who now hold the assets, can enter into negotiations directly with the entrepreneur. To quote Diamond and Rajan (2000), p. 2439: "If depositors have seized the loan, the banker is disintermediated, and the entrepreneur can directly initiate negotiations with depositors by making an offer." The threat of being disintermediated is what keeps the banker from extracting rents. However,

 $^{^{3}}$ See the discussion in Berger and Bouwman (2005).

⁴ Berger and Bouwman (2005) construct a liquidity measure by classifying asset, liabilities and off-balance sheet items into three categories: liquid, semi-liquid and illiquid. An example for an illiquid asset is a business loan, while cash and securities are classified as liquid assets. On the liability side, transaction deposits are classified as liquid, while, long-term debt and equity are illiquid items.

 $^{^5}$ Large banks are defined as banks with gross total assets over \$1 billion.

it seems not convincing, that depositors suffer a collective action problem vis-à-vis the bank but not vis-à-vis the entrepreneur.⁶ The assumption that the bank can be disintermediated by direct negotiations between (potentially large numbers of) depositors and creditors seems to be unrealistic and undermines the very reason for the existence of banks, namely their intermediary services.

To avoid these kinds of complications, we assume that there *always* exists a collective action problem among depositors and thus that no negotiations whatsoever are possible with this group. But on the other hand, the bank can issue bonds, and we assume that bondholders do not suffer from collective action problems. With respect to this group of investors, the bank can initiate negotiations to bargain about the surplus that can only be generated when bank managers contribute their specific abilities. The bargaining game takes place when bank owners decide to stop servicing the outstanding debt obligations. In this case, the debt holders have the choice between accepting or rejecting the bargaining offer. If they reject, liquidation will occur, and the former bank owners will hand over the assets. Thus, liquidation is assumed to occur on the basis of a debt-equity swap as e.g. in Fan and Sundaresan (2000). But the remaining assets without the specific know-how of the former bank manager will be worth less than as on a going-concern basis. Therefore, depending on the bargaining power, the surplus will be split between bank owners and debt holders. Although depositors are not part of this bargaining game, we assume that a high proportion of demand deposits will strengthen the position of debt holders vis-à-vis bank owners in the sense that it will increase the bargaining power of bondholders.

It will turn out, that bank owners are willing to voluntarily weaken their bargaining position by holding deposits in order to maximize ex ante firm value.

The remainder of the article is organized as follows. The next section sets up the model by starting with the general valuation framework, determining the optimal liquidation and bargaining values, and determination of the exogenous and endogenous thresholds. Section 3 solves the model and discusses the optimal debt mix and its implication. Section 4 concludes. Technical details are contained in the appendix.

2 Model framework

2.1 General valuation

This section will start by giving the outline of the general valuation framework. We assume that the central state variable upon which the corporate claims are defined is given by a flow variable, that we interpret as earnings before interest and taxes (EBIT). Denote the EBIT level at time t as x_t . We assume that the future stochastic evolution of the EBIT variable may be well described by a diffusion process of the

⁶ It may be argued, that it is essentially the sequential service constraint that creates the collective action problem, which is absent in the case of negotiations with the entrepreneur.

$$\mathrm{d}x_t = \mu \,\mathrm{d}t + \sigma \,\mathrm{d}W_t,\tag{1}$$

i.e. an Artihmetic Brownian Motion with drift and diffusion parameter μ and σ respectively, and where W_t is a standard Wiener process.⁷

Any claim we will introduce in the following can be interpreted as being contingent on the state variable, therefore being a function of x_t . The dynamics of a function $F(x_t)$ is easily found by an application of Itô's formula as

$$\mathrm{d}F = \left(F_t + \mu F_x + \frac{1}{2}\sigma^2 F_{xx}\right)\mathrm{d}t + \sigma F_x\,\mathrm{d}W_t$$

The absence of arbitrage opportunities implies that the total risk-neutral expected return on the claim has to equal the risk-free rate. The claims we will consider not only change their value due to a change in the underlying state variable, but also because of a continuous in-/outflow, which may be constant (c) or a fraction (m) of the state-variable. We get the following relationship

$$rF dt = \mathbb{E}^{Q}[dF + (mx + c) dt] = (F_{t} + \mu F_{x} + \frac{1}{2}\sigma^{2}F_{xx} + mx + c) dt, \qquad (2)$$

which gives the partial differential equation that has to be fulfilled by any claim. In the following, we will only consider claims with no stated maturity. This implies time-independence, so that things simplify to the following ordinary differential equation (ODE)

$$\frac{1}{2}\sigma^2 F_{xx} + \mu F_x - rF + mx + c = 0, \tag{3}$$

which has the general solution

$$F(x) = \Lambda_1 e^{\beta_1 x} + \Lambda_2 e^{\beta_2 x} + \frac{mx+c}{r} + \frac{\mu m}{r^2},$$
(4)

with Λ_1 and Λ_2 being constants that have to be determined by appropriate boundary conditions, and β_1 and β_2 are roots of the quadratic equation $Q(\beta) = \frac{1}{2}\sigma^2\beta^2 + \mu\beta - r = 0$ which are given by

$$\beta_1 = \frac{-\mu + \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2}, \qquad \beta_2 = \frac{-\mu - \sqrt{\mu^2 + 2\sigma^2 r}}{\sigma^2}.$$

Note that $\beta_1 > 0$ and $\beta_2 < 0$.

