Relational Contracts and the Theory of the Firm: A Renegotiation-Proof Approach

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Abstract

I analyze the effect of long-term relations on the property rights theory of the firm when agents's actions affect the structure of the game. In this setting, renegotiation proof relational contracts exists. Single ownership dominates joint ownership when the discount factor is very high or very low, however there exists a mid-region where joint ownership becomes optimal. Joint ownership has a higher chance of being optimal in industries where agents' actions affect the structure of the game, i.e. where reputation and network effects are important.

1 Introduction

The boundary of the firm is one of the essential topics in corporate finance, which has important implications for the organization of long-term partnerships and alliances. One of the prominent theories is the property rights theory of the firm introduced by (Grossman and Hart, 1986; Hart and Moore, 1990)[1]. In the property rights theory of the firm incomplete contracts combined with hold-up problems imply that the allocation of control over residual rights (ownership structure) is important in the ex-post sharing of revenues, which distorts the ex-ante investments. The boundary of the firm is determined by the optimal allocation of ownership that minimizes the ex-ante investment distortions. According to classical property rights theory, ownership of assets by a single agent always dominates joint ownership.

The classical property rights theory of the firm is based on the analysis of a single period interaction. However, agents may use repeated relationships to motivate cooperative behavior and mitigate hold-up problems. Holmstrom and Roberts (1998) argue that Japanese manufacturing firms' subcontracting practices exhibit the importance of repeated relationships in motivating the behavior of agents.[2] Therefore it is reasonable to explore whether repeated relations affect the optimal allocation of ownership in the property rights theory of the firm.

Recent literature analyzes how long-term relationships affect the optimal allocation of ownership. In a repeated setting, Halonen (2002) finds that relational contracts are important in determining the optimal ownership structure, and joint ownership can be optimal contrary to the predictions of the single period model. Baker, Gibbons, and Murphy (1999, 2001,

[1] An earlier theory is the transactions costs theory of Williamson (1975, 1985) and Klein, Crawford, and Alchian (1978).

[2] Firms in the supply chains get involved in long-run deals with each other, and features such as flexibility are not contracted but provided in certain cases. (Klein and Leffler, 1981; Telser, 1980). Voluntary transfers between agents can be made contingent on non-contractible performance measures and can be self-enforced (Bull, 1987; MacLeod and Malcomson, 1989; Baker, Gibbons, and Murphy, 1994).

2002) discuss the interaction of relational contracts and the choice of asset ownership; asset ownership affects the best relational contract that can be sustained in equilibrium.

The relational contracts literature relies on repeated game techniques to model the long-term interactions of agents[3]. Repeated interactions allow agents to use relational contracts to sustain cooperation, based on the idea that if the initial contract is violated agents can punish each other in the long-term relationship. The possibility of punishing each other introduces an additional tradeoff; while single ownership maximizes the incentive to invest joint ownership maximizes the punishment for the breach of the contract. Therefore it is possible to arrive at different predictions from Grossman and Hart (1986) and Hart and Moore (1990) (here in after GHM).

The common practice in the relational contracts literature is to assume that agents are capable of committing not to renegotiate the relational contract (Hart, 2001). It is difficult to remove this assumption because if agents are rational enough to conjecture that they will renegotiate in the case of breach, they will not cooperate in the first place. It is not certain whether agents can commit not to renegotiate in practice (see (Hart, 1995; Maskin and Tirole, 1999; Tirole, 1999) for this debate). Even if commitment through contracts is possible it may not be socially desirable to enforce contractual terms that prevent renegotiation (Schmitz, 2005). Therefore, it is important to analyze the effect of relational contracts on the property rights theory of the firm when agents are allowed to renegotiate.

I analyze the optimal ownership decision in a repeated setting where agents' actions affect future payoffs and agents are allowed to renegotiate relational contracts. For convenience, I call these games endogenously structured games. Endogenously structured games includes repeated games as a sub category. It has been acknowledged for a long time that the payoff structure of partnership games depend on the strategies of players (Radner, 1991) and these

[3]See Levin (2003) for a formal discussion.

type of games are commonly used to model partnerships. Natural places to look for endogenously structured economic environments are industries where reputation or network effects are important. Moreover, when network effects are important firms rely more on relational contracts (Robinson and Stuart, 2006), which further justifies using endogenously structured games to analyze the effect of relational contracts on the theory of the firm.

I define renegotiation-proofness in endogenously structured games and identify the necessary conditions for the existence of renegotiation-proof relational contracts. If the payoff structure of the game sufficiently depends on the history of play renegotiation-proof relational contracts exist. Consider a case where the payoff structure depends on the reputation of the agents; exerting suboptimal effort and producing low quality products may damage reputation, which will affect the future payoffs. Renegotiation-proof relational contracts exist because agents can restore reputation after deviation only by taking costly actions.

I show that the optimal ownership structure in a repeated relationship is the same as the optimal ownership structure in the one period GHM model when agents have very high or very low discount factors. However, there may be a mid-region of discount factors where joint ownership becomes optimal. Contrary to the common belief, the probability that joint ownership is optimal does not increase with the discount factor. When the discount factor is neither very high nor very low, agents care enough about future so that they cooperate in equilibrium, however they do not care enough about future to cooperate after deviation. This strategy profile is very similar to grim strategy profile in repeated games, however unlike for repeated games, this strategy profile will be renegotiation-proof as long as the cost of taking action to improve future payoffs is high. Joint ownership may become optimal because it provides stronger punishment off the equilibrium path, which helps to sustain cooperation. Joint ownership is optimal only in industries where the payoff structure sufficiently depends on the strategies of agents.

When agents do not care enough about the future, they basically play the single period game repeatedly. Therefore, it is not surprising that I find the same optimal ownership structure as GHM. When agents care sufficiently about the future, they prefer to cooperate even after deviation in order to improve the payoffs in the future. Agreeing to cooperate after deviation does not eliminate incentive to cooperate in the first place because taking action to improve future payoffs is costly. In this case, I find that GHM's predictions continue to hold because there is no additional tradeoff among ownership structures. Single ownership provides the best incentives to invest both on and off the equilibrium path.

This paper is related to the literature that analyzes the cases in which joint ventures are optimal. Cai (2003) shows that when agents can make general investments, which increase their outside options, joint ownership becomes optimal because joint ownership limits the incentives to invest outside the relationship. Maskin and Tirole (1999) show that joint ownership can lead to first best investment levels when combined with an option to sell contract. Matouschek (2004) introduces a theory of the firm based on minimizing the ex-post bargaining inefficiency and shows that joint ownership is optimal when expected gains from trade are large.

Gibbons (2005) provides a review of the theory of the firm literature.[4] The model in this paper is useful to analyze the effect of long-term relations on the various theories of the firm when agents are allowed to renegotiate.

[4]Holmstrom and Roberts (1998) argue that in a more general theory, agency problems (Holmstrom, 1991; Holmstrom and Milgrom, 1994; Holmstrom and Tirole, 1991; Holmstrom, 1999), market monitoring and knowledge transfers should also affect the incentives to invest. Hart and Holmstrom (2002) expand the property rights theory to be suitable to analyze the organization of large firms where managers and owners are different and there are private benefits and profit diversion. Several papers pointed out that ownership of assets may not always motivate relation specific investments (Rajan and Zingales, 1998; DeMeza and Lockwood, 1998; Chiu, 1998).

