Analyzing Risks and Returns in Emerging Equity Markets

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This study applies an operations research technique, Data Envelopment Analysis (DEA) on emerging equity market returns. Sharpe and Treynor measures focus only one risk aspect of portfolio return and in reality investors consider several alternative risk measures outside the traditional mean-variance framework. DEA is a multivariate approach that can incorporate multiple risk characteristics that may be equally important for the investor's decision to allocate assets to emerging markets, the risk and performance relationships are explored in a multivariate framework.

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This study applies an operations research technique, Data Envelopment Analysis (DEA) on emerging equity market returns. Sharpe and Treynor measures focus only one risk aspect of portfolio return and in reality investors consider several alternative risk measures outside the traditional mean-variance framework. DEA is a multivariate approach that can incorporate multiple risk characteristics that may be equally important for the investor's decision to allocate assets to emerging markets, the risk and performance relationships are explored in a multivariate framework.

Conventional univariate risk-adjusted performance metrics, such as the Sharpe and Treynor ratios, fail to contemporaneously capture multiple aspects of the risk and return relationship. Using an alternative approach, Data Envelopment Analysis, I analyze emerging market risk and returns. DEA, an innovative non-parametric, multi-criteria linear programming technique, allows for the joint and simultaneous analysis of multiple risk, return, and performance criteria. Using DEA in analyzing performance may capture relationships that the traditional approaches fail do detect, which improves the investment decision making process.

As a linear programming technique DEA relates inputs to outputs and evaluates the most efficient combination. This efficiency is different from classical market efficiency and is a technical efficiency. It quantifies how effectively inputs contribute to output factors, or how well output reflects the input. The efficiency score quantifies how efficiently each unit of input affects the output.

$$Efficiency = \frac{Output}{Input}$$

In analyzing market performance using DEA, risk variables are appropriate input variables and return or performance variables act as output variables. Here, the efficiency score quantifies how effectively risk factors contribute to performance, or how well performance reflects the risk variables. Financial applications of DEA investigate the risk and performance of major stock markets (Meric and Meric 2001), the performance of mutual funds (e.g., McMullen and Strong 1998, and Galagadera and Silvapulle, 2002), and hedge funds (Gregorhiou and Zhu, 2005). In this study, I analyze the risk and performance characteristics of 23 emerging markets between 1995 and 2005 using DEA. Due to their complex risk relationships, these markets may be particularly well suited for exploring the use alternative analytics.

In analyzing risks, DEA offers several benefits compared to the traditional regression based factor analysis.

- DEA allows for the simultaneous evaluation of multiple risk and performance variables, while regression based models are limited and can relate multiple risk variables to one performance variable.
- DEA evaluates all feasible combinations determined by the data and measures relative performance and not average performance. Regression based risk models relate individual risk to the average risk in the sample. The efficiency score evaluates and scores results relative to all possible combinations.
- 3. DEA is a non-parametric method, which does not require any specification, assumption or prior knowledge of the statistical or distributional properties of the underlying time-series data. Even smaller samples can be analyzed without skewing the statistical and distributional properties of the data.

With various risk measures as input variables and performance variables as outputs, I calculate a single measure that not only measures how efficiently risks translate into performance, but also relate each market's performance to the market with the highest efficiency. This allows for ranking of markets relative to each other and for quantifying the relative inefficiencies among markets regarding the relationships between risks and returns.

I use the commonly used measures of risk – variance of returns, beta and idiosyncratic risk – as input variables. In emerging markets downside risk is well-founded concern and lower partial moments, such as semi-deviation and semi-mean, capture the negative contribution

of returns to overall returns. In traditional multiple-regression model based models concurrently using lower partial moments and variance could bias the results due to possible multicollinearity. This problem does not exist in DEA. As performance variables, I use excess returns. Further, I also attempt quantify positive performance persistence. Positive performance persistence is measured by concurrently using four performance variables: the arithmetic average monthly excess return, the average geometric monthly excess returns, the proportion of positive excess returns to total returns, and the maximum number of months with consecutive monthly excess returns (c.f., Gregorhiou and Zhu, 2005).

Data and risk characteristics with summary statistics

Sample

Total return indexes from Morgan Stanley Capital International for emerging markets, denominated USD serve as data.² As several countries included in the dataset periodically suffered from galloping inflation, the use of USD denominated returns adjusts for high nominal, non-inflation adjusted returns.³ Four selected benchmarks proxy four distinct investment objectives.

- 1. MSCI Emerging Market Index proxies the investment opportunities in emerging markets
- 2. MSCI World Index ex USA proxies international investment opportunities outside the United States in both developed and emerging markets
- The S&P 500 Index proxies a return requirement imposed by a US domiciled investor
- 4. MSCI World Index proxies global investment opportunities

<u>Table 1</u> offers descriptive statistics for the excess returns and benchmark series, and the descriptive statistics corroborates previous findings on emerging equity market: return volatilities are high with skewed and leptokurtic distributions.⁴

INSERT TABLE 1 AROUND HERE.

DEA provides certain flexibility in selecting input and output variables. The empirical DEA literature in finance is generally in agreement that risk variables should be used as input variables with performance variables as output variables (cf. Gregoriou, et al, 2005).

Risk variables

As input factors for this study, I employ several different risk variables, including total risk, correlations, and beta as well as lower partial moments. The risk variables are listed in Table 2. Various lower moment risk variables as input variables expand the empirical findings of Estrada [2000] and Harvey [2003]. Table 3 provides correlation coefficients for the individual input and output variables.

INSERT TABLE 2 AROUND HERE.

Total risk

In the mean-variance framework, the total risk of a financial asset is measured by the variance of returns and is calculated on the average monthly return for each period, $\sigma j2$, with subscript j identifying the individual country returns.

Correlation

As correlation assesses the individual contribution of one financial asset to the overall risk of the portfolio, I calculate correlations relative the benchmarks, to yield $\rho(j, EM)$, $\rho(j, World ex US)$, $\rho(j, SP500)$ and $\rho(j, WORLD)$ Subscript EM refers to the USD denominated MSCI Emerging Market Index, World ex US refers to the USD denominated MSCI World Index ex USA, and SP500 refers to the S&P 500 index, and WORLD refers to the USD denominated MSCI World Index, respectively. The correlation cluster between the US and Global market indexes reflects the considerable proportion the US market accounts of the global markets.

