“MODELLING HIGHER MOMENTS OF ELECTRICITY PRICES”

Gregorio Serna* and Pablo Villaplana**

Abstract

It is crucial to model, quantify and understand the variables and dynamics that underlie the well-known extreme behaviour of spot electricity prices in wholesale markets. We explicitly model the conditional volatility and skewness of electricity prices. A GARCH-type model allowing for time-varying volatility and skewness, which is estimated assuming a Gram-Charlier expansion of the normal density function, is presented. This model is applied to data from Pennsylvania- New Jersey, NordPool and Victoria (Australia) markets. We document the existence of a rich structure in the conditional skewness of spot prices and we show the relationship between skewness and demand-supply related variables.

This version: May 2007

EFMA classification codes: 310, 420, 620, 630.

JEL classifications: G12; G13; C13; C14; Q4.

Keywords: Spot electricity prices; GRACH models; conditional volatility; conditional skewness; Gram-Charlier expansions.

---

* Universidad de Castilla – La Mancha. c/ Cobertizo de San Pedro Mártir, s/n – 45071 Toledo, Spain. Gregorio.Serna@uclm.es

** Comisión Nacional de Energía, c/ Alcalá, 47 – 28014 Madrid, Spain. pvc@cne.es

We have received valuable comments from Alfonso Novales, Eliseo Navarro, Vicente Meneu, Hipólit Torró and Tomeu Pascual. Gregorio Serna acknowledges the financial support provided by the Ministerio de Educación y Ciencia grant SEJ2005-08931-C02-01 and Junta de Castilla-La Mancha grant PAI05-074.

1. Introduction

The worldwide electrical power industry has faced a restructuring process over the last decades. It is well documented that spot electricity prices present a complex behavior in restructured (wholesale) markets. Some of the characteristics that have been noted in the literature are seasonality (intra-day, weekly, monthly, calendar effects), mean reversion, stochastic volatility and extreme behavior with fast-reverting spikes. It must be noted that electricity is non-storable, and demand and supply must be matched at every instant, and as a consequence spikes are a typical feature of electricity prices.

Recent econometric studies of spot market prices have tried to capture the main characteristics of electricity prices. Many authors have tried to capture the behavior of electricity prices either through mixtures of Gaussian distributions (Escribano et al. (2002), Goto and Karolyi (2004), Knittel and Roberts (2005), Tipping et al. (2004)) or through regime-switching models (Huisman and Mahieu (2001), Mount et al. (2006)). See also Geman and Roncoroni (2004) and Bunn and Karakatsani (2003) for a survey. One of the most interesting but difficult task is to model the spikes. The spikes being abrupt (positive) changes in electricity prices affect the conditional distribution of electricity prices, and in particular affect the conditional skewness of the process. Therefore if we claim that modeling spikes is a necessary task, we have to conclude that is obviously important to understand the dynamics and economic determinnats of the conditional skewness of electricity prices.

It must be stressed that the modeling of conditional skewness of spot prices is not only important to forecast the future distribution of spot prices, but also to understand the behavior of derivatives prices (derivatives valuation) and to quantify the risk of a given position (risk management). In particular, Bessembinder and Lemmon (2002) in their equilibrium model showed that "*skewness will affect the equilibrium forward premium and optimal forward positions*". Specifically, the skewness of the spot power price distribution increases the equilibrium forward premium. Therefore it is crucial to understand the dynamics and to identify determinants of electricity spot price skewness in order to understand and forecast the prices of electricity derivatives, the compensation required by the agents to face price risk (forward premium) and the hedging decisions made by the participants in the electricity market.

The goal of this paper is to model not only the (conditional) mean and volatility of electricity prices, but also the conditional skewness, which is needed to understand the dynamics and determinants of skewness. On one hand, understanding and forecasting the skewness of electricity prices is an important aspect by itself. The effects of time-varying skewness have been studied in the case of some financial assets like daily returns of stock indices or exchange rates, but they have not been deeply investigated in the case of daily spot electricity prices. On the other hand, time-varying skewness has also implications for instance, on the price of electricity derivatives, the behavior of forward risk premium, optimal strategies of execution of swing contracts, and estimation of at-risk measures.

There is a growing literature on stock and option price behavior dealing with the role of skewness, see Corrado and Su (1996), Harvey and Siddique (1999), Jondeau and Rockinger (2000), Premaratne and Bera (2003) and Leon et al. (2005) among others.

In this paper, we focus on the work by Leon et al. (2005). The authors propose a GARCH-type model allowing for time-varying volatility, skewness and kurtosis. The model is estimated assuming a Gram-Charlier series expansion of the normal density function for the error term. Our goal is to apply the model to electricity spot prices and to analyze which variables generate the observed skewness in electricity prices. Apart from analysing the temporal evolution of skewness (persistence) we are also very interested in introducing explanatory variables in the skewness process. Specifically, spikes, probably due to some demand and/or supply shocks, generate higher positive skewness. Therefore, we present an extended GARCH-type model allowing not only for time-varying volatility and skewness, but also for the potential effect of demand and supply shocks on price skewness.

Moreover, as pointed out by Leon et al. (2005), their model for time-varying moments is particularly useful for financial series characterized by high risk and pronounced departures from normality, which is particularly the case of spot electricity markets.

The rest of the paper is organized as follows. Section 2 presents the data and some preliminary results. In Section 3 we present the general GARCH-type model for estimating time-varying variance and skewness jointly. We also present the specific models we have analysed. Section 4 presents the empirical results regarding the estimation of the models and compares the models allowing for time-varying skewness and the standard models with constant third moment. Section 5 concludes with a summary and discussion.

2. Data and descriptive statistics

The data employed in the study are average daily prices for electricity from three different markets: NordPool (Scandinavia), Pennsylvania-New Jersey-Maryland market (henceforth PJM) and Victoria market (Australia). Table 1 summarizes the dataset. Table 2 presents some descriptive statistics. Figures 1 to 3 show the evolution of electricity prices and the evolution of the extra economic variable we have in our data set. As usual in electricity markets, we may observe electricity prices face some seasonal behavior (more clear in the case of NordPool), mean-reversion, non-constant volatility and occasional spikes. From Table 2 we see electricity price (and log-prices) are quite volatile, and highly non-normal. In fact the null of normality of price and log-price series is rejected with the Jarque-Bera test in all three markets.

Table 3 presents the (unconditional) skewness of log-price series per month, see also Figure 4. Table 3 illustrates that skewness is not constant across the year, suggesting that time-varying skewness should be incorporated in the model for the electricity price.

Scatter plots in Figures 5 to 7 show the relationship between prices and weekly hydro reservoirs in NordPool (figures 5a and 5b); the relationship between price and the ratio load/capacity in PJM (figures 6a and 6b) and the relationship between price and demand in Victoria (figure 7). We see that there exists a positive relationship between price and ratio load capacity and between price and demand in PJM and Victoria markets, respectively. Moreover the relationship is not necessarily linear, and it may resemble in some cases and inverted L. This “inverted L” pattern appears clearly in figure 6b. We will take into account this nonlinearity in the specification for the skewness process below.

3. Methodology

In this section we propose a model for conditional variance and skewness. Given a series of electricity prices $\{P_0, P_1, \ldots, P_T\}$, we denote the natural logarithm of the price by $y_t = log(P_t)$. Specifically, we present a GARCH(1,1)-type model for the conditional variance and also a GARCH (1,1) structure for conditional skewness. This model, denoted as GARCHS is given by[1]:

[1] See León, Rubio and Serna (2005).

$$y_t = f(t) + X_t \tag{1a}$$

$$X_t = \phi \cdot X_{t-1} + \varepsilon_t; \qquad \varepsilon_t \sim \left(0, \sigma_\varepsilon^2\right) \tag{1b}$$

$$\varepsilon_t = h_t^{1/2} \eta_t; \quad \eta_t \sim (0,1); \quad \varepsilon_t \mid I_{t-1} \sim \left(0, h_t\right) \tag{1c}$$

$$h_t = \beta_0 + \beta_1 \cdot \varepsilon_{t-1}^2 + \beta_2 \cdot h_{t-1} \tag{1d}$$

$$s_t = \gamma_0 + \gamma_1 \cdot \eta_{t-1}^3 + \gamma_2 \cdot s_{t-1} + \gamma_3 \cdot Z_t \tag{1e}$$

where I_{t-1} denotes an information set till period $t-1$ and h_t and s_t denote conditional variance and skewness respectively. Z_t accounts for the potential effect of demand and supply shocks on conditional skewness.

