Departures From Rational Expectations and Asset Pricing Anomalies

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Abstract

We investigate the potential of the consumption CAPM with pessimism, doubt, and the availability heuristic in the agent's beliefs to resolve the equity premium and risk-free rate puzzles. Using the nonlinear GMM estimation techniques, we find that doubt and the availability heuristic play an important role in explaining the cross-section of asset returns. However, when taken alone, these deviations from rational expectations can not resolve the equity premium and risk-free rate puzzles. This result is robust to the assumption that the expected value of an uncertain prospect is nonlinear in the subjective outcome probabilities.

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1 Introduction

Over the last two decades numerous studies have focused on a consumption-based capital asset pricing model (consumption CAPM). The standard Lucas (1978) consumption CAPM relates asset prices to the consumption and savings decisions of a single power utility maximizing representative investor that can freely trade in perfect capital markets without incurring transaction costs, limitations on borrowing or short sales, and taxes. Empirical evidence is that this model is inconsistent with data on consumption and asset returns in many respects. For example, a reasonably parameterized standard consumption CAPM generates a mean equity premium, which is substantially lower than that observed in data. The model can explain the mean excess return on the market portfolio over the risk-free rate only if the representative agent is assumed to be implausibly averse to risk. This is the equity premium puzzle discussed by Mehra and Prescott (1985) and Hansen and Jagannathan (1991) among others. Another anomaly with the standard consumption CAPM is that a time preference discount factor greater than one is required to fit the mean return on the risk-free asset. This is the risk-free rate puzzle as described in Weil (1989).

Since Mehra and Prescott's (1985) original investigation, several ways have been explored to improve the empirical performance of the standard consumption CAPM. One straightforward response to the difficulties with this model is to try different functional forms for utility (e.g., Aschauer 1985; Eichenbaum *et al.* 1988; Epstein and Zin 1989, 1991; Startz 1989; Sundaresan 1989; Abel 1990, 1999; Constantinides 1990; Harvey 1991; McCurdy and Morgan 1991; Chou *et al.* 1992; Bakshi and Chen 1996; Campbell and Cochrane 1999; Gordon and St-Amour 2004). Another suggestion is that market frictions, such as transactions costs and limits on borrowing or short sales, can make aggregate consumption in the economy an inadequate proxy for the consumption of stock market investors (e.g., Campbell and Mankiw 1990; Mankiw and Zeldes 1991; Basak and Cuoco 1998; Alvarez and Jermann 2000; Constantinides *et al.* 2002). Bewley (1982), Mehra and Prescott (1985), Mankiw (1986), Constantinides and Duffie (1996), Brav *et al.* (2002), and Kocherlakota and Pistaferri (2005) argue that consumers' heterogeneity induced by market incompleteness can be relevant for asset pricing. Another strand of the literature suggests that departures from rational expectations can help explain the equity premium and risk-free rate puzzles (e.g., Rietz 1988; Cecchetti *et al.* 2000; Abel 2002). Although substantial progress has been made, there is no yet a model that would be generally accepted.

Rietz (1988), Cecchetti *et al.* (2000), and Abel (2002) take an approach based on an assumption that the representative agent is not fully rational and argue that deviations from rational expectations enhance the empirical performance of the standard consumption CAPM. Rietz (1988) specifies the model of Mehra and Prescott (1985) to capture the effects of possible, though unlikely, crashes and shows that the addition of a crash state allows to explain both a high equity risk premium and a low risk-free rate with reasonable degrees of risk aversion.[1]

As in Mehra and Prescott's (1985) investigation, Cecchetti *et al.* (2000) assume that consumption growth follows a two-state Markov process and show that using simple rules of thumb to estimate the transition probabilities agents will form the subjective probabilities that are, on average, relatively pessimistic about the persistence of the expansion state and relatively optimistic about the persistence of the contraction state. These belief distortions allow to match the first moments of the equity premium and the risk-free rate with a positive relative risk-aversion coefficient less than 10 and a discount factor below one. Furthermore, their analysis shows that persistence in the subjective transition probabilities together with random fluctuations in beliefs about the persistence of low-growth states are sufficient to explain the volatility of asset returns, as well as persistence and predictability of excess returns exhibited in the data. However, the correspondence between the predicted cyclical pattern of the equity premium and that found in the data is rough.

Departing from the assumption of rational expectations, Abel (2002) explores the effect of pessimism and doubt in the agent's beliefs on the mean equity premium and risk-free rate. His finding is that pessimism and doubt may play an important role in explaining the mean equity premium and risk-free rate of return. Assuming lognormality of the objective and subjective distributions of the growth rate of aggregate consumption, Abel (2002) finds that the consumption CAPM with pessimism and doubt can match the historical average equity premium and risk-free rate with plausible values of the agent's coefficient of relative risk aversion and time preference discount factor. However, in some cases this model needs implausibly high levels of pessimism and doubt to explain the observed mean equity premium and risk-free rate.