As an application for the general solution and for further use, we consider in this section an **all equity claim**, i.e. the equity value of an unlevered firm. An all equity claim is not simply the asset value for two reasons. First, we consider

type

⁷ The choice of an Arithmetic Brownian Motion (ABM) has the advantage that negative values can realize, which is per definition impossible in the more common formulation of Geometric Brownian Motion (GBM). The drawback of ABM is the loss of a convenient homogeneity property which facilitates the analytical handling in the case of GBM. Since we will have to resort to numerical simulations anyway, this drawback is not relevant in our model.

a world with taxes,⁸ thus the equity holders claim is reduced by the tax burden. Second, contrary to the case where the state variable follows a GBM, it may be optimal to abandon even an unlevered firm. Denoting this abandonment level by x_a , the corporate tax rate by τ , and the all equity claim by A(x), we get

$$A(x) = (1 - \tau) \left(\left(\frac{x}{r} + \frac{\mu}{r^2}\right) - \left(\frac{x_a}{r} + \frac{\mu}{r^2}\right) e^{\beta_2(x - x_a)} \right).$$
(5)

This is found by setting c = 0, $m = (1 - \tau)$ and the boundary condition, that the claim is worthless at the abandonment level, i.e. $A(x_a) = 0$.

The solution is to be interpreted as the net present value of an income stream from now until some future abandonment time, which we define as the stopping time

$$\mathbf{T}^{x_a} = \inf\{t; x_t \le x_a\}$$

Therefore, the all equity claim can equivalently be written as the present value

$$A(x_0) = \mathbb{E}_0 \Big(\int_0^{T^{x_a}} e^{-rs} (1-\tau) x_s \, \mathrm{d}s \Big).$$
 (6)

Note, that in (5) the term $\left(\frac{x}{r} + \frac{\mu}{r^2}\right)$ is the value of an infinite income stream without the abandonment option, i.e. $\mathbb{E}_0\left(\int_0^\infty e^{-rs} x_s \, ds\right)$, while the term $e^{\beta_2(x-x_a)}$ is to be interpreted as a probability-weighted discount factor,⁹ i.e. $\mathbb{E}_0\left(\int_0^{T^{x_a}} e^{-rs} \, ds\right)$. For ease of notation, we introduce the following abbreviation

$$\mathcal{V}x = \left(\frac{x}{r} + \frac{\mu}{r^2}\right), \qquad \mathcal{D}_0^{x_a} = e^{\beta_2(x_0 - x_a)}.$$

With this notation, (5) simplifies to

$$A(x_0) = (1 - \tau) \left(\mathcal{V}x_0 - \mathcal{V}x_a \mathcal{D}_0^{x_a} \right).$$
(7)

Next, we introduce the **depositor claim**, which is defined by the following two characteristics. First, depositors receive a constant payment, denoted by d, as long as the bank is solvent. The payment is simply to be interpreted as the interest payment on the deposit account. Second, we consider demand deposits from which funds can be withdrawn at any time. We assume that depositors will withdraw their funds only when they fear that the bank cannot redeem their claims in whole. Since the group of depositors is assumed to be homogenous, everybody will want to withdraw at the same time, essentially triggering a bank run. We denote the level where this occurs by x_e and the corresponding stopping time by $T^{x_e} = \inf\{t; x_t \leq x_e\}$.

The bank run threshold x_e can be considered as an *exogenous* reason for bank default. Usually, in the capital structure literature, one assumes that managerowners will determine an *endogenous* default threshold, where they decide to stop servicing their debt obligations¹⁰ and enter into bankruptcy procedures. Call this

⁸ Only taxes at the corporate level are considered, while ignoring taxes at the investor level. It would be straightforward to also introduce a personal tax, but this would add no further insights for our purposes.

⁹ See e.g. Mella-Barral (1999), p. 541.

¹⁰ In case that the current cash flow is not sufficient to finance the debt payment, the only possibility to raise money, is through issuance of equity or injection of existing equity holders.

endogenous threshold x_b and its corresponding stopping time $T^{x_b} = \inf\{t; x_t \leq x_b\}$. Thus, two default scenarios are possible. Either a bank run occurs, or bank owners no longer honor their obligations, e.g. either x_e or x_b is binding. We denote the binding threshold by $\bar{x} = \max(x_e, x_b)$ and the corresponding stopping time by

$$\begin{aligned} \mathbf{T}^{\bar{x}} &= \mathbf{T}^{x_e} \vee \mathbf{T}^{x_b} \\ &= \inf\{t; \, x_t \leq \bar{x} = \max(x_e, x_b)\}. \end{aligned}$$

Further, denote the value of the depositor claim at the binding threshold by $\mathcal{L}^{D,11}$. Using the general solution, the depositor claim - denoted by D - is found to be of the form

$$D(x_0) = \frac{d}{r} + \left(\mathcal{L}^D - \frac{d}{r}\right) \mathcal{D}_0^{\bar{x}}.$$
(8)

Besides funding the business with deposits, the bank has the possibility to issue bonds. The valuation for the **bondholders claim** is analogous. As long as the bank is solvent, bondholders receive the fixed contracted coupon payment, denoted by b. Again, if the bank is no longer able or willing to service its debt, the bondholders get a - yet undetermined - liquidation value, denoted by \mathcal{L}^B . However, contrary to depositors, bondholders do not have the possibility to influence the timing of default.

