Evidence of Non-stationary Bias in Scaling by Square Root of Time: Implications for Value-at-Risk[1]

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Abstract

In this paper, we show that scaled conditional volatilities obtained by the square root formula applied to *i.i.d* residuals from a sample of Canadian stock market data for various time horizons and error distributions, typically underestimate the true conditional volatility; consistently have a higher standard deviation and exhibit nonstationary kurtosis. Furthermore, the bias produced by volatility scaling is nonstationary in mean and standard deviation and its magnitude is likely influenced by monetary policy regime shifts. Moreover, while VaR is risk-coherence for elliptical distributions, this bias remains even for this class of distributions.

Revised 2 September 2006

JEL classification: C30; G30

Keywords: Square-root-of-time formula; Value-at-Risk.

[1] The authors thank participants at the 55th Midwest Finance Association (MFA) meetings, Chicago, Illinois, 13th Multinational Finance Society (MFS) meetings, Edinburgh, and the 4th INFINITI Conference on International Finance, Dublin, for their helpful comments. We are also grateful to the editor and an anonymous referee for their constructive suggestions that have led to a linkage between monetary policy regime shifts and non-stationary bias in volatility scaling.

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1. Introduction

It is routine practice in the financial services industry to impute a longer-term volatility from *one*-day volatility via the square root formula. Operationally, an *n*-day volatility is derived from a one-day volatility by multiplying the later by $\sqrt{n}$. Indeed, the Basle 1996 "Amendment to the Capital Accord to Incorporate Market Risks" requires a 10-day holding period whereby a one-day Value-at-Risk (henceforth VaR) is converted to a 10-day equivalent that is "scaled up … by the square root of time" (p.44, section B.4, paragraph c). In addition, while financial services firms typically use a one-day VaR for internal risk control (Danielsson, Hartman and De Vries, 1998) and for the purpose of determining regulatory capital allocation against market risk, banks are obliged to assume that they cannot liquidate their trading portfolios before the end of ten business days. To facilitate this transition, there is reliance on the square root formula where $Var(n) = Var(1) * \sqrt{n}$.

Several studies reported in the econophysics literature have focused on the econometric implications of volatility scaling. Batten and Ellis (2001) apply fractal geometry on an example of spot of currency returns and provide evidence implying that linear scaling to estimate risk of long-term returns using observable short-term returns is not appropriate when returns series are not independent. Gencay, Selcuk and Whitcher (2001) using a wavelet methodology show that foreign exchange rate volatility exhibits different scaling properties at different time horizons, suggesting that utilizing a unique global scaling factor such as square root of *n* may lead to misleading results. A possible explanation for the observed scaling behaviour for the distribution of price changes may be attributed to the long range volatility correlations found by Liu *et al* (1999) who show that the probability density function of the volatility of the S&P 500 index is a mixture of a log-normal at the centre with the tail distribution described by a power law having an index outside the stable Levy range.

Diebold *et al* (1997) parametrize a GARCH (1,1) process and produce simulated data showing that scaling magnifies volatility relative to estimates obtained from he Drost-Nijman (1993) temporal aggregation formula. But we find important limitations with this approach.

First, as recognized by Diebold *et al*, if the *one*-day model is correctly specified as a GARCH(1,1) process, then *n*-day volatilities can be computed by the Drost-Nijman formula. But there is a preponderance of studies showing the GARCH (1,1) is inadequate to *fully* capture volatility clustering in typical financial time series. One possible reason for this is that GARCH(1,1) models only first order heteroskedasticity. Therefore, GARCH(1,1) fails to capture all the volatility clustering in presence of higher order heteroskedasticity. As results, models such as GARCH (2,2) should be examined. In addition, a major limitation to GARCH process is that it does not allow for leverage effect, which is known also as volatility asymmetry. Leverage effect means that volatility tends to rise in response to lower than expected returns and to fall in response to higher than expected returns. Failing to capture this fact, GARCH model may not produce accurate forecasts. To overcome this limitation models such as EGARCH, and PGARCH were suggested. Furthermore, GARCH does not account for long memory behavior observed in the volatility of financial assets returns; to deal with this shortcoming there is an increasing volume of studies that use FIEGARCH model. Hence reliance on the Drost-Nijman temporal aggregation model may be misleading.

Second, if the *one*-day return series does not follow a weak GARCH process, then Drost-Nijman will not apply, leaving no obvious benchmark comparison for the *n*-day volatility series obtained from the square root of time formula.

In this paper, we address these issues and contribute to the literature in several ways. First, we apply a clean econometric test of the adequacy of the square root formula on real rather than simulated data based on a pre-selected GARCH model. In particular, we consider daily data for the Toronto Stock Exchange Composite Index (S&P/TSX) for the period 1977-2004 and by means of a powerful test originally proposed by Brock, Dechert and Scheinkman (1987) (henceforth BDS) and designed by Brock *et al* (1996), we create *i.i.d* residuals.[2] Volatility clustering is specified by an EGARCH(1,1) with Hsieh's test for nonlinearity. Second, we consider three different distributions – normal, Student and generalized error distribution (GED) Nelson (1991) – as possible candidates for the error term. Third, we investigate the potential bias in scaling by square root of time over the full sample from 1997 through 2004 and a sample partition based on subperiods identified with three U.S. monetary regimes over the period 1977 through 1986 as well as shifts in Canadian monetary policy in January 1991 and January 1999 as found by Shambora, Choi and Jung (2006). The rationale for this sample partitioning is explained in greater detail in Section 5. In brief, we provide evidence implying a linkage between monetary policy and conditional volatility. Consequently, it is reasonable to hypothesize that monetary policy regime *shifts*, should affect the standard deviation of conditional volatility. Also, we refer to research which shows that U.S. monetary policy leads Canadian monetary policy. Accordingly, we partition the full sample on the basis of U.S. monetary policy regime shifts and evidence of endogenous monetary policy shifts in Canada. This gives us six sub-periods that permits a test of potential temporal variability in the mean and variance of possible bias created from volatility scaling

[2] The BDS test is a non-parametric test with the null hypothesis that the series in question is *i.i.d* against an unspecified alternative.

