CHARACTERISING NON-NORMALITY IN ASSET RETURNS USING THE GENERALISED SKEW STUDENT DISTRIBUTION

by

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Abstract

To investigate asymmetry and fat-tailedness in daily stock and portfolio returns we extend the Generalised Skew Student t distribution (GST) by separate parameterisation for observations above and below zero. A sequence of likelihood ratio tests examines progressive parameter restrictions on the extended GST. Empirical evidence comes from the UK, Japanese, South African and US markets. The main conclusions are: the Student t is the single most common model; the symmetric GST is a common model of UK and South African stocks; significantly asymmetrical returns are generally modelled by a 5 parameter GST; few Japanese or US stocks exhibit skewness.

Key words: Fat-tails, Kurtosis, Persistence, Skewness, VaR/CVaR.

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1. INTRODUCTION

It is well known that skewness and kurtosis are often present in asset returns. In general terms, there are two ways of dealing with these departures from normality. One is to modify utility by adding higher moments to the usual quadratic function. The other, the focus of this paper, is to consider other probability models for returns.

Numerous different probability distributions have been applied to the task of modelling asset returns to capture departures from normality. Use of the Student t distribution in finance dates back to Praetz (1972) and Blattberg and Gonedes (1974). Aparicio Acosta and Estrada (2001) apply it to returns on European stocks. The Edgeworth-Sargan distribution has been compared with the Student t by Mauleon and Perote (2000) and Mauleon (2006). Corrado and Su (1996) make use of Gram-Charlier series expansions. The use of stable distributions dates back to Mandelbrot (1963). Kon (1984) examines mixtures of normal distributions. Skewness specifically is investigated in Hansen (1994) and Harris, Kukukozmen and Yilmaz (2004), with both papers using an asymmetric generalisation of the Student t distribution.

More recently, the skew-normal distribution associated with the work of Azzalini (1985, 1986) has been applied in finance by Adcock and Shutes (2001), Adcock (2004a,b) and Harvey, Leichty, Leichty and Muller (2004). Fernandez and Steel (1998) present a model that accommodates both skewness and fat tails. Their distribution is based on a density function, $f(.)$ say, which is symmetric about zero and unimodal. A class of skewed distributions for a variable X is indexed by a scalar parameter, γ in their notation, and has a density function which is proportional to $f(\gamma x.)$ when $x \geq 0$ and to $f(x/\gamma.)$ otherwise. They choose $f(.)$ to be Student t and apply it to stock returns. There are many other examples of the use of non-normal distributions.

Here we present an extended version of the generalised skewed Student t distribution and apply it to modelling asset returns. This extended version is used to explore the nature of both fat-tails and asymmetry often observed in asset returns. The generalised skewed Student t distribution was introduced by McDonald and Newey (1988) and further described in McDonald and Nelson (1989) and McDonald and Xu (1995). It was extended and applied to modelling asset returns in Theodossiou (1998), henceforth PT. As PT points out, this is a rich family of probability distributions. Placing constraints on the parameters generates a wide range of special cases, including the Gaussian, Laplace, Student t itself and generalised error distributions.

We gain greater flexibility in modelling both fat-tails and asymmetry by extending the parameterisation of the distribution. Specifically, we consider a random variable, X say, which has a probability distribution with density function

$$f_X(x) = \frac{K}{\left(1 + \frac{1}{\upsilon_i}\left|\frac{x}{\sigma_i}\right|\right)^{\frac{\upsilon_i}{2}+\frac{1}{\omega_i}}}; i = 1 \text{ for } x \geq 0, i = 2 \text{ for } x < 0, -\infty < x < \infty . \qquad (1.)$$

The parameters σ_i, ν_i and ω_i, i =1,2 are all positive and K is the normalising constant. We refer to this as the generalised skewed t or GST distribution. The two ν parameters are referred to as degrees of freedom. The deviations of the two ω parameters from 2 measure the departure of the distribution from Student t, that is "Studentness" or the lack of it. The σs are scaling parameters. PT's model is a special case of the GST; when the degrees of freedom and Studentness parameters are equal, i.e. $\nu_1 = \nu_2$ and $\omega_1 = \omega_2$. When, in addition to the above, $\omega_1 = \omega_2 = 2$ and $\sigma_1 = \sigma_2$, the distribution is proportional to Student t.

In PT's model, skewness is driven only by differences in (σ_1, σ_2). The contribution of the GST distribution defined at equation (1.) is that both skewness and kurtosis can be generated by differences between the values of one or more of the pairs of parameters, (ν_1, ν_2), (ω_1, ω_2), (σ_1, σ_2). To illustrate the potential of the more general parameterisation of the GST distribution, Figure 1 shows two density functions, GST (a) and GST (b) say. Both distributions have mean zero, unit variance, a skewness of –1.6 and a small difference in excess kurtosis. However, the asymmetry depicted is noticeably different. It can be seen in Figure 1 that GST (a) is more peaked and, from inspection of *ln* (F (x)), that it has a fatter left hand tail than GST (b).

The contrast shown in Figure 1 explains why there are six parameters in the density function (1). For a parsimonious parameterisation, one could expect one parameter per moment, giving a maximum of four parameters to describe a random variable with non-zero skewness and excess kurtosis. The two further parameters of the GST distribution defined at (1.) provide different forms of asymmetry and tail behaviour that are visually different, even though the moments are similar.

Figure 1 about here

In addition to its relevance for general tasks in financial modelling, an accurate density forecast of the return on an asset is invaluable for the management of risk. Value at Risk (VaR) is a convenient description of a financial institution's market exposure and a widely used form of risk management. Regulators demand that banks inform them regularly of their exposure to market risk using VaR as the measure. For a bank holding *I* money units invested an asset, whose return, at time *t*, over a period *T*, is $X_{t,T}$ and for which $\Pr(X_{t,T} < -\xi_\alpha) = 1 - \alpha$, then the bank's VaR is $MI\xi_\alpha$ where M is a 'safety factor'. The bank's reserves would be required to exceed this amount. The Basle Accord (see, for example Jackson, Perraudin and Maude, 1997) suggests that $\alpha = 0.99$, T is ten days and the safety factor, *M*, is three. The literature on the estimation of Value at Risk is voluminous (for background, see Jorion, 1996), but two clear themes have emerged: one is the use of dynamic volatility models such as GARCH to give conditional probability estimates; the other theme is the computation of unconditional probability estimates. A review of relative merits of dynamic volatility models is given by Brooks and Persand (2003). The work here falls into the context of the second theme. Longin (2000) discusses the applications of extreme value theory to VaR. In Longin (2005) this work is further developed and used to inform the choice of distribution for asset returns. Using US data, he demonstrated that of the Gaussian, Student t and the stable Paretian density functions, only the Student t was acceptable as an unconditional model of returns. Knowledge of the appropriate GST model for returns for an asset permits the evaluation of the unconditional probability above.

The second aim of this paper is to investigate the persistence of non-normality. It is generally accepted that fat-tails are a permanent empirical feature of the probability distribution of asset returns. However the persistence of an appropriate model for the distribution is a issue of importance. Misspecification of the model may lead to errors in the computation of measures of risk or in portfolio selection. Evidence of the lack of persistence of a model leads to the need periodically to investigate not only model parameter values but also the specification of the model itself.

Skewness raises slightly different issues. A number of authors have considered the persistence of skewness over time. Singleton and Wingender (1986) consider the persistence of skewness using monthly returns data for the period 1961-80. They find that positively skewed assets are as likely to exhibit negative skewness in the next period as positive and vice versa. In a study of emerging markets, Bekaert, Harvey, Erb and Viskantam (1998) draw attention to the hypothesis that skewness and kurtosis may be time varying. An explanation for such temporal variation is that these forms of non-normality are artifacts of the process of emergence. An implication of this hypothesis, if correct, is that incidence of significant values of these moments will decrease as time progresses. For the mature markets studied in this paper, the implications are that the majority of stocks will not exhibit skewness but that for those that do it will be a transient phenomenon.

We will use the GST family of distributions to investigate the nature of skewness and kurtosis in the probability distributions of asset returns and the persistence of these departures from normality. As well as reporting results for individual securities, we also describe investigations into portfolios of assets. The motivation for this is that *a priori* it might be expected that, as a consequence of the central limit theorem, returns on portfolios would tend to exhibit low levels of both skewness and kurtosis. An additional motivation is that VaR is usually applied to portfolios rather than individual assets.

The paper is set out as follows. Section two gives a derivation of the skewed generalised Student t distribution and describes its properties. The aim of the derivation is to give insights into possible return generating processes by representing the GST as a mixture of distributions. Section three summarises the data and methods that are used in the empirical studies reported in the two following sections. Section four describes the application of the GST distribution to daily returns on UK stocks that are constituents of the UK FTSE 350 index. Section five describes the comparable results for portfolios of UK FTSE 350 stocks. Section six reports a comparative study based on stocks from the Japanese Nikkei 225, South African FTSE Johannesburg stock exchange all share index and the US S&P 500 index and for portfolios of stocks selected from the index constituents in each of these markets. Section seven concludes. In order to save space, only the main results are presented in the paper, more detailed results are available from the authors on request. The computations were performed in S-plus. Notation is that in common use.

2. DERIVATION AND PROPERTIES OF THE GENERALISED SKEW STUDENT DISTRIBUTION

In order to model the return generating process, we consider a non-negative random variable, X say, which represents the departure of the asset return from zero in absolute terms. This step allows for the possibility that the process creating positive returns may differ from that creating negative returns. We assume that the random variable X has a generalised Weibull distribution with probability density function given by

$$f_{X|\theta}(x) = \frac{\omega\theta^{\frac{1}{\omega}}}{\Gamma\left(\frac{1}{\omega}\right)} \exp\left(-\theta x^{\omega}\right);\ \theta, \omega > 0,\ 0 \le x < \infty . \qquad (2.)$$

The probability density function at (2.) is a special case of the generalized gamma distribution, which is due to Stacy and Mirham (1965). Further details of this distribution and its properties are in Johnson et al (1994, p689). The shape of the density function is controlled by ω, as demonstrated in Figure 2.

