Estimating Value at Risk with a Dynamical Conditional Range Model

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Abstract:

Value at risk (VaR) is a single, summary, statistical measure of possible asset losses. This paper explores the various VaR models that were proposed with different characteristics of financial data. Above all, we introduce the idea of dynamical range process to the VaR estimation. Accordingly, the Conditional Autoregressive Range (CARR) model proposed by Chou (2005) is used for the estimation of volatility process for single asset and extended to the modification of Engle's (2002) DCC model for a portfolio. Then the range-based VaR model is constructed. Our method can be applied to measure the VaR for single assets and for portfolios. Using empirical data of the stock indices of S&P 500 composite and the ten-year Treasury bond, we find that the range-based model with theory of extreme value have salient evaluation performance than other alternatives, such as the conventional return-based models in the estimation of VaR.

Keywords: Value at risk, CARR, Extreme value theory, DCC, Range.

1. Introduction

Value at risk (VaR, hereafter) has become the conventional measure of market risk employed by financial institutions for both control and risk management purposes. Specifically, VaR is a measure of losses due to normal market movement. Losses greater than the VaR are suffered only with a specified small probability. Subject to the simplifying assumptions used in its calculation, VaR aggregates all of the risks in an asset or a portfolio into a single number suitable for use in the boardroom, reporting to regulators, or disclosure in an annual report. Once one crosses the hurdle of using a statistical measure, the concept of VaR is really a straightforward to understand. It is simply a way to describe the magnitude of the likely losses on the asset.

Some financial institutions had been working on their own internal model and VaR software systems were also being developed by specialist companies that concentrated on software but were not in a position to provide data. The resulting systems differed quite considerably from each other. Even where they were based on broadly similar theoretical ideas, there were still considerable differences in terms of subsidiary assumptions, use of data, procedures to estimate volatility and correlation, and many other details. Besides, some systems were built using historical simulation approaches that estimate VaR from histograms of past profit and loss data, and other systems were developed using Monte Carlo simulation techniques.

VaR information can be used in many ways. First, senior management can use it to set their overall risk target, as well as from that determine risk targets and position limit down the line. If they want the firm to increase its risks, they would increase the overall VaR target, and vice versa. Second, VaR can be very useful for reporting and disclosing purposes, and firms increasingly make a point of reporting VaR information in their annual reports.[1] Third, VaR information can be used to implement portfolio-wide hedging strategies that are otherwise rarely possible. Fourth, since VaR tells us the maximum amount we are likely to lose, we can use it to determine asset allocation. Finally, VaR information can be used to provide new remuneration rules for traders, managers and other employees that take account of the risks they take, and so discourage the excessive risk taking that occurs when employees are rewarded on the basis of profits alone, without any reference to the risks they took to get those profits. In short, VaR can help provide for a more consistent and integrated approach to the management of different risks, leading also to greater risk transparency and disclosure, and better strategic management.

Despite its conceptual simplicity, the evaluation of VaR is a very challenging statistical problem and none of the approaches developed so far gives satisfactory solutions. Due to VaR is simply a specific quantile of future portfolio values, conditional on current information, and due to the distribution of portfolio return typically changes over time, the mission is to get an appropriate model for time varying conditional quantiles. In this paper we introduced a range-based model that proposed by Chou(2005) for volatility modeling and arrange the elegant results from CARR model into the estimation of VaR. Namely, we provide a scheme for estimating VaR_t as a function of variables known at time t-1.

The paper is organized as follows. In section 2, we briefly review the related

[1] For more on the use of VaR for reporting and disclosure purposes, see Dowd(2000), Jorion(2000) and Moosa and Knight(2001).

literature and current approaches to VaR estimation. Section 3 discusses the range based VaR model and data analysis. Section 4 presents an empirical application to real data for an asset and model identification. Selection 5 is model back testing. Namely, we diagnose the number of exceptions during the stage of back testing for each feasible model. Section 6 extends to the evaluation of portfolio's VaR for robust checking. Finally, section 7 concludes the paper.

2. Conventional VaR Models

The Basel Committee on Banking Supervision (1996) at the Bank for International Settlements uses VaR to require financial institutions such as investment corporations and banks to meet capital requirements to cover the markets that incur as a result of their normal operations. Regardless the VaR is overestimate or underestimate, the result is an inefficient allocation of resources. The existing models for estimating VaR differ in many aspects. The main differences among VaR models are related to the distribution of the portfolio returns. Several different methodologies have been employed. Some first estimate the volatility of the portfolio, perhaps by GARCH or exponential smoothing, and then calculate VaR from this, often assuming normality. Others use the logic of the extreme value theory. It is not difficult to comment each of these approaches. The volatility approach assumes that the negative extremes follow the same process as the rest of the returns and that the distribution of the returns divided by standard deviations will be independent and identically distributed, if not normal. Applications of extreme value estimation methods to VaR have been recently proposed, also see Danielsson and de Vries(2000). The intuition is to manipulate results from statistical extreme value theory and to pay attention to the asymptotic form of the tail, rather than modeling the whole distribution. Analogical criticism also can find by Engle (2002). Engle (2002) infers two possible conflicts behind this approach. First it uses merely for very low probability quantiles. Second, these approaches are constructed in a framework of i.i.d variables, which is not consistent with the properties of most financial datasets and consequently the risk of a portfolio may not vary with the conditional information set.[2] As to what approach is appropriate in data fitting and the performance is better for the evaluation of VaR should be an empirical issue.

2.1 Static (unconditional) VaR Model

We explore three popular approaches for unconditional VaR models: the first is historical simulation method. The second is unconditional variance and covariance approach. Finally, we illustrate the approach of the unconditional VaR-X method.

Historical simulation approach is an extremely popular method for many types of institutions for the estimate of VaR. One simply uses real historical data to build an empirical density for the asset. No assumption about the analytic form of this distribution is made at all. Here we use the hypothesis that the distribution is stationary. The VaR is then obtained by determining the asset return that corresponds to the confidence level chosen.