In analogy to (8), the bondholders claim, denoted by B is given by

$$B(x_0) = \frac{b}{r} + \left(\mathcal{L}^B - \frac{b}{r}\right) \mathcal{D}_0^{\bar{x}}.$$
(9)

Finally, the **equity holders claim** is determined by its characteristics as residual claim. Before $T^{\bar{x}}$, equity holders receive the after tax residual value of the current EBIT, i.e. $(1 - \tau)(x_t - (d + b))$. As usual, interest payments are tax deductible and represent a tax shield for the firm.

At $T^{\bar{x}}$, equity holders can enter into negotiations with bondholders but only if a bank run has not occurred, i.e. only if x_b is binding. In this case, the value of the equity claim is given by the outcome of the bargaining game. Denote this renegotiation gain for equity holders by \mathcal{L}^E . Similar reasoning as above, yields the valuation for the equity claim, denoted by E

$$E(x_0) = (1-\tau) \left(\left(\mathcal{V}x_0 - \frac{(d+b)}{r} \right) - \left(\mathcal{V}\bar{x} - \frac{(d+b)}{r} \right) \mathcal{D}_0^{\bar{x}} \right) + \mathcal{L}^E \mathcal{D}_0^{\bar{x}}.$$
(10)

The firm value is then the sum of (8), (9) and (10) and will be denoted by V(x) = D(x) + B(x) + E(x).

Before we can make the formulae operational, we need to specify the different thresholds x_a , x_e , x_b and $\bar{x} = \max(x_e, x_b)$, as well as the liquidation/negotiation values \mathcal{L}^D , \mathcal{L}^B and \mathcal{L}^E .

¹¹ We will give a precise characterization of x_e , x_b and \mathcal{L}^D later on.

2.2 Optimal liquidation and bargaining values

For the determination of the liquidation/negotiation values, it is crucial to define the asset value that is available at the time a certain threshold is attained. Since our state variable is a flow variable, there is not a unique way to do this, and different possibilities exist. One may choose to define the liquidation value as the unlevered asset value, i.e. as the value of an all equity claim, or one may want to include its leverage potential. In this case, the new owner of the assets can optimally lever the firm to realize a higher value. However, by levering up the firm a new default threshold has to be determined, which brings up again the problem of determining a liquidation value. Thus, one runs into a recursive problem.¹² Alternatively, to circumvent the recursion problem, one may take the unlevered asset value, i.e. the all equity value and scale this value up by a factor $\rho > 1$.

As in other contributions (see e.g. Mello and Parsons (1992), Morellec (2004) or Fan and Sundaresan (2000)), we choose to use the unlevered all equity value, since the proper determination of the liquidation value is not the core of our model. Thus, in general, the value available to claimants at the threshold \bar{x} is $A(\bar{x})$.

However, due to the assumption that it matters who holds the assets, things get a little more complicated. As in Diamond and Rajan (2000), we assume that the incumbent bank manager-owners have specific abilities that enables them to produce an EBIT flow of x_t . Without this specific know-how, anyone else will only be able to produce a smaller EBIT flow with the same assets, and we assume that this is a scaled down version of x and denote it by

$$\xi_t = \rho \, x_t,$$

where $\rho < 1$ is the scale factor, which measures the extent of the specific abilities. The smaller ρ , the more specific abilities do incumbent manager-owners have. Note, that ρ may also be interpreted as the liquidity of the bank assets.

As a consequence, in the case of liquidation, the available value for investors other than the former bank owners is smaller than for the incumbent bank managerowners. In fact, appendix A shows that that the following relation holds

$$\rho A_x(\bar{x}) = A_\xi(\bar{x}),$$

where A_x is the all equity claim defined on the process x and A_{ξ} is defined on ξ . In addition, as is usual, the liquidation value is reduced by proportional direct bankruptcy costs, that we denote by α .

The liquidation value for depositors, \mathcal{L}^D is then given by

$$\mathcal{L}^{D} = \min\{(1-\alpha)\rho A(\bar{x}), d/r\}.$$

In case the depositors initiate a bank run, they receive $(1-\alpha)\rho A(\bar{x})$ but never more than their riskless nominal value d/r. In fact, as we will see in the next section, the exogenous bank run threshold x_e will always be such, that $(1-\alpha)\rho A(\bar{x}) = d/r$ and

¹² Christensen et al. (2005) address a related problem and solve it numerically. In a different setup, Koziol (2006) finds an analytical solution.

therefore $\mathcal{L}^D = d/r$. This implies, that (8) reduces to $D(x_0) = d/r$, i.e. deposits are independent of the EBIT level and riskless.

We now turn to the determination of \mathcal{L}^B and \mathcal{L}^E , i.e. the threshold values for the bondholders and equity holders, which cannot be called 'liquidation' values in a strict sense. As mentioned above, as long as no bank run has occurred, equity holders can propose negotiations to the bondholders over the surplus that can be generated, when the incumbent manager-owners contribute their specific skills. \mathcal{L}^B and \mathcal{L}^E thus constitute the outcome of this bargaining game, which we will turn to now.

