Forecasting Time-varying Covariance with a Range-Based Dynamic Conditional Correlation Model

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Abstract

This paper proposes a range-based Dynamic Conditional Correlation (DCC) model, which is an extension of Engle's (2002) return-based DCC model. The efficiency of the range data in volatility estimation is documented in Parkinson (1980), Alizadeh, Brandt, and Diebold (2002), Brandt and Jones (2005), and Chou (2005, 2006), among others. It is hence natural to consider the implications of this result in estimation of multivariate GARCH models. In the DCC model, the conditional correlation coefficients are estimated using a dynamic model for producing pair-wise return series, each normalized by their conditional standard deviations. The conditional standard deviation is calculated using a univariate GARCH for the return series.

We use the Conditional Autoregressive Range (CARR) model of Chou (2005) as an alternative to the univariate GARCH in the DCC first-step estimation. We, therefore, construct a range-based DCC model. The substantial gain in efficiency of volatility estimation can induce an efficiency gain in estimation of the series of time-varying covariances. For the purpose of enforcing our conclusion in the empirical study, besides dissecting of return-based and range-based DCC models, models of MA100, EWMA, CCC and BEKK are also incorporated. This paper uses three data sets for empirical analyses: the S&P500 and Nasdaq stock indices, and the 10-year Treasury bond yield. Both in-sample and out-of-sample results indicate some consistent inferences. Of all the models considered, the range-based DCC model is largely supported in terms of precision in estimating and forecasting the covariance matrices.

Keywords: DCC model, CARR model, range, dynamic correlation, covariance, volatility

I. Introduction

It is of primary importance in the practice of portfolio management, asset allocation and risk management to have an accurate estimate of the covariance matrices for asset prices. When valuing derivatives, forecasts of volatilities and correlations over the whole life of the derivatives are usually required. Judging from the past literature, the univariate ARCH/GARCH family of models provides effective tools to estimate volatilities of individual assets' prices. Tailored to the needs of different asset classes, these various models have achieved remarkable success; please see Bollerslev, Chou, and Kroner (1992), and Engle (2004), for a review. However, estimating the covariance or correlation matrices of multiple variables, especially large sets of asset prices, is still an active research issue. Early attempts include the VECH model[1] of Bollerslev, Engle, and Wooldridge (1988), the BEKK model[2] of Engle and Kroner (1995), and the constant correlation model of Bollerslev (1990), among others. The constant correlation model is too restrictive in that it imposes stringent constraints whereby the dynamic structure of the covariance is completely determined by individual volatilities. The VECH and the BEKK models are, however, more flexible in that they allow time-varying correlations. While the BEKK parameterization for a bivariate model involves 11 parameters, for higher-dimensional systems, the additional parameters make in the BEKK model estimation very difficult.

In a series of papers, Engle and Sheppard (2001), Engle (2002), and Engle, Cappiello, and Sheppard (2003) provide another solution to this problem by using a model referred to as the Dynamic Conditional Correlation Multivariate GARCH (henceforth DCC). Intuitively, the conditional covariance estimation for two variables is simplified by estimating univariate GARCH models for each asset's variance process. Then, the estimation of the time-varying conditional correlation is made using the transformed standardized residuals. A meaningful and strong performance of this model is reported in these studies. Other econometric methods for estimating the time-varying correlation are proposed by Tsay (2002) and by Tse and Tsui (2002), too.

The objective of this article is to propose an alternative to return-based DCC

[1] The n-dimensional VECH model is written as vech(H_t)=A+B vech($\xi_{t-1}\xi_{t-1}^{'}$)+C vech(H_{t-1}), where H_t is the conditional covariance matrix at time t and vech(H_t) is the vector that stacks all the elements of the covariance matrix.

[2] A general parameterization that involves the minimum number of parameters while imposing no cross equation restrictions and ensuring positive definiteness for any parameter value is the BEKK model, named after Baba, Engle, Kraft, and Kroner who wrote the preliminary version of Engle and Kroner (1995).

analysis. In this paper, we consider a refinement of the return-based DCC model by utilizing the high/low range data of asset prices. In estimating the volatility of asset prices, there is a growing awareness of the fact that the range data of asset prices can provide sharper estimates and forecasts than the return data based on close-to-close prices. Studies that provide supporting evidence include Parkinson (1980), Garman and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1992) and, more recently, Gallant, Hsu, and Tauchen (1999), Yang and Zhang (2000), Alizadeh, Brandt, and Diebold (2002), Brandt and Jones (2005), Chou (2005, 2006) and Chou, Wu, and Liu (2004). Chou (2005) proposed the Conditional Autoregressive Range (henceforth CARR) model which can capture the dynamic volatility process and has obtained some insightful evidence in terms of real transactional data. In other words, a range-based volatility model serves as an useful alternative to the return-based volatility model in describing the process of volatility. In light of the success of the range-based univariate volatility models, it is natural to inquire whether this estimation efficiency can be extended and incorporated into a multivariate framework, in this case the DCC model.

The remainder of the study is laid out as follows. Section 2 introduces the framework of the bivariate models for estimating the covariance process, especially for the return-based and range-based DCC models. Section 3 describes the empirical data used and discusses the empirical results. The conclusion and directions for future studies are given in section 4.

II. Various Covariance Estimations and the DCC Model

This section provides a brief overview of methods used to describe the current level of covariance. Conventionally, the conditional covariance estimation between two random variables r_1 and r_2 with zero means have been defined as:

$$COV_{12,t} = E_{t-1}(r_{1,t} r_{2,t}) , \tag{1}$$

Time-varying covariance parameters between asset returns (ex: r_1 and r_2) are useful in financial analysis. For example, they can be used to estimate the time-varying beta of the market model of a return series. By this definition, the information of the conditional covariance is derived from previous trading data. However, such an expression may create some doubts. The latent questions can be divided into two segments, namely, too early or old data get injected; and equal weights are assigned for every previous leg of observation. To deal with the former, this paper works with a moving average with a 100-week window, MA100, which is rich enough to be relevant and yet simple enough to permit a streamlined exposition:

$$COV_{12,t} = \frac{1}{100} \sum_{s=t-100}^{t-1} r_{1,s} r_{2,s} , \tag{2}$$

Intuitively, it makes sense to attach more weight to recent data. Going by this, we introduce an exponentially weighted moving average (EWMA) model where the weights decrease exponentially as we move back through time. Exponential smoothing is used to model the unobservable variables for volatility in JP Morgan's RiskMetrics, too. The EWMA model has an attractive feature in that relatively little data need to be stored. Exponential averages assign the most weight to the most recent observations, with weights declining exponentially as observations go back in time. It turns out that the EWMA model for covariance estimation can be illustrated as follows.

$$COV_{12,t} = (1-\lambda) \sum_{s=1}^{t-1} \lambda^{t-s-1} r_{1,s} r_{2,s} , \tag{3}$$

Usually, the smoothing parameter λ lies between zero and unity. The value of λ governs how sensitive the estimate of the current variable is to percent changes in the most recent period. The RiskMetrics approach uses exponential moving averages[3] to estimate future volatility because it believes the method responds rapidly to market shocks.

Later, Bollerslev (1990) proposed the Constant Correlation Coefficient (henceforth CCC) multivariate GARCH specification, where univariate GARCH models are estimated for each asset and then the corresponding correlation matrix is estimated, using the standard MLE correlation estimator, by transforming the residuals using their estimated conditional standard deviations. An illustration of the CCC model is shown below, in brief. Assume that $k \times 1$ vector of asset returns r_t is conditionally normal with mean zero and covariance matrix H_t; hence, H_t can be decomposed as follows:

$$H_t = D_t R D_t , \tag{4}$$

Where R is the sample correlation matrix and D_t is the $k \times k$ diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{i,t}}$ on the i[th] diagonal and $\sqrt{h_{i,t}}$ is the square root of the estimated variance. The assumption of constant correlation makes estimating a large model feasible and ensures that the estimator is positive definite, simply requiring each univariate conditional variance to be non-zero and the correlation matrix to be of full rank. Under such a situation, we can obtain the estimate of the conditional covariance, based on information regarding the fixed correlation and the product of the two conditional standard deviations. The type of model is quite easy to estimate, by the quasi-maximum likelihood function.