In this paper, we explore the potential of pessimism, doubt, and the availability heuristic in the agent's beliefs to help resolve the equity premium and risk-free rate puzzles. To test the ability of the consumption CAPM with deviations from rational expectations to explain

[1] Mehra and Prescott (1985) assume that equity returns vary little between good and poor times. Rietz (1988) incorporates in Mehra-Prescott's (1985) model a low-probability, depression-like third state, which captures the effects of crashes, when equity returns are far below average.

the observed equity premium and risk-free rate, we use the nonlinear Generalized Method of Moments (GMM) estimation approach. This allows us to focus on the time-series properties of asset returns rather than on the historical averages as in Abel (2002). We start by assuming that the aversion to risk is determined solely by the curvature of the agent's utility function. Under this assumption, when calculating the expected value of an uncertain prospect, an agent takes a sum of outcomes weighted by their subjective probabilities of occurring. Then, in accordance with the cumulative prospect theory of Tversky and Kahneman (1992), we suppose that the agent's risk aversion is determined jointly by the utility function and the decision weights that are strictly increasing monotonic nonlinear functions of subjective probabilities. In this case, the expected value of an uncertain prospect is nonlinear in the subjective probabilities.

The rest of the paper is structured as follows. In Section 2, we analyze the effect of pessimism, doubt, and the availability heuristic on asset returns. In Section 3, we use the GMM estimation technique to explore the potential of the consumption CAPM with pessimism, doubt, and the availability heuristic in the agent's beliefs to explain the observed equity premium and risk-free rate. Section 4 concludes.

2 The Consumption CAPM with Distorted Beliefs

2.1 Asset Pricing in the Economy with Rational Expectations

One of the first-order conditions for the representative investor's intertemporal consumption and savings choice problem is

$$E_t\left[\delta \frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)} R_{i,t+1}\right] = 1, \tag{1}$$

where δ is the time preference discount factor, C_t is aggregate consumption per capita in period t, $u\left(C_t\right)$ is the representative-agent period utility function, and $R_{i,t+1}$ is the simple gross return on asset i between t and $t+1$. The expectation in (1) is taken across states at time $t+1$ given the information known to an investor at time t. Equation (1) is known as the consumption-based capital asset pricing model, or the consumption CAPM.

Assume that the representative agent maximizes a time-separable power utility function, so that $u\left(C_t\right) = (C_t^{1-\gamma} - 1)/(1-\gamma)$, where γ is the relative risk aversion coefficient. With this preference specification, marginal utility $u'\left(C_t\right) = C_t^{-\gamma}$ and therefore from (1)

$$E_t\left[\delta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{i,t+1}\right] = 1, \tag{2}$$

where $\delta \left(C_{t+1}/C_t \right)^{-\gamma}$ is the representative agent's intertemporal marginal rate of substitution.

By taking unconditional expectations of the left- and right-hand sides of (2), we get an unconditional version:

$$E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{i,t+1}\right] = 1. \tag{3}$$

For the excess return on asset i over the risk-free rate $R_{f,t+1}$, equation (3) becomes

$$E\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} (R_{i,t+1} - R_{f,t+1})\right] = 0. \tag{4}$$

For the risk-free rate, we get

$$E\left[\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R_{f,t+1}\right] = 1. \tag{5}$$

It is common in empirical research to use historical data when testing pricing conditions (4) and (5) and hence to implicitly assume that the next period joint distribution of the growth rate of average consumption and asset returns may be well approximated by their joint historical distribution. Suppose that at time t an investor can observe the most recent S realizations of the growth rate of average consumption and asset returns. This provides us with S different scenarios for what might happen at time $t+1$. According to the expectation principle, when calculating the expected value of an uncertain prospect, the agent takes a sum of outcomes weighted by their probabilities of occurring. Define $p_{s,t+1}$ as the probability that scenario s $(s = 1, ..., S)$ occurs at time $t+1$.

The assumption of rationality implies that the agent weights outcomes by their objective probabilities of occurring. When historical data are used, the objective probability equals $1/S$ and hence under rational expectations $p_{s,t+1} = 1/S$ $(s = 1, ..., S)$. Empirical evidence is that the consumption CAPM with rational expectations can jointly explain the historical excess return on the market portfolio and risk-free rate only if the representative agent is implausibly averse to risk (this is the equity premium puzzle discussed by Mehra and Prescott (1985) and Hansen and Jagannathan (1991) among others) and the time preference discount factor is greater than one (this is the risk-free rate puzzle as described in Weil (1989)).

2.2 Departures from Rationality and Asset Returns

Based on the experimental evidence that human choices are not always rational, a number of recent papers explore the role of departures from rational expectations (in an otherwise standard neoclassical framework) in explaining the equity premium and risk-free rate puzzles. One way to take non-rationality into account is to suppose that the representative agent weights possible outcomes in pricing conditions (4) and (5) by the subjective probabilities, which may differ from the objective probabilities, rather than by the objective probabilities as it is usually assumed in empirical research.