As usual in the literature, Lucía and Schwarzt (2002), we have incorporated a deterministic seasonal function, *f(t)* given by:

$$f(t) = \alpha_0 + \alpha_T \cdot t + \alpha_d \cdot wkd_t + \alpha_1 \cdot \sin\left(\left(t + \alpha_2\right) \cdot \frac{2\pi}{365}\right) + \alpha_3 \cdot \sin\left(\left(t + \alpha_4\right) \cdot \frac{4\pi}{365}\right) \tag{2}$$

where wkd_t is a dummy variable that takes a value of 1 if the observation is in weekday and zero otherwise[2] (weekend). Parameters α_1 and α_3 capture the amplitude of the cycle, while parameters α_2 and α_4 capture the location of the (local) peak of the cycle. With this general formulation for the sinusoidal function we allow for the possibility of having two cycles per year (two local maximum per year). In the case of a single annual cycle we should have $\alpha_3 = \alpha_4 = 0$. Equation (1b) captures the mean-reversion pattern usually observed in electricity prices. Equations (1c)-(1d) characterize the volatility process, in particular we have specified a GARCH(1,1) process for volatility of electricity prices, although another type of model of the GARCH family could be used[3]. The (nonnegative) parameters β_0, β_1 and β_2 characterize the dynamics of the volatility following a GARCH(1,1) process ($\beta_0 > 0$; β_1 , $\beta_2 \geq 0$). The non-negativity restrictions

[2] It should be noted that very similar results have been found with monthly dummy variables instead of sinusoidal cycles. However, the sinusoidal specification is preferably given its reduced number of parameters, which is important in the estimation process.

[3] In particular, some kind of non-linear GARCH process could be appealing in this case. In fact, Knittel and Roberts (2005) reported that at least for the California market, an "inverse leverage" effect could exist. We have estimated a modified version of the model incorporating a NAGARCH(1,1) process. It is found that the (inverse) leverage effect is only significant in the Victoria market.

are needed to guarantee that the conditional variance is positive and also ω has to be strictly positive for the process not to degenerate. If $\beta_1 + \beta_1 < 1$, then the variance reverts back to its unconditional mean $\sigma^2 = \beta_0 /(1 - \beta_1 - \beta_2)$.

Finally, equation (1e) captures the dynamics of skewness. We establish that $E_{t-1}(\eta_t) = 0$, $E_{t-1}(\eta_t^2) = 1$ and $E_{t-1}(\eta_t^3) = s_t$, where s_t is driven by a GARCH (1,1) structure. Hence, s_t represents the skewness corresponding to the conditional distribution of the standardized residual $\eta_t = \varepsilon_t h_t^{-1/2}$. Therefore, we propose to model the skewness process as a kind of GARCH(1,1) process with an extra variable Z_t related to demand and supply conditions. Given data availability, the variable Z_t will differ among markets. Although below we show the exact definition of variable Z_t for each market it is worth to mention that variable Z_t when applied to NordPool data is related to supply shocks ("hydro reservoir shocks"), while is related to "demand shocks" when estimating the model with prices from the Victoria market and finally is related to the Load / Capacity ratio when estimating the model with PJM prices.

Using a Gram-Charlier (GC) series expansion of the normal density function and truncating in the fourth moment[4], we obtain the following density function for the standardized residuals η_t conditional on information available in $t-1$:

$$g(\eta_t \mid I_{t-1}) = \phi(\eta_t)\left[1 + \frac{s_t}{3!}\left(\eta_t^3 - 3\eta_t\right) + \frac{k-3}{4!}\left(\eta_t^4 - 6\eta_t^2 + 3\right)\right] = \phi(\eta_t)\psi(\eta_t) \qquad (3)$$

where $\phi(\cdot)$ denotes the probability density function (henceforth pdf) corresponding to the standard normal distribution, k denotes standardized kurtosis, which is considered to be constant over time[5], and $\Psi(\cdot)$ is the fourth order polynomial in brackets in (3). Note that the pdf defined in (3) is not really a density function because it might be negative for some parameter values in (1) due to the component $\Psi(\cdot)$. Also, the integral of $g(\cdot)$ on $\Re$ is not equal to one. We propose a truly pdf, denoted as $f(\cdot)$, by transforming the

[4] See Jarrow and Rudd (1982) and also, Corrado and Su (1996).
[5] It should be noted that this procedure also allows for time-varying kurtosis (see for example Leon et. al, 2005). However, our main goal is to investigate the effects of time-varying skewness on electricity spot prices. Therefore, we present a model for time-varying skewness, allowing for non-normal (although constant) kurtosis. Furthermore, it is important in the estimation process to keep the number of parameters as small as possible.

density $g(\cdot)$ according to the methodology in Gallant and Tauchen (1989). Specifically, in order to insure positivity we shall square the polynomial part $\Psi(\cdot)$, and to insure that the density integrates to one[6] we shall divide by the integral of $g(\cdot)$ over $\Re$. Hence, the resulting pdf written in abbreviated form is:

$$f\left(\eta_t \left| I_{t-1}\right.\right) = \phi(\eta_t)\psi^2(\eta_t)/\Gamma_t \tag{4}$$

where

$$\Gamma_t = 1 + \frac{s_t^3}{3!} + \frac{(k-3)^2}{4!}$$

Therefore, after omitting unessential constants, the logarithm of the likelihood function for one observation corresponding to the conditional distribution $\varepsilon_t = h_t^{1/2}\eta_t$, whose pdf is $h_t^{-1/2} f\left(\eta_t \left| I_{t-1}\right.\right)$, is given by:

$$l_t = -\frac{1}{2}\ln h_t - \frac{1}{2}\eta_t^2 + \ln\left(\psi^2(\eta_t)\right) - \ln\left(\Gamma_t\right) \tag{5}$$

It is important to note that this likelihood function is smoother and easier to estimate than the one based on a non-central t proposed by Harvey and Siddique (1999). In fact, the likelihood function in (3) is the same as in the standard normal case plus two adjustment terms accounting for non-normal skewness and kurtosis. The density function based on a Gram-Charlier series expansion in equation (4) nests the normal density function (when $s_t = 0$ and $k = 3$), while the non-central t does not. Therefore, the restrictions imposed by the normal density function with respect to the more general density based on a Gram-Charlier series expansion can be easily tested.

Before presenting the estimation results, it is important to note that the likelihood function in (5) is highly nonlinear. Therefore the starting values of the parameters must be selected with care. As usual in these cases, we estimate our GARCHS model following several stages, using the parameters estimated from simpler

---

[6] See the appendix to prove that this nonnegative function is really a density function that integrates to one.

(nested) models as starting values for more complex ones. These nested models can be summarized, together with the distribution assumed for the unconditional standardized error, as follows[7]:

GARCH(1,1) – N(0,1)	$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}$ $s_t = 0$
GARCHS(1,1) / NAGARCH(1,1) – Gallant &Tauchen	$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}$ $h_t = \beta_0 + \beta_1 \left(\varepsilon_{t-1} + \beta_3 h_{t-1}^{1/2}\right)^2 + \beta_2 h_{t-1}$ $s_t = \gamma_0 + \gamma_1 \eta_{t-1}^2 + \gamma_2 s_{t-1}$
GARCHS(1,1) / NAGARCH(1,1) – Gallant & Tauchen with explanatory variable	$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}$ $h_t = \beta_0 + \beta_1 \left(\varepsilon_{t-1} + \beta_3 h_{t-1}^{1/2}\right)^2 + \beta_2 h_{t-1}$ $s_t = \gamma_0 + \gamma_1 \eta_{t-1}^2 + \gamma_2 s_{t-1} + \gamma_3 Z_t$

The GARCH(1,1) specification for the volatility process has been used in the NordPool and PJM markets, whereas the NAGARCH(1,1) specification has been used in the Victoria market (see footnote 3).

We have the following definitions for the variable Z_t:

<u>PJM market</u>

Z_t is a dummy variable that takes the value 1 when the ratio load / capacity is greater or equal than 0.7, and zero otherwise. In other words, Z_t is a dummy variable capturing those days in which the system is near full capacity, and in particular when demand is above 70% of the maximum available capacity. The idea is that the relationship between price and ratio load capacity is non-linear and have an inverted L shape.

In a regime-switching context Mount et al. (2006) found that a similar value of the ratio load capacity separates a low and high volatility state. In our paper, in order to decide the breaking point that defines this dummy variable Z, we have run regressions of the electricity price against the load/capacity ratio, including a dummy variable for the slope which takes the value 1 when the ratio is higher than a certain value, say *a*, and 0 otherwise, with *a* varying from 0.40 and 0.82 (the minimum and maximum of the

[7] In all models bellow the mean equation is given by expressions (1a) – (1c) and (2).

load/capacity ratio respectively). The breaking point has been chosen as the value of *a* that maximizes the R^2 of the regression, which is 0.70 (see Table 4)[8]. Therefore if the load / capacity ratio affects price skewness we should obtain that the parameter γ_3 is positive (and significantly different from zero).

