The Price-setting Behavior of Banks: An Analysis of Open-end Leverage Certificates on the German Market

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Working Paper

First Version: August 10, 2006
This Version: January 15, 2007

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[*] Parts of this research were done while Oliver Entrop was visiting the School of Banking and Finance, University of New South Wales. He thanks Terry Walter and the academic and administrative staff for their hospitality and support.

We are grateful to participations at the Australasian Banking and Finance Conference 2006, Sydney, for helpful comments and suggestions on an earlier draft of this paper.

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Abstract

This paper presents the first analysis of the pricing of exchange-traded open-end leverage certificates on the German retail market. The major innovations of these certificates are twofold. First, the issuers announce ex-ante a price-setting formula according to which they are willing to buy and sell the certificates over time. In particular, this formula is independent of the volatility of the underlying. Second, the product's lifetime is potentially endless. Our main findings can be summarized as follows: i) The price-setting formula strongly favors the issuers. ii) Issuers can hedge these certificates easily with a semi-static superhedge using spot market instruments. iii) The price-setting formula confirms the main outcome of the 'life cycle hypothesis' for structured financial products (e.g., Stoimenov and Wilkens, 2005), insuring that profits by issuers systematically increase in the course of product lifetimes. iv) The value of the 'mispricing by construction' depends on the volatility of the underlying and the so-called funding rate spread. Compared with these factors, the influence of interest rates and their dynamics was found to be negligible.

JEL: G13; G21; G24

Keywords: Structured products; Certificates; Hedging; German market; Pricing

1 Introduction

In many countries since the mid-nineties, exchange-traded innovative financial products (IFPs), such as equity-linked bonds and leverage products, have become increasingly important in the retail market (see Stoimenov and Wilkens, 2005, for a current overview of the German market as one of the most important markets for IFPs). Off-exchange trades of IFPs are usually settled by issuers, whereas exchange trades are primarily conducted by market makers. As issuing banks normally handle the market making by themselves, they de facto dominate not only the primary but also the secondary market. Since short selling of IFPs is virtually impossible, market makers can systematically quote prices that do not match fair theoretical values but favor themselves. In fact, the price-setting mechanism applied by issuers is generally kept hidden from investors. Since IFPs are often complex, it is frequently difficult for private investors to calculate their 'fair' values and, hence, to evaluate the 'fairness' of the quotes.

Due to this intransparency, a large body of empirical work has been carried out to analyze the price-setting behavior of issuers by comparing quoted prices and theoretical fair values. Chen and Kensinger (1990), Chen and Sears (1990), Baubonis et al. (1993), and Benet et al. (2006) report significant deviations for equity-linked products on the US market. Brown and Davis (2004) recently detected significant price deviations for endowment warrants on the Australian market. An analogue result was found for the Swiss market by Wasserfallen and Schenk (1996), Burth et al. (2001), and Grünbichler and Wohlwend (2005), and for the German market by Wilkens et al. (2003), Stoimenov and Wilkens (2005), and Baule et al. (2006). All these empirical studies reveal the pricing behavior of issuers: At issuance, they regularly sell IFPs for their theoretical value plus a positive premium, and later on they buy them back paying the theoretical value plus a decreased premium. As a result, issuers gain by diminishing overpricing in the course of product lifetimes. Wilkens et al. (2003) and Stoimenov and Wilkens (2005) analyze this behavior in detail for structured financial products and subsume decreasing premiums over time under 'life cycle hypothesis'.

In recent years, several banks have issued leverage products as a new type of IFP. Although they were not issued until October 2001, this market segment now replaces a substantial portion of the classical warrant market in Germany. The main characteristics of the first generation of leverage products are equivalent to those of one-sided barrier options. Muck (2006a, 2006b) and Wilkens and Stoimenov (2006) analyze the pricing of these certificates similarly to the above-mentioned studies. Muck (2006a) and Wilkens and Stoimenov (2006) report clear positive premiums that favor issuers. In contrast, Muck (2006b) finds that jump risk at least partially explains these premiums. Looking at the whole data set containing seven issuers, Muck (2006a) could not confirm decreasing premiums over the product lifetime, while he finds only weak evidence supporting the life cycle hypothesis on an individual issuer level. Wilkens and Stoimenov (2006) refrain from pursuing the life cycle hypothesis because the knock-out characteristic of leverage certificates yields stochastic lifetimes.

This paper is the first analyzing a new generation of leverage products on the German retail market, namely open-end leverage certificates.[1] In October 2002, banks started issuing this generation, beginning with 84 certificates in 2002 and reaching 14,030 during the first three quarters of 2006. Compared to financial products analyzed by the studies mentioned above, and in particular in contrast to the first generation of leverage certificates discussed in Muck (2006a, 2006b) and Wilkens and Stoimenov (2006), this new generation exhibits two main innovative features: i) Issuers announce ex-ante a relatively simple price-setting formula, according to which they are willing to sell and repurchase these certificates over time. ii) Open-end leverage certificates do not have a fixed product maturity, but a potentially perpetual lifetime. Feature i) removes the 'arbitrariness' of the issuers' quotes for IFPs, from the investors' point of view; this arbitrariness is normally present and only slightly limited through competition across issuing banks.

To the best of our knowledge this study is the first that does not have to rely on quotes collected on the primary and secondary market for analyzing the price-setting behavior of issuers. Since we focus directly on the price-setting formula used by issuers, we are able to fill a conspicuous gap in the present empirical literature.

The paper is organized as follows: Section 2 describes the construction of open-end leverage certificates and the price-setting formula applied by banks. Section 3 analyzes a semi-static superhedge of open-end leverage certificates using spot market instruments. Based on this superhedge we find the price-setting formula strongly favors issuers. Additionally, we show that the life cycle hypothesis clearly holds for open-end leverage certificates – a finding that could not be shown for the first generation of leverage certificates. Section 4 presents a comparative static analysis of the 'value of mispricing' from the issuers' point of view. This valuation is based on a Black and Scholes (1973) and Merton (1973) world with stochastic

[1] These certificates are sold under different names, see Table 3.

interest rates. In this context we discuss main factors influencing the theoretical value of open-end leverage certificates, in particular the volatility of the underlying and the so-called 'funding rate spread' set by the issuer. Section 5 discusses the impact of differing product features on our main findings. Section 6 concludes.

2 Main characteristics of open-end leverage certificates

Open-end leverage certificates are issued in two basic forms: as long certificates which benefit from the increasing prices of the underlying, and as short certificates which profit from decreasing prices. We focus on open-end long certificates (OELCs), which are more important considering the number of issues in Germany from October 2002 to September 2006 (see Table 1). However, the analysis can be transferred to open-end short certificates straightforwardly.[2] Table 2 reports the number of issues in relation to the specific underlying. Obviously stocks and stock indices are the most common underlyings.

– Insert Table 1 about here –

– Insert Table 2 about here –

Depending on the issuer of OELCs, we find slightly varying product features in practice. However, the main characteristics of these certificates match closely. An OELC is based on an underlying S and has a strike X, and a knock-out barrier B. To keep the illustration general and as intuitive as possible, the following analysis is based on a stylized definition of OELCs that is closely related to certificates of HSBC Trinkaus & Burkhardt.[3] We analyze the influence of differing product features in Section 5.

As already pointed out, OELCs do not have a fixed product maturity, but a potentially perpetual lifetime. However, they become due when the price of the underlying hits or falls below the barrier for the first time. This first passage time τ is given by

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[2] We discuss briefly the price-setting formula issuers apply for open-end short certificates in Section 3, Footnote 9.