JP Morgan made publicly available in 1994 a method to estimate VaR that is closely related to the method used internally. The approach is called RiskMetrics™ or

[2] McNeil and Frey (2000) proposed fitting a GARCH model to the time series of returns and then applying the extreme value theory to the standardized residuals, which are assumed to be i.i.d. Although it is one of modification over existing applications, this idea still suffers from the same comment applied to the volatility models.

unconditional variance covariance method. To estimate VaR, one has to forecast the fifth percentile of the distribution of the portfolio's return under the confidence levels of 95%. This approach usually assumes that the continuously compounded return of the asset over the next day is normally distributed. When an asset has a normally distributed return, the five percent VaR can be obtained by multiplying the forecast of the volatility of the return of the asset by 1.645. Consequently, to obtain an estimate of the VaR, the unconditional variance-covariance method has to come up with an estimate of the volatility of the asset. Since the distribution for the return of asset is not time varying, so we call such a method is Delta-Normal approach for the time being. In other words, under 100(c)% confidence level, assume the lowest value of asset is V_{t+1}^*, we can find the expected value X_{t+1}^* which corresponds to the V_{t+1}^*. Meanwhile, $X_{t+1}^* = -N^*\hat{\sigma} + \hat{\mu}$, where N^* denotes the critical value based on standard normal distribution, $\hat{\mu}$ is the estimate of asset historical average return and $\hat{\sigma}$ is a constant volatility which is estimated from historical trading data.

If we assume the return of asset follows normal distribution when it is actually fat-tailed, then we are likely to underestimate the value of VaR, and these underestimates are likely to be particularly large when dealing with VaR at high confidence levels. One way to accommodate excess kurtosis is to use a t distribution instead of a normal one. For example, a t distribution with υ degrees of freedom has a kurtosis of $\frac{3(\upsilon-2)}{(\upsilon-4)}$ which is greater than the situation of normal distribution, given $\upsilon \geq 5$. Besides, an alternative VaR approach is proposed by Huisman, Koedijk and Pownall(1998), they called VaR-X method. Instead of fitting the t distribution by matching the number of degrees of freedom, υ, to the empirical kurtosis, they make the number of degrees of freedom equal to the inverse of a Hill estimator of the tail index, modified to correct for small sample bias. The problem with this approach is that it produces an implied kurtosis. Namely, equal to $\frac{3(\upsilon-2)}{(\upsilon-4)}$ that may not equal the empirical kurtosis, and where it does match the empirical kurtosis, it is much easier to dispense with the tail index estimation and simply use the kurtosis matching method.

However, as stated by Jorion(2000), there are some estimation errors will incur when extreme value theory is used with ignoring the characteristic of heteroscedasticity. For the sake of such a problem, McNeil and Frey (2000) proposed that one can choice suitable volatility model for fitting the process of returns and collect standardized residual series for the estimation of the tail index. Here, we first introduced popular ARMA(m,n) model to character the return's process. As to the structure for conditional volatility, there are two approaches are considered, too. One is conventional GARCH(p,q) model suggested by Bollerslev (1986) and the other is crisp CARR(p,q) model proposed by Chou(2005). Hence, combining the mean equation and the volatility equation, we are able to establish the ARMA(m,n)-GARCH(p,q) and the CARR(p,q) models as the intermediate stage for the estimation of VaR. Below structures are their functional forms in general.

$$X_t = \mu + \sum_{i=1}^{m} \phi_i X_{t-i} + \sum_{j=1}^{n} \theta_j \varepsilon_{t-j} + \varepsilon_t \qquad \varepsilon_t \big| I_{t-1} \overset{iid}{\sim} N(0,\sigma_t^2)$$

$$\sigma_t^2 = \omega^G + \sum_{i=1}^{p} \alpha_i^G \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j^G \sigma_{t-j}^2 \qquad (1)$$

where X_t denotes log-return at time t, $\varepsilon_t = X_t - E(X_t \big| I_{t-1})$, μ, ϕ_i, θ_j, ω^G, α_i^G and β_i^G are parameters. Some constraints for these parameters are $\omega^G > 0$, $\alpha_i^G \geq 0$,

$\beta_j^G \geq 0$ and $\sum_{i=1}^{p}\alpha_i^G + \sum_{j=1}^{q}\beta_j^G < 1$, here $i = 1,\cdots,p$ and $j = 1,\cdots,q$. Equation (1) is called ARMA(m,n)-GARCH(p,q) and one can obtain the series of expected returns $\{\hat{\mu}_t\}$ and the series of standard deviation $\{\hat{\sigma}_t\}$ from this equation. Besides, one can obtain standardized residual series $\{z_t^G\}$ readily. Their outcomes are shown on equation (2).

$$\{z_{t-n+1}^G,\cdots,z_t^G\} = \{\frac{X_{t-n+1} - E(X_{t-n+1}|I_{t-n})}{\hat{\sigma}_{t-n+1}},\cdots,\frac{X_t - E(X_t|I_{t-1})}{\hat{\sigma}_t}\} \tag{2}$$

As to the structure of CARR(p,q) model can show as equation (3).

$$\Re_t = \lambda_t u_t \qquad u_t|I_{t-1} \overset{iid}{\sim} f(1,\xi_t)$$
$$\lambda_t = \omega^C + \sum_{i=1}^{p}\alpha_i^C \Re_{t-i} + \sum_{j=1}^{q}\beta_j^C \lambda_{t-j} \tag{3}$$

where $\Re_t = \ln(P_t^{high}) - \ln(P_t^{low})$ denotes range variable. λ_t is conditional mean of range given all available information set up to time t. The disturbance term is denoted by u_t. The coefficients (ω^C, α_i^C, β_j^C) in the conditional range variable equation are all positive to ensure positively of λ_t. Due to the range $\Re_t$ and its expected value λ_t are positively values, thus u_t belongs to positive field, too. A natural choice for the distribution is the exponential as it has non-negative possibility. Assuming that the distribution follows an exponential distribution with unit mean then the log likelihood function can be written as

$$L(\omega^C,\alpha_i^C,\beta_j^C;\Re_1,\ldots,\Re_T) = -\sum_{t=1}^{T}\left[\ln(\lambda_t) + \frac{\Re_t}{\lambda_t}\right] \tag{4}$$

Similar to the approach in equation (2), one can get a similar standardized residual series based on CARR model. The expression can be stated as equation (5).

$$\{z_{t-n+1}^C,\cdots,z_t^C\} = \{\frac{\Re_{t-n+1} - \hat{\lambda}_{t-n+1}}{\hat{\mu}_{t-n+1}},\cdots,\frac{\Re_t - \hat{\lambda}_t}{\hat{\mu}_t}\} \tag{5}$$

For the details in advance, one can consult Chou(2005) for more explanation.