<u>NordPool market</u>

For NordPool we have weekly data on the level of the hydro reservoir. Since prices have daily frequency we have imposed that all days in the same week have the same value of hydro reservoir. From weekly hydro reservoir we have substracted the predictable part in order to obtain a measure of supply shock. The variable supply shock is defined as the residuals obtained from this regression:

$$Supply_t = \beta_0 + \sum_{j=1}^{11} \beta_j^M \cdot D_{j,t}^M + \beta^{wkd} \cdot weekday_t + \beta^T \cdot trend_t + v_t$$

where $D_{j,t}^M$ is a set of monthly dummies and v_t are the residuals of the regression.

Instead of dealing with the variable v_t we have defined a measure of shocks in percentage terms, defined as: “supply shock” / Hydro Reservoir.

Finally since relationship between hydro reservoir and prices could be a non-linear, we have built a dummy that takes values 1 if “supply shock” / Hydro Reservoir is below -0.3 and 0 otherwise[9, 10].

<u>Victoria market</u>

As stated before, variable Z_t in the case of the Victoria market is related to demand shocks. We have defined demand shocks as the residuals from the following regression:

$$Demand_t = \beta_0 + \sum_{j=1}^{11} \beta_j^M \cdot D_{j,t}^M + \beta^{wkd} \cdot weekday_t + \beta^T \cdot trend_t + v_t$$

where $D_{j,t}^M$ is a set of monthly dummies and v_t are the residuals of the regression.

---

[8] We agree that other possible definitions for Z could be used. In particular another plausible definition for Z could be: Z = max { (Load / Capacity) – 0.7, 0}.

[9] This breaking point has been chosen in a similar way as in the PJM market, i.e. running regressions of the electricity price against “supply shock”/Hydro Reservoir.

But instead of dealing with the variable ν_t it has been defined a measure of shocks in percentage terms: "demand shocks" / demand. Finally we have built a dummy variable that takes the value 1 if "demand shock" / demand is higher than 0.05.[11]

4. Empirical results

Before we estimate our GARCHS/NAGARCHS models, we analyze the presence of unit roots in the level of the process. As pointed out by Boswijik (2001) and Kim and Schmidt (1993) among others, standard Dickey-Fuller tests based on LS estimators are often sensitive, in the presence of GARCH errors. This problem becomes serious when the volatility process is near integrated ($\beta_1 + \beta_2$ close to 1). Boswijk (2001) and Boswijik and Doornik (2005) propose tests for a unit root in models with GARCH errors, based on a likelihood ratio statistic, which substantially improves the asymptotic local power of the standard Dickey-Fuller tests, specially when the volatility process is near integrated and the short-run variation in volatility (β_1) is relatively high, which is particularly the case of spot electricity prices.

The likelihood ratio test will be based on the following model:

$$\begin{aligned}
&\Delta X_t = (\phi - 1)(X_{t-1} - \mu) + \varepsilon_t \\
&\varepsilon_t = h_t^{1/2}\eta_t; \\
&h_t^{1/2} = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}
\end{aligned}$$

Where $X_t = y_t - f(t)$. The parameter ϕ describes the degree of mean-reversion. The null hypothesis will be H_0: $(\phi - 1) = 0$, which is tested against the alternative H_1: $(\phi - 1) < 0$. The distribution for the likelihood ratio statistic (under the null) is approximated by a gamma distribution (see Boswijik and Doornik, 2005).

The results are presented in Table 5. The null hypothesis can be rejected for PJM and Victoria markets, but not for the NordPool market[12]. Therefore, only in the NordPool market, our GARCH/GARCHS model will be estimated with the variables in the mean equation in first differences.

---

[10] Other possible definition of Z could be used. Note that we just defined a supply shock in percentages and we have taken into account that the relationship between supply shocks and log-prices is nonlinear.
[11] This breaking point has been decided in a similar way as in the PJM and NordPool markets, i.e. running regressions of the electricity price against "demand shock"/demand.
[12] In fact the autorregresive parameter in a standard GARCH(1,1) model for the spot electricity price in the NordPool market is around 0.99.

Tables 6, 7 and 8 contain the results of the estimation of our three models (GARCH/NAGARCH–N(0,1), GARCHS/NAGARCHS–Gallant & Tauchen and GARCHS/NAGARCHS–Gallant & Tauchen with explanatory variable) for NordPool, PJM and Victoria respectively. Concerning the mean equation, it is found that all the coefficients associated to the seasonal effects are significant, except for α_T in the NordPool. Specifically, trading days are characterized by higher prices than non-trading days, since the coefficient α_d is positive and significant in all markets. We have obtained that only one annual seasonal cycle is needed in all markets.

As usual, volatility is found to be persistent since the coefficient of lagged volatility is positive and significant, indicating that high conditional variance is followed by high conditional variance. It is also interesting the relatively high value of the short-run variation in volatility found in the NordPool (β_1). Finally, an inverse leverage effect is found in the Victoria Market, indicating that positive shocks to the price result in larger increases in volatility than negative shocks. This inverse leverage effect has also been found by Knittel and Roberts (2005) in the California market.

Concerning the skewness equation, it is found that, for NordPool and PJM markets, days with high skewness are followed by days with low skewness, since the coefficient for lagged skewness (γ_2) is negative and significant, although its magnitude (in absolute value) is lower than the one for the variance case. The opposite result is found in the Victoria market, i.e., days high skewness are followed by days with high sekwness. Also, shocks to skewness are significant though they are less relevant than its persistence.

Next, we will briefly describe the results of the estimation of the GARCHS (1,1)/NAGARCH(1,1) model with explanatory variable, with the pdf in (4) for the error term. As argued above, the point is that jumps associated to demand/supply related variables could generate higher positive skewness. The results confirm this hypothesis, since the coefficient associated to the explanatory variable Z_t (γ_3) is positive and significant in all markets. Even though jumps generate higher skewness, it is important to model the time-varying skewness which is not due to jumps, since the coefficients associated to the GARCH(1,1) effects in the skewness equation (γ_1 and γ_2) are still significant.

Finally, it is worth noting that the value of the Schwarz Information Criterion (SIC), shown at the bottom of Tables 6, 7 and 8 rises monotonically in all cases when we move from the simpler models to the more complicated ones, with the GARCHS/NAGARCHS model with explanatory variable showing the highest figure.

Figure 8 shows the behavior of the conditional variance obtained with both the standard GARCH – N(0,l) model and with the GARCHS – Gallant & Tauchen with explanatory variable model, in the PJM market[13]. It is clear that conditional variance obtained with the model accounting for conditional skewness is smoother than the one obtained with the standard GARCH model. This is confirmed by the results in Table 9, which shows the main descriptive statistics for both conditional variances. In fact, the conditional variance obtained with the GARCHS model shows a lower standard deviation and less skewness and kurtosis than the variance obtained with the standard GARCH model. This is consistent with the findings of Harvey and Siddique (1999) and León, Rubio and Serna (2005).

The behavior of the conditional skewness obtained with the GARCHS – Gallant & Tauchen model with explanatory variable is also depicted in Figure 8. Looking at the three graphics in Figure 8 we can see that periods with high volatility in the conditional variances series are also characterized by high volatility in the conditional skewness series.

To determine how well the standard GARCH and the GARCHS with explanatory variable models perform in predicting spot electricity volatility, tree metrics have been calculated. The variable predicted is the squared forecast error (ε^2_t) and the predictors are the conditional variances (h_t) from, respectively, the standard GARCH and GARCHS with explanatory variable. The three metrics are:

Mean absolute error: $MAE = \frac{1}{T}\sum_{t=1}^{T}\left|\varepsilon_t^2 - h_t\right|$

Root mean squared error: $RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^2 - h_t\right)^2}$

$$R^2 = 1 - \frac{\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_t^2 - h_t\right)^2}{\frac{1}{T}\sum_{t=1}^{T}\varepsilon_t^2}$$

[13] In order to save space, only the results for the PJM market are presented.

The results for the PJM market are shown in Table 10. In the full sample period the model accounting for time-varying skewness with explanatory variable outperforms the standard GARCH model, using all three metrics. It is also the best performing model in all the annual periods considered in the Table. Interestingly, the best performance of the GARCHS with explanatory variable model is found in the first and last annual periods (1999 and 2003), which are characterized by higher volatility (see Figure 2).

Finally, Table 11 presents some descriptive statistics for a 30-day simple moving average skewness and also the statistics corresponding to the conditional skewness under the GARCHS – Gallant & Tauchen with explanatory variable model, for the PJM market. It is clear that conditional skewness shows less standard deviation than the 30-day moving average measure of skewness. Moreover, looking at the median statistic, we can conclude that the conditional measure seems to provide more pronounced positive skewness than the moving average one.