The value of the above approach is that we compare the actual (or true) *n*-day volatilities of *i.i.d* residuals obtained from real data against different economic contexts. We use this series as our benchmark for comparison with the *n*-day (scaled) volatility series created via the square root of time formula. The bias in the *n*-day scaled volatility series compared to the benchmark series is then computed. We find very interesting results when using *i.i.d* residuals as opposed to Diebold *et al* methodology of comparing (simulated) scaled volatilities to those obtained from Drost-Nijman temporal aggregation. First, unlike Diebold *et al* (1997) who found in favor of the square root formula for short time horizons no greater than 10 days, we find that this formula underestimates the true conditional volatility for both short and longer time horizons and for all three assumed error distributions.

Second, the bias introduced by the square root formula is apparently a concave function of time in that it increases at a decreasing rate. However, for a given time horizon, the average bias is relatively stable for both the normal and fat-tailed distributions. For example, we find that for 10-day horizons, the average bias is about 23%, implying that scaled formula results in estimates that are lower than the true value by about 23%. This is true whether we assume that the error distribution is normal, Student-t or GED. Similar results are found for 30-day and 90-day horizons but where the average bias is 28%. This is evidence of concavity where the bias increases for short horizons and quickly stabilizes – after 30 days in our empirical findings.

Third, we find a meaningful degree of temporal variation in the bias with its smallest value in the most recent period outside the three monetary regimes. This suggests that while the average bias in scaled volatilities is relatively stable over the full sample period of 1977-2004 and for each error distribution, there is significant temporal variability in the estimated bias that may be linked to monetary policy regime shifts.

Fourth, the scaled conditional volatility estimates consistently have a larger standard deviation than that of the true conditional values. The size of the difference is greatest for the fat-tailed distributions considered. Hence, not only does the square root formula produce under estimates of the true conditional volatilities, but they are also relatively more volatile especially for fat-tailed distributions. Given that there is overwhelming evidence that financial time series are fat-tailed, then there is meaningful estimation risk in using the scaled formula.

Fifth, the empirical distribution of scaled volatilities exhibits significantly greater kurtosis than the corresponding distribution of true conditional volatilities, indicating simultaneous peakedness and fat tail behavior. But, like the average bias, there is a great degree of temporal variation in the kurtosis estimates. The evidence of leptokurtosis is largely attributed to the most recent period outside the three monetary regimes with the distribution of true conditional volatilities showing evidence of platykurtosis – less peaked and thinner tails.

The rest of this paper is organized as follows. Section 2 describes the data used in this study with appropriate initial tests including normality and root tests. Section 3, presents a rationale for a partition of the full sample based on monetary regime shifts. Section 4 is full description of our methodology for producing *i.i.d* residuals using the BDS and Hsieh tests, respectively. Section 5 presents the results- both graphical and tabular-for the scaled volatility obtained from the square root formula and compares them with the true n-day conditional volatility for n =10, 30 and 90 days respectively. Section 6 discusses the results. Section 7 presents the implications for Value At Risk estimation while section 8 concludes the paper.

2. Data and initial diagnostics

We note that in the subsequent sections of this paper, we apply an econometric procedure that is differentiated from that in the literature as evidenced by Diebold *et al*. In particular, we model real *daily* return series by an AR(k)-EGARCH(p,q) model while ensuring that the residuals are *i.i.d* by the BDS and Hsieh tests. We then use the formula,

$$\textit{n-day conditional volatility} = \textit{(1-day conditional volatility)} * \sqrt{n}$$

We do this for n =10, 30 and 90 days under normal, Student-*t* and GED respectively. In addition, to obtain a benchmark true *n*-day conditional volatility, we compute the return *over n days***,** for *n* =10, 30 and 90 days respectively and for the same error distributions.

So first, we describe the real data series we use for this study.

The data considered in this paper is the set of daily S&P/TSX composite price index beginning on January 03, 1977 and ending on December 31, 2004 forming a sample of 7056 observations. The data is obtained from Toronto Stock Exchange - Canadian Financial Markets Research Center (CFMRC) database. Market index prices are transformed to daily returns as the natural logarithmic first difference of the daily closing price.

Table1 below provides various descriptive statistics for index returns. The distribution of daily returns is negatively skewed. The null hypothesis of skewness coefficient conforming to the normal distribution value of zero is rejected at 1% level. In addition, the null hypothesis of kurtosis coefficient conforming to the normal distribution value of three is rejected at 1% level. The daily returns are thus not normally distributed, a conclusion which is confirmed by *Jarque-Bera* test statistic.

To see whether the series is stationary, we employ the Augmented Dickey Fuller (ADF) test. Perron (1989) has demonstrated that ADF is subject to misspecification bias and size distortion when the series involved has undergone structural shifts leading to spurious acceptance of the unit root hypothesis. We overcome this limitation by also using Philips-Perron (PP) test which allows for a one-time structural break. The outcomes from conventional ADF and PP unit root tests (not reported here) suggest that the series is non-stationary in levels and stationary in first differences at one percent level of significance. To examine the linear dependence of the returns series, we use the modified Q-statistic of Ljung and Box (1978). Table 2 below provides the autocorrelations coefficients up to lag 50. The results suggest the existence of significant serial autocorrelation at all lags.

(Insert Table 1 about here)

(Insert Table 2 about here)

3. Monetary policy regime shifts and sample partitioning

In assessing whether potential bias arising from the scaled formula is temporally stable, we partition the full sample into sub-periods based on U.S. monetary policy regime shifts. The rationale underlying this choice is based on evidence that monetary regimes shifts may have significant impact on conditional volatility of several financial time series. For example, Lastrapes (1989) noted that monetary policy regimes significantly affect the mean and variance of nominal exchange rates. Choi and Kim (1991), using a GARCH model in a study covering the period 1975 through 1989, find that the foreign exchange risk premium depends on changes in the monetary regime. In the same vein, Hsieh (1991) finds that changes in operating procedures of the U.S. Federal Reserve Board (FRB) can shift the volatility of financial markets.

In identifying the monetary regimes within the period 1977-2004 we use U.S. data as proxies for the ensuing shifts in Canadian monetary policy as potential structure changes. Several authors have pointed out the high correlation between Canadian and U.S. interest rates (see among others Howitt, 1986; Pesando and Plourde, 1988; Mittoo, 1992). For instance, Mittoo (1992) finds the correlation between the 3-month T-Bill rate for the U.S. and Canada for the period January 1977 to December 1986 to be over 0.5. Thornton (1990) reports evidence suggesting that the U.S. monetary policy leads Canadian monetary policy. [3] Finally, Yamada (2002) shows there is a one-to-one long-run linkage between the U.S. and Canadian real interest rates.