Using the $\{z^G_{t-n+1},\cdots,z^G_t\}$ and $\{z^C_{t-n+1},\cdots,z^C_t\}$ with VaR-X method,[3] the tail index can be computed, respectively. In this paper, the former approach is called VaR-X after GARCH-filtered and the later one is called VaR-X after CARR-filtered.

2.2 Dynamical (conditional) VaR Model

The main difference between dynamical VaR model and static one is the procedure for estimating volatility process. Under static model, by and large, the probability distribution of asset's return is fixed. Hence, historical standard deviation of asset return amounts to the estimate of volatility in the future. As to the dynamical model, the probability distribution of asset's return is time varying with heteroscedasticity. Based on such a situation, it is necessary to using past trading data for inferring the possible volatility process and calculating a more accurate estimate of the VaR. Hence, we incorporate two types of dynamical VaR models, including conditional variance-covariance method and conditional VaR-X method. Recall the critical expression for the estimation of VaR under unconditional variance-covariance method is $X^*_{t+1} = -N^*\hat{\sigma} + \hat{\mu}$.[4] Corresponding to the conditional variance-covariance method, this equation has to be modified as $X^*_{t+1} = -N^*\hat{\sigma}_{t+1} + \hat{\mu}_{t+1}$. There are many approaches for the estimation of volatility $\hat{\sigma}_{t+1}$ in the related literature. According to the different methods for the estimation of volatility, one can induce various VaR models. For instance, the GARCH model and CARR model are appropriated for the estimation of volatility $\hat{\sigma}_{t+1}$ in our later discussion. For briefly, we call these two

[3] VaR-X method is proposed by Huisman, Koedijk and Pownall (1998).

[4] Where N^* denotes the critical value based on standard normal distribution, $\hat{\mu}$ is the estimate of asset historical average return and $\hat{\sigma}$ is a constant volatility which is estimated from historical trading data.

methods for the evaluations of VaR are GARCH-Normal approach and CARR-Normal approach, respectively.

Another type of conditional dynamic VaR model is conditional VaR-X method. There are at least two methods belong to this type. One is called GARCH-VaR-X model, and the other is called CARR-VaR-X method. Adjusting from the VaR-X after GARCH-filtered approach in the estimation of the $\hat{\sigma}$, which is fitted from historical standard deviation of asset's return. However, GARCH-VaR-X model replaces the estimate of $\hat{\sigma}$ for t+1 with $\hat{\sigma}_{t+1}$ which from the result of GARCH forecasting model. By the same token, CARR-VaR-X model modifies the VaR-X after CARR-filter approach in the estimation of $\hat{\sigma}$ with $\hat{\sigma}_{t+1}$ which is derived from the CARR model.

3. Data Analysis

We collect the daily index data of the S&P 500 composite and the U.S. 10-year treasure bond for the sample period from November 1, 1993 to March 17, 2006. Data are mainly collected from Yahoo Finance. It is worth taking a look at some descriptive statistics for the returns of the S&P 500 and the U.S. 10-year treasure bond. We illustrate them in Table 1.

Table 1

Descriptive statistics for the returns of the S&P 500 stock index and the U.S. 10-yearT-bond.
（1993/11/1~2006/3/17）

Descriptive statistics	S&P500	10-year T-bond
Mean（%）	0.033	0.005
Median（%）	0.051	0
Maximum（%）	5.574	5.090
Minimum（%）	-7.113	-5.972
St.d（%）	1.070	1.180
Skewness	-0.115	-0.381
Kurtosis	6.644	5.164
Jarque-Bera	1722.171***	680.110***
	（0）	（0）
＜-1.65 sample ratio（%）#	5.923%	5.962%
＜-1.96sample ratio（%）#	3.692%	4%
＜-2.33sample ratio（%）#	2.039%	2.269%
Ljung-Box statistics for autocorrelation		
$Q(6)$	10.321*(0.1)	16.032**(0.014)
$Q(12)$	24.105**（0.02）	26.274***（0.01）
$Q^2(6)$	598.84***（0）	232***（0）
$Q^2(12)$	948.47***（0）	408.93***（0）

Note:

1. Jarque-Bera=N/6(S^2+1/4$(K-3)^2$)~ $\chi^2(2)$ is useful in normality testing. The null hypothesis is that data follow normal distribution, N is sample size, S denotes skewness as well as K states the kurtosis for data selected. Based on the significant level is α=5%, then $\chi^2_{1-\alpha}(2)=5.99$. Namely, under the confidence level at 5% , if the value of JB is greater than the critical value of 5.99, then the assumption of normality is rejected.
2. The symbol # shows the percentage ratio of the standardized data less than the critical value.
3. Q is proposed by Ljung-Box (1978) for the testing of autocorrelation. i.e. Q=T(T+2)(Σ $\hat{\tau}_k^{\,2}$/T-k)~ χ^2_m , here $\hat{\tau}_k$ is the ACF for the k lag, T denotes the sample size and m is the largest lag period.
4. Numbers in parentheses are p-values as well as *, ** and *** represent the level of 10%, 5% and 1% are significant, respectively.