Systematic and idiosyncratic risk

Using the empirical specification of CAPM, I calculate the systematic risk of each emerging market relative the chosen benchmarks. In calculating the excess return for the four betas β EM, β World ex US, β SP500, β WORLD, I use the 3-month U.S. Treasury bill yield as the risk-free rate. The correlation cluster between US and Global market indexes continues in this risk measure as well. The idiosyncratic risk is the standard deviation of the residual of the empirical model, ϵ jt.

(2)
$$r_{jt} - Rf_t = \alpha_j + \beta_j (R_{mt} - Rf_t) + \varepsilon_{jt}$$

Value-at-risk

I use the average of monthly returns below the 5th percentile level, a variable used both by Estrada [2000] and Campbell [2003]. The VAR variable exhibits high correlation with idiosyncratic risk variables.

Semi-deviation and lower partial moments

Although portfolio theory uses variance of returns as its chief measure of total risk, there are several arguments why it may not be an appropriate measure. For instance, variance is only appropriate when the distributions are symmetric; evidence from descriptive statistics in Table 1 as well as from other studies suggest that equity market returns are non-symmetric. Moreover, kurtosis and skewness is reported to be considerably higher in emerging markets than in developed markets due to the excess volatility and persistence of runs.

For most investors downside risk is the major concern; therefore, I use semi-deviation which only considers deviations below the zero. This measure is non-symmetric as only negative returns increase the semi-deviation, but positive returns do not influence semi-deviation:

(3)
$$SEMI - Dev = \sqrt{\frac{1}{N} \sum_{t=1}^{T} (Min(0, r_{j,t} - \mu_{j,t}))^2}$$

µjt equals zero for each emerging market, (2) yields SEMI-Devj. N is the number of negative return months and not the total number of return months. As semi-deviation combines into one measure the information provided by two statistics: variance and skewness, it is useful in explaining market returns (e.g., Harvey 2000). Since investors generally prefer upside volatility and shun, when possible, downside volatility, using semi-deviation reflects these preferences.⁵ To complement semi-deviation, I also calculate MEAN-Down; the average return when monthly returns are negative.

(4)
$$MEAN - Down = \frac{1}{N} \sum_{t=1}^{T} Min(0, r_{j,t} - \mu_{j,t})$$

For completeness, I also include UP-months, the proportion of excess monthly returns exceeding zero. P is the number of positive return months and not the total number of return months. Setting µjt to equal zero, (5) yields the average excess positive monthly returns, or MEAN-Up.

(5)
$$MEAN - Up = \frac{1}{P} \sum_{t=1}^{T} Max(0, r_{j,t} - \mu_{j,t})$$

INSERT TABLE 3 AROUND HERE.

Performance variables

Performance variables serve as output variables and I use four measures of performance. The first variable, the average monthly excess returns, is the traditional measure of performance. The last three performance variables capture positive performance persistence: the magnitude, consistency and sustainability of positive, long-term positive excess returns generated by each market.

- 1. the arithmetic average of monthly excess returns for each country,
- 2. the geometric average of monthly excess returns over the period measures consistency of long-term returns in each market over the period studied

- 3. the longest number of consecutive months of positive returns or runs within each period, measures the overall persistence of positive returns
- the proportion of positive excess returns to total returns within each period identifies markets with the ability to sustain the greatest number of monthly positive excess returns

Findings

DEA offers two different approaches. The *input oriented DEA models* measure how efficiently inputs generate the existent output; to improve performance, inputs should be reduced. Inefficiencies quantify a slack, the needed reduction of inputs to maintain the existing levels of outputs. When the inefficiently used inputs are reduced, the unit in question becomes efficient. The *output oriented DEA models* measure the potential increase in outputs given the existent levels of inputs. Here inefficiencies quantify a slack as well: the needed increase in outputs to effectively use the existing levels of inputs to generate outputs. With inputs constant, the output increases to an efficient level, because currently they do not generate efficient performance relative to the levels of inputs used. ⁶

In analyzing the relationship between risks and performance, both input and output oriented models could be used. Previous studies argue for using input oriented models, since the calculated inefficiencies reveal the loss in performance given the level of risk. Additionally, the Markowitz model can be seen as a special case of input oriented DEA. Based upon these considerations, the analysis uses the input oriented approach. With input variables acting as various risk characteristics and the output variables as performance characteristics, the efficiency score identifies the best risk adjusted performance and ranks markets relative to the best risk-adjusted performance or how much higher relative risk is compared to the risk of the most efficient market (Brockman et al 2006).

The highest efficiency score of 1.00 indicates a market that offers the highest performance relative to risks.⁷ Since this is the most efficient combination of risk and performance, risk

is transferred into performance and not one part of risk is wasted. When the efficiency score is below 1.00, then the actual risk-adjusted performance is lower and shows inefficiencies. The distance from 1.00 measures the relative inefficiencies and in this case is a risk slack. *Risk slack* is the proportion of the risk that disappears and does not contribute to the returns. For instance, an efficiency score of 0.75 indicates 25% inefficiency; to achieve efficiency at the existing performance, risk should decrease by 25%, or 25% of the risk does not generate performance. Alternatively, the risk adjusted performance is only 75% of the best risk adjusted performance.

Additionally, it is generally accepted that the sample should be twice as large as the number of input and output variables used in the analysis. For consistency, I will keep the number of output variables constant (one or four) and only change the input variables. In exploring the use of DEA in assessing risk and performance in emerging markets, I

- 1. identify the individual efficiencies of the risk variables to performance, by relating each risk variable to the arithmetic return (univariate case)
- 2. augment the return variable with the three performance variables to explain the efficiency of individual risk variables to positive performance persistency in a multiple output variable framework (multivariate case).
- 3. measure the influence several risk variables jointly have on performance and positive performance persistence.