Since the mid-seventies, the U.S. Federal Reserve Board (FRB) has made substantial alterations to the way monetary policy is conducted. The literature has identified three non-overlapping monetary regimes over the period 1976-1986, each characterized by different operating procedures[4]: (1) January 7, 1976, to October 3, 1979; (2) October 10, 1979, to October 1, 1982; and (3) October 6, 1982, to November 19, 1986.[5]

Apart from these three time periods, there is evidence that Canadian monetary policy also experienced significant shifts in the post 1986 period. Shambora, Choi and Jung (2006) find that January 1991 and January 1999 represent structural shifts in monetary policy. For example, in January 1991, the Bank of Canada adopted a policy of inflation targeting and shifted away from this policy in January 1999 to a less restrictive monetary policy in order to boost corporate earnings growth. Consequently, we partition the full sample period into five sub-periods representing the three U.S. monetary regime shifts ending in 1986 coupled with two shifts in Canadian monetary policy in January 1991 and January 1999.

4. Methodology

To test whether the share price changes are *i.i.d* we use a powerful test designed by Brock *et al* (1996). The BDS test is a non-parametric test with the null hypothesis that the series in

[3] See Calvet and Rahman (1995) for a more detailed review on the linkage between U.S. monetary regimes, Canadian monetary policy, and Canadian stock returns.
[4] See Bergstrand (1983), Roley (1986) and Lastrapes (1989).
[5] Andolfatto and Gomme (2003) find evidence that October 1979 represented a significant shift in monetary policy in Canada. This was also confirmed by Duffy and Engle-Warnick (2006).

question is *i.i.d* against an unspecified alternative. The test is based on the concept of correlation integral, a measure of spatial correlation in *n*-dimensional space:

$$BDS(m,\varepsilon) = \frac{C_m(\varepsilon,T) - [C_1(\varepsilon)]^m}{\sigma_m(\varepsilon,T)/\sqrt{T}} \tag{1}$$

where m is the embedding dimension, T is the sample size, $\sigma_m(\varepsilon,T)/\sqrt{T}$ is the standard deviation of the difference between the two correlation measures $C_m(\varepsilon,T)$ and $[C_1(\varepsilon)]^m$. For large samples, the BDS statistic has a standard normal limiting distribution under the null of *i.i.d*. If asset price changes are not identically and independent random variables, then $C_m(\varepsilon) > C_1(\varepsilon)^m$.

It is important to note that the BDS test statistic is sensitive to the choice of the embedding dimension m and the bound ε. As mentioned by Scheinkman and LeBaron, (1989), if we attribute a value that is too small for ε, the null hypothesis of a random *i.i.d* process will be accepted too often irrespective of it being true or false. As well, it is not safe to choose too large a value for ε. To deal with this problem Brock et al. (1991) suggest that, for a large sample size (T > 500), ε should equal 0.5, 1.0, 1.5 and 2 times standard deviations of the data. As for the choice of the relevant embedding dimension *m*, Hsieh (1989) suggests consideration of a broad range of values from 2 to 10 for this parameter. Following recent studies of Barnett et al. (1995), we implement the BDS test for the range of m-values from 2 to an upper bond of 8.

In general, a rejection of the null hypothesis is consistent with some type of dependence in the returns that could result from non-stationarity, a linear stochastic process, a non-linear stochastic process, or a non-linear deterministic system.[6] Based on ADF and PP test the non-stationarity argument is rejected. According to Hsieh (1991), linear dependence can be ruled out by prior fitting of Akaike Information Criterion (AIC)-minimizing autoregressive moving

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[6] The Simulation studies of Brock et al. (1991) show that the BDS test has power against a variety of linear and non-linear processes, including for example GARCH and EGARCH processes.

average (ARMA) model. Therefore, a rejection of the *i.i.d* assumption using filtered data can be the result of a non-linear stochastic process or a non-linear deterministic system. However, BDS test is neither able to distinguish between stochastic and deterministic non-linearity, nor can it discriminate between additive and multiplicative stochastic dependence. Because we are concerned with a stochastic explanation of returns behavior, the latter issue matters in this case.

As stated earlier, in order to choose an appropriate non-linear model describing the returns series, it is crucial to know the source of non-linearity in the data. Non-linearity can enter through the mean of a return generating process (additive dependence) as in the case of threshold autoregressive model, or through the variance (multiplicative dependence), as in the case ARCH model proposed by Engle (1982). Non-linearity can be both additive and multiplicative as in the case of GARCH-M model. To determine the source of non-linearity in the returns series we use Hsieh's test (Hsieh, 1989).

Although the Hsieh's test provides us with the type of non-linearity underlying the data series, it does not tell what model to choose for the returns generating process. Still, the results of Hsieh's test provide the first step towards finding the best non-linear model to fit the data. For instance, if the source of non-linearity turns out to be the variance (a multiplicative dependence) then we should look into ARCH models. Engle (1982) was first to introduce these models, which are now very widely used in financial time series modeling. For example the generalized ARCH (GARCH) models, designed by Bollerslev (1986), are very successful in describing certain properties of high frequency financial time series such as excess kurtosis and volatility clustering. However there are some aspects of financial time series, such as leverage effect, that are not considered in the basic model. GARCH model is a symmetric variance process, in that the sign of the disturbance is ignored meaning that bad news have the same impact on volatility as good news which is not consistent with the *leverage effect*. Neslon (1991), among others, propose a model that allows for leverage effect known as exponential GARCH (EGARCH) model:

$$h_t = \eta + \sum_{i=1}^{p} \lambda_i \frac{|\varepsilon_{t-i}| + \rho_i \varepsilon_{t-1}}{\sigma_{t-i}} + \sum_{j=1}^{q} \theta_j h_{t-j} \tag{2}$$

where λ, θ, ρ, are the ARCH, GARCH, and leverage parameters respectively. $h_t = \log \sigma_t^2$ and σ_t^2 is the variance of an *i.i.d* random variable.

