Habit persistence and asset pricing: Evidence from Denmark

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Abstract

We use an iterated GMM approach to estimate and test the consumption based habit persistence model of Campbell and Cochrane (1999) on quarterly Danish stock and bond returns over the period 1985-2001. For comparative purposes we also estimate and test the standard time-separable model based on power utility. In addition, we compare the pricing errors of the different models using Hansen and Jagannathan's (1997) specification error measure. The empirical results, which are quite robust across different asset combinations and instrument sets, show that, *i*) neither the Campbell-Cochrane model nor the power utility model are statistically rejected by Hansen's J-test; *ii*) pricing errors are of the same magnitude for both models; *iii*) the risk-free rate is positive in the power utility model and mostly negative in the Campbell-Cochrane model; *iv*) in the Campbell-Cochrane model risk-aversion does not move countercyclically. These results suggest that - in contrast to what characterizes the US -, for Denmark the Campbell-Cochrane model does not perform better than the simple power utility model in explaining asset returns.

Keywords: Campbell and Cochrane model, GMM, Specification error.

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1 Introduction

Since Mehra and Prescott's (1985) seminal study, explaining the observed high equity premium within the consumption based asset pricing framework has occupied a large number of researchers in finance and macroeconomics. Despite an intense research effort, still no consensus has emerged as to why stocks have given such a high average return compared to bonds. At first sight the natural response to the equity premium puzzle is to dismiss the consumption based framework altogether. However, as emphasized by Cochrane (2005), within the rational equilibrium paradigm of finance, there is really no alternative to the consumption based model, since other models are not alternatives to - but special cases of - the consumption based model. Thus, despite its poor empirical performance, the consumption based framework continues to dominate studies of the equity premium on the aggregate stock market.

In a recent paper Chen and Ludvigson (2006) argue that within the equilibrium consumption based framework, habit formation models are the most promising and successful in describing aggregate stock market behaviour. The most prominent habit model is the one developed by Campbell and Cochrane (1999). In this model people slowly develop habits for a high or low consumption level, such that risk-aversion becomes time-varying and countercyclical. The model is able to explain the high US equity premium and a number of other stylized facts for the US stock market. A special feature of the model is that the average risk aversion over time is quite high, but the risk-free rate is low and stable. Thus, the model solves the equity premium puzzle by high risk aversion, but without facing a risk-free rate puzzle.

Campbell and Cochrane (1999) themselves, and most subsequent applications of their model, do not estimate and test the model econometrically. Instead they calibrate the model parameters to match the historical risk-free rate and Sharpe ratio, and then simulate a chosen set of moments which are informally compared to those based on actual historical data. Only a few papers engage in formal econometric estimation and testing of the model. Tallarini and Zhang (2005) use an Efficient Method of Moments technique to estimate and test the model on US data. They statistically reject the model and find that it has strongly counterfactual implications for the risk-free interest rate, although they also find that the model performs well in other dimensions. Fillat and Garduno (2005) and Garcia et al. (2005) use an iterated Generalized Method of Moments approach to estimate and test the model on US data. Fillat and Garduno strongly reject the model by Hansen's (1982) J-test. On the other hand Garcia et al. do not reject the model at conventional significance levels. However, Garcia et al. face the problem that their iterated GMM approach does not lead to convergence with positive values of the risk-aversion parameter.

To our knowledge, there have been no formal econometric studies of the Campbell-Cochrane model on data from other countries than the US. Our paper is a first attempt to fill this gap. We examine the Campbell-Cochrane model's ability to explain Danish stock and bond returns. Denmark is interesting because historically over a long period of time the average return on Danish stocks has not been nearly as high as in the US and most other countries, and at the same time the return on Danish bonds has been somewhat higher than in other countries, see e.g. Engsted and Tanggaard (1999), Engsted (2002), and Dimson et al. (2002). Thus, the Danish equity premium is not nearly as high as in most other countries, and might not even be regarded a puzzle.

On quarterly Danish data for the period 1985-2001 we estimate and test both the standard model based on power utility and the Campbell-Cochrane model based on habit formation. We basically follow the iterated GMM approach set out in Garcia et al. (2005). However, in contrast to Garcia et al., - who use Hansen's (1982) statistically optimal weighting matrix in the GMM iterations -, we follow Cochrane (2005)'s suggestion and use the identity matrix as weighting matrix. Thereby we attach equal weight to each of the assets in the application. The use of the identity matrix has the further advantage that in our application it leads to convergence with positive values of the risk-aversion parameter, in contrast to the case where we use Hansen's weighting matrix (Garcia et al. also face convergence problems, which might be due to their exclusive use of the Hansen weighting matrix). We also compute Hansen and Jagannathan's (1997) specification error measure based on the second moment matrix of returns as weighting matrix. This measure has an intuitively appealing percentage pricing error interpretation, and it allows for direct comparison of the magnitude of pricing errors across models.

Our main findings are as follows. First, neither the Campbell-Cochrane model nor the power utility model are statistically rejected by Hansen's J-test. Second, pricing errors are of the same magnitude for both models. Third, the quarterly real risk-free rate is estimated to be positive and around 2% in the power utility model, while it is mostly negative (and around -1%) in the Campbell-Cochrane model. Finally, in the Campbell-Cochrane model risk-aversion does not move countercyclically. These results are quite robust across different asset combinations and instrument sets, and suggest that - in contrast to what characterizes the US -, for Denmark the Campbell-Cochrane model does not perform better than the simple power utility model in explaining asset returns.