Figure 2 about here

The mean of a random variable with distribution given by (2.) is

$$\frac{\theta^{-\frac{1}{\omega}}\Gamma\left(\frac{2}{\omega}\right)}{\Gamma\left(\frac{1}{\omega}\right)},$$

indicating that θ acts as a scaling factor for the magnitude of the return. We further assume that there is heterogeneity in the return generating process and that the scaling factor θ is distributed as a gamma random variable with mean and variance given by

$$\left(\frac{\nu}{2\theta_0}\right), \left(\frac{\nu}{2\theta_0^2}\right),$$

respectively. The probability density function of the distribution of θ is

$$f_{\Theta}(\theta) = \frac{\theta^{\frac{1}{2}\nu-1}\theta_0^{\frac{1}{2}\nu}}{\Gamma\left(\frac{1}{2}\nu\right)} \exp\left(-\theta\theta_0\right);\ \theta_0, \upsilon > 0,\ 0 \le \theta < \infty .$$

Integration over the mixing distribution

$$\int_0^\infty f_{x|\theta}(x\mid\theta) f_\theta(\theta)\, d\theta,$$

gives the unconditional distribution of X, which has the probability density function

$$f_x(x) = \frac{\omega\theta_0^{-\frac{1}{\omega}}\Gamma\left(\frac{\nu}{2}+\frac{1}{\omega}\right)}{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\frac{1}{\omega}\right)} \frac{1}{\left[1+\frac{x^\omega}{\theta_0}\right]^{\frac{\nu}{2}+\frac{1}{\omega}}} \quad 0 \le x < \infty . \qquad (3.)$$

Since the variable X^ω/θ_0 has a beta type two distribution, it follows that the n^{th} moment of X about the origin is

$$E\left(X^n\right) = \frac{\theta_0^{\frac{n}{\omega}}\Gamma\left(\frac{n+1}{\omega}\right)\Gamma\left(\frac{\nu}{2}-\frac{n}{\omega}\right)}{\Gamma\left(\frac{1}{2}\nu\right)\Gamma\left(\frac{1}{\omega}\right)} \quad \text{for } \nu\omega > 2n . \qquad (4.)$$

To create the GST distribution, the density function for X defined at equation (4.) is generalised to allow positive and negative returns to behave differently. The probability density function of a GST variable is defined as

$$\mathrm{f}_\mathrm{X}(\mathrm{x}) = \frac{\mathrm{K}}{\left(1+\frac{|\mathrm{x}|^{\omega_i}}{\theta_{0i}}\right)^{\frac{\upsilon_i}{2}+\frac{1}{\omega_i}}}; \mathrm{i}=1 \text{ for } \mathrm{x} \ge 0, \mathrm{i}=2 \text{ for } \mathrm{x}<0, -\infty < \mathrm{x} < \infty . \qquad (5.)$$

To make the effect of the scaling parameter more direct and to make the correspondence with Student t more apparent, θ_{0i} is redefined as $\theta_{0i} = \nu_i \sigma_i^{\omega_i}; i = 1,2$. The probability density function is then as defined at equation (1.). The normalising constant K is given by

$$\frac{1}{\mathrm{K}} = \frac{1}{\mathrm{K}_1} + \frac{1}{\mathrm{K}_2} \text{ where } \mathrm{K}_i = \frac{\omega_i\Gamma\left(\frac{1}{2}\nu_i + \frac{1}{\omega_i}\right)}{\Gamma\left(\frac{1}{\omega_i}\right)\Gamma\left(\frac{1}{2}\nu_1\right)\sigma_i\nu_{0i}^{\frac{1}{\omega_i}}} . \qquad (6.)$$

As well as the general case of the distribution, there are 7 special cases in which one or more of the pairs of parameters are restricted to take equal values.

On initial perusal, it appears that this distribution is over-parameterised. In the tails of the distribution, the probability density function is of the form

$$f_X(x) \approx K'\left|\frac{x}{\sigma}\right|^{-\frac{1}{2}\upsilon\omega-1}, \qquad (7.)$$

where K' is a constant and the subscript i has been omitted. Since the tail values dominate the likelihood function, one could take the view that a more appropriate parameterisation would be in terms of the products $\omega_i\nu_i$, i =1,2. Furthermore, in the symmetric case, the implication of (7.) is that Student t distribution may be a more suitable model with degrees of freedom equal to $\omega\nu/2$. However, as well as the visual evidence presented in Figure 1, the empirical results that are described in sections four onwards support the role of the general parameterisation in equations (1.) and (6.) in many cases. As will be described, for the returns data studied in this paper it is rare for 5 or 6 parameters to be required although a small number of such cases do arise empirically. Furthermore, as demonstrated above, the use of ω_i and ν_i, i =1,2, as separate parameters gives rise to a model that offers some insights into the processes that generate returns.

In the following sections, the eight cases of the GST distribution are referred to using an abbreviation of the general form GST$\alpha\beta\gamma$. This shorthand is to be interpreted as follows. GST$\omega\upsilon\sigma$ means that the three pairs of parameters take equal values; that is the distribution is symmetric. GST$\omega\upsilon$-, for example, means that asymmetry is generated only through variation in the suppressed parameter, in this case σ. In general replacement of the Greek letter by a hyphen means that symmetry may be generated by variation of the suppressed parameter. The most general case is therefore GST---.

Two closely related families of distributions may be obtained by imposing fixed values on the degrees of freedom or the Studentness parameters. First, as ν_1 and ν_2 both increase without limit, a skewed version of the generalised error distribution is obtained. See Nelson (1991) for further details. Secondly, if ω_1 and ω_2 are both set equal to two the skewed Student distribution is obtained. In the following sections of the paper, we refer to these two distributions as GED and GT respectively. The same shorthand described above is used to refer to the various special cases. The expressions for the GED and GT probability density functions are omitted, as both may be obtained from equations (1.) and (6.) using standard manipulations. There are 4 cases for both GED and GT distributions, respectively, giving an overall total of 16 models.

The effects of different values for the parameters, ω_i, σ_i and ν_i, are shown in Figures 3, 4 and 5. The base, symmetric, model is given by $(\omega_1, \nu_1, \sigma_1, \omega_2, \nu_2, \sigma_2) = (3,2,1,3,2,1)$. For $\omega_1 \neq \omega_2$ (Figure 3) the asymmetry is manifested by a change in shape of the peak of the density function, by a change in location of the peak and by a shortening of the right-hand tail. For $\sigma_1 \neq \sigma_2$, (Figure 4) the asymmetry occurs due to a change in location of the peak of the density function, as the $\max(\sigma_1, \sigma_2)$ increases, the kurtosis increases and the right hand tail is shortened. For $\nu_1 \neq \nu_2$, (Figure 5), the kurtosis decreases as the $\max(\nu_1, \nu_2)$ increases and the location of the peak moves slightly compared to the movement caused by varying the other parameters.

Figures 3 – 5 about here

Moments about the origin can be derived from equations (1.) and (6.) and the n^{th} moment exists if

$$\min\left(\nu_i \omega_i \big/ 2 \right) > n.$$

As PT shows, analytical evaluation of the moments of this distribution is complicated. However, numerical evaluation of central moments and, hence, computation of mean, variance, skewness and kurtosis is straightforward. Cumulative probabilities may be routinely computed using the incomplete beta function.

3. DATA AND METHODS FOR THE EMPIRICAL STUDY

The empirical study reported in this paper is based on four sets of daily price data which were obtained from Datastream. The first data set, which is reported on in some detail in sections four and five, consists of 380 non-financial UK stocks which were members of the UK FTSE350 index during the period January 1990 to December 2002. The data set used in this study covers the 1500 day period from 3rd February 1998 until 3rd November 2003. The other three data sets, which are reported on more briefly in section 6, are based on the Japanese Nikkei 225, the FTSE Johannesburg stock exchange all share index and the US S&P500 index. Daily data was also obtained for the four respective market indices. All prices are in local currency. Daily returns are computed in the usual way by taking the difference of the natural logarithm of price. As well as price data, the corresponding daily market values were also obtained. This data set is used to compute the capitalisation weight of each stock in its respective index. The data set is summarised in Table 1.

Table 1 about here

As well as computations based on individual securities, this study reports results for simulated portfolios of stocks selected from these data sets. A portfolio contains 5, 10, 25, 50, 100, 200, 300 or more stocks depending on the number of stocks in the index. For each case, the requisite stocks are chosen according to one of three different sampling schemes. These are as follows: equal probability of selection, probability of selection proportional to market capitalisation and probability of selection inversely proportional to market capitalisation. The second method of selection gives portfolios of large capitalisation stocks and the third method portfolios of small capitalisation stocks. For each selection, portfolio return is computed in two ways; namely by assuming equal weights or weights proportional to market capitalisation. For the purpose of this study, market capitalisation is recomputed each day using the previous 500 days of data. For the UK, for example, the entire sampling scheme thus gives 7*3*2 = 42 different portfolios. This is repeated five times to give a sample of 210 portfolios. The time series of returns for each portfolio commences on the first date for which there are valid returns for at least one of the selected stocks. Thus, a portfolio with, for example, five stocks may start as a holding in a single stock, with stocks 2 through 5 being added as they become available. The number of such portfolios for each of the four markets is summarised in Table 1.

The parameters of the distributions specified in section 2 are estimated for a location parameter model for each stock and each portfolio. This is done using a sample of 500 days, ending on 3rd November 2003. This is referred to as period C. Those stocks which do not have 500 days of data available as at the last date are excluded from the analysis. Returns on the index itself are included with the data for each market. The number of stocks for which data is available for estimation is as listed in Table 1. To investigate persistence of both the 3rd and 4th moments and of the return distribution itself, the above exercise is repeated for two earlier time periods. These consist of two non-overlapping blocks of 500 days ending on 3rd January 2000 and 3rd December 2001 respectively. As above, stocks which do not have 500 days of data available as at the last date of the block in question are excluded from the analysis. These two periods are referred to as A and B respectively.

The maximum likelihood estimators of the location parameter models corresponding to each distribution are computed using a variation of the BHHH algorithm, Berndt, Hall, Hall and Hausman (1974). Following convention σ^2 is estimated rather than σ. To avoid the possibility of the algorithm computing negative estimates of ω_i, ν_i and σ_i^2; i = 1,2, the usual logarithmic transformation is used. The derivatives of the log-likelihood with respect to the parameters are computed numerically.

In order to analyse the extent and nature of asymmetry present in the stock returns of each market a sequence of likelihood ratio tests is performed. Each test examines a simple constraint on the parameters. The analysis commences with the estimation of the (unconstrained) GST model. The sequence of likelihood ratio tests is shown in Figure 6. If asymmetry is not caused by a particular parameter, for example there is no reason to doubt $\omega_1 = \omega_2$, then a further check for 'Studentness' is carried out. Similarly, if there is no reason to doubt $\upsilon_1 = \upsilon_2$, then the magnitude of the degrees of freedom is examined (with 5000 as a proxy for infinity). In the figure, if the hypothesis cannot be rejected, the analysis follows the left hand branch, if it can be rejected the right hand branch is followed. The process can be regarded as a classification tree for the nature of asymmetry in stock returns. The bottom line of 18 bins, identified alphabetically, in the figure contains several special cases. A is the normal distribution, B is Student t, D is the generalised error distribution and K is PT. The probability distribution associated with each of the 18 bins is defined in Table 2, in the next section.