5. Conclusions

It is well known that spot electricity prices in wholesale markets present a complex behaviour, i.e. high (time-varying volatility), fast reverting spikes and non-normality. There have been many papers trying to capture this extreme behaviour through mixtures of Gaussian distributions (see Knittel and Roberts, 2005, Goto and Karolyi, 2004 and Escribano et al., 2002, among others), or through regime-switching models (Huisman and Mahieu, 2001; Mount et al., 2006).

In this paper it has been presented an alternative way of capturing such extreme behaviour. Instead of introducing extra terms that generate jumps, which indirectly generate conditional skewness, we have presented a model allowing not only for time-varying volatility but also for time-varying skewness. Specifically, we propose a GARCH-type model for conditional volatility and skewness of electricity prices, assuming a Gram-Charlier series expansion of the normal density function for the error term. This model also allows for explanatory variables in the skewness process. Particularly, we have investigated the effect of jumps, associated to demand-supply related variables, on skewness. The point is that jumps due to demand-supply related variables could generate higher positive skewness. It is worth noting that understanding of the process for conditional volatility and skewness has consequences in terms of hedging, risk management and valuation of financial derivatives and real assets. In fact, as pointed out by Besembinder and Lemmon (2002), the equilibrium forward premium and the optimal forward positions are affected by the skewness of the spot price distribution.

The data employed in the study are average daily prices for electricity in the Pennsylvania-New Jersey-Maryland (PJM), Victoria and NordPool markets. The results indicate significant presence of conditional skewness. Additionally, the specification allowing for time-varying skewness outperforms the standard GARCH specification with constant third moment. It is also found that jumps associated to demand-supply variables, generate high skewness. However, even though jumps generate higher skewness, it is still important to model time-varying skewness which is not due to jumps.

REFERENCES

Bessembinder, H. and M.L. Lemmon, 2002. "Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets", *Journal of Finance* 57, 1347-82.

Boswijik, P., 2001. "Testing for a Unit Root with Near-Integrated Volatility". Timbergen Institute Discussion Paper # 01-077/4.

Boswijik, P. and J. A. Doornik, 2005. "Distribution Approximations for Cointegration Tests with Stationary Exogenous Regressors". *Journal of Applied Econometrics* 20, 797-810.

Bunn, D. W., 2004. "Structural and Behavioural Foundations of Competitive Electricity Prices", in *Modelling Prices in Competitive Electricity Markets*, edited by D.W. Bunn, John Wiley & Sons.

Bunn, D. and N. Karakatsani, 2003. "Forecasting Electricity Prices", working paper, London Business School.

Corrado, C. and T. Su, 1996. "Skewness and Kurtosis in S&P 500 index returns implied by option prices", *Journal of Financial Research* 19, 175-192.

De Vany, A.S. and W.D. Walls, 2000, "Price Dynamics in a network of decentralized power markets. *J. Regul. Econ.* 15, 123-140.

Escribano, A., J.I. Peña and P. Villaplana, 2002. "Modelling Electricity Prices: International Evidence", working paper, Universidad Carlos III de Madrid.

Gallant, A. R. and G. Tauchen (1989), "Seminonparametric Estimation of Conditionally Constrained Heterogeneous Processes: Asset Pricing Applications", *Econometrica* 57, 1091-1120.

Goto and Karolyi (2004), "Understanding Electricity Price Volatility Within and Across Markets", working paper.

Harvey, C. R. and A. Siddique, 1999. "Autoregresive Conditional Skewness", *Journal of Financial and Quantitative Analysis* 34, 465-487.

Huisman R. and R. Mahieu, 2001. "Regime Jumps in Power Prices", working paper.

Jondeau, E. and M. Rockinger, 2000. "Conditional Volatility, Skewness and Kurtosis: Existence and Persistence", *Working Paper, HEC School of Management*.

Kim, K. and P. Schmidt, 1993. "Unit Root Tests with Conditional Heteroscedasticity". *Journal of Econometrics* 59, 287-300.

Knittel, C. and M. Roberts, 2005. “An empirical examination of restructured electricity prices”. *Energy Economics* 27, 791-817.

León, A., G. Rubio and G. Serna (2005), “Autoregresive Conditional Volatility, Skewness and Kurtosis”, *The Quarterly Review of Economics and Finance* 45, 599-618.

Longstaff, F., and A. Wang, 2004. “Electricity Forward Prices: a high-frequency empirical analysis”, Journal of Finance

Lucia, J.J., and E.S. Schwartz, 2001. “Electricity Prices and Power Derivatives: Evidence for the Nordic Power Exchange”, *Working Paper, University of California Los Angeles*.

Mount, T., Y. Ning and X. Cai, 2006. “Predicting price spikes in electricity markets using a regime-switching model with time-varying parameters”, *Energy Economics* 28, 62-80.

Pirrong, C. and Jermakyan, M. (2000). “The Price of Power: the Valuation of Power and Weather Derivatives”, working Paper, Olin School of Business, Washington University.

Premaratne, G. and A. K. Bera, 2001. “Modeling Asymmetry and Excess Kurtosis in Stock Return Data”, *Working Paper, Department of Economics, University of Illinois*.

Robinson, T., 2000. “Electricity pool series: a case study in non-linear time series modeling. *Applied Economics* 32, 527-532.

Worthington, A., Kay-Spratley, A. and H. Higgs, 2003. “Transmission of prices and price volatility in Australian electricity spot markets: a multivariate GARCH analysis”, *Energy Economics*.

APPENDIX

Here we show that the nonnegative function $f\left(\eta_t \middle| I_{t-1}\right)$ in (4) is really a density function, that is, it integrates to one. We can rewrite $\psi\left(\eta_t\right)$ in (2) as:

$$\psi\left(\eta_t\right)=1+\frac{s_t}{\sqrt{3!}}H_3\left(\eta_t\right)+\frac{k_t-3}{\sqrt{4!}}H_4\left(\eta_t\right)$$

where $\{H_i(x)\}_{i\in N}$ represents the Hermite polynomials such that $H_0\left(x\right)=1,\ H_1\left(x\right)=x$ and for $i\geq 2$ they hold the following recurrence relation:

$$H_i\left(x\right)=\left(xH_{i-1}\left(x\right)-\sqrt{i-1}H_{i-2}\left(x\right)\right)/\sqrt{i}$$

It is verified that $\{H_i(x)\}_{i\in N}$ is an orthonormal basis satisfying that:

$$\int_{-\infty}^{\infty}H_i\left(x\right)\phi\left(x\right)dx=1,\quad \forall i \tag{A-1}$$

$$\int_{-\infty}^{\infty}H_i\left(x\right)H_j\left(x\right)\phi\left(x\right)dx=0,\quad \forall i\neq j \tag{A-2}$$

where $\phi(\bullet)$ denotes the N(0,1) density function. If we integrate the conditional density function in (4), given conditions (A-1) and (A-2):

$$\left(1/\Gamma_t\right)\int_{-\infty}^{\infty}\phi\left(\eta_t\right)\left[1+\frac{s_t}{\sqrt{3!}}H_3\left(\eta_t\right)+\frac{k_t-3}{\sqrt{4!}}H_4\left(\eta_t\right)\right]^2 d\eta_t$$

$$=\left(1/\Gamma_t\right)\left[\int_{-\infty}^{\infty}\phi\left(\eta_t\right)d\eta_t+\frac{s_t^2}{3!}\int_{-\infty}^{\infty}H_3^2\left(\eta_t\right)\phi\left(\eta_t\right)d\eta_t+\frac{\left(k_t-3\right)^2}{4!}\int_{-\infty}^{\infty}H_4^2\left(\eta_t\right)\phi\left(\eta_t\right)d\eta_t\right]$$

$$=\left(1/\Gamma_t\right)\left[1+\frac{s_t^2}{3!}+\frac{\left(k_t-3\right)^2}{4!}\right]$$

$$=1.$$

Table 1. Summary of available data

Market	In-sample period (dd/mm/yy)	Spot Price (daily)	Extra Variable
NP	01/01/95 - 19/11/03	X	Weekly Hydro Reservoir level
PJM	01/01/99 - 31/05/03	X	Ratio Load / Available Generation Capacity (Daily)
VIC	01/05/99 - 30/09/04	X	Daily Load (demand)

Table 2a. Descriptive statistics. Price Series

Price	N. Obs.	Mean	Min.	Max.	Std.Dev.	Skew.	Kurt.	CV	Jarque-Bera (p-value)
NP	3245	166.53	21.27	831.41	89.00	2.36	13.28	0.53	17302.17 (0.0000)
PJM	1612	29.42	8.19	397.34	23.79	7.47	78.04	0.81	421415.7 (0.0000)
VIC	1980	30.79	4.98	1014.60	39.66	15.23	324.33	1.29	8582505 (0.0000)

CV: Coefficient of variation: Standard Deviation / Mean

Table 2b. Descriptive statistics. Log-Price Series

Price	N. Obs.	Mean	Min.	Max.	Std.Dev.	Skew.	Kurt.	CV	Jarque-Bera (p-value)
NP	3245	5.00	3.06	6.72	0.48	-0.016	3.77	0.10	80.88 (0.0000)
PJM	1612	3.25	2.10	5.98	0.45	1.28	7.29	0.14	1683.50 (0.0000)
VIC	1980	3.26	1.61	6.92	0.47	1.84	11.12	0.14	6546.55 (0.0000)

CV: Coefficient of variation: Standard Deviation / Mean

Figure 1: NordPool Price, Log-price series and Weekly Hydro Reservoir Level.