The rest of the paper is organized as follows. The next section briefly presents the power utility and habit persistence models. Section 3 explains the iterated GMM approach used to estimate the models. Section 4 presents the empirical results based on Danish data. Finally, section 5 offers some concluding remarks.

2 The consumption based models

In this section we start by describing the standard power utility version of the consumption based model. Since this version of the model is well-known and familiar to most readers, the description will be very brief. Then we give a more detailed description of the Campbell-Cochrane habit based model.

2.1 The power utility model

Standard asset pricing theory implies that the price of an asset at time t, P_t, is determined by the expected future asset payoff, Y_{t+1}, multiplied by the stochastic discount factor, M_{t+1}: $P_t = E_t(M_{t+1}Y_{t+1})$. The payoff is given as prices plus dividends, $Y_{t+1} = P_{t+1} + D_{t+1}$, and the stochastic discount factor depends on the underlying asset pricing model. In consumption based models M_{t+1} is the intertemporal marginal rate of substitution in consumption. With power utility (constant relative risk aversion), $U(C_t) = \frac{C_t^{1-\gamma}-1}{1-\gamma}$, where $\gamma \geq 0$ is the degree of relative risk aversion, the stochastic discount factor becomes $M_{t+1} = \delta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$, where $\delta = (1+t_p)^{-1}$ and t_p is the rate of time-preference. Defining gross return as $R_{t+1} = \frac{P_{t+1}+D_{t+1}}{P_t}$, the asset pricing relationship can be stated as:

$$0 = E_t \left[\delta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} - 1 \right]. \tag{1}$$

Equation (1) captures the basic idea that risk-adjusted equilibrium returns are unpredictable. In the consumption based model, risk-adjustment takes place by multiplying the raw return with the intertemporal marginal rate of substitution in consumption. Risk-averse consumers want to smooth consumption over time, and for that purpose they use (dis)investments in the asset, thereby making a direct connection between consumption growth and the asset return. The correlation between consumption growth and returns then becomes crucial for the equilibrium expected return. From (1) expected returns are given as:

$$E_t\left[R_{t+1}\right] = \frac{1 - \text{Cov}_t\left[R_{t+1}, \delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]}{E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]} \tag{2}$$

The higher the correlation between consumption growth and returns (the lower the correlation between the stochastic discount factor and returns), the higher will be expected equilibrium return (*ceteris paribus*), because the higher the correlation, the less able the asset will be in helping to smooth consumption over time, which means that the asset will be considered riskier and thereby demand a higher return.

Equation (1) lends itself directly to empirical estimation and testing within the GMM framework, c.f. section 3. Empirically the consumption based power utility model has run into trouble because consumption growth and stock returns are not sufficiently positively correlated to explain the historically observed high return on common stocks, unless the degree of risk aversion γ is extremely high. The basic problem is that unless γ is very high, the variability of the intertemporal marginal rate of substitution cannot match the variability of stock returns. Perhaps people *are* highly risk-averse, but then the power utility model faces another problem, namely that with a high γ, the risk-free rate implied by the model becomes implausibly high. For the risk-free rate the covariance with the stochastic discount factor is zero, thus from (2):

$$R_{f,t+1} = \frac{1}{E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]} \tag{3}$$

Thus, within the standard power utility framework, the equity premium puzzle cannot be solved without running into a risk-free rate puzzle. This has led to the development of alternative utility models with a higher volatility of the stochastic discount factor, and with plausible implications for the risk-free rate. The habit persistence model described in the next subsection is one such model.

2.2 The Campbell-Cochrane model

Habit formation models differ from the standard power utility model by letting the utility function be time-nonseparable in the sense that the utility at time t depends not only on consumption at time t, but also on previous periods consumption. The basic idea is that people get used to a certain standard of living and thereby the utility of some consumption level at time t will be higher (lower) if previous periods consumption was low (high) than if previous periods consumption was high (low).

Habit formation can be modelled in a number of different ways. In the Campbell-Cochrane model utility is specified as

$$U(C_t, X_t) = \frac{(C_t - X_t)^{1-\gamma} - 1}{1-\gamma}, \quad C_t > X_t \tag{4}$$

where X_t is an external habit level that depends on previous periods consumption. Define the *surplus consumption ratio* as $S_t = \frac{C_t - X_t}{C_t}$. Then the stochastic discount factor can be stated as $M_{t+1} = \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}$ and the pricing equation becomes

$$0 = E_t \left[\delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} - 1 \right] \tag{5}$$

Compared to the standard power utility model in (1), the Campbell-Cochrane model implies a stochastic discount factor that not only depends on consumption growth but also on growth in the consumption surplus ratio. In this model relative risk-aversion is no longer measured by γ but as $\frac{\gamma}{S_t}$. This shows that relative risk-aversion is time-varying and counter-cyclical: when consumption is high relative to habit, relative risk-aversion is low and expected returns are low. By contrast, when consumption is low and close to habit, relative risk-aversion is high leading to high expected returns. Basically the model explains time-varying and counter-cyclical ex ante returns (which implies pro-cyclical stock prices) as a result of time-varying and counter-cyclical risk-aversion of people. From (5) expected returns are given as:

$$E_t \left[R_{t+1} \right] = \frac{1 - \mathrm{Cov}_t \left[R_{t+1}, \delta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right]}{E_t \left[\left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right]} \tag{6}$$