The conditional variance-covariance matrix may also be used to build a multivariate ARCH model. This approach has been adopted by Engle and Kroner(1995), who proposed the so-called BEKK (Baba-Engle-Kraft-Kroner) model. The parameters, however, easily diverge when a model of the type of the full-rank BEKK (FBEKK for short) model is adopted. In the related literature, the Diagonal BEKK (hereafter DBEKK) model is more popular due to its property of convergence

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[3] The RiskMetrics database uses the exponentially-weighted moving average model with λ =0.94 for updating daily volatility estimates. J.P. Morgan found that, across variant market variables, this value of λ results in forecasts of the volatility that come closest to the realized volatility. Following J.P. Morgan's suggestion, the variable λ equals 0.94 for the time being in the later empirical discussion.

of parameters used in empirical research. Particularly, the DBEKK is more well-organized in estimating than the FBEKK model, when the number of samples is a constraint[4]. Let us consider the bivariate for the diagonal BEKK model, shown as below:

$$H_t = \begin{bmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \quad (5)$$

Although the CCC model is meaningful, the setting of constant conditional correlations can sometimes be too restrictive and the estimators in the constant correlation setting, as proposed, do not offer a rule to construct consistent standard errors, using the multi-stage estimation process. Another drawback of the constant-correlation model is that the correlation coefficient tends to change over time in real applications. Engle (2002) extended the simple CCC model to the more complete DCC model. The DCC model is a new class of estimator that both preserves the ease of estimation of the CCC model and yet allows for non-constant correlations. The DCC model preserves the parsimony of the univariate GARCH model of individual assets' volatilities with a simple GARCH-like time varying correlation. The DCC model differs from the CCC model mainly in that it allows the correlation matrix R to be changed over time. Accordingly, we can write the DCC model as

$$H_t = D_t R_t D_t \,, \quad (6)$$

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2} \,. \quad (7)$$

Where, D_t is defined as in equation (4) and R_t is the possibly time-varying correlation matrix.

$$Q_t = S \circ (\iota\iota' - A - B) + A \circ Z_{t-1} Z_{t-1}' + B \circ Q_{t-1} \,. \quad (8)$$

In equation (8), A and B are parameters and o denotes the Hadamard matrix product operator, i.e. element-wise multiplication. The DCC model was constructed to

[4] In empirical result for this paper is inclined to support similar inference, too.

permit for two-stage estimation of the conditional covariance matrix H_t. Briefly speaking, during the first step, univariate volatility models are assigned for each of the assets and estimates of $h_{i,t}$ are obtained. In the second step, asset returns transformed by their estimated standard deviations estimating the parameters of the conditional correlation. The symbol ι is a vector of ones and S is the unconditional covariance of the standardized residuals. Finally, $Z_t = D_t^{-1} \times r_t$ denotes the standardized but correlated residual. The variable r_t is the symbol for the returns of the assets. The returns can be either mean zero or residuals from a filtered time series, i.e.

$$r_t \left| I_{t-1} \sim N(0, H_t) \right. . \tag{9}$$

Conditional variances of the components of Z_t are, in other words, equal to 1, but the conditional correlation matrix is given by variable R_t. If A and B are zero, then the DCC model can revert to the results of the CCC model. It is interesting and important to recognize that although the dynamics of the D_t matrix has usually been structured as a standard univariate GARCH model, it can be easily extended to many other types of models. For instance, one could adopt the EGARCH or GJR-GARCH models to capture the asymmetric phenomenon in the real volatility process or use the FIGARCH model to allow for the long memory volatility processes. In order to verify if the specification selected adequately fit the DCC model, the next paragraph will propose to adopt the Conditional Autoregressive Range (CARR) model of Chou (2005) as an alternative. Details will be given later in the section.

As A and B are diagonal parameter matrices, the condition of positive definite for covariance matrix will be satisfied. Literature shows that if A, B, and $(\iota\iota' - A - B)$ are positive semi-definite, then Q_t will also be positive semi-definite. If any one of the matrices is positive definite, then Q_t will also be so. For the ijth element of R_t, the conditional correlation matrix is given by $q_{ij,t} / \sqrt{q_{ii,t} q_{jj,t}}$. The conditional covariance can then be expressed using the product of conditional correlation between these two variables and their individual conditional standard deviations. For a good overview of properties of the DCC, please also see Engle (2002). The log-likelihood of this estimator is straightforward. One simply maximizes the log-likelihood:

$$
\begin{aligned}
L &= -\frac{1}{2}\sum_{t=1}^{T}\left(k\log(2\pi)+\log\left|H_t\right|+r_t{}'H_t^{-1}r_t\right)\\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(k\log(2\pi)+\log\left|D_tR_tD_t\right|+r_t{}'D_t^{-1}R_t^{-1}D_t^{-1}r_t\right)\\
&= -\frac{1}{2}\sum_{t=1}^{T}\left(k\log(2\pi)+2\log\left|D_t\right|+\log\left|R_t\right|+Z_t{}'R_t^{-1}Z_t\right)
\end{aligned}
\tag{10}
$$

Where $Z_t \sim N(0,R_t)$ denote the univariate GARCH standardized residuals. Following Engle (2002)'s argument, one can perform the estimation by means of a two-step process. Maximizing the likelihood function can yield consistent parameter estimates. This approach is called the quasi-maximum likelihood estimation (QMLE). The benefits of QMLE are its simplicity and consistency. Its shortcomings are that the estimates are inefficient, even asymptotically, and more importantly, its small-sample properties are suspect. (See Hafner and Franses (2003), for a review.) Let the parameters in D_t be denoted by θ and the additional parameters in R_t be denoted by the Greek letter ϕ. After doing so, one can separate the log-likelihood function into two parts:

$$
L(\theta,\phi)=L_V(\theta)+L_C(\theta,\phi)\,. \tag{11}
$$

The former term represents the volatility part:

$$
L_V(\theta)=-\frac{1}{2}\sum_{t}\left(n\log(2\pi)+\log\left|D_t\right|^2+r_t{}'D_t^{-2}r_t\right). \tag{12}
$$

The latter term can be viewed as the correlation component:

$$
L_C(\theta,\phi)=-\frac{1}{2}\sum_{t}\left(\log\left|R_t\right|+Z_t{}'R_t^{-1}Z_t-Z_t{}'Z_t\right). \tag{13}
$$

Following the recipe for the first stage, we can pick up a suitable θ easily, which satisfies equation (12) and is maximized after the estimate of $\hat{\theta}$ is computed. Subsequently, in the second stage, the correlation part in equation (13) can be maximized with respect to optimized θ and ϕ simultaneously. Consequently, the formidable task of maximizing equation (11) is attainable. Estimates for $\hat{\theta}$ and $\hat{\phi}$ are useful in subsequent dissections.

When the specific GARCH model is fitted, the term of volatility in the likelihood function can demonstrated as below:

$$L_V(\theta) = -\frac{1}{2}\sum_t \sum_{i=1}^{k}\left(\log(2\pi) + \log(h_{i,t}) + \frac{r_{i,t}^2}{h_{i,t}} \right). \qquad (14)$$

By the same token, if D_t is determined by a CARR specification, then the likelihood function of the volatility term will be

$$L_V(\theta) = -\frac{1}{2}\sum_t \sum_{i=1}^{k}\left(\log(2\pi) + 2\log(\lambda_{i,t}^{*}) + \frac{r_{i,t}^2}{\lambda_{i,t}^{*2}} \right), \qquad (15)$$

Where $\lambda_{i,t}^{*}$ denotes the conditional standard deviation as computed from a scaled expected range, using the CARR model of Chou (2005).

The second part of the likelihood function will be used to estimate the parameters for correlations. As the squared residuals are not dependent on these parameters, they will not appear in the first-order conditions and can be neglected. A simple transformation of the two-stage framework to maximize the likelihood function is achieved. Apparently,

$$\hat{\theta} = \arg\max\{L_V(\theta)\}, \qquad (16)$$

and then extract this value $\hat{\theta}$ as given, into the second step,

$$\max_{\phi}\{L_C(\hat{\theta},\phi)\}. \qquad (17)$$

It is shown in Engle and Sheppard (2001) that under some regularity conditions, the condition for consistency will be satisfied. Maximization of equation (13) will be a function of the parameter estimates from equation (12). These conditions are similar to those given in White (1994), where the asymptotic normality and the consistency of the two-step QMLE estimator are established.

Another theoretical justification for the above result appeared in Engle (2002). Engle (2002) referred to the work of Newey and McFadden (1994), whereby in Theorem 6.1, a formulated two-step GMM problem can be applied to this model. Consider the moment condition corresponding to the first step as being $\nabla_{\theta}\{L_v(\theta)\}$=0. The moment condition corresponding to the second step is $\nabla_{\phi}\{L_C(\hat{\theta},\phi)\}$. Under standard regularity conditions, which are given as conditions i) to v) in Theorem 3.4

of Newey and McFadden (1994), the parameter estimates will be consistent and asymptotically normal, with a covariance matrix of a familiar form. This matrix is the product of two inverted Hessians around an outer product of scores. Details of this proof can be found in Engle and Sheppard (2001).

The DCC model is a new type of multivariate and can fit the GARCH or CARR model in the first stage, which is particularly convenient for complex dynamic systems. The DCC method first estimates volatilities for each asset and computes the standardized residuals. For bivariate cases, one can use the following GARCH and CARR structures to perform the first step. The covariance series is then estimated between these, using a maximum likelihood criterion and one of several models for the correlations.