2 Repeated versus Endogenously Structured Games

A simple example is useful to understand the difference between endogenously structured games and repeated games. In this example the quality of the product depends on the effort levels of both agents. There are two effort levels: cooperate, C and deviate, D. In a repeated game, the structure of the game does not depend on the history of play. Table 1 describes the payoffs of agents for different effort levels, where agents are playing a repeated prisoner's dilemma game with a double-sided moral hazard problem (Holmstrom, 1982).

Table 1: **Repeated Game**

		Agent1	
		Cooperate	Deviate
Agent2	Cooperate	10,10	20,-10
	Deviate	-10,20	5,5

The grim trigger strategy profile, which is commonly used in the relational contracts literature, prescribes playing (C,C) if both agents played cooperation in the past and prescribes playing (D,D) if any of the agents deviated in the past. The cooperation outcome, (C,C), can be supported as the subgame perfect Nash equilibrium (SPNE) of the game by the grim-trigger strategy if $\delta > 1/2$, where $\delta \in (0,1]$ is the discount factor. However, if renegotiation is allowed agents can renegotiate after deviation and agree on playing (C,C) without playing the self-imposed punishment. If agents can conjecture this outcome, possibility of renegotiation will destroy agents' incentives to cooperate in the first place. Therefore, the problem does not have a simple solution.

The solution to the renegotiation problem is to incorporate the broader picture into the game. Suppose that consumers do not know the quality of the product at the time of purchase, but they can accurately judge the quality after using the product. Consumers believe that the product quality in the current period is the same as the product quality in the previous period. One can use different belief updating procedures. I just need consumers to pay significantly less for the current period product when they discover that the previous period product is of low quality.

After any history of the game, the product quality is high if both agents have exerted effort in the previous period, otherwise it is low. Table 2 shows the agents' payoffs when the consumers play endogeneously structured game. Note that when the consumers believe that the product is of low quality, cooperation is not the social best for a single period. On the other hand, in a multi-period game exerting effort could be seen as an investment in reputation that will pay off once the consumers' beliefs are updated.

Table 2: **Endogenously Structured Game**

		Belief: High Quality		Belief: Low Quality	
		Agent1		Agent1	
		Cooperate	Deviate	Cooperate	Deviate
Agent2	Cooperate	10,10	20,-10	0,0	3,-10
	Deviate	-10,20	5,5	-10,3	-5,-5

Let's assume that in the low-quality game agents agree to play deviation (D,D) forever. In that case, the minimum discount factor supporting cooperation in the high-quality game as the SPNE is 1/2. If playing cooperation is the Nash Equilibrium (NE) in the low-quality game, the minimum discount factor supporting cooperation as the SPNE in the high-quality game is 2/3.

Now let's summarize what happens as we increase the discount factor. If $\delta < 1/2$, cooperation in the high-quality game cannot be maintained as the SPNE of the game. If $1/2 < \delta < 2/3$, by using a simple grim-trigger strategy, agents can sustain cooperation as the SPNE of the high-quality game. I will later show that this equilibrium is also renegotiation-proof. The grim-trigger strategy becomes renegotiation-proof when agents do not care enough about the future and do not want to upgrade the consumers' beliefs from low quality to high-quality.

If $\delta > 2/3$, to change the consumers' beliefs, it is worth exerting effort, producing good quality products and selling them at the low quality price for one period. Since agents would like to change consumers' beliefs once they are in the low quality game, the grim-trigger strategy is no longer renegotiation-proof. On the other hand, I will show that a strategy profile that always prescribes cooperation becomes the renegotiation-proof equilibrium of the game.

3 Renegotiation in Endogenously Structured Games

3.1 Model

In this section, I introduce a formal model and define renegotiation-proofness in the endogenously structured games. The number of players is denoted by $I = 1,2$. Each player exerts effort $e_i \in E_i$, which is not observable by third parties. As a result, effort levels are not contractible. For now, I assume that there are only two effort levels, $E = (C,D)$. Strategy S_i for player i is a function that defines an effort level (action) for every possible history h of play, $S_i : h \rightarrow E_i$.

State variable is $x \in H,L$.[5] H and L denote high and low states of the world, respectively. The general assumption is that in the high state, agents can sell more goods, sell goods at a higher price or both. The next period state will be determined by the state transition function $g : x^{t+1} = g(S^t, x^t)$, where x^0 is assumed to be the high state H. By including this state variable,

[5] If the state variable is continuous then the number of games that could be played is infinite. Representing the game in the normal form is not be possible and solving optimal strategies requires recursive solution techniques. Radner and Benhabib (1992) provide an example for a continuous case and analyze the joint exploitation of a productive asset.

I relax the repeated-game assumption that the physical environment is memoryless. In this game, players change the state of the world by playing certain strategies. State transition can be modeled in various ways; agents may switch from high state to low state if they cannot maintain a certain level of output, quality or reputation. In this simple game, I assume that if one agent deviates, agents start playing the game in the low state. I will discuss this assumption in detail when I introduce continuous effort levels. Agents can upgrade from low to high state if they can coordinate their actions. For example, agents may be required to increase sales or quality to a certain level to upgrade the state. In this game, if both agents play cooperation in the low state, the next period state will be high.

The agents have von Neumann-Morgenstern utility and payoff functions, $u_i(S,x)$ for the stage game (one period game) in state x. The total utility is the net present value of stage game utilities $U_i(S,x^0,\delta)$, where δ is the discount factor. A game is defined by $\Gamma(E,x^0,g,I,u_i,\delta)$.

Table 3 summarizes the payoffs. The letters n, d, c and b respectively denote the stage game NE payoff, deviation payoff, cooperation payoff and the betrayed agent's payoff. The ordering of payoffs in the high state could be described as follows: $d_i(H) > c_i(H) > n_i(H) > b_i(H)$ and $d_i(H)+b_j(H) < c_i(H)+c_j(H)$. This ordering is similar to the ordering of payoffs in a standard prisoners' dilemma game. In the low state, the stage game payoffs are ordered as follows: $d_i(L), n_i(L) > c_i(L) > b_i(L)$. In the low state, cooperation payoffs are lower than the NE payoffs since producing a good quality product and selling it at the price of a low quality product is costly. In order to make the game more general, I do not specify the ordering of $d_i(L)$ and $n_i(L)$.

Table 3: **Payoffs**

		High State		Low State	
		Agent1			
		C	D	C	D
Agent2	C	$c_i(H), c_i(H)$	$d_i(H), b_i(H)$	$c_i(L), c_i(L)$	$d_i(L), b_i(L)$
	D	$b_i(H), d_i(H)$	$n_i(H), n_i(H)$	$b_i(L), d_i(L)$	$n_i(L), n_i(L)$

3.2 The Definition of Renegotiation-Proofness

In repeated games, if the players are sufficiently patient, any feasible and individually rational payoff can be enforced in a SPNE equilibrium. Equilibrium arises as an initial agreement between the players. It is also possible that agents can renegotiate to play another strategy profile at an arbitrary point in time. If the initial agreement is Pareto dominated by the new one, it is possible for agents to mutually agree on this change. The restriction that eliminates this sort of mutual deviation is called renegotiation-proofness.