In this section, we first consider three particular departures from rationality (pessimism, doubt, and the availability heuristic) and show how these departures can be taken into account when forming the subjective probabilities that consumers assign to different outcomes. When consumers are not rational, under the assumption that the aversion to risk is determined solely by the curvature of the agent's utility function the expected value of an uncertain prospect is linear in the subjective outcome probabilities. Then, we focus on the cumulative prospect theory of Tversky and Kahneman (1992), according to which the agent's risk aversion is determined jointly by the utility function and the decision weights. Within this framework, the agent calculates the expected value of an uncertain prospect as a sum of outcomes weighted by the decision weights that are strictly increasing monotonic nonlinear functions of subjective probabilities.

We start by defining pessimism, doubt, and the availability heuristic and exploring the effects of these particular departures from rationality on asset returns.

Pessimism. An agent that is pessimistic about the distribution of the aggregate per capita consumption growth rate weights heavier lower growth rates and assigns smaller probabilities to high consumption growth rates relative to the case of rational expectations. Pessimism leads the agent to reduce his current period consumption and to save more in order to self-insure against a negative shock to his next period consumption and therefore the demand for both the safe asset and the risky asset increases. This increased demand requires a decrease in the equilibrium rate on both assets below the rate that would prevail under rational expectations. Thus, pessimism in beliefs can help resolve the risk-free rate puzzle.

To understand the effect of this additional demand for assets on the predicted equity premium, we must determine to which asset, bond or equity, this increased savings is predominantly allocated. Intuitively, if consumers are pessimistic about the growth rate of their consumption, and therefore about the growth rate of dividends, then they expect equity to be less attractive than they would expect under rational expectations.[2] This suggests that the demand for equity

[2] In the equilibrium of the Lucas (1978) economy, the average consumption growth rate is identical to the growth rate of dividends per capita.

increases less than that for the risk-free asset what results in an increasing equity premium. Because pessimism in the agent's beliefs increases the expected equity premium, it can help resolve the equity premium puzzle.

Abel (2002) uses the term pessimism to mean that the subjective distribution of average consumption growth rates is first-order stochastically dominated by the objective distribution. The risk-free rate from period t to period $t+1$ is know at time t and therefore from (5) $R_{f,t+1} = \{E[\delta\,(C_{t+1}/C_t)^{-\gamma}]\}^{-1}$. When the subjective distribution of average consumption growth rates is first-order stochastically dominated by the objective distribution, the expectation of the intertemporal marginal rate of substitution under pessimism is greater than the expectation of the intertemporal marginal rate of substitution under rational expectations at any arbitrary values of δ and $\gamma > 0$. It follows that pessimism reduces the equilibrium risk-free rate of return and therefore the consumption CAPM with pessimistic consumers has the potential to resolve the risk-free rate puzzle.[3] To examine the extent, to which pessimism can help explain the excess return on a risky asset over the risk-free rate of return, Abel (2002) defines uniform pessimism as a leftward translation of the objective distribution of the logarithm of the average consumption growth rate. He shows that uniform pessimism increases the expectation of the equity premium and hence can help resolve the equity premium puzzle.

Doubt. People exhibit doubt when they think that their knowledge is less accurate than it is in fact. Doubt implies that consumers assign to events the subjective probabilities, which are lower than the objective probabilities when they believe that the event probably will occur and higher than the true frequency when they believe that the event will not occur.

To explore whether doubt can help resolve the equity premium and risk-free rate puzzles, Abel (2002) defines the subjective distribution of growth rates of aggregate consumption per capita to be characterized by doubt if it is a mean preserving spread of the objective distribution or, equivalently, is second order stochastically dominated by the objective distribution of average consumption growth rates.[4] Under this assumption, the both distribution functions are defined on the same state space and have identical means. However, the subjective distribution function has a larger variance and therefore is riskier. This additional risk makes consumers more willing to acquire the risk-free asset what increases the demand for the risk-free asset and therefore puts downward pressure on the risk-free interest rate. Thus, doubt, as defined in Abel (2002), has

[3] See Abel (2002).
[4] See Rothschild and Stiglitz (1970).

the potential to explain the risk-free rate puzzle. Since doubt increases consumers' perceived risk associated with equity, it decreases the demand for equity and consequently drives up the equity return. The increase in the equity return accompanied by the decrease in the risk-free rate leads to the expectation of the equity premium, which is greater than the expectation of the equity premium that would prevail under the assumption of rationality. This shows that doubt can help resolve the equity premium puzzle.