For the GARCH volatility structure (return-based conditional volatility model), the function form can be illustrated as below:

$$r_{k,t} = \varepsilon_{k,t} \qquad \varepsilon_{k,t} \mid I_{t-1} \sim N(0, h_{k,t}) \quad , \qquad k=1,2$$

$$h_{k,t} = \omega_k + \alpha_k \varepsilon^2_{k,t-i} + \beta_k h_{k,t-1}, \tag{18}$$

$$z^a_{k,t} = r_{k,t} / \sqrt{h_{k,t}} \; .$$

If the volatility model is replaced by the CARR framework (range-based conditional volatility model), the structure can be expressed as:

$$\Re_{k,t} = u_{k,t} \qquad u_{k,t} \mid I_{t-1} \sim \exp(1;\cdot)\,,\; k=1,2$$

$$\lambda_{k,t} = \omega_k + \alpha_k \Re_{k,t-1} + \beta_k \lambda_{k,t-1}, \tag{19}$$

$$z^c_{k,t} = r_{k,t} / \lambda^*_{k,t} \text{ , where } \lambda^*_{k,t} = adj_k \times \lambda_{k,t} \text{ , } adj_k = \frac{\overline{\sigma}}{\overline{\hat{\lambda}_k}},$$

Where $\Re_{k,t}$ is the high/low range in logarithm of the k^{th} asset during the time interval t, $\overline{\sigma}$ and $\overline{\hat{\lambda}_k}$ are the unconditional variance of the return series and the sample mean of the estimated conditional range of the series k respectively. This is a special case of the multiplicative error model of Engle (2002). The specification of the exponential distribution of the disturbance term provides a consistent, if inefficient, estimator of the parameters. For specific discussions, see Chou (2005); also for a review.

In the next analysis, we explore the structure of the DCC model under the bivariate condition. For the bivariate case, the DCC model can be expressed by the following equation. According to statements from Engle (2002), the model setting below is called as the standard DCC with mean reversion. This is most easily seen by adjusting the general expression in equation (6).

Under the framework of the bivariate case, the DCC can be constructed by the following equation.

$Q_t = S \circ (\iota\iota' - A - B) + A \circ Z_{t-1}Z_{t-1}' + B \circ Q_{t-1}$, or

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z_{2,t-1}^2 \end{bmatrix} + b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1} \end{bmatrix} \quad (20)$$

where $\bar{q}_{12} = \frac{1}{T}\sum_{t=1}^{T} z_{1,t} z_{2,t}$.

As another type of DCC is called the Integrated DCC, we arrange its dynamic structure at footnote[5] for reference.

Since financial risk is commonly assessed in terms of covariance, the ability of providing accurate forecasts of future risks acquires great importance. Like the specific property of volatilities, the covariance matrices are also unobservable. Here we use daily data to construct the proxies for the weekly-realized covariance observations. The concept of the realized volatility has been used productively by French, Schwert, and Stambaugh (1987) and Andersen et al. (2001). The realized volatility is nothing more than the sum of squared high-frequency returns over a given sampling period. For instance, one calculates a daily realized variance series by summing up, each day, a sequence of squared intraday returns. The purpose behind doing this is to extract the values of the so-called “measured covariances”, denoted by MCOV, as one kind of benchmark for determining the relative performance of the return-based DCC model and the range-based DCC model, for the time being. For

[5] For the bivariate I_DCC can shown briefly as $Q_t = A \circ Z_{t-1}Z_{t-1}' + (1-A) \circ Q_{t-1}$, or

$$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{12,t} & q_{22,t} \end{bmatrix} = a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{1,t-1}z_{2,t-1} & z_{2,t-1}^2 \end{bmatrix} + (1-a)\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{12,t-1} & q_{22,t-1} \end{bmatrix}$$

distinguishing the forecasting abilities of the return-based DCC model and the range-based DCC model, as in Taylor (2004), we still use root mean square error (RMSE) and mean absolute error (MAE) as two criterions for comparison.

In performing a comparison of the in-sample data during subsequent empirical analysis of the covariance matrices, several related and conventional models are included - MA100, EWMA[6] with $\lambda = 0.94$, CCC, and BEKK models.

For completeness, we also perform out-of-sample forecast comparisons. It is very straightforward to derive the formulation for computing the out-of-sample conditional correlation for a DCC specification. Out of sample forecasts of the DCC-type models for correlation can be obtained using the standard backward iterative approach; given T as the sample size, T+1st observation will be obtained.

At time T, the out of sample forecast for conditional correlation in the period (T+1) is presented by

$$\begin{bmatrix} q_{1,T+1} & q_{12,T+1} \\ q_{12,T+1} & q_{2,T+1} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + a\begin{bmatrix} z_{1,T}^2 & z_{1,T}z_{2,T} \\ z_{1,T}z_{2,T} & z_{2,T}^2 \end{bmatrix} + b\begin{bmatrix} q_{1,T} & q_{12,T} \\ q_{12,T} & q_{2,T} \end{bmatrix} \tag{21}$$

where $\rho_{T+1} = q_{12,T+1} / \sqrt{q_{1,T+1}q_{2,T+1}}$.

The out of sample prediction for correlation for the period (*T*+*h*), where $h \geq 2$, can be expressed as below:

$$\begin{bmatrix} q_{1,T+h} & q_{12,T+h} \\ q_{12,T+h} & q_{2,T+h} \end{bmatrix} = (1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1 \end{bmatrix} + (a+b)\begin{bmatrix} q_{1,T+h-1} & q_{12,T+h-1} \\ q_{12,T+h-1} & q_{2,T+h-1} \end{bmatrix} \tag{22}$$

In addition to the range-based and return-based DCC models, MA100 , EWMA, CCC and BEKK models are introduced for an out-of-sample predictive comparison[7]. Empirically speaking, the indices of RMSE and MAE are still used as indications for comparison of preciseness, too.

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[6]The estimate of λ is 0.94 approximately for three different time series data that we adopted in this study.

[7] It is also intuitively clear that the out-of-sample forecasts for the covariance are all constant in the EWMA model.

III. Comparison of Various Conditional Covariance Forecasts

The main data employed in this study comprise 626 weekly observations on the S&P500 Composite (henceforth S&P500), the Nasdaq stock market index, and the yield for 10-year treasury bond (Tbond) spanning the period from January 3, 1994 to December 30, 2005.[8] Furthermore, daily market observations are used to construct the series of the measured, or the so-called realized, covariances in the past literature. We have retrieved range and return data for the entire period from Yahoo's database (www.yahoo.com/finance).

It is worth taking a look at some descriptive statistics. Panels A, B, and C in Figure 1 demonstrate the weekly data patterns for the time-series of the S&P500 stock market index, the Nasdaq index and the yield to maturity for the 10-year Tbond over the sample period. We compute Tbond returns to be the negative changes in the 10-year benchmark yield to maturity as in Engle (2002). Additionally, Table 1 provides summary statistics for weekly continuously compounded returns and the weekly ranges for these indices.

Initially, the sample period for daily data from 1/3/1994 to 12/30/2005 is extracted. Totally, have 3023 daily data for model fitting with the return-based DCC, the range-based DCC and the Diagonal BEKK (DBEKK), respectively. In the meanwhile, each daily implied covariance is collected in this stage. Sequentially, it is easy to get the individual weekly implied estimates for covariance series, followed by the computation below.

$$MCOV_t = \sum_{i=1}^{\tau} \text{cov}_t^i \qquad (23)$$

Where cov_t^i denotes daily implied covariance at time t with model i. Here, we introduce different models - return-based DCC model, range-based DCC model and DBEKK model.

The realized covariance can be expressed as:

$$MCOV_t = \sum_{i=1}^{\tau} (r_{1t}^i \times r_{2t}^i) \qquad (24)$$

Where r_{kt}^i denotes return for k^{th} asset on i^{th} day during the corresponding week t^{th}.

[8] Due to the initial bond data for daily high and low prices offered by yahoo finance database is collected from October 29, 1993. So we choose January 1, 1994 as the beginning for our analysis.

This expression is a direct extension of the concept of the realized volatility of Andersen et al. (2000), too.

Checking the Figure 2 to 4, we depict the different covariance patterns between S&P 500, Tbond and Nasdaq series for DBEKK model, return-based DCC model and range-based DCC model, respectively. Some useful insights can be obtained from these figures. We can find that both the realized volatility and implied volatility are able to play a proxy for realized covariance. Observing from these figures, there seem to exist strong interactions around these variables. Intuitively speaking, the oscillation of the realized volatility pattern will be an informative and useful explanatory variable for these models.

A. In-sample forecast comparison for covariance

In this section, we present the empirical results of using the in-sample data, that is, the forecast performances are constructed and measured using the same database. Mainly, we exhibit the in-sample forecasting ability of the return-based DCC model, the ranged-based DCC model and some related models for the purpose of performance comparison. As for the parameters fitted for DCC model, we have estimated and arranged them in Table 2. Due to the procedure for parameters estimating under the DCC setting, we have to cope with two inherent stages. In the first stage, one can utilize the GARCH model fitted by return, or the CARR model fitted by range, with individual assets, for standardized residuals. Afterwards, bring these standardized residuals series into the second stage for dynamic conditional covariance estimating.

Table 3 illustrates some brief results of covariances estimated for in-sample prediction, based on different econometrical models that we have mentioned previously. We drew clear inference from the three panels in Table 3 to the effect that they all appeared to be more accurate in the range-based DCC model than in the other five models, regardless of what criterion[9] was adopted. This appears to be consistent not only in RMSE but in MAE too. The worst performance in predicting of covariance under in-sample analysis, one can judge from the results of RMSE and MAE in Table 3, is the MA100.

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[9] Here we follow the conventional evaluated indicators with RMSE and MAE as benchmark for the time being.

We refer to some insightful versions associated with various forecasting models for covariance in-sample situations hereinbelow.