[3] We choose this issuer, as he explicitly declares a constant funding rate spread and a constant relative difference between barrier and strike over time (which will be defined more precisely later) in sales prospects (see HSBC Trinkaus & Burkhardt, 2006). Furthermore, OELCs of this issuer show daily changing strikes and barriers as well.

$$\tau = \inf\{t: S_t \leq B_t\}, \tag{1}$$

where S_t denotes the price of the underlying and B_t the barrier in t. In the case of a knock-out in t, the investor receives a settlement amount (rebate) P_t, which is the difference between the price of the underlying S_t and the strike X_t:[4]

$$P_t = S_t - X_t. \tag{2}$$

In practice HSBC Trinkaus & Burkhardt, for example, determine the settlement amount P_t of OELCs within one hour following the knock-out based on the prices they get from terminating their hedging instruments (see HSBC Trinkaus & Burkhardt, 2006). Therefore the exact price of the underlying S_t at knock-out in t is not necessarily relevant for the settlement amount. Inter alia this characteristic transfers the liquidity risk in context with the hedging instruments to the investor. However, we will abstract from this in our analysis.

Barrier and strike are not constant over time. The initial strike X_0 increases over time, related to the so-called 'funding rate'. This funding rate consists of a (variable) short-term money market interest rate r'_t such as EONIA (Euro OverNight Index Average) and a 'funding rate spread' $z > 0$. The barrier is designed to permanently exceed the strike by the factor $a > 0$: $B_t = (1 + a)\, X_t$.[5] Assuming r'_t and z are continuously compounded we have:

$$X_t = X_0 \exp\left(\int_0^t (r'_s + z)\, ds\right) = X_0 \exp\left(\int_0^t r'_s\, ds + z\, t\right), \tag{3}$$

$$B_t = (1 + a)\, X_t = (1 + a)\, X_0 \exp\left(\int_0^t r'_s\, ds + z\, t\right). \tag{4}$$

Substituting (3) into (2), the settlement amount in the case of a knock-out in t is given by:

[4] The conversion ratio is often not one, thus the investor receives a part or a multiple of this difference.

[5] As a rule, factor a is large enough to ensure that, in general, the price of the underlying is higher than the strike, even in the case of an occasional illiquidity of the underlying after the knock-out event. Changing market conditions, in particular clearly increasing volatilities, could cause issuers to increase the factor a to restore a close-to-zero probability of a negative settlement amount (see also Section 5). Only in the most unlikely event of surprisingly high illiquidity of the underlying or large negative jumps of the underlying price – not only beneath the barrier but also beneath the strike – could the settlement amount according to (2) be negative. Since, in general, the settlement amount of real certificates is defined as non-negative, issuers take this most unlikely, practically negligible risk.

$$P_t = S_t - X_0 \exp\left(\int_0^t r'_s \, ds + z\,t\right). \tag{5}$$

Equation (5) not only describes the settlement amount, it is also used by the issuer as a price-setting formula for the secondary market.[6] At any time during the lifetime of a certificate, the issuer is willing to sell or buy back the certificate for a price according to (5).[7] Hence, the price of the certificate in t only depends on the stock price in t, the initial strike, past money market rates, and the funding rate spread. We emphasize that Equation (5) is independent of the knock-out barrier and the volatility of the underlying.

3 Semi-static superhedge and life cycle hypothesis

What is the intuition behind the price-setting formula (5)? When the issuer sells the certificate for $P_0 = S_0 - X_0$ in $t = 0$, he can at the same time purchase the underlying for S_0 and issue revolving short-term debt for the notional amount X_0 which comes out to a total payment of zero. As banks can refinance themselves at short-term money market interest rates r'_t, such as EONIA in the inter-bank market,[8] the value of this hedge position, the 'leveraged underlying', LU_t, in t is given by:

$$LU_t = S_t - X_0 \exp\left(\int_0^t r'_s \, ds\right), \tag{6}$$

where we assume that interest rate payments are accrued. A decomposition of the price-setting formula for OELCs according to (5) clarifies the relation between this formula and the leveraged underlying:

$$P_t = \underbrace{S_t - X_0 \exp\left(\int_0^t r'_s \, ds\right)}_{\substack{\text{leveraged} \\ \text{underlying } LU_t}} \underbrace{- X_0 \left(\exp\left(\int_0^t r'_s \, ds + z\,t\right) - \exp\left(\int_0^t r'_s \, ds\right)\right)}_{\text{profit potential } PP_t}. \tag{7}$$

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[6] Investors can often also exercise certificates daily (e.g., Goldman Sachs, 2006) or monthly (e.g., HSBC Trinkaus & Burkhardt, 2006). In that case they receive a settlement amount according to the particular price-setting formula based on the closing-price of the underlying.

[7] In the following we abstract from bid-ask spreads existing in practice.

[8] Additionally, the issuer can use the purchased underlying to collateralize other debt which reduces his interest rate payments. In this respect, he can refund the underlying at a net interest rate that is even lower than r'_t.

The price of the OELC in t equals the value of the leveraged underlying minus a term PP_t which is positive by definition at any $t > 0$. Accordingly, the value of the leveraged underlying is always higher than the quoted price of the OELC. Hence, buying the leveraged underlying in $t = 0$ represents a semi-static superhedging strategy which has to be terminated by the issuer in the case of a knock-out or a repurchase.[9] If the investor returns the certificate in t or, alternatively, if the OELC is knocked out in t, the bank can sell the leveraged underlying and settle the investor's claim, which yields a bank's positive (gross[10]) profit PP_t. Therefore, we denote PP_t as banks' 'profit potential'. The term 'potential' is used because the point in time of a knock-out or repurchase is ex-ante unknown. As the price according to (7) always develops more poorly than the superhedge, the price-setting formula clearly favors the issuer. Note that the semi-static superhedge is based on spot market instruments and can easily be implemented, as it works without potentially illiquid derivatives.

The issuer's profit potential PP_t from OELCs equals zero at issuance and increases over time. This finding is clearly in line with the main outcome of the life cycle hypothesis for IFPs, quoting systematically rising gains for issuers in the course of product lifetimes. However, the above-mentioned studies dealing with the first generation of leverage certificates could not confirm the life cycle hypothesis. Additionally, in contrast to papers analyzing the life cycle hypothesis for other IFPs, our result can be derived analytically based on the price-setting formula rather than indirectly by observing quoted prices and calculating fair theoretical values relying on option valuation models. Hence, in contrast to earlier studies our conclusions are not subject to model risk.

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9 The decomposed price-setting formula for open-end short certificates is:
$P_t^{short} = X_0 \exp\left(\int_0^t r'_s\, ds\right) - S_t - X_0 \left(\exp\left(\int_0^t r'_s\, ds\right) - \exp\left(\int_0^t r'_s\, ds - z\, t\right)\right)$. Obviously, a semi-static superhedging strategy for open-end short certificates contains a short position in the underlying S and an investment of X_0 in a short-term money market account.

10 The costs of structuring, distribution etc. have to be deducted from the gross profit (potential) to get to the net profit (potential). As long as there is no danger of confusion, the addition 'gross' will be abandoned in the following.

4 Valuation from the bank's perspective

4.1 Valuation algorithm

What are the fair theoretical values of OELCs? Although we do not have to apply valuation models to confirm the life cycle hypothesis for OELCs in Section 3, we still need to derive a valuation algorithm to compute the fair theoretical values of OELCs and the value of issuers' mispricing. Based on this algorithm and the later comparative static analysis, investors and issuers can assess how attractive OELCs are depending on differing underlyings, product features, and market conditions. Moreover, comparing OELCs and other IFPs, in terms of the differences between market prices and fair theoretical values, reveals the comparatively extraordinary gains for banks issuing OELCs.