Figure 6 about here

4. STUDY OF THE UK FTSE350 INDEX

As noted in the introduction, only the main results of the empirical study are described here. More detailed results are available on request, including the usual sample statistics. The main results described in this section are based on the data set for period C which ends on 3rd November 2003.

The results of carrying out the series of likelihood ratio tests as described in section 3 for UK FTSE350 stocks are shown in Table 2. The table shows the number of stocks classified in each bin at four levels of probability, ranging from 10% to 0.1%. As the table shows, over 95% of the stocks are classified in one of six bins, regardless of the level of probability. The other twelve bins account for the remaining 5% of securities. Of these twelve bins, seven have no stocks at any of the four given levels of probability. In the rest of this section, the numbers of stocks in each of the six bins are taken from the 1% column of Table 2. Some general conclusions are presented first. The main features of Student t distribution and the five other models are then described in turn. This is followed by the main results concerning the persistence of return distributions and, by implication, of both skewness and kurtosis.

For daily returns, the normal distribution accounts for very few stocks in the UK FTSE350 index. Even at the 0.1% level of significance, only 11 securities have returns that are normal. Models with few or no parameter restrictions are selected for a small number of stocks. The generalised error distribution is never selected as a model. The skew Student model in which skewness is generated only by unequal values of the degrees of freedom parameters is selected, but other members of the GT class are not.

Table 2 about here

With a small number of exceptions, the most complicated non-symmetric models that are selected for this data set are the GSTω-σ, GST-$\upsilon\sigma$ and GST$\omega\upsilon$- distributions, which each have five free parameters. In these cases, skewness is generated by variation in degrees of freedom, υ, Studentness, ω, or scale, σ, respectively. It is interesting to note that PT's version of the GST distribution, GST$\omega\upsilon$-, in which asymmetry is generated only by variations in the scaling parameter σ, is not a widely selected model. For the 92 stocks for which skewness is a significant feature of returns, only 4 employ the PT model. For the remainder of this section, attention is generally restricted to the six most common models.

Table 3 about here

Table 3 shows an analysis of the six most common models based on the capitalisation weights of each stock in the index. Since the weights change each day, an average weight is computed based on the same 500 days used for parameter estimation. First, the daily returns on the UK FTSE350 index follows a Student t distribution. For the remaining 183 stocks classified as having Student t distribution, the average of the average capitalisation weight is 0.45%, the smallest average weight is 0.007% and the maximum is 8.61%

From Table 3, it is clear that, on average, returns on the larger capitalisation stocks are characterised by Student t, the symmetric version of the GST distribution and the skewed Student distribution in which asymmetry is generated only by variation in degrees of freedom. Skewness, as modelled by the GST distribution, is generally associated with smaller capitalisation stocks. However, inspection of the columns labelled Max% and Min% indicate that there are exceptions to this. For example, the row labelled GTω-σ and GSTωυσ both contain stocks with very small weights, as well as those with large capitalisation. An interesting point to note is that the average capitalisation of GTω-σ stocks is about 0.6%, which is larger than the average for Student t. Overall, stocks with symmetric return distributions account for about 84% of total market capitalisation. The 92 stocks with skewed returns account for the remaining 16%.

The constituents of the contemporaneous FTSE100 index will be present in the FTSE350. It is therefore reasonable to infer from the analysis based on capitalisation in Table 3 that daily returns of many FTSE100 securities will be distributed as Student t. Indeed, out of the 100 stocks in this analysis with the largest average capitalisation weight, 88 are classified as Student t.

4.1 Student t distribution

For the 184 stocks, including the index, which are classified at Student t, the most significant feature is the behaviour of the tail probabilities. Table 4 shows an analysis of the effect of using the normal distribution as a model for returns when the correct model is Student t. The table entries are computed as follows. The critical values corresponding to the eight nominal probabilities shown in the table are computed using estimated parameters based on an assumed normal distribution. The actual probability corresponding to each critical value is then computed using the estimated parameters based on Student t distribution. The computed probability is treated as being the same as the nominal value if the absolute difference between them is 0.0001 or less.

Perusal of the columns in section A of Table 4 shows that an assumption of normality in the case of Student returns causes under-estimation of the tail probabilities at the two-sided 1% and 5% levels for the vast majority of stocks. Conversely, the two sided 10% tail probabilities are over estimated. It may also be noted that extreme tail probabilities, two-sided 0.1%, are also over-estimated.

Table 4 about here

The columns in section B repeat the analysis, but show the percentage of the index capitalisation weight in each category. As these columns show, under-estimation of tail probabilities at the 1% and 5% levels occurs for stocks accounting for between 75% and 82% of the total capitalisation.

The distribution of the estimated degrees of freedom is shown in Figure 7. There are 4 stocks with degrees of freedom less than 1 and for which the tails are therefore fatter than those of the Cauchy distribution. The maximum of the estimated degrees of freedom is less than 10. As noted above, the returns on UK FTSE350 index are classified as having a Student t distribution, with estimated degrees of freedom being about 3.9. The small values of the degrees of freedom for stocks classified at Student t is not a surprise. This is because the probability distribution increasingly resembles the normal as the degrees of freedom increase above 10.

Figures 7 & 8 about here

A graphical view of the effect on the tail probabilities is in Figure 8. This figure shows the estimated Value at Risk (VaR) at 1% for the stocks which are classified as Student t. The VaR is computed as a percentage and the graphs are plotted in the order of decreasing risk according to the value given by the correctly specified Student t distribution. As the graph shows, use of the normal distribution generally causes an under-estimation of the VaR. It may also be noted that as the Student VaR increases the corresponding estimates based on a normal distribution become progressively more unreliable.

4.2 The Symmetric Generalised Student Distribution

This corresponds to bin E, the GST$\omega\upsilon\sigma$ model. This is a symmetric distribution, but the Studentness parameter ω is unrestricted. According to Table 2, at the 1% significance level there are 87 stocks in this category. Table 5 shows an analysis of the effect of using Student t distribution as a model for returns when the correct model is the GST$\omega\upsilon\sigma$. The table entries are computed in the same way as those in Table 5. The critical values corresponding to the eight nominal probabilities are computed using estimated parameters based on an assumed t distribution. The actual probability corresponding to each critical value is then computed using the estimated parameters based on the GST$\omega\upsilon\sigma$ distribution.

Table 5 about here

Similar to the results shown in Table 4, use of Student t distribution when the correct model is the symmetric generalised Student distribution cause serious under-estimation of the tail probabilities at the (two sided) 5% levels for the majority of stocks. Under-estimation also occurs at the (two-sided) 1% level, although for a smaller number of stocks.

When these results are viewed using market capitalisation (section B of the table), a similar picture emerges. However, the analysis based on capitalisation weights makes it clear that stocks which are classified as having the GSTωυσ model only account for about 4% of the total weight of the index. From a purely statistical perspective, one could argue that this is a small percentage and therefore use of Student t distribution would represent a specification error that is of a minor consequence in practice. From the portfolio selection perspective, however, these specification errors could be of greater significance. This is because it is common practice in the investment industry to construct portfolios for which the holdings are generally similar in percentage terms to those of the index weight. Portfolio managers then seek to obtain excess return over and above that of the index by over-weighting (under-weighting) stocks that are expected to out-perform (under-perform) the index. Such stocks are likely to be those which exhibit unusual return characteristics. Thus it is possible that stocks with the GST distribution may assume an importance in the portfolio that is greater than their index weight alone might suggest. To put it another way: the 3.23% of market capitalisation whose 2.5% (left hand tail) probability is underestimated may comprise precisely those stocks which could lead to excess profits or losses!

Examination of the estimated values of ω and ν for the symmetric generalised Student distribution provides further evidence of the differences with the Student t model.

Table 6 about here

Table 6 has two panels. Panel (i) summarises the estimated values of ω and υ for the 87 stocks which are classified as GSTωυσ . Also shown in panel (i) are the corresponding summaries for the estimated degrees of freedom υ under the assumption that the 87 stocks are Student t. As panel (i) of the table shows, the estimated values of ω vary between 0.97 and 1.70. The estimated degrees of freedom υ vary between 0.6 and 4.3. Panel (ii) of the table shows the same information for the 184 stocks that are classified as Student t. It is clear from the table that most of these securities have an estimated value of ω that is numerically close to 2. The average value of the degrees of freedom are similar under Student t and the GSTωυσ models.

In order to investigate the possibility, as suggested in section 2 after equation (7.), that the symmetric GSTωυσ model may be replaced by a Student distribution with degrees of freedom equal to ωυ/2 an OLS regression is performed. This uses the appropriate estimates for the 87 stocks classified as GSTωυσ. The dependent variable is the estimated degrees of freedom for Student t and the independent variable is the product of the estimates of ω and υ, estimated for the GSTωυσ model. The null hypothesis is that the intercept in the regression equals zero and the slope coefficient equals 0.5. There is little evidence to support the view that there is a relationship between the parameters of the kind postulated in section 2. By contrast, for the 184 stocks that are classified as Student t, a similar OLS regression supports the proposition that, for such stocks, the degrees of freedom under Student t is very well approximated by ωυ/2,where the ω and υ are estimated under the GSTωυσ model. That is, if returns are indeed Student t there is no advantage from using the GSTωυσ model, but equally little harm results. By contrast, if returns are indeed GSTωυσ then it is wise to use the correct model. Details of these OLS regressions are omitted, but are available on request.

The effects of using the GSTωυσ model when the correct model is in fact Student t is summarised in Table 7. The first column of the table shows the probabilities computed under the GSTωυσ model when the nominal Student t probability is 1%. Panel (i) shows the four stocks for which the computed probabilities are the smallest. Panel (ii) shows the four stocks for which the computed probabilities are closest to the nominal value and panel (iii) shows the four stocks for which the computed probabilities are the largest. In panel (iii) the error in probability is not great; at worst 9%. In both panels (ii) and (iii) the estimated values of the Studentness parameter ω are all numerically close to 2 and the computation 0.5ωυ gives values close to the estimated degrees of freedom under the correct model. By contrast, in panel (i) the estimated values of ω are between 1.1 and 1.4. However, it is only these four stocks which may have been mis-classified by the method described in Section 3.