Apparently, excepting the MAE indicator of forecasting realized covariance for Nasdaq and Tbond in Panel C, the range based DCC model is more appropriate than the others, no matter what RMSE or MAE indicators are adopted. Generally speaking, there are no significant differences in covariance forecasting performance between return-based DCC model and DBEKK model under the in-sample context. In addition to the range-based DCC approach, predicting results of the CCC model proposed by Bollerslev (1990) are relatively precise in Panel A. One reasonable conjecture is the correlation relationship between S&P and Nasdaq - both of them are stock composites and hence the pattern of their correlation over time is more stable than other combinations. One can observe an analogous phenomenon from Figure 2 to Figure 5. In the composition of S&P and Tbond or Nasdaq and Tbond, we can easily find their individual patterns, which appear to be reverting to the phenomenon. Under such a situation, the method of EWMA is better than that of the CCC model in covariance forecasting. Accordingly, it seems inappropriate to assume that the correlation parameter between different assets is constant over time. Besides, regardless of any composition, the empirical results of MA100 are the worst in depicting the pattern of covariance. We also plotted corresponding covariance patterns with various compositions from Figure 5 to Figure 7 for completeness.

B. Out-of-sample forecast comparison

In the same way, we can assess the out-of-sample forecasting performance for different models by using RMSE and MAE, discussed in the previous in-sample comparison. Given that the data set contains a total of 626 usable observations, it is possible to use a holdback period of observations. This way, there are 400 observations in each estimated model and 223 out-of-sample forecasting values for comparison. Here, the rolling sample approach for out-of-sample measurement is adopted. We show one period ahead of out-of-sample forecasting results for covariance in Table 4. We also consider two periods and four periods ahead out-of-sample forecast performance and illustrates the results in Table 5 and Table 6.

We obtain a consistent conclusion for covariance prediction’s performance based on different competitive models. Various forecasting results for covariance with different selected trading markets are presented in Panel A, B and C from Table 4 to Table 6. Almost all of the inferences demonstrate an overwhelming phenomenon, namely, the range-based DCC approach dominates other methods in accuracy in out-of-sample forecasting. Except for the realized volatility with the model of MA100 as forecasted benchmark, the results in Table 4 to Table 6 appear a trend, namely, that the forecasting errors are proportion to the forecasted periods. Most results are similar to the inference drawn from in-sample forecasting. Roughly speaking, from observing Figure 8 to Figure 10, it seems that the range-based DCC model has the most appropriate realized covariance fitting, at the first glance.

Exploring other latent characteristics of out-of-sample forecasting, the MA100, among these competitive models, is the worst one again. For the CCC model used in out-of-sample forecasting, the prediction performance looks the same as in-sample's situation, though the range-based DCC model outperforms the return-based DCC model in prediction abilities. However, the return-based DCC model, in covariance prediction, seems better than the DBEKK model. It is surprising that the EWMA model, holding constant out-of-sample forecast, still performs well in different period ahead forecasts. Besides, in the combination of S&P and Nasdaq or S&P and Tbond, the EWMA model performs better than other forecasting models, in addition to the range-based DCC approach.

In view of in-sample and out-of-sample empirical results, we can not clearly put all forecasting models in a proper order. However, it is undoubted that the range-based DCC model possesses the optimum forecasting power in covariance.

IV. Conclusions

In this paper, a brand-new estimator of the time-varying covariance matrices is proposed, utilizing the range data that combines the CARR model proposed by Chou (2005) with the framework provided by Engle (2002)'s DCC model. The advantage of this range-based DCC model, in terms of its ability to outperform the standard return-based DCC model, hinges on the relative efficiency of the range data over the return data in estimating volatilities. Using weekly data of S&P500, Nasdaq and 10-year treasury bond, we find consistent results - the range-based DCC model outperforms the return-based models in estimating and forecasting covariance matrices, both in-sample and out-of-sample analysis.

In addition to using conventional realized covariance for the purpose of comparison, we introduce the concepts of implied covariance, which is derived from DCC and DBEKK models for benchmarking robustness. Nonetheless, no matter what realized covariance or implied covariance is embedded in DCC or DBEKK models, we obtain a consistent conclusion that the range-based DCC approach is the best one for covariance prediction.

Although we just apply this estimator to the bivariate systems, it can be applied to larger systems in a manner similar to the application of the return-based DCC model structures; this has already been demonstrated in Engle and Sheppard (2001). Future research that adopts more diagnostic statistics or tests based on value-at-risk calculations as proposed by Engle and Manganelli (2004) will surely be useful. Other applications such as estimating the optimal portfolio weighting matrices and calculating the dynamic hedge ratio in the futures market will also bear fruit.

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Table 1: Summary Statistics for the Returns and Ranges of Weekly S&P500, Nasdaq Indices, and Tbond Yield, 1994-2005

	S&P500		Nasdaq		Tbond	
	Return	Range	Return	Range	Return	Range
Mean	0.156	3.263	0.166	4.844	0.041	3.893
Median	0.281	2.707	0.316	3.881	0.000	3.533
Maximum	7.492	14.534	17.377	31.499	7.756	16.593
Minimum	-12.330	0.707	-29.175	0.927	-12.625	0.657
Std. Dev.	2.241	1.818	3.623	3.384	2.647	1.933
Skewness	-0.506	1.697	-1.049	2.286	-0.597	1.798
Kurtosis	5.978	7.507	11.405	11.893	4.673	8.507
Jarque-Bera	257.547	828.895	1954.311	2604.271	110.025	1126.350

Notes: The ranges and returns for stock indices are computed by $100\times\log(p^{high}/p^{low})$ and $100\times\log(p_t^{close}/p_{t-1}^{close})$, respectively. The ranges and returns for the 10-year Treasury bond are inferred by $100\times\log(p^{high}/p^{low})$ and $-100\times\log(p_t^{close}/p_{t-1}^{close})$, respectively. Jarque-Bera is the statistic for normality. There are 626 weekly sample observations. All data are extracted from Yahoo/Finance. The computation of the returns of the bond yield follows Engle (2002).

Table 2: Estimation of Bivariate Return-based and Range-based DCC Model Using Weekly S&P500 and Nasdaq , and Tbond , 1994-2005

Step1: $$h_{k,t}=\omega_k+\alpha_k\varepsilon_{k,t-i}^2+\beta_k h_{k,t-1}\qquad \varepsilon_{k,t}\mid I_{t-1}\sim N(0,h_{k,t})$$

$$\lambda_{k,t}=\omega_k+\alpha_k\Re_{k,t-1}+\beta_k\lambda_{k,t-1},\qquad R_{k,t}\mid I_{t-1}\sim \exp(1;\cdot)$$

Step2: $$\begin{bmatrix} q_{11,t} & q_{12,t} \\ q_{21,t} & q_{22,t}\end{bmatrix}=(1-a-b)\begin{bmatrix} 1 & \bar{q}_{12} \\ \bar{q}_{12} & 1\end{bmatrix}+a\begin{bmatrix} z_{1,t-1}^2 & z_{1,t-1}z_{2,t-1} \\ z_{2,t-1}z_{1,t-1} & z_{2,t-1}^2\end{bmatrix}+b\begin{bmatrix} q_{11,t-1} & q_{12,t-1} \\ q_{21,t-1} & q_{22,t-1}\end{bmatrix}$$

Step1	S&P500		Nasdaq		Tbond	
	GARCH	CARR	GARCH	CARR	GARCH	CARR
ω	0.032	0.157	0.111	0.124	0.285	0.231
	(1.316)	(3.239)	(1.315)	(2.615)	(1.605)	(2.565)
α	0.051	0.280	0.089	0.257	0.082	0.190
	(3.346)	(8.816)	(2.363)	(7.271)	(2.258)	(4.973)
β	0.944	0.673	0.905	0.717	0.878	0.750
	(62.305)	(18.375)	(29.240)	(19.289)	(16.636)	(14.372)
Step2	S&P500 and Nasdaq		S&P500 and Tbond		Nasdaq and Tbond	
DCC	Return-based	Range-based	Return-based	Range-based	Return-based	Range-based
a	0.044	0.038	0.054	0.087	0.030	0.052
	(4.923)	(3.317)	(5.314)	(5.125)	(4.116)	(3.654)
b	0.948	0.960	0.937	0.902	0.961	0.938
	(70.120)	(69.435)	(70.876)	(42.761)	(92.107)	(44.599)

Notes: Numbers in parentheses are t-values.