5. Results

To test for the *i.i.d* assumption we employ the BDS test. It is a powerful test frequently used to detect several non-linear structures and to test for the adequacy of a variety of models. Table 3 below reports the BDS statistic for embedding dimension 2 to 8 and for epsilon values starting from 0.5 to 2 times the standard deviation of the returns series. The results strongly reject the null hypothesis of independently and identically distributed index price changes at 5% and 1% significance level.

(Insert Table 3 about here)

Since the BDS test has a good power against linear as well as non-linear system, we use a filter to remove the serial dependence in the return series and the resulting residuals series are re–tested for possible non-linear hidden structures. We use an autoregressive AR(k) model to take out all the linearity in the series. Empirical studies show that non-synchronous trading causes a deviation of the observed index returns from the true index returns. An advantage of using the residuals of AR(k) model is that it reduces the effect of infrequent trading, which is more pronounced in price indices of thinly traded stock markets.[7] The identification of the order of autoregression, k, is based on the lowest AIC. The Modified Q-statistics (not tabulated herein) show that the residuals of an AR(12) are white noise, suggesting that the model accounts for all the linearity dependence in the series.

To test whether linear dependence is the reason of rejecting the *i.i.d* assumption, we employ the BDS test on the residuals of the AR(12) model. The results (not tabulated here) still reject the *i.i.d* assumption. Hence, given that we can rule out the non-stationarity and linearity as causes of the rejection of the *i.i.d* assumption, we can say that S&P/TSX returns index exhibits some inherent non-linearity which is either stochastic or deterministic.

Although the results from the BDS test strongly support the existence of inherent non-linearity, it does not tell us whether it enters through the mean or variance of the returns series. To uncover the source of non-linear behaviour, we calculate the third-order moment test statistics of Hsieh (1989). None of the values of the approximately normally distributed Hsieh test statistic (not tabulated herein) are significant, implying a failure to reject the null hypothesis of multiplicative dependence. This supports the view expressed above that volatility clustering is responsible for the rejection of *i.i.d* in index returns series. Therefore, a GARCH model is most likely to succeed in describing the return generating process than a GARCH-M model.

Given the results of Hsieh's test, we have examined several GARCH (p,q) type models. Using the AIC and BIC as tools for model selection, it turns out that an EGARCH (1,1) is the best model to fit the series. Table 4 reports the estimation results of a EGARCH(1,1) process under the assumption that the innovations follow four distributions: Normal, Student-t,

[7] To proxy for the true but unobserved index returns Stoll and Whaley (1990) have used the residuals from an ARMA regression.

generalized error distribution (GED) proposed by Nelson (1991). All the model selection criteria show that EGARCH(1,1) under Student-t is the best in describing our data.

The results of the diagnostic tests show that the models (under the three distributions) are correctly specified. The modified Q-statistics (not reported here) for the standardized residuals and standardized squared residuals are both insignificant, suggesting the chosen EGARCH process is successful at modeling the serial correlation structure in the conditional mean and conditional variance. JB test for normality fail to reject the null hypothesis that the standardized residuals are normally distributed. To see whether the model has captured all the volatility clustering in the series we calculate the Lagrange-multiplier (LM) test proposed by Engle (1982). The null hypothesis that the residuals lack ARCH effect is not rejected, which shows that the EGARCH (1,1) has indeed counted for all the volatility clustering in the data.

(Insert Table 4 about here)

To examine whether the EGARCH model has succeeded in capturing all the nonlinear structure in the data, we employ the BDS test to its standardized residuals. A rejection of the *i.i.d* hypotheses will imply that the conditional heteroskedasticity is not responsible for all the nonlinearity in index returns, and there is some other hidden structure in the data possibly deterministic. Table 5 displays the BDS statistics on the standardized residuals from the EGARCH process. The BDS test fails to reject the null hypothesis that the standardized residuals are *i.i.d* random variables at 5% and 1% degree of significance. This confirms that he EGARCH process indeed captures all the non-linearity in the series, and that the conditional heteroscedasticity is the cause of the non-linearity structure uncovered in the returns series.

Figure 1 below plots the conditional volatility obtained from the EGARCH model under Student-t distribution. It is obvious from the graph that the conditional variance varies over time.

The series is characterized by significant heteroscedasticity, which manifests by changes in volatility of S&P/TSX index over the period of investigation.

(Insert Table 5 about here)

(Insert Figure 1 about here)

5.1. Testing the accuracy of scaling by the square root of time formula

To test the relative accuracy of scaling by the square root formula, we first present the details of the calculation of the true and scaled volatilities. That is, we model linear dependence and volatility clustering via an AR(k)-EGARCH model and account for potential non-linear dependence by the Hsieh test.

The comparison will be based on graphics and also descriptive statistics for the whole sample as well as four sub-samples based on monetary regime shifts. Figures 2 displays the scaled and true condition volatilities for n = 10 under normal error distributions: normal. Table 6 reports the descriptive statistics.[8]

(Insert Figure 2 about here)

6. Discussion of results

First, we discuss the results for the full sample period 1977:1 through 2004:12. The main results are presented in Table 6 for n=10 days. To facilitate the interpretation of the results in Table 6 we define additional statistics as follows:

$$BIAS = 1 - \frac{Scaled\ \ Volatility}{True\ \ Volatility}$$

[8] For sake of brevity, the figures and tables reporting the results for n =30 and n = 90 under three error distributions (normal, Student and GED) are not reported here, but are available upon request from the authors.

Clearly, if the scaled volatility is lower (higher) than the true volatility, the bias will be positive (negative). We also define ΔSD as follows:

ΔSD= Standard Deviation of True Conditional Volatility – Standard Deviation of Scaled Conditional Volatility

Clearly, if ΔSD is positive (negative), then the standard deviation of the true conditional volatility is greater (less) than the standard deviation of the scaled conditional volatility.

(Insert Table 6 about here)

From Table 7, we see that for the total sample (1977-2004) and n =10 days, the bias is positive with a mean value of 23% irrespective of the error distribution – normal, Student and GED. A similar result holds for the longer time horizon (n = 30 and 90 days) where the average bias is 28%. Hence, the mean bias increases over time but quickly stabilizes revealing a degree of concavity.