To investigate whether the consumption CAPM with doubt can better match the risk-free return, as in Abel (2002), consider equation $R_{f,t+1} = \{E[\delta\,(C_{t+1}/C_t)^{-\gamma}]\}^{-1}$. Because the intertemporal marginal rate of substitution is a decreasing and convex function of the average consumption growth rate, the expected value of the intertemporal marginal rate of substitution is larger and therefore the equilibrium risk-free rate is lower under doubt than under the assumption of rational expectations. Abel (2002) also shows that doubt increases the required equity premium and that this increase in the equity premium is proportional to the agent's coefficient of relative risk aversion and the difference in the squared coefficient of variation of the average consumption growth rate between the subjective and objective probability distributions.

The Availability Heuristic. The agent's judgement about the probability of an outcome often depends upon how easy that outcome is to imagine. Since the more recent experiences or reports are easier to imagine, people consider the more recent events as being more likely and hence assign to them higher probabilities of occurring in the future. For example, after a news feature about a shark attack, many people will be more nervous about swimming in waist-deep ocean waters. The phenomenon when the more recent events have a more significant effect on the agent's decisions is called the availability heuristic and was first reported by Tversky and Kahneman (1973).[5]

Under the assumption that the aversion to risk is determined solely by the curvature of the agent's utility function, the agent calculates the expected utility as a sum of outcomes weighted by the subjective probabilities of occurring. Tversky and Kahneman (1992) however argue that the agent's risk aversion is rather determined jointly by the utility function and the decision weights that the agent assigns to different subjective probabilities. This follows that the agent calculates the expected value of an uncertain prospect as a sum of outcomes weighted by some nonlinear functions of subjective probabilities rather than by the subjective probabilities themselves.

[5] See also Tversky and Kahneman (1974) and Carroll (1978), e.g.

Cumulative Prospect Theory. Experimental evidence is that under uncertainty decision makers systematically overweight low probabilities and underweight high probabilities of occurring. It is observed that the impact of a given change in the probability of winning a given prize diminishes with the distance from the endpoints of the certainty scale. Allais (1953), e.g., shows that the difference between probabilities of 0.99 and 1.00 has more impact on preferences than the difference between 0.10 and 0.11.[6]

To account for the observed nonlinearity in outcome probabilities, Tversky and Kahneman (1992) propose to weight each outcome by a decision weight rather than by its probability. The cumulative prospect theory of Tversky and Kahneman (1992) asserts that decision weights π_s are strictly increasing monotonic nonlinear functions of outcome probabilities, $\pi_{s,t+1} = \pi\left(p_{s,t+1}\right)$, from the unit interval into itself satisfying $\pi(0) = 0$ and $\pi(1) = 1$. Diminishing sensitivity implies that the weighting function must be concave near zero and convex near one.

Tversky and Kahneman (1992) define decision weights $\pi\left(p_{s,t+1}\right)$ as

$$\pi\left(p_{s,t+1}\right) = \frac{p_{s,t+1}^{\lambda}}{\left(p_{s,t+1}^{\lambda} + \left(1 - p_{s,t+1}\right)^{\lambda}\right)^{1/\lambda}} \tag{6}$$

implying that, in accord with the principle of diminishing sensitivity, the weighting function $\pi\left(p_{s,t+1}\right)$ is steepest near the endpoints of the certainty scale and shallower in the middle of the range. If $\lambda = 1$, then $\pi\left(p_{s,t+1}\right)$ is simply the probability of scenario s, $\pi\left(p_{s,t+1}\right) = p_{s,t+1}$.

In contrast to the expected utility theory, in which risk aversion and risk seeking are determined solely by the utility function, in the cumulative prospect theory of Tversky and Kahneman (1992) they are determined jointly by the utility function and the weighting functions. Within the cumulative prospect theory framework, a risk averse agent tends to underweights the outcome probabilities, while a risk seeking agent overweights the probabilities. If an agent is risk neutral, then the decision weight that he assigns to an outcome equals its probability of occurring. Risk aversion for gains is further enhanced by the curvature of the agent's utility function. Kachelmeier and Shehata (1992) find risk seeking for gains of low probability and risk aversion for gains of moderate and high probability of winning. The weighting function (6) exhibits the characteristic pattern of risk aversion for moderate and high probabilities and risk seeking for small probabilities when λ is close to but less than 1.

[6] See also Camerer and Ho (1991).

3 Empirical Investigation

In this section, we test empirically whether pessimism, doubt, and the availability heuristic, either separately or in combination, can help resolve the equity premium and risk-free rate puzzles. We start by assuming that risk aversion is determined solely by the curvature of the agent's utility function and investigate if the consumption CAPM with pessimism, doubt, or the availability heuristic can explain the observed equity premium and risk-free rate with plausible values of the agent's risk aversion and time preference discount factor. Then, in accordance with the cumulative prospect theory of Tversky and Kahneman (1992), we suppose that the agent's risk aversion is determined jointly by the utility function and the decision weights that are strictly increasing monotonic nonlinear functions of subjective probabilities obtained under the assumption that there is pessimism, doubt, or the availability heuristic. We use the nonlinear GMM estimation approach to test the empirical performance of the proposed models in explaining asset returns. The use of the GMM estimation techniques allows us to focus on the time-series properties of consumption and asset returns rather than on the historical averages of asset returns as in Abel (2002).