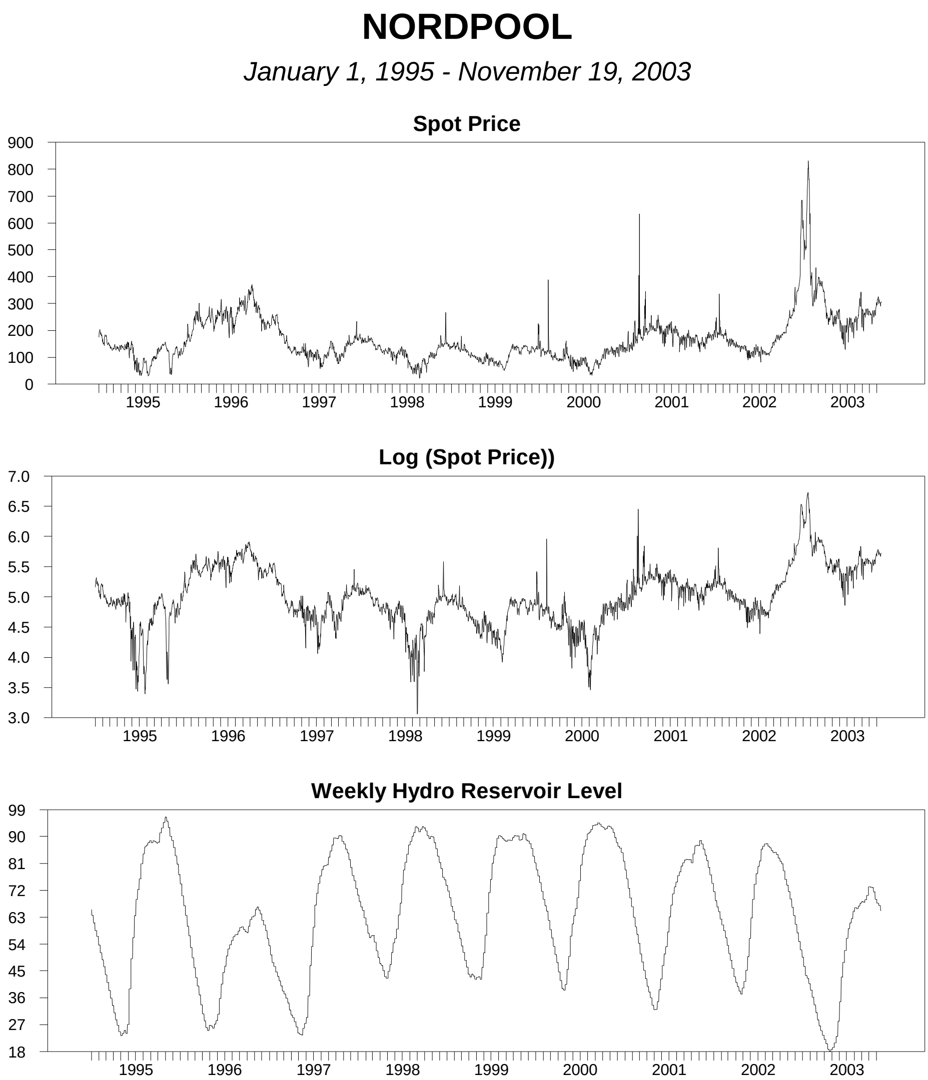


Figure 2: PJM Price and Log-price series and Daily Ratio Load / Capacity

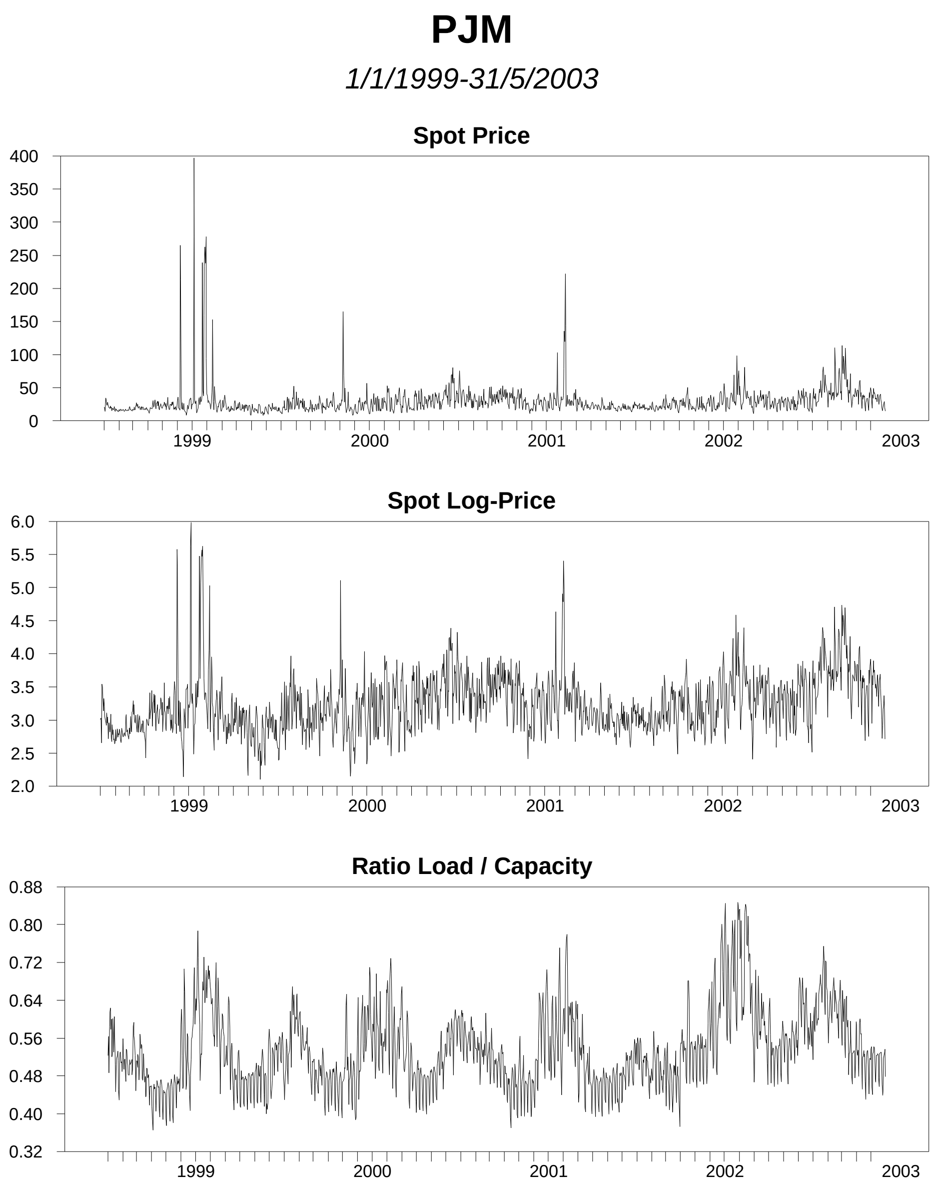


Figure 3: Victoria (Australia) price, log-price and load (demand) series.