Table 7 about here

4.3 Asymmetric Distributions

This section covers the three most commonly selected asymmetric distributions. These are the GTω-σ (16 stocks), GSTω-σ (31) and GST-υσ (29). Table 8 shows an analysis of the tail probabilities constructed in a similar manner to that shown in Table 5. Critical values are computed using estimated parameters that assume a Student t distribution. Actual probabilities corresponding to the critical values are computed using the selected asymmetric distribution. Panels (i), (ii) and (iii) show results for the GTω-σ, GSTω-σ and GST-υσ distributions. Panel (iv) shows the total. The data in section A shows the extent to which use of Student t causes tail probabilities to be under or over estimated. Depending on the nominal probability, under or over estimation occurs for a substantial proportion of stocks for which the correct model is GSTω-σ or GST-υσ. However, as shown in section B, these stocks account for a very small percentage of market capitalisation. For the GTω-σ model, accounting for about 9.8% of market capitalisation, the majority of the computed probabilities are acceptably close the nominal values.

Table 8 about here

There is a small number of FTSE350 stocks, 9 out of 365, which require 6 or even 7 free parameters. It is difficult to argue either on statistical or, in this case, investment grounds that there is a case for using these models. Overall, therefore, the data used for this study supports the use of parsimonious versions of the GST family. Furthermore, the implication of the results shown in Table 8 is that symmetric members of the GST family are adequate to model returns on most of the FTSE350 stocks included in the study if capitalisation is taken into account. This view is further supported by the two graphs of Value at Risk which are shown in Figure 9. This shows the VaR as a percentage at the 1% level of probability for all stocks for which the selected model is one of the three skewed models above. The VaR is computed using both the correctly specified model and Student t distribution. To aid visualization, the results are shown sorted in increasing order of VaR based on the selected model.

Figure 9 about here

As Figure 9 makes clear, the differences in VaR at the 1% level are generally small. However, there is a small number of securities, between 10 and 20 depending on one's point of view, for which the VaR computed using Student t underestimates the value based on the selected skewed model.

4.4 Persistence of Distributions

As described in Section 3 of the paper, the estimation exercise was also carried out for two earlier contiguous but non-overlapping blocks of 500 days, periods A and B. The detailed results from this exercise are omitted but are available on request. The two following tables summarise the extent of persistence in the models selected for each stock.

Table 9 summarises the dynamics in the return distributions for the 327 stocks for which data was available for all three 500 days blocks. The table shows only entries for the three selected symmetric models (Normal, Student and GSTωυσ) and the three most common skewed models (GTω-σ, GSTω-σ and GST-υσ). Panel (i) shows the dynamics between the end of periods A and B. Panel (ii) shows the same data between the end of periods B and C. The two sets of row totals show the situation at the end of periods A and B. The column totals show the situation at the end of periods B and C. Note that the column totals in the first panel do not agree exactly with the row totals in the second panel because of some migration to less common asymmetric models not shown in the table.

Table 9 about here

It is clear from the table that about two thirds of stocks have symmetric distributions at the end of each period. Of these between 10 and 20% will cease to be classified as symmetric by the end of the following period. For stocks which are non-symmetric, around half become symmetric at the end of the following period.

The issue of migration between symmetric models is also of interest. Table 10 shows the number of stocks that migrate between the two symmetric classes and any other class. Panel (i) shows numbers of stocks in each category. Panel (ii) shows the results as percentages. The rows of the table may be interpreted as follows: of the 136 stocks that were classified as Student t at the end of period A, 99 remained Student stocks for both periods B and C, 6 became GSTωυσ and remained so, 9 moved between the two symmetric categories and 22 were skewed for at least one 500 day period. The implication of this row of the table is that the majority of Student stocks remain so. By contrast, the second row of the table shows that the distribution of returns on stocks which were classified as GSTωυσ at the end of period A showed a greater propensity to change.

Table 10 about here

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5. PORTFOLIOS OF UK STOCKS

The results of carrying out the series of likelihood ratio tests as described in section 3 for portfolios of UK FTSE350 stocks are shown in Table 11 based on data for period C. The table shows the number of portfolios classified in each bin at four levels of probability, ranging from 10% to 0.1%. Bins containing no portfolios at all four levels of probability are omitted. As the table shows, the vast majority of portfolios follow a Student t distribution. Very few, and at the 1% level none, follow the symmetric GST distribution. There is a small number of portfolios which exhibit skewed returns and which are classified as GSTω-σ.

Table 11 about here

In view of the pre-eminence of Student t as the model for portfolio returns, a detailed analysis similar to that in section 4 is omitted. The following results are noteworthy. First, an analysis similar to that in Table 4 shows that an erroneous assumption of normality results in errors in the computation of tail probabilities. Secondly, Student t distribution is remarkably persistent. At the end of periods B and C, over 180 portfolios were Student t and the majority of the remainder were normal. The fact that 200 were Student t at the end of period A reflects some change in market conditions but does not undermine the conclusion that on the basis of this data the Student model is the single most important model for portfolio returns

Thirdly, the estimated degrees of freedom of portfolios classified as Student varies between 3 and 9. These low values are interesting for two reasons. They offer empirical evidence that suggests that the central limit theorem does not hold for portfolios of FTSE350 stocks. The range of degrees of freedom for portfolios is consistent with that for stocks and with the FTSE350 index itself. This lends some support to the use of the multivariate Student distribution as a model for stock returns. See for example Johnson & Kotz (1972, page 162 et sec) or Bernardo and Smith (1994, page 435 and 441) for details. The multivariate Student distribution is characterised by a single degree of freedom parameter. Under this model, individual stocks follow a Student t distribution with the same degrees of freedom. More importantly, any portfolio has a distribution that is proportional to Student t with the same degrees of freedom.

6. COMPARATIVE STUDY OF JAPAN SOUTH AFRICA AND THE UNITED STATES

The computations done for UK FTSE350 stocks were repeated for the selected stocks in the Japanese Nikkei225, FTSE Johannesburg stock exchange and US S&P500 indices. The results reported in this section are a comparative study of the four markets. Japan was selected as an example of a mature stock market. South Africa was selected as a representative of markets with a large number of securities which, because of economic change, might exhibit different patterns of a symmetry from the UK. The USA was selected because *a priori* it might be expected that the high level of market efficiency would preclude the existence of many stocks with skewness.

Table 12 shows the results of carrying out the series of likelihood ratio tests for all four markets. The significance level used is 1%. The table has two sections, the first for stocks and the second for portfolios. Panel (i) shows the number of stocks and portfolios with symmetric return distributions. Panel (ii) shows the numbers of stocks and portfolios with the most common asymmetric distributions.

Table 12 about here

The most striking feature of the table is that 98% of Japanese stocks and 97% of US stocks have symmetric distributions, almost all of which are the normal or Student t. In South Africa, 54% of stocks have a symmetric distribution. Of these, 1 is normal, 36, including the index, follow Student t and 42 follow the GSTωυσ model. The GSTωυσ model is only selected for a substantial number of stocks in South Africa and the UK. The returns on all four indices are Student t.

For stocks which have asymmetric returns, the models selected are more or less the same as those selected for the UK. However, the presence of asymmetry only occurs for a significant number of stocks in the UK and South Africa. In Japan and the USA asymmetry occurs in two and three percent of stocks respectively. As is the case for the UK, the GED model is never selected and the GT model is only selected in the GTω-σ form. There are further comments on these results in the conclusions to this paper.

For portfolios of stocks, asymmetry in the return distribution virtually disappears. Furthermore, the symmetric GSTωυσ distribution is never selected. In Japan, the majority of portfolios have a normal distribution. In South Africa about 20% of portfolios are normally distributed. By contrast, and somewhat curiously, in the UK and the US, the majority of portfolios have return distributions that are Student t.

The analysis of market value corresponding to that in Table 3 is omitted. This confirms the main findings from Table 3, namely that on average the larger capitalisation stocks follow a Student t distribution and that the average size of stocks with other return distributions is small.

Table 13 about here

Table 13 contains two panels which summarise values of the estimated degrees of freedom and Studentness parameters for stocks in all four markets which are classified as Student t or as GSTωυσ. Panel (i) summarises stocks which are classified as Student t. For the UK, South Africa and the USA the average degrees of freedom is broadly equal to 5. In Japan it is closer to 7. In both the UK and South Africa, the minimum is less that one implying distributions that have fatter tails than the Cauchy. The maximum degrees of freedom is about 10 for the UK and South Africa and about 12 for Japan and the USA. For portfolios, the distribution of the estimated values of υ does not change greatly. The average increases somewhat. In the Japan, the UK and the USA all portfolios have variances which exist, that is the estimated value of υ is always greater than two. In South Africa this is not the case. Panel (ii) of Table 13 shows summary statistics for UK and South African stocks which are classified as GSTωυσ. The interesting result is that the average estimated value of ω is about 1.35 with a minimum of 0.97 (found in the UK) and a maximum of 1.95 (South Africa).

An analysis of the tail probabilities computed using the methods described in Section 4 confirms the finding already reported above. It is important to classify correctly those stocks whose returns are distributed as Student t, but it is less important from a capitalisation perspective to correctly classify other stocks.

Table 14, which is similar to Table 9, summarises the dynamics in the return distributions for South African stocks for which data was available for all three 500 days blocks. The table entries show the percentage of stocks classified in each cell. As the table shows, over the past three 500 day periods, there has been a substantial number of stocks for which the selected model is not persistent. Furthermore, there is lack of persistence in skewness per se. Many stocks exhibit skewness in one period but not the next and vice versa. The dynamics for Japan and the USA are omitted. These show some migration between Student t and normality, but the number of stocks which migrate between symmetric and asymmetric distributions is very small. For portfolios there is a small amount of migration between normality and Student t.

Table 14 about here

Table 15 presents summary of 1% VaR computations for stocks and portfolios in all four markets which are classified as Student t. The table summarises the VaR computations based on an assumed normal distribution and the correctly specified Student t distribution. Panel (i) shows results for stocks and panel (ii) for portfolios. The column entitled *Avg* shows that on average use of the normal distribution results in the 1% VaR being under estimated. The other columns of the table present other summary statistics which confirm that in general use of the normal distribution results in under-estimation of VaR at the 1% level. For stocks, these differences are substantial. For portfolios, the differences are smaller. For example, for UK portfolios the average 1% VaR is about 3% under the normal model and about 3.5% under

Student t. However, in monetary terms this could still be substantial; about £5,000 per million. Computations for conditional expected loss (CEL), the expected loss givenm that it is worse than the VaR, are omitted, but give similar results; use of the normal distribution causes CEL to be under estimated.

Table 15 about here

Table 16 shows the equivalent results for the UK and South Africa. Panel (i) shows a comparison between a correctly specified GSTωυσ model and Student t. Panel (ii) shows a comparison between all skewed models and Student t. These show results which are the opposite of those presented in Table 15; the use of Student t when the correct model is a skewed distribution results in 1% VaR being over-estimated.

