Jumps and Recovery Rates Inferred from Corporate CDS Premia*

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Abstract

We provide a thorough investigation of the US corporate credit default swap market. We take a full parametric approach with an observable, multi-factor, affine reduced-form model which accomodates jumps in both riskless and defaultable discount rates. Our empirical results reveal that a multi-factor formulation with a multivariate jump size distribution is important for fitting both the time-series and in particular the cross-section of CDS premia. Incorporating jumps also significantly improves the model's capability to capture empirical stylized facts. Market-implied loss given default is well identified. We find that rating and industry affiliation have significant explanatory power on the cross-section of CDS-implied loss given default rates.

Keywords: credit default swaps, credit risk, recovery risk, stochastic intensity, jump-diffusion, Markov chain Monte Carlo estimation

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1 Introduction

There are two important questions concerning the pricing of credit default swaps (CDS) which have not received enough attention in the academic literature so far. One question is related to the recovery value of the reference asset in a credit event, and the other relates to the nature of the stochastic process used for the default intensity of an obligor when modeling credit risk within the reduced-form framework.

Credit default swaps are derivative contracts aimed at transferring default risk of a third party, termed the reference entity, from one market participant to another. The protection seller assumes the credit risk of the underlying entity by committing to compensate the protection buyer for the loss suffered in case of a default of the entity on its outstanding debt, in return for a regular protection fee paid by the CDS buyer. Following a credit event, and assuming physical settlement, the seller makes a payment to the buyer equalling the notional value of the contract, and in turn receives defaulted obligations of equivalent notional value.

From the mechanism of a CDS it becomes clear that the seller is additionally exposed to recovery risk over and above the default risk of the underlying entity since his loss given default, LGD, equals the difference between the protection payment of par (100%) and the post-default market value of the delivered obligation(s), i.e. the recovery rate. Naturally, the seller wants to price this risk in already at the inception of the CDS contract. Holding the underlying term structure of default probabilities fixed, the more the seller expects to receive as recovery in default, the less premium he will demand for protection. By symmetric reasoning, the cost of protection for the CDS buyer should be lower if a high recovery rate is likely because it makes his benefit lower. This simple argument is also reflected in the pricing formula for CDS: In its rudimentary form the CDS premium can be expressed as PD $\times$ LGD, where PD denotes the (risk-neutral) probability of default during the life of the contract. Conversely, if loss given default is fixed, the CDS spread is driven exclusively by default probabilities, forcing them to behave unrealistically.

The first contribution of the present paper (Section 6.2) provides insight into the distribution of loss given default implied by market CDS premia across industries. Moreover, we investigate the relationship between loss given default and probability of default by looking at its behavior across ratings as a crude proxy for credit quality. Our analyses reveal that market-implied LGDs are particularly dependent on the investment grade of the obligor; industry affiliation has some explanatory power in the cross-section of implied LGDs, though coefficients exhibit high standard errors relative to their size.

The vast majority of empirical literature namely restricts the recovery rate to an exogenously specified, mostly arbitrary level, which is normally held constant regardless of the type of business the obligor pursues. The reasons put forward for doing this are that CDS prices purportedly do not react to the recovery parameter (e.g. Houweling and Vorst (2005), Longstaff et al. (2005)), that it is market practice to fix the recovery rate at some particular level (e.g. 25% in Pan and Singleton (2006), 40% in Chen et al. (2005)), or because the papers focus only on the (most liquid) 5-year CDS maturity and otherwise could not disentangle recovery from credit risk (e.g. Berndt et al. (2005), Longstaff et al. (2005)).

Since the recovery of face value (RFV) assumption is satisfied in the CDS market, Pan and Singleton (2006) demonstrate that the recovery rate (equivalently, the loss given default) can actually be estimated from CDS data if the calibration is performed on the full term structure of spreads. The authors take approximately 5.5 years of recent sovereign CDS data for Mexico, Korea and Turkey with maturities of 1, 2, 3, 5 and 10 years, and find LGD estimates of 23 and 83 percent, in contrast to the standard assumption of 75 percent. Our analysis confirms that implied loss given default rates can be identified from the term structure of CDS spreads. Pan and Singleton (2006) further illustrate that long-maturity CDS premia should be essential for identification since the effect of changes in the recovery rate on short-maturity CDS premia turns out relatively low. In contrast to the CDS market, recovery rates are poorly identified when considering the bond market (see e.g. Frühwirth and Sögner (2006)).

Of course there exist surveys of historically realized recovery rates, such as Batterman et al. (2005) for US defaults in the 2000 to 2004 period, as well as Altman and Kishore (1996), Altman et al. (2003), Altman et al. (2004) and Acharya et al. (2004) based on US defaults data from the 70ies, 80ies and 90ies. The most recent survey on recovery rates, Batterman et al. (2005), covering the years 2000 to 2004, reports an average annual recovery rate[1] of 34% (33% par-weighted) for senior unsecured bonds, with substantial variability among industries. The following factors are pointed out as the most likely determinants of recovery rates: debt seniority, macroeconomic variables, and industry characteristics (such as competitiveness, leverage, nature of assets, or regulation). Controlling for default risk through the rating, a comparison of recovery rates and CDS spreads between sectors reveals that while the corresponding spreads hardly differ, realized recoveries vary tremendously. The authors conjecture that recovery is usually disregarded as long as the probability of default is considered low, but can dominate CDS valuation for high-yield and distressed obligors.

Altman and Kishore (1996) is one of the first papers to investigate bond prices at default stratified by industry affiliation and seniority of the issue. They report an average recovery rate of 41% in US default data from 1978 to 1995, with a standard deviation of 25.56%, and conclude that issuer-specific knowledge is still indispensable. Altman et al. (2003) find an average annual recovery rate of 41.8% in (mostly) US data between 1982 and 2001 (37.2% if weighted by the market value of defaulted corporate bonds). Using a regression model it is shown that (the logarithm of) the default rate, the changes in the default rate, the amount of high-yield bonds outstanding, and the changes in the gross domestic product significantly impact the recovery rate. Altman et al. (2004) point out the effect of debt seniority on recovery rates from US defaults data between 1974 and 2003. As expected, the recovery rates decrease with subordination, however the differences are insignificant due to huge standard deviations: The mean recovery rates at default[2] (standard deviations in parentheses) in this period were 52.84% (23.05) for senior secured bonds, 34.89% (26.62) for senior unsecured bonds, 30.17% (24.97) for senior subordinated bonds, and 29.03% (22.53) for junior subordinated bonds. Similar figures on historical recovery rates are also cited in Schönbucher (2003, pg. 160). In addition to seniority, Acharya et al. (2004) identify the industry as the second important determinant of recovery rates based on US data from 1982 to 1999. Again, due to high standard deviations of the estimates, the differences are insignificant. A decrease of 10 to 12 percent in the recovery rate is estimated when an industry is in distress (defined by aggregate equity returns being below -30%).

The second contribution of the present paper (Sections 4.2 and 6.2) concerns the nature of the dynamics employed for the default intensity of the latent Cox process governing the default and survival of an obligor. Our specification of the stochastic intensity is embedded in the affine framework of Duffie et al. (2000). Our analyses generate the following results concerning the specification of an affine model for the default intensity. First, as conjectured by Pan and Singleton (2006) for the sovereign market, a two-factor specification significantly improves the cross-sectional as well as the time-series fit for corporate obligors as well, in particular for the shortest- and longest-maturity contracts. Second, we find no linear relationship between the default intensity and the long-run mean and the stochastic variance of the risk

[1] Based on bond prices one month after default, which is the value relevant for the CDS market.

[2] Based on bond prices just after default, which is the value relevant for the CDS market.

free short rate to support correlation between the risk free term structure and the default intensity. Third, incorporating jumps significantly improves the model's capability to reproduce the time-series behavior of CDS premia, and captures the following empirical stylized facts inferred from an exhaustive cross-section of US corporate obligors:

A If a discontinuity occurs, it mostly affects both the short end and the long end of the CDS maturity spectrum.

B When a discontinuity occurs, the change in the premia is mostly positive, and the jump size is related to the time to maturity.

C The 1-year CDS premium exhibits a unique variation pattern.

Furthermore, intensities of jumps in the credit risk components under the physical measure increase with deteriorating rating.

The academic literature testing alternative specifications of stochastic intensity-based models is rather scarce[3] compared to the vast number of empirical studies on affine models for the term structure of riskless interest rates. Empirical evidence for jumps in risk free discount rates is mixed. While the majority of papers specifies the risk free term structure by means of an $A_n(m)$ model (see Dai and Singleton (2002)) or an unspanned stochastic volatility setting (see Collin-Dufresne et al. (2004)), Johannes (2004) inferred jumps even in the risk free term structure.

Pan and Singleton (2006) estimate a log-normal diffusion model using extended affine market prices of risk (see Cheridito et al. (2006)). The estimates of the default rates are persistent under the physical measure and partially explosive under the pricing measure. Moreover, their risk-neutral default intensities exhibit substantial pairwise correlation and correlation with the VIX option volatility index.

As a further contribution (Sections 4.1 and 6.1) we estimate an Andersen et al. (2004)-type model for the risk free term structure. To the best of our knowledge we are the first to estimate this model on a panel of risk free zero yields. The parameter estimates are comparable to the ones in Andersen et al. (2004), but we find a substantial risk premium on the standard deviation of the jump size. As already observed in an earlier version of Pan and Singleton (2006), our sophisticated risk free model negligibly affects model-implied CDS premia.

Our final contribution (Section 3.2) is an investigation of the pricing differences resulting from continuous vs. discrete monitoring of default events. Using a one-factor jump-diffusion intensity model, we find that a discretized formulation results in substantial underpricing for several parameters.

2 CDS Data

The CDS history is obtained from Markit, a leading data provider specializing in mark-to-market CDS pricing, among other products. Their data is sourced from a broad range of dealers[4] contributing on a daily basis. The data points in the history represent neither trades nor quotes; the contributed values are based on the dealers' books of record or feeds to automated trading systems, and are subsequently aggregated by Markit. The data is commonly used by global financial institutions for price verification and risk management purposes.

[3]Moreover, most of these studies do not use CDS, but corporate bond data.

[4]There are 62 contributors as of April 2006.

Though the credit derivatives market, and especially the CDS market, have grown tremendously in the past ten years, the quality and availability of data still present a considerable restriction to empirical research: There is a clear trade-off between the length of the time period and the number of spreads in the cross-section of maturities – the further one reaches into the past, the less data points are available per day. For this reason our data set contains daily spreads spanning two years from October 19, 2004 to October 19, 2006 (totaling 523 days).

There are approximately 1500 names altogether at our disposal, but for the results to be comparable to other studies our focus is exclusively on obligors based in the United States. At the same time, by restricting the analysis to US obligors only, we reduce the effect of the delivery option on CDS spreads. Namely, CDS contracts with physical settlement never specify one single obligation to be solely deliverable, but rather admit a basket of deliverable obligations. At default, the protection buyer thus has the option to deliver the cheapest obligation from this basket. Deliverable obligations are defined by a set of characteristics, the most relevant of which are currency and seniority.

There are six standard specified currencies[5], and obligations are normally accepted for delivery if denominated in one of these. In conversations with practitioners we have learned that a higher value is attached to the delivery option if the reference entity has debt outstanding in several currencies because of the foreign exchange risk and foreign interest rate risk induced. Since the US corporate bond market is globally the most developed, we conjecture that it is more probable for US obligors – than for those based e.g. in Europe – not to issue debt in foreign currencies. We therefore restrict our analysis to USD-denominated CDS contracts and do not take into account foreign interest rate risk or foreign exchange risk when modeling the recovery rate.

Let us now turn to the seniority characteristic. The debt acceleration clause[6] in bond indentures and bankruptcy codes essentially establishes that the principal amount of all specified debt becomes immediately due and payable if a credit event applies to any single obligation involved. This provision allows CDS contracts to be written on a certain tier of the obligor's debt (e.g. secured, senior unsecured, subordinated or junior subordinated) without the explicit need for specifying one concrete reference obligation. The most widely referred to tier is senior unsecured, which is also adopted for our analysis.

Though the existence of the debt acceleration clause reduces the value of the delivery option, it can never completely remove it. In a broad study of bond prices following actual credit events from the recent past Guha (2002) investigates whether obligations of the same seniority (but possibly having different maturities or coupons) trade at the same price after the credit event, which would be a consequence of the debt acceleration clause. The evidence he finds is ambiguous (cf. Guha (2002, Table VIII)). Nevertheless, this provision theoretically justifies the recovery of face value (also called the recovery of par) assumption[7] we employ in CDS pricing.

Furthermore, the CDS restructuring clause, which restricts the maturity of deliverable obligations in case of a restructuring credit event, needs to be taken into account for comparability of CDS spreads. The modified restructuring clause is the US market standard, where it e.g. accounts for approximately 90% of investment-grade CDS contracts (cf. Reyfman and Toft (2004)).

These four criteria (USD-denominated CDS contracts with the modified restructuring clause on senior

[5]The Standard Specified Currencies are the currencies of Canada, Japan, Switzerland, the United Kingdom and the United States, and the euro, cf. Section 2.19.(b)(ii) of the 2003 ISDA Credit Derivatives Definitions.

[6]Sometimes also termed the cross-default clause.

[7]For detailed comparisons of pricing properties for different recovery assumptions cf. Bakshi et al. (2006), Delianedis and Lagnado (2002) and Schönbucher (2003, Section 6).

unsecured debt issued by US-domiciled obligors) leave us with 675 obligors. Another 7 names must be excluded because of taking part in mergers or demergers during our sample period. For each obligor we have at our disposal a panel $\{\bar{s}_t(M) : M = 1y, 3y, 5y, 7y, 10y,\ t = 1, \ldots, T\}$ of CDS premia:

$$\begin{pmatrix} \bar{s}_1(1y) & \bar{s}_1(3y) & \bar{s}_1(5y) & \bar{s}_1(7y) & \bar{s}_1(10y) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \bar{s}_t(1y) & \bar{s}_t(3y) & \bar{s}_t(5y) & \bar{s}_t(7y) & \bar{s}_t(10y) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \bar{s}_T(1y) & \bar{s}_T(3y) & \bar{s}_T(5y) & \bar{s}_T(7y) & \bar{s}_T(10y) \end{pmatrix} \tag{2.1}$$

where the subscript $t = 1, \ldots, T$ with $T = 523$ indicates the observation day, and the index M the maturity of the CDS. Only the five canonical CDS maturities (cf. Brigo and Mercurio (2006, pg. 719)) of 1 year, 3 years, 5 years, 7 years and 10 years are used in our analysis since these are known to be the most frequently quoted and traded. Though Markit's approach to collecting data mitigates the problem of missing and stale values to a large extent, it does not completely eliminate it. CDS spreads for individual maturities, or even whole days, are missing in some panels, and there is a non-negligible proportion of stale spreads, which we treat as if they were missing. Since high-quality time-series are necessary for a meaningful analysis, we set high thresholds on data quality: First, the overall percentage of missing spreads per obligor (which in our definition includes stale spreads) must not exceed 10%, and second, the length of the longest series of consecutive missing spreads must be 5 days or shorter. The former requirement guarantees enough data points for estimation, and the latter ensures that the missing data points are not clustered together. These two quality criteria are satisfied by 282 of the selected 668 obligors.

The processed set of obligors is classified into industry sectors according to the ICB[8] scheme provided to us by Dow Jones. The ICB classification system consists of four layers, the first of which is the Industry layer. With only ten categories in total it is probably too rough a classification because the resulting groups of obligors are still too heterogeneous with respect to their businesses (assets), but it suffices to obtain preliminary results. On the other hand there exists a trade-off between the level of partition and the number of obligors in each sector. We are currently investigating the effects of using finer classification layers (the so-called Supersector or even Sector level).

3 CDS Valuation

3.1 CDS Pricing

Consider the standard intensity-based setup (for details consult standard references, such as Lando (2004) or Schönbucher (2003)), where a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{Q})$ is given, with $\mathbb{Q}$ being a risk-neutral or pricing measure. The intensity-based framework postulates a latent Cox process N whose first jump time τ determines the default time of the obligor. Let λ be the non-negative stochastic process for the jump intensity of N (concrete specifications are discussed in Section 4.2). There also exists a risk

[8] The Industry Classification Benchmark (ICB) is a four-tiered industry classification system launched by Dow Jones and FTSE in 2005. It aims at providing standardized and comprehensive coverage of the global corporate universe, and has been adopted to date by most of the leading financial institutions, exchanges and data providers. Please visit `www.icbenchmark.com` for details.

free short rate r such that the time-t price of a risk free zero-coupon bond maturing at time T may be expressed as

$$P(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\int_t^T r_s\, ds\right\}\right], \tag{3.1}$$

where the expectation is taken under the measure $\mathbb{Q}$ and conditional on information available at time t. Then, assuming zero recovery, the price of an equivalent defaultable zero-coupon bond is

$$\overline{P}(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{-\int_t^T (r_s + \lambda_s)\, ds\right\}\right], \tag{3.2}$$

where the riskless short rate is replaced by a default-adjusted short rate $\widetilde{r} := r + \lambda$.

There are two sides to a CDS contract: the fixed leg, comprising the fee payments by the protection buyer, and the default leg, containing the contingent payment by the protection seller[9]. The exact cash flow structure of the fixed leg in a standard contract, as laid down in the 2003 ISDA Credit Derivatives Definitions, is specified as follows: Premium payment dates are fixed and do not depend on the specific contract date. They are quarterly and happen on the 20th of March, June, September and December. Thus, if a CDS is contracted between those dates, the first period is not a full quarter and the first premium payment is adjusted accordingly. In addition, we account for the now variable maturity of CDS contracts: As a result of fixing the premium payment dates, the length of the protection period varies and depends on the contract date since the quoted CDS maturity begins on the first premium payment date. Furthermore, the accrued premium in case of default must be taken into account. We assume absence of any transaction costs.

Consider at time t a CDS with outstanding premium payments s at times $T_1 < T_2 < \ldots < T_N$, $T_1 > t$, maturity at $T = T_N$ and notional normalized to 1. As discussed in Section 2, the recovery of face value assumption is employed, and the recovery parameter under the pricing measure $\mathbb{Q}$ denoted by $R^{\mathbb{Q}}$. Denoting the time-t value of the fixed leg by $V_t^{\text{fix}}(T;s)$ and the time-t value of the default leg (with the discounted payment normalized to 1) by $V_t^{\text{def}}(T)$, then the time-t value of the CDS contract to the buyer is $(1 - R^{\mathbb{Q}})\, V_t^{\text{def}}(T) - V_t^{\text{fix}}(T;s)$. Since CDS contracts are priced at initiation only, the values of the fixed and the floating legs are considered at the time of initial offering, meaning that time-t values correspond to contracts initiated at time t. This paper analyzes time-series of spreads, so the values of the legs and of the spreads with initiation at time t are called time-t values.