Table 3: In-sample Forecasting Errors for Covariances between the S&P500 and Nasdaq, S&P500 and Tbond, and Nasdaq and Tbond, 1994-2005

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(FCOV_t - MCOV_t)^2}\ ,\quad MAE = \frac{1}{T}\sum_{t=1}^{T}|\,FCOV_t - MCOV_t\,|$$

Panel A: S&P500 and Nasdaq

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs				MAE of Forecasting Implied/Realized MCOVs			
	Return DCC	Range DCC	DBEKK	Realized	Return DCC	Range DCC	DBEKK	Realized
MA100	11.2883	13.6724	11.2641	14.4930	8.4991	10.2663	8.5649	8.4784
EWMA	5.4352	7.2354	5.1455	10.2515	2.9868	3.9244	2.8216	4.9966
Return-based DCC	5.0536	6.8808	4.7915	10.0517	2.7238	3.6953	2.6069	4.8364
Range-based DCC	3.1291	4.2128	3.0214	9.1064	1.6843	2.3039	1.6540	4.6055
CCC	5.0685	6.9146	4.8139	10.0114	2.7058	3.7134	2.6055	4.8024
DBEKK	5.1082	6.8649	4.8004	10.2039	2.9246	3.8445	2.7472	5.0799

Panel B: S&P500 and Tbond

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs				MAE of Forecasting Implied/Realized MCOVs			
	Return DCC	Range DCC	DBEKK	Realized	Return DCC	Range DCC	DBEKK	Realized
MA100	3.1374	3.5097	3.2024	5.2280	2.1223	2.3864	2.1150	3.0857
EWMA	1.8984	2.0796	2.0917	4.3755	1.3250	1.3967	1.4506	2.6789
Return-based DCC	1.7624	1.9892	1.9653	4.3278	1.1934	1.3203	1.3238	2.5987
Range-based DCC	1.3117	1.4156	1.5103	4.1151	0.9794	1.0151	1.1120	2.5776
CCC	2.9202	3.2685	2.9959	4.8678	2.0177	2.2723	2.0326	2.8478
DBEKK	1.7810	2.0093	1.9146	4.3389	1.1652	1.2840	1.2423	2.6355

Panel C: Nasdaq and Tbond

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs				MAE of Forecasting Implied/Realized MCOVs			
	Return DCC	Range DCC	DBEKK	Realized	Return DCC	Range DCC	DBEKK	Realized
MA100	3.8283	4.6628	4.0731	7.6538	2.5139	3.0914	2.5954	4.3087
EWMA	2.4148	2.7837	2.8305	6.5481	1.6108	1.7612	1.8565	3.7181
Return-based DCC	2.1554	2.7297	2.6537	6.5250	1.3969	1.7428	1.7118	3.5710
Range-based DCC	1.7500	2.0133	2.2398	6.3314	1.1595	1.3145	1.4364	3.5772
CCC	3.0885	3.8217	3.4246	6.8767	2.1228	2.6312	2.3553	3.8568
DBEKK	2.3138	2.8551	2.5999	6.5359	1.4522	1.7669	1.6169	3.6285

Notes: The number with an underline stands for the smallest estimating error in each MCOV column. While weekly implied/realized measured covariances are built from the daily data, the forecasting models, MA100, EWMA, return-/range-based DCC, CCC and DBEKK, are estimated from the weekly data.

Table 4: One Period Ahead Out-of-sample Forecasting Errors for Covariances between the S&P500 and Nasdaq , S&P500 and Tbond, and Nasdaq and Tbond, 1994-2005

$$RMSE = \sqrt{\frac{1}{223}\sum_{t=T+1}^{T+223}(FCOV_t - MCOV_t)^2}\ , \quad MAE = \frac{1}{223}\sum_{t=T+1}^{T+223}|FCOV_t - MCOV_t|$$

Panel A: S&P500 and Nasdaq

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	10.0320	12.6749	10.0122	11.6286	7.5302	9.3476	7.6002	7.3431
EWMA	4.9520	6.9700	4.8070	8.0246	2.6979	3.6557	2.6238	4.3508
Return-based DCC	5.1180	7.2078	4.9745	8.0893	2.9547	3.8139	2.9023	4.6323
Range-based DCC	<u>2.6403</u>	<u>4.2542</u>	<u>2.6543</u>	<u>6.6307</u>	<u>1.5516</u>	<u>2.4632</u>	<u>1.5767</u>	<u>3.7831</u>
CCC	5.0825	7.2751	4.9428	8.0392	2.9045	3.8605	2.8517	4.5429
DBEKK	5.3983	7.0671	5.3358	8.4548	3.4496	4.0051	3.4297	5.1474

Panel B: S&P500 and Tbond

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	4.0522	4.5289	4.1338	6.2178	2.5936	2.9375	2.5591	3.7136
EWMA	2.2711	2.4909	2.5990	5.2098	1.6612	1.7209	1.9509	3.4361
Return-based DCC	2.3588	2.6305	2.6427	5.2923	1.6899	1.8234	1.9373	3.4332
Range-based DCC	<u>1.5495</u>	<u>1.6577</u>	<u>1.8521</u>	<u>4.8244</u>	<u>1.1870</u>	<u>1.2324</u>	<u>1.4576</u>	<u>3.3154</u>
CCC	3.9834	4.4367	4.0850	6.1864	2.6654	2.9933	2.6903	3.7290
DBEKK	2.3636	2.6734	2.5426	5.3584	1.6667	1.8773	1.8050	3.4377

Panel C: Nasdaq and Tbond

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	4.7287	5.6697	4.9060	8.0418	3.1770	3.8875	3.2096	5.0928
EWMA	2.6121	2.8532	3.1809	6.8019	1.9748	2.0196	2.4486	4.6951
Return-based DCC	2.2770	2.8180	2.8508	6.8034	1.6959	1.9846	2.1365	<u>4.5082</u>
Range-based DCC	<u>1.8996</u>	<u>1.9771</u>	<u>2.3652</u>	<u>6.4507</u>	<u>1.4528</u>	<u>1.5397</u>	<u>1.8030</u>	4.5176
CCC	3.9340	4.8145	4.2201	7.5815	2.5930	3.2158	2.8797	4.7704
DBEKK	2.9788	3.6218	3.2204	6.9603	2.1415	2.6559	2.2870	4.7122

Notes: There are 400 observations, about 8 years, in each of the estimated models. Additionally, the rolling sample method provides 223 one period ahead out-of-sample forecasting values for comparison. The first forecasted value occurs the week of September 3, 2001. The number with a underline stands for the smallest estimating error in each MCOV column. While weekly implied/realized measured covariances are built from the daily data, the forecasting models, MA100, EWMA, return-/range-based DCC, CCC and DBEKK, are estimated from the weekly data.

Table 5: Two Periods Ahead Out-of-sample Forecasting Errors for Covariances between the S&P500 and Nasdaq , S&P500 and Tbond, and Nasdaq and Tbond, 1994-2005

$$RMSE = \sqrt{\frac{1}{223}\sum_{t=T+1}^{T+223}(FCOV_t - MCOV_t)^2}\ ,\quad MAE = \frac{1}{223}\sum_{t=T+1}^{T+223}|FCOV_t - MCOV_t|$$

Panel A: S&P500 and Nasdaq

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	10.0209	12.6501	10.0013	11.6138	7.5054	9.3048	7.5764	7.3041
EWMA	5.1588	7.1954	4.9853	8.1626	2.8573	3.7846	2.7751	4.4787
Return-based DCC	5.2614	7.3452	5.0987	8.1326	3.1668	3.9516	3.0825	4.7778
Range-based DCC	3.0250	4.9838	2.8728	7.0416	1.8082	2.7501	1.7350	4.1742
CCC	5.2080	7.3920	5.0496	8.0785	3.0874	3.9658	3.0198	4.6696
DBEKK	5.7708	7.4828	5.6593	8.7537	3.8107	4.3520	3.7652	5.4251

Panel B: S&P500 and Tbond

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	4.0502	4.5268	4.1322	6.1994	2.5848	2.9280	2.5523	3.6855
EWMA	2.4112	2.6242	2.7411	5.2543	1.7664	1.8274	2.0619	3.4620
Return-based DCC	2.5104	2.7709	2.8035	5.3294	1.8024	1.9202	2.0612	3.4563
Range-based DCC	1.8043	1.9891	2.1152	4.9762	1.3317	1.4210	1.6349	3.3842
CCC	4.0033	4.4561	4.1082	6.1881	2.6878	3.0127	2.7175	3.7295
DBEKK	2.5918	2.8906	2.7875	5.4728	1.7719	1.9926	1.9170	3.5269

Panel C: Nasdaq and Tbond

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	4.7260	5.6664	4.9042	8.0410	3.1664	3.8745	3.2003	5.0854
EWMA	2.7232	2.9295	3.3058	6.8461	2.0928	2.1070	2.5780	4.7314
Return-based DCC	2.4507	2.9404	3.0273	6.8715	1.8349	2.0725	2.3038	4.5910
Range-based DCC	2.0274	2.2012	2.5453	6.5780	1.5854	1.7034	2.0085	4.6282
CCC	3.9638	4.8367	4.2528	7.5942	2.6184	3.2298	2.9035	4.7792
DBEKK	2.9349	3.5540	3.2597	7.1486	2.1484	2.6495	2.3864	4.7398

Notes: There are 400 observations, about 8 years, in each of the estimated models. Additionally, the rolling sample method provides 223 two periods ahead out-of-sample forecasting values for comparison. The first forecasted value occurs the week of September 10, 2001. The number with a underline stands for the smallest estimating error in each MCOV column. While weekly implied/realized measured covariances are built from the daily data, the forecasting models, MA100, EWMA, return-/range-based DCC, CCC and DBEKK, are estimated from the weekly data.