3.1 Data

To construct a time series of the aggregate per capita consumption growth rate, we use data on real, monthly personal consumption of nondurables and services from the U.S. Bureau of Economic Analysis (BEA). Monthly per capita consumption is obtained by dividing the real aggregate consumption by the total population (including armed forces overseas) from BEA.

As nominal, monthly gross returns on individual assets, we take the nominal, monthly returns (capital gain plus dividends) on the market capitalization-based decile portfolios of all stocks listed on the NYSE, AMEX, and NASDAQ obtained from the Center for Research in Security Prices (CRSP) of the University of Chicago. The nominal, monthly risk-free rate is the one-month Treasury bill return from CRSP. The real, monthly returns are calculated as the nominal returns divided by the one-month inflation rate based on the deflator defined for nondurables and services consumption.

3.2 GMM Estimation Results

The sampling period is from 1959:01 to 2003:12. The unconditional equations for the equity premia on the market capitalization-based decile portfolios (4) and the risk-free rate (5) are estimated jointly using the iterated GMM approach. Table I reports the estimation results.

The Standard Consumption CAPM. This is our benchmark model. In this model, different scenarios have equal probabilities of occurring. We find that the model is rejected statistically at the 5% significance level. The obtained estimate of the relative risk aversion coefficient, $\widehat{\gamma} = 83.74$, is high and significantly positive at the 5% level of significance. Mehra and Prescott (1985) and Hansen and Jagannathan (1991) call this the equity premium puzzle.[7] The estimate of the time preference discount factor, $\widehat{\delta} = 1.11$, is significantly greater than one at the 5% significance level. This result illustrates the risk-free rate puzzle emphasized by Weil (1989).

The Consumption CAPM with Departures from Rational Expectations.

To investigate whether the poor empirical performance of the standard consumption CAPM is due to the assumption of rationality, in what follows we assume that the agent weights possible outcomes by the subjective rather than by the objective probabilities of occurring. We start by restricting the expected values in pricing conditions (4) and (5) to be linear in outcome probabilities.

When assuming the representative agent to be pessimistic about the future consumption growth rate, we arrange the observed historical consumption growth rates in increasing order and assign to different observations probabilities of occurring that decline exponentially in the consumption growth rate:

$$p_{s,t+1} = \frac{\theta^{s-1}(1-\theta)}{1-\theta^S}, \; 0 < \theta < 1, \; s = 1, ..., S. \tag{7}$$

where $s = 1$ corresponds to the scenario with the smallest consumption growth rate, $s = 2$ corresponds to the scenario with the next greater rate and so on. Finally, the Sth scenario is that corresponding to the highest growth rate of aggregate consumption per capita.

We estimate the equations for the risk premia on the market capitalization-based decile portfolios and the risk-free rate jointly for the values of θ decreasing from 0.9999 to 0.9800 with

[7] As economically realistic, one recognizes the values of the relative risk aversion coefficient that do not exceed 10.

decrements of 0.0001.[8] We find that the marginal significance level associated with Hansen's test of over-identifying restrictions as well as the estimate of the relative risk aversion coefficient and the associated t-statistic decrease as the value of θ decreases. The lowest significantly positive at the 10% significance level value of the relative risk aversion, $\widehat{\gamma} = 64.17$, is obtained at $\theta = 0.9995$. At this value of the parameter θ, $\widehat{\delta} = 1.07$. The null hypothesis that the estimate of the time preference discount factor is greater than one is not rejected statistically at the 5% significance level. The model with pessimism in the agent's beliefs is rejected statistically at the 5% level of significance according to Hansen's J-statistic at any value of the parameter θ.

Under the assumption that the representative agent exhibits doubt, the scenarios are sorted in decreasing order according to the distance from the observed value of the aggregate per capita consumption growth rate to the mean of the historical distribution of the average consumption growth rate. To calculate the subjective probability assigned to each observation, we use formula (7). The highest probability of occurring is assigned to the average consumption growth rate with the greatest distance from the mean of the historical distribution and the smallest probability is assigned to the average consumption growth rate that is closest to the mean.

The unconditional equations for the risk premia on the market capitalization-based decile portfolios and the risk-free rate are estimated jointly for the values of θ decreasing from 0.9999 to 0.9800 with decrements of 0.0001. Our result is that the estimate of the relative risk aversion coefficient and the associated t-statistic decrease, while the marginal significance level associated with Hansen's test of over-identifying restrictions increases as the value of θ decreases. The lowest significantly positive at the 10% significance level value of the relative risk aversion, $\widehat{\gamma} = 32.73$, is obtained for $\theta = 0.9910$. The corresponding estimate of the time preference discount factor, $\widehat{\delta} = 1.03$, is not significantly greater than one at the 5% significance level. According to Hansen's test of the over-identifying restrictions, at $\theta = 0.9910$ the consumption CAPM with doubt is not rejected statistically at the 5% level of significance.