If the former manager-owners contribute their skills, the asset value at \bar{x} is assumed to be the all equity claim $A(\bar{x})$. This value must be greater than d/r, otherwise a bank run would have occurred. Thus, the value, equity and bondholders can bargain about is $A(\bar{x}) - d/r > 0$. The outcome of the bargaining game is an optimal sharing rule, denoted by θ , how to split this value. Thus, \mathcal{L}^B and \mathcal{L}^E are determined by

$$\mathcal{L}^B = (1-\theta)(A(\bar{x}) - d/r) \mathbf{1}_{\{x_b > x_e\}}$$
$$\mathcal{L}^E = \theta(A(\bar{x}) - d/r) \mathbf{1}_{\{x_b > x_e\}}.$$

The Nash solution for the bargaining game, i.e. the optimal sharing rule θ^* depends crucially on the assumed bargaining power, which is formalized by the parameter η , and is found by the following reasoning.¹³ For equity holders the incremental value of accepting the offer is

$$(\theta(A(\bar{x}) - d/r) - 0),$$

while for bondholders it is

$$\left((1-\theta)(A(\bar{x})-d/r)-((1-\alpha)\rho A(\bar{x})-d/r)\right).$$

The optimal sharing rule is therefore characterized as¹⁴

$$\theta^{*} = \arg \max_{\theta} \left\{ \left(\theta(A(\bar{x}) - d/r) - 0 \right)^{\eta} \left((1 - \theta)(A(\bar{x}) - d/r) - ((1 - \alpha)\rho A(\bar{x}) - d/r) \right)^{(1 - \eta)} \right\} \\
= \frac{A(\bar{x}) r \eta (1 - \rho(1 - \alpha))}{A(\bar{x}) r - d}.$$
(11)

As an example, $\eta = 0$ would give full bargaining power to bondholders who could make a take-it-or-leave-it offer to equity holders. In this case, equity holders would get nothing from the negotiation value. In general, the choice of the bargaining power is exogenous, and in general, we will assume that the bondholders have a stronger position if the bank has also a substantial amount of deposits outstanding. Thus, we let η be a function of d, i.e.

$$\eta(d) = 1 - \frac{d}{d+b}.$$
(12)

¹³ See e.g. Fan and Sundaresan (2000), p. 1063.

 $^{^{14}}$ See appendix B for the derivation.

Thus, financing only with bonds (d = 0) will give full bargaining power to equity holders $(\eta = 1)$, while increasing amounts of deposits weaken the position of equity holders. The motivation for this assumption is the following reasoning. If depositors initiate a run, no bargaining is possible. Thus, bond holders bargaining position is strengthened by a large amount of deposits that potentially threaten the negotiation to take place. Bond holders might then threaten with the possibility to spread rumours about the true asset value of the bank, that makes a bank run more likely.¹⁵

We now turn to the optimal determination of the threshold values.

2.3 Optimal determination of the thresholds

We already characterized the threshold x_e as being exogenous¹⁶ or as being the bank run threshold. We consider a world with no information asymmetries, thus, depositors have full knowledge about the true level of EBIT. They will act as to maximize the value of their claim, which implies that they will want to withdraw their funds whenever they realize that the asset value of the bank, or more precisely, the value they can get when initiating a bank run, is just enough to satisfy their claim. It follows, that the optimal threshold x_e^* is implicitly defined by the relationship $(1 - \alpha)\rho A(x_e^*) = \frac{d}{r}$, which solves for

$$x_e^* = A^{-1} \left(\frac{d}{r \left(1 - \alpha \right) \rho} \right),\tag{13}$$

where A^{-1} denotes the inverse function of A^{17} . Note, that due to the convexity of $A(\cdot), x_e^*(d)$ is not linear in d.

The all equity claim, that we use as liquidation value involves the threshold x_a , which we interpret as the abandonment level, i.e. the level where even an unlevered firm will be shut down. This abandonment level is determined by maximizing the all equity claim. The corresponding optimality conditions are given by value-matching and smooth-pasting requirements.¹⁸ In the present case, the all equity claim at x_a is zero as well as its derivative at that point, i.e. $A(x_a) = 0$ and $\frac{\partial A}{\partial x}|_{x=x_a} = 0$, from which one deduces the optimal threshold x_a^* ,

$$x_a^* = \frac{1}{\beta_2} - \frac{\mu}{r}.$$
 (14)

Note, that since $\beta_2 < 0$ and $\mu \ge 0$, this is always negative.

¹⁵ It has to be acknowledged that this reasoning is at the present stage not fully model-consistent, since we assume full knowledge about the true EBIT level. However, a further extension of this basic framework is inteded to incorporate asymmetric information on the part of depositors, which makes this assumption valid.

¹⁶ We call it exogenous in the sense that its determination is not within the power of decision of firm owners, and not in the sense that it is not model-inherent.

¹⁷ Although A(x) involves terms of the form $x e^x$, the inverse function A^{-1} can be given a quasiclosed form solution by using the so-called Lambert W-function.

¹⁸ See e.g. Dixit (1993), Dixit and Pindyck (1994).