Table 6: Four Periods Ahead Out-of-sample Forecasting Errors for Covariances between the S&P500 and Nasdaq , S&P500 and Tbond, and Nasdaq and Tbond, 1994-2005

$$RMSE = \sqrt{\frac{1}{223}\sum_{t=T+1}^{T+223}(FCOV_t - MCOV_t)^2}\ ,\quad MAE = \frac{1}{223}\sum_{t=T+1}^{T+223}|\,FCOV_t - MCOV_t\,|$$

Panel A: S&P500 and Nasdaq

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	9.9384	12.5278	9.9326	10.9551	7.4248	9.1916	7.5044	7.0575
EWMA	5.4703	7.4334	5.2721	7.7152	3.0789	3.9020	2.9804	4.2977
Return-based DCC	5.4723	7.3991	5.2934	7.6426	3.4886	4.0222	3.3987	4.6995
Range-based DCC	<u>3.9475</u>	<u>5.9944</u>	<u>3.6582</u>	<u>6.9068</u>	<u>2.4983</u>	<u>3.2709</u>	<u>2.3640</u>	<u>4.2905</u>
CCC	5.4032	7.4394	5.2278	7.5812	3.3911	4.0207	3.3097	4.5842
DBEKK	6.4161	8.0126	6.2415	8.6573	4.3837	4.8316	4.3110	5.6351

Panel B: S&P500 and Tbond

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	4.0166	4.5060	4.0961	6.0158	2.5478	2.8966	2.5156	3.5887
EWMA	2.5924	2.8247	2.9166	5.1686	1.9274	1.9980	2.2325	3.4691
Return-based DCC	2.6846	2.9708	2.9777	5.2213	1.9618	2.0769	2.2338	3.4559
Range-based DCC	<u>2.1849</u>	<u>2.4616</u>	<u>2.4854</u>	<u>4.9455</u>	<u>1.6185</u>	<u>1.7431</u>	<u>1.8979</u>	<u>3.3612</u>
CCC	4.0058	4.4730	4.1117	6.0103	2.7001	3.0312	2.7344	3.6538
DBEKK	2.9526	3.3002	3.1526	5.5049	2.0352	2.2675	2.1982	3.5668

Panel C: Nasdaq and Tbond

Forecasting Model	RMSE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized	MAE of Forecasting Implied/Realized MCOVs: Return DCC	Range DCC	DBEKK	Realized
MA100	4.6884	5.6449	4.8613	7.7705	3.1251	3.8393	3.1545	4.9519
EWMA	2.8514	3.0233	3.4438	6.6537	2.2778	2.2529	2.7671	4.6719
Return-based DCC	2.6398	3.0991	3.2059	6.6433	2.0679	2.2368	2.5450	4.5126
Range-based DCC	<u>2.2393</u>	<u>2.4987</u>	<u>2.7943</u>	<u>6.3692</u>	<u>1.8008</u>	<u>1.9312</u>	<u>2.2642</u>	<u>4.4420</u>
CCC	3.9711	4.8553	4.2560	7.3273	2.6325	3.2528	2.9223	4.6733
DBEKK	3.2078	3.8322	3.5380	6.9793	2.4081	2.8222	2.6711	4.6900

Notes: There are 400 observations, about 8 years, in each of the estimated models. Additionally, the rolling sample method provides 223 four periods ahead out-of-sample forecasting values for comparison. The first forecasted value occurs the week of September 17, 2001. The number with a underline stands for the smallest estimating error in each MCOV column. While weekly implied/realized measured covariances are built from the daily data, the forecasting models, MA100, EWMA, return-/range-based DCC, CCC and DBEKK, are estimated from the weekly data.

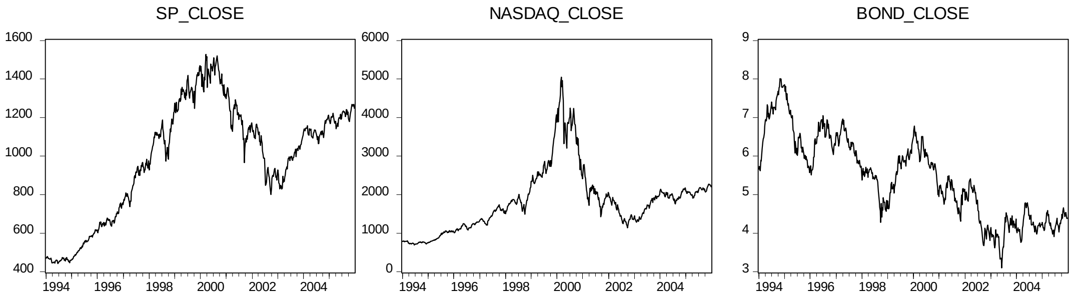

Figure 1: S&P500, Nasdaq Indices, and Tbond Yield Weekly Closing Prices, 1994-2005.

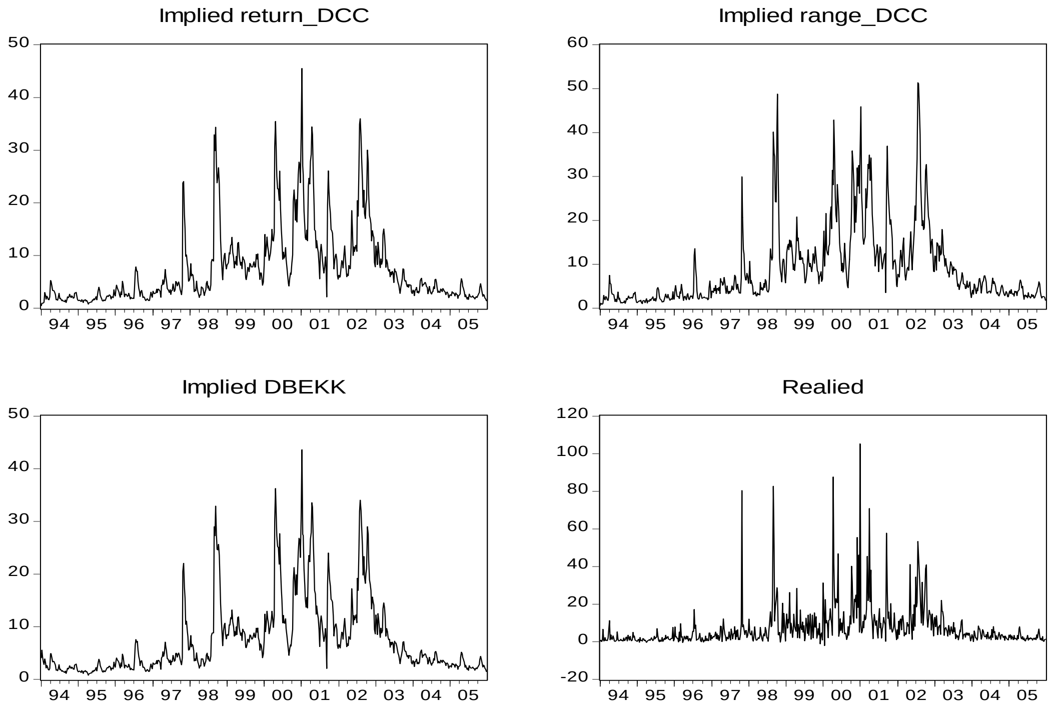

Figure 2: Four Measured Covariances for S&P500 and Nasdaq Indices, 1994-2005.

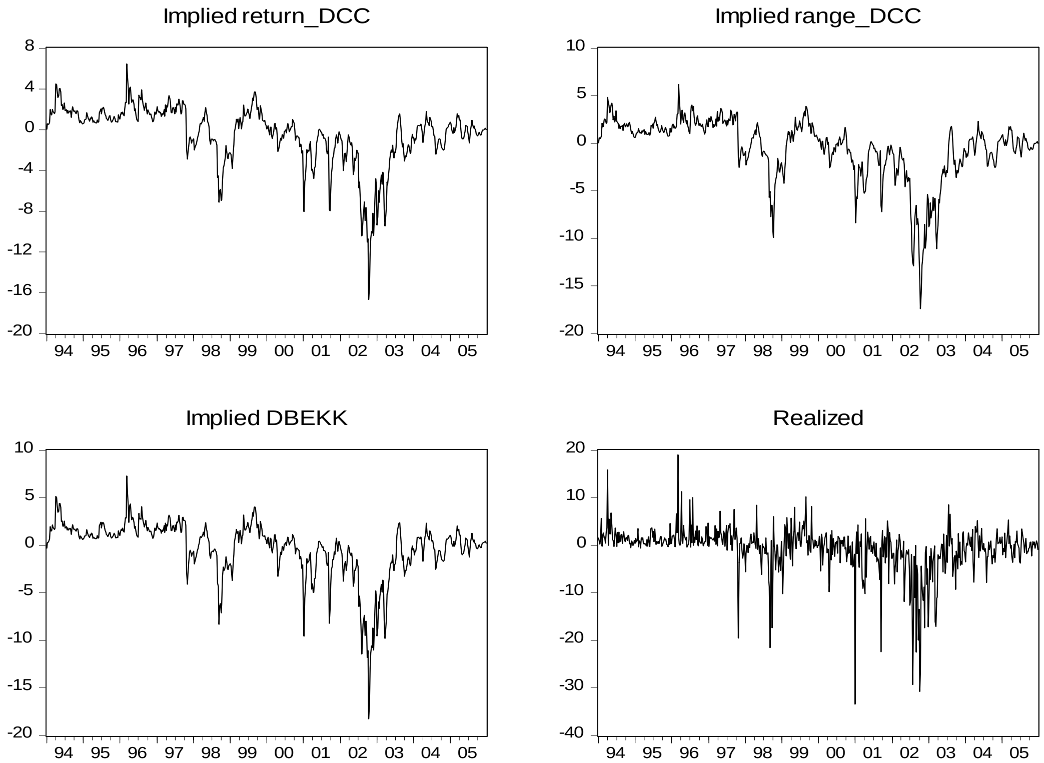

Figure 3: Four Measured Covariances for S&P500 Index and Tbond Yield, 1994-2005.