Mecir and Mecir [2001] use a single input variable, univariate, approach in explaining the risk characteristics of major stock markets. Both in McMullen and Strong [1998] and Galagadera and Silvapulle [2002] the multiple input variables were risk variables and return and other performance related variables were the output variables. Gregorhiou and Zhu [2005] apply a multiple input and output variable approach in their DEA specifications when assessing the efficiency of hedge funds and CTAs, investment vehicles with performance characteristics not dissimilar to emerging markets.

Efficiency scores in Table 4 provide efficiency scores for individual risk measures to the average monthly return of the market. Here, in the input oriented analysis I only use one input and output variable; the analysis yields one efficiency score.

INSERT TABLE 4 AROUND HERE.

Performance

In the case of total risk, σ_{j2} , Israel offers the best performance, and the Chilean market, with an efficiency score of 0.93, is nearest in efficiency. The only market on the efficient frontier is Israel and all other markets are inefficient in terms or risk and return. The 0.93 score indicates that the Chilean market's input inefficiency relative the Israeli market is 7%. Were the Chilean total market risk decline by 7%, the performance of the Chilean market would become efficient as well. The Thai market offers the lowest relative return for total risk, 0.12. In efficiency terms, for the total risk investors assumed in the Thai market, their reward was 88% worse than investors in the Israeli equity market and 81% worse than the Chilean market. Differently put, the Thai equity market should have had an 88% lower variance to be considered efficient or their risk adjusted performance is 88% less than the Chilean market.⁸ The Colombian market's correlation relative to its performance is the highest, with all other combinations inferior; the worst existing combination is Thailand. If investors consider high correlation to given returns as positive, the Colombian market offers the best relationship. However, if low correlations to returns are preferable, then investing in the Thai market is beneficial. An investor focusing on diversifiable risk with βWorld ex. US as the benchmark would have received the best compensation for this risk in the Mexican market. Israel provided the highest performance efficiency for most diversifiable risk or β measures. In terms of idiosyncratic risk, Colombia is the most efficient for all specifications, except for *\varepsilon*, where the Sri Lankan market is the most efficient. For this measure, the Colombian market is near efficiency (0.99). The other risk measures indicate that India is the most efficient market in terms of VAR 95%.

Focusing on the relationship between lower partial moments and performance, the Czech Republic offered the highest efficiency in terms semi-deviation; Thailand offers the best efficiency in MEAN-Down. Israel offers the highest efficiency in MEAN-Up. For an investor focusing on lower partial moments, investment in these markets would have provided the best opportunity in terms of the partial risks and total excess returns.

INSERT TABLE 5 AROUND HERE.

Performance persistence

One of the advantages of DEA is the concurrent use of multiple input and output variables to calculate efficiencies. By adding three additional performance variables to average monthly returns, I attempt to capture the persistence of positive performance. With multiple input and output variables, the number of possible efficient combinations increases and multiple market can provide efficiencies. In this multivariate case, the efficiency score maximizes the investor's utility (e.g., McMullen and Strong 1998, and Galagadera and Silvapulle, 2002, Gregorhiou and Zhu, 2005). ⁹ The efficiency scores then distinguish between those markets that are efficient and the relative degree of inefficiencies between the remaining markets relative risk and performance. ¹⁰ Identifying the markets that are not on efficient frontier can be used to steer investments into markets with higher efficiencies. <u>Table 5</u> contains the efficiency score for individual risk measures to positive performance persistence.

For instance, after introducing positive performance persistence, the Israeli market lost its highest efficiency. Both Brazil and India are efficient in terms of total risk, σ j2. Investors looking at performance persistence would reap the best benefits if investing in either of these two markets; investing in other markets might not be as beneficial. The inefficiency of the Israeli market is 14% relative to either of these markets. Other risk variables indicate similar changes in their efficiencies;

Venezuela is the only market efficient in semi-deviations and Taiwan is the only market efficient relative MEAN-Down. This finding is unusual: multiple input and output variables often lead to multiple efficiencies. Neither of these markets demonstrate any other efficiencies, but they have several near efficiencies in the univariate and multivariate analysis. Brazil, India, Philippines and Venezuela are all efficient in MEAN-Up. Investors seeking emerging market exposure and demanding positive performance persistence could shift part of their assets into Venezuela; the market has a higher combined proportion of positive return months and average positive returns than markets that a less efficient.

Multiple risk measures and performance

Creating composite input and output variables offers an additional benefit over regression models that are limited by relating a single or a pre-defined combination of weighted inputs to one single output. In DEA, input and output weights do not need to be specified initially and the certain model specifications would select a combination of output variables that is most efficient relative to input variables.

INSERT TABLE 6 AROUND HERE.

From the investment benchmark perspective and to analyze efficiencies from investment opportunity sets, I calculate efficiencies of multiple input and output variables. Table 6 uses benchmark risk variables, correlation, beta, and idiosyncratic risk, as input variables with performance as the output variable. Colombia, Israel, and the Czech Republic offer the highest efficiency from the MSCI Emerging Market Index based risk variables. When including performance variables, these three markets remain efficient and two new markets become efficient: Chile and South Africa. Of the 23 markets, these 5 markets provide the best combination of risk and positive performance persistence.

Using the MSCI World ex US Index as benchmark, Mexico, Colombia and the Czech Republic are efficient in performance terms. After including positive performance persistence variables, Chile and South Africa, two near-efficient markets in the single output variable specification become efficient in the multiple input specification. Comparing the results for these two benchmarks suggest Colombia, Czech Republic, Chile and South Africa all offer combinations that would high risk adjusted returns for investors seeking exposure to both emerging markets and developed markets outside the US. Using the Standard and Poor's 500 Index as benchmark, Colombia, Israel, Sri Lanka, and the Czech Republic are efficient in the single output specification. Further considering the persistence of positive performance Chile, Mexico and South Africa are efficient.

Using the MSCI World Index as benchmark for the risk variables, Colombia, Israel and the Czech Republic are efficient in average performance terms. Augmenting a multiple output specification, Chile and Mexico become efficient as well. Overall, the Czech Republic has provided multiple efficiencies across various benchmarks and specifications. An investor seeking exposure to these multiple investment objectives would have benefited from investing in Czech market compared to another market such as Hungary or Argentina.

INSERT TABLE 7 AROUND HERE.