To make allowance for that consumers consider the more recent events as being more likely, we assign to different scenarios probabilities declining exponentially through the past according to formula (7) with θ decreasing from 0.9999 to 0.9800 with decrements of 0.0001. The first scenario here is where the growth rate of average consumption and the return on asset i in period t are assumed to occur. The values of the growth rate of average consumption and the

[8] As $\theta \to 1$, $p_{s,t+1} \to 1/S$ (i.e., the subjective probabilities get equivalent to the objective probabilities) and therefore the consumption CAPM with distorted beliefs becomes equivalent to the standard consumption CAPM, in which different scenarios are weighted by the objective probabilities.

asset return in the second scenario are the same as in period $t-1$ and so on. Finally, the Sth scenario is where the growth rate of average consumption and the asset i return are the same as they were in period $t+1-S$.

As the value of θ decreases, the estimate of the relative risk aversion coefficient and the associated t-statistic decrease, while the marginal significance level associated with Hansen's test of over-identifying restrictions increases. We find that the lowest significantly positive at the 10% significance level estimate of the relative risk aversion, $\widehat{\gamma} = 59.01$, is obtained for $\theta = 0.9974$. The corresponding estimate of the time preference discount factor is $\widehat{\delta} = 1.09$. The hypothesis that this estimate is greater than one is rejected statistically at the 5% significance level. According to Hansen's test of the over-identifying restrictions, at $\theta = 0.9974$ the consumption CAPM with the availability heuristic is rejected statistically at the 5% level of significance.

The next step is to investigate if various combinations of pessimism, doubt, and the availability heuristic can help resolve the equity premium and risk-free rate puzzles. Empirical evidence is that under the assumptions that (i) there are pessimism and the availability heuristic in the agent's beliefs and (ii) the agent exhibits pessimism and doubt, there is no significantly positive at the 10% significance level estimate of the agent's coefficient of relative risk aversion. However, when the subjective distribution is characterized by doubt and the availability heuristic, we find that the lowest significantly positive at the 10% level of significance estimate of the relative risk aversion coefficient, $\widehat{\gamma} = 24.84$, is obtained when $\theta = 0.9935$ for both doubt and the availability heuristic. The corresponding estimate of the time preference discount factor is $\widehat{\delta} = 1.03$ and is significantly greater than one at the 5% level of significance. At these values of the parameters, the model with doubt and the availability heuristic is not rejected according to Hansen's test of over-identifying restrictions at the 5% significance level.

A more general approach is to assume that the expected values in pricing conditions (4) and (5) may be nonlinear in the subjective probabilities. The cumulative prospect theory of Tversky and Kahneman (1992) asserts that an agent usually assigns to possible outcomes decision weights that are strictly increasing monotonic nonlinear functions of outcome probabilities. Following Tversky and Kahneman (1992), we assume that the weighting function $\pi_{s,t+1} = \pi\left(p_{s,t+1}\right)$ is given by formula (6) with $p_{s,t+1}$ being the subjective probability of an outcome. When calculating the subjective probabilities $p_{s,t+1}$, we assume that the representative agent exhibits either pessimism, or doubt, or the availability heuristic, or some combination of them. As before, the subjective probabilities are calculated using formula (7). In the special case when $\lambda = 1$, $\pi\left(p_{s,t+1}\right) = p_{s,t+1}$

and hence expectations are linear in the subjective probabilities $p_{s,t+1}$. When λ differs from one, risk aversion and risk seeking are determined jointly by the utility function and the weighting functions.

As we mentioned above, experimental evidence is that people usually overestimate low probabilities (risk seeking) and underestimate moderate and high probabilities of winning (risk aversion). For $0.9800 \leqslant \theta < 1$ and $S = 539$, the weighting function (6) exhibits the characteristic pattern of risk aversion for moderate and high probabilities and risk seeking for small probabilities when $0.3 \leqslant \lambda < 1$. In what follows, we assume $\lambda = 0.5$ and $\lambda = 0.8$ and test whether the assumption that risk aversion and risk seeking are determined jointly by the utility function and the weighting functions can improve the empirical performance of the model, in which the representative agent is assumed to exhibit either pessimism, or doubt, or the availability heuristic, or some combination of these departures from rationality. We find that the estimation results are similar to those obtained under the assumption that the aversion to risk is determined solely by the curvature of the agent's utility function and therefore expectations are linear in the subjective probabilities $p_{s,t+1}$.

4 Concluding Remarks

In this paper, we explored the potential of pessimism, doubt, and the availability heuristic in the agent's beliefs to enhance the empirical performance of the standard representative agent consumption CAPM with power utility. First, we assumed that risk aversion is determined solely by the curvature of the agent's utility function and therefore that the expected value of an uncertain prospect is linear in the subjective probabilities of outcomes. Then, we supposed that risk aversion and risk seeking are determined jointly by the utility function and the decision weights, which are strictly increasing monotonic nonlinear functions of outcome probabilities. In this case, the expected value is nonlinear in the subjective probabilities.