Table 16 about here

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7. CONCLUSIONS

The main conclusions of the study are as follows. A well known stylised fact concerning returns is confirmed. The normal distribution is not in general a useful model for daily returns. It is only appropriate in a few cases for stocks with large market capitalisation. Student t distribution is the single most common model of daily returns. This is particularly true in Japan and the United States, where almost all stocks have the t distribution. The symmetric version of the GST distribution is a common model for UK FTSE350 and South African stocks, but not for Japanese or American securities. As noted in Section 4 the returns on the majority of FTSE100 stocks follow a Student t distribution. Other members of the GST family are therefore more common among the next 250 securities ranked by capitalisation.

Analysis of the tail probabilities indicates that it is important to use the correctly specified model when returns are Student t. Incorrect use of the normal distribution results in errors in computation of tail probabilities and hence in the calculation of risk measures like VaR and CEL. Using Student t when the correct model is the symmetric GST distribution will also result in errors in the computation of tail probabilities. This is true even when the estimated moments are similar in value. On a market capitalisation basis, this may be viewed as a less serious problem because such stocks have small weights in their index. However, the designer of a small cap portfolio might disagree. Overall, the analysis of the tail probabilities confirms the findings of Longin (2005).

Returns for stocks with significant asymmetry are generally modelled with a member of the GST class with 5 free parameters. This is in agreement with the study reported in Theodossiou (1998). However in his paper, asymmetry is generated by variations of the scale parameter only. Our findings indicate that asymmetry is more often generated by variations in the degrees of freedom or by the parameter that reflects

Studentness. As is the case with symmetric returns, failure to use the correctly specified model will result in errors in the computation of tail probabilities. From a statistical perspective, it can be argued that such errors will be of minor practical consequence since such stocks typically have small capitalisation. However, when viewed from an investment perspective, such stocks may play an important role in a portfolio. The errors in the computation of tail probabilities that result from mis-specification may not then be ignored so easily.

The general version of the GST distribution with 6 or 7 free parameters is selected only rarely as an appropriate model for daily stock returns. The data thus supports a reasonable degree of parsimony even though it is clear from the results of this study that four parameters are often required in the symmetric case and five parameters generally needed when asymmetry is present.

The study of persistence shows that there is some migration between the symmetric classes. For the UK and South Africa, the analysis of stocks with asymmetric distributions shows that skewness is not generally persistent. There is a substantial degree of migration within members of the class of asymmetric distributions and between symmetric and asymmetric distributions. These findings are consistent with those reported by Singleton and Wingender (1986) and Bekaert et al (1998). The presence of asymmetry in returns on many South African stocks is not a surprise in view of the economic changes that have taken place. However, the incidence of skewness in the returns in UK FTSE350 stocks is a surprising finding when contrasted with the almost total lack of asymmetry in returns in Japan and the United States.

Very few portfolios, even those with only 5 stocks, exhibit skewness. The normal and Student t are the distributions selected in the vast majority of cases. The symmetric GST$\omega\upsilon\sigma$ model is never selected. These findings are consistent with the fact that index returns are Student t. As noted above, this suggests that the central limit theorem does not hold for daily returns on stocks. Furthermore it provides support for the use of the multivariate Student distribution as a coherent model for stock returns.

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Table 1 - Definition of the Data Sets

Country	No. of Stocks Index	Period A	Period B	Period C	No. of Portfolios
Japan	225	214	216	221	180
South Africa	162	122	138	145	150
United Kingdom	380	326	340	364	210
United States	500	453	473	484	240

Table 2 – Analysis of the Categorisation of UK FTSE350 Stocks Using the Decision Tree in Figure 6

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

Bin	Model	Probability level (%) 10	5	1	0.1
A	Normal	0	0	2	11
B	Student t	159	173	184	187
C	GTω-σ	32	19	16	15
D	GEDωυσ	0	0	0	0
E	GSTωυσ	76	81	87	88
F	GSTω-σ	25	25	31	37
G	GED-υσ	0	0	0	0
H	GST-υσ	46	43	29	19
I	GST--σ	4	4	3	2
J	GEDωυ-	0	0	0	0
K	GTωυ-	0	0	0	0
L	GTω--	0	0	0	0
M	GEDωυ-	0	0	0	0
N	GSTωυ-	9	7	4	3
O	GSTω--	3	3	2	0
P	GED-υ-	0	0	0	0
Q	GST-υ-	8	7	4	2
R	GST---	3	3	3	1

NOTE: The bin labels are as shown in the diagram in Figure 6. The nomenclature for the models is defined in section 2. Each column shows the number of stocks falling in each bin at the specified level of probability.

Table 3 – Analysis of Capitalisation Weights for Commonly Selected Models for UK FTSE350 Stocks

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

Bin	Model	No. of stocks	Min%	Avg%	Max%
B	Student t	183	0.0070	0.4500	8.6103
C	GTω-σ	16	0.0004	0.6073	6.6936
E	GSTωυσ	87	0.0025	0.0452	0.2846
F	GSTω-σ	31	0.0111	0.0429	0.1864
H	GST-υσ	29	0.0082	0.0299	0.0748
N	GSTωυ-	4	0.0144	0.0226	0.0353

NOTE: The bin labels are as shown in the diagram in Figure 6. The nomenclature for the models is defined in section 2. The column No. of stocks shows the number of stocks falling in each bin at the 1% probability level. The bins and models shown are those that contain the most stocks. The main table entries are based on the percentage capitalisation weights for each stock averaged over the same 500 days used for estimation. Avg% is the average weight, Min% and Max% are respectively the minimum and maximum.

Table 4 – Analysis of Tail Probabilities Student t Distribution vs the Normal Distribution for UK FTSE350 Stocks

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

Nominal Prob (%)	Section A			Section B		
	> Prob%	Same	≤ Prob%	> Prob%	Same	≤ Prob%
0.05%	4	0	180	0.06	0.00	82.09
0.50%	180	0	4	81.97	0.00	0.19
1.00%	174	2	8	81.09	0.37	0.70
2.50%	146	1	37	74.66	1.17	6.33
5.00%	7	0	177	1.33	0.00	80.83
95.00%	180	0	4	82.09	0.00	0.06
97.50%	35	4	145	5.95	1.04	75.17
99.00%	9	1	174	0.86	0.21	81.09
99.50%	2	2	180	0.14	0.05	81.97
99.95%	180	0	4	82.09	0.00	0.06

NOTE: Critical values corresponding to the eight nominal probabilities are computed using estimated parameters based on an assumed normal distribution. The actual probability corresponding to each critical value is then computed using the estimated parameters based on Student t distribution. In section A, the entries in the columns headed >Prob% (≤Prob%) are the numbers of stocks for which the tail probability based on Student t is greater (less) than the nominal values shown in the rows of the table plus (minus) 0.0001. The computed probability is treated as being the same as the nominal value if the absolute difference between them is 0.0001 or less. Section B presents the same analysis using the average capitalisation weights of each stock shown as a percentage correct to 2 decimal places.

Table 5 – Analysis of Tail Probabilities for GSTωυσ Distribution vs Student t Distribution for UK FTSE350 Stocks

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

	Section A			Section B		
Nominal Prob (%)	**> Prob%**	**Same**	**≤ Prob%**	**> Prob%**	**Same**	**≤ Prob%**
0.50%	8	9	70	0.27	0.43	3.23
1.00%	24	5	58	1.07	0.13	2.72
2.50%	64	4	19	3.23	0.20	0.50
5.00%	73	0	14	3.60	0.00	0.33
95.00%	10	0	77	0.23	0.00	3.70
97.50%	20	2	65	0.54	0.06	3.33
99.00%	56	4	27	2.49	0.18	1.26
99.50%	70	9	8	3.31	0.39	0.23

NOTE: Critical values corresponding to the eight nominal probabilities are computed using estimated parameters based on an assumed Student t distribution. The actual probability corresponding to each critical value is then computed using the estimated parameters based on the GSTωυσ distribution. In section A, the entries in the columns headed >Prob% (≤Prob%) are the numbers of stocks for which the tail probability based on Student t is greater (less) than the nominal values shown in the rows of the table plus (minus) 0.0001. The computed probability is treated as being the same as the nominal value if the absolute difference between them is 0.0001 or less. Section B presents the same analysis using the average capitalisation weights of each stock shown as a percentage correct to 2 decimal places.

Table 6 – Summary Of The Estimated Values Of ω And ν For The GSTωυσ Distribution For UK FTSE350 Stocks

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

Parameter	Avg	Vol	Min	LowerQ	Median	UpperQ	Max
(i) Stocks classified as GSTωυσ							
GSTωυσ-ω	1.3438	0.1305	0.9684	1.2691	1.3435	1.3435	1.6895
GSTωυσ-υ	4.9569	6.7091	0.5931	1.6963	3.8322	3.8322	13.5303
Student t-υ	1.9185	1.0499	0.4108	0.9900	1.8868	1.8868	4.8142
(ii) Stocks classified as Student t							
GSTωυσ-ω	1.9562	0.1188	1.1247	1.9716	1.9847	1.9847	2.0053
GSTωυσ-υ	4.8426	1.7351	1.3048	3.4985	4.4817	4.4817	11.9338
Student t-υ	4.7330	1.7250	0.3979	3.4596	4.4588	4.4588	9.8528

NOTE: Panel (i) contains summary data for stocks which are classified as GSTωυσ. The first two rows summarise the estimated values of the ω and υ parameters. The third row summarises the estimated values of the degrees of freedom υ for GSTωυσ stocks when the Student t distribution is estimated instead. Panel (ii) contains the same information for stocks which are classified as Student t. The column headed *Avg* is the average value for stocks in each of the four markets. The other columns are the usual summary statistics.

Table 7 – Examples of Nominal Probabilities and Model Parameters for a selection of Stocks with Student t Distribution

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

Prob.	ω	υ	0.5ωυ	Studt υ
(i) 4 stocks with lowest computed probabilities				
0.0011	1.3677	2.5806	1.7647	0.4277
0.1201	1.3137	1.3200	0.8670	0.4438
0.1483	1.1247	1.3048	0.7338	0.3979
0.1483	1.1247	1.3048	0.7338	0.3979
(ii) 4 stocks closest to the nominal value				
0.9962	2.0014	3.5007	3.5032	3.5007
0.9978	2.0007	6.4494	6.4517	6.4494
1.0007	1.9998	6.8880	6.8873	6.8880
1.0013	1.9996	8.1561	8.1545	8.1561
(iii) 4 stocks with highest computed probabilities				
1.0702	1.9755	3.9753	3.9266	3.9753
1.0740	1.9200	2.9967	2.8768	2.9078
1.0763	1.9219	3.4943	3.3578	3.3994
1.0931	1.8543	3.0902	2.8651	2.8888

NOTE: The first column of the table shows the probabilities computed under the GSTωυσ model when the nominal Student t probability is 1%. Panel (i) shows the four stocks for which the computed probabilities are the smallest. Panel (ii) shows the four stocks for which the computed probabilities are closest to the nominal value and panel (iii) shows the four stocks for which the computed probabilities are the largest. The columns headed ω and υ show the estimated values of the corresponding parameters under the GSTωυσ distribution. The column headed Studt υ shows the estimated degrees of freedom under Student t distribution.