If default happens within the protection period, the protection buyer has made $I(\tau) = \max\{1 \leq n \leq N : T_n \leq \tau\}$ premium payments, the remaining ones $I(\tau)+1, \ldots, N$ being no longer due, except for an accrual payment of $s \times (\tau - T_{I(\tau)})$ at the default time τ. Hence, the value of the fixed leg of a CDS contract initiated at time t with maturity T is given by

$$V_t^{\text{fix}}(T;s) \;=\; \sum_{n=1}^{N} s\,(T_n - T_{n-1})\,\overline{P}(t,T_n) \;+\; \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{T_N} s\,(u - T_{I(u)})\,\lambda_u \exp\left\{-\int_t^u \widetilde{r}_v\, dv\right\} du\right], \tag{3.3}$$

[9] Our valuation approach ignores counterparty risk on both sides of the contract. More precisely, the assumption is that during the life of the contract the counterparties either maintain the credit rating underlying generic (e.g. A-rated) CDS or have symmetric default probabilities (credit quality), cf. Duffie and Singleton (1997). We presume that this aspect has a relatively low impact on the spreads of typical CDS contracts.

where $T_0 = t$ and $T_N = T$. On the other hand, the time-t value of the default leg (with the discounted payment normalized to 1) is given by

$$V_t^{\text{def}}(T) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T \lambda_u \exp\left\{-\int_t^u \widetilde{r}_v \, dv\right\} du\right]. \tag{3.4}$$

At initiation of a CDS the premium $s_t(T)$ is chosen such that the contract value to both parties is zero, and since the value of the fixed leg is homogenous of degree 1 in s, it follows that

$$s_t(T) = (1 - R^{\mathbb{Q}}) \, \frac{V_t^{\text{def}}(T)}{V_t^{\text{fix}}(T; 1)} . \tag{3.5}$$

3.2 Discrete vs. Continuous CDS Pricing

The valuation described in the preceding section represents the most general approach, taking into account all cash flows generated by a CDS contract and making no more assumptions than absolutely necessary: absence of transaction costs, counterparty risk and the delivery option. The widely employed discretized valuation formula (as presented in standard textbooks like Lando (2004, Section 8) or Schönbucher (2003, Section 3)) is based upon several simplifying assumptions regarding the payoff structure of a CDS (cf. Schönbucher (2003, Section 2) or Duffie (1999)). The exact payment structure of the fixed leg is ignored, and premia are paid in regular (quarterly) intervals starting at the initiation date, so there is no maturity shift. The fixed leg is additionally reduced by neglecting the payment of the accrued premium at default. As for the default leg, an equally-spaced grid on the time interval $\langle t, T]$ is assumed, interpreted as the time points at which the obligor is monitored and the occurrence of a default since the previous observation point is detected (for example monthly or weekly). As a consequence, the termination payment is made at the observation date immediately following actual default. Technically speaking, the grid $t = S_0 < S_1 < \ldots < S_K = T$ on $\langle t, T]$ induces a discretization of the integral in (3.4), which is then numerically approximated via finite differences. In most cases the risk free short rate r is additionally assumed at least independent of the default intensity λ, which simplifies equation (3.4) to:

$$\begin{aligned} V_t^{\text{def}}(T) &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T \lambda_s \exp\left\{-\int_t^s \widetilde{r}_v \, dv\right\} ds\right] \\ &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T \exp\left\{-\int_t^s r_v \, dv\right\} dQ(t,s)\right] \\ &\approx \sum_{k=1}^{K} P(t, S_k) \left(Q(t, S_{k-1}) - Q(t, S_k)\right), \end{aligned} \tag{3.6}$$

where $Q(t, S) = \mathbb{E}_t^{\mathbb{Q}}[\exp\{-\int_t^S \lambda_v \, dv\}]$ denotes the (cumulative) survival probability over the interval $\langle t, S]$.

The aim of this section is to compare the magnitude of the pricing differences resulting from these two formulations when individual parameters of the model specification are varied over a plausible spectrum of values. As described in the preceding section, the general approach in particular involves continuously discounting the LGD amount, whereas this integral is discretely approximated in the approach described above. For this reason we call the former pricing continuous, and the latter discrete. Since the continuous formulation involves a considerably greater computational effort, it is important to assess the trade-off when choosing the discrete formula. For example, one expects the accuracy of the discretization to increase by raising the default observation frequency governed by $\delta = S_k - S_{k-1}$.

Not to be distracted by a possible influence of a varying risk free short rate r, we fix it at a constant value in order to focus only on the effects of the valuation formula. For the sake of simplicity we use a one-factor specification for the default intensity, which is assumed to follow a square-root jump-diffusion:

$$d\lambda_t = \kappa\,(\theta - \lambda_t)\,dt + \sigma\sqrt{\lambda_t}\,dW_t + dZ_t\,, \tag{3.7}$$

where Z is a compound Poisson process with jump intensity ν and exponentially distributed jump sizes with mean μ, independent of the Brownian motion W. The absolute pricing errors in Figure 1 are computed by holding all parameters fixed at plausible values ($\delta = 1/8$, $\lambda_0 = 0.3\%$, $\kappa = 0.5$, $\theta = 0.3\%$, $\sigma = 7\%$, $\mu = 0.2\%$, $\nu = 3$, and $r = 3\%$), while varying each of the parameters one by one over a reasonable range of values.

The time step δ is allowed to vary only up to one quarter of a year because a value larger than 0.25 would mean that the protection buyer is paying premia without knowing whether the obligor has defaulted or not. Varying this parameter yields only minor absolute pricing differences (0.1 bp order of magnitude), though only for "nice" values of the remaining parameters. For example, as soon as the long-run mean θ is set to a higher value of e.g. 0.5 – which frequently happens when fitting persistent time-series like riskless interest rates or CDS spreads – the pricing difference shoots up to 70 bp for a 10-year maturity.

Discretization is harmless for non-explosive values of the speed-of-mean-reversion parameter κ; explosive values ($\kappa < 0$), which are necessary for a reasonable fit to CDS spreads according to Pan and Singleton (2006), induce huge pricing differences of up to 20 bp and worse the longer the maturity of the CDS and the more negative the parameter. The effect of the long-run-mean parameter θ is severe for high values (up to 60 bp for a 10-year CDS), which (similarly to κ) frequently appear when fitting persistent time-series like CDS spreads (cp. to our results in Section 6). Even worse is the pricing error induced by the expected jump size parameter μ, ranging up to 400 bp for longer-dated contracts.

All other model parameters (the starting value λ_0, instantaneous volatility σ, jump intensity ν, and the riskless interest rate r) produce negligible pricing errors. A final striking observation can be made with respect to the sign of the pricing errors. All large deviations highlighted above exhibit a positive sign; a discretization of the general, continuous pricing formula therefore yields underpricing for this model.

4 Model Specification

4.1 Riskless Model

We employ an observable stochastic volatility jump-diffusion model for riskless discount rates, where the panel of zero yields is subject to diffusive risk generated by Brownian motions, as well as jump risk generated by a compound Poisson process with normally $N(0, \sigma_J^2)$ distributed jump size. A parsimonious version of the model below is introduced and tested extensively with respect to its time-series capabilities in Andersen et al. (2004). To the best of our knowledge we are the first to estimate this model on a panel of zero yields. The dynamics under $\mathbb{Q}$ are:

$$\begin{aligned} dV_t &= (\kappa\theta_V - \kappa_V\,V_t)dt + \sigma_V\sqrt{V_t}\,dW_V^{\mathbb{Q}}(t) \\ d\mu_t &= (\kappa\theta_\mu - \kappa_{V\mu}\,V_t - \kappa_\mu\,\mu_t)dt + \sigma_\mu\sqrt{\mu_t}\,dW_\mu^{\mathbb{Q}}(t) \\ dr_t &= \kappa_r(\mu_t - r_t)dt + \sqrt{V_t}\,dW_r^{\mathbb{Q}}(t) + d\hat{Z}_r^{\mathbb{Q}}(t). \end{aligned} \tag{4.1}$$

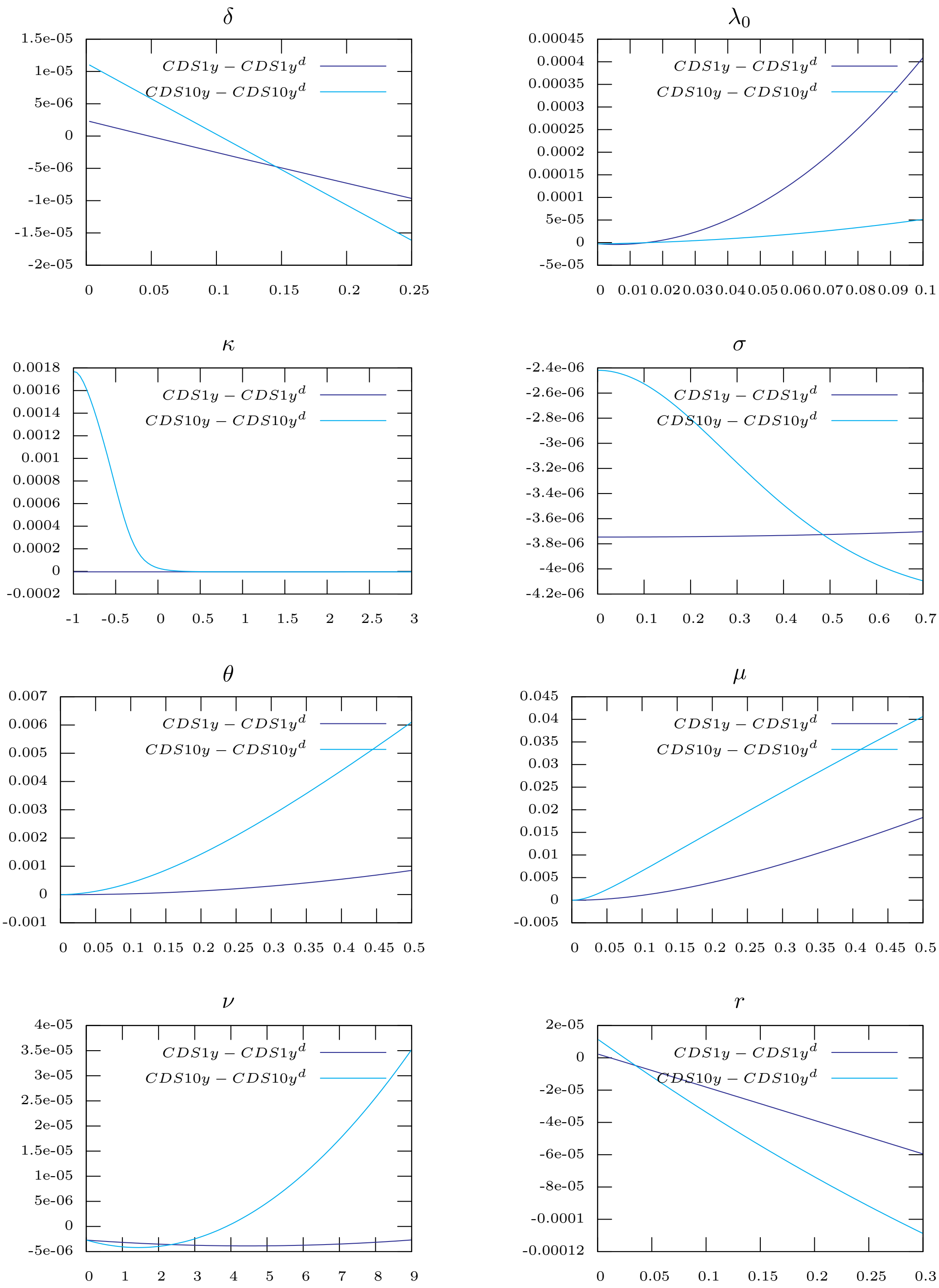

Figure 1: Pricing Discretization Error: This figure shows absolute deviations computed as the difference between the general pricing formula (3.5) and its discretized version (3.6). Errors are computed for CDS with maturities 1 year and 10 years to cover the maturity spectrum.

This formulation admits an interpretation of the state variables as stochastic volatility, long-run short rate mean and short rate. In subsequent sections we use $r^\star \equiv (V, \mu, r)$ to lighten notation. With extended affine market prices of risk from Cheridito et al. (2005) one may estimate the drift coefficients under $\mathbb{P}$ separately from the $\mathbb{Q}$ parameters. The dynamics under $\mathbb{P}$ are assumed to be determined by:

$$\begin{aligned} dV_t &= (\kappa\theta_V^{\mathbb{P}} - \kappa_V^{\mathbb{P}}\, V_t)dt + \sigma_V\sqrt{V_t}\, dW_V^{\mathbb{P}}(t) \\ d\mu_t &= (\kappa\theta_\mu^{\mathbb{P}} - \kappa_{V\mu}^{\mathbb{P}}\, V_t - \kappa_\mu^{\mathbb{P}}\, \mu_t)dt + \sigma_\mu\sqrt{\mu_t}\, dW_\mu^{\mathbb{P}}(t) \\ dr_t &= \kappa_r^{\mathbb{P}}(\mu_t - r_t)dt + \sqrt{V_t}\, dW_r^{\mathbb{P}}(t) + d\hat{Z}_r^{\mathbb{P}}(t). \end{aligned} \tag{4.2}$$

The Feller condition is imposed for the short rate variance process and the long-run mean process under both probability measures. Riskless zero-coupon yields y_t^ν, $\nu = (3m, 2y, 5y, 1m, 1y, 7y, 10y)$, are bootstrapped from Libor rates (for maturities under one year: 1, 3, 6 and 9 months) and swap rates (for maturities between 1 and 10 years: 1, 2, 3, 4, 5, 7 and 10 years). All riskless data is taken from DataStream.

4.2 Default Risk Specifications

Observation **A** from the Introduction has intuitive appeal, since contracts with a longer time to maturity are equally affected by a credit event which the market anticipates in the short run. Observation **B** reflects the fact that financial distress, or rumours of financial distress, cause sudden upward moves in CDS premia. On the other hand the market tends to forgive slowly or not at all after such an event. Therefore premia move down slowly – if they move down at all. Observation **C**, together with preliminary experiments with one-factor models[10], as well as empirical evidence in Pan and Singleton (2006) and Chen et al. (2005), suggests the use of a two-factor model for a firm's default intensity:

$$d\gamma_t = (\kappa\theta_\gamma - \kappa_\gamma\, \gamma_t)dt + \sigma_\gamma\sqrt{\gamma_t}dW_\gamma^{\mathbb{Q}}(t) + d\left(Z_{\gamma\eta}^{\mathbb{Q}}(t) + Z_{\gamma\eta r}^{\mathbb{Q}}(t)\right) \tag{4.3}$$

$$d\eta_t = \kappa_\eta(\gamma_t - \eta_t)dt + \sigma_\eta\sqrt{\eta_t}dW_\eta^{\mathbb{Q}}(t) + d\left(Z_{\gamma\eta}^{\mathbb{Q}}(t) + Z_{\gamma\eta r}^{\mathbb{Q}}(t)\right), \tag{4.4}$$

where η is interpreted as the default intensity and γ as its stochastic long-run mean. We found the following parameterization under $\mathbb{P}$ to be reliable for estimation:

$$\begin{aligned} d\gamma_t &= (\kappa\theta_\gamma - \kappa_\gamma^{\mathbb{P}}\, \gamma_t)dt + \sigma_\gamma\sqrt{\gamma_t}dW_\gamma^{\mathbb{P}}(t) + d\left(Z_{\gamma\eta}^{\mathbb{P}}(t) + Z_{\gamma\eta r}^{\mathbb{P}}(t)\right) \\ d\eta_t &= (\kappa_\eta\gamma_t - \kappa_\eta^{\mathbb{P}}\eta_t)dt + \sigma_\eta\sqrt{\eta_t}dW_\eta^{\mathbb{P}}(t) + d\left(Z_{\gamma\eta}^{\mathbb{P}}(t) + Z_{\gamma\eta r}^{\mathbb{P}}(t)\right) \end{aligned}. \tag{4.5}$$

The Feller condition is imposed on the stochastic long-run mean process under both probablity measures. To model observation **A** we assume that jumps in the driving state variables may occur as mutually exclusive events

1. to the risk free term structure only,

[10] Our experiments result in parameter estimates similar to Pan and Singleton (2006), even though they investigate the sovereign market. Some firms admit a reasonable fit only by letting the default intensity explode under $\mathbb{Q}$. Some firms cannot be reasonably fitted even with an explosive process under $\mathbb{Q}$, when the short end of the premia deviates too much from the long end.

2. to the default intensity of the firm and its long-run mean simultaneously,

3. to both the risk free term structure, the default intensity, and its long-run mean (all three state variables at the same time).

The jump process $\hat{Z}_r^{\mathbb{Q}}(t)$ from (4.1) is therefore shorthand for the sum $Z_r^{\mathbb{Q}} + Z_{\gamma\eta r}^{\mathbb{Q}}$. Jump times are governed by exponentially distributed inter-arrival times $\tau_r \sim \mathcal{E}xp(l_r)$, $\tau_{\gamma\eta} \sim \mathcal{E}xp(l_{\gamma\eta})$, and $\tau_{r\gamma\eta} \sim \mathcal{E}xp(l_{r\gamma\eta})$. The expected waiting times between two jumps of the corresponding Poisson processes are therefore l_r, $l_{\gamma\eta}$, and $l_{r\gamma\eta}$, respectively.

To accomodate observation **B** our model uses a multivariate jump-size distribution for jumps in η and γ. Recall from Section 4.1 that jumps in the riskless short rate are normally distributed with $N(0, \sigma_J^2)$. The moment generating function for a mean-zero normal distribution is known to be $\Phi(u) = \exp\left(\frac{1}{2}\sigma_J^2 u^2\right)$. Here we follow Andersen et al. (2004), who also normalize expected jump size to zero. For the default risk components we employ a bivariate exponential distribution with exponential marginals as developed in Marshall and Olkin (1967). It is characterized by the parameters ζ_γ, ζ_η, and $\zeta_{\gamma\eta}$. The (marginal) expected jump size for γ is $\frac{1}{1/\zeta_\gamma + 1/\zeta_{\gamma\eta}}$, and $\frac{1}{1/\zeta_\eta + 1/\zeta_{\gamma\eta}}$ for η. Marshall and Olkin (1967) also provide the moment generating function:

$$\Psi(u, v) = \frac{(\zeta - u - v)(1/\zeta_\eta + 1/\zeta_{\gamma\eta})(1/\zeta_\gamma + 1/\zeta_{\gamma\eta}) + uv/\zeta_{\gamma\eta}}{(1/\zeta - u - v)(1/\zeta_\eta + 1/\zeta_{\gamma\eta} - u)(1/\zeta_\gamma + 1/\zeta_{\gamma\eta} - v)}\ , \tag{4.6}$$

where $\zeta := 1/\zeta_\eta + 1/\zeta_\gamma + 1/\zeta_{\gamma\eta}$. I.e. $dZ_\eta^{\mathbb{P}}(t), dZ_\eta^{\mathbb{P}}(t) > 0$ if we observe a jump in the risky term structure only ($\eta\gamma$-jump) or in the case of a joint jump of the risk free term structure and credit risk ($\eta\gamma r$-jump). In both cases, the jump sizes of the components (η) and (γ) have a bivariate exponential distribution.