All studies mentioned in Section 1 analyzing quotes of IFPs assume arbitrage-free markets and apply risk-neutral valuation techniques to calculate fair theoretical values of financial products. In contrast, it is evident that IFPs on the whole, and OELCs in particular, offer arbitrage opportunities to banks as already discussed in Sections 1 and 3. This seeming conflict can be resolved by a market segmentation hypothesis similar to that of Jarrow and van Deventer (1998) in the context of credit cards loans and demand deposits: There are a number of banks with access to capital markets, whereas this access is limited for individual investors for several reasons, such as legal restrictions, large entry barriers or excessive transaction costs. It can be assumed that markets are (nearly) arbitrage-free for issuers when they hedge such IFPs, but private investors cannot buy the replicating portfolio or the payoff profile of IFPs, at least not without additional costs. Additionally, they cannot take short-positions in IFPs to benefit from unfair quotes, i.e. to make arbitrage gains. Hence, when risk-neutral valuation techniques are applied, the resulting values are fair theoretical values for banks and lower boundaries for the value out of the perspective of private investors. The following analysis calculates the value of OELCs from the bank's point of view.

Assumption 1: *For banks, capital markets are arbitrage-free, frictionless, and complete.*

Under mild regularity conditions this ensures the existence of a unique equivalent risk-neutral measure Q (Harrison and Pliska, 1981, Heath et al., 1992). Further, we assume that the underlying of the OELC is a stock (index) following a geometric Brownian motion and allow the default-free short rate to be stochastic (Merton, 1973). For simplicity, we assume the stock to be non-dividend paying. However, this could easily be relaxed.

Assumption 2: *The default-free short rate r_t is stochastic. Under Q, the stock price satisfies the equation*

$$dS_t = r_t \; S_t \, dt + \sigma S_t \, dW_t, \tag{8}$$

where $\sigma > 0$ denotes the constant volatility and W_t a standard Brownian motion.

Studying leverage certificates of the first generation in the German market with finite lifetimes and without an openly communicated price-setting formula, Muck (2006b) allows for stochastic volatility and jumps. While it turns out that stochastic volatility has only a marginal effect on theoretical values of these leverage certificates, jump risk exercises a substantial impact. The reason for this is that the type of leverage certificates analyzed in Muck (2006b) exhibits time-invariant and coinciding barrier and strike which, in the case of a knock-out event, yields a settlement amount of zero. Therefore, possible negative jumps do not necessarily harm investors severely because if the stock price jumps beneath the barrier (and the strike) which yields a knock-out of the certificate, its value becomes zero regardless of the amount the strike is undershot. In contrast, positive jumps always fully benefit investors. Since the issuer has to bear this negative jump risk, the feature ‘barrier equals strike‘ has a positive impact on the theoretical value of leverage certificates in the presence of jump risk, as Muck (2006b) shows. However, the innovative OELCs studied in this paper exhibit a barrier that permanently exceeds the strike by a factor *a,* which is usually large enough to make it extremely unlikely that a stock price will undershoot not only the barrier but also the strike. Therefore, the strong positive impact of jump risk on theoretical values of leverage certificates analyzed in Muck (2006b) does not transfer to the OELCs considered here. Consequently, we ignore possible jump risk.

For simplicity, we disregard credit spreads in the money market and assume the bank’s short-term refinance rate r_t' equals the default-free short rate r_t. Additionally, we assume the issuer is default-free.[11]

Assumption 3: *The issuer of the certificate is default-free. The bank’s short-term refinance rate r_t' equals the default-free short rate r_t.*

[11] The influence of the issuer’s credit risk could, for example, be analyzed along the lines of Hull and White (1995) or Baule et al. (2006).

Clearly, theoretical fair OELC values depend on the holding period T of investors, since the realized profit potential of the bank increases with the length of time investors hold these certificates. The above-mentioned studies deal with IFPs exhibiting a fixed maturity. Furthermore, it is implicitly assumed that investors hold these products until maturity, although in general they have the opportunity to sell them back to the issuer at any time. To obtain comparable results, we assume that investors plan to hold OELCs for a certain time period T. Therefore, a payment prior to the expiration of this holding period only occurs in the case of a knock-out. To show the impact of different assumed holding periods on theoretical values, we later report the results for various choices of T in a comparative static analysis und discuss further extensions in Section 6.

Assumption 4: *Investors plan to hold the OELC for a finite holding period T.*

According to the risk-neutral valuation technique, the present value of a security results from the expected value of the discounted payoffs. Taking a possible knock-out into account, the point in time when the investor receives a payment according to the price-setting formula is $\tau^T = \min(\tau, T)$ given a planned holding period T. Based on Equation (7), today's fair theoretical value $OELC_0^T$ of open-end long certificates can be calculated as:

$$
\begin{aligned}
OELC_0^T &= E_Q\left(\exp\left(-\int_0^{\tau^T} r_s\, ds\right) P_{\tau^T}\right) \\
&= E_Q\left(\exp\left(-\int_0^{\tau^T} r_s\, ds\right)\left(S_{\tau^T} - X_0 \exp\left(\int_0^{\tau^T} r_s\, ds\right) - X_0\left(\exp\left(\int_0^{\tau^T} r_s\, ds + z\,\tau^T\right) - \exp\left(\int_0^{\tau^T} r_s\, ds\right)\right)\right)\right) \qquad (9) \\
&= S_0 - X_0 - X_0\left(E_Q(\exp(z\,\tau^T)) - 1\right) \\
&= \underbrace{S_0 - X_0}_{LU_0} \underbrace{- X_0\left(\exp(z\,T)\,(1 - Q(\tau \le T)) + E_Q\left(1_{\{\tau \le T\}} \exp(z\,\tau)\right) - 1\right)}_{VPP_0^T}
\end{aligned}
$$

where $E_Q(\cdot)$ denotes expectation with respect to Q.

In the Appendix we derive closed-form solutions for the risk-neutral cumulative knock-out probability $Q(\tau \le t)$ and the expression $E_Q\left(1_{\{\tau \le t\}} \exp(z\,\tau)\right)$ for every $t > 0$. We have

$$
Q(\tau \le t) = N(h_1(t)) + \left(\frac{B_0}{S_0}\right)^{\frac{-2\,(\sigma^2/2 + z)}{\sigma^2}} N(h_2(t)), \qquad (10)
$$

$$E_Q\left(1_{\{\tau \le t\}} \exp(z\,\tau)\right) = \left(\frac{B_0}{S_0}\right)^{\frac{-2\,z}{\sigma^2}} N(h_3(t)) + \left(\frac{B_0}{S_0}\right)^{-1} N(h_4(t))$$

$$\text{with } h_1(t) = \frac{\ln(B_0 / S_0) + (\sigma^2 / 2 + z)\,t}{\sigma\sqrt{t}},\; h_2(t) = \frac{\ln(B_0 / S_0) - (\sigma^2 / 2 + z)\,t}{\sigma\sqrt{t}},$$

$$h_3(t) = \frac{\ln(B_0 / S_0) + (\sigma^2 / 2 - z)\,t}{\sigma\sqrt{t}},\text{ and } h_4(t) = \frac{\ln(B_0 / S_0) - (\sigma^2 / 2 - z)\,t}{\sigma\sqrt{t}},$$

where $N(\cdot)$ denotes the standard Gaussian cumulative distribution function. Since we have $\sigma^2 / 2 + z > 0$, $Q(\tau \le t)$ converges to 1 for large t. The valuation of OELCs according to (9) discloses that their values do not depend on the short rate and its dynamics. This is a natural result, since the short rate enters both the drift of the stock price process (8) and the time-varying barrier (4).