There are a number of definitions for renegotiation-proof equilibrium that are not necessarily in line with each other (Pearce, 1992). It is beyond the scope of this paper to review this literature in detail. The approach in this paper is taken from Farrell and Maskin (1989) and conforms to that of Bernheim and Ray (1989). However, Farrell and Maskin's definition does not exactly fit into this game. In repeated games agents can freely switch equilibrium, whereas in this game they have to consider the state as well. That's why I need to modify the definition of Farrell and Maskin's weakly renegotiation-proof equilibrium.

Definition 1. *Define the game the high and the low game if we start playing the game in the high and low state, respectively. A subgame perfect equilibrium S in the high and low game is weakly renegotiation-proof if continuation equilibria S_i and S_j of S such that S_i strictly Pareto dominates S_j do not exist after any history in the high and low game, respectively.*

The weakly renegotiation-proof criterion is not strong enough because any strategy profile that prescribes only one continuation equilibrium for each state is trivially weakly renegotiation-proof. Since agents can also mutually agree on another weakly renegotiation-proof equilibrium, a stronger refinement is needed.

Definition 2. *A weakly renegotiation-proof equilibrium S is strongly renegotiation-proof if none of its continuation equilibria in the high and low game is strictly Pareto dominated by another weakly renegotiation-proof equilibrium in the high and low game respectively.*

The strongly renegotiation-proof criterion eliminates cases where agents can jointly switch between two weakly renegotiation-proof equilibria. In the rest of the paper, renegotiation-proof means strongly renegotiation-proof.

3.3 Renegotiation-Proof Strategy Profiles

I consider only the forgiving strategy and the grim strategy profiles, which are defined below. Later, it will be clear that no other strategy profile exists that sustains cooperation in the high game and that is not Pareto dominated by one of these two strategy profiles.

Forgiving Strategy Profile: If the state is high cooperate, play (C,C). If the state is low cooperate, play (C,C).

Grim Strategy Profile: If the state is high cooperate, play (C,C). If the state is low, play (D,D).

The grim strategy profile is very similar to the grim-trigger strategy in repeated games, which is commonly employed in the relational contracts literature. The forgiving strategy profile is more interesting because it sustains cooperation in the high game even if agents continue to cooperate after deviation. I am interested in finding the minimum discount factor that is necessary to make a specific strategy profile that sustains cooperation in the high state to be the renegotiation-proof equilibrium of the game (sufficiency requires the discount factor to be less than 1). First, I derive the minimum discount factors for the strategy profiles to be the SPNE of the game.

Proposition 1. *In the high state game, define* $\delta^i_{H,F}$ *and* $\delta^i_{H,G}$ *for agent i as the minimum discount factors that make forgiving and grim strategy profiles incentive compatible, respectively.*

$$\delta^i_{H,F} = \frac{d_i(H) - c_i(H)}{c_i(H) - c_i(L)}, \tag{1}$$

$$\delta^i_{H,G} = \frac{d_i(H) - c_i(H)}{d_i(H) - n_i(L)}. \tag{2}$$

In the low state game, define $\delta^i_{L,F}$ *and* $\delta^i_{L,G}$ *for agent i as the minimum discount factors that make the forgiving and grim strategy profiles incentive compatible, respectively.*

$$\delta^i_{L,F} = \frac{d_i(L) - c_i(L)}{c_i(H) - c_i(L)}, \tag{3}$$

$$\delta^i_{L,G} = 0. \tag{4}$$

Define δ_F *and* δ_G *as the minimum discount factors that make the forgiving and grim strategy profiles the SPNE of the game, respectively.*

$$\delta_F = max_i(\delta^i_{H,F}, \delta^i_{L,F}), \tag{5}$$

$$\delta_G = max_i(\delta^i_{H,G}). \tag{6}$$

In order to find the minimum discount factors for a strategy profile to be the SPNE of the game, I check single person deviations from the strategy profile. Basically, a player deviating from the equilibrium calculates the continuation payoffs assuming that the other player will follow the strategy profile.

In order to check whether a strategy profile is the renegotiation-proof equilibrium, we need to consider all possible joint deviations. This task is not that difficult in this simple game. The simplicity of the proof arises from the fact that cooperation payoffs in the high state cannot be Pareto dominated by the payoffs of any other renegotiation-proof strategy profile, and the Nash Equilibrium payoffs in the low state can only be Pareto dominated by the payoffs of a renegotiation-proof strategy profile by upgrading the state.

Only one of the strategy profiles can be the renegotiation-proof equilibrium of a given game because agents can jointly switch from one equilibrium to the other. The proposition below summarizes the conditions for each strategy profile to become the renegotiation-proof equilibrium of the game. The grim strategy profile is Pareto dominated by the forgiving strategy profile if the discount factor is larger than the threshold value that is given below. If agents care enough about the future, producing a high quality product and selling it at the low quality price pays off in the long run.

Proposition 2. *Given that the forgiving strategy profile is the SPNE, the forgiving strategy profile is the renegotiation-proof equilibrium of the game if the grim strategy profile is not the SPNE or if the grim strategy profile is the SPNE and* $\delta_i > \delta_T$ *for both agents. Given that the grim strategy profile is the SPNE, the grim strategy profile is the renegotiation-proof equilibrium of the game if the forgiving strategy profile is not the SPNE or if* $\delta_i < \delta_T$ *for at least one agent.*

$$\delta_T = \frac{n_i(L) - c_i(L)}{c_i(H) - c_i(L)}. \tag{7}$$

4 The Theory of the Firm

In this section, I determine the optimal ownership structure under different scenarios and compare my results with the predictions of the relational contracts literature and the property rights theory of the firm. I use a simple version of the Hart and Moore (1990) model, similar to that of Halonen (2002), in order to compare my results with those of the literature. My goal is to understand the effect of repeated relationships on the optimal allocation of ownership when we allow for the possibility of renegotiation.

4.1 The Model

In this model, there are two agents who contribute to the production of the same good and sell it to the market. One or more assets are used in the production of this good. Both agents invest in human capital to specialize in the use of these assets. Because the effort levels of agents are not observable by third parties, agents cannot write enforceable contracts that depend on the effort levels or on the total output. We maintain the assumptions about the game structure in the previous model.

I introduce continuous effort levels $e_i \in R^+$. Ownership structure is defined by w, where w=0 denotes joint ownership or non-owner, w=1 and w=2 denotes ownership by agent1 and agent2, respectively. Assuming that the assets under consideration are highly complementary and the joint payoff is always larger than the sum of the outside options, I do not consider the separate ownership of assets. However, the analysis can easily be extended to divisible assets, where each agent can own a separate asset.