From Table 1, we find that the average return for the S&P 500 stock index is greater than the U.S. 10-year T-bond, nearly six times. As to the normality test, both of them are skew to the left in terms of skewness and the coefficients of kurtosis are more than three. Additionally, the values for JB are 1722.171 and 680.11 for the S&P 500 stock index and the 10-year T-bond respectively. Both of them reject the null hypothesis for normality testing. For comparison, we compute the descriptive statistics for range variable data for the S&P 500 stock index and the 10-year T-bond in Table 2.

Table 2

Descriptive statistics for the ranges variable of the S&P 500 stock index and the U.S. 10-year T-bond.

（1993/11/1~2006/3/17）

Descriptive statistics	S&P500	10-year T-bond
Mean（%）	1.318	1.399
Median（%）	1.111	1.193
Maximum（%）	8.479	7.259
Minimum（%）	0.239	0.001
St.d（%）	0.821	0.873
Jarque-Bera	13593.940***（0）	5008.238***（0）
$Q(6)$	4395.3***（0）	1279.8***（0）
$Q(12)$	7870.4***（0）	2343.3***（0）

Note:

1. $\text{Range} = 100 \times [\ln(P_t^{high}) - \ln(P_t^{low})]$, where P_t^{high} is the highest price and P_t^{low} is the lowest price at time t period.
2. $Q(6)$ is Q statistics for the daily range until to lag sixth period.
3. Numbers in parentheses are p-values as well as *, ** and *** represent the level of 10%, 5% and 1% are significant, respectively.

Similar to the results in the JB values of return-based data for normality testing. In other words, regardless range-based or return-based variable, all of them are not belong to the normal distribution in statistics conventionally. However, when compare the statistics of daily standard deviation for range-based data and return-based data. No matter the S&P 500 stock index or the U.S. 10-year T-bond, it is salient that the dispersion for daily range-based data smaller than the daily return-based data. Intuitively, the range-based data seems more appropriate in capturing the volatility process than the return-based data.

From the Q-statistics in Table 1 and Table 2, we find that both of the return-based or range-based series, the S&P 500 stock index and the U.S. 10-year T-bond data have the phenomenon of autocorrelation. For solving such problem in econometric, first, we adopted the approach of Berkowitz and O'Brien (2002) in return-based series similarly. They fit the structure of conventional ARMA(1,1) model into mean equation for the expected value of asset returns. Besides, one can try various GARCH structures for the modeling of volatility process. There are GARCH(1,1), GARCH(1,2), GARCH(2,1) and GARCH(2,2) frameworks are examined later. Meanwhile, selecting the better one judging from the criticism of SBC(Schwartz Baysian information criterion). We demonstrate the output in the upper panel of Table 3.

Table 3:

ARMA(1, 1)－GARCH(p, q) and CARR(p, q) models for the S&P 500 stock index and the U.S. 10-year T-bond data with SBC statistics
（1993/11/1~2006/3/17）

ARMA(1, 1)－GARCH(p, q) model:

$$X_t = \mu + \phi_1 X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \qquad \varepsilon_t \left| I_{t-1} \overset{iid}{\sim} N(0, \sigma_t^2) \right.$$

$$\sigma_t^2 = \omega^G + \sum_{i=1}^{p} \alpha_i^G \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j^G \sigma_{t-j}^2$$

CARR(p,q) model:

$$\Re_t = \lambda_t u_t \qquad u_t \left| I_{t-1} \overset{iid}{\sim} f(1, \xi_t) \right.$$

$$\lambda_t = \omega^C + \sum_{i=1}^{p} \alpha_i^C \Re_{t-i} + \sum_{j=1}^{q} \beta_j^C \lambda_{t-j}$$

Model types	SBC	
ARMA(1, 1)－GARCH(p, q)	S&P500	10-year T-bond
ARMA(1, 1)－GARCH(1, 1)	2.7076	3.0544
ARMA(1, 1)－GARCH(1, 2)	2.7081	3.0569
ARMA(1, 1)－GARCH(2, 1)	2.7104	3.0569
ARMA(1, 1)－GARCH(2, 2)	2.7089	3.0593
	SBC	
CARR(p, q)	S&P500	10-year T-bond
CARR(1, 1)	2.7062	3.1455
CARR(1, 2)	2.7084	3.1479
CARR(2, 1)	2.7079	3.1480
CARR(2, 2)	2.7094	3.1504

Note: SBC（Schwartz Baysian information criterion）$= \ln(\hat{s}^2) + L/T \ln(T)$, $\hat{s}^2$ is variance of residuals, L is the number of parameter to be estimated. T denotes sample size.

4. Model Fitting and Selection

No matter what the return series of the S&P 500 stock index or the U.S. 10-year T-bond data, judging by the conventional SBC criticism for model fitting, ARMA(1,1)-GARCH(1,1) corresponds to the smallest one. Hence, it is reasonable to choose ARMA(1,1)-GARCH(1,1) model as representation of GARCH family for the basis of the comparison of VaR later. As to the appropriate model structure for the range-based data, we find that the CARR(1,1) is relatively suitable one under the SBC criterion, too. Data fitting results are shown in the lower panel of Table 4. Likewise, the CARR(1,1) model can be used as the representation of the CARR family in the comparison of the evaluation of VaR.

For the ARMA(1,1)-GARCH(1,1) and CARR(1,1) models with the S&P 500 stock index and the U.S. 10-year T-bond data, their parameters estimates are demonstrated in Table 4. Empirical results from Table 4, one can get an apparent demonstration by coefficients' significance that these two models can indeed act as a pivotal role to depict stock and bond markets volatility patterns.