A similar reasoning leads to the determination of x_b . With respect to the valuematching and smooth-pasting conditions, we have to take into account the bargaining outcome. Bargaining can only take place if a bank run has not yet occurred, therefore x_b must be binding and we can write (10) as

$$E(x) = (1-\tau) \left(\left(\mathcal{V}x - \frac{(d+b)}{r} \right) - \left(\mathcal{V}x_b - \frac{(d+b)}{r} \right) \mathcal{D}^{x_b} \right) + \mathcal{L}^E \mathcal{D}^{x_b}.$$
(15)

At x_b , $\mathcal{D}^{x_b} = 1$, thus the first term vanishes and equity is worth \mathcal{L}^E . Therefore, the value-matching as well as the smooth-pasting conditions are

$$E(x_b) = \mathcal{L}^E = \theta^* (A(x_b) - d/r)$$
(16)

$$\left. \frac{\partial E}{\partial x} \right|_{x=x_b} = \frac{\partial}{\partial x_b} \mathcal{L}^E.$$
 (17)

From (16) and (17), x_b^* turns out to be

$$x_b^* = \frac{1}{\beta_2} - \frac{\mu}{r} + \frac{d+b}{1 - \eta \left(1 - (1-\alpha)\rho\right)}.$$
(18)

The result shows, that the optimal threshold depends upon the bargaining power as was shown in Fan and Sundaresan (2000). However, due to our assumption, that η is endogenous and depends on d and b (see (12)), this can also be written as $x_b^* = \frac{1}{\beta_2} - \frac{\mu}{r} + \frac{(d+b)^2}{(1-\alpha)\rho \, b+d}$.

Note, that if there were nothing to negotiate about (i.e. $\alpha = 0$, $\rho = 1$), x_b^* would be just $x_a^* + d + b$ and linear in d and b. As long as there is something to negotiate about and equity holders have some bargaining power (i.e. $\eta > 0$), x_b^* is ceteris paribus higher as without negotiation. An earlier default time is c.p. detrimental to firm value, since it implies that the tax shield is exploited to a lesser extent.

The last step to close the model would be to determine the optimal amount of deposits and bonds, represented by d and b, the bank owners want to issue. In principle, the optimal choice of d and b is found by maximizing the firm value with respect to these two dimensions, i.e.

$$(d^*, b^*) = \arg \max_{(d,b\geq 0)} \left\{ E(d, b, x_0, \bar{x}) + D(d) + B(b, x_0, \bar{x}) \right\},\tag{19}$$

where we have indicated only those arguments, that have direct dependencies. Unfortunately, the optimization problem of (19) cannot be given a closed-form solution, and we have to resort to numerical simulation in the following section.

3 Discussion and implications

For numerical solutions, we assume the following base case parameter constellation, unless otherwise mentioned. The current EBIT level, x_0 is assumed to be $x_0 = 1000$. The EBIT process parameters are chosen as $\mu = 5$ and $\sigma = 50$. The riskless interest rate is assumed to be 5%, i.e. r = 0.05. The corporate tax rate is assumed to be 35%, i.e. $\tau = 0.35$. The direct bankruptcy costs are assumed to be 5%, i.e. $\alpha = 0.05$.

3.1 Financing with bonds

As a first step, we analyze the situation when the bank can only issue bonds. This serves as a reference point and as a contrast to the results that will be obtained when the bank can additionally issue demand deposits. In particular, we show how the firm value and the leverage will vary with different choices of the bargaining power parameter, as well as the special-abilities parameter ρ .



Figure 1: Equity, bond and firm value as functions of η (left panel) and ρ (right panel).

Figure 1 graphs the firm value (solid line), the bond value (dashed line) and the equity value (dotted line) as functions of η and ρ .

The left panel shows the impact of the bargaining power, while the scale parameter is fixed at $\rho = 0.7$. A parameter value of $\eta = 0$ means that equity holders have no bargaining power, and the bondholders can make a take-it-or-leave-it offer. At the other extreme, $\eta = 1$ gives full bargaining power to equity holders. Full bargaining power does, however, not mean that the full negotiation value goes to equity holders. The optimal share for equity holders that results out of the Nash solution is $\theta^* =$ $\eta(1 - (1 - \alpha)\rho)$, i.e. $\theta^* = 0.24$ with the assumed parameter values.

As one might expect, the left panel of figure 1 shows that the equity value increases with η , while the bond value decreases, which is due to the stronger bargaining position of equity holders. However, the increase in equity value is smaller than the decrease of the bond value as can be seen from the declining graph of the firm value.

The right panel shows the influence of the special-abilities parameter ρ for a fixed bargaining power of $\eta = 0.5$, i.e. an equally balanced bargaining game. The higher ρ , the smaller is the surplus that bank manager-owner can generate through their special abilities. It can be seen that a smaller ρ is associated with higher equity but lower bond values. In sum, the firm value is increasing with ρ .

Figure 2 shows the leverage as function of η and ρ . The current leverage L is defined as $L(x_0) = \frac{B(x_0)}{B(x_0) + E(x_0)}$. Note, that it is a market-value based definition. The left panel shows L as a function of η for three different choices for ρ . A significant the left panel shows L as a function of η for three different choices for ρ .

nificant decline can be observed, whereby the slope is more pronounced for low values of ρ . The explanation follows directly from figure 1, but the magnitude of the leverage change is remarkable.