Table 8 – Analysis of Tail Probabilities for Selected Asymmetric GST Distributions vs Student t Distribution for UK FTSE350 Stocks

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

	Section A			Section B		
Nominal Prob (%)	**> Prob%**	**Same**	**< Prob%**	**> Prob%**	**Same**	**< Prob%**
(i) Bin C GTω-σ vs Student 16 stocks						
1%	0	14	2	0.00	9.70	0.02
2.50%	0	13	3	0.00	9.62	0.09
5%	2	12	2	0.09	9.53	0.09
95%	3	13	0	0.10	9.61	0.00
97.50%	3	13	0	0.10	9.61	0.00
99%	2	14	0	0.02	9.70	0.00
(ii) Bin F GSTω-σ vs Student 31 stocks						
1%	9	0	22	0.26	0.00	1.07
2.50%	13	1	17	0.47	0.05	0.81
5%	19	0	12	0.71	0.00	0.62
95%	4	1	26	0.11	0.05	1.17
97.50%	12	0	19	0.39	0.00	0.94
99%	17	1	13	0.54	0.03	0.76
(iii) Bin H GST-υσ vs Student 29 stocks						
1%	1	0	28	0.07	0.04	0.76
2.50%	2	0	27	0.25	0.05	0.56
5%	7	0	22	0.37	0.00	0.50
95%	13	0	16	0.20	0.03	0.64
97.50%	15	1	13	0.43	0.02	0.41
99%	19	0	10	0.51	0.00	0.35
(iv) Totals						
1%	10	14	52	0.33	9.74	1.85
2.50%	15	14	47	0.72	9.72	1.46
5%	28	12	36	1.17	9.53	1.21
95%	20	14	42	0.41	9.69	1.81
97.50%	30	14	32	0.92	9.63	1.35
99%	38	15	23	1.07	9.73	1.11

NOTE: See Table 5 for explanation of method of computation.

Table 9 – Model Specification Dynamics for FTSE350 Stocks

Estimated using 500 daily returns in each of three contiguous non-overlapping blocks.

	Normal	Student t	GSTωυσ	GTω-σ	GSTω-σ	GST-υσ	Totals
(i) Periods A & B							
Normal	0	1	0	0	0	0	1
Student t	2	112	14	4	3	1	136
GSTωυσ	0	31	36	0	10	7	84
GTω-σ	0	3	3	12	4	6	28
GSTω-σ	0	1	15	0	1	5	22
GST-υσ	0	1	16	0	2	7	26
Totals	2	149	84	16	20	26	297
(i) Periods B & C							
Normal	0	2	0	0	0	0	2
Student t	1	127	11	8	5	1	153
GSTωυσ	0	23	36	0	15	13	87
GTω-σ	1	2	5	5	1	2	16
GSTω-σ	0	6	8	0	3	1	18
GST-υσ	0	2	12	1	3	8	26
Totals	2	162	72	14	27	25	302

NOTE: The table shows only entries for the three selected symmetric models (Normal, Student and GSTωυσ) and the three most common skewed models (GTω-σ, GSTω-σ and GST-υσ). Panel (i) shows the dynamics between the end of periods A and B. Panel (ii) shows the same data between the end of periods B and C. The two sets of row totals show the situation at the end of periods A and B. The column totals show the situation at the end of periods B and C. Note that the column totals in the first panel do not agree exactly with the row totals in the second panel because of some migration to less common asymmetric models not shown in the table. Period A ends on 3rd January 2000, B on 3rd December 2001and C on 3rd November 2003

Table10 – Model Specification Dynamics for Stocks with Symmetrically Distributed Returns for UKFTSE350 Stocks

Estimated using 500 daily returns in each of three contiguous non-overlapping blocks.

Inital model	No. of stocks	S&S	S&GST	GST&S	GST&GST	Symmetric	Other
(i) Number of stocks							
Student	136	99	3	6	6	114	22
GSTωυσ	87	22	5	11	13	51	36
All other	104	6	3	6	17	32	72
(ii) Percentages							
Student	41.59	30.28	0.92	1.83	1.83	34.86	6.73
GSTωυσ	26.61	6.73	1.53	3.36	3.98	15.60	11.01
All other	31.80	1.83	0.92	1.83	5.20	9.79	22.02

NOTE: Panel (i) shows numbers of stocks in each category. Panel (ii) shows the results as percentages. The rows of the table may be interpreted as follows: of the 136 stocks that were classified as Student t at the end of period A, 99 remained Student stocks (column titled S&S) for both periods B and C, 6 became GSTωυσ and remained so (GST&GST), 9 moved between the two symmetric categories and 22 were skewed for at least one 500 day period.

Table 11 – Analysis of the Categorisation of Portfolios of UK FTSE350 Stocks Using the Decision Tree in Figure 6

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

Bin	Model	Probability level (%)			
		10	5	1	0.1
A	Normal	0	0	2	5
B	Student t	166	181	200	201
C	GTω-σ	41	27	8	4
E	GSTωυσ	1	2	0	0
F	GSTω-σ	1	0	0	0
H	GST-υσ	1	0	0	0

NOTE: The bin labels are as shown in the diagram in Figure 6. The nomenclature for the models is defined in section 2. Each column shows the number of stocks falling in each bin at the specified level of probability.

Table 12 – Comparative Analysis of Skewness And Kurtosis For UK FTSE350 Japanese Nikkei225 South African FTSE Johannesburg and S&P500 Stocks Using the Decision Tree in Figure 6

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003.

	Stocks				**Portfolios**			
	UK	**JPN**	**RSA**	**USA**	**UK**	**JPN**	**RSA**	**USA**
(i) Symmetric Models								
Normal	2	47	1	17	2	143	32	8
Student t	184	166	36	455	201	37	117	231
GSTωυσ	87	4	42	0	0	0	0	0
No. of stocks	273	217	79	472	203	180	149	239
%'age of total	*75*	*98*	*54*	*97*	*96*	*99*	*99*	*99*
(ii) Asymmetric Models								
GTω-σ	16	1	22	8	8	0	2	2
GSTω-σ	31	0	21	0	0	0	0	0
GST-υσ	29	3	17	5	0	1	0	0
GST--σ	3	1	3	0	0	0	0	0
GSTωυ-	4	0	3	0	0	0	0	0
GST---	3	0	0	0	0	0	0	0
No. of stocks	86	5	66	13	8	1	2	2
%'age of total	*24*	*2*	*45*	*3*	*4*	*1*	*1*	*1*
(iii) Other Asymmetric								
No. of stocks	6	0	1	0	0	0	0	0
%'age of total	*2*	*0*	*1*	*0*	*0*	*0*	*0*	*0*

NOTE: See Table 2 for explanation of contents. The level of probability used in this table is 1%.. The country abbreviations are UK for FTSE350 stocks. JPN for Nikkei225, RSA for South African FTSE and USA for S&P500.

Table 13 – Summary of the Estimated Degrees of Freedom And Studentness Parameters For Stocks Distributed As Student t Or GSTωυσ

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003. Table entries rounded to two decimal places.

	Avg	Vol	Min	LowerQ	Median	UpperQ	Max
(i) Stocks Classified as Student t							
Stocks							
UK	4.73	1.73	0.40	3.46	4.46	5.48	9.85
Japan	6.63	2.09	2.17	5.09	6.39	7.95	11.56
South Africa	4.97	2.44	0.35	3.37	5.43	6.91	9.88
USA	4.64	1.58	1.54	3.55	4.41	5.41	12.09
Portfolios							
UK	4.71	1.07	2.99	3.90	4.66	5.15	8.98
Japan	8.40	1.87	3.48	7.45	8.82	9.85	11.08
South Africa	7.24	2.15	1.52	5.74	7.98	8.83	11.02
USA	6.45	1.30	2.33	5.92	6.39	6.89	10.02
(ii) Stocks Classified at GSTωυσ							
UK							
ω	1.34	0.13	0.97	1.26	1.34	1.42	1.69
υ	4.96	6.71	0.59	1.68	3.83	6.26	60.37
υ-Studt	1.92	1.05	0.41	0.99	1.89	2.79	4.81
South Africa							
ω	1.35	0.15	1.09	1.24	1.33	1.43	1.95
υ	6.28	5.26	0.65	2.74	4.56	8.66	23.89
υ-Studt	2.31	1.28	0.36	1.34	2.31	3.10	5.85

NOTE: The columns of the table summarise the distribution of the estimated Studentness and degrees of freedom parameters for stocks classified as Student t or GSTωυσ. The column headed *Avg* is the average value for stocks in each of the four markets. The other columns are the usual summary statistics.

Table 14 – Model Specification Dynamics for FTSE Johannesburg Stocks Using the Decision Tree in Figure 6

Estimated using 500 daily returns in each of three contiguous non-overlapping blocks. Table entries are shown as percentages of the total number of stocks for which data is available for all three periods.

	Normal	Student t	GSTωυσ	GTω-σ	GSTω-σ	GST-υσ	GSTωυ-	Totals
(i) Periods A & B								
Normal	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Student t	0.00	12.20	0.00	1.63	3.25	1.63	0.00	18.71
GSTωυσ	0.00	9.76	18.70	0.81	4.07	0.81	0.00	34.15
GTω-σ	0.00	0.00	4.07	11.38	2.44	0.81	0.00	18.70
GSTω-σ	0.00	4.07	4.07	0.00	2.44	1.63	0.81	13.02
GST-υσ	0.00	2.44	2.44	1.63	0.81	0.00	0.81	8.13
GSTωυ-	0.00	0.00	1.63	0.00	0.00	0.00	0.00	1.63
Totals	0.00	28.47	30.91	15.45	13.01	4.88	1.62	94.34
(i) Periods B & C								
Normal	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Student t	0.81	17.07	7.32	0.81	1.63	0.81	0.00	28.45
GSTωυσ	0.00	4.88	9.76	0.81	8.13	6.50	0.00	30.08
GTω-σ	0.00	0.81	1.63	9.76	0.00	2.44	0.00	14.64
GSTω-σ	0.00	1.63	6.50	1.63	3.25	0.81	0.00	13.82
GST-υσ	0.00	0.00	2.44	0.81	0.00	1.63	0.00	4.88
GSTωυ-	0.00	0.00	0.81	0.00	0.00	0.00	0.00	0.81
Totals	0.81	24.39	28.46	13.82	13.01	12.19	0.00	92.68

NOTE: Constructed in the same manner as Table 9, except that cell entries are the number of stocks shown as a percentage, correct to 2 decimal places.