Using the nonlinear GMM estimation approach, we found that, when the expected value is linear in the subjective probabilities, the consumption CAPM with doubt and the availability heuristic explains the observed equity premia on the market capitalization-based decile portfolios and the risk-free rate much better than the conventional consumption CAPM with the rational representative agent. However, the obtained estimates of the agent's preference parameters are still too high to be recognized as economically plausible. The assumption of nonlinearity of the expected value in the subjective probabilities did not affect the estimation results. This suggests that doubt and the availability heuristic have some potential to help explain better asset returns, but, taken alone, can not resolve the equity premium and risk-free rate puzzles.

References

[1] Abel, Andrew B., 1990, Asset Prices under Habit Formation and Catching Up with the Joneses, American Economic Review 80, Papers and Proceedings, 38-42.

[2] Abel, Andrew B., 1999, Risk Premia and Term Premia in General Equilibrium, Journal of Monetary Economics 43, 3-33.

[3] Abel, Andrew B., 2002, An Exploration of the Effects of Pessimism and Doubt on Asset Returns, Journal of Economic Dynamics and Control 26, 1075-1092.

[4] Allais, Maurice, 1953, Le Comportement de l'Homme Rationnel devant le Risque: Critiques des Postulats et Axioms de l'Ecole Americaine, Econometrica 21, 503-546.

[5] Alvarez, Fernando, and Urban J. Jermann, 2000, Efficiency, Equilibrium, and Asset Pricing with Risk of Default, Econometrica 68, 775-797.

[6] Aschauer, David Alan, 1985, Fiscal Policy and Aggregate Demand, American Economic Review 75, 117-127.

[7] Bakshi, Gurdip S., and Zhiwu Chen, 1996, The Spirit of Capitalism and Stock-Market Prices, American Economic Review 86, 133-157.

[8] Basak, Suleyman, and Domenico Cuoco, 1998, An Equilibrium Model with Restricted Stock Market Participation, Review of Financial Studies 11, 309-341.

[9] Bewley, Truman F., 1982, Thoughts on Tests of the Intertemporal Asset Pricing Model, working paper, Evanston, Ill.: Northwestern University.

[10] Brav, Alon, George M. Constantinides, and Christopher C. Géczy, 2002, Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence, Journal of Political Economy 110, 793-824.

[11] Camerer, Colin E, and Teck-Hua Ho, 1991, Nonlinear Weighting of Probabilities and Violations of the Betweenness Axiom. Unpublished manuscript, The Wharton School, University of Pennsylvania.

[12] Campbell, John Y., and John H. Cochrane, 1999, By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior, Journal of Political Economy 107, 205-251.

[13] Campbell, John Y., and N. Gregory Mankiw, 1990, Permanent Income, Current Income, and Consumption, Journal of Business and Economic Statistics 8, 265-278.

[14] Carroll, John S., 1978, The Effect of Imagining an Event on Expectations for the Event: An Interpretation in Terms of the Availability Heuristic, Journal of Experimental Social Psychology 14, 88-96.

[15] Cecchetti, Stephen G., Pok-Sang Lam, and Nelson C. Mark, 2000, Asset Pricing with Distorted Beliefs: Are Equity Returns Too Good To Be True?, American Economic Review 90, 787-805.

[16] Chou, Ray Y., Robert F. Engle, and Alex Kane, 1992, Measuring Risk Aversion from Excess Returns on a Stock Index, Journal of Econometrics 52, 201-224.

[17] Constantinides, George M., 1990, Habit Formation: A Resolution of the Equity Premium Puzzle, Journal of Political Economy 98, 519-543.

[18] Constantinides, George M., John B. Donaldson, and Rajnish Mehra, 2002, Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle, Quarterly Journal of Economics 117, 269-296.

[19] Constantinides, George M., and Darrell Duffie, 1996, Asset Pricing with Heterogeneous Consumers, Journal of Political Economy 104, 219-240.

[20] Eichenbaum, Martin S., Lars Peter Hansen, and Kenneth J. Singleton, 1988, A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty, Quarterly Journal of Economics 103, 51-78.

[21] Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework, Econometrica 57, 937-969.

[22] Epstein, Larry G., and Stanley E. Zin, 1991, Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: An Empirical analysis, Journal of Political Economy 99, 263-288.

[23] Gordon, Stephen, and Pascal St-Amour, 2004, Asset Returns and State-Dependent Risk Preferences, Journal of Business and Economic Statistics 22, 241-252.

[24] Hansen, Lars Peter, and Ravi Jagannathan, 1991, Implications of Security Market Data for Models of Dynamic Economies, Journal of Political Economy 99, 225-262.