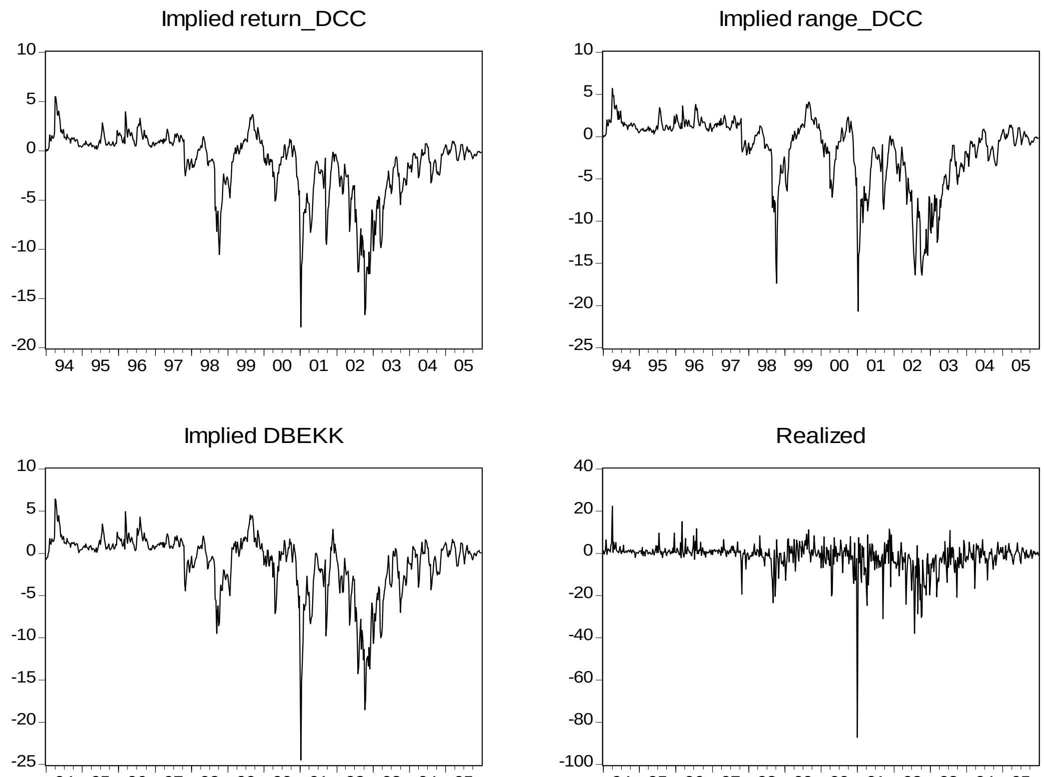

Figure 4: Four Measured Covariances for Nasdaq Index and Tbond Yield, 1994-2005.

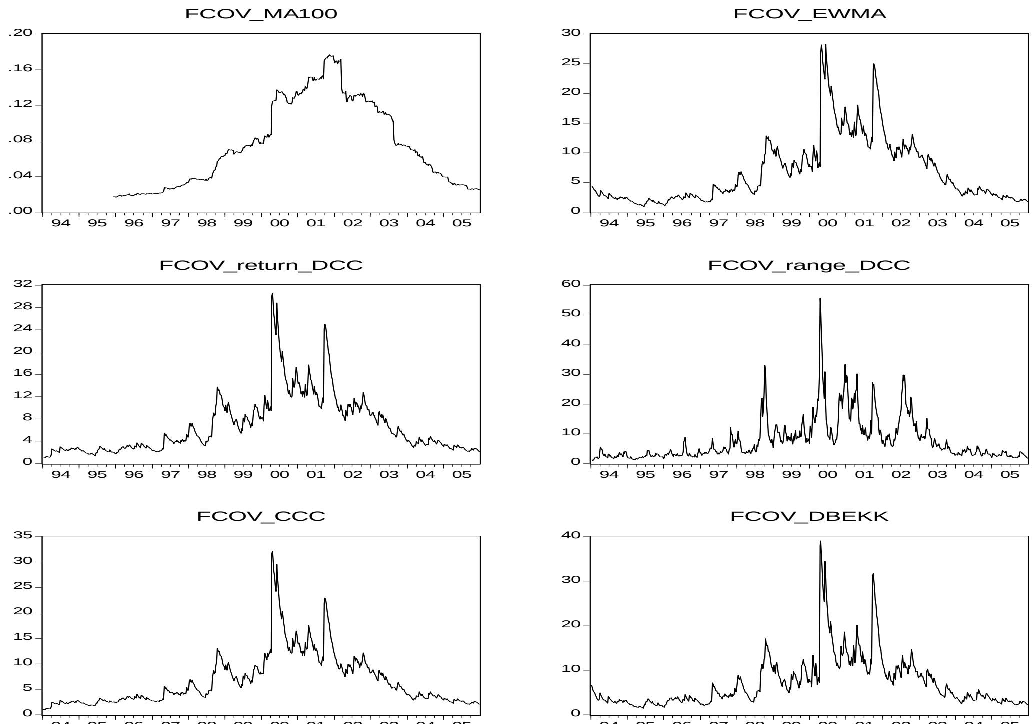

Figure 5: In-sample Forecasting Covariances using 6 models for S&P500 and Nasdaq Indices, 1994-2005.

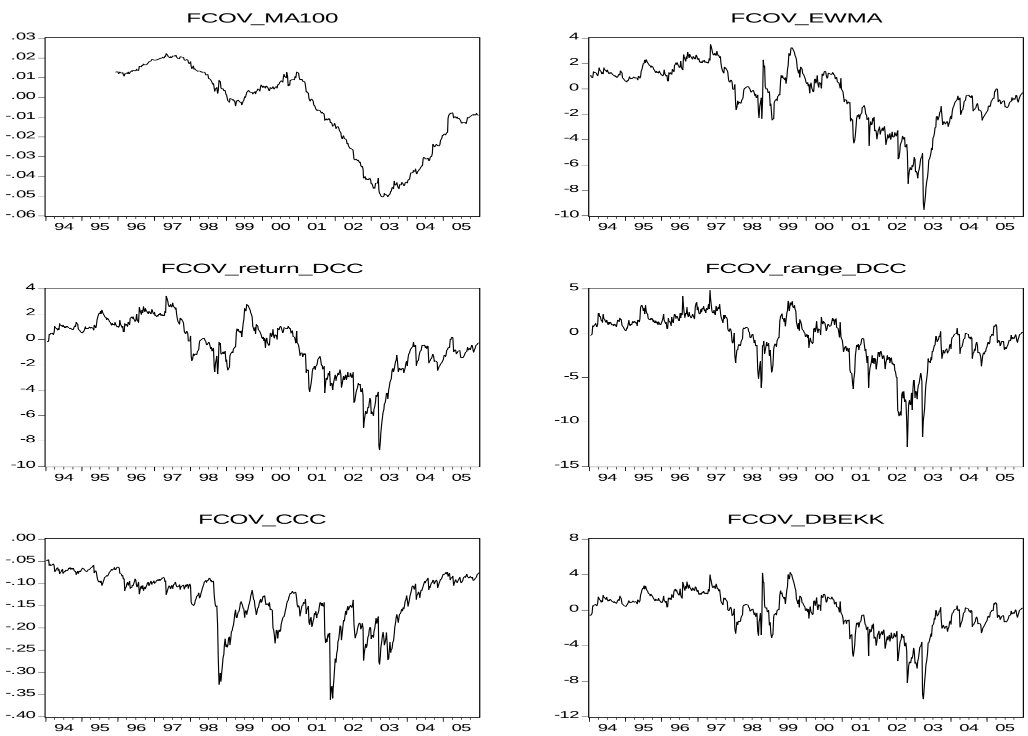

Figure 6: In-sample Forecasting Covariances using 6 models for S&P500 Index and Tbond Yield, 1994-2005.