An advantage of DEA is the simultaneous evaluation of multiple risk and performance variables, while regression based models are limited and can relate multiple risk variables to one performance variable. Table 7 contains several combinations of risk variables, where both total risk and other downside risk variables are inputs. Israel and the Czech Republic are efficient in total risk and semi-deviation specification of input variables relative to average return. Including the variables for positive performance persistence, Chile becomes efficient as well. Including total risk and VAR 95%, India, Israel and Peru are efficient in the average return case and South Africa and Chile are efficient in the positive performance persistence.

Conclusions

Using Data Envelopment Analysis, I examine the contemporaneous influence risk variables have on the performance of 23 emerging markets. DEA is a non-parametric, multi-criteria linear programming method that offers distinct advantages over traditional regression based risk analysis. One of the benefits of DEA is that several investment performance variables can be assessed vis-à-vis various risk characteristics. The method evaluates relative performance, ranking the strength of the relationships. The findings indicate that several

markets exhibit multiple efficiencies across different specification, while other markets do not exhibit any at all.

Considering one risk variable and return, the Colombian market showed multiple efficiencies – correlation and idiosyncratic risk – between 1995 and 2005. Israel showed the highest efficiency in terms of total and market risk. Considering lower partial risk moments and returns, the Czech market was the most efficient. Considering one risk variable and a proxy for positive performance persistence several markets demonstrated efficiencies. Analyzing semi-deviation and positive performance persistence, Venezuela was efficient; average negative mean and performance persistence terms Taiwan was efficient. These findings are unusual; using multiple input and output variables often lead to multiple efficiencies. Analyzing the efficiencies of investment benchmarks identifies Colombia and Czech Republic as efficient in return terms; in positive performance persistence Chile, Israel, and Mexico are also efficient.

Using DEA can provide additional insights in the portfolio construction and selection process as well as in the evaluation of performance relative to various investment risks. The core benefit of this approach, the multiple inputs and outputs, allow the investors to pinpoint those combinations that for the given level or combination input variables offer the best possible combination of outputs.

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Appendix - DEA

I use an input-oriented, variable-returns-to-scale, VRS, specification in calculating the efficiency score of risk variables. In this specification, DEA captures the relationship between the risk variables as inputs and performance variables as outputs to yield an efficiency score. This score captures the efficiency is which the market is able to generate performance.

For a general DEA model, yrj is the known positive output level of country j, r = 1,2,...,s where s is the number of outputs, xrj is known positive input level of country j, r = 1,2,...,s where s is the number of inputs, and n is total number of countries. Thus, the relative efficiency of a country "A" is

)
$$Max\left\{\frac{\sum\limits_{r=1}^{s}u_{r}y_{rA}}{\sum\limits_{i=1}^{m}u_{i}y_{iA}}\right\}$$

subject to

(A1)

(A2)
$$\begin{array}{l} \sum_{i=1}^{s} u_{i} y_{ij} \\ \sum_{i=1}^{m} u_{i} y_{ij} \\ (A3) \quad r = 1, 2, ..., s; \text{ and } i = 1, 2, ..., m. \end{array}$$

In the above model, the variables are input and output weights of ur and vi, respectively. The objective function (A1) defines the ratio of weighted sum of outputs to weighted sum of inputs. Here the weights are the optimal values of the variables ur and vi to be determined. The model can be transformed into an equivalent linear programming model. This linear program determines the relative efficiency score, θ , of fund of a country A by (A4) $Max\theta$

subject to

(A5)
$$\sum_{j=1}^{n} \lambda_{j} y_{rj} \leq y_{rA_{i}}, r = 1, 2, ..., s;$$

 $\theta x_{iA} \leq \sum_{j=1}^{n} \lambda_{j} x_{ij}, i = 1, 2, ..., m;$
(A6) $\lambda_{j} \geq 0, j = 1, 2, ..., n;$

(A8)
$$\sum_{j=1}^{j} \lambda_j = 1$$

The variables of the model are θ and λ , which are both non-negative. θ is the proportional reduction required in each input of the specific country fund to achieve efficiency. The model contains constraints; their function is to ensure that relative efficiency of the fund cannot exceed one. The sufficient condition for efficiency is that the optimum value of θ equals one. If that is not the case, the country is inefficient compared to the other countries in the sample. Consequently, a DEA produces relative efficiency scores and a set of λ_{J} ,

j=1, 2, ..., n; values for each country. The set of λ_j values defines a point on the envelopment surface. For an inefficient country, λ_j values establish a benchmark. Introducing the convexity requirement, (A8) in the linear programming model outlined in (A4-A7), distinguishes the variable return-to-scale approach. The Markowitz efficient frontier can be derived from this model.

Table 1 – Descriptive statistics

Univariate descriptive statistics of the USD denominated excess returns for emerging markets and benchmarks used in the study between January 1995 and December 2005. Average is the arithmetic average of monthly excess return, standard deviation is calculated the same return series. For the higher moments, skewness is centered at 0. Ljung-Box is the Ljung-Box statistics with 12 lagged correlations. The Jarque-Berra is the test for normality. The geometric average is the geometric average of monthly excess return. Runs is the longest consecutively positive monthly excess return during the period.