A crucial aspect in operalizing the model is the modelling of the risk-free rate. Campbell and Cochrane specify the model in such a way that the risk-free rate is constant and low by construction. First, assume that consumption is lognormally distributed such that consumption growth is normally distributed and *iid*:

$$\Delta c_{t+1} = g + v_{t+1}, \quad v_{t+1} \sim niid(0, \sigma_v^2) \tag{7}$$

where $c_t \equiv \log(C_t)$. g is the mean consumption growth rate. Next, specify the log surplus consumption ratio $s_t = \log(S_t)$ as a stationary first-order autoregressive process

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)v_{t+1} \tag{8}$$

where $0 < \phi < 1$, $\bar{s}$ is the steady state level of s_t, and $\lambda(s_t)$ is the sensitivity function to be specified below. Note that shocks to consumption growth are modelled to have a direct impact on the surplus consumption level, and for ϕ close to one, habit responds slowly to these shocks.

The sensitivity function $\lambda(s_t)$ is specified as follows:

$$\lambda(s_t) = \begin{Bmatrix} \frac{1}{\bar{S}}\sqrt{1-2(s_t-\bar{s})} - 1 & \text{if } s_t \leq s_{\max} \\ 0 & \text{else} \end{Bmatrix} \tag{9}$$

where

$$\bar{S} = \sigma_v \sqrt{\frac{\gamma}{1-\phi}}, \qquad s_{\max} \equiv \bar{s} + \frac{1}{2}(1-\bar{S}^2), \qquad \bar{s} = \log(\bar{S})$$

Specifying $\lambda(s_t)$ in this way implies the following equation for the log risk-free rate:

$$r_{f,t+1} = -\log(\delta) + \gamma g - \frac{\gamma^2\sigma_v^2}{2}\left(\frac{1}{\bar{S}}\right)^2 \tag{10}$$

As seen, no time-dependent variables appear in (10), thus the risk-free rate is constant over time. Economically this property of the model is obtained by letting the effects of intertemporal substitution and precautionary saving - which have opposite effects on the risk-free rate - cancel each other out, see Campbell and Cochrane (1999) for details.

Campbell and Cochrane calibrate their model with parameters chosen to match post war US data: mean real consumption growth rate (g), mean real risk-free rate (r_f), volatility (σ_v), etc. Then, based on the calibrated model, simulated time-series for returns, price-dividend ratios, etc., are generated and their properties are compared to the properties of the actually observed post war data. In the present paper we instead follow Garcia et al. (2005) and estimate the model parameters in a GMM framework. The next section describes how.

3 GMM estimation of the models

The GMM technique developed by Hansen (1982) estimates the model parameters based on the orthogonality conditions implied by the model. Let the asset pricing equation be $0 = E_t\left[M_{t+1}(\theta)R_{t+1} - 1\right]$, where M_{t+1} is the stochastic discount factor, R_{t+1} is a vector of asset returns, and the vector θ contains the model parameters. In the present context this equation corresponds to either (1) or (5) with $\theta = (\delta \quad \gamma)'$. Define a vector of instrumental variables, Z_t, observable at time t. Then the asset pricing equation implies the following orthogonality conditions $E\left[(M_{t+1}(\theta)R_{t+1} - 1) \otimes Z_t\right] = 0$. GMM estimates θ by making the sample counterpart to these orthogonality conditions as close to zero as possible, by minimizing a quadratic form of the sample orthogonality conditions based on a chosen weighting matrix. Define $g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T}(M_{t+1}(\theta)R_{t+1} - 1) \otimes Z_t$ as the sample orthogonality conditions based on T observations. Then the parameter vector θ is estimated by minimizing

$$g_T(\theta)' W g_T(\theta) \tag{11}$$

where W is the weighting matrix. The statistically optimal (most efficient) weighting matrix is obtained as the inverse of the covariance matrix of the sample orthogonality conditions. Other weighting matrices can be chosen, however, and often a fixed and model-independent weighting matrix (the identity matrix, for example) is used in order to make it possible to compare the magnitude of estimated pricing errors across different models. Such a comparison cannot be done if the statistically optimal weighting matrix is used because this matrix is model-dependent.

GMM estimation of the standard power utility model (1) is straightforward. However, estimation of the Campbell-Cochrane model, equation (5), is complicated by the fact that the surplus consumption ratio, S_t, is not observable in the same way as returns, R_t, and consumption, C_t, are directly observable. Garcia et al. (2005) suggest to generate a process for s_t by initially estimating the parameters ϕ and σ_v, and setting γ to some initial value, which then gives $\overline{s}$, from which s_t can be constructed using (8) and a starting value for s_t at $t = 0$. Garcia et al. set $s_o = \overline{s}$. Having obtained a series for the surplus consumption ratio, GMM can be applied directly. Since the surplus consumption ratio depends on γ, however, the resulting GMM estimate of γ may not correspond to the value initially imposed in generating s_t. Therefore, Garcia et al. iterate over γ by estimating the model in each iteration using GMM with the statistically optimal weighting matrix. Unfortunately, this procedure does not lead to convergence with a positive value of γ in their application. Instead they do a grid search that implies an estimated value of γ close to the initiallly picked value.