Analogously, we observe a significant increase in the leverage in the right panel,



Figure 2: Leverage (L) as function of η (left panel) and ρ (right panel).

where the impact of ρ is shown for three levels of the bargaining power parameter. Again, the explanation follows directly from figure 1, but in this case the magnitude of the leverage change is even more pronounced.

Table 1 summarizes the main numerical results for the base case scenario.

| | b^* | x_b^* | $E(x_0)$ | $B(x_0)$ | $V(x_0)$ | $L(x_0)$ |
|--------------|--------|---------|----------|----------|----------|----------|
| $\eta = 1$ | 728.01 | 742.07 | 4959.4 | 13886.9 | 18846.3 | 0.736 |
| $\eta = 0.5$ | 842.95 | 742.07 | 3484.6 | 16079.6 | 19564.2 | 0.822 |
| $\eta = 0$ | 957.9 | 742.07 | 2009.7 | 18272.3 | 20282.0 | 0.901 |

Table 1: Base case numerical results ($\rho = 0.8$).

Both figures convey more or less the same message. The more severe is the agency problem, either because of a stronger bargaining power of equity holders (high values of η), or due to a higher negotiation stake (low value of ρ), the smaller is the debt capacity in terms of the amount of bonds a bank can issue. A smaller debt capacity implies that only a smaller tax shield can be realized, which in turn diminishes overall firm value.

In other words, for a given negotiation stake, the firm value could be increased if the bank manager-owners were able to credibly weaken their bargaining power. We will show in the next section, that financing with demand deposits will actually provide such a commitment device and is able to increase firm value.

3.2 Optimal debt mix

We start by showing that financing the bank only with deposits is clearly an inferior solution, since deposits represent a hard claim in the sense that manager-owners give up their option to time the default. Therefore, the debt capacity is substantially smaller, as can be seen from figure 3. At the base case parameter value $\rho = 0.8$, the firm value when only deposits are outstanding is 11.5% smaller than when only bonds are used. Note, that in the latter case, we assume that equity holders have full bargaining power, which is a lower bound for the firm value in that case. Now that we have shown that a corner solution (only deposits) is clearly not optimal, we go on to characterize the optimal mix of bonds and deposits.



Figure 3: Dept capacity (left panel) and firm value (right panel) in the case that the bank is only financed by deposits (solid line) or bonds (dashed line) as a function of ρ .

In order to do this, consider the firm value for an increasing amount of deposits, i.e. for increasing d, while for every given d, the amount of bonds is optimized, i.e. b^* . This is shown in the upper and lower left panel of figure 4. Vice versa, the upper and lower right panel graph the firm value for increasing b and the corresponding optimal d^* . The solid lines represent the base case, with $\rho = 0.8$, but the



Figure 4: Firm value and optimal debt level.

graphs for $\rho = 0.7$ (dashed line) and $\rho = 0.9$ (dotted line) are similar. Whether we let d or b increase, we observe in both cases a maximum firm value in the two upper panels. The lower panels graph the corresponding optimal amount of bonds for a given amount of deposits (lower left), and the optimal amount of deposits for a given amount of bonds (lower right). The jumps that can be observed are due to the discontinuities that occur when the binding threshold switches from the endogenous to the exogenous (bank run) threshold.

An even more convincing illustration of the optimum can be given in a threedimensional graphic, where the firm value is plotted as function of d and b. This is shown in figure 5. In the foreground, the amount of deposits d is increased from



Figure 5: Firm value as function of d and b.

left to right, while the amount of bonds increases on the axis leading backwards. The figure clearly shows that the function has a maximum. The numerical values are found to be $d^* = 416.78$ and $b^* = 426.27$, which lead to an optimal firm value of $V^* = 19556.1$. We summarize the results and compare them to the case of only bond-financing (d = 0) in table 2. The possibility to include demand deposits

| | b^* | \bar{x}^* | $E(x_0)$ | $B(x_0) + D(x_0)$ | $V(x_0)$ | $L(x_0)$ |
|-----------------|--------|-------------|----------|-------------------|----------|----------|
| d = 0 | 728.01 | 742.07 | 4959.4 | 13886.9 | 18846.3 | 0.736 |
| $d^{*} = 416.8$ | 426.3 | 743.65 | 3485.0 | 16071.0 | 19556.1 | 0.822 |

Table 2: Base case numerical results ($\rho = 0.8$).

has the potential to increase the firm value by 3.8% in our base case scenario. The value of outstanding debt rises from 13886.9 to 16071, which is an increase of 15.7%, while the equity value decreases by 29.7%. Obviously, this increases the leverage from 73.6% to 82.2%.

We will not present further numerical results, but it is straightforward to see that the lower the specific-abilities parameter ρ , the more severe is the agency problem, and the higher is the potential to increase the firm value by using deposits as commitment device. Thus, the firm value increase (i.e. $V(d^*) - V(d = 0)$) is a monotonically decreasing function of ρ , while the leverage is monotonically increasing with ρ .