Both the normal and the bivariate exponential moment generating functions serve as ingredients to the differential equations from Duffie et al. (2000) which we solve to evaluate the conditional expectations in equations (3.3) and (3.4). To engineer our jump transform we proceed analogously to Duffie et al. (2000, Section 4 and Appendix B) and specify

$$\Omega(t, u, v) = \frac{1}{l_r + l_{\gamma\eta} + l_{\gamma\eta r}} \left(l_r \Phi(t) + l_{\gamma\eta}\Psi(u, v) + l_{\gamma\eta r}\Psi(u, v)\Phi(t)\right). \tag{4.7}$$

Evidence for dependence between risless rates and credit-risky rates is rather mixed. Some studies find a positive relation, others find a negative relation. Feldhütter and Lando (2005) provide a discussion of evidence from the recent literature. We are not aware of any studies on the role of the riskless short rate volatility or the expected short rate in credit-risky markets, however. To empirically investigate this issue we perform preliminary estimations with a default intensity specification $\lambda = \eta + c\mu + eV$, where μ and V are the riskless stochastic long-run mean and stochastic variance, respectively, from system (4.1). This does not cause an additional computational burden since the above formulation preserves affinity and only the initial conditions of the pricing ODEs change. We deliberately do not include the riskless short rate r directly since it is a Gaussian process. Estimates for e and c turn out to be close to zero and exhibit high variability, somewhat similar to the results in Feldhütter and Lando (2005) for the short rate; this indicates only weak evidence for dependence, if at all, and we therefore perform the reported estimations with $c = e = 0$.

5 Methodology

Estimation is performed in two steps. First, model (4.1) is estimated on the panel data of riskless zero yields. In a second step the CDS model is estimated firm by firm. For the first step we assume that three zero yields are observed without error. This assumption is commonly not made in MCMC studies, but frequently appears in the ML literature on term structure modeling (see for example Aït-Sahalia and Kimmel, 2005; Duffee, 2002; Cheridito et al., 2006, for studies that estimate with some yields observed without error). With this assumption the model-implied and parameter-implied state variables V_t, μ_t and r_t can be inverted from observed zero yields. Appendix A.1 shows the link between observed zero yields and state variables. We find this measure necessary to separate stochastic volatility from jumps. For both steps (4.5) and (4.2) we employ an Euler discretization. Equations from system (4.2) in discretized form read:

$$\begin{aligned} V_{t+1} &= V_t + (\kappa\theta_V^{\mathbb{P}} - \kappa_V^{\mathbb{P}} V_t)\Delta + \sigma_V \sqrt{V_t \Delta}\, \epsilon_V(t+1), \\ \mu_{t+1} &= \mu_t + (\kappa\theta_\mu^{\mathbb{P}} - \kappa_{V\mu}^{\mathbb{P}} V_t - \kappa_\mu^{\mathbb{P}} \mu_t)\Delta + \sigma_\mu \sqrt{\mu_t \Delta}\, \epsilon_\mu(t+1), \\ r_{t+1} &= r_t + \kappa_r^{\mathbb{P}}(\mu_t - r_t)\Delta + \sqrt{V_t \Delta}\, \epsilon_r(t+1) + Z_r(t+1) J_r(t+1). \end{aligned} \tag{5.1}$$

The step width Δ is taken to be constant at 1/250. Innovations $\epsilon_{V,\mu,r}$ are $N(0,1)$-distributed random variables, $J_r(t+1)$ has a Bernoulli distribution $\mathcal{B}(l_r/250)$, and $Z_r(t+1) \sim N(0, \sigma^2_{J\mathbb{P}})$. The bias introduced by this discretization at a daily step width is small (see Johannes, 2004; Johannes and Polson, 2003, in a very similar context). System (4.5) is treated analogously.

Our model is comprised of latent risk free state variables V, μ, and r, a latent default intensity η, its stochastic long-run mean γ and a constant, yet unknown recovery rate $R^{\mathbb{Q}} = 1 - \text{LGD}$. From this model we compute model-implied CDS premia s, where

$$s_t = (s_t(1), s_t(3), s_t(5), s_t(7), s_t(10))^\top = \text{LGD} \cdot \left(\frac{V^{\text{def}}(1)}{V^{\text{fix}}(1)}, \frac{V^{\text{def}}(3)}{V^{\text{fix}}(3)}, \frac{V^{\text{def}}(5)}{V^{\text{fix}}(5)}, \frac{V^{\text{def}}(7)}{V^{\text{fix}}(7)}, \frac{V^{\text{def}}(10)}{V^{\text{fix}}(10)} \right)^\top. \tag{5.2}$$

In our panel of CDS premia $\bar{s} = \{\bar{s}_t\}_{t=1}^T$, where $\bar{s}_t = (\bar{s}_t(1), \bar{s}_t(3), \bar{s}_t(5), \bar{s}_t(7), \bar{s}_t(10))^\top$, there are both observed values, denoted by $\bar{s}_t^o(\cdot)$, as well as missing values, denoted by $\bar{s}_t^m(\cdot)$. It holds that $\{\bar{s}^o, \bar{s}^m\} = \bar{s}$. We assume that our panel is observed with an additive i.i.d. observation error $\varepsilon_t \sim MVN(0, \Sigma_\varepsilon(t)), \forall t$ such that for each t we have

$$\bar{s}_t = s_t + \varepsilon_t. \tag{5.3}$$

The covariance matrix $\Sigma_\varepsilon(t)$ is assumed to be a diagonal matrix where the i-th diagonal entry is given by $[\Sigma_\varepsilon(t)]\,(ii) = \exp\left(a_0 + a_1(T_i - t) + a_2(T_i - t)^2\right)$. $T_i - t$ is the time to maturity of CDS contract $i = 1, \dots 5$ at time t.

Letting θ comprise the parameters of the stochastic processes, Σ_ε and LGD, then the joint posterior distribution of the parameters with the latent state variables and missing CDS prices is given by

$$\begin{aligned} p(s^m, r^\star, \gamma, \eta, \theta \mid s^o, y^\nu) &\propto p(s^o, s^m \mid r^\star, \gamma, \eta, \theta)\, p(y^\nu, \lambda, \theta) \\ &\propto p(s \mid r^\star, \gamma, \eta, \theta)\, p(y^\nu, \gamma, \eta \mid \theta)\, p(\theta) \\ &= p(s \mid r^\star, \lambda, \theta)\, \frac{1}{\det |H|}\, p(r^\star \mid \theta)\, p(\gamma, \eta \mid \theta)\, p(\theta) \end{aligned} \tag{5.4}$$

by independence of the observation errors. For estimation on CDS premia we do not take into account $p(r^\star \mid \theta)$ since it is determined by the risk free market. Note that the processes of the risk free term structure $r^\star$ need not be simulated, since by our assumption of riskless zero yields observed without error $r^\star$ is given by the model parameters and riskless zero yields y_t^ν; the functional relationship is described in Appendix A.1. The density $p(s \mid r^\star, \lambda, \theta)$ is a multivariate normal distribution arising from 5.3 and the specification of Σ_ε. $p(\lambda \mid \theta)$ is defined from our specification of the process λ_t.

Since we perform a Bayesian analysis we also specify the priors $\pi(\theta)$. We use uninformative priors for all elements of θ, normal priors for variables with support on the real line and gamma priors for variables with support on the positive real line, with very high variances. Proposal densities for the Metropolis steps are random walk, except for l_r and $l_{r\gamma\eta}$, where we use normal proposals with mean $l_r^{\mathbb{P}}$ and $l_{r\gamma\eta}^{\mathbb{P}}$, respectively. We further restrict our parameterization of the jump size parameters to $\zeta_\eta = \zeta_\gamma$ and $\zeta_\eta^{\mathbb{P}} = \zeta_\gamma^{\mathbb{P}}$.

6 Results

6.1 Risk Free Term Structure

This section presents and discusses the results from estimation of the risk free term structure model (4.1) and (4.2) on zero yields bootstrapped from US swap rates. Table 1 in Appendix B presents the parameter estimates. The respective parameter paths from the Metropolis-Gibbs sampler are reported in Figures 3 through 6. Estimation is based on 10 years of daily data from January 1997 to October 2006.

Andersen et al. (2004) (henceforth ABL) find no evidence for correlation between the state variables and the "level effect". The level effect stands for the empirical stylized fact that interest rate volatility is high when interest rates are high. Our model differs from the ABL model in that it is able to accomodate (positive) correlation between the stochastic long-run mean and stochastic variance via the $\kappa_{\mu V}$ parameter under both probability measures. Yet it is not able to reproduce the level-effect itself, but instead it may induce the effect that interest rate volatility is high when *expected* interest rates are high, with similar economic appeal. Under $\mathbb{Q}$, tight confidence bands deem the parameter $\kappa_{\mu V}$ well identified, while $\kappa_{V\mu}^{\mathbb{P}}$ displays high variability.

Johannes (2004) finds that $\mathbb{Q}$ jump parameters are poorly identified from yields only. In our estimation procedure we therefore propose the jump intensity under $\mathbb{Q}$, l_r, from a normal distribution with mean $l_{r\mathbb{P}}$ instead of a more standard random walk proposal. This includes the assumption into our proposal density that the market for zero bonds does not put a risk premium on the jump intensity. As a consequence the resulting point estimate of l_r at 3.56 is only slightly higher than $l_{r\mathbb{P}}$ at 3.53, which is approximately equal to the ABL estimate. In turn, Figure 5 shows that the standard deviation of the jump size under $\mathbb{Q}$ is very well identified at a very high 0.09, with a standard random walk proposal. This implies a large risk premium on jump size risk, since the standard deviation of the jump size under $\mathbb{P}$ is estimated at 0.003, which is very close to the ABL estimates.

From Table 1 we observe evidence for high persistency in the processes, under both the empirical measure $\mathbb{P}$ and the equivalent martingale measure $\mathbb{Q}$, respectively. This is a standard finding in the empirical term structure literature. For example the coefficient of mean reversion for the short rate variance and the stochastic long-run mean are both approximately 1 under $\mathbb{P}$, such that in discrete time we have an autoregressive coefficient of around $\exp(-1/250) \approx 0.996$ on a daily time scale. The confidence band for $\kappa_V^{\mathbb{P}}$ contains the ABL estimate, while our estimate for $\kappa_\mu^{\mathbb{P}}$ is slightly higher. Under $\mathbb{Q}$ the processes are even more persistent.

The most striking difference to ABL concerns our estimate for the standard deviation of the stochastic short rate variance at 0.001, which is much lower than the 0.011 estimated by ABL. This may be due to our much smaller sample, which does not contain the turbulent period at the beginning of the 80's, or it may be related to the fact that we estimate both on the time-series and the cross-section, an estimation practice forcefully suggested by Chernov and Ghysels (2000).

The posterior distribution of the latent jump-time indicators $\mathbf{J}_r$ (cf. Appendix A for details) provides us with the posterior distribution of the jump events. The results are presented in Figure 2. The upper panel reads as follows: On the horizontal axis we have the time scale t, while the vertical axis represents the posterior probability of a jump at t. For values larger than 0.5 we infer a jump. The lower plot in Figure 2 displays the posterior estimates of the jump sizes and thus clarifies why we use normally distributed jumps for the risk free term structure since both upward and downward jumps in the short rate are observed.

6.2 Analysis of Credit-Risky Components

This section presents and discusses our parameter estimates of models (4.3) and (4.5). Since we consider 282 obligors and parameters are estimated on a firm wise basis, we group firms either with respect to their rating or with respect to their industry classification. The mean values of the point estimates, of their quantiles and standard deviations for the model parameters are presented in Tables 2 to 8 in Appendix B. Table 9 contains means and standard deviations of the latent instantaneous intensity process η and its stochastic long-run mean process γ. The point estimates are taken to be the multivariate median from the posterior distribution of the parameters, conditional on the data (we use the same multivariate median as Collin-Dufresne et al., 2004). The quantiles and the standard deviation are also estimated on the basis of the posterior distribution. Out of 5,000,000 draws from the MCMC sampler only every 1000th draw is recorded to remedy high autocorrelation in the parameter paths. The first 3000 samples are discarded, and the remaining 2000 used in the computation of the point estimate, the quantiles, and the standard deviation.

The estimates are very similar to what one would expect from an analysis of the risk free term structure. Since κ_γ is small, the stochastic long-run mean γ is extremely persistent. Figure 8 shows posterior estimates for the trajectory of the intensity and its long run mean for ticker FDX as an example. While the intensity process varies between 0.001 and 0.005, the long run mean process has its minimum at 0.06 and reaches 0.24 as its maximum level. It can also be seen that persistency is much more pronounced under $\mathbb{Q}$ compared to the situation under $\mathbb{P}$. Nevertheless, it is not necessary to let these processes explode, neither under the pricing measure nor the physical measure, which would be necessary when fitting a panel of CDS premia with a one-factor model. Autocorrelations of the pricing/observation errors are high and reminiscent of what is seen in the riskless market.

Another interesting parameter is θ_γ, the long-run mean of the stochastic long-run mean. Especially under the $\mathbb{Q}$ measure we expect an increasing θ_γ with deteriorating rating. At the same time we are aware that persistency in the processes, which is more pronounced under $\mathbb{Q}$, will drive up variability of the parameter estimates. The fifth and sixth column of Table 8 present our estimates. While the parameter estimates for $\theta_\gamma^{\mathbb{Q}}$ are slightly increasing from AAA to A, we observe a decrease in these parameters from A to BBB followed by a further drastic decrease from BBB to BB and B. For CCC the estimates rise to a level above the AA and BB level. We have to remark that the standard deviations are often larger than the point estimates due to persistency, such that no significant differences with respect to the rating categories can be inferred on statistical grounds. This result is confirmed by running a regression of the parameter estimates for $\theta_\gamma^{\mathbb{Q}}$ on rating and industry. Using either only rating or only industry no significant parameters with p-values smaller than 10% can be observed. On the other hand, if both rating *and* industry are used, the Industrials sector (p-value 10.5%) is close to being significant, but all other variables are insignificant. The same effects are observed under the $\mathbb{P}$ measure, where Industrials exhibit a p-value of 1.8%. Summing up, with respect to the long-run mean of the credit spread neither the rating nor the industry provides us with sufficient information on the credit spreads across sectors. The differences across sectors are too large due to persistency. The only exception are Industrials where significantly higher $\theta_\gamma^{\mathbb{Q}}$ and $\theta_\gamma^{\mathbb{P}}$ are observed.

We observe jumps in different CDS maturities with size approximately proportional to time to maturity. The process η determines the short end, while γ controls the long end of the credit spread term structure. It is therefore economically meaningful to expect similar jump sizes for the long and for the short end. Additionally, preliminary estimates of ζ_η and ζ_γ with both parameters freely varying are poorly identified; we therefore impose the restriction $\zeta_\eta = \zeta_\gamma$, thereby greatly improving the stability of the Metropolis-Gibbs sampler. Joint jump sizes $\zeta_{\eta\gamma}$ are much larger than individual jump sizes under $\mathbb{Q}$. This indicates that empirical fact **B** (see the Introduction) is priced into CDS contracts. Under $\mathbb{P}$ the results are mixed (see Tables 4 and 7). For ζ_η we observe a similar structure with respect to the rating as for θ_γ. On the other hand, significant coefficients for Financials (p-value 0.3%), AA (p-value 11.8%), BB (p-value 1.1%) and B (p-value 9.4%) are observed under the $\mathbb{P}$ measure.

Our model accomodates separate jumps in the risk free and risky processes, as well as simultaneous jumps in the risk free and risky components. The intensity of jumps specific to credit risk components, $l_{\eta\gamma}$, is very much alike across issuers under the $\mathbb{Q}$ measure, while under the $\mathbb{P}$ measure we observe an increase with deteriorating rating. Under the empirical measure we expect between 2 and 10 $\gamma\eta$-jumps in the credit risk processes per year. For the joint jumps we observe a homogenous structure across ratings. The $\gamma\eta r$-jumps arrive at a much lower frequency, averaging about 0.2 to 0.3 jumps per year under both the $\mathbb{Q}$ and $\mathbb{P}$ measures. A regression of $l_{\eta\gamma r}$ on industry and rating shows a significant effect for BB, all other industries and ratings being insignificant. The investment-grade variable has a negative and significant effect on the jump intensities. For $l_{\eta\gamma r}^{\mathbb{Q}}$, p-values are smaller than 10% only for the BB rating. For $l_{\eta\gamma r}^{\mathbb{P}}$, p-values are smaller than 10% for Technology (p-value 8.7%), Telecommunications (p-value 9.3%) and Utilities (p-value 2.3%) sectors. Using investment grade instead of rating slightly alters the p-values under $\mathbb{P}$, but the results remain stable, while investment grade is highly significant under $\mathbb{Q}$.

LGD is very well identified, which confirms the claim in Pan and Singleton (2006) that it is possible to disentangle recovery from default risk when using the recovery of face value formulation. This can be inferred from extremely small confidence bands from the posterior distribution of the LGD parameters. Our estimates shed light on the relation between realized LGDs from the studies in Batterman et al. (2005), Altman and Kishore (1996), Altman et al. (2003), Altman et al. (2004), Acharya et al. (2004) and Singh (2004), and market-implied LGDs from traded CDS premia. Our estimates indicate that there are common determinants behind realized and market-implied LGDs. A Tobit regression analysis of LGD on industry and rating shows a positive and significant effect for the BB, B and CCC ratings. This is in line with the conjecture from Batterman et al. (2005) that recovery is usually disregarded as long as the probability of default is considered low, but can dominate CDS valuation for high-yield and distressed obligors. The coefficient of determination is about 34%. Using investment grade instead of rating results in a highly significant and negative effect, LGD is small for investment-grade issuers (R^2 is 25%). No significant relationship can be shown to industry, however, in a Tobit regression on a constant, industry, and rating. The BB, B and CCC ratings remain highly significant with positive coefficients, while coefficients for industry are too close to zero, with too high standard errors. The constant is estimated at 0.28 with a p-value of 0.0058. To check whether industry should be dropped from the Tobit regression we perform a likelihood-ratio test between a model with a constant and the ratings, and a model with constant, industry and ratings. The test decides for the unrestricted model with constant, industry, and ratings, even though the industry coefficients have too high standard errors for common significance levels. A Wald test rejects the hypothesis that the constant from the Tobit regression is 0.75, a standard setting, at all common significance levels. Adding $\theta_\gamma^{\mathbb{Q}}$ and $\theta_\gamma^{\mathbb{P}}$ to the regression parameters on LGD, we observe a positive and significant impact of the BB, B and CCC ratings and a negative and significant effect for $\theta_\gamma^{\mathbb{P}}$. I.e. the higher the long-run mean of the credit spread $\theta_\gamma^{\mathbb{Q}}$, the smaller the LGD, which is in contrast to existing literature, but is easily explained by the persistency the data exhibit.

Finally, we investigate the posterior trajectories of the instantaneous spread process η and its long-run mean processes γ. Table 9 presents descriptive statistics of these time-series. We perform a panel regression with $\hat{\eta}_t$ resp. $\hat{\gamma}_t$ regressed on lagged terms, the rating history, and the VIX implied volatility index for the corresponding time period. Despite the econometric model being a first-order autoregressive model, the data demands an order-two specification in the regression analysis. A p-value of 12.62% is observed for the rating histories in the regression with γ, the stochastic long-run mean. For the instantaneous default intensity η, rating history is highly significant; the regression parameters are positive. The VIX index is positive and highly significant in both cases. These findings are in line with existing literature. Worse ratings induce higher instantaneous spreads and higher long-run means. Volatility measured by the VIX increases the spreads.