The outside option of the agent i is denoted by $O_i(e_i, w_i)$, which depends on the ownership structure and the effort level of the agent i. The outside option increases with the investment in human capital, which helps to better utilize the asset. I normalize non-owner outside option to be zero. I assume that for one of the agents, when the ownership is given to the agent, the derivative of the outside option with respect to the effort level $\frac{\partial O_i}{\partial e_i}$ is higher than that of the other agent for every effort level. Given that the outside options affect the ex-post sharing of revenues, the agent with the higher derivative of outside option with respect to effort has higher incentive to invest to utilize the asset. In the single-ownership case the derivative is zero for the agent who is not the owner. In the joint ownership case the derivative is zero for both agents because they cannot utilize the asset outside without the consent of the other party. Except for their ownership rights and their derivative of outside options with respect to effort, agents are symmetric.

The joint output is shared according to the sharing rule β. In GHM, β_i represents the share of agent i according to the Nash Bargaining solution. On the other hand, in the relational contracts literature agents are free to agree on an optimal sharing rule. The sharing rule can be enforced if deviation from the sharing rule results in downgrading of the state. I will discuss both cases. The payoff functions are defined as follows:

$$u_i(S,x) = \beta_i[P_x(e_i + e_j)] - \theta(e_i). \tag{8}$$

This formulation is in line with Halonen (2002) to allow direct comparison. P_x is the price of the good when the market's belief of quality is x. The price in the high state P_h is higher than the price in the low state P_l. The cost of effort, $\theta(e)$, is a twice differentiable convex function. A game is defined by $\Gamma(g, x^0, I, u_i, \delta_i)$.

4.2 Continuous Effort Levels and the Normal Form Game

The effort levels in the previous section were discrete. However, in the theory of the firm literature effort levels are continuous. The difference in effort structures does not introduce difficulty in using the results of the previous section in analyzing the theory of the firm. There are only two states and the state transition is governed by a function that downgrades the state after a threshold effort level. These two assumptions allow me to calculate the cooperation, deviation and the NE effort levels and their corresponding payoffs in both states. Other possible effort levels and payoffs are not relevant because they are not individually rational or cannot be part of a renegotiation-proof equilibrium (proof in the appendix). Therefore, I can represent the game in a simple 2x2 form and I can use results from the previous section.

The cooperation effort level e^c is the threshold effort level determined by the structure of the game exogenously. If any of the agents exert less effort than e^c the product quality will be low. The cooperation effort level does not change with the state or with the agent. The cooperation effort level could be lower than the first-best effort levels, however it is larger than the NE effort levels of the stage game. First, I assume agents may agree on a sharing rule β, which is relaxed later. The payoff of agent i in state $x \in H, L$ when both agents cooperate is as follows.

$$c_i(x) = \beta_i[P_x(e^c + e^c)] - \theta(e^c). \tag{9}$$

In the deviation period, the sharing rule is always the Nash bargaining solution. The Nash bargaining solution gives each agent their outside options and allocates the rest of the output equally. According to the Nash bargaining solution, the equation below shows the payoff of the deviating agent. Given that the other agent is cooperating and even slight deviation from the threshold effort level changes the state, the deviating agent determines its optimal effort level by maximizing its one period payoff:

$$e_i^{d*}(x) \in argmax \quad d_i(x) = O_i(e_i^d, w_i) + \frac{1}{2}[P_x(e_i^d + e_j^c) - O_i(e_i^d, w_i) - O_j(e_j^c, w_j)] - \theta(e_i^d). \tag{10}$$

The first order condition determines the optimal deviation effort level from the cooperation equilibrium. The optimal deviation effort level depends both on the ownership structure and the state. I ignore the superscript * to simplify notation. All deviation effort levels are determined by the optimization problem above.

After deviation payoffs of the betrayed and deviating agents are determined according to the Nash bargaining rule, the payoff of the deviating agent in state x is $d_i(x)$ and the payoff of the betrayed agent is $b_i(x)$. The deviating agent and the betrayed agent first receive an amount equal to their outside options $O_i(e_i^d(x), w_i)$ and $O_i(e_i^c, w_i)$, respectively. The remaining output is shared equally. The payoffs of the deviating agent and the betrayed agents are:

$$d_i(x) = O_i(e_i^d(x), w_i) + \frac{1}{2}[P_x(e_i^d(x) + e_j^c) - O_i(e_i^d(x), w_i) - O_j(e_j^c, w_j)] - \theta(e_i^d(x)), \tag{11}$$

$$b_i(x) = O_i(e_i^c, w_i) + \frac{1}{2}[P_x(e_i^c + e_j^d(x)) - O_i(e_i^c, w_i) - O_j(e_j^d(x), w_j)] - \theta(e_i^c). \tag{12}$$

Let's calculate the optimal effort levels when agents are playing NE. This time, the deviating agent determines optimal effort level by assuming that the other agent also deviates:

$$e_i^{n*}(x) \in argmax \quad n_i(x) = O_i(e_i^n, w_i) + \frac{1}{2}[P_x(e_i^n + e_j^n) - O_i(e_i^n, w_i) - O_j(e_j^n, w_j)] - \theta(e_i^n). \quad (13)$$

The total revenue is again shared according to the Nash bargaining solution as shown below.

$$n_i(x) = O_i(e_i^n(x), w_i) + \frac{1}{2}[P_x(e_i^n(x) + e_j^n(x)) - O_i(e_i^n(x), w_i) - O_j(e_j^n(x), w_j)] - \theta(e_i^n(x)). \quad (14)$$

After calculating cooperation, deviation, NE and the betrayed agent payoffs in both states, I can represent the game in the normal form and use the definitions and propositions introduced in the previous section.

4.3 Relational Contract

The agents agree on a relational contract that describes the strategy profile, which will in turn determine the optimal ownership structure and the optimal sharing rule. That is why I exclude the optimal sharing rule and the ownership structure from the definition of the relational contract. From now on, I use relational contract and strategy profile interchangeably. I require relational contracts to be renegotiation-proof as defined in the previous section.

Definition 3. *For a given discount factor, the optimal relational contract is the strategy profile that constitutes the renegotiation-proof equilibrium of the supergame. For a given relational contract, the optimal ownership structure and the optimal sharing rule are the ones that mini-*

mize the discount factor that makes the relational contract the renegotiation-proof equilibrium of the game.

There could be multiple ownership structures and sharing rules that make a relational contract the renegotiation-proof equilibrium of the game. The optimal sharing rule and the optimal ownership structure maximize the range of discount factors in which a given relational contract is the renegotiation-proof equilibrium of the game. This ensures that the relational contract will be unaffected by slight changes in the structure of the game and the implicit discount factors of agents.

4.4 The Optimal Sharing Rule

For a given relational contract, the optimal sharing rule should minimize the maximum discount factor that is necessary for both agents to comply with the relational contract. This can only be achieved by equalizing the minimum discount factor of both agents. If there are two different discounts factors, one can minimize the maximum of the discount factors by making a transfer from the agent who has the lower discount factor to the agent who has the higher discount factor. This relationship allows us to derive the optimal sharing rule in terms of stage game payoffs.