Table 4:

Estimating parameters for ARMA(1, 1)－GARCH(1, 1) and CARR(1, 1)models
（1993/11/1~2006/3/17）

ARMA(1, 1)－GARCH(1,1) model:

$$X_t = \mu + \phi_1 X_{t-1} + \theta_1 \varepsilon_{t-1} + \varepsilon_t \qquad \varepsilon_t | I_{t-1} \overset{iid}{\sim} N(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega^G + \alpha_1^G \varepsilon_{t-1}^2 + \beta_1^G \sigma_{t-1}^2$$

CARR(1,1) model:

$$\Re_t = \lambda_t u_t \qquad u_t | I_{t-1} \overset{iid}{\sim} f(1, \xi_t)$$

$$\lambda_t = \omega^C + \alpha_1^C \Re_{t-1} + \beta_1^C \lambda_{t-1}$$

	ARMA(1, 1)－GARCH(1, 1)	
	S&P500	10-year T-bond
$\hat{\mu}$	0.0561***	0.0154
	(3.9315)	(0.8147)
$\hat{\phi}_1$	-0.8810***	-0.3867*
	(-3.8841)	(-1.6963)
$\hat{\theta}_1$	0.8730***	0.4282*
	(3.7317)	(1.9306)
$\hat{\omega}^G$	0.0069***	0.0139***
	(2.7161)	(2.4017)
$\hat{\alpha}_1^G$	0.0714***	0.0449***
	(6.1137)	(4.8773)
$\hat{\beta}_1^G$	0.9245***	0.9451***
	(86.0830)	(81.0940)
	CARR(1, 1)	
	S&P500	10-year T-bond
$\hat{\omega}^C$	0.0150***	0.0164***
	(3.7937)	(3.2064)
$\hat{\alpha}_1^C$	0.1372***	0.0683***
	(11.3993)	(8.0726)
$\hat{\beta}_1^C$	0.8514***	0.9198***
	(66.3967)	(87.1971)

Note: 1. $\hat{\mu}$, $\hat{\phi}_1$, $\hat{\theta}_1$, $\hat{\omega}^G$, $\hat{\alpha}_1^G$ and $\hat{\beta}_1^G$ are parameter estimates in ARMA(1, 1)-GARCH(1, 1) model. Meanwhile, $\hat{\omega}^C$, $\hat{\alpha}_1^C$ and $\hat{\beta}_1^C$ are parameter estimates in CARR(1, 1) model.
2. Numbers in parentheses are t-values.
3. *, ** and *** represent the level of 10%, 5% and 1% are significant, respectively.

After the parameters estimation for the ARMA(1, 1)－GARCH(1, 1) and

CARR(1,1) models are achieved. Using the series of $\{z_{t-n+1}^G, \cdots, z_t^G\}$ and $\{z_{t-n+1}^C, \cdots, z_t^C\}$ with VaR-X method which derived by equation (2) and (5), the tail index can be computed, respectively. Again, the former approach is called VaR-X after GARCH-filtered and the later one is called VaR-X after CARR-filtered in this paper. Both of these two models have incorporated the characteristic of thick tail of data. Reviewing the descriptive statistics for the residuals with these two series after GARCH and CARR models filtered on Table 5. Clearly, we improve most of the problems about the phenomenon of autocorrelation from original returns and ranges series as well as capture the patterns of heteroscedasticity and volatility clustering. Even the value of JB is smaller than the original series apparently, but the null hypothesis of normal distribution still be rejected. Data still appear fat-tail than the normal distribution for the S&P 500 stock index and the 10-year T-bond yield rate. One of reasonable explanation is downside risk or some special extreme events lead to these results. Hence, it is worth introducing the concept of extreme value theory to dispose such a situation.

Table 5

Descriptive statistics for the residuals of the S&P 500 stock index and the U.S. 10-year T-bond after GARCH and CARR models filtered.

（1993/11/1~2006/3/17）

	Residuals after GARCH(1, 1) filtered		Residuals after CARR(1, 1) filtered	
Descriptive statistics	S&P500	10-year T-bond	S&P 500	10-year T-bond
Mean（%）	-0.003	0.012	-0.002	0.009
St.d（%）	0.9995	0.9988	0.9225	0.9753
Skewness	-0.419	-0.161	-0.333	-0.185
Kurtosis	4.579	4.506	3.965	4.653
Jarque-Bera	412.94***（0）	306.49***（0）	177.45***（0）	370.81***（0）
$Q(6)$	6.226（0.398）	5.465（0.486）	3.935（0.685）	6.22（0.399）
$Q(12)$	17.144（0.144）	13.299（0.348）	15.857（0.198）	13.877（0.309）
$Q^2(6)$	4.358（0.628）	6.548（0.365）	4.082（0.666）	10.409（0.108）
$Q^2(12)$	5.507（0.939）	9.471（0.662）	6.791（0.871）	13.484（0.335）

Note:

1. In the second column are descriptive statistics for the residuals after GARCH(1, 1) filtered.

   i.e. $\{z_{t-n+1}^G,\cdots,z_t^G\}=\{\frac{X_{t-n+1}-E(X_{t-n+1}|I_{t-n})}{\hat{\sigma}_{t-n+1}},\cdots,\frac{X_t-E(X_t|I_{t-1})}{\hat{\sigma}_t}\}$, where $\hat{\mu}_t$ and $\hat{\sigma}_t$ is estimates by the fitting of GARCH(1, 1) model.

2. In the third column are descriptive statistics for residuals after CARR(1, 1) filtered.

   i.e. $\{z_{t-n+1}^C,\cdots,z_t^C\}=\{\frac{\Re_{t-n+1}-\hat{\lambda}_{t-n+1}}{\hat{\mu}_{t-n+1}},\cdots,\frac{\Re_t-\hat{\lambda}_t}{\hat{\mu}_t}\}$, where $\hat{\mu}_t$ and $\hat{\lambda}_t$ is estimates by the fitting of CARR(1, 1) model.

3. Numbers in parentheses are p-values.
4. *, ** and *** represent the level of 10%, 5% and 1% are significant, respectively.

The degree of thick or thin about the tail distribution can be quantity by tail index indirectly. Here we using modified Hill estimator to get the tail index for the S&P 500 stock index and the 10-year bond yield rate.