The last row in (9) allows for an economic interpretation of the theoretical value of OELCs. Today's certificate value $OELC_0^T$ consists of the value of the leveraged underlying LU_0 (= P_0 = $S_0 - X_0$) minus today's theoretical value of the profit potential VPP_0^T given the holding period T. In other words, VPP_0^T represents today's value of the difference between the semi-static superhedging strategy and the price of the OELC. Clearly, banks are most interested in increasing this difference, as it presents the value of their arbitrage gains.

The essential target analyzed in earlier studies is the relative price deviation between the price set by the issuer and the fair theoretical value of the IFP. This relative price deviation can be interpreted as issuers' percentage profit under the standard assumption of investors being invested into the product until maturity. An analogue proceeding is possible in the context of the OELCs analyzed here:

$$RPD_0^T = \frac{P_0 - OELC_0^T}{P_0} = \frac{VPP_0^T}{P_0}, \tag{11}$$

where the numerator denotes the absolute price deviation between the quoted price P_0 and the value of the certificate $OELC_0^T$. In contrast to other studies, here we relate this difference to the current price P_0 of the OELC and not to its theoretical value. This means that according to (11), the relative price deviation is related to the price of the hedging instruments in $t = 0$. In the case of OELCs, the value of this hedge position exactly equals the current price, whereas the value of common hedge positions of classical IFPs (e.g., discount certificates) matches the theoretical value of the product. With this in view, we relate the price deviation to the value of the hedge position, like in other empirical studies.

4.2 Comparative static analysis

In this Section, we analyze the impact of different product designs and market conditions on the theoretical values of OELCs. From the perspective of banks or investors, this shows which product design is especially profitable or disadvantageous depending on market conditions. As a starting point, we examine a notional OELC on the German blue-chip stock index DAX. Since the DAX is a performance index, no adjustments for dividends are needed. The main characteristics of the certificate match real OELCs offered by HSBC Trinkaus & Burkhardt. Initially, the certificate has a strike X_0 of 5,370.00 and a barrier exceeding the strike by a = 1.5 %. Thus at issuance, the barrier amounts to B_0 = 5,450.55. According to the price-setting formula (5) the certificate's strike is continuously compounded based on the funding rate $r_t + z = r_t + 1.5$ %. For the current DAX of S_0 = 5,700.00, the price of the certificate at issuance is P_0 = 330.00.

Given a constant short rate of r_t = 3 %, the left ordinate of Figure 1 shows the issuer's profit potential PP_t. Initially, the price of the certificate and the value of the leveraged underlying match at 330.00. Consequently, the profit potential and its value are zero. The black-labeled line shows the profit potential PP_t almost linearly increasing in t. For example, given a one-year holding period, the profit potential PP_1 already reaches 83.63. That is exactly the amount the issuer gains if the certificate has not been knocked-out before and the investor still holds the certificate in one year. With respect to the current price of the certificate, the relative profit potential is as huge as 25.34 % (= PP_1 / P_0 = 83.63 / 330.00) – much more than for other IFPs analyzed in the studies mentioned above.

– Insert Figure 1 about here –

The grey plotted line in Figure 1 shows, at the same time, the today´s value of the profit potential VPP_0^T (left ordinate) and the relative price deviation RPD_0^T (right ordinate). Due to the possibility of an early knock-out, both increase much more flatly than the profit potential. Obviously, the risk-neutral probability of a premature knock-out of the certificate has a strong influence on the value of the profit potential VPP_0^T. Besides the ratio of the initial barrier and the price of the underlying at issuance (B_0 / S_0), the essential factors determining the risk-neutral knock-out probability are the volatility of the underlying σ and the funding rate spread z (see Equation (10)).[12] Figure 2 shows the cumulative risk-neutral knock-out probability of the notional certificate depending on time t and volatility σ. Given a volatility of $\sigma = 0$, the exemplary certificate is going to be knocked-out at time $\tau = 2.98$ (= $\ln(S_0 / B_0) / z$) because in the risk-neutral world the barrier grows faster than the underlying due to the funding rate spread z.[13] Consequently, there is a knock-out probability of zero for any moment before. At first, higher volatilities increase the knock-out probability quite steeply, but it flattens later on. It converges to 1 for large t.

– Insert Figure 2 about here –

Figure 3 shows the impact of the volatility on the value of the profit potential VPP_0^T for a planned holding period of $T = 1$ and for various choices of the funding rate spread z of 1.5 %, 2.5 %, and 3.5 %, and current DAX values S_0 of 5,700.00 and 6,000.00. Again, given a volatility of zero, the risk-neutral knock-out probability of the certificate is zero within the first year for all three choices of the funding rate spread z. Hence, the high profit potential PP_1 will definitely be realized by the bank. The value of the profit potential VPP_0^T decreases with increasing volatility. Consequently, a higher volatility is disadvantageous for the bank, as it increases the value of the certificate. This is caused by the fact that the investor is more likely to be 'forced out' of the certificate earlier by a higher volatility since it causes a higher probability of premature knock-out, which implies a lower value of the profit potential VPP_0^T. A higher current DAX of 6,000.00 causes a higher value of the profit potential, as it increases the difference between underlying and barrier and thus lowers the probability of a knock-out.

– Insert Figure 3 about here –

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12 The sensitivities of the risk-neutral knock-out probability can be derived analytically. The positive dependence of the risk-neutral knock-out probability on the volatility of the underlying holds for $\ln(B_0 / S_0) + t\,z < 0$. This condition is equivalent to the probability of a knock-out until time t converges to zero, given $\sigma \to 0$.

13 Given a volatility of zero, the price of the underlying in t is $S_t = S_0 \exp\left(\int_0^t r_s\, ds\right)$ while the barrier is $B_t = B_0 \exp\left(\int_0^t r_s\, ds + z\, t\right)$. This yields the non-stochastic knock-out time $\tau = \ln(S_0 / B_0) / z$.

In practice, banks issue various certificates on the same underlying with different strikes. For example, when BNP Paribas entered the market for OELCs in June 2006, they at once offered 25 certificates on the DAX with a wide range of predominantly equidistant strikes (BNP Paribas, 2006). Figure 4 shows the relative profit potential (PP_t / P_0) and the relative price deviation RPD_0^T for the notional OELC for different strikes as well as various choices of the funding rate spread z of 1.5 %, 2.5 %, and 3.5 % and, again, a holding period of $T = 1$ and a short rate of constantly 3 %. Clearly, the relative profit potential (black lines) increases with higher strikes. On the other hand, the grey plotted lines of the relative price deviations RPD_0^1 rise at first, but reach a peak at about 7 %, 11 %, and 15 %, respectively. Finally, the relative price deviations decrease to zero at an initial strike of 5,615.76. Given this strike, the barrier just matches the current DAX; $B_0 = (1 + a)\ X_0 = 1.015 \cdot 5{,}615.76 = 5{,}700.00 = S_0$. Therefore, the certificate is instantly knocked-out and thus the issuer cannot create any profit. The difference between the relative profit potential and the relative price deviation is again mainly determined by the knock-out probabilities. Certificates with a higher strike create a higher relative profit potential for the issuer over time. However, the knock-out probability increases as well, especially at initial strikes higher than about $X_0 = 4{,}000$ – which counteracts the simultaneous increase in the relative profit potential.