Proposition 3. *The optimal sharing rule for the forgiving strategy profile in state x is:*

$$\beta_{i,F}(x) = \frac{d_i(x) + \theta(e^c)}{d_i(x) + d_j(x) + 2\theta(e^c)}. \tag{15}$$

The optimal sharing rule for the grim strategy profile in the high state H is given below. The sharing rule for the grim strategy profile in the low state L is Nash bargaining.

$$\beta_{i,G}(H) = \frac{2P_h e^c (n_i(L) - d_i(H)) + n_j(L)d_i(H) - n_j(L)d_i(H) - \theta(e^c)(n_i(L) - n_j(L) - d_i(H) + d_j(H)}{2P_h e^c (n_i(L) + n_j(L) - d_i(H) - d_j(H))}. \tag{16}$$

The optimal sharing rule for the forgiving strategy profile depends only on the deviation payoffs. If the deviation payoff of agent1 is larger, the optimal sharing rule assigns more output to agent1 in order to prevent him from deviating. In the grim strategy profile, only the Nash bargaining solution can be implemented after deviation. In the high state, the sharing rule for the grim strategy profile depends both on deviation payoffs and on the payoffs in the punishment period.

4.5 The Optimal Ownership Structure

By replacing the sharing rule in the incentive compatibility equations of agents, the minimum discount factor that is required to make the strategy profile the renegotiation-proof equilibrium of the game can be derived. For the forgiving relational contract, the optimal ownership structure should minimize the maximum of $(\delta^i_{H,F}, \delta^i_{L,F})$, and for the grim relational contract the optimal ownership structure should minimize the maximum of $\delta^i_{H,G}$. We do not have to worry about the discount factor δ_T at which the forgiving strategy profile dominates the grim strategy profile. Given that $d_i(L) > n_i(L)$ $\forall$i, the forgiving strategy profile always dominates the grim strategy profile when it is the SPNE of the game because δ_T is always smaller than $\delta^i_{L,F}$.

Proposition 4. *The minimum discount factor that supports forgiving strategy profile in state x as the renegotiation-proof equilibrium of the game is:*

$$\delta_{x,F} = \frac{[d_1(x) + d_2(x)] - [2P_h e^c - 2\theta(e^c)]}{2e^c(P_h - P_l)}. \tag{17}$$

The minimum discount factor that supports the grim strategy profile as the SPNE equilibrium of the game in the low state is zero and in the high state is given below. $\delta_{H,G}$ supports the grim strategy profile as the renegotiation-proof equilibrium of the game when the forgiving strategy profile is not the SPNE.

$$\delta_{H,G} = \frac{[d_1(H) + d_2(H)] - [2P_h e^c - 2\theta(e^c)]}{[d_1(H) + d_2(H)] - [n_1(L) + n_2(L)]}. \tag{18}$$

The proposition gives us important information about the optimal ownership structure in the forgiving and grim strategy profiles. First, let us consider the forgiving strategy profile and delay the discussion of the optimal ownership structure implied by the grim strategy profile. The denominator in $\delta_{x,F}$ depends on the structure of the game but not on the ownership structure. On the other hand, the numerator depends on the ownership structure, which determines the outside options and hence the deviation payoffs of agents. In order to minimize $\delta_{x,F}$, one should select the ownership structure that minimizes the numerator. I summarize these conclusions in the following proposition.

Proposition 5. *The optimal ownership structure in the forgiving relational contract (strategy profile) is the one that minimizes the total deviation payoffs $d_1(x) + d_2(x)$. Ownership is given to the agent with the highest $\frac{\partial O_i}{\partial e_i}$. Joint ownership is never optimal.*

Allocating the ownership to the agent who has the highest derivative of the outside option with respect to effort minimizes total deviation payoffs. The agent with the highest $\frac{\partial O_i}{\partial e_i}$ invests more on the deviation path when ownership is given to him because he captures more of the surplus. On the other hand, ownership does not increase tendency to deviate because on the equilibrium path the sharing rule equalizes agents' tendencies to deviate. Therefore, it is easier to maintain cooperation in a game where the owner invests at a higher level off the equilibrium path. In joint ownership, however, no one has the incentive to invest at higher levels off the equilibrium path. Since this maximizes the total deviation payoffs, joint ownership is never optimal when the relational contract is the forgiving strategy profile.

Now, let's compare the optimal ownership structure in the forgiving relational contract with that of a spot contract. From the first-order condition of an agent's optimization problem we see that an agent's effort level increases as the derivative of the outside option, with respect to the effort level, increases. Therefore to maximize the joint output, ownership is assigned to the agent who has the highest $\frac{\partial O_i}{\partial e_i}$. Joint ownership, which does not provide incentive to anyone, can never be optimal in the spot game.

Proposition 6. *In a spot contract (one period contract) the optimal ownership structure is the one that maximizes the total surplus* $(n_1(x)+n_2(x))$. *Ownership is given to the agent with the highest* $\frac{\partial O_i}{\partial e_i}$. *Joint ownership is never optimal.*

Optimal ownership structures in the forgiving relational contract and the spot contract are equal. This important result shows that the predictions of GHM hold in the multi-period analysis when forgiving strategy profile is the renegotiation-proof equilibrium of the game.

When grim relational contract is the renegotiation-proof equilibrium of the game, the optimal ownership structure could be different than what is predicted by GHM. Both the denominator and the numerator in $\delta_{H,G}$ depend on the ownership structure, and it is not immediately clear which ownership structure is optimal. Giving ownership to the agent with the highest derivative of outside option with respect to effort minimizes the total deviation payoffs, while increasing the NE payoffs off the equilibrium path. This ownership choice minimizes the numerator but also minimizes the denominator. On the other hand, joint ownership maximizes the total deviation payoffs while minimizing the NE payoffs off the equilibrium path in the low state game. Joint ownership minimizes the numerator but also minimizes the denominator. This additional tradeoff, which is absent in the GHM type one period models, introduces the possibility that joint ownership can be optimal.

Proposition 7. *Define* $a_i = [d_1(H) + d_2(H) - c_1(H) - c_2(H)]$ *and* $b_i = [d_1(H) + d_2(H) - n_1(L) - n_2(L)]$ *when ownership is given to agent i. Assuming that agent1 has higher derivative of outside option with respect to agent2,* $a_1 < a_2 < a_0$ *and* $b_1 < b_2 < b_0$*. Given that grim relational contract is the renegotiation-proof equilibrium of the game, joint ownership is optimal if* $a_0/b_0 < a_1/b_1$ *and* $a_0/b_0 < a_2/b_2$*, otherwise ownership by one agent is optimal.*

This result is similar to the findings of the relational contract literature. Joint venture may become optimal because it provides higher punishment off the equilibrium path. However, the probability of having joint ownership is not an increasing function of the discount factor, as commonly claimed.

4.6 When Sharing Rule is not Enforceable

In the repeated game version of relational contracts, the punishment period starts regardless of whether cheating was in the effort levels or in the sharing rule. In the previous section, I made the same implicit assumption, which allowed agents to agree on the optimal sharing rule. In this section, I relax this assumption. Since agents can cheat from the sharing rule without affecting the state, no ex-ante sharing rule can be enforced ex-post. As a result, agents always share the revenue according to the Nash Bargaining solution. This modeling approach is inline with the GHM models.