Our procedure will differ from Garcia et al.'s in the following way: we will iterate over γ in order to minimize the objective function (11), and by using a fixed and prespecified weighting matrix across all GMM estimations, we attach equal weight to each asset.[1] The details of our estimation procedure is as follows:

Step 1: Following Campbell and Cochrane (1999) and Garcia et al. (2005) we estimate ϕ as the first-order autocorrelation parameter for the log price-dividend ratio:

$$p_t - d_t = \alpha + \phi(p_{t-1} - d_{t-1}) + \varepsilon_t \tag{12}$$

This is feasible since in the Campbell-Cochrane model the surplus consumption ratio is the only state variable, whereby the log price-dividend ratio, $p_t - d_t$ will inherit its dynamic properties from the log surplus consumption ratio, s_t.

Step 2: g and σ_v are estimated from (7), and the implied process for v_t is obtained.

Step 3: An initial value of γ is chosen to obtain a process for S_t. Given ϕ, g, γ, σ_v, and v_t, the parameters $\overline{S}$ and $\overline{s}$ can be determined, and the s_t process is obtained from (8). We follow Garcia et al. (2005) at set $s_t = \overline{s}$ at $t = 0$. Given s_t, S_t is obtained as $\exp(s_t)$.

Step 4: Given the observed time-series for asset returns and consumption growth, and given the time-series for the surplus consumption ratio generated in step 3, equation (5) can be estimated by minimizing (11) based on a chosen weighting matrix. We follow Cochrane (2005) and use the identity matrix. This gives GMM estimates of δ and γ. We repeat this procedure until convergence of γ and δ.

Since the chosen weighting matrix in step 4 is not the efficient Hansen (1982) matrix but the identity matrix I, the formula for the covariance matrix of the parameter vector is (c.f. Cochrane (2005), chpt. 11):

$$Var(\widehat{\theta}) = \frac{1}{T}(d'Id)^{-1}d'ISId(d'Id)^{-1} \tag{13}$$

where $d' = \partial g_T(\theta)/\partial\theta$, and the spectral density matrix $S = \sum_{j=-\infty}^{\infty} E[g_T(\theta)g_{T-j}(\theta)']$ is computed with the usual Newey and West (1987) estimator with a lag truncation. Similarly, the J-test of overidentifying restrictions is computed based on the general formula (c.f. Cochrane (2005) chpt. 11):

$$J_T = Tg_T(\widehat{\theta})' \left[(I - d(d'Id)^{-1}d'I)S(I - Id(d'Id)^{-1}d')\right]^{-1} g_T(\widehat{\theta}) \tag{14}$$

[1] We use a GMM programme written in MatLab. The programme is available upon request.

J_T has an asymptotic χ^2 distribution with degrees of freedom equal to the number of overidentifying restrictions. (14) involves the covariance matrix $Var(g_T(\widehat{\theta})) = \frac{1}{T}(I - d(d'Id)^{-1}d'I)S(I - Id(d'Id)^{-1}d')$, which is singular, so it is inverted using the Moore-Penrose pseudo-inversion.

In addition to formally testing the model using the J-test, we also compute the Hansen and Jagannathan (1997) misspecification measure, HJ, as

$$HJ = \left[E(M_{t+1}(\theta)R_{t+1} - 1)'(E(R_{t+1}R'_{t+1}))^{-1}E(M_{t+1}(\theta)R_{t+1} - 1)\right]^{\frac{1}{2}} \tag{15}$$

HJ measures the minimum distance between the candidate stochastic discount factor M_{t+1} and the set of admissible stochastic discount factors. HJ can be interpreted as the maximum pricing error per unit payoff norm. Thus, it has an intuitively appealing percentage pricing error interpretation. It is a measure of the magnitude of pricing errors that gives a useful *economic* measure of fit, in contrast to the statistical measure of fit given by Hansen's J-test. In addition, since the HJ measure is based on a model-independent weighting matrix, it can be used to compare pricing errors across models. The HJ measure is computed at the GMM estimates of δ and γ. We compute the asymptotic standard error of $\widehat{HJ}$ using the Hansen et al. (1995) procedure.[2]

4 Empirical results

We estimate the models on quarterly data, spanning the period from 1985:2 to 2001:4. Consumption is measured as per capita expenditures on non-durables and services. For asset returns, we use the return on the Danish stock market index constructed by Belter et al. (2005), which is a dividend-adjusted version of the official Danish KFX index, the return on long-term government bonds (7-10 years), and the return on intermediate-term bonds (1-3 years). Excess returns are computed by subtracting the short-term (3 month) interest rate from these returns. As instrument variables we use lags of these returns, the term premium defined as the spread between the long-term and intermediate term bond yields, the dividend-price ratio (where dividends are accumulated over the past year), and consumption growth. We use the consumption deflator to convert nominal returns and nominal consumption into real returns and real consumption.