Table 15 – Summary of Value at Risk Computations for Stocks and Portfolios Classified as Student t

Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003. Table entries rounded to two decimal places.

	Avg	Vol	Min	LowerQ	Median	UpperQ	Max
I Stocks							
(i) UK							
Normal	-5.75	2.50	-17.65	-7.01	-5.14	-4.04	-1.77
Student	-8.36	13.82	-99.87	-7.88	-5.92	-4.62	-2.46
(ii) Japan							
Normal	-5.61	1.63	-10.72	-6.54	-5.29	-4.49	-2.28
Student	-6.27	2.02	-15.14	-7.25	-5.78	-4.98	-2.48
(iii) South Africa							
Normal	-5.48	1.95	-11.45	-6.13	-5.16	-4.24	-2.75
Student	-16.63	29.51	-100.00	-9.49	-6.07	-4.76	-2.94
(iv) USA							
Normal	-5.62	2.45	-21.40	-6.27	-4.97	-4.12	-2.20
Student	-6.40	2.87	-26.14	-7.10	-5.67	-4.64	-2.53
II Portfolios							
(i) UK							
Normal	-3.01	0.79	-6.23	-3.50	-2.97	-2.35	-1.86
Student	-3.48	0.98	-7.78	-4.12	-3.37	-2.65	-2.12
(ii) Japan							
Normal	-4.08	0.73	-6.29	-4.54	-3.89	-3.57	-2.98
Student	-4.40	0.86	-7.14	-4.89	-4.12	-3.78	-3.27
(iii) South Africa							
Normal	-2.60	1.00	-5.30	-3.31	-2.70	-1.78	-0.97
Student	-2.88	1.13	-5.94	-3.72	-2.94	-1.96	-1.06
(iv) USA							
Normal	-3.40	0.56	-7.65	-3.45	-3.24	-3.16	-2.54
Student	-3.77	0.63	-8.48	-3.81	-3.60	-3.51	-2.80

NOTE: Panel I shows summaries of the VaR computed for stocks in each of the four markets shown. The *Normal* row shows summaries computed using an assumed normal distribution. The *Student* row shows summaries based on the correctly specified Student t distribution. Panel II presents the same information for portfolios. The column entitled *Avg* is the average VaR for all stocks in the market in question with available data computed using each distribution. The other columns of the table present other standard summary statistics computed in the usual way.

Table 16 – Summary of Value at Risk Computations for UKFTE 350 and South African Stocks Classified With GSTωυσ Or A Skewed Distribution

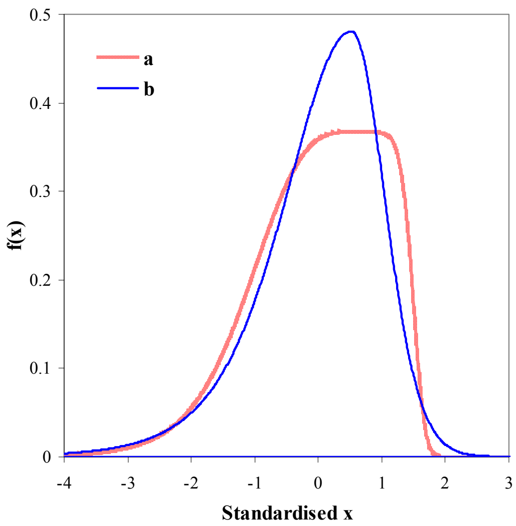
Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003. Table entries rounded to two decimal places.

	Avg	Vol	Min	LowerQ	Median	UpperQ	Max
I GSTωυσ							
(i) UK							
Student	-15.78	20.16	-99.54	-14.71	-8.70	-5.73	-1.85
GSTωυσ	-13.83	18.38	-100.00	-12.16	-8.07	-5.28	-1.85
(ii) South Africa							
Student	-18.78	29.86	-100.00	-11.71	-7.87	-5.30	-3.13
GSTωυσ	-16.41	25.17	-99.97	-10.35	-7.46	-5.20	-4.21
II Skewed							
(i) UK							
Student	-24.81	30.57	-100.00	-31.73	-10.83	-5.37	-2.41
Skewed	-18.12	23.05	-100.00	-16.86	-9.29	-5.16	-2.26
(ii) South Africa							
Student	-26.03	35.81	-100.00	-41.11	-6.06	-3.13	-3.13
Skewed	-18.13	27.82	-100.00	-13.17	-5.52	-3.13	-3.13

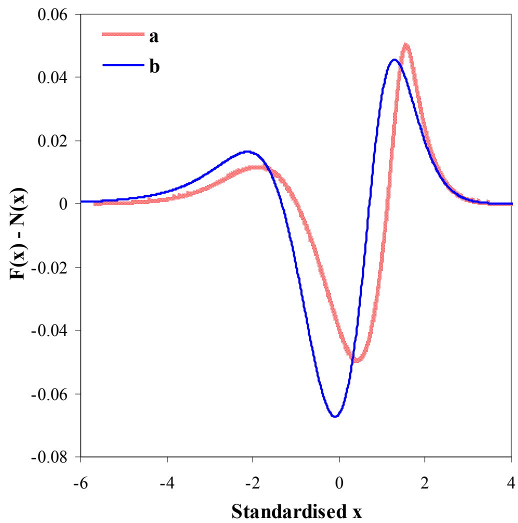
NOTE: Panel I shows summaries of the VaR computed for stocks in each of the four markets shown. The *Student* row shows summaries computed using an assumed Student distribution. The GSTωυσ row shows summaries based on the correctly specified GSTωυσdistribution. The column entitled *Avg* is the average VaR for all stocks in the market in question with available data computed using each distribution. The other columns of the table present other standard summary statistics computed in the usual way. Panel II presents the same results for stocks which have one of the four main skewed distributions.

Figure 1. Two GSTs both with skewness of –1.6 but displaying different forms of asymmetry

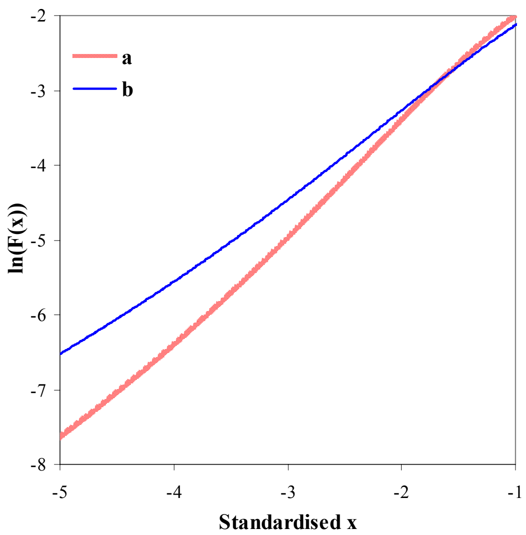
Density functions f (x)



Distribution function F (x) – Gaussian N (x)



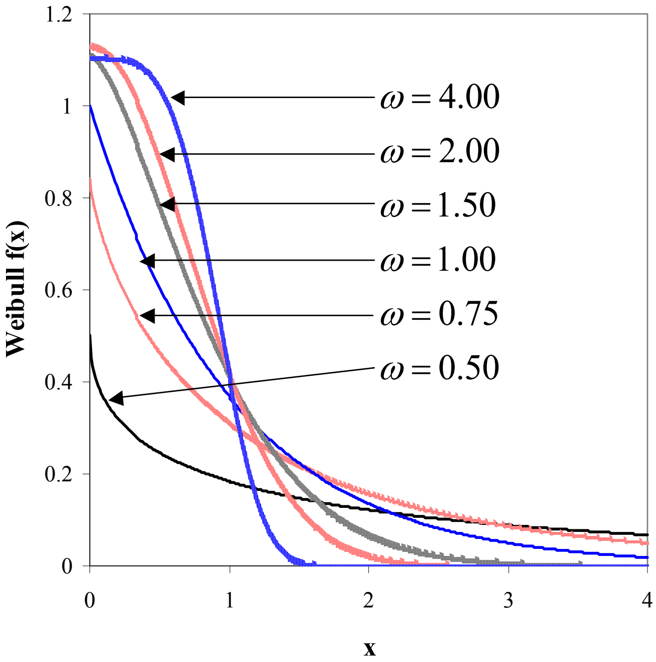
ln (F (x))



Parameter values

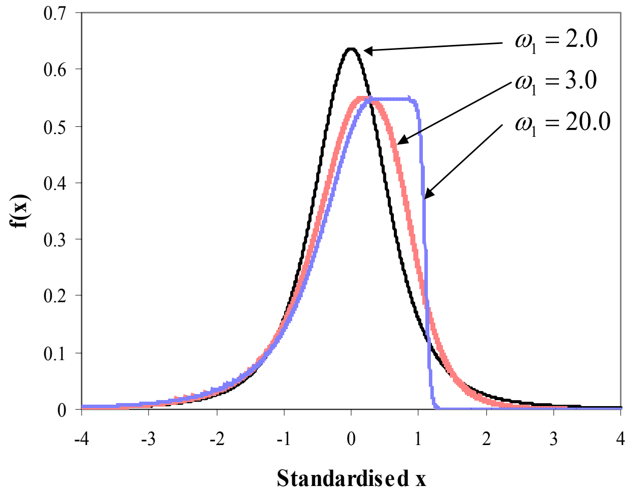
GST	ν_1	ω_1	σ_1	ν_2	ω_2	σ_2	Skewness	Excess kurtosis
a	9.5	8.0	1.5	4.5	3.0	2.4	-1.6	7.9
b	25.0	2.0	0.6	6.0	2.0	1.2	-1.6	7.3

Figure 2. The Effect Of The Shape Parameter, ω, On The Generalised Weibull

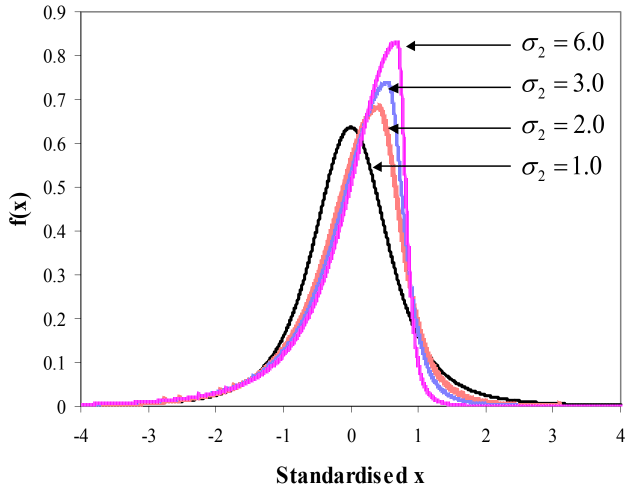


Figures 3 – 5. The Effects Of Asymmetric Parameter Values On The Shape Of The GST