Since the profit potential is zero for an initial strike of $X_0 = 0$ and for an initial strike of $X_0 = S_0 / (1 + a)$, an optimal initial strike exists yielding a maximum relative price deviation. To maximize their (relative) gains, issuers could prefer to issue OELCs exhibiting this "most profitable" initial strike. In contrast, banks regularly offer a variety of strikes (see, e.g., the issues of BNP Paribas in June 2006). This is because banks realize profits over time regardless of the strikes of OELCs. Therefore, they offer various strikes to attract a large number of investors. Since a higher strike yields a lower price according to (2) and a higher leverage of the certificate, certificates with different strikes can be attractive for investors preferring different risk-profiles.

– Insert Figure 4 about here –

5 Impact of differing product features

The open-end leverage long certificates analyzed in the previous sections are closely related to the product design by HSBC Trinkaus & Burkhardt, but exist in very similar forms for other issuers. Table 3 provides an overview of the characteristics of OELCs on the DAX for different banks. Whereas all banks adjust the strike daily, most rely on monthly adjustments of the barrier. This ceteris paribus leads to slightly lower knock-out probabilities. Additionally, it suggests that interest rates influence knock-out probabilities, as the short rate no longer vanishes in the formula for the knock-out probabilities (see the derivation of the knock-out probability in the Appendix). However, the resulting effect on the value of OELCs should be negligible. Furthermore, factor a, the relative difference between the strike and the barrier, need not be fixed over the product's lifetime. Some banks state they might change it in extraordinary market conditions such as strongly increasing volatilities (see, e.g., Sal. Oppenheim, 2006). Higher volatilities result in higher knock-out probabilities and a higher probability that the difference between the price of the underlying and the strike is non-positive when the investors' claim is settled following a knock-out. This would be unfavorable for issuers. Enhancing a increases the probability of a positive difference but simultaneously decreases the value of the banks' profit potential. Some banks additionally state in their product brochures that they might vary the funding rate spread z over the product's lifetime as well. In the context of a possible enhancement of a, increasing the funding rate spread z at the same time could allow issuers to keep the value of their profit potential stable.

– Insert Table 3 about here –

How are our findings affected by non-fixed relative differences between strike and barrier and funding rate spreads over time? As the value of the superhedge portfolio is at any time $t > 0$ above the price-setting formula (see (7)), as long as z is positive, regardless of a, the price setting formula favors the issuer in any situation, even if z is stochastic. Analog considerations hold for the life cycle hypothesis since the profit potential increases over time, even for a stochastic z. However, the valuation model had to be modified.

6 Conclusion

This paper presents the first analysis of the pricing and valuation of open-end leverage certificates on the German retail market. In contrast to earlier studies focusing on the price-setting behavior of banks issuing IFPs, we do not have to rely on prices collected from primary and secondary markets since issuers communicate their price-setting formulas for open-end leverage certificates. By applying a semi-static superhedge based on spot market instruments, it turned out that the issuers' price-setting formula strongly favors themselves. This 'mispricing by construction' does not cause arbitrage activities of other market participants, because short positions in the certificates are practically impossible. Furthermore, our findings clearly confirm the life cycle hypothesis for leverage products for the first time, meaning systematically increasing profits for issuers over the product's lifetime.

Applying standard valuation techniques and assuming fixed planned holding periods, we determine the value of the issuers' mispricing of OELCs. Given a holding period of one year and assuming realistic parameters for the DAX, respective open-end leverage long certificates are regularly sold at (least) about 5 to 10 % above their theoretical values. Clearly, this analysis depends on the assumed behavior of investors. This could be derived from empirical data about the buying and selling decisions of investors. However, due to a lack of data at present, this challenging analysis will be the topic of a subsequent study.

Moreover, issuers can easily gain ex-post realized potential profits of about 20 to 30 % related to the initial price, if the DAX increases to a certain level over time, so that value and price of the certificate rise as well and neither a knock-out nor a repurchase occurs. Therefore a positive price development of the underlying of open-end long leverage certificates can be at the same time beneficial to issuers and investors. Furthermore, these relatively high profits over time are also possible for issuers, if the underlying develops in a negative direction which may cause certificates to be prematurely knocked-out. It is only necessary for certificates that were knocked-out to be substituted through new ones for the total volume of outstanding open-end certificates to remain roughly constant.

By purchasing open-end leverage certificates, investors participate to a disproportionately high extent in changes of the underlying. However, banks and institutional investors can also attain equal or similar payoff profiles at lower costs using forwards or futures contracts traded on the EUREX (European Exchange) or over-the-counter. Private investors, however, often have no access to these markets. Hence the existence of open-end leverage certificates is justified by incomplete capital markets or imperfections, such as market access barriers, transaction costs, and information asymmetries. Private investors interested in the payoff profiles of open-end leverage certificates therefore normally rely on purchasing those certificates from banks.

Since banks regularly apply the ‘unfair’ price-setting formula discussed in this paper, they produce an enormous profit potential due to the funding rate spread included in the price. However, in determining the net profit of issuers, an adequate payment for the issuer’s service to retail costumers should be incorporated; this should at least cover the cost of structuring, distribution etc. Considering the simple semi-static superhedging strategy shown above, the identified profit potentials for issuers are still noteworthy. However, in the future we do expect decreasing issuers’ profits due to the rising competition in this segment of the German retail market, which will probably be reflected in lower funding rate spreads.

Appendix

The derivations of the cumulative risk-neutral knock-probability $Q(\tau \leq t)$ and the expression $E_Q(1_{\{\tau \leq t\}} \exp(z\,\tau))$ are based on the following two lemmas:

Lemma 1 (e.g., Bielecki and Rutkowski, 2002, p. 67): For every $t > 0$ let the stochastic process Y_t be given by

$$Y_t = y_0 + \nu\, t + \sigma\, W_t$$

for some constants $y_0 > 0$, ν, $\sigma > 0$ and a standard Brownian motion W_t under the probability measure Q. The stopping time τ is defined by $\tau = \inf\{t : Y_t \leq 0\}$.

For any $s > 0$, we have

$$Q(\tau \leq s) = N(h_1(s)) + \exp(-\,2\,\nu\,\sigma^{-2}\,y_0)\,N(h_2(s))$$

where $N(\cdot)$ stands for the standard Gaussian cumulative distribution function, and $h_1(s) = \dfrac{-\,y_0 - \nu\, s}{\sigma\sqrt{s}}$, $h_2(s) = \dfrac{-\,y_0 + \nu\, s}{\sigma\sqrt{s}}$.

Lemma 2 (Bielecki and Rutkowski, 2002, p. 74): Let a, b, c be constants with $b < 0$ and $c^2 > 2\,a$. For every $y > 0$, we have

$$\int_0^y \exp(a\,x)\; dN\!\left(\frac{b - c\,x}{\sqrt{x}}\right) = \frac{d + c}{2\,d}\,g(y) + \frac{d - c}{2\,d}\,f(y),$$

where $d = \sqrt{c^2 - 2\,a}$, $g(y) = \exp(b\,(c - d))\,N\!\left(\dfrac{b - d\,y}{\sqrt{y}}\right)$ and $f(y) = \exp(b\,(c + d))\,N\!\left(\dfrac{b + d\,y}{\sqrt{y}}\right)$.

Derivation of $Q(\tau \leq t)$

Let the process S_t, $t \geq 0$, satisfy

$$dS_t = r_t\,S_t\,dt + \sigma\,S_t\,dW_t$$

under the risk-neutral probability measure Q, where r_t denotes the short rate in t, $\sigma > 0$ is a constant and W_t denotes a standard Brownian motion. By definition we have $S_t = S_0 \exp\!\left(\int_0^t r_s\,ds - \sigma^2\,t\,/\,2 + \sigma\,W_t\right)$. Let the barrier in t be given by $B_t = B_0 \exp\!\left(\int_0^t r_s\,ds + z\,t\right)$ for some constant z and $0 < B_0 < S_0$. The first passage time is defined by $\tau = \inf\{t\text{: } S_t \leq B_t\}$.