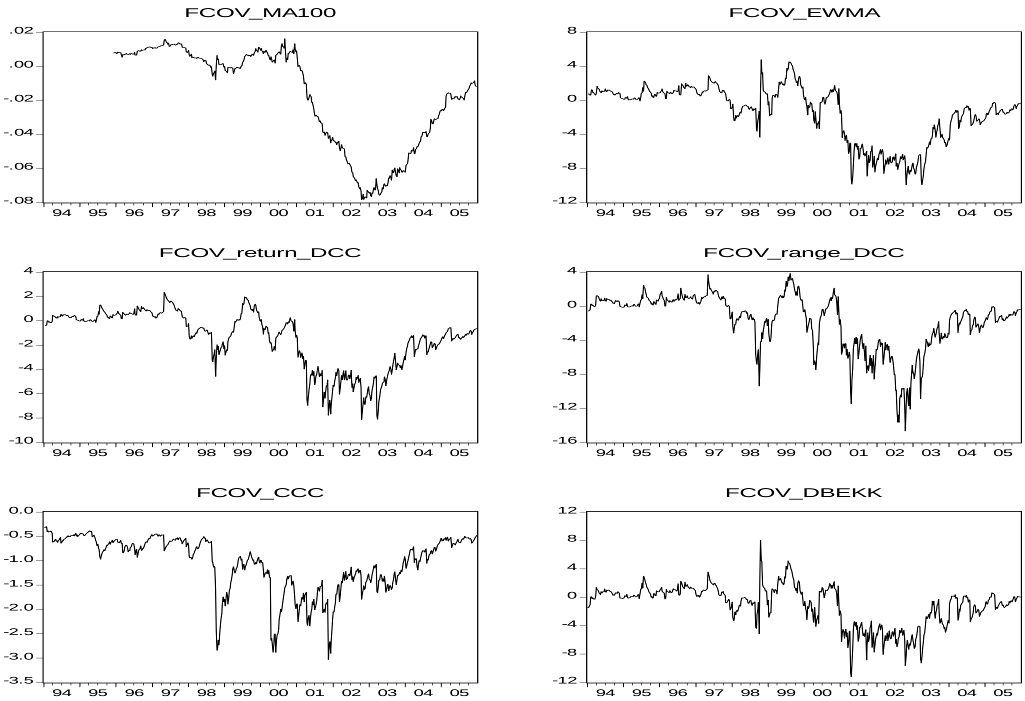

Figure 7: In-sample Forecasting Covariances using 6 models for Nasdaq Index and Tbond Yield, 1994-2005.

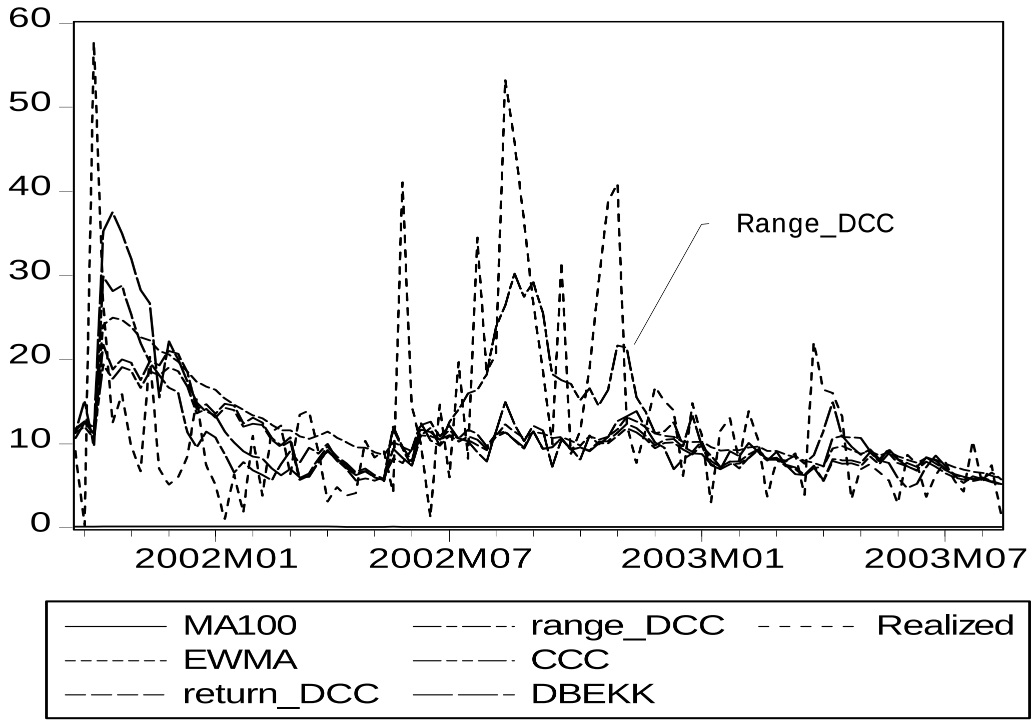

Figure 8: One Period Ahead Out-of-sample Forecasting Realized Covariances using 6 models for S&P500 and Nasdaq Indices, 1994-2005. For convenience in distinguishing, we may reserve the first 100 observations (in all we have 223) with one step ahead of out-of-sample forecasting for covariances.

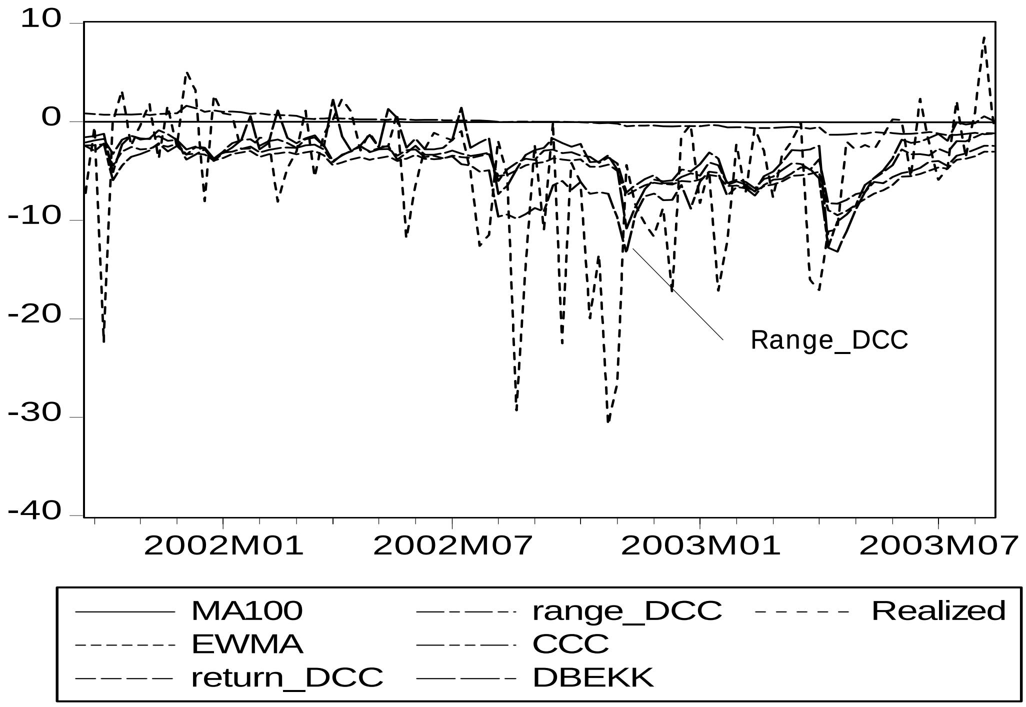

Figure 9: One Period Ahead Out-of-sample Forecasting Realized Covariances using 6 models for S&P500 Index and Tbond Yield, 1994-2005. For convenience in distinguishing, we may reserve the first 100 observations (in all we have 223) with one step ahead of out-of-sample forecasting for covariances.

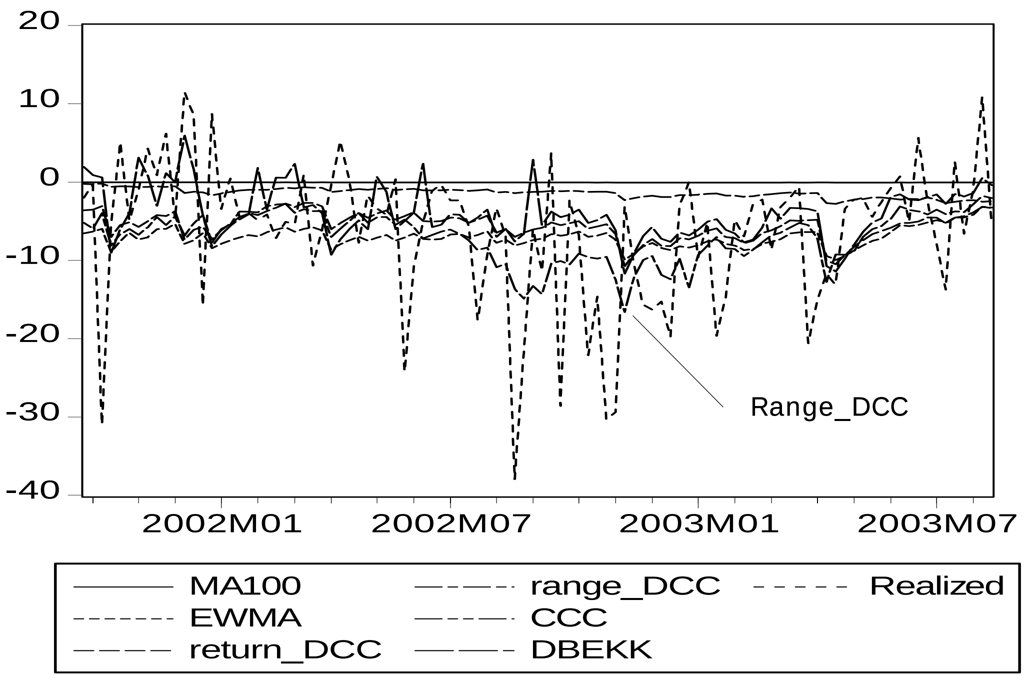

Figure 10: One Period Ahead Out-of-sample Forecasting Realized Covariances using 6 models for Nasdaq Index and Tbond Yield, 1994-2005. For convenience in distinguishing, we may reserve the first 100 observations (in all we have 223) with one step ahead of out-of-sample forecasting for covariances.