When we consider the standard deviation of the conditional scaled volatility estimates, a different picture emerges. While the mean of the scaled volatilities systematically underestimate the true mean volatilities for all time horizons (n = 10, 30 and 90 days) and all error distributions (normal, Student and GED), the standard deviation of the scaled volatilities are persistent overestimates. This is observed in Table 7 where ΔSD is negative indicating that the conditional volatility of scaled estimates has a higher standard deviation when compared to the conditional volatility of true estimates. In particular, Table 7 shows that for n = 10 days, the standard deviation of the scaled volatilities are highest for the fat-tailed distributions (-11% for normal and -53% for Student-t error distributions.) This pattern of persistent overestimates worsens for longer time intervals with bias exceeding 100% for n=30 and all error distributions.

We now examine the results for each of the six sub-periods identified by U.S. and Canadian monetary policy regime shifts. Like the full sample, BIAS is positive and ΔSD is negative for each sub-period, for all three time horizons and error distributions. However, there is evidence of significant temporal variability in these statistics. For n=10 and each error distribution, the highest mean bias is in the first monetary regime and the lowest mean bias is found in the last sub-period. But the difference between the standard deviation of the true conditional volatilities and the scaled conditional volatilities varies randomly across each sub-period. If we consider longer time periods, (n=30, 90 days), the same random variation exist but also for the mean bias. The location of the highest and lowest mean bias is not predictable as is apparent in the case where n=10 days. While the mean bias is non-stationary but has the lowest value in the 1999-2004 sub-period, the bias in the standard deviation of the scaled volatilities is also nonstationarity and highest in the last three sub-periods While the general pattern of results (positive mean bias and negative ΔSD remains across sub-periods, we observe that temporal variability in both biases is likely influenced by monetary policy regime shifts.

The empirical distribution of the scaled conditional volatilities relative to that of the true conditional volatilities exhibits some interesting relationships. In Table 6 we see that the distribution of scaled volatilities typically show significantly higher levels of kurtosis. For example, for n = 10 and a normal error distribution, the kurtosis of the distribution of scaled volatilities is 18.62 for the entire period relative to 10.98 for the distribution of true volatilities. However, there is also significant nonstationarity in the kurtosis statistics with the highest value (almost an outlier value) for all error distributions in the 1986-1990 subperiod.

For the longer time periods, the nonstationarity in kurtosis is even more acute. For n = 30 the kurtosis for the distribution of scaled volatilities averages 18.00 for all error distributions as compared to an average of zero for the distribution of true values. A similar picture emerges for n = 90 with evidence of temporal variability. As for the case of n=10 days, the highest value of the kurtosis is found in the period 1986-1990.

We may conclude that scaled conditional volatilities obtained by the square root formula and applied to *i.i.d* residuals from a sample Canadian stock market data for various time horizons, typically underestimate the true conditional volatility; have a higher standard deviation and exhibit nonstationary kurtosis. Furthermore, the mean bias arising from the scaled formula is concave over time but also exhibits nonstationarity. Finally, the degree of temporal variability of the estimated bias is likely influenced by monetary policy regime shifts.

We now examine the implications of the square root formula for estimation typical risk measures such as Value at Risk.

(Insert Table 7 about here)

7. Implications for value-at-risk estimation

It is known that Value at Risk is not a coherent risk measure in the sense that subadditivity is violated. In particular, for two portfolios A and B, subadditivity is stated as VaR (A+B) ≤ Var(A) + Var (B). The lack of subaddivity can give rise to regulatory arbitrage in the sense if capital requirements are based on VaR, a firm could create artificial subsidiaries in order to save on regulatory capital. However, to guarantee subadditivity and hence to restore the risk coherence of VaR, Breuer, Krenn and Pistovcak (2002) of parametric VaR, the portfolio value must be a linear function of risk factors whose changes are elliptically distributed. Classical examples of elliptical distributions are multivariate normal and Student t distributions. Indeed, So and Yu (2006), emphasize that “although most return series show fat-tailed distributions and satisfy long memory properties, it is more important to consider a model with fat-tailed error in estimating VaR.

The results of this paper add another dimension to the problems of VaR as a risk measure. While the coherence of VaR is restored for normal and Student –distributions with the latter being fat-tailed, the evidence in this paper shows that the nonstationary bias found for the scaled conditional volatilities still remains. Hence it is likely that VaR estimates based on the square root of time formula will underestimate the true trading risk that a financial institution faces. To compound this problem, VaR estimates are likely to be unreliable given the evidence of overestimation of the standard deviation of the true conditional volatilities. Since VaR is typically used as basis for allocating regulatory capital and economic capital to cover possible losses from trading activities, the implication for optimal capital allocation is evident. That is, capital allocation may cover trading risk only randomly.

8. Conclusion

In this paper, we conduct a clean econometric test of the accuracy of the square of time formula that is commonly used to create longer-term conditional volatilities from those derived for shorter time horizons. Unlike previous studies in the literature that assume particular GARCH models and simulate data against the Drost-Nijman temporal aggregation model, we consider daily data for the Toronto Stock Exchange Composite Index (S&P/TSX) for the period 1977-2004 and by means of a powerful test designed by Brock *et al* (1996), we create *i.i.d* residuals. The BDS test is a non-parametric test with the null hypothesis that the series in question is *i.i.d* against an unspecified alternative. Volatility clustering is specified by an AR(k)-EGARCH with Hsieh's test for nonlinearity. In addition, we consider three different distributions – normal, Student and generalized error distribution (GED) – as possible candidates for the error term.

Our main finding may be summarized thus: scaled conditional volatilities obtained by the square root formula applied to *i.i.d* residuals from real stock market data for various time horizons, typically underestimate the true conditional volatility; have a higher standard deviation and exhibit nonstationary kurtosis.[9] Furthermore, Furthermore, the bias arising from the scaled formula is concave over time and its magnitude is likely influenced by monetary policy regime shifts. Furthermore, while VaR is risk-coherent for elliptical distributions, we show that the bias indicated above still remains.

There are likely important economic effects arising from linear rescaling of risk. Batten and Ellis (2001) employ a simple Black-Scholes model for pricing currency options and find that this when scaling by $\sqrt{n}$, there is significant underpricing of call and put options. Since, *ceteris paribus,* a lower conditional volatility leads to option values, the Batten and Ellis result is supported by our findings. However, since we find that the mean bias is also nonstationarity, then the option underpricing is likely random and influenced by monetary policy changes and extreme economic events. Finally, our study reports evidence of nonstationarity bias for volatility scaling for the Canadian stock market. While some research in the econophytics literature has been conducted for selected samples of currency exchange rate volatility, there is need for more research on the econometric and economic impacts of volatility scaling for various financial time series in different markets. The implications for pricing of contingent claims beyond the generic Black-Scholes model or the allocation of economic capital based on VaR-type procedures may be profound.