7 Conclusion

In this article we investigate two important questions concerning the pricing of credit default swaps which have not received enough attention in the academic literature so far. One question is related to the nature of the stochastic process used for the default intensity of an obligor when modeling credit risk within the reduced-form framework. The second question relates to the recovery value of the reference asset in a credit event, and its determinants.

For this purpose we estimate a five-factor specification of an affine intensity-based model. The formulation is comprised of a three-factor Andersen et al. (2004)-type observable jump-diffusion model for the riskless short rate of interest and a two-factor jump-diffusion model for the default intensity of an obligor. The model accomodates simultaneous as well as individual jumps in the risk free and credit-risky state variables. The jump size distribution for the credit-risky state variables is multivariate exponential, a feature that is called for by the data. The dataset at our disposal consists of two years of daily observations for CDS spreads in the five most liquid maturities (1y, 3y, 5y, 7y, and 10y). The estimation is engineered by means of Bayesian simulation methods. This methodology reliably provides a distribution of the parameters conditional on the data even in relatively small samples such as ours (only two years in the time-series dimension).

Estimates for the risk free model are similar to the ones reported in Andersen et al. (2004) with respect to the parameters under the physical probablity measure. However, the cross-section of zero yields identifies an "expected" level effect, dependency between short rate variance and the long-run mean of the short rate under the martingale measure. Furthermore we find a large risk premium on the standard deviation of jumps in the short rate.

We find no evidence for linear dependence of credit spreads and stochastic long-run mean and variance of the short rate. The stochastic long-run mean of the intensity itself is extremely persistent, as well as the default intensity itself. Nevertheless, it is not necessary to let the processes explode, neither under the pricing measure nor under the physical measure. Autocorrelations of the pricing errors are very high, similar to pricing errors reported from the risk free literature. By estimating a panel regression of implied stochastic long-run mean processes on the VIX index and rating, both the rating and the VIX have a positive impact on the long-run mean, i.e. the spread increases when the rating deteriorates or the VIX increases. Exactly the same relationship is inferred for the default intensity itself.

Regarding jumps, we estimate two to fifteen events per year when the credit risky components jump. In the risk free term structure we estimate approximately four jumps per year. Simultaneous jumps of the riskfree, and the credit-risky components arrive at a rate of approximately 0.3 jumps per year.

Parameters for loss given default are very well identified, which confirms the claim in Pan and Singleton (2006) that it is possible to disentangle recovery from default risk when using the recovery of face value formulation. This can be inferred from the extremely small confidence band from the posterior distribution of the LGD parameters. On the other hand, the values themselves are widely dispersed within sectors, similar to realized LGDs from earlier studies. Nevertheless, in a regression analysis we observe that 30% of the variance in the LGDs is explained by rating and industry; a likelihood-ratio test supports a significant dependence of the LGDs on rating and industry.

Comparing the mean CDS spread levels with the LGD estimates a positive dependence is detected – the higher the spreads, the higher the LGD (resp. the lower the recovery). The observation is further strengthened by looking at ratings as a crude proxy for credit quality. This finding supports the conjecture by Batterman et al. (2005) that market participants usually do not take into consideration the prospective recovery as long as there is a seemingly low chance of default, but that it dominates CDS pricing once the issuer is in distress. We obtain that a clear-cut distinction in the levels of the LGD shows between investment-grade and speculative-grade issuers.

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A Conditional Densities for MCMC estimation

Most of the full conditional densities in this appendix can be found in Johannes and Polson (2003) and Jones (1998) but we state them here for completeness.

Before we describe the conditional densities necessary to perform a MCMC analysis, we investigate the parameterization of the jump times. Equivalently to Johannes and Polson (2003) and Johannes (2004), we use a time discretization for the jump process. With this approximation jumps are only permitted at the grid points $t = 1, \ldots, T$. More than one jump event of an individual process and joint jumps of exclusive jump component *and* the common jump component are excluded by this approximation. In this setting the jump times are parameterized by means of the latent indicators $J_r(t)$, $J_{\gamma\eta}(t)$ and $J_{r\gamma\eta}(t)$, $t = 1, \ldots, T$. If, only the short rate jumps at t, then $J_r(t) = 1, J_{\gamma\eta}(t) = 0, J_{r\gamma\eta}(t) = 0$, while an exclusive jump in the default intensity results in $J_r(t) = 0, J_{\gamma\eta}(t) = 1, J_{r\gamma\eta}(t) = 0$. For common jumps the indicators are $J_r(t) = 0, J_{\gamma\eta}(t) = 0, J_{r\gamma\eta}(t) = 1$, while with no jumps we get $J_r(t) = 0, J_{\gamma\eta}(t) = 0, J_{r\gamma\eta}(t) = 0$.

Using this parameterization the jump time indicators $\mathbf{J_r} = (J_r(1), \ldots, J_r(T))$, etc. have the binomial distributions $\mathcal{B}(T, l_r)$, $\mathcal{B}(T, l_{\gamma\eta})$ and $\mathcal{B}(T, l_{r\gamma\eta})$, respectively; the corresponding densities are abbreviated by $p(\mathbf{J}_r|l_r)$, $p(\mathbf{J}_{\gamma\eta}|l_{\gamma\eta})$, $p(\mathbf{J}_{r\gamma\eta}|l_{r\gamma\eta})$ where the joint density $p(\mathbf{J}|l_r, l_{\gamma\eta}, l_{r\gamma\eta})$ is the product of the former densities. The correspond jump sizes are $\mathbf{Z}$, where $\mathbf{Z} = (Z_r(1), Z_\gamma(1), Z_\eta(1), \ldots, Z_r(T), Z_\gamma(T), Z_\eta(T))$.

Let θ comprise the parameters of the stochastic processes, Σ_ε, c, e and LGD. Furthermore, we augment the parameter space by the latent processes $\eta = (\eta_1, \ldots, \eta_K)$ and $\gamma = (\gamma_1, \ldots, \gamma_K)$ and the initial values X_0 for the processes η, γ, V, μ, r, such that the conditional density of the first period can be computed. By (5.4) the joint posterior distribution of the parameters is given by

$$
\begin{aligned}
p(s^m, r^\star, \gamma, \eta, \theta \mid s^o, y^\nu) &\propto p(s^o, s^m \mid r^\star, \gamma, \eta, \theta, X_0)\, p(y^\nu, \lambda, \theta) && \text{(A.1)}\\
&\propto p(s \mid r^\star, \gamma, \eta, \theta, X_0)\, p(y^\nu, \gamma, \eta \mid \theta, X_0)\, p(\theta) p(X_0) && \text{(A.2)}\\
&= p(s \mid r^\star, \lambda, \theta)\, \frac{1}{\det |H|} p(r^\star \mid \theta, X_0)\, p(\gamma, \eta \mid \theta, X_0)\, p(\theta) p(X_0), && \text{(A.3)}
\end{aligned}
$$

Note that the first densities are determined by the model assumption while by $p(\theta)$ and $p(X_0)$ have to determined exogenously. For these density we use uninformative priors.

A.1 Riskless Model

We assume that the 6m, 2y, and 5y zero yields are observed without error. The remaining zero yields are assumed to be observed with normally distributed i.i.d errors $\varrho_t \sim MVN(0, \Sigma_\varrho)$. In total we use 7 yields for maturities $\nu = (6m, 2y, 5y, 1m, 1y, 7y, 10y)$. We stack the solutions to the pricing ODE's for our risk free model (4.1) into matrices $G^\star, H^\star$ and G, H, where

$$
\begin{aligned}
G = -\begin{pmatrix} \alpha(\nu_1)/\nu_1 \\ \alpha(\nu_2)/\nu_2 \\ \alpha(\nu_3)/\nu_3 \end{pmatrix}, H = -\begin{pmatrix} \beta_V(\nu_1)/\nu_1 & \beta_\mu(\nu_1)/\nu_1 & \beta_r(\nu_1)/\nu_1 \\ \beta_V(\nu_2)/\nu_2 & \beta_\mu(\nu_2)/\nu_2 & \beta_r(\nu_2)/\nu_2 \\ \beta_V(\nu_3)/\nu_3 & \beta_\mu(\nu_3)/\nu_3 & \beta_r(\nu_3)/\nu_3 \end{pmatrix}\\
G^\star = -\begin{pmatrix} \alpha(\nu_4)/\nu_4 \\ \vdots \\ \alpha(\nu_N)/\nu_N \end{pmatrix}, H^\star = -\begin{pmatrix} \beta_V(\nu_4)/\nu_4 & \beta_\mu(\nu_4)/\nu_4 & \beta_r(\nu_4)/\nu_4 \\ \vdots & & \\ \beta_V(\nu_N)/\nu_N & \beta_\mu(\nu_N)/\nu_N & \beta_r(\nu_N)/\nu_N \end{pmatrix}.
\end{aligned} \qquad \text{(A.4)}
$$

Using the matrices from (A.4) it is now possible to write

$$\begin{pmatrix} V_t \\ \mu_t \\ r_t \end{pmatrix} = H^{-1} \left[\begin{pmatrix} y_t^{\nu_1} \\ y_t^{\nu_2} \\ y_t^{\nu_3} \end{pmatrix} - G \right].$$

A.1.1 Drawing $\mathbb{Q}$ Parameters for the Riskless Model

We draw the $\mathbb{Q}$ parameters with Metropolis steps. The target density for σ_V and σ_μ is determined by the likelihood of the observation errors and the transition densities of the yields that are observed without error:

$$\frac{1}{\det |H|} p(V_t, \mu_t, r_t \mid V_{t-1}, \mu_{t-1}, r_{t-1}, \theta) p(\varrho_t \mid \theta).$$

The likelihood for the remaining parameters is determined by $p(\varrho_t \mid \theta)$ by the independence assumption.

A.1.2 Drawing $\mathbb{P}$ Parameters for the Riskless Model

Our data is available on a daily basis. Many studies show that an Euler discretization is innocuous in this case (see for example Jones, 2003; Duffee and Stanton, 2004; Schneider, 2006, for empirical studies), in particular with affine systems like (4.1). Conditioning on jump times and jumpsizes we can write the dynamics in regression form and draw the parameters with Gibbs steps.

A.2 Default Risky model

A.2.1 Drawing the Recovery Rate

From equation (5.3) one observes that, conditional on other parameters and all state variables, as well as the data, missing and observed, the LGD is a coefficient in a panel regression model. We assume a truncated normal prior, where the truncation function is $\mathbb{1}_{\{LGD \in [0,1]\}}$. A standard Gibbs sampler is employed to sample from the posterior distribution.

A.2.2 Drawing Missing CDS Premia

The target density is

$$p(s^m \mid s^o, r^\star, \eta, \gamma, \theta) \propto p(s^m, s^o \mid r^\star, \eta, \gamma, \theta) = p(s \mid r^\star, \eta, \gamma, \theta),$$

which is a normal density from equation (5.3). Proposing from a random walk we draw missing prices $s^m_{(g+1)}$ one by one with probability

$$\alpha = \min \left\{ \frac{p(s^o, s^m_{(g+1)} \mid r^\star, \eta, \gamma, \theta)}{p(s^o, s^m_{(g)} \mid r^\star, \eta, \gamma, \theta)}, 1 \right\}.$$

A.2.3 Drawing Intensity Realizations

Since intensities cannot be directly inverted from CDS prices as a consequence of the observation error in equation (5.3), we draw realizations from the latent time-series with a Metropolis step conditional on the parameters, CDS prices, risk free state variables, jump times, jump sizes and LGD. The target density is given by:

$$\begin{aligned}
p(\eta_t, \gamma_t \mid \eta_{\backslash t}, \gamma_{\backslash t}, r^\star, \bar{s}, J, Z, \theta) &\propto p(\eta_t, \gamma_t, \bar{s}_t \mid \eta_{\backslash t}, \gamma_{\backslash t}, \bar{s}_{\backslash t}, r^\star, J, Z, \theta) \\
&\propto p(\bar{s}_t \mid \eta_t, \gamma_t, r^\star, \theta)\; p(\eta_t, \gamma_t \mid \eta_{t+1}, \gamma_{t+1}, \eta_{t-1}, \gamma_{t-1}, r^\star, J, Z, \theta) \\
&\propto p(\bar{s}_t \mid \eta_t, \gamma_t, r^\star, \theta)\; p(\eta_{t+1}, \gamma_{t+1} \mid \eta_t, \gamma_t, r^\star, J_{t+1}, Z_{t+1}, \theta) \times \\
&\quad \times p(\eta_t, \gamma_t \mid \eta_{t-1}, \gamma_{t-1}, r^\star, J_t, Z_t, \theta).
\end{aligned}$$

From the Euler discretization the above density is a product of normal densities and is thus easy to sample. We employ random-walk proposals and accept $(\eta_t^{(g+1)}, \gamma_t^{(g+1)})$ with probability

$$\begin{aligned}
\alpha = \min \Bigg\{ 1, \; & \frac{p(\bar{s}_t \mid \eta_t^{(g+1)}, \gamma_t^{(g+1)}, r, \theta)\; p(\eta_{t+1}, \gamma_{t+1} \mid \eta_t^{(g+1)}, \gamma_t^{(g+1)}, r^\star, J_{t+1}, Z_{t+1}, \theta)}{p(\bar{s}_t \mid \eta_t^{(g)}, \gamma_t^{(g)}, r^\star, \theta)\; p(\eta_{t+1}, \gamma_{t+1} \mid \eta_t^{(g)}, \gamma_t^{(g)}, r^\star, J_{t+1}, Z_{t+1}, \theta)} \times \\
& \times \frac{p(\eta_t^{(g+1)}, \gamma_t^{(g+1)} \mid \eta_{t-1}, \gamma_{t-1}, r^\star, J_t, Z_t, \theta)}{p(\eta_t^{(g)}, \gamma_t^{(g)} \mid \eta_{t-1}, \gamma_{t-1}, r^\star, J_t, Z_t, \theta)} \Bigg\}.
\end{aligned}$$

The target density for the first realization of the default intensity, (η_0, γ_0), is (the first CDS premia are observed at $t = 1$):

$$p(\eta_0, \gamma_0 \mid \eta_{\backslash 0}, \gamma_{\backslash 0}, r^\star, \bar{s}, J, Z, \theta) \propto p(\eta_1, \gamma_1 \mid \eta_0, \gamma_0, J_1, Z_1, \theta)\; p(\eta_0, \gamma_0 \mid \theta).$$

We employ random-walk proposals and accept the $(g+1)$-st Metropolis step with probability

$$\alpha = \min \left\{ \frac{p(\eta_1, \gamma_1 \mid \eta_0^{(g+1)}, \gamma_0^{(g+1)}, J_1, Z_1, \theta)\; p(\eta_0^{(g+1)}, \gamma_0^{(g+1)} \mid \theta)}{p(\eta_1, \gamma_1 \mid \eta_0^{(g)}, \gamma_0^{(g)}, J_1, Z_1, \theta)\; p(\eta_0^{(g)}, \gamma_0^{(g)} \mid \theta)}, 1 \right\}.$$

The target density of (η_T, γ_T) is given by

$$\begin{aligned}
p(\eta_T, \gamma_T \mid r^\star, \bar{s}, \eta_{\backslash T}, \gamma_{\backslash T}, J, Z, \theta) &\propto p(\eta_T, \gamma_T, \bar{s}_T \mid \eta_{\backslash T}, \gamma_{\backslash T}, r^\star, \bar{s}_{\backslash T}, \theta) \\
&= p(\bar{s}_T \mid \eta_T, \gamma_T, r_T^\star, \theta)\; p(\eta_T, \gamma_T \mid \eta_{T-1}, \gamma_{T-1}, J_T, Z_T, \theta).
\end{aligned}$$

Proposing from $p(\eta_T, \gamma_T \mid \eta_{T-1}, \gamma_{T-1}, J_T, Z_T, \theta)$ we accept the $(g+1)$-th Metropolis step with probability

$$\alpha = \min \left\{ \frac{p(\bar{s}_T \mid \eta_T^{(g+1)}, \gamma_T^{(g+1)}, r_T^\star, \theta)}{p(\bar{s}_T \mid \eta_T^{(g)}, \gamma_T^{(g)}, r_T^\star, \theta)}, 1 \right\}.$$

A.2.4 Drawing Jump Times and Jump Sizes

For the jump times use a proxy based on the parameterization on the discrete grid (see Johannes and Polson (2003) and Johannes (2004)). It is necessary to split-up the estimation into an estimation of the risk free and the risky term structure, respectively. Due to our model assumptions, we consider separate jumps and joint jumps in both structures. When only the risk free term structure is investigated we cannot identify a pure jump in the risk free term structure or a joint jump. Thus, with exponentially distributed arrival intervals with parameters l_r and $l_{\eta\gamma r}$, the distribution of inter-arrival times is derived as follows. The arrival times are τ_1 and τ_2. From the properties of our arrival times, both arrival times do not depend on past history. Therefore, the probability of a jump before t (at least one) corresponds to the event $Prob(\tau_1 \vee \tau_2 \leq t) = Prob(min(\tau_1, \tau_2) \leq t) = 1 - Prob(\tau_1 \wedge \tau_2 \geq t)$. For independent arrival times, like on our case, this results in $Prob(\tau_1 \wedge \tau_2 \leq t) = Prob(\tau_1 \geq t)Prob(\tau_2 \geq t) = \bar{F}_{l_r}(t)\bar{F}_{l_{\eta\gamma r}}(t)$. For exponentially distributed random variables $\bar{F}_{l_r}(t) = \exp(-l_r t)$. This results in $Prob(min(\tau_1, \tau_2) \leq t) = 1 - \exp(-(l_r + l_{\eta\gamma r})t)$. I.e. the arrival time are exponential with parameter $l_r + l_{\eta\gamma r}$. This sum can be estimated in the first estimation step.

Now the updates can be performed by the Metropolis Hastings algorithm. For each t we propose a jump event $J_.^{(g+1)}(t)$ from a Bernoulli distribution, i.e. $q(J_.(t)) \sim \mathcal{B}(0.5)$. If $J_.^{(g)}(t) = J_.^{(g+1)}(t)$ we continue with $t+1$. Otherwise we accept $J_.(t)$ with probability

$$\min\left\{\frac{p(\mathbf{J}_.^{(g+1)}(t) \mid l_r, l_{\gamma\eta}, l_{r\gamma\eta})p(V_t, \mu_t, r_t, \gamma_t, \eta_t \mid V_{t-1}, \mu_{t-1}, r_{t-1}, \gamma_{t-1}, \eta_{t-1}, Z_t, \mathbf{J}_.^{(g+1)}(t), \theta)}{p(\mathbf{J}_.^{(g)}(t) \mid l_r, l_{\gamma\eta}, l_{r\gamma\eta})p(V_t, \mu_t, r_t, \mid V_{t-1}, \mu_{t-1}, r_{t-1}, \gamma_{t-1}, \eta_{t-1}, Z_t, \mathbf{J}_.^{(g)}(t), \theta)}, 1\right\} .$$

Updates of blocks of jump times work in the same way. By using a $Beta(\alpha_0, \beta_0)$ prior for l_r, $l_{\gamma\eta}$, and $l_{r\gamma\eta}$, samples can be obtained by means Gibbs sampling, e.g. if N_{rJ} is the number of exclusive jumps in short rate, then $\pi(l_r|\mathbf{J_r}) = Beta(\alpha_0 + N_{rJ}, T - N_{rJ} + \beta_0)$. Equivalently we sample l_λ and $l_{r\lambda}$.