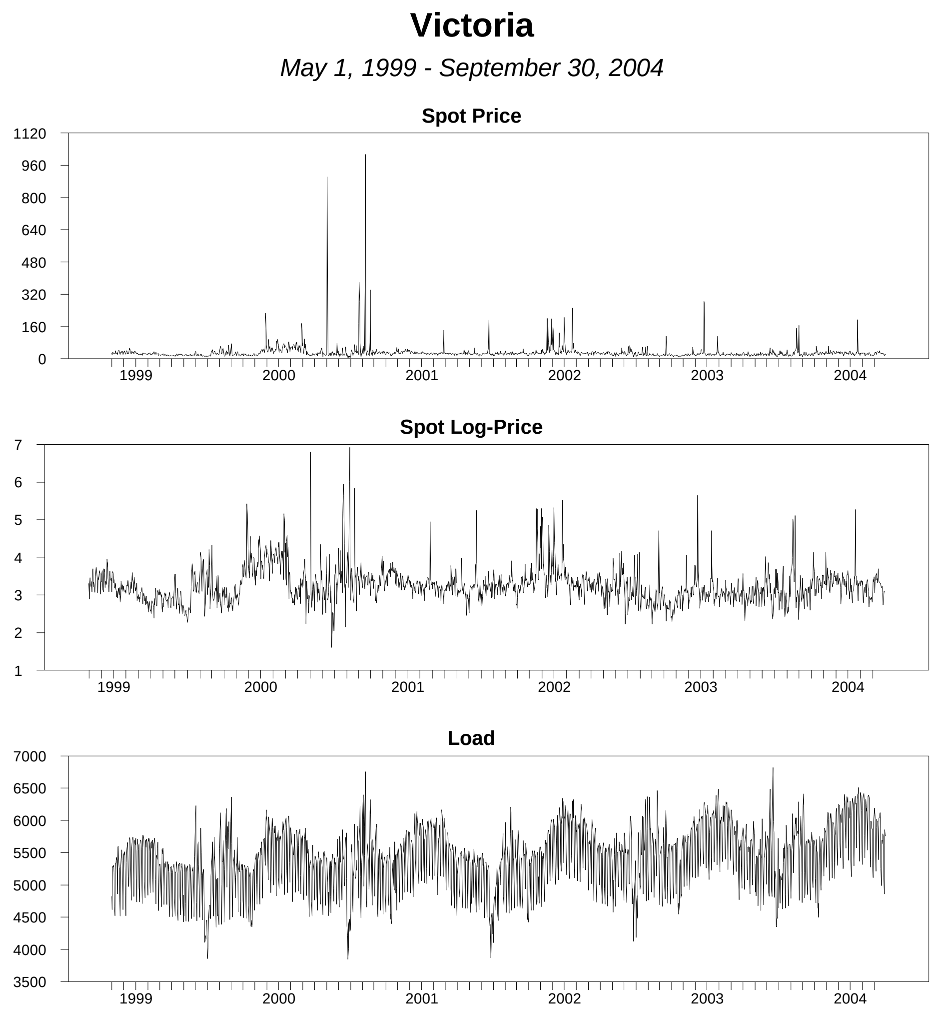


Table 3: Log-price skewness per month (p-value for the null of zero skewness in parenthesis)

	NORDPOOL	PJM	VICTORIA
January	1.617 (0.00)	0.487 (0.01)	2.034 (0.00)
February	0.261 (0.05)	0.902 (0.00)	2.0428 (0.00)
March	0.186 (0.14)	0.884 (0.00)	0.793 (0.00)
April	-0.239 (0.06)	0.103 (0.61	0.248 (0.18)
May	-1.02 (0.00)	0.716 (0.00)	1.777 (0.00)
June	-0.548 (0.00)	1.875 (0.00)	1.785 (0.00)
July	-0.526 (0.00)	1.728 (0.00)	1.737 (0.00)
August	-1.043 (0.00)	1.177 (0.00)	1.662 (0.00)
September	-1.64 (0.00)	0.149 (0.51)	1.533 (0.00)
October	-1.11 (0.00)	-0.006 (0.98)	-0.225 (0.26)
November	0.427 (0.00)	-0.11 (0.61)	4.663 (0.00)
December	1.296 (0.00)	0.501 (0.02)	0.560 (0.00)

Figure 4: Skewness of log-prices per month

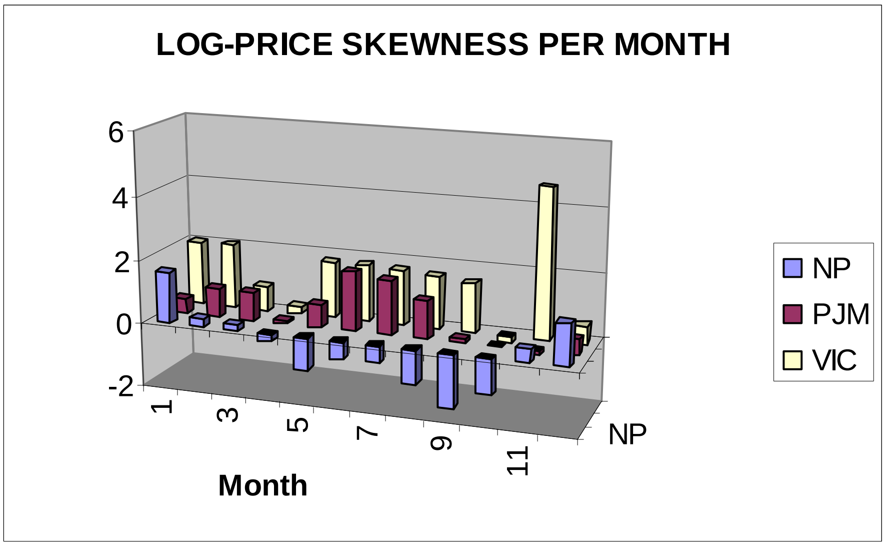


FIGURE 5a: NordPool. Scatter Plot: Log-Price (vertical axis) vs. Weekly Hydro Reservoirs (horizontal axis)

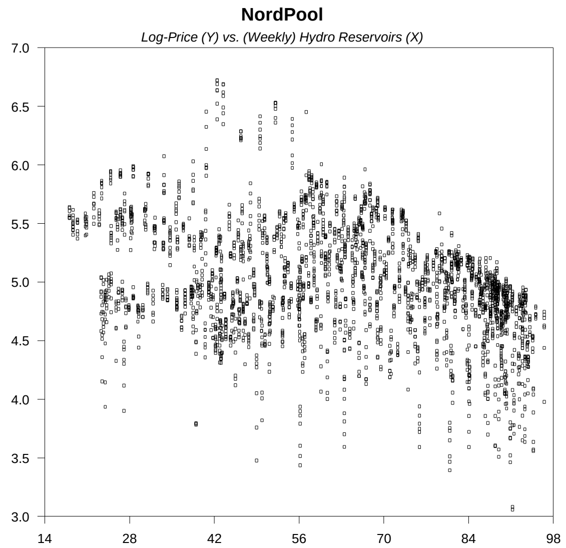


FIGURE 5b: NordPool. Scatter Plot: Price (vertical axis) vs. Weekly Hydro Reservoirs (horizontal axis)

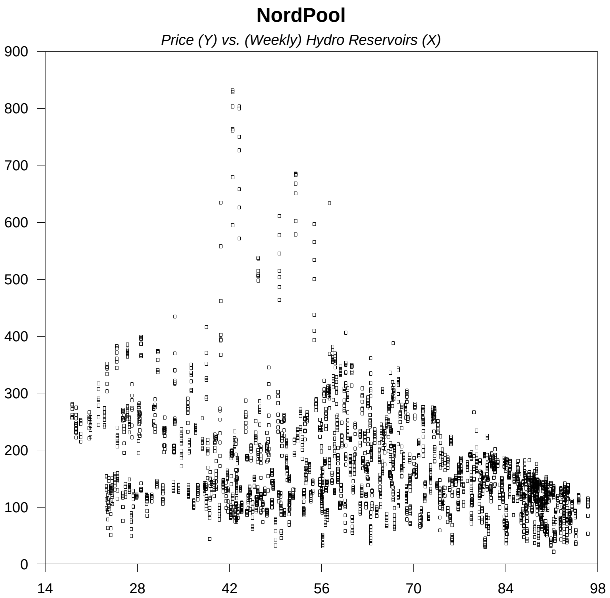


FIGURE 6a: PJM market. Scatter Plot: Log-Price (vertical axis) vs. Ratio Load capacity (horizontal axis)

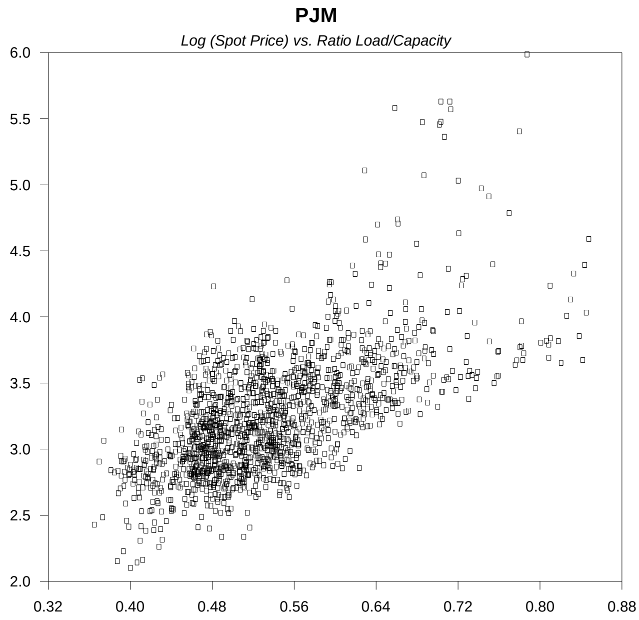


FIGURE 6b: PJM market. Scatter Plot: Price (vertical axis) vs. Ratio Load capacity (horizontal axis)