[9] These results are similar to those found by Danielsson and Zigrand (2004) who examine the time scaling of risk when returns follow a lognormal stochastic process with a Poisson jump. Their objective is centered around rare events. They found that the square root formula underestimates risk and the bias increases with time horizon, jump intensity and confidence interval.

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Tables and Figures

Table 1: Descriptive statistics for daily S&P/TSX returns index

Statistic	r_t
Mean	0.0003
Standard Error	0.0001
Median	0.0006
St-Dev	0.0086
Kurtosis (*K*)	14.41**
Skewness (*S*)	-0.99 **
Minimum	-0.1201
Maximum	0.0865
Sample size	7055
Jarque-Bera (*JB*)	62138**

Note: ** Significant at the 1% level. Sample is formed from daily returns index beginning on January 03, 1977 and ending on December 31, 2004

Table 2: Test for serial correlation of the daily returns: modified Q-statistic

MQ(5)	MQ(10)	MQ(20)	MQ(30)	MQ(40)	MQ(50)
201.72**	205.02**	234.82**	249.37**	263.97**	284.02**

Note: **Significance at the 1% level. MQ(k) is the modified Q-statistic at lag k defined as the modified Q-statistic of Ljung and Box (1978) is defined as:

Table 3: BDS test statistic for raw data

m	ϵ/σ		ϵ/σ		ϵ/σ		ϵ/σ	
2	0.5	19.210**	1	19.736**	1.5	20.946**	2	23.565**
3	0.5	23.001**	1	22.000**	1.5	22.458**	2	24.452**
4	0.5	29.022**	1	24.127**	1.5	23.541**	2	25.654**
5	0.5	35.824**	1	29.395**	1.5	24.076**	2	26.357**
6	0.5	47.125**	1	32.456**	1.5	25.536**	2	24.951**
7	0.5	58.454**	1	38.881**	1.5	26.081**	2	24.852**
8	0.5	75.042**	1	41.125**	1.5	29.123**	2	24.545**

Note. m is embedding dimension, ε is the bound, * Significant at the 5% level.,** Significant at the 1% level. The critical values for BDS test are 1.96 for 5% and 2.58 for 1%.

Table 4. Modeling conditional heteroscedasticity in S&P/TSX daily returns.

AR(12)-EGARCH(1,1)	*Normal*		*Student-t*		*GED*	
Coefficient	*Value*	*p-value*	*Value*	*p-value*	*Value*	*p-value*
$\beta 0$	0.0002	0.0011	0.0004	0.0000	0.0004	0.0000
$\beta 1$	0.2316	0.0000	0.2304	0.0000	0.2221	0.0000
$\beta 2$	-0.0164	0.0862	-0.0326	0.0041	-0.0420	0.0000
$\beta 3$	0.0238	0.0274	0.0200	0.0533	0.0167	0.0479
$\beta 4$	-0.0172	0.0799	0.0014	0.4550	0.0107	0.1404
$\beta 5$	0.0244	0.0228	0.0277	0.0103	0.0295	0.0014
$\beta 6$	-0.0072	0.2796	-0.0121	0.1549	-0.0075	0.2215
$\beta 7$	-0.0230	0.0225	-0.0177	0.0656	-0.0098	0.1567
$\beta 8$	0.0045	0.3486	0.0015	0.4470	-0.0034	0.3617
$\beta 9$	-0.0081	0.2503	0.0036	0.3781	-0.0062	0.2588
$\beta 10$	0.0292	0.0044	0.0247	0.0136	0.0205	0.0155
$\beta 11$	-0.0263	0.0112	-0.0199	0.0374	-0.0141	0.0670
$\beta 12$	0.0438	0.0000	0.0327	0.0012	0.0252	0.0029
η	-0.4929	0.0000	-0.5031	0.0000	-0.4084	0.0000
λ_1	0.2134	0.0000	0.2184	0.0000	0.2065	0.0000
θ_1	0.9666	0.0000	0.9662	0.0000	0.9735	0.0000
ρ_1	-0.2155	0.0000	-0.1684	0.0001	-0.1728	0.0023
LM Test	9.906	0.6242	9.501	0.6599	13.198	0.3548
JB	6705	0.0000	7226	0.0000	8085	0.0000

Note: λ, θ, ρ, are the ARCH, GARCH, and leverage parameters respectively.

Table 5. BDS Test Statistics for Standardized Residuals from FIEGARCH Model

m	ϵ/σ		ϵ/σ		ϵ/σ		ϵ/σ	
2	0.5	0.509	1	0.664	1.5	0.105	2	-0.128
3	0.5	0.135	1	0.160	1.5	-0.323	2	-0.513
4	0.5	0.152	1	0.131	1.5	-0.122	2	-0.231
5	0.5	0.314	1	0.425	1.5	0.077	2	0.126
6	0.5	0.492	1	0.609	1.5	0.211	2	0.222
7	0.5	0.711	1	0.846	1.5	0.434	2	0.278
8	0.5	1.014	1	1.195	1.5	0.565	2	0.524

Note. Critical values for BDS test are 1.96 for 5% and 2.58 for 1%

Figure 1: Conditional volatility obtained from the EGARCH model under Student-t distribution.