Next we investigate the update of the jump sizes $\mathbf{Z}$. Uncorrelated jump sizes, i.e. $\zeta_{\gamma\eta} = 0$, or the single factor setting where the jump sizes of the default intensity are $\mathcal{E}xp(\zeta_\eta)$ are special cases of the following. Jump sizes in the risk free model are normally distributed $N(0, \sigma^2_{J\mathbb{P}})$. Updates of $Z_r(t)$ and $\sigma^2_{J\mathbb{P}}$ by means of the Metropolis Hasting algorithm are straightforward, i.e. we accept jumpsize $Z_r^{(g+1)}(t)$ with probability

$$\min\left\{\frac{p(Z_r^{(g+1)}(t) \mid \sigma^2_{J\mathbb{P}})p(V_t, \mu_t, r_t \mid V_{t-1}, \mu_{t-1}, r_{t-1}, J_r(t), Z_r^{(g+1)}(t), \theta)}{p(Z_r^{(g)}(t) \mid \sigma^2_{J\mathbb{P}})p(V_t, \mu_t, r_t \mid V_{t-1}, \mu_{t-1}, r_{t-1}, J_r(t), Z_r^{(g)}(t), \theta)}, 1\right\},$$

where $p(Z_t \mid \sigma^2_{J\mathbb{P}})$ is a normal $N(0, \sigma^2_{J\mathbb{P}})$ and $p(Z_r^{(g+1)}(t) \mid \sigma^2_{J\mathbb{P}})p(V_t, \mu_t, r_t \mid V_{t-1}, \mu_{t-1}, r_{t-1}, J_r(t), Z_r^{(g+1)}(t), \theta)$ is also normal from the Euler approximation. By using a gamma conjugate prior, samples of $\sigma^2_{J\mathbb{P}}$ are derived by Gibbs-sampling.

In contrast to independent exponential random variables, where an application of the Metropolis Hastings algorithm is straightforward, the bivariate exponential density exhibits a singular part such that we have to deal with a non-standard Metropolis-Hastings problem; in Gottardo and Raftery (2006) it is shown, however, that for mixtures of mutually singular distributions the MH scheme can be applied. The joint density of the jump sizes in the η and γ processes is given by

$$\pi(\mathbf{Z}|\zeta_\eta, \zeta_\gamma, \zeta_{\gamma\eta}) = \Pi_{t=1}^T \mathcal{E}_{BVE}(Z_\gamma(t), Z_\eta(t); \zeta_\eta, \zeta_\gamma, \zeta_{\gamma\eta}) . \tag{A.5}$$

Fist, the distribution function is given by:

$$
\begin{aligned}
F_{BVE}(Z_\gamma(t), Z_\eta(t); \zeta_\eta, \zeta_\gamma, \zeta_{\gamma\eta}) &= 1 - \left(\frac{1/\zeta_\eta + 1/\zeta_\gamma}{\zeta} F_\alpha(Z_\gamma(t), Z_\eta(t)) + \frac{1/\zeta_{\gamma\eta}}{\zeta} F_c(Z_\gamma(t), Z_\eta(t)) \right) \\
&= 1 - (w F_\alpha(Z_\gamma(t), Z_\eta(t)) + (1-w) F_c(Z_\gamma(t), Z_\eta(t))) \qquad \text{(A.6)} \\
&\quad \text{with} \\
\zeta &= 1/\zeta_\eta + 1/\zeta_\gamma + 1/\zeta_{\eta\gamma} \text{ and } w = \frac{1/\zeta_\eta + 1/\zeta_\gamma}{\zeta}
\end{aligned}
$$

$$
\begin{aligned}
F_\alpha(Z_\gamma(t), Z_\eta(t)) &= \frac{1/\zeta_\eta + 1/\zeta_\gamma + 1/\zeta_{\gamma\eta}}{1/\zeta_\eta + 1/\zeta_\gamma} \exp(-1/\zeta_\eta Z_\gamma(t) - 1/\zeta_\gamma Z_\eta(t) - 1/\zeta_{\gamma\eta} \max\{Z_\gamma(t), Z_\eta(t)\}) \\
&\quad - \frac{1/\zeta_{\gamma\eta}}{1/\zeta_\eta + 1/\zeta_\gamma} \exp(-1/(\zeta_\eta + \zeta_\gamma + \zeta_{\gamma\eta}) \max\{Z_\gamma(t), Z_\eta(t)\}) \\
F_c(Z_\gamma(t), Z_\eta(t)) &= \exp(-1/(\zeta_\eta + \zeta_\gamma + \zeta_{\gamma\eta}) \max\{Z_\gamma(t), Z_\eta(t)\}) \qquad \text{(A.7)}
\end{aligned}
$$

where F_α is the absolute continuous part (with respect to the Lebesque measure Λ) and F_c is the singular part of this distribution. Note, that the support of the singular part of this distribution is the diagonal $Z_\gamma(t) = Z_\eta(t)$. I.e. in addition to $(Z_\gamma(t), Z_\eta(t)) \in \mathrm{R}^{2+}$ where $Prob(Z_\gamma(t) = Z_\eta(t) = 0$ we have a set where $(Z_\gamma(t) = Z_\eta(t))$ with a probability of $\zeta_{\gamma\eta}/\zeta > 0$. Since $\max\{Z_\gamma(t), Z_\eta(t)\}= Z_\gamma(t) = Z_\eta(t) = O$ on the diagonal, the jump sizes are exponentially distributed with parameter ζ. The density of the jump sizes with respect to $(\delta + \Lambda)$ is now given by

$$
\mathcal{E}_{BVE}(Z_\gamma, Z_\eta; \zeta_\eta, \zeta_\gamma, \zeta_{\gamma\eta}) = (1-w)\delta(Z_\gamma, Z_\eta) \exp(-\zeta Z_\gamma) + w(1 - \delta(Z_\gamma, Z_\eta)) f_\alpha(Z_\gamma, Z_\eta) \qquad \text{(A.8)}
$$

$$
f_\alpha(Z_\gamma, Z_\eta; \zeta_\eta, \zeta_\gamma, \zeta_{\gamma\eta}) = \begin{cases} \frac{1/\zeta_\eta(1/\zeta_\gamma + 1/\zeta_{\gamma\eta})}{w}(1 - F_{BVE}(Z_\gamma, Z_\eta; \zeta_\eta, \zeta_\gamma, \zeta_{\gamma\eta})) & \text{iff } Z_\gamma > Z_\eta \\ \frac{1/\zeta_\gamma(1/\zeta_\eta + 1/\zeta_{\gamma\eta})}{w}(1 - F_{BVE}(Z_\gamma, Z_\eta; \zeta_\eta, \zeta_\gamma, \zeta_{\gamma\eta})) & \text{iff } Z_\gamma < Z_\eta \end{cases}
$$

where δ is the Dirac measure; $\delta(Z_\gamma, Z_\eta) = 1$ iff $Z_\gamma = Z_\eta$ otherwise it is zero and $f_\alpha(.)$ is the density of the absolutely continuous part.

According to Gottardo and Raftery (2006) the Metropolis-Hastings algorithm can now be applied, where we have to ensure that draws from the absolutely continuous as well as the singular part are possible. We construct our proposal as follows:

1. We perform a Bernoulli trial with probability w_q, i.e. we a probability of w_q we sample from the absolutely continuous part, with $1 - w_q$ from the singular. If the latter case is the result of the Bernoulli trail $\delta(Z_\gamma, Z_\eta)) = 0$, in the former case it is one. For our application we choose $w_q = 1/2$

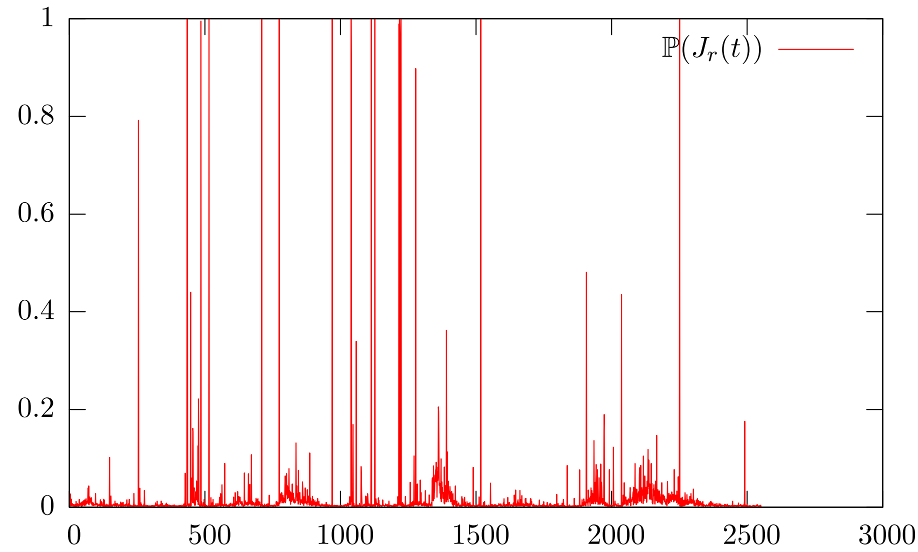
2. When sampling from the singular part we propose $Z_\gamma^{(g+1)} = Z_\eta^{(g+1)}$ from $\log(0.5(Z_\gamma + Z_\eta)) + c_z\varepsilon$, where ε is standard normal

3. When sampling from the absolutely continuous part we propose $Z_\gamma^{(g+1)}, Z_\eta(g+1)$ from $\log(Z_\gamma)+c_z\varepsilon_1$ and $\log(Z_\eta) + c_z\varepsilon_2$

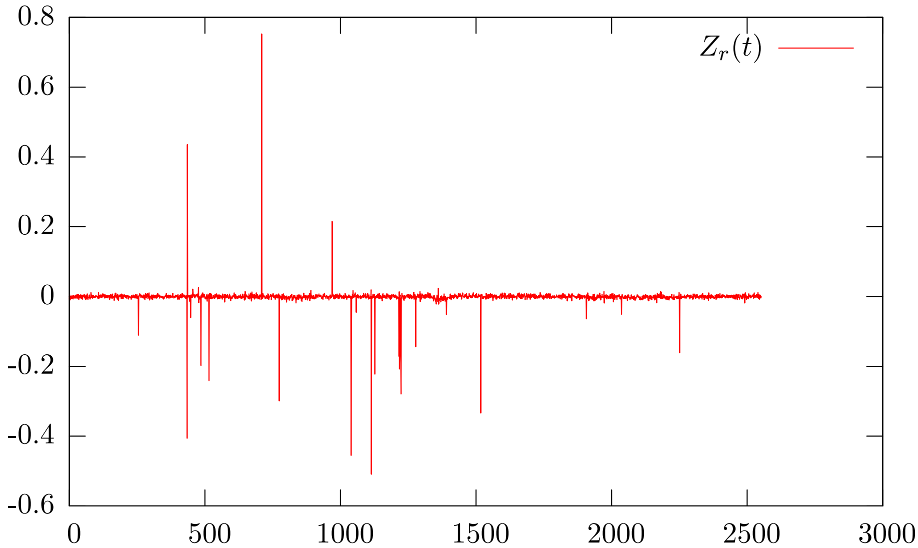
B Parameter Estimates

Parameter	2.5% Quantile	Point Estimate	97.5% Quantile
σ_V	0.00139602	0.00156328	0.00168477
σ_μ	0.105893	0.111559	0.12109
κ_V	0.0448812	0.0516368	0.0609556
$\kappa\theta_V$	2.18186e-06	2.48114e-06	2.83792e-06
$\kappa_{\mu V}$	-947.79	-887.084	-801.309
$\kappa\theta_\mu$	0.00597344	0.0073798	0.00901734
κ_μ	0.516936	0.54546	0.559314
κ_r	1.84877	1.93549	2.03173
l_r	1.86648	3.56638	13.0233
σ_J	0.046547	0.0933689	0.126679
a_0	-13.4039	-13.34	-13.2835
a_1	-1.06222	-1.01301	-0.97078
a_2	0.0889003	0.0933726	0.0985766
$\kappa_V^{\mathbb{P}}$	0.277158	0.958854	1.83104
$\kappa\theta_V^{\mathbb{P}}$	7.47689e-06	2.69525e-05	5.42897e-05
$\kappa_{\mu V}^{\mathbb{P}}$	-1727.58	-1061.4	-42.0052
$\kappa\theta_\mu^{\mathbb{P}}$	0.00742807	0.0179177	0.0485912
$\kappa_\mu^{\mathbb{P}}$	0.521018	1.03673	1.69241
$\kappa_r^{\mathbb{P}}$	0.254746	7.33498	15.6641
$l_{r\mathbb{P}}/250$	0.00790119	0.0141274	0.0549086
$\sigma_{J\mathbb{P}}$	0.00224649	0.00301166	0.00383766

Table 1: Posterior Estimates for Model (4.1) **and** (4.2)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for definition of multivariate posterior median). Estimates are based on 10 years of daily panel data of zero youpon yields bootstrapped from US swap rates. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5000 draws only the last 3000 are taken into the computation.



(a) Jump Flags $J_r(t)$



(b) Jump Sizes $Z_r(t)$ in %

Figure 2: Posterior Estimates for Jump Times and Jump Flags for Model (4.2)**:** This figure displays the posterior mean of daily realizations of the discretized processes $J_r(t)$ and $Z_r(t)$ from (5.1). From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw is recorded to remedy high autocorrelation in the parameter sample paths. From the remaining 5000 draws only the last 3000 are taken into the computation.

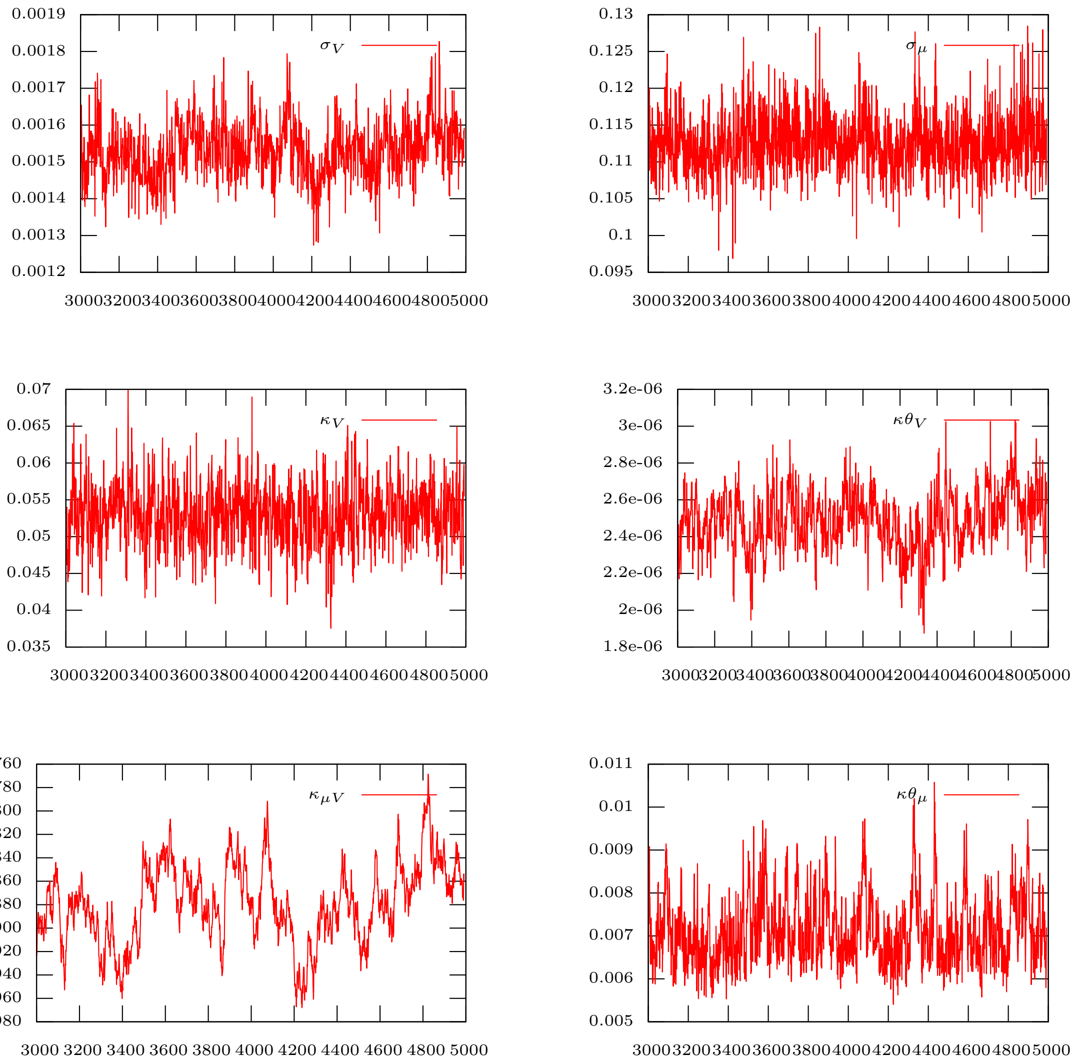


Figure 3: Posterior Estimates for Model (4.1) **and** (4.2)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for definition of multivariate posterior median). Estimates are based on 10 years of daily panel data of zero youpon yields bootstrapped from US swap rates. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5000 draws only the last 3000 are taken into the computation of the quantiles and the multivariate posterior median.