	Average	Standard dev	Skewness	Kurtosis	Ljung-Box	p-value	Jarque- Berra	p-value	Geometric average	Runs
Argentina	-0.0005	0.05	-0.31	4.92	10.95	0.5336	22.41	0	0.015544	7
Brazil	0.001	0.052	-0.96	5.27	5.57	0.9361	48.77	0	0.01621	10
Chile	-0.0013	0.0304	-0.95	6.76	9.74	0.6391	97.49	0	0.015289	9
China	-0.0051	0.0477	0.24	4.71	22.07	0.0367	17.33	0.0002	-0.0035	9
Colombia	0.0024	0.0428	-0.22	3.44	22.52	0.0321	2.16	0.3401	0.012216	8
Czech Republic	0.0026	0.0369	-0.56	4.6	16.38	0.1746	21.07	0	0.013998	9
Hungary	0.0042	0.0456	-0.74	7.18	13.37	0.3426	108.25	0	0.017483	10
India	-0.0005	0.0364	-0.14	2.37	15.37	0.2217	2.63	0.2688	0.006579	8
Indonesia	-0.004	0.0662	-0.38	4.63	23.55	0.0234	17.83	0.0001	0.006852	6
Israel	0.0009	0.0329	-0.45	3.66	8.93	0.7088	6.84	0.0328	0.009211	8
Korea	-0.0008	0.055	0.28	5.41	7.16	0.8469	33.62	0	0.006465	7
Malaysia	-0.0038	0.0437	-0.09	6.55	28.6	0.0045	69.41	0	0.005358	7
Mexico	0.0016	0.0402	-1.24	6.08	7.64	0.8124	85.97	0	0.018767	7
Peru	0.0009	0.0361	-0.87	7.94	9.56	0.6541	150.99	0	0.010991	6
Philippines	-0.0068	0.0427	0.01	5.02	16.3	0.1777	22.34	0	0.003626	6
Poland	0.0004	0.0463	-0.3	4.8	13.56	0.3299	19.86	0	0.009541	5
Russia	0.0043	0.0813	-1.08	7.49	12.09	0.4383	136.55	0	0.018455	6
South Africa	-0.0001	0.0354	-1.13	5.88	9.87	0.6274	73.81	0	0.008082	9
Sri Lanka	-0.0026	0.0451	0.44	5.28	18.99	0.0888	33.01	0	0.001432	5
Taiwan	-0.0035	0.039	0.09	3.24	16.98	0.1504	0.5	0.7775	0.005154	6
Thailand	-0.006	0.0576	-0.29	4.29	26.26	0.0099	11.02	0.0041	0.004975	10
Turkey	0.0027	0.073	-0.24	4.26	10.05	0.6117	9.99	0.0068	0.010844	9
Venezuela	-0.0012	0.0617	-0.82	7.92	7.14	0.848	148.13	0	0.004687	7
MSCI EM	0.0047	0.0688	-1.29	6.9	6.79	0.8709	120.31	0		
MSCI WORLD	0.0069	0.0408	-0.79	4.01	7.79	0.8014	19.14	0.0001		
MSCI WORLD EXUS	0.0057	0.0425	-0.65	3.61	7.2	0.8444	11.27	0.0036		
S&P 500	0.0086	0.0433	-0.61	3.56	8.48	0.7463	9.94	0.0069		

Variable	Definition
σj2	Total risk or the variance of monthly excess returns
ρ(j, EM),	Correlation with MSCI Emerging Market Index
ρ(j, WORLD),	Correlation with MSCI World Market Index
ρ(j, SP),	Correlation with S&P 500 Index
ρ(j, World ex US)	Correlation with MSCI World ex US Index
β(j, EM),	Market risk, beta using MSCI Emerging Market Index as market proxy
β(j, WORLD),	Market risk, beta using MSCI World Market Index as market proxy
β(j, SP),	Market risk, beta using S&P 500 Index as market proxy
β(j, World ex US)	Market risk, beta using MSCI World ex US Index as market proxy
ε(j, EM),	Idiosyncratic risk, standard deviation of residuals using MSCI Emerging Market Index as the market proxy
ε(j, WORLD),	Idiosyncratic risk, standard deviation of residuals using MSCI World Market Index as the market proxy
ε(j, SP),	Idiosyncratic risk, standard deviation of residuals using S&P 500 Index as the market proxy
ε(j, World ex US)	Idiosyncratic risk, standard deviation of residuals using MSCI World ex US Index as the market proxy
SEMI-Dev	Semi-deviation of excess returns less than zero (negative returns)
MEAN-Down	Average return when monthly excess return is less than zero
MEAN-Up	Average return when monthly excess return is greater than zero
VAR 95%	Monthly excess return Value-at-risk below the 5th percentile

Table 2 – Risk variables used as input factors in the DEA model

Table 3 – Correlation coefficients for input and output variables

Negative values for input and output variables are adjusted according to the translation invariance property of the variables. Input variables are risk variables found in table 2, and the output variables are the arithmetic average monthly excess return, the geometric average excess return, the longest consecutively positive monthly excess return, and positive to total is the proportion of positive excess returns to the total number of returns.

	aj2	ρ(j, EM),	o(j, WORLD),	p(j, SP),	, World ex US)	β(j, EM),	ŝ(j, WORLD),	β(j, SP),	, World ex US)	VAR 95%	ε(j, EM),	ɛ(j, WORLD),	ε(j, SP),	, World ex US)	EMI-DevZero	AN-DownZero	IEAN-UpZero	verage return	ometric return	Longest run
			4		ΡÚ		4		β(j			-		ε(j	S	ME	2	Ā	ő	
ρ(j, EM),	0.96	1.00	4.00																	
ρ(J, WORLD),	0.99	0.99	1.00	1 00																
ρ(J, SP),	0.92	0.91	0.93	1.00	1 00															
p(j, world ex US)	0.55	0.54	0.54	0.05	1.00	1 00														
	0.50	0.55	0.50	0.07	1.00	1.00	1 00													
R(i SP)	0.33	0.51	0.52	0.04	0.08	0.07	0.08	1 00												
B(i World ex US)	0.47	0.43	0.40	0.31	-0.30	-0.30	-0.30	-0.48	1 00											
VAR 95%	0.40	0.40	0.40	0.40	-0.32	-0.00	-0.25	-0.31	0.83	1 00										
ε(i FM)	0.63	0.73	0.69	0.51	-0.15	-0.15	-0.18	-0.24	0.79	0.98	1 00									
ε(i, WORLD).	0.63	0.61	0.63	0.45	-0.29	-0.25	-0.30	-0.36	0.85	0.97	0.91	1.00								
ε(i, SP).	-0.09	-0.12	-0.11	0.04	-0.14	-0.13	-0.13	-0.22	0.22	-0.03	-0.06	0.01	1.00							
ε(j, World ex US)	0.78	0.76	0.77	0.86	0.92	0.93	0.92	0.85	0.00	0.10	0.15	0.05	-0.06	1.00						
SEMI-DevZero	-0.11	-0.08	-0.10	0.01	-0.20	-0.21	-0.21	-0.28	0.23	0.03	0.03	0.04	0.64	-0.10	1.00					
MEAN-DownZero	0.66	0.64	0.66	0.69	0.95	0.96	0.95	0.94	-0.24	-0.07	-0.02	-0.12	-0.21	0.91	-0.23	1.00				
MEAN-UpZero	0.04	0.07	0.06	-0.04	0.15	0.14	0.14	0.20	-0.22	-0.06	-0.02	-0.10	-0.30	0.06	-0.47	0.20	1.00			
Average return	0.24	0.18	0.22	0.14	0.04	0.06	0.04	0.05	0.04	0.16	0.10	0.20	-0.20	0.18	0.25	0.18	0.07	1.00		
Geometric return	0.30	0.27	0.29	0.30	-0.02	-0.01	-0.03	-0.09	0.36	0.31	0.26	0.34	-0.01	0.20	0.20	0.06	0.20	0.78	1.00	
Longest run	0.19	0.23	0.21	0.09	-0.18	-0.18	-0.19	-0.19	0.28	0.36	0.37	0.35	0.01	-0.05	0.17	-0.09	0.05	0.19	0.26	1.00
Positive to total	0.31	0.24	0.28	0.23	-0.12	-0.09	-0.12	-0.15	0.32	0.38	0.31	0.44	0.04	0.15	0.38	-0.06	-0.17	0.81	0.77	0.26