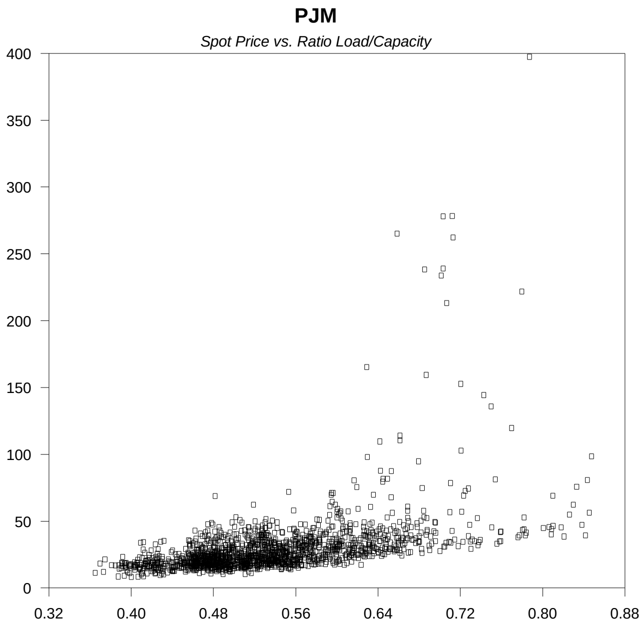


FIGURE 7: Victoria. Scatter Plot: Log-Price (vertical axis) vs. Demand (horizontal axis)

TABLE 4

REGRESSIONS OF PRICE AGAINST LOAD/CAPACITY RATIO (PJM)

L/C	R^2	L/C	R^2
0.40	0.23053336	0.62	0.25237065
0.41	0.23143723	0.63	0.25187283
0.42	0.23302725	0.64	0.25560971
0.43	0.23442824	0.65	0.2559233
0.44	0.23487498	0.66	0.2544859
0.45	0.23518622	0.67	0.26185767
0.46	0.23626816	0.68	0.27388471
0.47	0.23848113	0.69	0.26968033
0.48	0.24100554	0.70	0.29011417
0.49	0.24436078	0.71	0.24646167
0.50	0.24565862	0.72	0.23149419
0.51	0.24687302	0.73	0.22952251
0.52	0.24813424	0.74	0.23628863
0.53	0.25134046	0.75	0.23242932
0.54	0.25356158	0.76	0.23949901
0.55	0.25289631	0.77	0.23643459
0.56	0.25293611	0.78	0.23301318
0.57	0.2528751	0.79	0.23362085
0.58	0.25286022	0.80	0.23362085
0.59	0.25305734	0.81	0.23166277
0.60	0.25206418	0.82	0.23059204
0.61	0.2519667		

TABLE 5

BOSWIJK (2001) UNIT ROOT TEST

	NordPool	PJM	Victoria
LR	5.514	254.6608	146.6158
p-value	0.2401	0.0000	0.0000

TABLE 6. NORDPOOL ESTIMATION RESULTS

The reported coefficients shown in each row of the table are ML estimates of the model:

$$y_t = \alpha_0 + \alpha_T \cdot trend + \alpha_d \cdot weekday + \alpha_1 \cdot \sin\left(\left(trend + \alpha_2\right) \cdot \frac{2\pi}{365}\right) + x_t$$

$$x_t = \phi\, x_{t-1} + \varepsilon_t$$

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}$$

$$\mathrm{s_t} = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 s_{t-1} + \gamma_3 z_t$$

where y_t is the log-price of the electricity in NordPool. $h_t = var(y_t \mid y_{t-1}, y_{t-2}, \ldots)$, $s_t = skewness(y_t \mid y_{t-1}, y_{t-2}, \ldots)$, $\eta_t = \varepsilon_t h_t^{-1/2}$, $\varepsilon_t \mid \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots$ follows a distribution based on a Gram-Charlier series expansion of the standard normal density, weekday is a dummy variable which takes the value 1 for trading days and 0 otherwise, and Z is a dummy variable related to supply shocks and defined in the main text. All models have been estimated using the Brendt-Hall-Hall-Hausman algorithm (quasi-maximum likelihood p-values in parenthesis). The mean equation has been estimated with the variables in first differences (unit root).

	Parameter	Estimated value (p-value)		
		GARCH(1,1) – N(0,1)	GARCHS(1,1) – GT	GARCHS(1,1) – GT EV
Mean equation	α_0	–	–	–
	α_T	0.0001 (0.9681)	0.0008 (0.5126)	-0.00005 (0.9779)
	α_d	0.0729 (0.0000)	0.0748 (0.0000)	0.0704 (0.0000)
	α_1	0.6282 (0.0000)	0.2114 (0.0025)	0.4014 (0.0143)
	α_2	873.0643 (0.0000)	825.3590 (0.0000)	844.6245 (0.0000)
	ϕ	–	–	–
Variance equation	β_0	0.0010 (0.0064)	0.0013 (0.0007)	0.0015 (0.0247)
	β_1	0.5241 (0.0000)	0.3403 (0.0000)	0.5247 (0.0000)
	β_2	0.4659 (0.0000)	0.4702 (0.0000)	0.3463 (0.0000)
Skewness equation	γ_0	–	0.0136 (0.7278)	0.0925 (0.2858)
	γ_1	–	0.0498 (0.0000)	0.0000 (0.0000)
	γ_2	–	-0.4165 (0.0000)	-0.9225 (0.0000)
	γ_3	–	–	0.5575 (0.0000)
Kurtosis	k	–	3.6810 (0.0000)	3.7690 (0.0000)
Log-Likelihood	–	6886.54	6964.21	7060.5423
SIC	–	6858.24	6919.74	7012.0368

TABLE 7. PJM ESTIMATION RESULTS

The reported coefficients shown in each row of the table are ML estimates of the model:

$$y_t = \alpha_0 + \alpha_T \cdot trend + \alpha_d \cdot weekday + \alpha_1 \cdot \sin\left(\left(trend + \alpha_2\right) \cdot \frac{2\pi}{365}\right) + x_t$$

$$x_t = \phi\, x_{t-1} + \varepsilon_t$$

$$h_t = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}$$

$$\mathrm{s}_t = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 s_{t-1} + \gamma_3 z_t$$

where y_t is the log-price of the electricity in PJM. $h_t = var(y_t \mid y_{t-1}, y_{t-2}, \ldots)$, $s_t = skewness(y_t \mid y_{t-1}, y_{t-2}, \ldots)$, $\eta_t = \varepsilon_t\, h_t^{-1/2}$, $\varepsilon_t \mid \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots$ follows a distribution based on a Gram-Charlier series expansion of the standard normal density, weekday is a dummy variable which takes the value 1 for trading days and 0 otherwise, and Z is a dummy variable related to supply shocks and defined in the main text.All models have been estimated using the Brendt-Hall-Hall-Hausman algorithm (quasi-maximum likelihood p-values in parenthesis).

Parameter		Estimated value (p-value)		
		GARCH(1,1) – N(0,1)	GARCHS(1,1) – GT	GARCHS(1,1) – GT EV
Mean equation	α_0	2.8061 (0.0000)	2.7945 (0.0000)	2.8188 (0.0000)
	α_T	0.0002 (0.0000)	0.0003 (0.0000)	0.00025 (0.0000)
	α_d	0.2519 (0.0000)	0.2445 (0.0000)	0.2472 (0.0000)
	α_1	-0.1111 (0.0153)	-0.1029 (0.0102)	-0.1033 (0.0065)
	α_2	109.8944 (0.0000)	114.6934 (0.0000)	116.3827 (0.0000)
	ϕ	0.6909) (0.0000)	0.7085 (0.0000)	0.6991 (0.0000)
Variance equation	β_0	0.0019 (0.1367)	0.0028 (0.1316)	0.0030 (0.0632)
	β_1	0.0827 (0.0000)	0.0986 (0.0000)	0.0404 (0.0000)
	β_2	0.8997 (0.0000)	0.8764 (0.0000)	0.9179 (0.0000)
Skewness equation	γ_0	–	0.0859 (0.0363)	0.1140 (0.0206)
	γ_1	–	0.0248 (0.0002)	0.0110 (0.0023)
	γ_2	–	-0.3036 (0.0000)	-0.4709 (0.0000)
	γ_3	–	–	0.4072 (0.0283)
Kurtosis	k	–	3.3102 (0.0000)	3.2800 (0.0000)
Log-Likelihood	–	1273.4604	1298.5588	1307.7483
SIC	–	1240.2324	1250.5629	1256.0604