Given the same assets, financing with deposits enables the bank to use more debt, and the firm value improvement is then due to a better exploitation of the tax shield. On the other hand, the higher debt proportion means that the bank can pledge a larger share of its assets to outside investors, than in the case when only bonds can be used. In other words, with deposits, the bank can increase its pledgable assets, and it is in this sense that Diamond and Rajan (2000) speak about the liquidity creation function of banks. As the results in table 2 show, we can confirm their results within our model. As figure 5 shows, the maximum occurs at the intersection of two surfaces. This is further demonstrated by a corresponding contour plot shown in figure 6. Both



Figure 6: Contour plots of the firm value.

panels show the firm value depending on d and b on a different scale. The left panel plots $350 \le d, b \le 450$, while the right panel plots $200 \le b \le 550$ and $200 \le d \le 500$. The contour lines show the discontinuity that occurs when the binding threshold switches from x_b (upper left part) to x_e (lower right part). According to this change, the firm value function is characterized by two intersecting surfaces. Thereby, the firm value maximum lies on the surface, where x_b is binding.¹⁹ The reason is that in the case of a bank run, the surplus by the specific abilities of bank managerowners is lost. However, the maximum occurs right at the edge of the x_b -surface, thus confirming in some sense the result of Diamond and Rajan (2000) that fragility of banks are a reason of optimization. The reason in our model is, that the bank wants to avoid a bank run, but uses a maximum of deposits to credibly weaken their bargaining position.

To conclude the discussion, we will focus on the optimal debt mix between deposits and bonds. At our numerical base case scenario (see table 2), we find a deposit ratio (defined as $\mathcal{R} = D/(D+B)$) of 51.8%, thus roughly half of the debt consists of deposits. Given their role as commitment device, it is interesting to see how this ratio is influenced by the extent of the specific abilities, which we also interpreted as the liquidity of the underlying asset, i.e. how \mathcal{R} varies with ρ . Figure 7 plots this relation, together with the influence of the riskiness of the assets, measured by σ . In the upper left panel, the deposit ratio is plotted as a function of ρ for different risk levels. We observe that \mathcal{R} is increasing for high values of ρ . However, \mathcal{R} is not monotonically increasing, and we observe a decrease for very low levels of ρ . The explanation for the graph of the deposit ratio follows from the lower left panel, where the corresponding graphs for the deposit and the bond claim (at $\sigma = 50$) are plotted. The deposit claim is nearly linearly increasing in ρ , which may be counterintuitive at first sight, since one might expect that the

¹⁹ Therefore, we call it the x_b -surface.



Figure 7: Optimal debt mix. The deposit ratio \mathcal{R} depending on ρ and σ .

higher the agency conflict in terms of a lower ρ , the more valuable are deposits as commitment device to mitigate this agency problem. While this is true, the result follows from the more fundamental constraint given by the exogenous bank run threshold x_e , which - according to (13) - is a (decreasing) function of ρ . The lower ρ , the higher c.p. x_e , and thus the less deposits can be issued in the optimum. On the other hand, the bond claim graph is concave in ρ . The reason that the optimal bond value even decreases for high ρ is essentially due to a similar concave relation for the bond liquidation value \mathcal{L}^B with respect to ρ , which is due to two offsetting effects. For low values of ρ , \mathcal{L}^B increases, because a higher proportion of deposits strengthens the bargaining power of bond holders, which gives them a higher share in the bargaining outcome. However, as ρ increases, the total surplus of the bargaining game $((1-(1-\alpha)\rho) A(\bar{x})))$ gets smaller, which then diminishes $\mathcal{L}^{B}.^{20}$

Analogously, the upper and lower right panels in figure 7 show the deposit ratio, deposit and bond values depending on the riskiness of assets. A clear negative relationship for the deposit ratio can be observed. I.e. the model predicts that banks with more risky assets will use a smaller proportion of deposits in their optimal debt mix. However, with respect to deposit and bond values (at $\rho = 0.8$), the dependencies are not that clear-cut. While the value of deposits are monotonically decreasing in σ (again, this follows from (13)²¹), the bond claim is convex in σ . For low risks, the optimal bond value is decreasing, while for high levels of σ it

²⁰ The higher ρ value and the higher amount of deposits *d* also work towards lowering the binding threshold \bar{x} , but the change is small, since *d* increases the numerator in (13), while at the same time also the denominator is increased by ρ .

²¹ To see this, note that c.p. $A(x; \sigma_1) < A(x; \sigma_2)$ for $\sigma_1 < \sigma_2$. Thus the inverse function decreases, i.e. shifts downwards.

increases. Again, the reason for this non-monotonic relation is due to two offsetting effects. On the one hand, a higher σ decreases the binding threshold (due to a lower amount of deposits) which decreases the value of the probability-weighted discount factor $\mathcal{D}^{\bar{x}}$. This in turn increases the bond value. On the other hand, a higher risk decreases the liquidation value \mathcal{L}^B , because due to the higher σ less deposits are outstanding which weakens the bargaining power of bond holders. Thus, they obtain a smaller share in the bargaining game. A smaller liquidation value obviously diminishes c.p. the bond value.

The relationships just discussed may serve as testable implications of our model. Although the usual caveat applies, that the proposed framework is a stylized model, that leaves out a number of issues one may consider to be important in practice, our results predict that banks where manager-owners have a high degree of specific abilities and/or risky assets will tend to use less deposits in their optimal debt mix.