Average return as output variable 1995 – 2003	aj2	ρ(j, EM),	p(j, WORLD),	p(j, SP),	p(j, World ex US)	β(j, EM),	β(j, WORLD),	β(j, SP),	ß(j, World ex US)	VAR 95%	ε(j, EM),	ε(j, WORLD),	ε(j, SP),	ε(j, World ex US)	SEMI-DevZero	MEAN- DownZero	MEAN-UpZero
Argentina	0.38	0.35	0.32	0.34	0.44	0.53	0.51	0.51	0.49	0.02	0.50	0.38	0.35	0.39	0.07	0.04	0.62
Brazil	0.40	0.25	0.22	0.24	0.37	0.71	0.65	0.69	0.71	0.04	0.44	0.28	0.26	0.29	0.08	0.04	0.66
Chile	0.93	0.37	0.32	0.35	0.48	0.90	0.85	0.87	0.99	0.01	0.34	0.24	0.21	0.26	0.09	0.05	0.81
China	0.22	0.22	0.14	0.18	0.23	0.28	0.29	0.28	0.27	0.05	0.25	0.20	0.15	0.24	0.08	0.01	0.32
Colombia	0.67	1.00	1.00	1.00	1.00	0.76	0.74	0.73	0.66	0.02	1.00	1.00	0.99	1.00	0.10	0.05	0.77
Czech Republic	0.92	0.71	0.87	0.79	0.86	0.94	0.88	0.89	0.84	0.02	0.74	0.67	0.73	0.60	1.00	0.20	0.96
Hungary	0.68	0.47	0.39	0.43	0.65	0.92	0.89	0.90	0.85	0.05	0.68	0.45	0.39	0.48	0.13	0.05	0.89
India	0.72	0.55	0.69	0.60	0.66	0.72	0.68	0.68	0.65	1.00	0.56	0.50	0.57	0.45	0.06	0.03	0.67
Indonesia	0.14	0.18	0.17	0.18	0.22	0.25	0.24	0.24	0.22	0.08	0.34	0.27	0.25	0.27	0.05	0.05	0.29
Israel	1.00	0.48	0.37	0.42	0.88	1.00	1.00	1.00	0.80	0.03	0.66	0.31	0.26	0.35	0.21	0.09	1.00
Korea	0.30	0.25	0.24	0.25	0.39	0.50	0.46	0.48	0.43	0.02	0.50	0.30	0.29	0.30	0.65	0.33	0.47
Malaysia	0.32	0.28	0.27	0.28	0.32	0.40	0.38	0.38	0.38	0.01	0.32	0.28	0.26	0.28	0.05	0.04	0.48
Mexico	0.71	0.36	0.30	0.33	0.49	0.93	0.90	0.93	1.00	0.05	0.45	0.30	0.26	0.33	0.13	0.07	0.88
Peru	0.83	0.59	0.72	0.63	0.65	0.84	0.79	0.80	0.81	0.30	0.54	0.52	0.58	0.48	0.16	0.07	0.89
Philippines	0.17	0.14	0.11	0.13	0.17	0.21	0.21	0.21	0.20	0.01	0.17	0.12	0.11	0.13	0.03	0.05	0.22
Poland	0.48	0.31	0.29	0.31	0.47	0.68	0.64	0.65	0.61	0.04	0.50	0.32	0.30	0.33	0.17	0.09	0.64
Russia	0.21	0.30	0.24	0.27	0.35	0.50	0.49	0.49	0.49	0.04	0.66	0.50	0.43	0.54	0.12	0.05	0.54
South Africa	0.78	0.35	0.35	0.36	0.49	0.90	0.80	0.84	0.90	0.09	0.40	0.28	0.28	0.28	0.06	0.03	0.88
Sri Lanka	0.36	0.57	0.93	0.72	0.65	0.43	0.42	0.41	0.36	0.02	0.68	0.76	1.00	0.60	0.05	0.04	0.47
Taiwan	0.42	0.27	0.22	0.24	0.32	0.48	0.47	0.47	0.49	0.02	0.28	0.21	0.19	0.24	0.05	0.03	0.45
Thailand	0.12	0.11	0.09	0.10	0.14	0.20	0.20	0.20	0.19	0.01	0.19	0.13	0.12	0.14	0.03	1.00	0.22
Turkey	0.24	0.26	0.22	0.24	0.42	0.52	0.50	0.51	0.44	0.02	0.71	0.39	0.35	0.42	0.22	0.11	0.48
Venezuela	0.23	0.38	0.32	0.36	0.49	0.38	0.37	0.37	0.32	0.02	0.69	0.49	0.43	0.53	0.35	0.77	0.40

Table 4 – DEA efficiency scores for risk variables using average monthly return as output variableInput variables are defined in Table 2, above. The efficiency scores are calculated using a VRS input-oriented DEA model.