TABLE 8. VICTORIA ESTIMATION RESULTS

The reported coefficients shown in each row of the table are ML estimates of the NAGARCH model:

$$y_t = \alpha_0 + \alpha_T \cdot trend + \alpha_d \cdot weekday + \alpha_1 \cdot \sin\left((trend + \alpha_2) \cdot \frac{2\pi}{365} \right) + x_t$$

$$x_t = \phi\, x_{t-1} + \varepsilon_t$$

$$h_t = \beta_0 + \beta_1 \left(\varepsilon_{t-1} + \beta_3 h_{t-1}^{1/2} \right)^2 + \beta_2 h_{t-1}$$

$$\mathrm{s}_t = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 s_{t-1} + \gamma_3 z_t$$

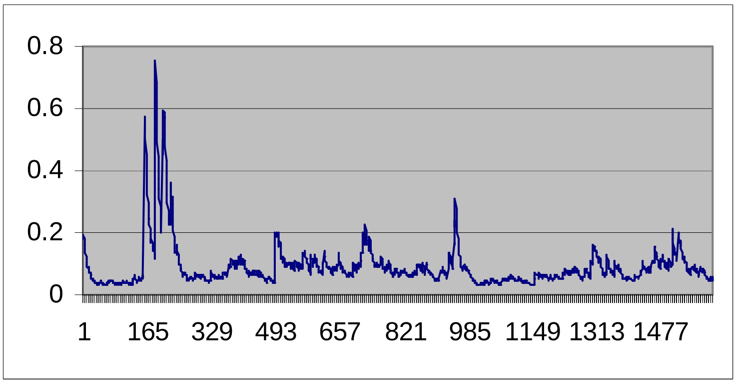
where y_t is the log-price of the electricity in Victoria. $h_t = var(y_t \mid y_{t-1}, y_{t-2}, \ldots)$, $s_t = skewness(y_t \mid y_{t-1}, y_{t-2}, \ldots)$, $\eta_t = \varepsilon_t h_t^{-1/2}$, $\varepsilon_t \mid \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots$ follows a distribution based on a Gram-Charlier series expansion of the standard normal density, weekday is a dummy variable which takes the value 1 for trading days and 0 otherwise, and Z is a dummy variable related to demand shocks and defined in the main text. All models have been estimated using the Brendt-Hall-Hall-Hausman algorithm (quasi-maximum likelihood p-values in parenthesis).

Parameter		Estimated value (p-value)		
		NAGARCH (1,1) – N(0,1)	NAGARCHS (1,1) – GT	NAGARCHS (1,1) – GT EV
Mean equation	α_0	3.3233 (0.0000)	3.2432 (0.0000)	3.2291 (0.0000)
	α_T	-0.00007 (0.0000)	-0.00003 (0.0000)	-0.00004 (0.0000)
	α_d	0.1726 (0.0000)	0.1530 (0.0000)	0.1533 (0.0000)
	α_1	-0.2065 (0.0062)	-0.2018 (0.0003)	-0.1909 (0.0002)
	α_2	961.7241 (0.0000)	948.4570 (0.0000)	958.4993 (0.0000)
	ϕ	0.8824 (0.0000)	0.8608 (0.0000)	0.8421 (0.0000)
Variance equation	β_0	0.0056 (0.0014)	0.0058 (0.0050)	0.0062 (0.0214)
	β_1	0.1147 (0.0069)	0.0703 (0.0122)	0.0714 (0.0625)
	β_2	0.5294 (0.0000)	0.5690 (0.0000)	0.6638 (0.0000)
	β_3	1.7779 (0.0000)	2.1603 (0.0000)	1.7402 (0.0030)
Skewness equation	γ_0	–	0.0998 (0.0069)	0.0357 (0.0663)
	γ_1	–	0.0059 (0.0727)	-0.0001 (0.7518)
	γ_2	–	0.5172 (0.0000)	0.8118 (0.0000)
	γ_3	–	–	1.0389 (0.0010)
Kurtosis	k	–	3.7615 (0.0000)	3.7745 (0.0000)
Log-Likelihood	–	1514.7434	1661.4719	1666.5361
SIC	–	1476.7942	1608.3430	1609.6123

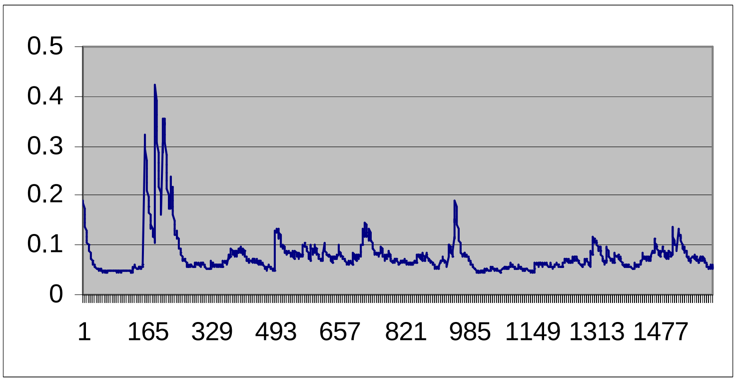
FIGURE 8. PJM

ESTIMATED CONDITIONAL MOMENTS

CONDITIONAL VARIANCE GARCH – N(0,1)



CONDITIONAL VARIANCE GARCHS – G&T WITH EXPLAN. VARIABLE



CONDITIONAL SKEWNESS G&T WITH EXPLAN. VARIABLE

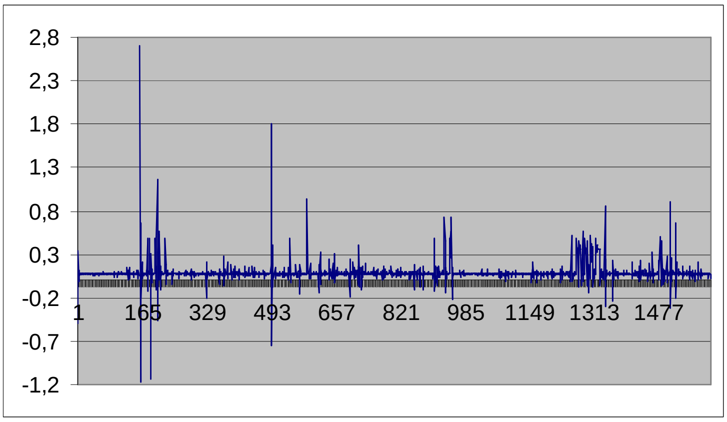


TABLE 9
DESCRIPTIVE STATISTICS FOR CONDITIONAL VARIANCES. PJM

The table shows the main descriptive statistics for the conditional variances obtained from the standard GARCH – N(0,01) model and from the GARCHS – Gallant & Tauchen with explanatory variable model.

STATISTIC	ht – GARCH	ht – GARCHS
Sample size	1610	1610
Mean	0.0907	0.0795
Median	0.0718	0.0687
Maximum	0.7550	0.4224
Minimum	0.0298	0.0442
Standard Deviation	0.0762	0.0431
Skewness	4.2065	3.8512
Kurtosis	22.1987	18.3394

TABLE 10
IN-SAMPLE PREDICTIVE POWER.PJM

The variable predicted is the squared forecast error (ε^2_t) and the predictors are the conditional variances (h_t) from, respectively, the standard GARCH and the GARCHS with explanatory variable models. The predictive ability of these models is compared through tree metrics: mean absolute error (MAE), root mean squared error (RMSE) and the coefficient R^2.

PERIOD	MODEL	MAE	RMSE	R^2
1999-2003	GARCH	0.0857	0.1621	0.6558
	GARCHS	0.0802	0.1601	0.6628
1999	GARCH	0.1084	0.2307	0.3293
	GARCHS	0.0979	0.2284	0.3444
2000	GARCH	0.0920	0.1570	0.7338
	GARCHS	0.0864	0.1554	0.7382
2001	GARCH	0.0745	0.1244	0.7670
	GARCHS	0.0711	0.1239	0.7675
2002	GARCH	0.0609	0.0908	0.8621
	GARCHS	0.0604	0.0909	0.8633
2003	GARCH	0.1027	0.1884	0.6186
	GARCHS	0.0926	0.1815	0.6351

TABLE 11
DESCRIPTIVE STATISTICS FOR MOVING AVERAGE AND CONDITIONAL SKEWNESS.PJM

The table shows the main descriptive statistics for 30-day simple moving average and conditional skewness obtained from the GARCHS – Gallant & Tauchen with explanatory variable.

STATISTIC	Mov.-Aver. S_t	Conditional S_t
Sample size	1582	1582
Mean	0.1402	0.0952
Median	0.0270	0.0777
Maximum	3.6786	3.7151
Minimum	-1.3504	-1.1694
Standard Deviation	0.7047	0.1330