We describe the well-known Hill approach to modeling the tails of heavy tailed distributions. For this method we assume that the underlying loss distribution is in the maximum domain of the Frechet distribution so that it has a tail structure,

$$\bar{F}(x) = L(x)x^{-\alpha} \quad (6)$$

for a slowly varying function L with a positive parameter α. Traditionally, in the Hill approach, interest centers on the tail index α, rather than its reciprocal ξ. The goal is to find an estimator of α based on identically distributed data $x_1,\ldots,x_n$. The Hill estimator can be derived in various ways. Perhaps the most elegant is to consider the mean excess function of the generic logarithmic loss ln(x), where x is a random variable with definition in equation (6). Writing e^* for the mean excess function of ln(x) and using integration by parts we find that

$$e^*(\ln\ \mu) = E(\ln\ X - \ln\ \mu \mid \ln\ X > \ln\ \mu) = \frac{1}{\bar{F}(\mu)}\int_{\mu}^{\infty}(\ln\ x - \ln\ \mu)dF(x)$$

$$= \frac{1}{\bar{F}(\mu)}\int_{\mu}^{\infty}\frac{\bar{F}(x)}{x}dx = \frac{1}{\bar{F}(\mu)}\int_{\mu}^{\infty}L(x)x^{-(\alpha+1)}dx \quad (7)$$

For μ sufficiently large, the slowly varying function L(x) for $x \geq \mu$ can essentially be treated as a constant and taken outside the integral. We expect that $e_n^*(\ln\ X_{k,n}) \approx \alpha^{-1}$ for n large and k sufficiently small, where $X_{n,n} \leq \ldots \leq X_{1,n}$ are the order statistics as usual. Evaluating $e_n^*(\ln\ X_{k,n})$ gives us the estimator is obtained by a minor modification:

$$\hat{\alpha}_{k,n}^{(H)} = (\frac{1}{k}\sum_{j=1}^{k}\ln X_{j,n} - \ln X_{k,n})^{-1}, 2 \leq k \leq n \quad (8)$$

The Hill estimator is one of the best studied estimators in the extreme value theory literature. The asymptotic properties (consistency, asymptotic normality) of this estimator[5] have been extensively investigated under various assumed models for the data. We concentrate on the use of the estimator in practice and, in particular, on

5 As sample size $n \to \infty$, number of extreme $k \to \infty$ and the so called tail fraction $k/n \to 0$.

its performance relative to the generalized Pareto distribution (GPD).

In calculating one day risk, using 500 days trading data as a moving window is approved in general literature. In Table 6, we demonstrate the results of the tail index in the left tail for the S&P 500 stock index and the 10-year T-bond yield rate after GARCH and CARR models are filtered. No matter what year data for both markets, all of the estimates of tail index (γ) are between 0.1653 and 0.2216. Seeming we can infer that these data can be viewed as Frechet distribution with the property of fat tail. Namely, these two asset returns have thick tail than normal type.[6] By Koedijk, Schafgans and de Vries (1990), when the reciprocal of tail index[7] is between zero and two, the distribution belongs to Pareto stable. If the reciprocal of tail index is greater than two, then the distribution belongs to t distribution. We show the estimates of tail index in Table 6.

Except for the result of the tail index in 10-year bond of 1997, all of them are smaller than 0.33. In other words, their reciprocal are greater than three that seems the proposed of fat tail distribution are supported, again. It is reasonable to inject the concept of extreme value theory for the discussion of VaR later.

Table 6:

[6] The tail index for normal distribution is zero.
[7] Conventionally, literature call the reciprocal of tail index is shape parameter.

The estimates of tail index for the residuals of the S&P 500 stock index and the U.S. 10-year T-bond after GARCH and CARR models filtered.

	S&P 500		10-year T-bond		
	γ (GARCH)	γ (CARR)	γ (GARCH)	γ (CARR)	Sample size
1995	0.207	0.202	0.238	0.242	43
1996	0.250	0.257	0.306	0.293	251
1997	0.281	0.285	0.336	0.314	250
1998	0.287	0.287	0.262	0.270	250
1999	0.226	0.223	0.233	0.227	250
2000	0.149	0.141	0.192	0.196	252
2001	0.122	0.099	0.185	0.210	248
2002	0.127	0.091	0.156	0.171	250
2003	0.146	0.137	0.158	0.137	252
2004	0.107	0.095	0.228	0.212	252
2005	0.062	0.053	0.169	0.172	250
2006	0.068	0.055	0.161	0.173	52
Total	0.174	0.165	0.222	0.220	2600

Note:

1. γ (GARCH) denotes the tail index after GARCH model filtered and γ (CARR) denotes the tail index after CARR model filtered.

2. Conventionally, literature call the reciprocal of tail index is shape parameter.

3. Due to the sake of data extraction, the sample size in 1995 and 2006 are only 43 and 52 respectively.

5. Back Testing

Whatever any method used for calculating VaR, an important reality check is back testing. The main purpose of this test is to make sure that the probability distribution is consistent with actual losses. It involves testing how well the VaR estimates would have performed in the past. Back testing compares the loss on any given day with the VaR predicted for that day. We will compare dynamical and static risk models as well as under the ranged based CARR model and return based GARCH model for VaR estimation. Suppose that we are computing a 1-day 95% VaR. Back testing would involve looking at how often the loss in a day exceeded the 1-day 95% VaR that would have been computed for that day. If this happened on about 5% of the days, we can feel reasonably comfortable with the methodology for computing VaR. If it happened on, say, 10% of the days, the methodology is suspect.

When estimating the tail index, we choice 500 days as a moving window. For consistency, we still extract the same time span for model fitting, get parameters and obtain the corresponding one-day VaR. Then rolling over the moving window for another one-day VaR. Repeat again and again, collecting the value of VaRs to the number of 2,600. Later, using these results and compare to the real market returns data. Hence, which model's performance is better can be compared by the procedure of back testing. Thus, we illustrate the empirical results for the historical simulation approach, static (unconditional) variance-covariance method (i.e. Delta-Normal) and extreme value method under the idea of VaR-X. According to the different disposition of standardized, there are two types for VaR-X. One is after GARCH model filtered and another is after CARR model filtered. All of the results for various static VaR models are listed on Table 7.