Table 5 – DEA efficiency scores for risk variables using performance variables as output variable

Input variables are defined in Table 2, above. The efficiency scores are calculated using a VRS input-oriented DEA model. The output variable are the arithmetic average monthly excess return, the geometric average excess return, the longest consecutively positive monthly excess return, and positive to total is the proportion of positive excess returns to the total number of returns.

All performance variables as output variables 1995 – 2003	aj2	ρ(j, EM),	ρ(j, WORLD),	p(j, SP),	p(j, World ex US)	β(j, EM),	β(j, WORLD),	β(j, SP),	β(j, World ex US)	VAR 95%	ε(j, EM),	ε(j, WORLD),	ε(j, SP),	ε(j, World ex US)	SEMI-DevZero	MEAN- DownZero	MEAN-UpZero
Argentina	0.43	0.35	0.31	0.34	0.51	0.77	0.73	0.75	0.71	0.07	0.62	0.39	0.36	0.41	0.10	0.04	0.75
Brazil	1.00	0.60	0.51	0.56	0.77	1.00	1.00	1.00	1.00	0.03	0.54	0.39	0.35	0.41	0.13	0.05	1.00
Chile	0.22	0.36	0.28	0.35	0.43	0.42	0.40	0.40	0.34	0.14	0.69	0.50	0.39	0.54	0.10	0.05	0.49
China	0.69	1.00	1.00	1.00	1.00	0.79	0.78	0.76	0.66	0.03	1.00	1.00	1.00	1.00	0.10	0.05	0.80
Colombia	0.73	0.71	0.90	0.78	0.86	0.75	0.72	0.72	0.65	1.00	0.73	0.65	0.70	0.59	0.07	0.03	0.73
Czech Republic	0.70	0.52	0.42	0.48	0.71	0.96	0.95	0.94	0.85	0.07	0.75	0.49	0.43	0.53	0.13	0.05	0.93
Hungary	0.22	0.31	0.25	0.28	0.37	0.53	0.53	0.51	0.49	0.06	0.69	0.51	0.45	0.56	0.12	0.05	0.56
India	1.00	0.57	0.42	0.50	1.00	1.00	1.00	1.00	0.81	0.03	0.80	0.36	0.29	0.42	0.24	0.09	1.00
Indonesia	0.30	0.30	0.29	0.30	0.46	0.51	0.47	0.48	0.43	0.02	0.61	0.37	0.33	0.37	0.76	0.33	0.48
Israel	0.86	0.66	0.80	0.70	0.72	0.88	0.85	0.84	0.81	0.49	0.61	0.58	0.65	0.54	0.17	0.07	0.96
Korea	0.48	0.51	0.48	0.51	0.55	0.60	0.57	0.57	0.52	0.02	0.58	0.50	0.44	0.51	0.08	0.04	0.73
Malaysia	0.75	0.47	0.38	0.42	0.63	1.00	1.00	1.00	1.00	0.09	0.58	0.38	0.34	0.42	0.16	0.07	0.99
Mexico	0.43	0.42	0.33	0.39	0.49	0.53	0.52	0.52	0.45	0.01	0.50	0.37	0.30	0.41	0.08	0.06	0.56
Peru	0.48	0.37	0.30	0.36	0.52	0.69	0.64	0.66	0.61	0.05	0.59	0.36	0.30	0.39	0.20	0.09	0.64
Philippines	0.57	0.45	0.31	0.40	0.50	0.67	0.66	0.66	0.62	0.02	0.47	0.34	0.26	0.40	0.08	0.03	0.62
Poland	0.84	0.49	0.50	0.50	0.69	1.00	0.91	0.94	0.91	0.10	0.57	0.40	0.37	0.39	0.07	0.03	1.00
Russia	0.43	0.35	0.31	0.34	0.51	0.77	0.73	0.75	0.71	0.07	0.62	0.39	0.36	0.41	0.10	0.04	0.75
South Africa	0.42	0.82	1.00	1.00	0.88	0.51	0.50	0.49	0.40	0.02	0.98	1.00	1.00	0.86	0.07	0.04	0.56
Sri Lanka	0.23	0.47	0.39	0.44	0.60	0.39	0.39	0.38	0.32	0.02	0.85	0.61	0.51	0.66	0.39	0.77	0.42
Taiwan	0.31	0.42	0.35	0.39	0.55	0.55	0.56	0.54	0.47	0.04	0.74	0.51	0.43	0.55	0.12	1.00	0.64
Thailand	0.24	0.28	0.24	0.26	0.46	0.53	0.52	0.52	0.44	0.03	0.78	0.43	0.38	0.47	0.22	0.11	0.50
Turkey	0.41	0.63	0.41	0.52	0.65	0.55	0.60	0.56	0.48	0.11	0.72	0.56	0.41	0.68	0.20	0.01	0.69
Venezuela	0.95	0.79	0.96	0.87	0.95	0.98	0.94	0.93	0.84	0.03	0.82	0.74	0.81	0.67	1.00	0.20	1.00

Table 6 – DEA efficiency scores for benchmarks

The efficiency scores are calculated using a VRS input-oriented DEA model. The input variables are correlation, beta, and idiosyncratic risk for each of the four respective benchmarks, i.e., MSCI Emerging Market Index, MSCI World ex US Index, Standard and Poor's 500 Index and MSCI World Index. Beta and idiosyncratic risk are calculated using all four benchmarks. The output variable(s) are either the arithmetic average monthly excess return, or performance variables. The performance variables are the arithmetic average monthly excess return, the longest consecutively positive monthly excess return and positive to total is the proportion of positive excess returns to the total number of returns.