The results about the optimal ownership structure remain unaffected when the sharing rule cannot be enforced. However, I need to change the definition of the optimal ownership structure slightly. Previously, the agents were trying to minimize the maximum discount factor to make a given relational contract the strongly renegotiation-proof equilibrium of the game. This time, I do not have the luxury of designing a sharing rule that equalizes the deviation tendencies of agents. Therefore, the discount factor that makes the relational contract the renegotiation-proof equilibrium of the game is not the same for both agents. Still, I try to maximize the probability that a given relational contract becomes renegotiation-proof. By allocating ownership, I can only affect the minimum discount factor, which makes the strategy profile incentive compatible with the owner. Therefore, I give ownership to the agent or agents who will have the lowest discount factor to make a given strategy profile the renegotiation-proof equilibrium of the game once the ownership is allocated.

Definition 4. *For a given relational contract, the optimal ownership structure is the one that maximize the probability that the relational contract will be the renegotiation-proof equilibrium of the game. Ceteris paribus, ownership is given to the agent(s) who has(have) the lowest discount factor that makes the relational contract the renegotiation-proof equilibrium of the game for that agent.*

Our optimal ownership predictions do not change when agents cannot agree on a sharing rule ex-ante. Ownership is given to the agent with the highest $\frac{\partial O_i}{\partial e_i}$ when forgiving strategy profile is the renegotiation-proof equilibrium of the game. Joint ownership may become optimal when grim strategy profile is the renegotiation-proof equilibrium of the game.

Proposition 8. *Given that the forgiving relational contract is the renegotiation-proof equilibrium of the game, the ownership is given to the agent, whose outside option has the highest derivative with respect to the effort level. Joint ownership is never optimal. Given that the* *grim relational contract is the renegotiation-proof equilibrium of the game the optimal ownership structure minimizes δ_G for agent i. Define $x_i = d_i(H) - c_i(H)$ and $y_i = d_i(H) - n_i(L)$ when i is the owner; Joint ownership is optimal if $x_0 y_1 < y_0 x_1$ and $x_0 y_2 < y_0 x_2$, otherwise ownership by one agent is optimal.*

In fact, the results about the optimal ownership structure are robust to various modifications of the sharing rule. If both agents know their discount factors, we may want to design a sharing rule that satisfies the incentive compatibility constraints of agents instead of minimizing the maximum of the agents' discount factors. Although the optimal sharing rule may change, propositions regarding the implied ownership structures for the strategy profiles will not be affected since the mutually-agreed sharing rule is not enforced off the equilibrium path and has no effect on the deviation payoffs. My results may change if revenue is shared differently off the equilibrium path (Rajan and Zingales, 1998; DeMeza and Lockwood, 1998; Chiu, 1998).

4.7 The Optimal Relational Contract

In the relational contracts literature any relational contract (equilibrium effort levels) can be enforced if agents sufficiently care about the future. The renegotiation-proofness criterion eliminates the multiplicity of relational contracts. For a given game and the discount factor of agents the optimal relational contract is unique.

Figure 1 exhibits the relationship among the discount factor of agents, the optimal relational contract and the allocation of ownership for one of the many possible outcomes. When agents' discount factors are very low, agents do not care about the future and agree on the consecutive spot contracts. Since they cannot maintain the cooperation investment levels, they play the NE strategies of the game L. If both agents have discount factors that are higher than

$\delta_{H,G}$ and at least one agent has discount factor lower than $\delta_{L,F}$, they can agree on the grim strategy profile and maintain cooperation in the game H. In this case, the optimal ownership structure can be joint ownership. However, if both agents have higher discount rates than $\delta_{L,F}$, they agree on forgiving strategy profile and ownership is given to the agent who has the highest derivative of outside option with respect to effort.

5 The Robustness of Results

In this section, I discuss the implications of some of the assumptions and the robustness of results. First, I assumed that agents cannot renegotiate the ownership structure. This assumption is realistic to the extent that ownership renegotiations are costly in real life. When agents are playing the spot contract or the forgiving strategy profile, the ex-ante allocation of the ownership is optimal and agents cannot Pareto dominate the total payoff by switching to any other ownership structure. Hence, the renegotiation of the ownership cannot introduce any problems.

The renegotiation of ownership constitutes a problem for the grim strategy profile, because it may prescribe an inefficient ownership structure. For example, grim strategy may prescribe joint ownership in the low state, however agents may like to renegotiate and give the ownership to the agent with the highest derivative of outside option with respect to effort. As a result, joint ownership can become optimal only if the renegotiation of ownership is costly.

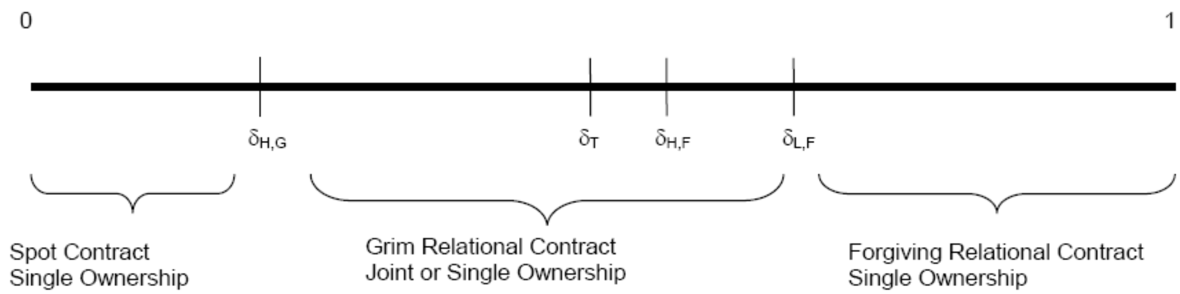
In real life, it is rarely the case that effort levels are perfectly observable, no uncertainty exists, agents are unboundedly rational and agents do not make mistakes. Repeated games are affected by the introduction of all these imperfections because the non-deviating agent has to consider the fact that bad outcomes could be the result of bad luck. One has to balance the loss due to the unnecessary punishment and the gain from curbing the incentive to cheat. In equilibrium, knowing that all incidences will not be punished, the agents will have tendency to cheat. On the contrary, the endogenously structured games introduced in this paper are quite robust to imperfect monitoring, mistakes and uncertainty. Given that expected payoffs from following a strategy profile remain the same, agents will not have higher tendencies to cheat because punishment is unavoidable.

6 Conclusion

The paper integrates the results of the property rights theory of the firm with those of the relational contracts literature by identifying the cases where different ownership structures become optimal in a world where agents repeatedly interact and renegotiate. The paper shows that, contrary to common belief, when the agents are allowed to renegotiate the long-term contracts, the probability of having joint ownership does not linearly increase with the discount factor. The optimal ownership structure predicted by the property rights theory of the firm may become optimal for both very low and very high discount factors. On the other hand, there may be a region where joint ownership and other ownership structures, which are not optimal in the property rights theory, can be optimal when agents prefer not to cooperate after deviation.