Table 1 reports summary statistics for the real stock and bond returns

[2] The asymptotic distribution of $\widehat{HJ}$ is degenerate when $HJ = 0$. Thus, the asymptotic standard error of $\widehat{HJ}$ cannot be used to test whether $HJ = 0$. Instead, the standard error gives a measure of the precision of the estimate of HJ.

and the instruments. As seen, the average quarterly arithmetic real stock return, R_S, over the 1985:2 - 2001:4 period is 2.08%, while the long-term, R_{LB}, and intermediate-term, R_{IB}, real bond returns are 1.94% and 1.34%, respectively. The corresponding standard deviations are 9.75%, 3.24%, and 1.32%. Thus, stocks give higher average returns than bonds, but are also more volatile. The approximate average *yearly* real returns are, for stocks, long-term bonds, and intermediate-term bonds (with standard deviations in parentheses): 8.32% (19.50%), 7.76% (6.48%), and 5.36% (2.64%). The average quarterly short-term real interest rate, R_{SB}, is 1.25% with standard deviation 0.89%. The average ex post yearly equity premium, i.e. the yearly stock return in excess of the 3-month interest rate, is 4 x (2.64 - 1.25) = 3.32%, with a standard deviation of 19.64%. Thus, the Danish equity premium is much lower than in most other countries, and in the US in particular, but it is just as volatile as in other countries (in fact, the Danish equity premium is not statistically significant: the standard error of the average premium is 2.46%). This is similar to what Engsted and Tanggaard (1999), Engsted (2002), and Dimson et al. (2002) have found using data over a longer period with annual data. However, Table 1 also reports summary statistics for a sample period that begins in 1986:4. The reason is that during the short period from 1985:2 to 1986:3, stock returns in Denmark were strongly negative which has a significant impact on the computed average. As seen, over this slightly smaller sample period the average yearly equity premium is 4 x (2.64% - 1.24%) = 5.60%, which is somewhat higher than the average of 3.32% for the full sample. Table 1 also shows that quarterly real stock returns are slightly positively autocorrelated, though not statistically significantly so. Bond returns, on the other hand, show stronger positive autocorrelation.

In a qualitative sense, the consumption based model implies that the stochastic discount factor should be negatively correlated with stock returns in order to generate a positive equity-premium. Table 2 reports correlations between M_{t+1} and real stock returns $R_{S,t+1}$, where M_{t+1} is either equal to $\delta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ (i.e. the standard power utility model, CRRA), or $\delta\left(\frac{S_{t+1}}{S_t}\frac{C_{t+1}}{C_t}\right)^{-\gamma}$ (i.e. the Campbell-Cochrane model), and where S_{t+1} has been constructed as in step 1-3. γ takes values from 1 to 20 in the power utility case, and from 0.25 to 1.00 in the Campbell-Cochrane case corresponding to values of relative risk-aversion γ/S_t ranging from 15-30, which is consistent with the GMM estimates reported below. For both models - and across the different values for risk-aversion - stock returns are negatively correlated with the stochastic discount factor. However, all correlations are close to 0, so although in a qualitative sense this is consistent with the basic consumption-based framework, the evidence does not strongly support it and certainly does not allow us to discriminate between the standard power utility model and the Campbell-Cochrane model.

Now we turn to formal estimation of the model parameters and statistical tests of the models. We first estimate the quarterly consumption growth rate, g, the innovations variance σ_v^2, and the persistence parameter, ϕ, from equations (7) and (12). This results in the following estimates, with standard errors in parentheses: $\widehat{\phi} = 0.834$ (0.085), $\widehat{g} = 0.0022$ (0.0010), and $\sigma_v = 0.0112$. Thus, the price-dividend ratio and, hence, the surplus consumption ratio, are stationary but highly persistent. The average yearly real per capita consumption growth rate is a little less than 1%, and consumption growth has a yearly standard deviation of just above 2%.

Next, based on these estimates and an initial value of γ equal to one, we construct an initial time-series for the surplus consumption ratio, and then we estimate the Euler equation (5) using GMM, c.f. steps 3 and 4 in section 3. For the standard power utility model, the Euler equation to be estimated is (1). We report results based on various combinations of returns and with different instrument sets (see the notes to Table 3 for the precise definitions of the instrument sets).

Table 3 shows results where the vector of returns include real returns on stocks and long-term bonds. For the standard power utility model the quarterly subjective discount factor δ is precisely estimated at around 0.99, which seems reasonable. The estimated risk-aversion parameter γ has the correct sign and ranges from 5 to 17, depending on which instrument set is used, but this parameter is very imprecisely estimated as seen by the large standard errors. This is a common finding in the literature. The J-test does not in any case reject the power utility model at conventional significance levels, and the HJ measure indicates pricing errors of around 6%. The quarterly real risk-free rate implied by these estimates is around 2.2%. The estimates in the lower part of Table 3 do not indicate that the Campbell-Cochrane model performs better than the simple power utility model. As with the power utility model, the Campbell-Cochrane model is not statistically rejected and implies quite low percentage pricing errors. However, the estimate of δ of around 0.95 (implying a quarterly rate of time-preference of 5%) seems unreasonably low. The estimate of γ ranges from 0.3 to 0.8, implying an average degree of risk-aversion in the range from 16 to 25. For two of the instrument sets, the real risk-free rate is estimated to be positive, but in most cases r_f is negative, although not nearly as low as what Tallarini and Zhang (2005) find for the US. Table 4 reports results where the only difference is that now stock returns are measured in excess of the short-term interest rate. We include this case in order to see whether the models better fit the equity premium rather than the stock return itself. As seen, this does not seem to be the case. HJ pricing errors increase to 10-11%, γ estimates increase for both models (and remain strongly insignificant), and the Campbell-Cochrane model now gives negative real risk-free rates for every instrument set.