4 Conclusion

Recent work in the banking theory literature stresses the role of uninsured deposits as commitment device to discipline bank manager-owners not to extract rents from their specific abilities in running the business. By using this credible commitment, the bank can issue more debt, and in turn increase the firm value. The aim of this article was to show if and how this finding can be incorporated in a continuous-time contingent claims valuation framework. It is important to stress, that in Diamond and Rajan (2000) depositors can negotiate with the entrepreneur after a run has occurred. Since we consider this to be implausible, we assume that depositors always suffer a collective action problem, which means that no negotiations whatsoever are possible with this group of claimants. However, negotiations are possible with bond holders, whereby the outcome of the bargaining game depends on the bargaining power, which we let depend on the proportion of deposits a bank has outstanding. The motivation for this is, that the bargaining position of bond holders vis-à-vis equity holders is stronger when there is another group of debt holders.

Within this setup, we can show that a bank can improve its firm value by using demand deposits as part of its debt mix, compared to financing only with bonds. The reason for this firm value increase is the fact, that deposits strengthen the bargaining position of bond holders in the case of renegotiation. Thus equity holders are willing to voluntarily weaken their bargaining position, because this enables them to issue more debt and to exploit the associated additional tax shield. In the sense, that the bank can increase its pledgable assets by using deposits, the bank creates liquidity as this has been stressed by Diamond and Rajan (2000).

The continuous-time setup enables us to derive quantitative results that yield testable implications. Our model predicts that banks where manager-owners have more specific abilities will use less deposits, than banks with less specific abilities. Analogously, banks with riskier asset will use more bonds in their debt mix.

A Appendix 1

It is assumed, that the incumbent manager-owner have specific abilities in running the bank business and given that they contribute their know-how, the assets in place produce a cash-flow of x_t . Anybody else holding these assets can only extract a fraction $\xi_t = \rho x_t$ of these cash-flows. We will show in this appendix, that the following relation holds

$$A_{\xi}(\xi) = \rho A_x(x),$$

where A_x , A_ξ denotes an all equity claim defined upon the process x_t and ξ_t respectively. It is shown in the text, that the all equity claim on the process $dx_t = \mu dt + \sigma dW_t$ has the solution

$$A_x(x) = (1 - \tau) \left(\left(\frac{x}{r} + \frac{\mu}{r^2}\right) - \left(\frac{x_a}{r} + \frac{\mu}{r^2}\right) e^{\beta_2(x - x_a)} \right).$$

To find the solution for an all equity claim defined upon the process ξ , first note, that the dynamics of ξ are given by

$$\mathrm{d}\xi_t = \hat{\mu}\,\mathrm{d}t + \hat{\sigma}\,\mathrm{d}W_t,$$

where $\hat{\mu} = \rho \mu$ and $\hat{\sigma} = \rho \sigma$. Therefore, we can apply the general solution by making appropriate replacements. This yields

$$A_{\xi}(\xi) = (1 - \tau) \left(\left(\frac{\xi}{r} + \frac{\hat{\mu}}{r^2}\right) - \left(\frac{\xi_a}{r} + \frac{\hat{\mu}}{r^2}\right) e^{\hat{\beta}_2(\xi - \xi_a)} \right).$$

where

$$\hat{\beta}_2 = \frac{-\hat{\mu} - \sqrt{\hat{\mu}^2 + 2\hat{\sigma}^2 r}}{\hat{\sigma}^2}, \qquad \xi_a = \frac{1}{\hat{\beta}_2} - \frac{\hat{\mu}}{r}.$$

However, $\hat{\beta}_2$ and ξ_a are simply

$$\hat{\beta}_2 = \frac{1}{\rho} \beta_2, \qquad \xi_a = \rho \, x_a.$$

Therefore, $e^{\hat{\beta}_2(\xi-\xi_a)} = e^{\beta_2(x-x_a)}$ and thus

$$A_{\xi}(\xi) = \rho A_x(x)$$

as asserted.

B Appendix 2

To solve the maximization problem in (11), rewrite it as

$$\theta^* = \arg \max_{\theta} \left\{ U(\theta)^{\eta} V(\theta)^{(1-\eta)} \right\}$$

where

$$\begin{split} U(\theta) &= \left(\theta(A(\bar{x}) - d/r) - 0\right) \\ V(\theta) &= \left((1 - \theta)(A(\bar{x}) - d/r) - ((1 - \alpha)\rho A(\bar{x}) - d/r)\right). \end{split}$$

Taking derivatives of U and V with respect to θ yields

$$U_{\theta} = \left(A(\bar{x}) - \frac{d}{r}\right) \eta \left(\left(A(\bar{x}) - \frac{d}{r}\right) \theta\right)^{-1+\eta}$$
$$V_{\theta} = \frac{\left(-A(\bar{x}) + \frac{d}{r}\right) (1-\eta)}{\left((1-\theta) \left(A(\bar{x}) - \frac{d}{r}\right) - (1-\alpha) \rho A(\bar{x}) + \frac{d}{r}\right)^{\eta}}$$

Therefore, the first order condition is

$$U_{\theta} V + U V_{\theta} = 0.$$

Solve for θ to obtain

$$\theta^* = \frac{A(\bar{x}) r \eta \left(1 - \rho(1 - \alpha)\right)}{A(\bar{x}) r - d}.$$

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