The existence of the renegotiation-proof relational contracts requires high dependency of the game structure on agents' effort levels. This provides the empirical prediction that renegotiation-proof relational contracts will be used more often in industries where the future payoff structure depends on the strategies of agents. Although it is difficult to come up with a precise measure of the dependence of the game structure on the strategies of agents, one criterion could be the importance of the network and reputation effects to the fate of the firm. Robinson and Stuart (2006) provide empirical evidence from biotech alliances. Because the punishment stemming from the network structure cannot be renegotiated, network structure enables the use of relational contracts. Approaching relational contracts from the perspective of endogenously structured games opens the possibility of explaining the differences in employing relational contracts across industries.

Figure 1. Optimal Contract and Ownership Structure for various Discount Factors



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7 Appendix: Proofs

I use additional abbreviations and notation in the appendix. SF is the forgiving strategy profile, SG is the grim strategy profile. WRP means weakly renegotiation-proof and SRP means strongly renegotiation-proof. I use underscore 1 and 2 to identify payoffs and the effort levels of agent1 and agent2, respectively. I use in parenthesis 0 and 1 to identify payoffs and the effort levels of non-owner and owner, respectively. I also use 0 for both agents when they jointly own the asset.

7.1 Proposition 1

Given that agents are following the strategy profile SF, V(H) and V(L) are the continuation value of the game (for the agent under consideration) given that we start from game H and L respectively. If agents are in state H, then the incentive compatibility constraint of the agent is as follows:

$$c(H)+\delta V_F(H) >= d(H)+\delta V_F(L) \tag{19}$$

Given that:

$$V_F(H)=\frac{c(H)}{1-\delta}, \quad V_F(L)=c(L)+\delta\frac{c(H)}{1-\delta} \tag{20}$$

We can write the incentive compatibility constraint at state H as follows:

$$\delta^i_{H,F} >= \frac{d_i(H)-c_i(H)}{c_i(H)-c_i(L)} \tag{21}$$

Similarly off the equilibrium path incentive compatibility constraint is:

$$\delta^i_{L,F} >= \frac{d_i(L)-c_i(L)}{c_i(H)-c_i(L)} \tag{22}$$

If both agents incentive compatibility constraints are satisfied then SF is the SPNE of the game since no agent can gain by unilaterally deviating on the equilibrium and off the equilibrium path. Proof is similar for the SG. If agents are in state H, then the incentive compatibility constraint of the agent is as follows:

$$c(H) + \delta V_G(H) >= d(H) + \delta V_G(L) \tag{23}$$

Given that:

$$V_G(H) = \frac{c(H)}{1-\delta}, \quad V_G(L) = \frac{n(L)}{1-\delta} \tag{24}$$

We can write the incentive compatibility constraint at state H as follows:

$$\delta^i_{H,G} >= \frac{d_i(H) - c_i(H)}{d_i(H) - n_i(L)} \tag{25}$$

Off the equilibrium path incentive compatibility constraint is:

$$\delta^i_{L,F} >= 0 \tag{26}$$

If both agents incentive compatibility constraints are satisfied then SG is the SPNE of the game.

7.2 Proposition 2

Given that SF is SPNE and there is only one continuation equilibrium in each state, SF is trivially WRP. In order to show that SF is SRP we need to show that SF cannot be dominated by any other WRP equilibrium. We need to check all joint deviations, which can Pareto dominate SF and constitute a WRP equilibrium of the game.

Starting with state H, there does not exist a different WRP strategy profile that can strictly Pareto dominate SF because $d_i(H) + b_j(H) < c_i(H) + c_j(H)$ and $c_i(H) + c_j(H) > n_i(L) + n_j(L)$. This rules out any type of WRP equilibrium, which requires agents to alternate in deviating in state H or any other type of strategy that prescribes deviating in state H.

Consider another WRP strategy profile that prescribes (C,C) in the game H but prescribes a different strategy profile from (C,C) in the game L. Since SF is SPNE unilateral deviation is not optimal. Therefore I only need to show that SF dominates playing (D,D) in state L. The incentive compatibility equation for joint deviation is as follows:

$$c(L) + \delta V_F(H) >= n(L) + \delta V_F(L) \tag{27}$$

Given that:

$$V_F(H) = \frac{c(H)}{1-\delta}, \quad V_F(L) = c(L) + \delta\frac{c(H)}{1-\delta} \tag{28}$$

$$\delta_i > \delta_T = \frac{n_i(L) - c_i(L)}{c_i(H) - c_i(L)} \tag{29}$$

This condition is automatically satisfied if $d_i(L) >= n_i(L)$ given that SF is SPNE. If $\delta > \delta_F$ and $\delta > \delta_T$ for both agents, playing (C,C) dominates playing (D,D) in the game L. Since there does not exist any other WRP strategy profile that can Pareto dominate SF. SF is SRP.

When SG is SPNE and there is only one continuation equilibrium for each game hence SG is trivially WRP. SG prescribes cooperation in the game H and no other strategy profile can dominate it in the game H. Therefore we only need to check joint deviation in state L. In order to prevent joint deviation the incentive compatibility condition below has to be satisfied.

$$c(L) + \delta V_G(H) >= n(L) + \delta V_G(L) \tag{30}$$

Given that:

$$V_G(H) = \frac{c(H)}{1-\delta}, \quad V_G(L) = \frac{n(L)}{1-\delta} \tag{31}$$

We can write the constraint as:

$$\delta_i =< \delta_T = \frac{n_i(L) - c_i(L)}{c_i(H) - c_i(L)} \tag{32}$$

This condition has to be satisfied only for one agent. We understand that when both SF and SG is SPNE δ_T determines which strategy profile is SRP. If $\delta > \delta_G$ for both agents and $\delta < \delta_T$ for at least one agent, playing (D,D) dominates playing (C,C) in the game L. Since there does not exist any other strategy profile that can Pareto dominate SG, SG is SRP.

The proof for the case when effort levels are continuous is similar. If we can show that only the cooperation, deviation and Nash equilibrium effort levels are relevant, we can map continuous effort levels to the 2x2 normal form game. In state H, exerting effort levels higher than e_i^f cannot be a part of SRP equilibrium because cheating from these effort levels cannot be punished since the state will not change. Off the equilibrium path exerting an effort level which is different than $e_i^d(H)$ is not individually rational, it does not prevent state from changing and cannot be enforced as a part of SRP equilibrium In state L, exerting effort levels higher than e_i^f is Pareto dominated by SF. Any effort level larger than Nash Equilibrium effort levels but lower than the threshold effort level, which changes the state, cannot be a part of WRP equilibrium because agents cannot sustain cooperation without the non-credible threat of punishment in state L. There is no lower state, which prevents agents to achieve higher payoffs than the Nash equilibrium payoffs by staying in state L in a renegotiation-proof way. Therefore we can map continuous effort levels to 2x2 normal form game and use the conditions above for renegotiation-proofness.