Table 7:

A back testing comparison of different static (unconditional) VaR models and actual losses for the S&P 500 stock index and the U.S. 10-year T-bond under the significant level of 95%, 97.5%, 99% and 99.5%

（1993/11/1~2006/3/17）

	HS	Delta-Normal	VaR-X	
			After GARCH filtered	After CARR filtered
95% significant level				
S&P 500	139	138	159	166
10-year T-bonds	141	131	174	176
Exceptions			130	
97.5% significant level				
S&P 500	78	81	80	88
10-year T-bonds	74	89	86	90
Exceptions			65	
99% significant level				
S&P 500	39	53	34	37
10-year T-bonds	38	58	44	43
Exceptions			26	
99.5% significant level				
S&P 500	19	33	15	18
10-year T-bonds	20	45	25	25
Exceptions			13	

Note:

1. We define an 'exception' to mean any day in which the actual data fell below the calculated confidence interval for the daily VaR under the full sample size of 2600.

2. HS denotes the method of historical simulation. Delta-Normal method is based on static normal assumption. VaR-X after GARCH filtered represents VaR after GARCH model fitting and VaR-X after CARR filtered represents VaR after CARR model fitting. Each number denotes the number of exception under specific model.

Under 95% confidence levels, no matter stock market or bond market, the better one for VaR valuation is the Delta-Normal approach. The accumulated times of exception is most near to 130. The second best is the method of historical simulation. From these results imply that based on relative lower confidence level (ex: 95%), the extreme value methods are not better than the conventional approaches. One reasonable explanation maybe the extreme value approach magnified dramatically too much in the tail area. So that the accumulated times of exception are higher than the real accumulated times. However, when the confidence level is increasing to 99%, the distinguished model is VaR-X with CARR model filtered for S&P 500 stock index. Its exception times are most near to the critical ones. By and large, the VaR-X approach is relative better than others for higher confidence levels for the S&P 500. Thus, introducing the theory of extreme value in the model of VaR can more firmly depict the dispersion of fat tail for the S&P 500 index. Also the performance in estimating the value of VaR is better than the conventional types. As to the 10-year T-bond market, the VaR estimation for the method of historical simulation is better than the others under the confidence levels of 99% and 99.5%. It is worth noting that the method of historical simulation assumes the volatility is the same now and in the future. Meanwhile, a longer data collected is necessary for historical simulation. Above all, the more confidence levels, the longer data collected is required. Thus, whether the historical extreme value is good enough for VaR estimation and the returns distribution is still unchanged now and in the future makes the two key factors for the accurateness of the historical simulation approach. However, in comparing static VaR evaluation model, we believe that the method of historical simulation can be incorporated into consideration. But, under data collection and reasonability, this proposition still need more evidence to support.

Considering the time varying property of volatility of trading data, we incorporate several well-known dynamical VaR models for comparison. The first model is EWMA (exponential weighted moving average) approach proposed by the RiskMetrics of J.P. Morgan. The second model is under normal distribution assumption for asset return, combined Delta-Normal method with GARCH model, we get a VaR model called GARCH-Normal approach. The third model is similar to the second model, but GARCH structure is replaced by CARR model, we name it as

CARR-Normal approach. Additionally, introducing the concepts of theory of extreme value, combined VaR-X and GARCH or CARR model, we obtain the GARCH-VaR-X model and the CARR-VaR-X model for the evaluation of VaR. All of the empirical results for the measurement of VaR under these five different methods are illustrated in Table 8 below.

Table 8:

The number of exceptions for dynamic(conditional) VaR model for the S&P 500 stock index and

10-year T-bond under the confidence levels of 95%, 97.5%, 99% and 99.5%.

(1995/10/30~2006/3/17)

	Normal assumption			Under the theory of extreme value	
	EWMA	GARCH-Normal	CARR-Normal	GARCH-VaR-X	CARR-VaR-X
95% confidence levels (the number of exception)		130		130	
S&P 500	144	144	137	163	167
10-year T-bond	130	128	131	150	157
97.5% confidence levels (the number of exception)		65		65	
S&P 500	96	87	73	84	77
10-year T-bond	78	83	77	81	78
99% confidence levels (the number of exception)		26		26	
S&P 500	51	46	39	30	31
10-year T-bond	52	52	45	37	28
99.5% confidence levels (the number of exception)		13		13	
S&P 500	34	29	27	12	13
10-year T-bond	36	36	31	20	15

Note:

1. EWMA represents a dynamic conditional VaR model by EWMA under the variance-covariance of RiskMetrics method. GARCH-Normal denotes a dynamic conditional VaR model by GARCH model get a variance-covariance estimates. CARR-Normal denotes a dynamic conditional VaR model by CARR model get a variance-covariance estimates. GARCH-VaR-X refer to a VaR-X model based on GARCH volatility model with the theory of extreme value. CARR-VaR-X refers to a VaR-X model based on CARR volatility model with the theory of extreme value.
2. All the numbers in Table 8 represent the number of exceptions based on various VaR methods.