In Tables 5 and 6 we report results where we in addition to stocks and long-term bonds also include intermediate-term bonds (1-3 years maturity). The models are still not rejected statistically, but HJ pricing errors increase to around 27% for both the power utility model and the Campbell-Cochrane model. This is an illustration of the fact emphasized by Hansen and Jagannathan (1997), Cochrane (2005), and others, that a statistical non-rejection by the J-test does not necessarily imply low pricing errors. The estimates of γ in most cases remain very high and statistically insignificant, and the Campbell-Cochrane model again produces a quite high rate of time-preference and negative risk-free rates. By contrast, the power utility model produces reasonable estimates of δ and r_f.

Figure 1 shows a graph of relative risk-aversion γ/S_t in the Campbell-Cochrane model, produced from the estimates based on instrument set 1 in Table 3 (graphs based on the other estimates in the tables look very similar). Relative risk-aversion should move counter-cyclically, i.e. γ/S_t should be high during the cyclical downturn in 1987-1993, and γ/S_t should be low during the cyclical upswing in 1994-2000. These general business cycle trends are not convincingly reflected in Figure 1, since we don't observe any difference in the level of relative risk aversion in the downturn period of 1987-1993 compared to the upswing period of 1994-2000. This again indicates that the Campbell-Cochrane model does not explain the Danish data very well.

5 Concluding remarks

The habit persistence model developed by Campbell and Cochrane (1999) has become one of the most prominent consumption based asset pricing models, in particular with respect to aggregate stock market returns. It explains procyclical stock prices, time-varying and countercyclical expected returns, and high and time-varying equity premia as a result of high but time-varying and countercyclical risk aversion, and it does this while keeping the risk-free rate low and stable.

When the Campbell-Cochrane model is calibrated to actual historical data from the US, the model is found to match a number of key aspects of the data. However, only a few attempts have been made to formally estimate and test the model, and only on US data. These formal estimations and tests generally have led to statistical rejection of the model. Thus, while there is evidence that the Campbell-Cochrane model has empirical content on US data, and that it clearly outperforms the standard power utility model, it is also clear that the model does involve significant pricing errors.[3]

In this paper we have performed a formal econometric estimation and testing of both the standard power utility model and the Campbell-Cochrane model using Danish stock and bond market returns and aggregate consumption. The results are quite different from the US results: neither model is statistically rejected at conventional significance levels; however, Hansen-Jagannathan pricing errors are economically important and of equal magnitude for both models. Thus, the Campbell-Cochrane model does not seem to perform better than the power utility model. In addition, in contrast to the power utility model, the Campbell-Cochrane model produces implausible values for the rate of time-preference and the risk-free rate. Finally, the model does not involve countercyclical risk aversion.

These results lead to the conclusion that for Denmark the Campbell-Cochrane model does not seem to explain asset returns any better than the standard power utility model. We are awaiting research that compares these two models using asset returns from other countries than the US and Denmark.

6 References

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[3]As noted by Campbell and Cochrane (1999) themselfes (p.236), the worst performance of the model occurs during the end of their sample period, i.e. the first half of the 1990s. It would be interesting to see the model calibrated on more recent data that includes the stock market boom and bust period since 1995.

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7 Tables and figures

	Mean (std.dev)	Autocorr. (std.dev)
1985:2 - 2001:4		
R_S	1.0208 (0.0975)	0.161 (0.123)
R_{LB}	1.0194 (0.0324)	0.399 (0.123)
R_{IB}	1.0134 (0.0132)	0.240 (0.123)
R_{SB}	1.0125 (0.0089)	0.577 (0.123)
C/C_{-1}	1.0022 (0.0111)	-0.255 (0.123)
$TERM$	0.0063 (0.0088)	0.912 (0.123)
1986:4 - 2001:4		
R_S	1.0264 (0.0960	0.113 (0.128)
R_{LB}	1.0189 (0.0301)	0.367 (0.128)
R_{IB}	1.0130 (0.0118)	0.297 (0.128)
R_{SB}	1.0124 (0.0086)	0.711 (0.128)
C/C_{-1}	1.0022 (0.0112)	-0.230 (0.128)
D/P	0.0187 (0.0086)	0.846 (0.128)
$TERM$	0.0064 (0.0091)	0.919 (0.128)

Notes: R_S, R_{LB}, R_{IB}, and R_{SB} are real quarterly gross returns on stocks, long-term bonds, intermediate-term bonds, and short-term bonds. C/C_{-1} is the real per capita gross consumption growth rate. $TERM$ is the spread between the yields on long-term bonds and intermediate-term bonds. D/P is the dividend-price ratio.