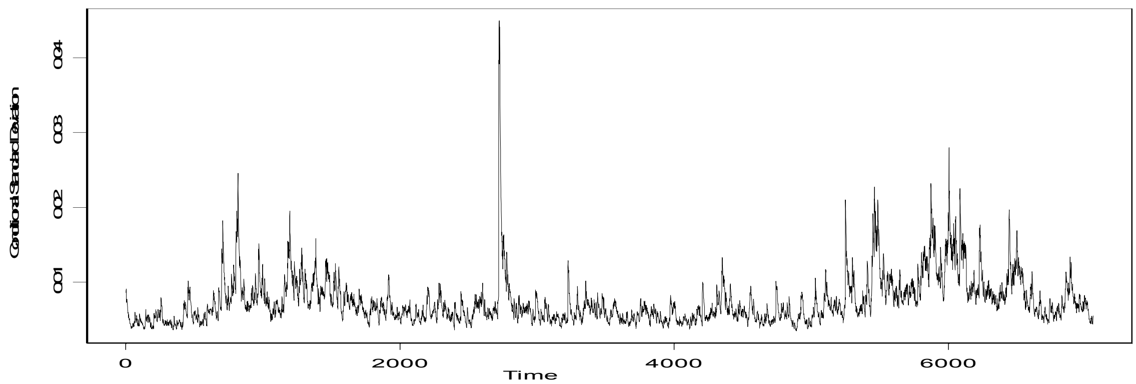
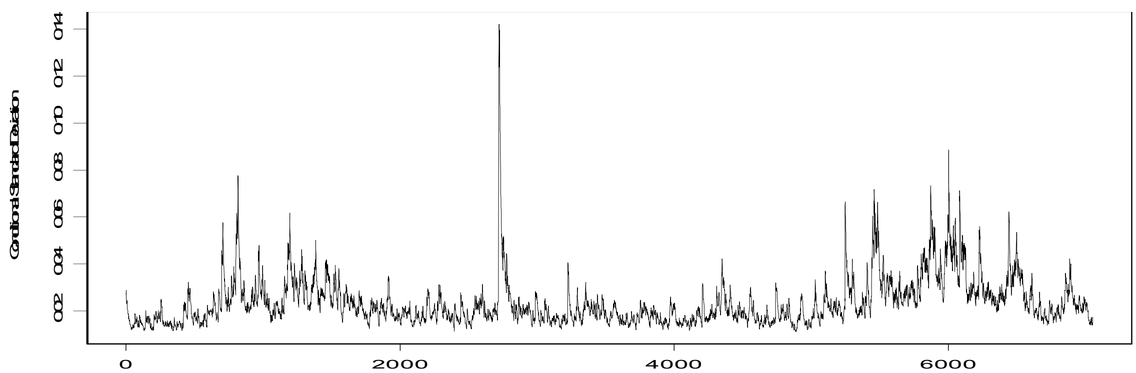


Figure 2: Scaled and true conditional volatility under normal distribution for n = 10

Scaled Conditional Volatility, n=10, Normal Distribution



True Conditional Volatility, n=10, Normal Distribution

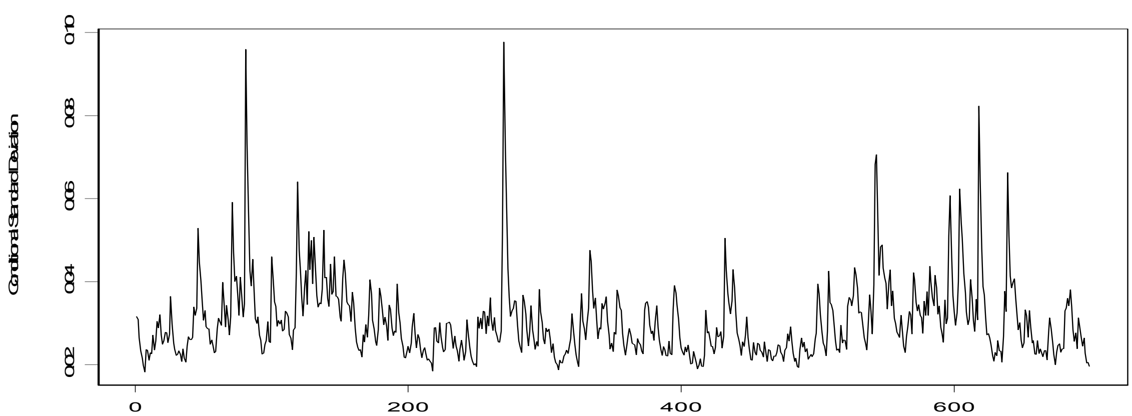


Table 6: 10-day scaled and true conditional volatilities: Sample partition based on monetary regime shifts over the period 1977-2004.

Scaled 10-day volatilities are obtained from daily volatilities multiplied by $\sqrt{10}$. True 10-day volatilities are estimated over a 10-day period.

All calculated are based on *i.i.d* residuals.

Panel A: Normal distribution.

	Total Sample		1977:1-1979:10		1979:10-1982:10		1982:10-1986:11		1986:12-1990:12		1991:1-1998:12		1999:12-2004:12	
	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True
Mean	0.0234	0.0304	0.0169	0.0279	0.0290	0.0378	0.0210	0.0284	0.0220	0.0303	0.0206	0.0282	0.0301	0.0322
St-Error	0.0001	0.0003	0.0001	0.0007	0.0003	0.0014	0.0002	0.0006	0.0004	0.0010	0.0002	0.0005	0.0003	0.0008
Median	0.0207	0.0281	0.0158	0.0267	0.0270	0.0352	0.0199	0.0272	0.0193	0.0286	0.0184	0.0255	0.0275	0.0309
St-Dev	0.0102	0.0092	0.0060	0.0040	0.0118	0.0091	0.0058	0.0054	0.0132	0.0104	0.0080	0.0078	0.0106	0.0097
Kurtosis	18.6200	10.9800	1.3346	4.0315	2.9934	7.7918	1.6504	0.4299	40.7579	18.6756	9.1718	7.4969	1.8880	5.9337
Skewness	3.0000	2.5200	1.2400	1.5675	1.4829	2.1962	1.2375	0.8905	5.7890	3.5645	2.6840	2.2329	1.2221	1.9716
Range	0.1310	0.0794	0.0208	0.0346	0.0627	0.0733	0.0303	0.0275	0.1299	0.0788	0.0607	0.0515	0.0746	0.0626