Under the confidence levels of 95%, regard to the S&P 500 stock index, the best dynamic VaR model is CARR-Normal model during the time horizon from 1995/10/30 to 2006/3/17, which the accumulated number of exceptions is 137, most close to the theoretical number of 130. Other priorities are GARCH-Normal method, EWMA method, GARCH-VaR-X method and CARR-VaR-X in order. For the 10-year T-bond market, we can observe that the best one is EWMA method. Meanwhile, the evaluation performance for VaR about CARR-Normal method and GARCH-Normal method are similar to the EWMA. The number of exceptions for these models is quite close to the theoretical value. In relatively, the more complex GARCH-VaR-X model and CARR-VaR-X model are not good enough under such a loose confidence levels (ex: 95%). Increasing the confidence levels to 99%, we can find that the evaluation performance for VaR is reverted apparently. Regardless the S&P 500 stock index or the 10-year T-bond market, the GARCH-VaR-X method and the CARR-VaR-X method have better performance than others in the evaluation of VaR. These inferences are convinced and proved again when we boost the confidence levels to 99.5%. Under the frame of extreme value theory, the group of VaR-X performs more accurate than others. Above all, for the S&P 500 stock index, the number of exceptions for the CARR-VaR-X method is just matched with the number of theory. Based on various significant levels, one can dissect the performance priority in evaluating VaR for different models. Under more strict confidence levels, their accurateness can rank as follows roughly. They are CARR-VaR-X method, GARCH-VaR-X method, CARR-Normal method, GARCH-Normal method and EWMA method.

The 1996 Amendment to the Basel accord describes the form of back testing that must be undertaken by firms wishing to use a VaR model for the calculation of market risk capital requirements. Regulators recommend to back-test the 1% 1-day VaR that is predicted by an internal model. The model should be back-test against both theoretical and actual profit and loss. Whether or not actual profit and loss gives rise to more exceptions during back-tests than theoretical profit and loss will depend on the nature of trading. From the viewpoint of Basel, they suggest the standard confidence level is 99% in the evaluation of VaR. Combined with the discussion above, we propose that VaR-X methods with the concept of extreme value theory can express more efficiency than only VaR-X structures and EWMA model.

6. Robust testing for the VaR of a portfolio

Suppose your portfolio is composed of two assets. For example, place half your funds in the S&P 500 stock index and half in the 10-year T-bond. In theory, the correlation of the two-asset portfolio is time varying and is a function of covariance for these two assets. Engle (2002) provide another solution to dynamical correlation process by using a model entitled the Dynamic Conditional Correlation Multivariate GARCH (henceforth DCC). Intuitively, the conditional covariance estimation for two variables is simplified by estimating univariate GARCH models for each asset's variance process. Then, carrying on by using the transformed standardized residuals from this stage, and estimating a time-varying conditional correlation estimator in the next stage, the DCC model is not linear, but can be estimated simply with the two-stage methods based on the maximum likelihood method.

Chou, Wu, and Liu (2005) consider a refinement of the return-based DCC model by using the high/low range data of asset prices. Meanwhile, they introduce the idea of CARR model into DCC structure and call it range-based DCC model. Range-based DCC model can replace the original return-based DCC model in the estimation of correlation process. In other words, the DCC model is a new type of multivariate and can fit the GARCH or CARR model in the first stage, which is particularly convenient for complex dynamic systems. The DCC method first estimates volatilities for each asset and computes the standardized residuals. For bivariate cases, one can use the

GARCH and CARR structures to perform the first step, respectively. The covariance processes are then obtained readily using maximum likelihood estimation. For more information, one can consult Engle (2002), Chou (2005) and Chou, Wu, and Liu (2005).

In order to discuss the performance of different VaR models for portfolio, here illustrates five methods for the valuation of VaR for the purpose of comparison. They are EWMA model of J.P. Morgan, GARCH-Normal with return-based DCC model, CARR-Normal with range-based DCC, GARCH-VaR-X with return based DCC model and CARR-VaR-X with range based DCC in order.

Table 9

The number of exceptions about the portfolio of the S&P 500 stock index and the 10-year T-bond for conditional VaR model under the confidence levels of 95% and 99%.

（1995/10/30~2006/3/17）

	Normal assumption			Under the theory of extreme value	
Confidence levels	EWMA	GARCH-Normal-DCC	CARR-Normal-DCC	GARCH-VaR-X-DCC	CARR-VaR-X-DCC
95%					
Theoretical valve		130		130	
portfolio	145	126	126	151	151
99%					
Theoretical valve		26		26	
portfolio	58	51	46	31	27

Note:

1. EWMA represents a dynamic conditional VaR model by EWMA proposed by J.P. Morgan. GARCH-Normal-DCC denotes a dynamic conditional VaR model by GARCH structure with return-based DCC model. CARR-Normal-DCC denotes a dynamic conditional VaR model by CARR structure with range-based DCC model. GARCH-VaR-X-DCC refers to a VaR-X model based on return-based GARCH-DCC model with the theory of extreme value. CARR-VaR-X-DCC refers to a VaR-X model based on range-based CARR-DCC model with the theory of extreme value.
2. All the numbers in Table 9 represent the number of exceptions based on various VaR methods.

From Table 9, the GARCH-Normal-DCC model and the CARR-Normal-DCC model perform relative well in estimating the number of exceptions for the VaR of portfolio under 95% confidence levels. Other models underestimate the real situation of risk mostly. When the confidence levels, however, lift into the level of 99%, it is observed clearly that the exception number of back testing for CARR-VaR-X-DCC model is the nearest to the theoretical threshold. The second priority is the GARCH-VaR-X-DCC model. Other models are inclined to overestimate the number of exceptions.

7. Conclusions

One approach to calculating VaR is historical simulation, but its performance in estimating VaR is less precise than considering the dynamic of volatility process. When calculating VaR, we are most interested in the current levels of volatilities because we are assessing possible changes in the value of an asset over a very short period of time. Beside, the theory of extreme value has some advantages over the standard approach to risk management. Empirical study has shown that the results for the measurement of VaR with extreme value methods and appropriate volatility model which provides a more precise approach for risk management and value at risk calculations.

We have considered as possible as many approaches for the evaluation of VaR for single asset and a portfolio in this paper. The most remarkable results are in evaluation of the higher confidence levels performance of the extreme value method with range-based CARR model. The empirical results demonstrate that the VaR calculated using tails of extreme distributions with range-based CARR model is significantly more suitable than the conventional return-based GARCH standard approach. Meanwhile, considering the information implied by tail distribution with range-based DCC model are more robust and inducing more accurate estimates of the number of exceptions for the S&P 500 stock index and the 10-year T-bond market mixed portfolio.

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