We have

$$\{S_t \le B_t\} = \left\{ S_0 \exp\left(\int_0^t r_s\, ds - \sigma^2 t / 2 + \sigma W_t \right) \le B_0 \exp\left(\int_0^t r_s\, ds + z\, t \right) \right\}$$

$$= \{S_0 \exp(-\sigma^2 t / 2 + \sigma W_t) \le B_0 \exp(z\, t)\}$$

$$= \{\ln(S_0 / B_0) + (-\sigma^2 / 2 - z)\, t + \sigma W_t \le 0\}.$$

By applying Lemma 1 to $y_0 = \ln(S_0 / B_0)$ and $\nu = -\sigma^2 / 2 - z$ we obtain for every $t > 0$

$$Q(\tau \le t) = N(h_1(t)) + \left(\frac{B_0}{S_0}\right)^{\frac{-2\,(\sigma^2/2 + z)}{\sigma^2}} N(h_2(t)),$$

$$h_1(t) = \frac{\ln(B_0 / S_0) + (\sigma^2 / 2 + z)\, t}{\sigma\sqrt{t}},\ h_2(t) = \frac{\ln(B_0 / S_0) - (\sigma^2 / 2 + z)\, t}{\sigma\sqrt{t}}.$$

Derivation of $E_Q(1_{\{\tau \le t\}} \exp(z\, \tau))$

Based on the above representation of $Q(\tau \le t)$ we can conclude

$$E_Q(1_{\{\tau \le t\}} \exp(z\, \tau)) = \int_0^t \exp(z\, x)\, dQ(\tau \le x)$$

$$= \int_0^t \exp(z\, x)\, dN(h_1(x)) + \left(\frac{B_0}{S_0}\right)^{\frac{-2\,(\sigma^2/2 + z)}{\sigma^2}} \int_0^t \exp(z\, x)\, dN(h_2(x)).$$

By applying Lemma 2 to each summand in the above equation separately with $y = t$, $a = z$, $b = \dfrac{\ln(B_0 / S_0)}{\sigma}$, $c = -(\sigma / 2 + z / \sigma)$ and $= (\sigma / 2 + z / \sigma)$, respectively, we obtain for $\sigma^2 \ne 2\, z$ ($\Leftrightarrow c^2 - 2\, a > 0$) after rearranging and collecting terms:

$$E_Q(1_{\{\tau \le t\}} \exp(z\, \tau)) = \left(\frac{B_0}{S_0}\right)^{\frac{-2\, z}{\sigma^2}} N(h_3(s)) + \left(\frac{B_0}{S_0}\right)^{-1} N(h_4(s)),$$

$$h_3(t) = \frac{\ln(B_0 / S_0) + (\sigma^2 / 2 - z)\, t}{\sigma\sqrt{t}},\ h_4(t) = \frac{\ln(B_0 / S_0) - (\sigma^2 / 2 - z)\, t}{\sigma\sqrt{t}}.$$

The same formula holds for $\sigma^2 = 2\, z$ as $E_Q(1_{\{\tau \le t\}} \exp(z\, \tau))$ is a continuous bounded function of σ.

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Table 1: Number of open-end leverage certificates issued by banks on the German market from 2002 to September 2006

	2002		2003		2004		2005		1-9/2006	
	Long Short	all	Long Short	all	Long Short	all	Long Short	all	Long Short	all
ABN Amro Bank	35	77	323	561	586	971	905	1,455	1,489	2,406
	42		238		385		550		917	
BNP Paribas									61	109
									48	
Citigroup							35	64	635	1,129
							29		494	
Commerzbank			133	240	870	1,340	1,814	2,972	2,412	3,390
			107		470		1,158		978	
Deutsche Bank	3	5	68	119	67	117	234	320	654	1,040
	2		51		50		86		386	
Dresdner Bank			2	5	6	11	6	13	7	13
			3		5		7		6	
Goldman Sachs			59	104	43	82	47	87	1,379	2,805
			45		39		40		1,426	
HSBC Trinkaus & Burkhardt			30	57	369	557	545	892	635	1,044
			27		188		347		409	
Lang & Schwarz							5	9	17	29
							4		12	
Raiffeisen Centrobank	1	2	4	6	40	55	12	22	26	49
	1		2		15		10		23	
Sal. Oppenheim					14	28	146	272	346	715
					14		126		369	
Société Générale			113	194	1,034	1,720	1,164	1,581	933	1,301
			81		686		417		368	
all	39	84	732	1,286	3,029	4,881	4,913	7,687	8,594	14,030
	45		554		1,852		2,774		5,436	

The table shows the number of open-end leverage certificates issued on the German market from 2002 to September 2006, listed according to issuers. Source: Deriva GmbH Financial IT and Consulting.

Table 2: Underlyings of open-end leverage certificates issued by banks on the German market from 2002 to September 2006

	2002		2003		2004		2005		1-9/2006	
Underlying	Long Short	all	Long Short	all	Long Short	all	Long Short	all	Long Short	all
Stocks			48 32	80	1,828 903	2,731	3,018 1,178	4,196	5,385 2,676	8,061
Stock indices	36 42	78	563 453	1,016	628 485	1,113	962 710	1,672	1,649 1,389	3,038
Exchange rates			43 25	68	249 211	460	406 445	851	370 305	675
Precious metals	3 3	6	40 13	53	152 102	254	164 99	263	441 384	825
Others			38 31	69	172 151	323	363 342	705	749 682	1,431
all	39 45	84	732 554	1,286	3,029 1,852	4,881	4,913 2,774	7,687	8,594 5,436	14,030

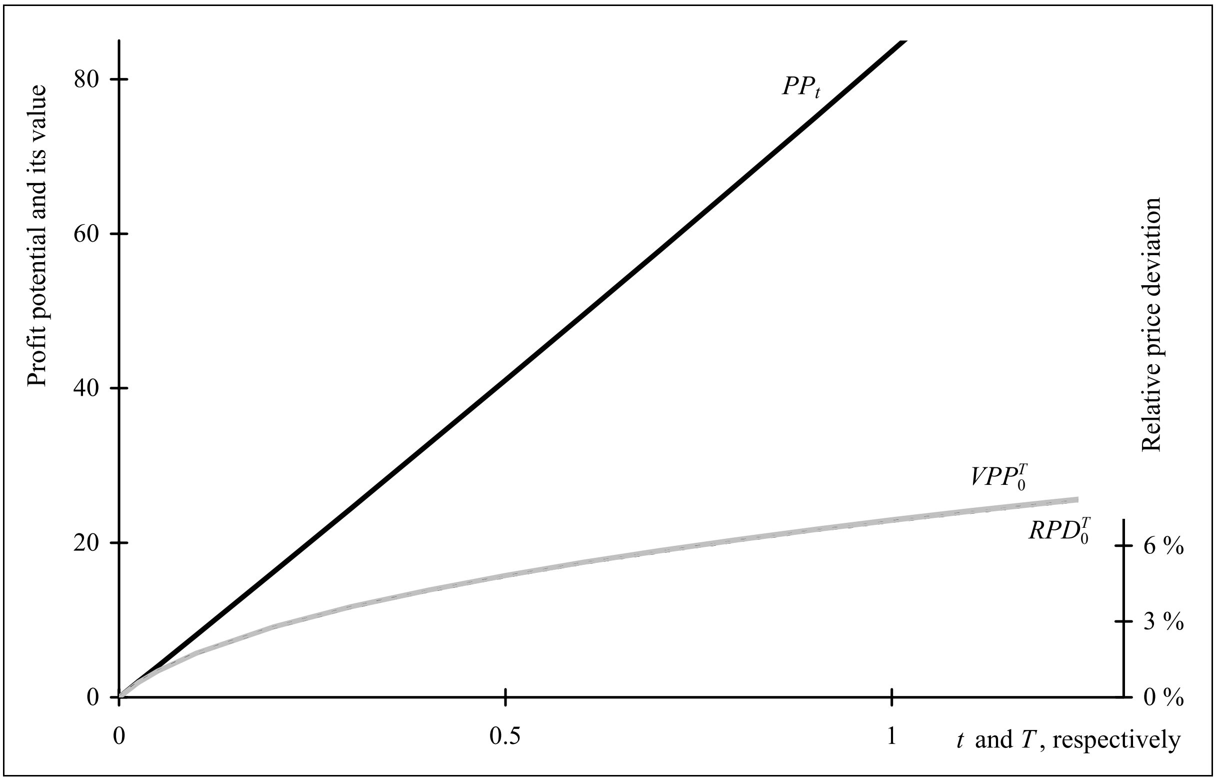
The table shows the number of open-end leverage certificates issued on the German market from 2002 to September 2006, listed according to the respective underlying. ‘Others’ contains open-end leverage certificates on, e.g., commodities, interest rates futures, and foods such as cacao, orange juice, coffee, and sugar. Source: Deriva GmbH Financial IT and Consulting.