Table 1: Summary statistics for asset returns and instruments

	$\gamma = 1$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
Corr(R_S, M^{CRRA})	-0.12	-0.11	-0.11	-0.10

	$\gamma = 0.25$	$\gamma = 0.50$	$\gamma = 0.75$	$\gamma = 1.00$
Corr(R_S, M^{CC})	-0.15	-0.08	-0.17	-0.14

Notes: M^{CRRA} and M^{CC} are the stochastic discount factors in the power (CRRA) utility model and Campbell-Cochrane model, respectively.

Table 2: Correlations between stock returns and the stochastic discount factor

Instrument set	1	2	3	4	5	6
Power utility						
δ	0.995	0.988	0.996	0.990	0.995	0.996
	(0.020)	(0.016)	(0.021)	(0.018)	(0.020)	(0.020)
γ	13.219	5.430	17.181	7.181	13.227	15.093
	(26.602)	(10.314)	(32.999)	(13.280)	(26.568)	(30.566)
J-test	1.084	6.393	2.321	7.605	3.055	3.364
	(0.897)	(0.603)	(0.888)	(0.815)	(0.802)	(0.910)
HJ	0.062	0.071	0.057	0.069	0.062	0.059
	(0.140)	(0.142)	(0.139)	(0.141)	(0.140)	(0.140)
r_f	2.32%	2.22%	2.35%	2.23%	2.32%	2.28%
Campbell-Cochrane						
δ	0.952	0.964	0.943	0.963	0.952	0.952
	(0.098)	(0.060)	(0.130)	(0.065)	(0.096)	(0.101)
γ	0.664	0.345	0.802	0.368	0.658	0.684
	(1.346)	(0.761)	(1.638)	(0.824)	(1.321)	(1.396)
J-test	1.064	5.535	1.977	6.296	3.722	4.031
	(0.900)	(0.700)	(0.922)	(0.900)	(0.714)	(0.854)
HJ	0.047	0.064	0.035	0.063	0.047	0.045
	(0.137)	(0.138)	(0.136)	(0.137)	(0.137)	(0.137)
r_f	-0.46%	0.91%	-0.62%	0.83%	-0.45%	-0.57%
γ/S	22	16	25	17	22	23

Notes: The Table reports estimates of δ and γ in the power utility and Campbell-Cochrane models using the iterated GMM approach described in section 3, with asymptotic standard errors in parentheses. J-test is Hansen's test of overidentifying restrictions, computed as in (14), with asymptotic p-value in parenthesis. HJ is the Hansen-Jagannathan specification error measure, computed as in (15), with asymptotic standard error in parenthesis. r_f is the real risk-free rate, computed from (3) and (10). S in γ/S is the average value of S over the sample. The instrument sets are:

1: Constant, R_S, D/P.
2: Constant, R_S, D/P, and their lags.
3: Constant, R_S, R_{LB}, D/P.
4. Constant, R_S, R_{LB}, D/P, and their lags.
5. Constant, R_S, D/P, $TERM$.
6. Constant, R_S, D/P, $TERM$, C/C_{-1}.

Table 3: GMM estimation of the power utility and Campbell-Cochrane models, with returns on stocks and long-term bonds.

Instrument set	1	2	3	4	5	6
Power utility						
δ	0.985	1.001	0.979	0.999	0.985	0.982
	(0.072)	(0.022)	(0.095)	(0.027)	(0.072)	(0.086)
γ	31.484	19.227	34.616	22.402	31.474	33.495
	(37.749)	(19.845)	(42.243)	(22.854)	(37.688)	(41.350)
J-test	1.964	6.945	3.954	8.317	3.136	5.654
	(0.742)	(0.543)	(0.683)	(0.760)	(0.792)	(0.686)
HJ	0.113	0.122	0.111	0.119	0.113	0.112
	(0.150)	(0.150)	(0.151)	(0.150)	(0.150)	(0.151)
r_f	2.18%	1.80%	2.22%	1.87%	2.18%	2.10%
Campbell-Cochrane						
δ	0.926	0.941	0.923	0.937	0.926	0.925
	(0.125)	(0.096)	(0.132)	(0.106)	(0.125)	(0.127)
γ	1.126	0.915	1.152	0.973	1.124	1.142
	(1.456)	(1.213)	(1.493)	(1.299)	(1.447)	(1.468)
J-test	1.817	5.624	3.933	7.522	2.955	4.643
	(0.769)	(0.689)	(0.686)	(0.821)	(0.815)	(0.795)
HJ	0.098	0.103	0.098	0.102	0.098	0.098
	(0.146)	(0.145)	(0.146)	(0.145)	(0.146)	(0.146)
r_f	-1.46%	-1.33%	-1.36%	-1.38%	-1.46%	-1.50%
γ/S	31	27	32	29	31	32

See the notes to Table 3.

Table 4: GMM estimation of the power utility and Campbell-Cochrane models, with excess stock returns and long-term bond returns.