7.3 Proposition 3

We want to minimize the maximum of the discount factors $\delta^i_{H,F}$, $\delta^i_{L,F}$ or $\delta^i_{H,G}$ Whatever the discount factor we choose to minimize, the optimal sharing rule should make the critical discount factors of both agents equal, otherwise by making a transfer we can decrease the maximum of the two. Therefore equalizing the discount factors of agents we can back out the optimal sharing rule.

7.4 Proposition 4

Just plug the optimal sharing rule in the formulas of $\delta^i_{H,F}$, $\delta^i_{L,F}$ or $\delta^i_{H,G}$.

7.5 Proposition 5

If we are following SF we want to minimize $d_1(x)+d_2(x)$. Since the state does not matter for the proof, I only do it for state H. Assume agent 1 has a higher derivative of outside option with respect to effort level. If agent1 is the owner then $d_1(H)+d_2(H)$ is shown below. I surpass notation for simplicity, state is H and all efforts levels are determined when agent1 is the owner.

$$d_1(H,1)+d_2(H,0)=\frac{1}{2}O_1(e_1^d)-\frac{1}{2}O_1(e_1^c)+\frac{1}{2}[P_h(2e^c+e_1^d+e_2^d(0)]-\theta(e_1^d)-\theta(e_2^d(0)) \quad (33)$$

If agent 2 is the owner then $d_1(H,0)+d_2(H,1)$ is equal to:

$$d_1(H,0)+d_2(H,1)=\frac{1}{2}O_2(e_2^d)-\frac{1}{2}O_2(e_2^c)+\frac{1}{2}[P_h(2e^c+e_2^d+e_1^d(0)]-\theta(e_2^d)-\theta(e_1^d(0)) \quad (34)$$

Since the derivative of outside option with respect to the effort level of agent1 is higher for all effort levels, we can show that ownership should be assigned to agent1. Take the difference of total deviation payoffs when agent1 is owner and agent2 is owner. Note that the deviation effort levels of agents are equal when they are both non-owner because the outside option of the non-owner is zero, and cooperation effort levels are equal.

$$= \frac{1}{2}O_1(e_1^d) - \frac{1}{2}O_1(e_1^c) - \frac{1}{2}O_2(e_2^d) + \frac{1}{2}O_2(e_2^f) + \frac{1}{2}[P_h(e_1^d - e_2^d] - \theta(e_1^d) + \theta(e_2^d) \tag{35}$$

Add the term below and separate the above equation into two parts:

$$\frac{1}{2}O_2(e_1^d) - \frac{1}{2}O_2(e_1^d) = 0 \tag{36}$$

$$Part1 = \frac{1}{2}O_1(e_1^d) - \frac{1}{2}O_1(e_1^c) - [\frac{1}{2}O_2(e_1^d) - \frac{1}{2}O_2(e_2^f)] \tag{37}$$

Given that the cooperation effort levels are the same $e_1^c = e_2^c$ part1 is negative because $\frac{\partial O_1}{\partial e} > \frac{\partial O_2}{\partial e}$ for all effort levels.

$$Part2 = -\frac{1}{2}O_2(e_2^d) + \frac{1}{2}O_2(e_1^d) + \frac{1}{2}[P_h(e_1^d - e_2^d] - \theta(e_1^d) + \theta(e_2^d) \tag{38}$$

Since e_d^2 is the effort level that maximizes the deviation payoff for agent2 we know that:

$$\frac{1}{2}O_2(e_2^d) + \frac{1}{2}[P_h(e_2^d)] - \theta(e_2^d) > \frac{1}{2}O_2(e_1^d) + \frac{1}{2}[P_h(e_1^d)] - \theta(e_1^d) \tag{39}$$

Therefore Part2 is also negative. As a result, $d_1 + d_2$ is minimized if we give ownership to agent1, who has higher derivative of outside option with respect to effort level. Now, let's compare single ownership with joint ownership. When agent's jointly own the assets.

$$d_1(H,0) + d_2(H,0) = [P_h(e^c + e^d(0)] - 2\theta(e^d(0)) \tag{40}$$

If we subtract this term from total deviation payoffs when agent 2 is the owner we get:

$$\frac{1}{2}O_2(e_2^d) - \frac{1}{2}O_2(e_2^f) + \frac{1}{2}[P_h(e_2^d - e_1^d(0)] - \theta(e_2^d) + \theta(e_1^d(0)) \tag{41}$$

Since the outside option is increasing in the effort levels summation of the first two terms are negative. The summation of remaining terms is also negative. Because, at the effort level $e_1^d(0)$ the marginal cost is equal to marginal revenue. Increasing the effort level beyond this point decreases total payoff since cost function is convex and revenue function is linear. Therefore, joint ownership cannot be optimal.

7.6 Proposition 7

We know that $d_1(H) + d_2(H)$ is the smallest when agent1 is the owner and the largest when agents have joint ownership. We also know that $c_1(H) - c_2(H)$ does not depend on the ownership structure therefore $a_1 < a_2 < a_0$. From Proposition 8 we know that $n_1(L) + n_2(L)$ is the largest when agent1 is the owner and smallest when agents have joint ownership. Therefore $b_1 < b_2 < b_0$. Joint ownership is optimal if $a_0/b_0 < a_1/b_1 and a_0/b_0 < a_2/b_2$.

7.7 Proposition 8

Given that the relational contract is SF, the optimal ownership minimizes δ_F for agent i. Assume that we are trying to minimize $\delta^i_{H,F}$, the proof is the same for $\delta^i_{L,F}$. We want to minimize $d_i(H) - c_i(H)$ to minimize $\delta^i_{H,F}$. Assume agent1 has a higher derivative of outside option with respect to effort level at all effort levels. Take the difference between $d_i(H) - c_i(H)$ when agent1 is owner and agent2 is owner.

$$= \frac{1}{2}O_1(e_1^d) - \frac{1}{2}O_1(e_1^c) - \frac{1}{2}O_2(e_2^d) + \frac{1}{2}O_2(e_2^c) + \frac{1}{2}[P_h(e_1^d - e_2^d] - \theta(e_1^d) + \theta(e_2^d) \qquad (42)$$

We showed that this term is negative in the proof of Proposition 5. Now let's compare single ownership to joint ownership. Subtract $d_i(H) - c_i(H)$ when we have joint ownership from the case when agent2 is the owner.

$$\frac{1}{2}O_2(e_2^d) - \frac{1}{2}O_2(e_2^c) + \frac{1}{2}[P_h(e_2^d - e_1^d(0)] - \theta(e_2^d) + \theta(e_1^d(0)) \qquad (43)$$

In proposition 7, I showed that this term is also negative meaning that allocating ownership to agent2 minimizes $d_i(H) - c_i(H)$ compared to joint ownership. Therefore if SF is the SRP equilibrium of the game, we allocate ownership to the agent who has the highest derivative of outside option with respect to effort at all effort levels. When SG is the SRP equilibrium of the game ownership affects both the numerator and the denominator. We know that $x_1 < x_2 < x_0$ from the first part of the proof. The exact ordering of y_1, y_2, y_0 depends on further conditions. Joint ownership can be optimal if $x_0/y_0 < x_1/y_1$ and $x_0/y_0 < x_2/y_2$.