Table 3: Specification of open-end long leverage certificates on the DAX, listed according to the issuing bank in August 2006

Issuer	Product name	Reference interest rate	Funding rate spread z	Factor a	Adjustment Barrier	Adjustment Strike
ABN Amro	Mini Future Certificate; DAX Index Mini Long Certificate	money market rate	at issuance: 3.0 %, possible changes over time, max 3.0 %	at issuance: 1.5 %, possible changes over time, min 1.5 %, max 5.0 %	monthly	daily
BNP Paribas	Open-end Turbo Long Warrant	1-month EURIBOR	at issuance: 2.5 % possible changes over time, min 0 %, max 5.0 %	normally: 1.5 %, for certain certificates: 2.0 % or 3.0 %	daily	daily
Citigroup	Open-end Stop Loss Bull Turbo; Open-end Turbo Stop Loss Knock-out Warrant	1-month EURIBOR	currently about 2.0 %, possible changes over time	at issuance: about 1.5 %, possible changes over time	monthly	daily
Commerzbank	Unlimited Turbo Bull Certificate	1-month EURIBOR	currently about 3.0 %, possible changes over time	currently about 1,5 %, possible changes over time	monthly	daily
Deutsche Bank	Wave XXL; Call Wave XXL Knock-out Warrant	EONIA	at issuance: 3.75 %, possible changes over time	currently 2.0%, possible changes over time, min 2.0 %, max 10.0 %	monthly	daily
Dresdner Bank	Call Open-end Knock-out Warrant	EONIA	at issuance: 1.5 %, possible changes over time	currently about 2.0 %, possible changes over time, min 2.0 %, max 10.0 %	monthly	daily
Goldman Sachs	Mini Future Turbo Warrant	EUR LIBOR Overnight	at issuance: 2.0 %, possible changes over time, max 4.0 %	at issuance: 2.0 %, possible changes over time, max 5.0 %	monthly	daily
HSBC Trinkaus & Burkhardt	Mini Future Certificate	EONIA	1.5 %	1.5 % (older certificates: 3.0 %)	daily	daily
Lang & Schwarz	Open-end Turbo Call	1-month EURIBOR	currently 1.5 %, possible changes over time	about 1.75 %	monthly	daily
Sal. Oppenheim	Turbo Open-end Warrant	1-month EURIBOR	at issuance: 2.0 %, possible changes over time	at issuance: 3.0 %, possible changes over time	daily	daily
Société Générale	Open-end Turbo Long Knock-out Warrant	EUR LIBOR Overnight	2.5 %	at issuance: 2.0 %, possible changes over time, max 7.0 %	monthly	daily

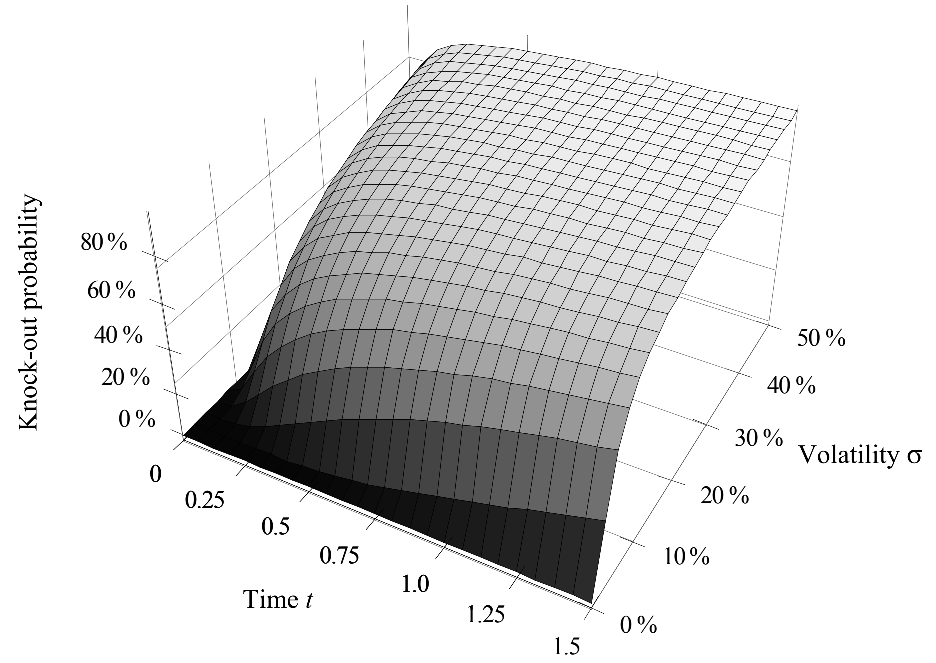
The table shows the specification of open-end leverage products on the DAX, listed according to the issuer. German product names are translated into English. The data were collected from internet-published documents such as product brochures and sales prospectuses of issuers on August 20, 2006.

Figure 1: Profit potential of banks and its value, and relative price deviation of an open-end long leverage certificate, depending on the point in time t and the investor's holding period T