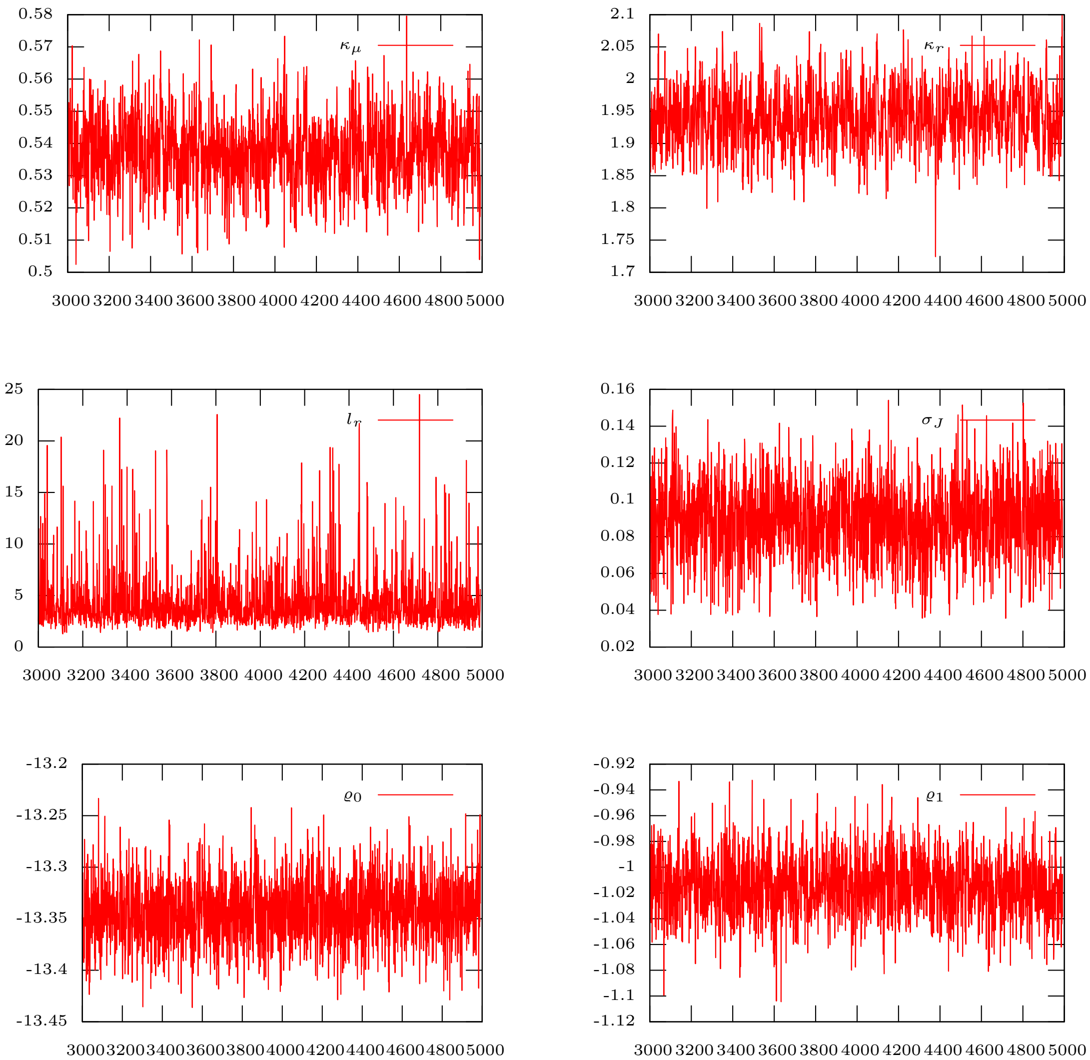


Figure 4: Posterior Estimates for Model (4.1) **and** (4.2)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for definition of multivariate posterior median). Estimates are based on 10 years of daily panel data of zero youpon yields bootstrapped from US swap rates. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5000 draws only the last 3000 are taken into the computation of the quantiles and the multivariate posterior median.

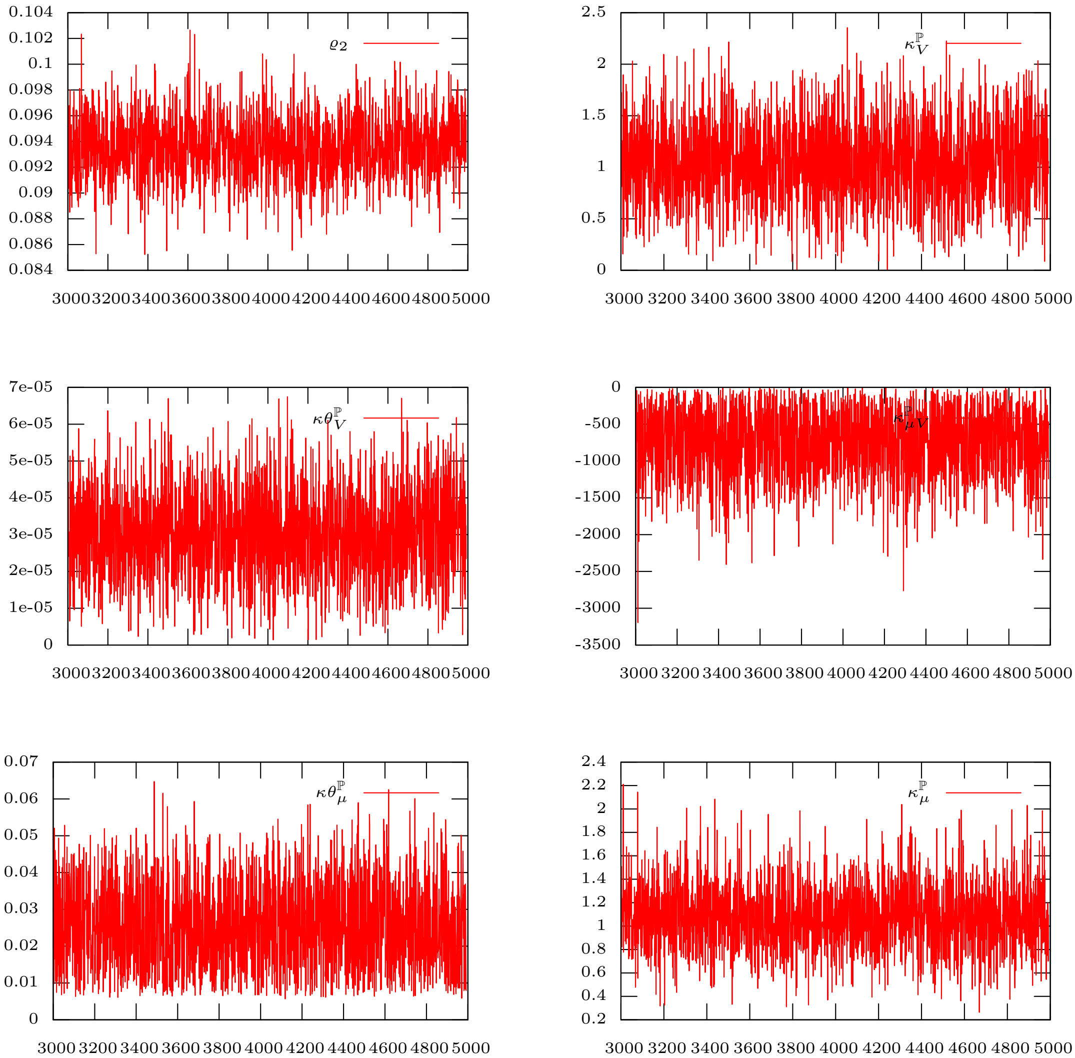


Figure 5: Posterior Estimates for Model (4.1) **and** (4.2)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for definition of multivariate posterior median). Estimates are based on 10 years of daily panel data of zero youpon yields bootstrapped from US swap rates. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5000 draws only the last 3000 are taken into the computation of the quantiles and the multivariate posterior median.

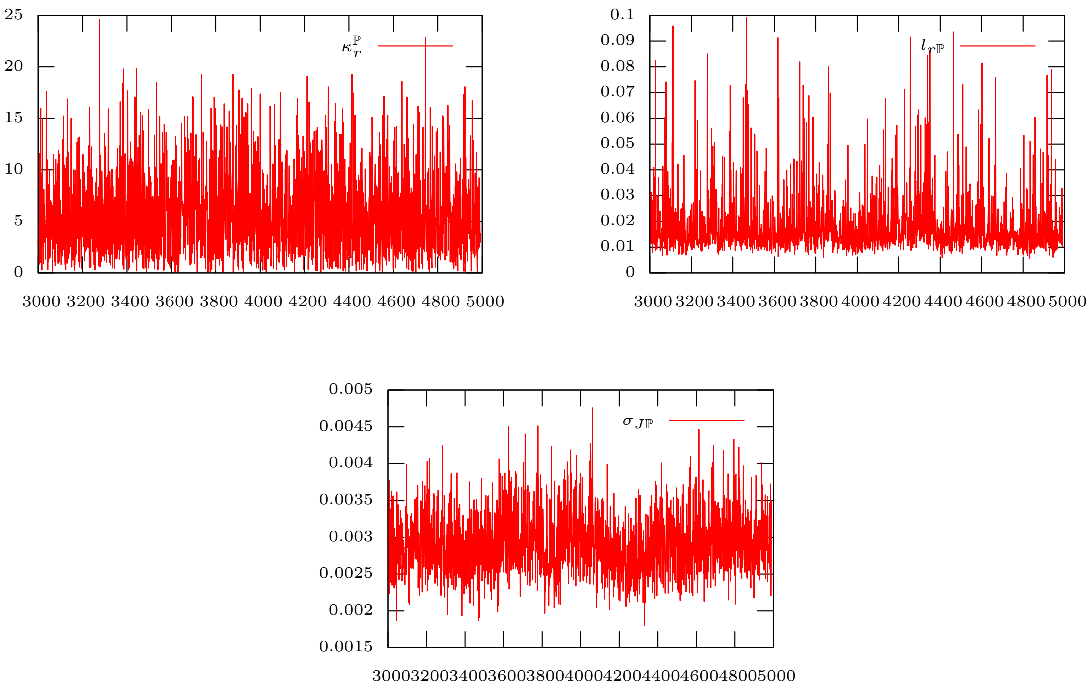
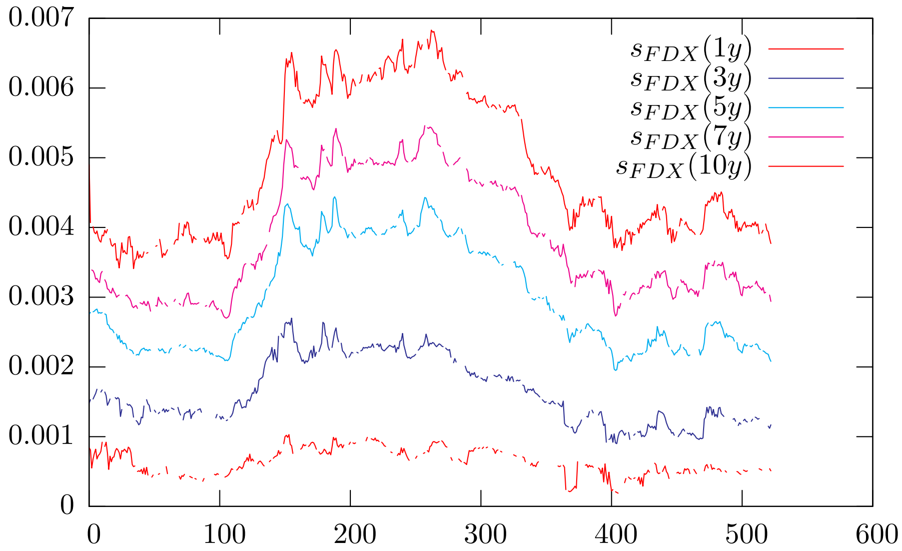
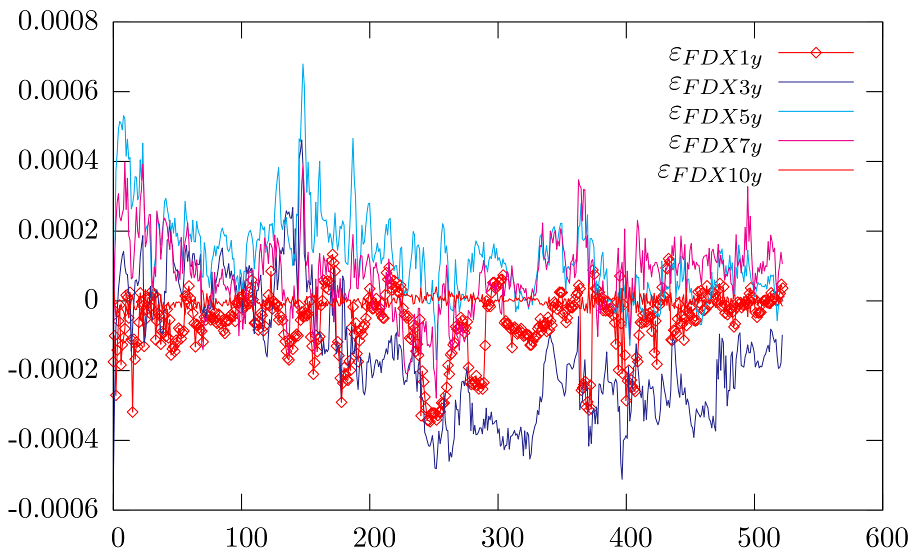


Figure 6: Posterior Estimates for Model (4.1) and (4.2): This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for definition of multivariate posterior median). Estimates are based on 10 years of daily panel data of zero youpon yields bootstrapped from US swap rates. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5000 draws only the last 3000 are taken into the computation of the quantiles and the multivariate posterior median.

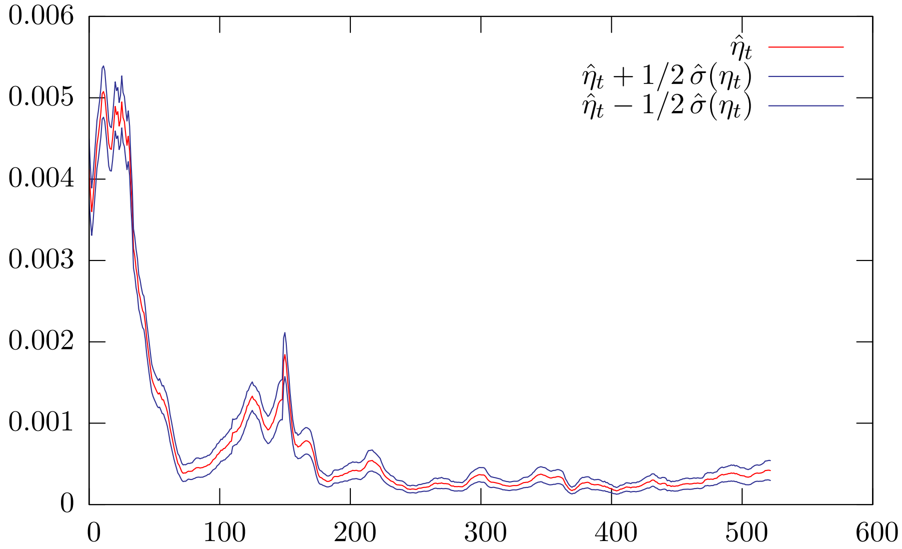


(a) CDS premia

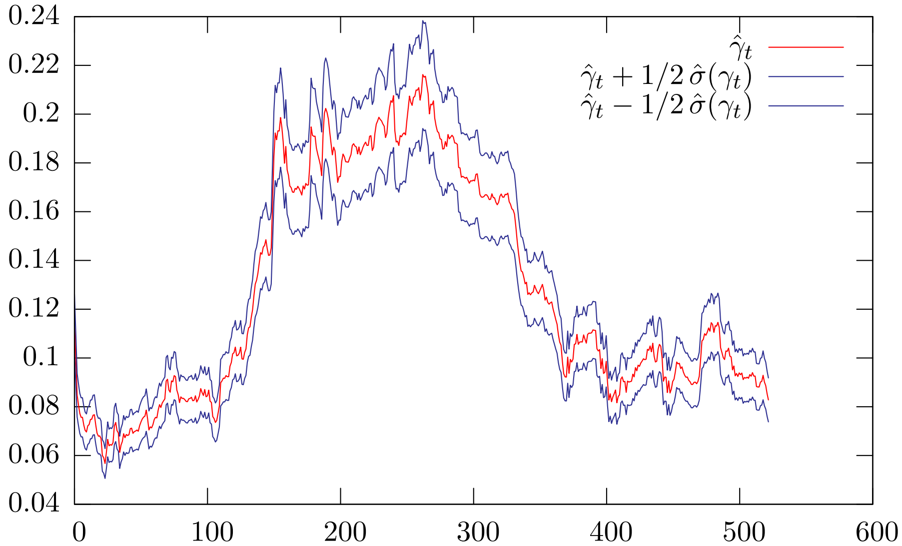


(b) Pricing Errors

Figure 7: CDS premia and Pricing Errors: The figures display observed CDS premia for FDX and posterior pricing errors implied by specification (4.3) through (4.5). Missing and stale premia are treated as missing values. They exhibit strong autocorrelation.



(a) Trajectory of η process for FDX



(b) Trajectory of γ process for FDX

Figure 8: Posterior Estimates for Realizations of γ and η processes conditional on FDX CDS premia and Missing Data: The figures display posterior realizations of γ and η processes conditional on FDX CDS premia, missing values and specification (4.3) through (4.5). The blue lines depict the standard deviation of the $\hat{\gamma}$ and $\hat{\eta}$ draws. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw is recorded to remedy high autocorrelation in the parameter paths. From the remaining 5000 draws only the last 3000 draws from the time-series are used for the computation.

	$Q(\sigma_\gamma;0.025)$	σ_γ	$Q(\sigma_\gamma;0.975)$	$Q(\kappa\theta_\gamma;0.025)$	$\kappa\theta_\gamma$	$Q(\kappa\theta_\gamma;0.025)$	$Q(\kappa_\gamma;0.025)$	κ_γ	$Q(\kappa_\gamma;0.975)$
AAA	0.0491	0.0542	0.0611	0.0025	0.0029	0.0032	0.0045	0.0058	0.0089
	(0.0315)	(0.0366)	(0.0443)	(0.0026)	(0.0033)	(0.0038)	(0.0077)	(0.0085)	(0.0093)
AA	0.0829	0.1011	0.1193	0.0076	0.0108	0.0146	0.0069	0.0106	0.0207
	(0.0641)	(0.0722)	(0.0764)	(0.0102)	(0.0142)	(0.0167)	(0.0220)	(0.0232)	(0.0254)
A	0.1735	0.2020	0.2315	0.0403	0.0507	0.0612	0.0068	0.0097	0.0173
	(0.1010)	(0.1190)	(0.1342)	(0.0356)	(0.0442)	(0.0531)	(0.0170)	(0.0194)	(0.0232)
BBB	0.1658	0.1912	0.2170	0.0403	0.0502	0.0634	0.0228	0.0278	0.0351
	(0.1333)	(0.1562)	(0.1827)	(0.0583)	(0.0705)	(0.0902)	(0.0361)	(0.0404)	(0.0441)
BB	0.0861	0.0963	0.1058	0.0091	0.0119	0.0146	0.0402	0.0447	0.0520
	(0.0872)	(0.1037)	(0.1192)	(0.0241)	(0.0321)	(0.0392)	(0.0407)	(0.0429)	(0.0447)
B	0.0917	0.1064	0.1288	0.0059	0.0082	0.0129	0.0518	0.0680	0.0786
	(0.0567)	(0.0700)	(0.0948)	(0.0077)	(0.0099)	(0.0169)	(0.0420)	(0.0441)	(0.0533)
CCC	0.1127	0.1233	0.1329	0.0082	0.0101	0.0129	0.0540	0.0578	0.0649
	(0.0667)	(0.0784)	(0.0889)	(0.0086)	(0.0112)	(0.0152)	(0.0483)	(0.0517)	(0.0478)
without	0.1749	0.2013	0.2303	0.0326	0.0425	0.0543	0.0072	0.0141	0.0267
	(0.0850)	(0.1071)	(0.1238)	(0.0288)	(0.0435)	(0.0581)	(0.0154)	(0.0172)	(0.0225)
Basic Materials	0.1708	0.1951	0.2215	0.0357	0.0441	0.0544	0.0187	0.0218	0.0253
	(0.1270)	(0.1480)	(0.1687)	(0.0474)	(0.0572)	(0.0720)	(0.0336)	(0.0374)	(0.0393)
Consumer Goods	0.1155	0.1325	0.1497	0.0188	0.0243	0.0296	0.0255	0.0295	0.0365
	(0.0950)	(0.1114)	(0.1244)	(0.0343)	(0.0426)	(0.0496)	(0.0315)	(0.0344)	(0.0363)
Consumer Services	0.1720	0.1955	0.2205	0.0463	0.0557	0.0690	0.0165	0.0205	0.0252
	(0.1488)	(0.1730)	(0.2002)	(0.0648)	(0.0762)	(0.0958)	(0.0296)	(0.0329)	(0.0358)
Financials	0.1077	0.1247	0.1431	0.0156	0.0196	0.0251	0.0255	0.0315	0.0409
	(0.0785)	(0.0962)	(0.1174)	(0.0244)	(0.0306)	(0.0384)	(0.0331)	(0.0391)	(0.0438)
Government	0.0363	0.0398	0.0434	0.0016	0.0016	0.0017	0.0133	0.0155	0.0197
	-	-	-	-	-	-	-	-	-
Health Care	0.1059	0.1248	0.1435	0.0137	0.0183	0.0246	0.0154	0.0246	0.0424
	(0.0636)	(0.0722)	(0.0832)	(0.0172)	(0.0198)	(0.0245)	(0.0283)	(0.0373)	(0.0459)
Industrials	0.2120	0.2425	0.2793	0.0563	0.0699	0.0861	0.0066	0.0107	0.0165
	(0.1177)	(0.1372)	(0.1564)	(0.0491)	(0.0619)	(0.0765)	(0.0191)	(0.0218)	(0.0244)
Oil & Gas	0.1583	0.1991	0.2300	0.0312	0.0465	0.0602	0.0252	0.0297	0.0422
	(0.1056)	(0.1354)	(0.1585)	(0.0323)	(0.0484)	(0.0636)	(0.0434)	(0.0463)	(0.0493)
Technology	0.1265	0.1436	0.1566	0.0192	0.0235	0.0271	0.0527	0.0576	0.0650
	(0.0734)	(0.0916)	(0.1025)	(0.0246)	(0.0305)	(0.0343)	(0.0635)	(0.0652)	(0.0682)
Telecommunications	0.2257	0.2625	0.3126	0.0611	0.0767	0.1036	0.0051	0.0183	0.0268
	(0.1428)	(0.1653)	(0.2011)	(0.0733)	(0.0905)	(0.1286)	(0.0121)	(0.0381)	(0.0512)
Utilities	0.0957	0.1101	0.1244	0.0135	0.0174	0.0212	0.0422	0.0495	0.0600
	(0.0772)	(0.0938)	(0.1107)	(0.0270)	(0.0351)	(0.0424)	(0.0420)	(0.0448)	(0.0440)

Table 2: Posterior Estimates for Model (4.3) **and** (4.5)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for details). Estimates are based on 2 years of daily panel data CDS premia written on 282 US firms. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw was recorded to remedy high autocorrelation in the parameter paths. From the remaining 3000 draws only the last 2000 were taken into the computation.