Instrument set	1	2	3	4	5	6
Power utility						
δ	0.999	0.993	0.998	0.995	0.999	0.999
	(0.019)	(0.015)	(0.025)	(0.017)	(0.019)	(0.021)
γ	15.866	6.448	20.086	8.468	15.866	17.979
	(23.848)	(9.254)	(29.556)	(11.782)	(23.816)	(27.379)
J-test	7.090	12.188	11.036	16.519	8.284	16.167
	(0.420)	(0.512)	(0.355)	(0.622)	(0.601)	(0.240)
HJ	0.276	0.276	0.277	0.276	0.276	0.276
	(0.181)	(0.179)	(0.182)	(0.179)	(0.181)	(0.182)
r_f	2.04%	1.90%	2.07%	1.91%	2.04%	2.00%
Campbell-Cochrane						
δ	0.946	0.964	0.942	0.959	0.946	0.946
	(0.099)	(0.052)	(0.113)	(0.065)	(0.098)	(0.099)
γ	0.796	0.401	0.868	0.565	0.792	0.802
	(1.267)	(0.643)	(1.405)	(0.887)	(1.251)	(1.278)
J-test	6.524	8.014	13.898	17.940	8.576	12.848
	(0.480)	(0.843)	(0.178)	(0.527)	(0.573)	(0.460)
HJ	0.275	0.266	0.279	0.266	0.275	0.276
	(0.186)	(0.184)	(0.187)	(0.184)	(0.186)	(0.186)
r_f	-0.92%	0.47%	-1.06%	-0.34%	-0.92%	-0.97%
γ/S	25	17	27	21	25	25

See the notes to Table 3.

Table 5: GMM estimation of the power utility and Campbell-Cochrane models, with returns on stocks, long-term bonds and intermediate-term bonds.

Instrument set	1	2	3	4	5	6
Power utility						
δ	0.997	1.003	0.993	1.003	0.997	0.995
	(0.038)	(0.019)	(0.055)	(0.020)	(0.038)	(0.047)
γ	25.221	13.748	28.573	16.519	25.211	27.212
	(30.121)	(14.713)	(34.910)	(17.349)	(30.077)	(33.459)
J-test	6.840	11.567	10.977	16.228	7.881	17.104
	(0.446)	(0.564)	(0.359)	(0.642)	(0.641)	(0.195)
HJ	0.277	0.274	0.279	0.274	0.277	0.278
	(0.184)	(0.181)	(0.185)	(0.181)	(0.184)	(0.185)
r_f	1.82%	1.53%	1.86%	1.58%	1.82%	1.77%
Campbell-Cochrane						
δ	0.942	0.947	0.939	0.947	0.942	0.941
	(0.101)	(0.091)	(0.114)	(0.091)	(0.100)	(0.102)
γ	0.913	0.832	0.964	0.835	0.911	0.932
	(1.253)	(1.169)	(1.384)	(1.179)	(1.244)	(1.273)
J-test	6.948	11.647	15.008	21.288	8.647	13.283
	(0.434)	(0.557)	(0.132)	(0.321)	(0.566)	(0.426)
HJ	0.279	0.276	0.281	0.276	0.279	0.280
	(0.187)	(0.187)	(0.187)	(0.187)	(0.187)	(0.187)
r_f	-1.40%	-1.29%	-1.47%	-1.37%	-1.40%	-1.51%
γ/S	27	26	28	26	27	28

See the notes to Table 3.

Table 6: GMM estimation of the power utility and Campbell-Cochrane models, with excess stock returns and returns on long-term bonds and intermediate-term bonds.

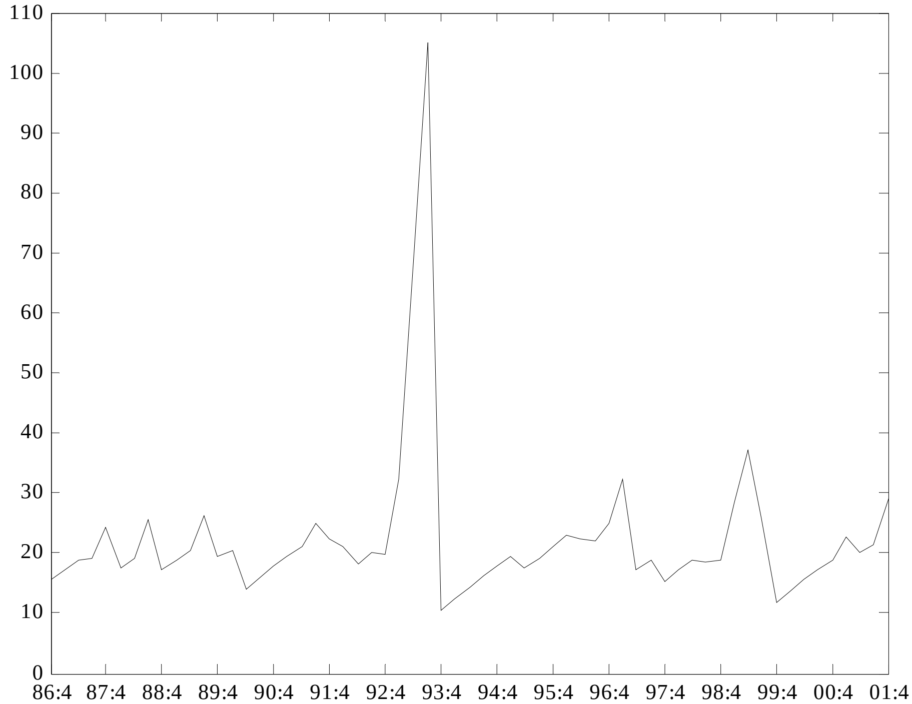

Figure 1:

Figure 1: Relative risk-aversion γ/S_t in the Campbell-Cochrane model