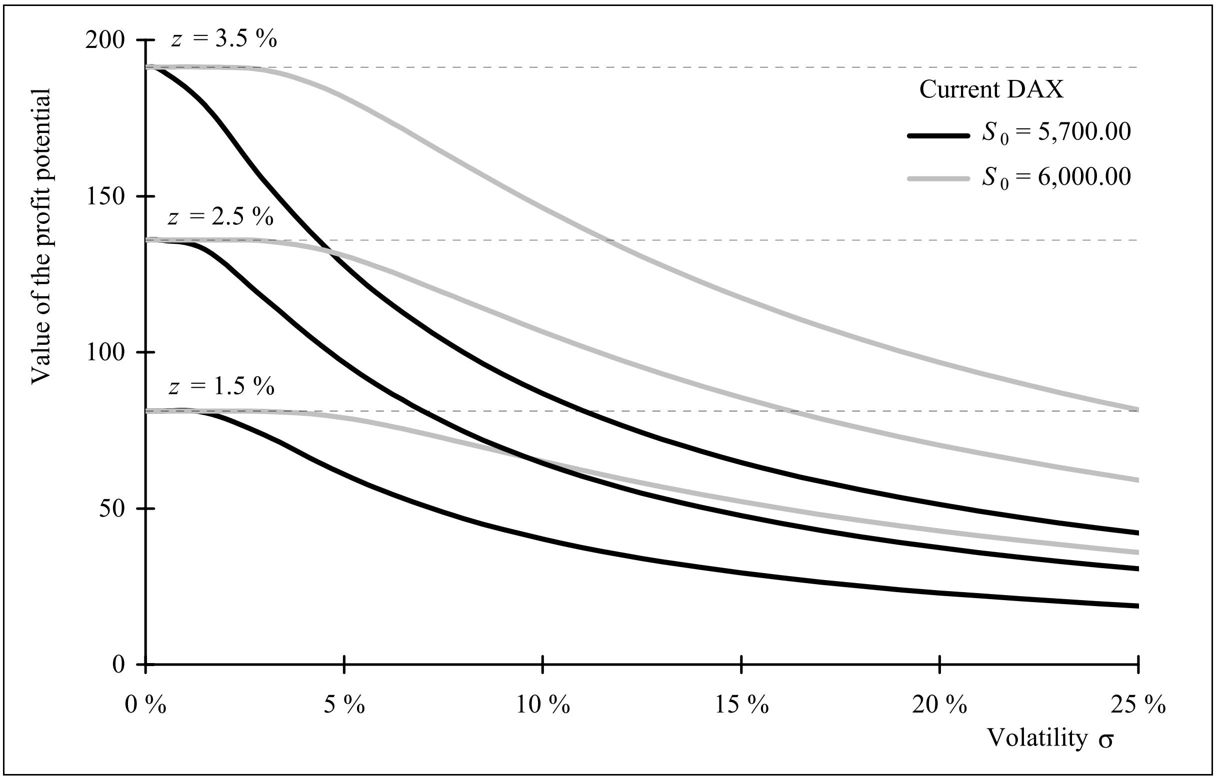
For a notional open-end long leverage certificate on the DAX, this figure shows the profit potential PP_t (see (7)) as well as its value VPP_0^T (see (9)), and the relative price deviation RPD_0^T (see (11)) as a function of the point in time t and the investor's holding period T, respectively. The left ordinate shows the profit potential and its value, the right ordinate the relative price deviation. The main features of the open-end leverage certificate on the DAX are: initial strike $X_0 = 5{,}370.00$, initial barrier $B_0 = 5{,}450.55$, relative difference between barrier and strike $a = 1.5$ %, and funding rate spread $z = 1.5$ %. The short rate is constantly $r_t = 3$ % and the parameters of the DAX are: $S_0 = 5{,}700.00$, $\sigma = 20$ %.

Figure 2: Cumulative risk-neutral knock-out probability of an open-end long leverage certificate, depending on the point in time t and the volatility σ



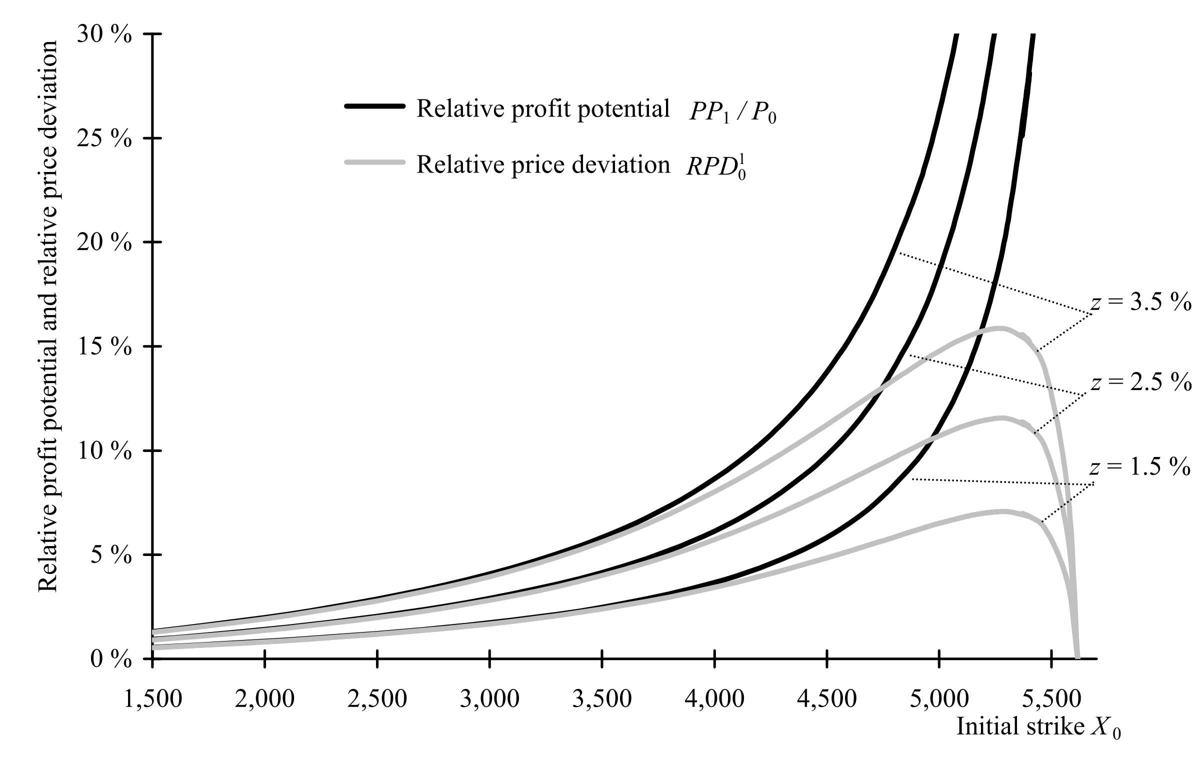
For a notional open-end long leverage certificate on the DAX, this figure shows the risk-neutral knock-out probability (see (10)) as a function of the point in time t and the volatility σ of the DAX. The main features of the open-end leverage certificate on the DAX are: initial strike $X_0 = 5{,}370.00$, initial barrier $B_0 = 5{,}450.55$, relative difference between barrier and strike $a = 1{,}5$ %, and funding rate spread $z = 1.5$ %. The current DAX is $S_0 = 5{,}700.00$.

Figure 3: Value of the banks' profit potential from an open-end long leverage certificate, depending on the volatility σ of the underlying for different funding rate spreads z



For a notional open-end long leverage certificate on the DAX, this figure shows the value of the potential profit VPP_0^T (see (9)) as a function of the volatility σ of the DAX for funding rate spreads of $z = 1.5$ %, $z = 2.5$ %, and $z = 3.5$ %. The other main features of the open-end leverage certificate on the DAX are: initial strike $X_0 = 5{,}370.00$, initial barrier $B_0 = 5{,}450.55$, and relative difference between barrier and strike $a = 1{,}5$ %. The planned holding period of the investor is $T = 1$, the current DAX is $S_0 = 5{,}700.00$ and $S_0 = 6{,}000.00$, respectively.

Figure 4: Relative profit potential of banks and relative price deviation of an open-end long leverage certificate, depending on the initial strike X_0 for different funding rate spreads z



For a notional open-end long leverage certificate on the DAX, this figure shows the relative profit potential PP_t / P_0 (see (7)) and the relative price deviation RPD_0^T (see (11)) as a function of the initial strike X_0 for funding rate spreads of $z = 1.5$ %, $z = 2.5$ %, and $z = 3.5$ %. The relative difference between barrier and strike is $a = 1.5$ %. The planned holding period of the investor is $T = 1$, the short rate is constantly $r_t = 3$ %, and the parameters of the DAX are: $S_0 = 5{,}700.00$, $\sigma = 20$ %.