	$Q(\sigma_\eta; 0.025)$	σ_η	$Q(\sigma_\eta; 0.975)$	$Q(\kappa_\eta; 0.025)$	κ_η	$Q(\kappa_\eta; 0.975)$	$Q(l_{\gamma\eta}; 0.025)$	$l_{\gamma\eta}$	$Q(l_{\gamma\eta}; 0.975)$
AAA	0.0283	0.0317	0.0363	0.1098	0.1132	0.1185	0.8152	1.1544	1.7463
	(0.0033)	(0.0044)	(0.0043)	(0.0704)	(0.0712)	(0.0702)	(1.2209)	(1.0404)	(0.8577)
AA	0.0478	0.0587	0.0695	0.0744	0.0876	0.1042	0.4477	1.3551	2.9542
	(0.0270)	(0.0307)	(0.0344)	(0.0329)	(0.0361)	(0.0394)	(0.6660)	(0.8171)	(1.0448)
A	0.0818	0.0950	0.1084	0.0805	0.0906	0.1034	0.6079	1.4560	2.9481
	(0.0336)	(0.0378)	(0.0416)	(0.0721)	(0.0764)	(0.0808)	(0.9519)	(1.0537)	(1.0839)
BBB	0.0783	0.0920	0.1064	0.1379	0.1481	0.1605	1.4383	2.3292	3.5739
	(0.0400)	(0.0443)	(0.0479)	(0.1019)	(0.1023)	(0.1018)	(2.3333)	(2.3632)	(2.2672)
BB	0.1039	0.1179	0.1324	0.2411	0.2521	0.2657	2.0023	2.8119	4.1149
	(0.0628)	(0.0687)	(0.0746)	(0.1158)	(0.1125)	(0.1106)	(3.2609)	(3.2998)	(3.2071)
B	0.1739	0.1938	0.2140	0.2754	0.2962	0.3127	1.8064	2.8704	4.2676
	(0.0963)	(0.1019)	(0.1107)	(0.1332)	(0.1297)	(0.1286)	(3.0282)	(2.8722)	(2.7447)
CCC	0.1863	0.2004	0.2328	0.3296	0.3525	0.3845	1.2732	1.8163	3.5080
	(0.1376)	(0.1545)	(0.1909)	(0.0784)	(0.1083)	(0.1523)	(1.9040)	(1.5553)	(1.4634)
without	0.1605	0.1780	0.1997	0.1494	0.1669	0.1847	0.4411	1.5459	3.4668
	(0.1538)	(0.1549)	(0.1560)	(0.1265)	(0.1390)	(0.1475)	(0.8734)	(0.7010)	(0.9471)
Basic Materials	0.0845	0.0968	0.1092	0.1221	0.1293	0.1403	0.5687	1.6624	2.9382
	(0.0362)	(0.0375)	(0.0390)	(0.1174)	(0.1169)	(0.1167)	(0.9840)	(0.9636)	(1.0178)
Consumer Goods	0.1089	0.1228	0.1379	0.1692	0.1839	0.1973	1.3367	2.1707	3.4927
	(0.1018)	(0.1062)	(0.1121)	(0.1134)	(0.1169)	(0.1201)	(2.5928)	(2.5819)	(2.3137)
Consumer Services	0.0876	0.0990	0.1128	0.1636	0.1721	0.1818	1.6454	2.5135	3.8009
	(0.0464)	(0.0507)	(0.0558)	(0.1272)	(0.1253)	(0.1232)	(2.9056)	(2.9569)	(2.8889)
Financials	0.0849	0.0981	0.1112	0.1574	0.1691	0.1841	1.4217	2.2729	3.4161
	(0.0662)	(0.0729)	(0.0782)	(0.1301)	(0.1317)	(0.1340)	(1.4254)	(1.3996)	(1.4666)
Government	0.0285	0.0310	0.0349	0.1708	0.1755	0.1804	0.1306	0.3559	0.8141
	-	-	-	-	-	-	-	-	-
Health Care	0.0682	0.0847	0.0992	0.1610	0.1819	0.2006	1.5256	2.3603	3.6930
	(0.0299)	(0.0319)	(0.0352)	(0.1145)	(0.1182)	(0.1226)	(3.7182)	(3.5908)	(3.3936)
Industrials	0.1000	0.1162	0.1321	0.1040	0.1135	0.1255	0.5633	1.3522	3.1262
	(0.0332)	(0.0356)	(0.0372)	(0.1152)	(0.1174)	(0.1202)	(1.2485)	(1.2231)	(1.3363)
Oil & Gas	0.0994	0.1163	0.1325	0.1071	0.1157	0.1338	1.0192	1.8634	3.1263
	(0.0535)	(0.0583)	(0.0628)	(0.0955)	(0.0961)	(0.0919)	(1.9023)	(1.8225)	(1.7111)
Technology	0.0995	0.1134	0.1311	0.1915	0.2064	0.2241	1.0204	1.9519	3.5596
	(0.0836)	(0.0892)	(0.1034)	(0.1236)	(0.1321)	(0.1436)	(1.3278)	(1.7771)	(1.8463)
Telecommunications	0.0944	0.1120	0.1301	0.0934	0.1081	0.1197	1.2362	2.8589	4.8648
	(0.0456)	(0.0618)	(0.0789)	(0.0818)	(0.0905)	(0.0946)	(3.3471)	(3.8080)	(3.7501)
Utilities	0.0776	0.0922	0.1080	0.1587	0.1708	0.1861	1.6960	2.4500	3.6506
	(0.0407)	(0.0477)	(0.0526)	(0.0895)	(0.0884)	(0.0862)	(1.9529)	(1.8138)	(1.6957)

Table 3: Posterior Estimates for Model (4.3) **and** (4.5)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for details). Estimates are based on 2 years of daily panel data CDS premia written on 282 US firms. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw was recorded to remedy high autocorrelation in the parameter paths. From the remaining 3000 draws only the last 2000 were taken into the computation.

rating	$Q(\zeta^{\mathbb{Q}}_{\eta}; 0.025)$	$\zeta^{\mathbb{Q}}_{\eta}$	$Q(\zeta^{\mathbb{Q}}_{\eta}; 0.975)$	$Q(\zeta^{\mathbb{Q}}_{\gamma\eta}; 0.025)$	$\zeta^{\mathbb{Q}}_{\gamma\eta}$	$Q(\zeta^{\mathbb{Q}}_{\eta\gamma}; 0.975)$	$Q(l_{\gamma\eta r}; 0.025)$	$l_{\gamma\eta r}$	$Q(l_{\gamma\eta r}; 0.975)$
AAA	0.0001	0.0001	0.0002	0.0771	0.1559	0.2703	0.0418	0.3372	1.3208
	(0.0001)	(0.0001)	(0.0001)	(0.1262)	(0.1146)	(0.0833)	(0.0232)	(0.1209)	(0.4381)
AA	0.0031	0.0047	0.0059	0.0899	0.2390	0.3699	0.0447	0.3689	1.3283
	(0.0099)	(0.0150)	(0.0167)	(0.1089)	(0.1759)	(0.1841)	(0.0157)	(0.1084)	(0.2197)
A	0.0027	0.0057	0.0094	0.1046	0.2110	0.3417	0.0529	0.3923	1.4098
	(0.0086)	(0.0133)	(0.0176)	(0.1545)	(0.2248)	(0.2680)	(0.0089)	(0.1624)	(0.2054)
BBB	0.0032	0.0062	0.0096	0.0815	0.1914	0.3249	0.0624	0.3696	1.3548
	(0.0098)	(0.0158)	(0.0195)	(0.1242)	(0.1893)	(0.2336)	(0.1108)	(0.1819)	(0.3147)
BB	0.0046	0.0076	0.0108	0.1008	0.2034	0.3173	0.2139	0.6034	1.5306
	(0.0125)	(0.0169)	(0.0207)	(0.1489)	(0.1916)	(0.2570)	(0.4781)	(0.4271)	(0.4727)
B	0.0018	0.0034	0.0056	0.0834	0.1657	0.2877	0.1336	0.4932	1.3941
	(0.0057)	(0.0107)	(0.0129)	(0.1479)	(0.1851)	(0.2480)	(0.1889)	(0.2350)	(0.5909)
CCC	0.0010	0.0023	0.0040	0.1504	0.2209	0.3476	0.0574	0.3533	1.4750
	(0.0016)	(0.0030)	(0.0033)	(0.1309)	(0.1919)	(0.3370)	(0.0064)	(0.2529)	(1.0502)
without	0.0001	0.0003	0.0065	0.0454	0.2440	0.4165	0.0528	0.4464	1.4314
	(0.0001)	(0.0004)	(0.0096)	(0.0555)	(0.1813)	(0.1831)	(0.0141)	(0.1944)	(0.2505)
Basic Materials	0.0001	0.0018	0.0034	0.1056	0.2083	0.3527	0.0927	0.4160	1.5506
	(0.0001)	(0.0061)	(0.0087)	(0.1104)	(0.1471)	(0.1804)	(0.1656)	(0.2671)	(0.5516)
Consumer Goods	0.0029	0.0066	0.0096	0.0604	0.1668	0.2959	0.1350	0.4534	1.3952
	(0.0098)	(0.0183)	(0.0210)	(0.0939)	(0.1715)	(0.1997)	(0.3319)	(0.3270)	(0.3277)
Consumer Services	0.0028	0.0060	0.0093	0.0856	0.2032	0.3158	0.1261	0.4338	1.3501
	(0.0081)	(0.0125)	(0.0159)	(0.1541)	(0.2141)	(0.2755)	(0.3276)	(0.3263)	(0.3592)
Financials	0.0014	0.0031	0.0071	0.0931	0.1756	0.2941	0.0505	0.3768	1.3387
	(0.0046)	(0.0082)	(0.0121)	(0.1343)	(0.1715)	(0.2073)	(0.0186)	(0.1693)	(0.2878)
Government	0.0000	0.0001	0.0002	0.0043	0.0608	0.1861	0.0286	0.2497	0.9182
	-	-	-	-	-	-	-	-	-
Health Care	0.0032	0.0068	0.0080	0.0810	0.2374	0.3856	0.0893	0.4653	1.4608
	(0.0090)	(0.0175)	(0.0190)	(0.1249)	(0.1978)	(0.2584)	(0.1271)	(0.1969)	(0.4247)
Industrials	0.0049	0.0079	0.0115	0.1129	0.2344	0.3767	0.0516	0.3864	1.4155
	(0.0120)	(0.0173)	(0.0230)	(0.1775)	(0.2436)	(0.2947)	(0.0125)	(0.1487)	(0.2438)
Oil & Gas	0.0062	0.0092	0.0139	0.0617	0.1822	0.3195	0.0539	0.3802	1.4200
	(0.0145)	(0.0184)	(0.0241)	(0.0977)	(0.1717)	(0.2600)	(0.0248)	(0.1361)	(0.3987)
Technology	0.0042	0.0062	0.0101	0.1133	0.2311	0.3467	0.0568	0.4908	1.5669
	(0.0151)	(0.0180)	(0.0225)	(0.1258)	(0.2126)	(0.2485)	(0.0163)	(0.2146)	(0.3372)
Telecommunications	0.0006	0.0024	0.0087	0.0699	0.1775	0.3126	0.0590	0.4797	1.3833
	(0.0012)	(0.0050)	(0.0115)	(0.1328)	(0.2275)	(0.2582)	(0.0170)	(0.0617)	(0.3522)
Utilities	0.0040	0.0062	0.0084	0.1424	0.2330	0.3741	0.0495	0.4049	1.3763
	(0.0108)	(0.0164)	(0.0198)	(0.1615)	(0.2257)	(0.2672)	(0.0169)	(0.1722)	(0.3232)

Table 4: Posterior Estimates for Model (4.3) **and** (4.5)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for details). Estimates are based on 2 years of daily panel data CDS premia written on 282 US firms. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw was recorded to remedy high autocorrelation in the parameter paths. From the remaining 3000 draws only the last 2000 were taken into the computation.

	$Q(a_0; 0.025)$	a_0	$Q(a_0; 0.975)$	$Q(a_1; 0.025)$	a_1	$Q(a_1; 0.975)$	$Q(a_2; 0.025)$	a_2	$Q(a_2; 0.975)$
AAA	-22.4600	-22.2000	-21.8041	1.1471	1.2908	1.4261	-0.1311	-0.1179	-0.1051
	(4.0219)	(4.0185)	(3.8429)	(2.0814)	(2.1428)	(2.1603)	(0.2101)	(0.2067)	(0.2007)
AA	-19.7241	-19.4245	-19.0881	-0.4151	-0.2616	-0.1304	0.0236	0.0348	0.0487
	(2.1454)	(2.1023)	(2.0612)	(0.9706)	(1.0002)	(1.0091)	(0.0947)	(0.0934)	(0.0896)
A	-19.6519	-19.2766	-18.9543	0.2012	0.3416	0.5100	-0.0469	-0.0318	-0.0188
	(1.6537)	(1.6583)	(1.6358)	(0.7936)	(0.8150)	(0.8330)	(0.0904)	(0.0877)	(0.0842)
BBB	-17.4538	-17.1036	-16.7699	0.2682	0.4212	0.5828	-0.0846	-0.0687	-0.0534
	(2.1922)	(2.1880)	(2.1841)	(0.6502)	(0.6622)	(0.6769)	(0.0797)	(0.0765)	(0.0736)
BB	-13.6258	-13.2995	-12.9829	0.0888	0.2372	0.3996	-0.0961	-0.0795	-0.0644
	(3.5385)	(3.4572)	(3.3969)	(0.7448)	(0.7552)	(0.7676)	(0.0882)	(0.0853)	(0.0819)
B	-11.3452	-11.0221	-10.7129	-0.7055	-0.5550	-0.3994	-0.0351	-0.0192	-0.0031
	(2.8251)	(2.7821)	(2.7433)	(1.1236)	(1.1405)	(1.1574)	(0.1163)	(0.1138)	(0.1111)
CCC	-9.1747	-8.9324	-8.6576	-0.7892	-0.6300	-0.5233	-0.0373	-0.0262	-0.0090
	(2.0299)	(1.9872)	(1.9754)	(0.6951)	(0.6992)	(0.7212)	(0.0739)	(0.0724)	(0.0687)
without	-16.6596	-16.3214	-16.0173	-0.1267	0.0096	0.1545	-0.0221	-0.0089	0.0034
	(3.1346)	(3.1191)	(3.1493)	(0.8120)	(0.8369)	(0.8491)	(0.0934)	(0.0900)	(0.0867)
Basic Materials	-17.2829	-16.9361	-16.6211	0.2544	0.3935	0.5506	-0.0804	-0.0661	-0.0519
	(3.3851)	(3.3551)	(3.3416)	(0.8813)	(0.8983)	(0.9032)	(0.0979)	(0.0967)	(0.0927)
Consumer Goods	-15.4135	-15.0528	-14.7290	-0.2384	-0.0810	0.0913	-0.0504	-0.0333	-0.0175
	(3.6402)	(3.5628)	(3.5315)	(0.8527)	(0.8685)	(0.8873)	(0.0934)	(0.0911)	(0.0885)
Consumer Services	-16.1089	-15.7916	-15.4809	0.1049	0.2472	0.3964	-0.0756	-0.0607	-0.0465
	(3.6621)	(3.6289)	(3.6136)	(0.5798)	(0.5946)	(0.6067)	(0.0743)	(0.0718)	(0.0685)
Financials	-18.5191	-18.1986	-17.8913	0.2373	0.3818	0.5332	-0.0727	-0.0580	-0.0434
	(3.1639)	(3.1678)	(3.1463)	(0.9747)	(0.9865)	(0.9941)	(0.1004)	(0.0985)	(0.0960)
Government	-26.0832	-25.7764	-25.1747	2.7430	2.9699	3.1243	-0.2897	-0.2728	-0.2528
	-	-	-	-	-	-	-	-	-
Health Care	-17.3120	-17.0431	-16.7352	-0.4206	-0.2873	-0.1556	-0.0023	0.0102	0.0224
	(3.5559)	(3.5269)	(3.5194)	(0.6029)	(0.6159)	(0.6384)	(0.0869)	(0.0837)	(0.0804)
Industrials	-18.2365	-17.9022	-17.5702	0.1943	0.3388	0.4958	-0.0585	-0.0434	-0.0296
	(2.5377)	(2.5231)	(2.5241)	(0.6375)	(0.6595)	(0.6759)	(0.0814)	(0.0775)	(0.0736)
Oil & Gas	-17.6641	-17.2181	-16.8238	0.0730	0.2452	0.4506	-0.0661	-0.0470	-0.0303
	(1.5802)	(1.7162)	(1.7612)	(0.6405)	(0.6641)	(0.6958)	(0.0875)	(0.0813)	(0.0775)
Technology	-16.3104	-15.8250	-15.4449	0.3016	0.4652	0.6601	-0.0912	-0.0743	-0.0589
	(3.1256)	(3.0827)	(3.0715)	(0.9319)	(0.9444)	(0.9948)	(0.1091)	(0.1037)	(0.1008)
Telecommunications	-18.0627	-17.6550	-17.3070	0.5803	0.7271	0.8940	-0.0965	-0.0815	-0.0674
	(4.3053)	(4.3261)	(4.3265)	(1.2727)	(1.3084)	(1.3270)	(0.1279)	(0.1242)	(0.1183)
Utilities	-17.7289	-17.3947	-17.0819	0.2857	0.4354	0.5890	-0.0845	-0.0692	-0.0546
	(2.3097)	(2.2788)	(2.2564)	(0.8272)	(0.8387)	(0.8579)	(0.0875)	(0.0843)	(0.0825)