	Average retu	rn			Average retu	Average return with performance variables							
	Emerging Market Index	World ex U.S. Index	S&P 500 Index	World Index	Emerging Market Index	World ex U.S. Index	S&P 500 Index	World Index					
Argentina	0.62	0.61	0.57	0.58	0.87	0.85	0.77	0.80					
Brazil	0.71	0.75	0.72	0.69	0.87	0.82	0.80	0.79					
Chile	0.90	0.99	0.87	0.86	1.00	1.00	1.00	1.00					
China	0.32	0.35	0.31	0.32	0.84	0.92	0.74	0.81					
Colombia	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00					
Czech Republic	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00					
Hungary	0.96	0.97	0.96	0.97	0.97	0.97	0.98	0.99					
India	0.77	0.77	0.77	0.78	0.91	0.90	0.91	0.93					
Indonesia	0.34	0.32	0.30	0.31	0.69	0.64	0.59	0.61					
Israel	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00					
Korea	0.59	0.51	0.53	0.52	0.67	0.59	0.60	0.59					
Malaysia	0.43	0.45	0.42	0.43	0.73	0.79	0.73	0.74					
Mexico	0.93	1.00	0.94	0.91	1.00	1.00	1.00	1.00					
Peru	0.89	0.93	0.89	0.89	0.94	0.98	0.95	0.94					
Philippines	0.22	0.23	0.22	0.23	0.63	0.67	0.62	0.65					
Poland	0.71	0.69	0.70	0.69	0.72	0.75	0.75	0.74					
Russia	0.66	0.69	0.57	0.62	0.69	0.70	0.57	0.63					
South Africa	0.90	0.93	0.87	0.83	1.00	0.99	1.00	0.96					
Sri Lanka	0.68	0.65	1.00	0.93	0.98	0.93	1.00	1.00					
Taiwan	0.49	0.54	0.50	0.50	0.72	0.82	0.74	0.75					
Thailand	0.24	0.23	0.22	0.22	0.85	0.81	0.73	0.76					
Turkey	0.71	0.58	0.56	0.57	0.78	0.60	0.56	0.57					
Venezuela	0.69	0.53	0.48	0.50	0.85	0.66	0.57	0.62					

Table 7 – DEA efficiency scores for total and downside risk variables

The efficiency scores are calculated using a VRS input-oriented DEA model. The input variables are total risk with a short-fall measure. The three shortfall measures are: SEMI-DevZero, the semi-deviation of excess returns less than zero (negative returns); SEMI-DevMean, the semi-deviation of excess returns less than the average return, and VAR 95%, the monthly excess return Value-at-risk below the 5th percentile. The output variable(s) are either the arithmetic average monthly excess return, or performance variables. The performance variables are the arithmetic average monthly excess return, the longest consecutively positive monthly excess return and positive to total is the proportion of positive excess returns to the total number of returns.

	Average return			Average return and performance variables						
	σj2 with SEMI-DevZero	σj2 with DOWN	σj2 with VAR 95%	σj2 with SEMI-DevZero	σj2 with DOWN	σj2 with VAR 95%				
Argentina	0.53	0.55	0.41	0.67	0.66	0.52				
Brazil	0.58	0.59	0.46	0.65	0.64	0.62				
Chile	0.93	0.93	0.93	1.00	1.00	1.00				
China	0.32	0.34	0.26	0.69	0.68	0.60				
Colombia	0.91	0.90	0.68	0.91	0.90	0.72				
Czech Republic	1.00	1.00	0.92	1.00	1.00	0.96				
Hungary	0.91	0.91	0.75	0.91	0.91	0.79				
India	0.81	0.85	1.00	0.90	0.91	1.00				
Indonesia	0.26	0.27	0.18	0.46	0.47	0.30				
Israel	1.00	1.00	1.00	1.00	1.00	1.00				
Korea	0.55	0.58	0.33	0.60	0.61	0.34				
Malaysia	0.43	0.45	0.34	0.68	0.70	0.51				
Mexico	0.77	0.77	0.80	0.84	0.82	0.91				
Peru	0.90	0.92	1.00	0.93	0.95	1.00				
Philippines	0.23	0.25	0.18	0.62	0.64	0.44				
Poland	0.68	0.70	0.55	0.72	0.72	0.56				
Russia	0.49	0.50	0.25	0.49	0.50	0.25				
South Africa	0.79	0.79	0.90	0.88	0.88	1.00				
Sri Lanka	0.55	0.57	0.40	0.69	0.71	0.47				
Taiwan	0.52	0.56	0.45	0.77	0.80	0.62				
Thailand	0.21	0.21	0.14	0.60	0.59	0.42				
Turkey	0.53	0.54	0.27	0.53	0.54	0.28				
Venezuela	0.45	0.46	0.26	0.48	0.49	0.27				

ENDNOTES

- ¹ At the 2005 Global Finance Conference in Dublin, Ireland, seminar participants offered constructive suggestions that greatly improved an earlier version of the present study. The usual disclaimer applies.
- ² The data is freely downloadable from the Morgan Stanly Capital International website <u>http://www.msci.com</u>. Due to overall low market capitalization concentrated to a handful closely-held major companies, comparatively infrequent trading, and structural problems within the exchanges and the economy, Jordan, Egypt, Pakistan, and Morocco are excluded. Results including these markets are available upon request.
- ³ Selecting MSCI World Index ex USA as a proxy excludes for the considerable weight of the US markets' capitalization and captures the broader international markets, while MSCI World Index captures global investment opportunities.
- ⁴ The Jaques-Berra test rejects the (log)normality of distributions. Additionally, Ljung-Box statistics, with 12 lagged lengths, suggest the presence of autocorrelation and heteroskedasticity in most returns.
- ⁵ While I performed the analysis with additional upper partial moments as risk variables, I do not present these results here; if requested, I will make them available.
- ⁶ In mutual fund parlance and most research in finance using DEA has been done on mutual funds – the input efficiency measure assesses whether the fund has had excessive loads, expenses, risk for the returns earned. The output efficiency measure assesses whether the returns have been adequate in terms of loads, expenses and risks. Simply, when focusing on mutual funds, the performance can examined from two different angles. Are there inefficiencies in generating outputs or are there input inefficiencies.
- ⁷ When needed, variables are linearly transformed, Zhu (2001).
- ⁸ The model examines the relationship between average return and variance of returns, and ranking by the variance to mean ratio would yield qualitatively similar results. Using the highest inverted variance-to-mean ratio as a base and then dividing each inverted variance-to-mean ratio with the base, would generate the same efficiency scores.
- ⁹ Efficiency scores of multiplicative DEA models could quantify the relative or preferential weight each output variable has.
- ¹⁰ Theoretically with sufficiently large number of input and output variables, it is possible to achieve efficiencies in all markets.