Asymmetry caused by different omegas



Asymmetry caused by different sigmas



Asymmetry caused by different nus

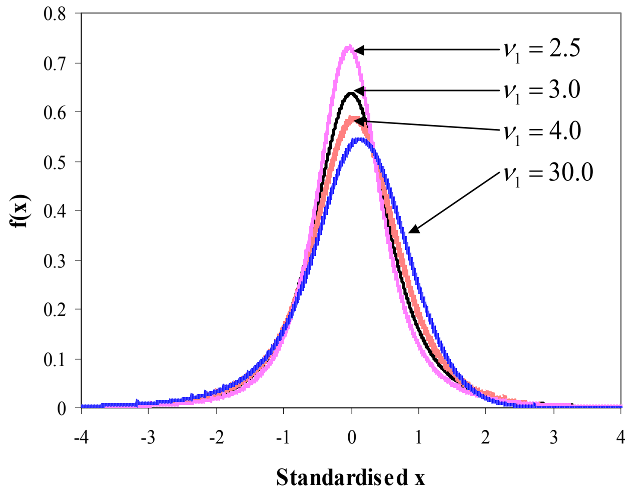


Figure 6. Tree of likelihood ratio tests applied to each series of stock returns

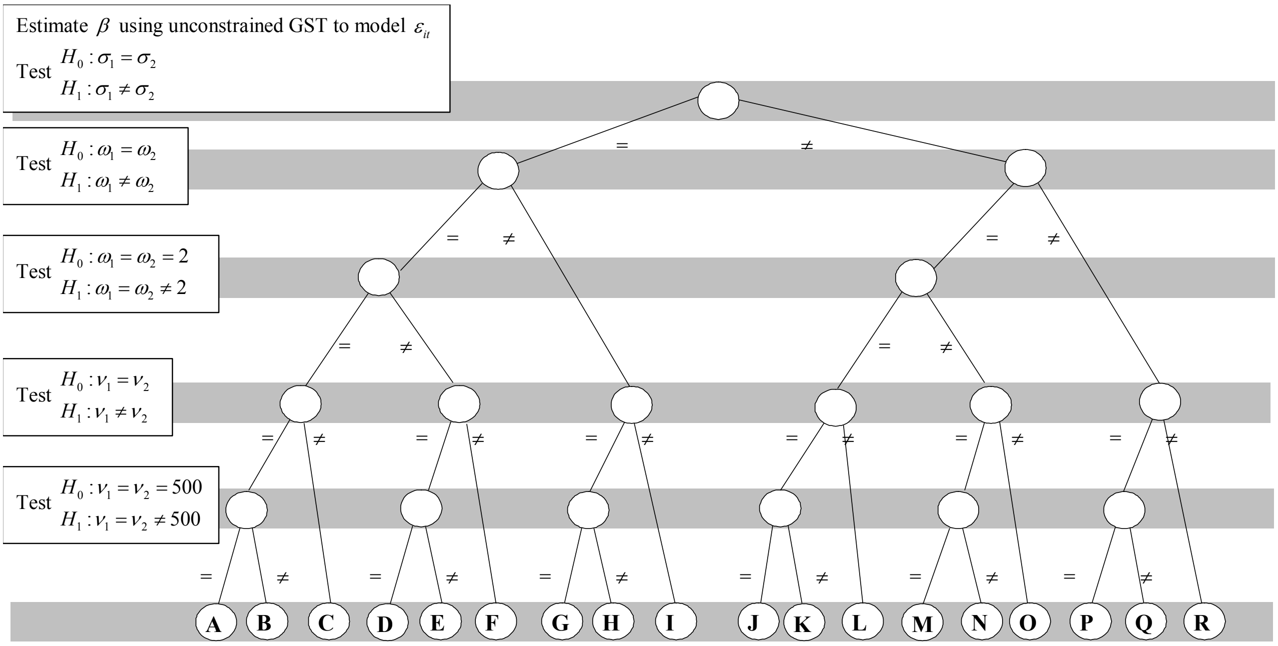


Figure 7. Histogram of the estimated degrees of freedom for stocks with Student t distribution for UK FTSE350 Stocks (Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003)

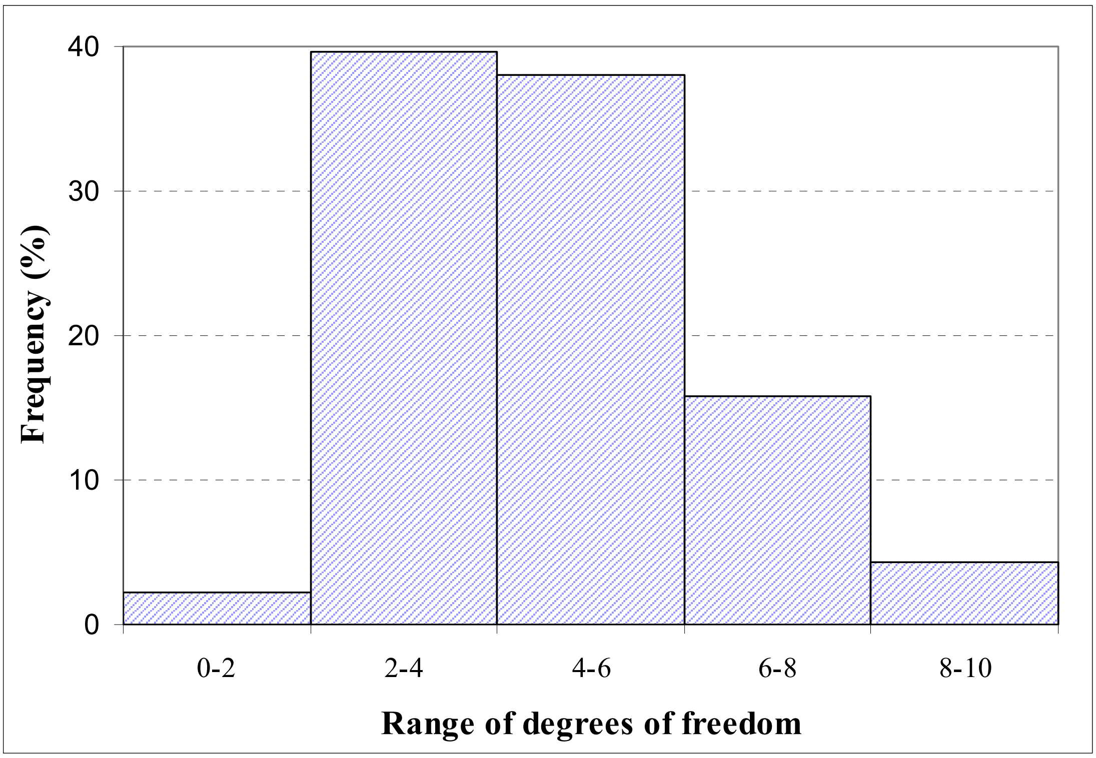
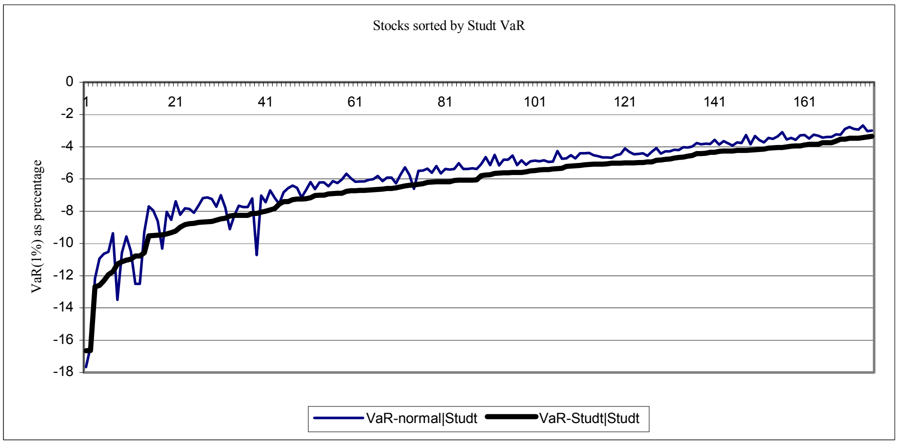
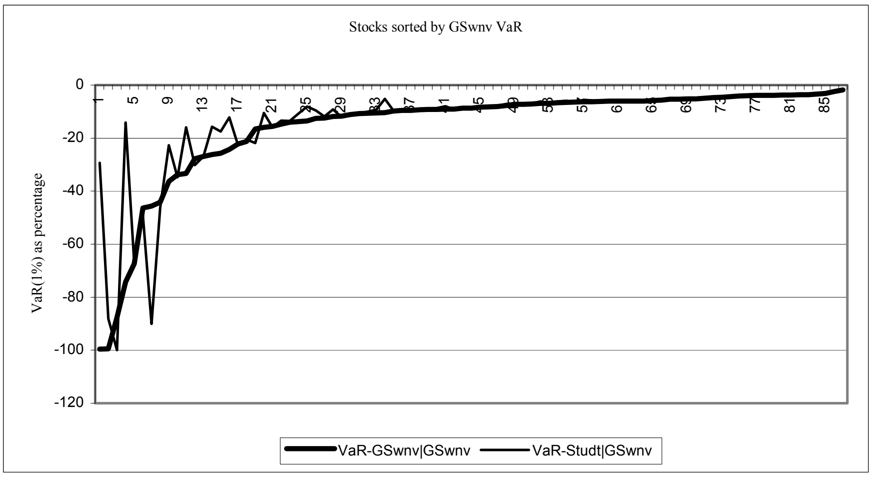


Figure 8 Graph of the Value at Risk for stocks with Student t distribution for UK FTSE350 Stocks (Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003)



The VaR is computed as a percentage, the graphs are plotted in the order of decreasing risk according to the correctly specified Student t distribution. value The curve entitled VaR-Studt|Studt is the correctly specified Student t distribtion. The curve VaR-normal|Studt shows the corresponding values if a normal distribution is assumed.

Figure 9 Graph of the Value at Risk For Stocks With GSωνσ Distribution for UK FTSE350 Stocks (Estimated using 500 daily returns from 4th December 2001 to 3rd November 2003)



The VaR is computed as a percentage, the graphs are plotted in the order of decreasing risk according to the correctly specified GSωνσ distribution. value The curve entitled VaR-GSwnv|GSwnv is the correctly specified distribution. The curve VaR-Studt|GSwnv shows the corresponding values if Student t distribution is assumed.