Panel B: Student-*t* distribution

	Total Sample		1977:1-1979:10		1979:10-1982:10		1982:10-1986:11		1986:12-1990:12		1991:1-1998:12		1999:12-2004:12	
	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True
Mean	0.0233	0.0301	0.0178	0.0297	0.0310	0.0390	0.0223	0.0305	0.0219	0.0300	0.0205	0.0281	0.0300	0.0319
St-Error	0.0001	0.0003	0.0002	0.0006	0.0003	0.0010	0.0002	0.0005	0.0004	0.0007	0.0002	0.0004	0.0003	0.0005
Median	0.0206	0.0286	0.0168	0.0288	0.0293	0.0385	0.0210	0.0291	0.0192	0.0286	0.0183	0.0262	0.0274	0.0310
St-Dev	0.0101	0.0066	0.0050	0.0040	0.0094	0.0089	0.0057	0.0051	0.0130	0.0074	0.0080	0.0058	0.0106	0.0065
Kurtosis	18.3700	3.5800	1.1451	2.4643	2.9537	3.2523	1.8508	0.1499	41.3673	9.4898	9.2482	4.6964	1.8224	0.8053
Skewness	2.9800	1.5000	1.1795	1.2675	1.4395	1.3373	1.2907	0.8281	5.8321	2.5171	2.6982	1.8749	1.2139	0.8916
Range	0.1304	0.0478	0.0211	0.0267	0.0649	0.0491	0.0315	0.0238	0.1289	0.0472	0.0601	0.0363	0.0731	0.0321

Panel C: Generalized Error Distribution (GED)

	Total Sample		1977:1-1979:10		1979:10-1982:10		1982:10-1986:11		1986:12-1990:12		1991:1-1998:12		1999:12-2004:12	
	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True	Scaled	True
Mean	0.0249	0.0321	0.0169	0.0280	0.0289	0.0364	0.0209	0.0287	0.0233	0.0321	0.0217	0.0300	0.0322	0.0339
St-Error	0.0001	0.0003	0.0001	0.0005	0.0003	0.0008	0.0002	0.0005	0.0004	0.0009	0.0002	0.0005	0.0003	0.0006
Median	0.0219	0.0303	0.0158	0.0273	0.0270	0.0361	0.0198	0.0277	0.0205	0.0304	0.0194	0.0278	0.0295	0.0333
St-Dev	0.0108	0.0078	0.0042	0.0039	0.0089	0.0070	0.0054	0.0047	0.0136	0.0088	0.0086	0.0068	0.0113	0.0079
Kurtosis	15.0200	5.8800	1.3359	1.3073	3.1483	1.8083	1.8722	0.2027	36.0812	13.0124	8.8929	5.6096	1.3214	2.1130
Skewness	2.7600	1.8600	1.2264	1.0612	1.4903	0.9709	1.2860	0.8950	5.4725	2.9657	2.6773	1.9710	1.1170	1.2536
Range	0.1301	0.0619	0.0207	0.0204	0.0634	0.0356	0.0310	0.0222	0.1284	0.0612	0.0630	0.0448	0.0736	0.0424

Table 7: Estimates of bias in scaling by square root of time as percentage error: Sample partition based on monetary regime shifts over the period 1977-2004.

An estimate of the bias as percentage error is defined as follows: $BIAS = 1 - \frac{Scaled\ Volatility}{True\ Volatility}$. This Table shows the mean bias (Mean), the median bias (Median). A positive value indicates that the conditional scaled volatility underestimates the true conditional volatility. ΔSD is the difference between the standard deviation of the true conditional volatility and the standard deviation of the scaled conditional volatility. A negative value of ΔSD indicates that the standard deviation of the scaled volatility overestimates the standard deviation of the true volatility.

		Total Sample	1977:1-1979:10	1979:10-1982:10	1982:10-1986:11	1986:12-1990:12	1991:1-1998:12	1999:12-2004:12
	10-Day							
Normal	Mean	0.2303	0.3924	0.2315	0.2603	0.2728	0.2690	0.0636
	Median	0.2633	0.4075	0.2329	0.2698	0.3236	0.2809	0.1084
	ΔSD	-0.1087	-0.336	-0.2303	-0.0716	-0.2593	-0.0360	-0.0965
Student-t	Mean	0.2259	0.4008	0.2064	0.2691	0.2711	0.2712	0.0605
	Median	0.2797	0.4155	0.2393	0.2765	0.3301	0.3036	0.1164
	ΔSD	-0.5303	-0.1945	-0.0589	-0.1086	-0.7556	-0.3682	-0.6204
GED	Mean	0.2243	0.3973	0.2077	0.271	0.2739	0.2757	0.0501
	Median	0.2772	0.4209	0.2512	0.2852	0.3266	0.3021	0.1135
	ΔSD	-0.3846	-0.0785	-0.2779	-0.1396	-0.5374	-0.2577	-0.4329
	30-Day							
Normal	Mean	0.2807	0.4641	0.5473	0.1318	0.3359	0.2841	0.1681
	Median	0.3571	0.4871	0.5592	0.1546	0.4155	0.3549	0.2346
	ΔSD	-1.25	-0.7043	-0.0201	-0.7807	-3.0056	-2.4595	-2.2062
Student-t	Mean	0.2962	0.4667	0.2653	0.3734	0.3412	0.2883	0.1719
	Median	0.3643	0.4873	0.2925	0.3791	0.4201	0.3579	0.2378
	ΔSD	-1.1084	-0.6812	-1.256	-0.0632	-2.9461	-2.4415	-2.1817
GED	Mean	0.2491	0.4372	0.2114	0.3334	0.2975	0.2443	0.1102
	Median	0.3232	0.4548	0.233	0.3409	0.3810	0.3180	0.1805
	ΔSD	-1.241	-0.7383	-1.3738	-0.1135	-3.1389	-2.6935	-2.4028
	90-Day							
Normal	Mean	0.2848	0.4898	0.2582	0.4335	0.3169	0.2186	0.1521
	Median	0.3788	0.5173	0.308	0.4515	0.3907	0.3019	0.2088
	ΔSD	-0.8155	-0.495	-1.3072	-0.4423	-3.1774	-2.5935	-1.7203
Student-t	Mean	0.2889	0.4922	0.2622	0.4357	0.3169	0.2186	0.1521
	Median	0.3818	0.5175	0.3073	0.4537	0.3907	0.3019	0.2088
	ΔSD	-0.7976	-0.4747	-1.2706	-0.4543	-3.1774	-2.5935	-1.7203
GED	Mean	0.2889	0.4642	0.2081	0.3997	0.3169	0.2186	0.1521
	Median	0.3818	0.487	0.249	0.4201	0.3907	0.3019	0.2088
	ΔSD	-0.7976	-0.5247	-1.3892	-0.5231	-3.1774	-2.5935	-1.7203