[25] Harvey, Campbell, 1991, The World Price of Covariance Risk, Journal of Finance 46, 111-157.

[26] Kachelmeier, Steven J., and Mohamed Shehata, 1992, Examining Risk Preferences Under High Monetary Incentives: Experimental Evidence from The People's Republic of China, American Economic Review 82, 1120-1141.

[27] Kocherlakota, Narayana R., and Luigi Pistaferri, 2005, Asset Pricing Implications of Pareto Optimality with Private Information, working paper.

[28] Lucas, Robert E., Jr., 1978, Asset Prices in an Exchange Economy, Econometrica 46, 1429-1445.

[29] Mankiw, N. Gregory, 1986, The Equity Premium and the Concentration of Aggregate Shocks, Journal of Financial Economics 17, 211-219.

[30] Mankiw, N. Gregory, and Stephen P. Zeldes, 1991, The Consumption of Stockholders and Non-Stockholders, Journal of Financial Economics 29, 97-112.

[31] Mehra, Rajnish, and Edward C. Prescott, 1985, The Equity Premium: A Puzzle, Journal of Monetary Economics 15, 145-162.

[32] McCurdy, Thomas H., and Ieuan G. Morgan, 1991, Tests for a Systematic Risk Component in Deviations from Uncovered Interest Rate Parity, Review of Economic Studies 58, 587-602.

[33] Rietz, Thomas A., 1988, The Equity Premium Puzzle: A Solution, Journal of Monetary Economics 22, 117-131.

[34] Rothschild, Michael, and Joseph E. Stiglitz, 1970, Increasing Risk: I. A Definition, Journal of Economic Theory 2, 225-243.

[35] Startz, Richard, 1989, The Stochastic Behavior of Durable and Non-Durable Consumption, Review of Economics and Statistics 71, 356-363.

[36] Sundaresan, Suresh M., 1989, Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth, Review of Financial Studies 2, 73-88.

[37] Tversky, Amos, and Daniel Kahneman, 1973, Availability: A Heuristic for Judging Frequency and Probability, Cognitive Psychology 5, 207-232.

[38] Tversky, Amos, and Daniel Kahneman, 1974, Judgment Under Uncertainty: Heuristics and Biases, Science 185, 1124-1130.

[39] Tversky, Amos, and Daniel Kahneman, 1992, Advances in Prospect Theory: Cumulative Representation of Uncertainty, Journal of Risk and Uncertainty 5, 297-323.

[40] Weil, Philippe, 1989, The Equity Premium Puzzle and the Risk-Free Rate Puzzle, Journal of Monetary Economics 24, 401-421.

Table I
GMM Estimation and Test Results

The sampling period is from 1959:01 to 2003:12. The unconditional Euler equations for the equity premia on the market capitalization-based decile portfolios and the risk-free rate are estimated jointly using the iterated GMM approach. The t-statistics are given in parentheses below the estimated coefficients. The J-statistic is Hansen's test of over-identifying restrictions. The P-value is the marginal significance level associated with the J-statistic. $\pi\left(p_{s,t+1}\right) = p^{\lambda}_{s,t+1}/(p^{\lambda}_{s,t+1} + (1 - p_{s,t+1})^{\lambda})^{1/\lambda}$.

Model	γ	δ	P-value
A. Expectations Linear in the Objective Outcome Probabilities			
The Standard Consumption CAPM	83.7407	1.1056	0.0373
	(1.7410)	(26.0685)	
B. Expectations Linear in the Subjective Outcome Probabilities			
Pessimism	64.1740	1.0708	0.0362
	(1.3123)	(26.5567)	
Doubt	32.7289	1.0344	0.1845
	(1.2839)	(43.5651)	
The Availability Heuristic	59.0132	1.0893	0.0377
	(1.2841)	(19.7355)	
Doubt and the Availability Heuristic	24.8424	1.0303	0.3547
	(1.2892)	(62.0356)	
C. Expectations Nonlinear in the Subjective Outcome Probabilities			
$\lambda = 0.8$			
Pessimism	65.0681	1.0722	0.0363
	(1.3316)	(26.5511)	0.9994
Doubt	30.2826	1.0183	0.6746
	(1.2825)	(38.1056)	0.974
The Availability Heuristic	59.44	1.0898	0.0371
	(1.2942)	(19.8361)	0.9968
Doubt and the Availability Heuristic	25.5344	1.0313	0.3457
	(1.3183)	(61.5650)	

Table I (continued)

Model	γ	δ	P-value
	$\lambda = 0.5$		
Pessimism	63.9304	1.0704	0.0362
	(1.3067)	(26.5547)	0.9989
Doubt	32.9847	1.0359	0.1642
	(1.2839)	(43.7819)	0.9813
The Availability Heuristic	59.2092	1.0897	0.0371
	(1.2896)	(19.7932)	0.9944
Doubt and the Availability Heuristic	24.8189	1.0303	0.3539
	(1.2861)	(61.8248)	