Table 5: Posterior Estimates for Model (4.3) **and** (4.5)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for details). Estimates are based on 2 years of daily panel data CDS premia written on 282 US firms. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw was recorded to remedy high autocorrelation in the parameter paths. From the remaining 3000 draws only the last 2000 were taken into the computation.

rating	$Q(\kappa_\gamma^{\mathbb{P}}; 0.025)$	$\kappa_\gamma^{\mathbb{P}}$	$Q(\kappa_\gamma^{\mathbb{P}}; 0.975)$	$Q(\kappa_\eta^{\mathbb{P}}; 0.025)$	$\kappa_\eta^{\mathbb{P}}$	$Q(\kappa_\eta^{\mathbb{P}}; 0.975)$	$Q(l_{\gamma\eta}^{\mathbb{P}}/250; 0.025)$	$l_{\gamma\eta}^{\mathbb{P}}/250$	$Q(l_{\gamma\eta}^{\mathbb{P}}/250; 0.975)$
AAA	0.1506	1.2476	2.5544	2.2976	4.8107	6.9316	0.0028	0.0079	0.0185
	(0.1345)	(0.0429)	(0.2950)	(0.8561)	(0.8830)	(0.9609)	(0.0009)	(0.0021)	(0.0036)
AA	0.1836	0.8102	1.8903	2.5318	5.2802	8.1073	0.0025	0.0086	0.0201
	(0.3072)	(0.4810)	(0.7126)	(1.9576)	(3.2599)	(3.9995)	(0.0026)	(0.0066)	(0.0105)
A	0.2011	0.6611	1.3025	4.0280	7.1817	10.3952	0.0033	0.0098	0.0224
	(0.2283)	(0.4605)	(0.7452)	(3.4828)	(4.3672)	(5.0126)	(0.0034)	(0.0062)	(0.0099)
BBB	0.1942	0.6798	1.3565	8.1925	11.9203	15.8461	0.0065	0.0141	0.0285
	(0.3343)	(0.6061)	(0.8130)	(8.0296)	(9.2898)	(9.8365)	(0.0099)	(0.0138)	(0.0161)
BB	0.1697	0.6391	1.3514	6.7727	10.0263	13.3820	0.0167	0.0296	0.0480
	(0.2758)	(0.4771)	(0.5699)	(4.1747)	(5.0186)	(6.1731)	(0.0153)	(0.0197)	(0.0226)
B	0.1233	0.4890	1.0036	5.2382	8.3635	11.1134	0.0265	0.0400	0.0593
	(0.1974)	(0.3743)	(0.4466)	(5.6869)	(7.9964)	(9.0945)	(0.0260)	(0.0317)	(0.0323)
CCC	0.0369	0.2942	0.6142	1.9779	3.4106	5.1775	0.0152	0.0271	0.0514
	(0.0527)	(0.1423)	(0.1956)	(0.5215)	(1.3491)	(2.2056)	(0.0169)	(0.0214)	(0.0304)
without	0.3963	0.8574	1.3144	3.1945	6.5004	9.0995	0.0127	0.0255	0.0431
	(0.4331)	(0.4749)	(0.5558)	(2.1934)	(2.9225)	(3.4192)	(0.0183)	(0.0273)	(0.0331)
Basic Materials	0.1169	0.4139	1.0080	5.7802	9.0010	12.2588	0.0093	0.0186	0.0322
	(0.1679)	(0.3384)	(0.5097)	(3.3075)	(4.1155)	(4.5527)	(0.0167)	(0.0195)	(0.0240)
Consumer Goods	0.1511	0.5787	1.2339	6.6362	9.8737	13.1500	0.0170	0.0285	0.0440
	(0.2844)	(0.4381)	(0.6272)	(7.7334)	(8.3022)	(9.4556)	(0.0217)	(0.0278)	(0.0289)
Consumer Services	0.2032	0.6508	1.2739	7.4381	10.9932	14.5520	0.0109	0.0205	0.0363
	(0.3263)	(0.5396)	(0.7197)	(8.3576)	(9.5411)	(10.3987)	(0.0159)	(0.0215)	(0.0258)
Financials	0.2392	0.8674	1.7573	6.1376	9.7291	13.1788	0.0064	0.0147	0.0310
	(0.2635)	(0.4163)	(0.6402)	(6.2226)	(7.5416)	(8.1527)	(0.0067)	(0.0099)	(0.0150)
Government	0.0996	1.2939	2.8471	2.7930	5.1074	7.2576	0.0035	0.0097	0.0198
	-	-	-	-	-	-	-	-	-
Health Care	0.4573	1.1331	2.0040	4.3072	7.0457	10.1806	0.0059	0.0129	0.0260
	(0.5458)	(0.8618)	(0.9970)	(4.4891)	(5.1698)	(6.0245)	(0.0085)	(0.0138)	(0.0200)
Industrials	0.1642	0.5182	1.0091	6.0735	9.9101	13.6509	0.0049	0.0118	0.0247
	(0.2130)	(0.4101)	(0.6198)	(6.4818)	(9.1549)	(9.5289)	(0.0069)	(0.0111)	(0.0157)
Oil & Gas	0.1338	0.5454	1.1827	5.3891	8.5946	12.3271	0.0048	0.0120	0.0269
	(0.2404)	(0.4760)	(0.7250)	(4.4442)	(5.6040)	(6.1645)	(0.0042)	(0.0077)	(0.0100)
Technology	0.2359	0.8732	1.5620	7.0962	10.4832	14.1918	0.0063	0.0147	0.0299
	(0.2516)	(0.6416)	(0.8872)	(5.6618)	(6.9300)	(8.2019)	(0.0050)	(0.0079)	(0.0117)
Telecommunications	0.2216	0.6803	1.2171	3.3309	6.2828	9.3937	0.0026	0.0092	0.0240
	(0.3827)	(0.9416)	(1.1458)	(1.9499)	(2.7585)	(3.3954)	(0.0029)	(0.0063)	(0.0106)
Utilities	0.0751	0.5947	1.3073	6.0851	9.2376	12.6696	0.0047	0.0118	0.0266
	(0.1228)	(0.3283)	(0.5158)	(3.7297)	(4.5313)	(5.1744)	(0.0043)	(0.0091)	(0.0125)

Table 6: Posterior Estimates for Model (4.3) and (4.5): This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for details). Estimates are based on 2 years of daily panel data CDS premia written on 282 US firms. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw was recorded to remedy high autocorrelation in the parameter paths. From the remaining 3000 draws only the last 2000 were taken into the computation.

	$Q(l^{\mathbb{P}}_{\gamma\eta r}/250; 0.025)$	$l^{\mathbb{P}}_{\gamma\eta r}/250$	$Q(l^{\mathbb{P}}_{\gamma\eta r}/250; 0.975)$	$Q(\zeta^{\mathbb{P}}_{\eta}; 0.025)$	$\zeta^{\mathbb{P}}_{\eta}$	$Q(\zeta^{\mathbb{P}}_{\eta}; 0.975)$	$Q(\zeta^{\mathbb{P}}_{\gamma\eta}; 0.025)$	$\zeta^{\mathbb{P}}_{\gamma\eta}$	$Q(\zeta^{\mathbb{P}}_{eta\gamma}; 0.975)$
AAA	0.0000	0.0016	0.0058	0.0014	0.0027	0.0045	0.7299	1.8509	2.4898
	(0.0000)	(0.0007)	(0.0002)	(0.0016)	(0.0032)	(0.0057)	(0.5838)	(1.0780)	(1.4010)
AA	0.0000	0.0009	0.0060	0.0027	0.0047	0.0097	1.6453	2.9777	4.0709
	(0.0000)	(0.0009)	(0.0002)	(0.0051)	(0.0084)	(0.0160)	(1.4852)	(3.4421)	(4.1141)
A	0.0000	0.0010	0.0059	0.0060	0.0111	0.0239	2.3978	4.1830	5.9201
	(0.0000)	(0.0005)	(0.0002)	(0.0080)	(0.0134)	(0.0243)	(2.0834)	(3.1847)	(3.9446)
BBB	0.0000	0.0011	0.0059	0.0049	0.0090	0.0183	1.9735	3.2908	4.8681
	(0.0000)	(0.0006)	(0.0002)	(0.0087)	(0.0154)	(0.0250)	(2.2097)	(2.9723)	(4.0832)
BB	0.0000	0.0010	0.0058	0.0025	0.0043	0.0090	1.4251	2.5986	3.5697
	(0.0000)	(0.0005)	(0.0009)	(0.0037)	(0.0070)	(0.0137)	(1.2730)	(2.0593)	(2.4388)
B	0.0000	0.0012	0.0059	0.0053	0.0088	0.0190	2.5411	3.7976	5.6791
	(0.0000)	(0.0005)	(0.0002)	(0.0064)	(0.0096)	(0.0225)	(1.9760)	(2.4518)	(4.1268)
CCC	0.0000	0.0011	0.0060	0.0040	0.0075	0.0138	2.0725	3.0077	4.6000
	(0.0000)	(0.0004)	(0.0003)	(0.0030)	(0.0071)	(0.0127)	(2.3330)	(2.9615)	(2.9303)
without	0.0000	0.0012	0.0059	0.0120	0.0188	0.0316	1.5156	3.6910	5.8757
	(0.0000)	(0.0004)	(0.0002)	(0.0135)	(0.0222)	(0.0360)	(1.1960)	(2.9785)	(4.9725)
Basic Materials	0.0000	0.0012	0.0059	0.0073	0.0135	0.0257	2.3442	3.6330	6.0677
	(0.0000)	(0.0005)	(0.0002)	(0.0098)	(0.0183)	(0.0308)	(1.9936)	(2.6883)	(4.4600)
Consumer Goods	0.0000	0.0011	0.0059	0.0051	0.0083	0.0158	1.8194	3.1786	4.5355
	(0.0000)	(0.0005)	(0.0002)	(0.0085)	(0.0139)	(0.0234)	(1.6319)	(2.7830)	(3.6698)
Consumer Services	0.0000	0.0011	0.0059	0.0054	0.0096	0.0190	1.7761	3.2226	4.7898
	(0.0000)	(0.0007)	(0.0002)	(0.0100)	(0.0175)	(0.0274)	(2.0464)	(2.8913)	(3.7901)
Financials	0.0000	0.0010	0.0058	0.0023	0.0043	0.0102	1.7846	3.0639	4.2273
	(0.0000)	(0.0006)	(0.0002)	(0.0029)	(0.0063)	(0.0143)	(2.1621)	(3.1359)	(3.8731)
Government	0.0000	0.0023	0.0056	0.0006	0.0012	0.0016	1.2936	2.7158	3.7941
	-	-	-	-	-	-	-	-	-
Health Care	0.0000	0.0010	0.0060	0.0033	0.0057	0.0132	1.5398	3.1003	4.2309
	(0.0000)	(0.0004)	(0.0002)	(0.0045)	(0.0070)	(0.0137)	(1.2772)	(2.8615)	(3.4482)
Industrials	0.0000	0.0010	0.0059	0.0068	0.0123	0.0256	2.3495	4.0379	5.9974
	(0.0000)	(0.0005)	(0.0002)	(0.0083)	(0.0141)	(0.0260)	(2.7094)	(3.6219)	(4.8509)
Oil & Gas	0.0000	0.0010	0.0059	0.0063	0.0118	0.0243	2.7287	4.4546	6.4562
	(0.0000)	(0.0005)	(0.0003)	(0.0074)	(0.0136)	(0.0236)	(2.2057)	(2.8307)	(3.8000)
Technology	0.0000	0.0009	0.0060	0.0058	0.0093	0.0179	2.1418	3.5033	4.7844
	(0.0000)	(0.0005)	(0.0003)	(0.0074)	(0.0111)	(0.0204)	(1.6322)	(2.3243)	(2.9164)
Telecommunications	0.0000	0.0009	0.0059	0.0072	0.0120	0.0286	2.9159	4.8411	6.8688
	(0.0000)	(0.0006)	(0.0002)	(0.0115)	(0.0192)	(0.0307)	(1.6630)	(2.5654)	(4.2377)
Utilities	0.0000	0.0008	0.0059	0.0021	0.0055	0.0131	1.7651	2.7477	3.8204
	(0.0000)	(0.0004)	(0.0003)	(0.0025)	(0.0065)	(0.0162)	(1.6423)	(2.0929)	(2.7649)

Table 7: Posterior Estimates for Model (4.3) **and** (4.5)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for details). Estimates are based on 2 years of daily panel data CDS premia written on 282 US firms. From 5,000,000 draws from the Gibbs-Metropolis sampler only every 1000-th draw was recorded to remedy high autocorrelation in the parameter paths. From the remaining 3000 draws only the last 2000 were taken into the computation.

rating	Q(LGD;0.025)	LGD	Q(LGD;0.975)	$\theta^{\mathbb{Q}}_{\gamma}$	$\theta^{\mathbb{P}}_{\gamma}$	# obs.
AAA	0.3456	0.3584	0.3783	0.4997	0.0023	3
	(0.0569)	(0.0509)	(0.0523)	(0.3919)	(0.0773)	
AA	0.1927	0.2357	0.2713	1.0212	0.0133	11
	(0.1272)	(0.1382)	(0.1362)	(0.6115)	(0.0295)	
A	0.1624	0.1767	0.1916	5.2477	0.0767	75
	(0.1933)	(0.2064)	(0.2134)	(2.2746)	(0.0961)	
BBB	0.3305	0.3455	0.3656	1.8047	0.0738	123
	(0.2988)	(0.3050)	(0.3144)	(1.7440)	(0.1163)	
BB	0.6369	0.6599	0.6910	0.2661	0.0186	38
	(0.2623)	(0.2639)	(0.2653)	(0.7479)	(0.0672)	
B	0.5939	0.6448	0.7007	0.1199	0.0167	22
	(0.2572)	(0.2149)	(0.1768)	(0.2250)	(0.0265)	
CCC	0.7186	0.7519	0.7730	0.1748	0.0344	3
	(0.2983)	(0.2888)	(0.2854)	(0.2172)	(0.0789)	
without	0.1869	0.2127	0.2315	3.0117	0.0000	7
	(0.1258)	(0.1524)	(0.1740)	(2.5275)	(0.0000)	
Basic Materials	0.3290	0.3409	0.3577	2.0264	0.1066	20
	(0.3213)	(0.3231)	(0.3314)	(1.5288)	(0.1691)	
Consumer Goods	0.4362	0.4642	0.4957	0.8227	0.0419	47
	(0.3145)	(0.3177)	(0.3225)	(1.2385)	(0.0974)	
Consumer Services	0.4223	0.4396	0.4574	2.7200	0.0855	54
	(0.3482)	(0.3547)	(0.3619)	(2.3126)	(0.1411)	
Financials	0.3054	0.3242	0.3428	0.6228	0.0226	43
	(0.2250)	(0.2310)	(0.2337)	(0.7819)	(0.0734)	
Government	0.3274	0.3314	0.3387	0.1047	0.0013	1
	-	-	-	-	-	
Health Care	0.2754	0.2999	0.3416	0.7451	0.0162	15
	(0.3104)	(0.3060)	(0.3047)	(0.5322)	(0.0230)	
Industrials	0.1850	0.2005	0.2155	6.5252	0.1349	35
	(0.2591)	(0.2662)	(0.2772)	(2.8403)	(0.1509)	
Oil & Gas	0.2690	0.2870	0.3172	1.5653	0.0852	20
	(0.2829)	(0.2911)	(0.3016)	(1.0459)	(0.1017)	
Technology	0.5285	0.5536	0.5855	0.4079	0.0269	14
	(0.3460)	(0.3605)	(0.3767)	(0.4670)	(0.0475)	
Telecommunications	0.2776	0.3046	0.3347	4.1864	0.1127	10
	(0.3138)	(0.3260)	(0.3485)	(2.3781)	(0.0961)	
Utilities	0.3453	0.3707	0.3983	0.3518	0.0293	23
	(0.2303)	(0.2373)	(0.2525)	(0.7832)	(0.1068)	

Table 8: Posterior Estimates for Model (4.3) **and** (4.5)**:** This table shows point estimates, taken to be the multivariate median, as well as quantiles from the posterior distribution of the parameters conditional on the data (see Collin-Dufresne et al., 2004, for details). The table further reports mean point estimates for the parameters $\theta^{\mathbb{Q}}_{\gamma}$ as well as $\theta^{\mathbb{P}}_{\gamma}$ averaged over industry, as well as rating classification. The last columns gives the number of firms contained in the respective rating and industry class.

	η	γ
AAA	0.0004	0.012
	(0.0004)	(0.0091)
AA	0.0011	0.0399
	(0.0018)	(0.0638)
A	0.0026	0.1621
	(0.0032)	(0.1823)
BBB	0.0024	0.1685
	(0.004)	(0.2296)
BB	0.0051	0.0914
	(0.0125)	(0.1657)
B	0.0194	0.1823
	(0.0495)	(0.2086)
CCC	0.0355	0.165
	(0.0384)	(0.0928)
Basic Materials	0.0034	0.1536
	(0.0044)	(0.1711)
Consumer Goods	0.0024	0.1064
	(0.0061)	(0.1601)
Consumer Services	0.0068	0.1695
	(0.0302)	(0.2323)
Financials	0.0032	0.1572
	(0.0107)	(0.2311)
Government	0.0008	0.1268
	(0.0011)	(0.0459)
Health Care	0.0078	0.2094
	(0.0212)	(0.2365)
Industrials	0.0054	0.1944
	(0.0132)	(0.2408)
Oil & Gas	0.0021	0.1307
	(0.003)	(0.1649)
Technology	0.0084	0.1189
	(0.0228)	(0.1226)
Telecommunications	0.0024	0.131
	(0.0041)	(0.2094)
Utilities	0.0036	0.1149
	(0.005)	(0.1201)

Table 9: Posterior Mean Estimates of latent processes (η) **and** (γ) This table shows the empirical means of the posterior trajectories of the default intensity γ and its stochastic long-run mean η averaged over the respective rating classes, as well as industries, respectively. The values reported in brackets are standard deviations for the values reported above.