Coordinated and Uncoordinated Liquidation Decisions of Creditors

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Abstract

Whether or not creditors can coordinate their actions will have an effect on when and how they decide to liquidate a firm that cannot meet its interest payments. In the context of a structural model of corporate debt, similar to e.g. the model presented by Leland (1994), this paper shows that coordinated creditors will have incentives to liquidate too early as they do not internalize the effects of their actions on the value of equity. Unlike coordinated creditors, uncoordinated creditors only care about the payoffs in their coordination games. Although the outcomes of such games are unlikely to be socially optimal from the point of view of creditors, they can actually benefit holders of equity, and raise firm value.

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1 Introduction

This paper addresses the question of how the liquidation decisions of coordinated and uncoordinated creditors differ, for situations in which creditors can individually seize assets of a firm that fails to make its interest payments. It also considers the effects of bankruptcy codes and automatic stays in this context. The argument is phrased in terms of a firm-value based model related to the model of Leland (1994) and the real-option approach described e.g. by Dixit and Pindyck (1994).

It is shown that coordinated creditors can be too trigger-happy, i.e. liquidate when it is not optimal from a social point of view, where the social optimum is defined as the maximum achievable firm value. This happens whenever holders of equity want to gamble for resurrection, i.e. the value of equity is maximized for very late liquidation. In this case, the coordinated creditors do not internalize the negative effect on the value of equity of early liquidation. This problem can be viewed as an instance of filtering failures as described by White (1994).

Uncoordinated creditors on the other hand only consider payoffs in a coordination game. In general, they will therefore liquidate at a point which is not optimal from the point of view of coordinated creditors. This leads to lower prices of debt. It can, however, increase the value of equity. For some parameter combinations and payoffs in the coordination game, the point at which uncoordinated creditors will liquidate can therefore be closer to the social optimum, and produce a higher firm value.

Even in the presence of bankruptcy codes that protect debtors from uncoordinated liquidation, the payoffs of the coordination game (which will not be played in equilibrium) can still determine the point at which the company enters bankruptcy.

Similar issues have been addressed in the literature: Bolton and Scharfstein (1996), for example, examined a model in which inefficiencies in liquidation or renegotiation that are the result of having a large number of creditors can have desirable effects. This is also the case here. Bolton and Scharfstein (1996) focus on moral hazard, however, which is not dealt with here.

Décamps and Faure-Grimaud (2002) look at a situation in which holders of equity have incentives to gamble for resurrection in the context of a structural Geske (1977)-type model of corporate debt. In their model, the fact that the owner of the firm can default on repayment of debt in the future creates excessive incentives to continue. Here, for the benchmark case of the liquidation decision of coordinated creditors, a model that also exhibits incentives to the owner to gamble for resurrection is used. However, the model is formulated in terms of perpetual debt and a continuous coupon, so that issues of compound optionality are avoided.

Morris and Shin (2004) (cf. also Morris and Shin, 2000) present a static model in which the possibility that in bad times, creditors of a firm might fail to roll over debt in a coordinated way has an effect on the price of debt. The coordination game presented here will closely mirror their setup. However, the decisions of agents are interpreted as seizing assets rather than not rolling over a loan. Also, the game presented here will be dynamic. To be able compare the coordination failure case to the continuous-time benchmark model of coordinated creditors, it is necessary to set up a sequence of discrete time games and take limits to get to a continuous time version. In doing so, close attention has to be paid to issues of information sets and timing assumptions, since the game of Morris and Shin (2004) is a private information game, and pricing in the structural model proceeds under the standard assumption of a single filtration.

The advantage of having a model of coordinated and uncoordinated creditors in continuous time is that debt, equity and the value of the firm can be priced and the model can be calibrated reasonably well.

Games in the context of structural models have been examined by Anderson and Sundaresan (1996); Mella-Barral and Perraudin (1997), who look at strategic decisions of holders of equity to reduce their debt service as a way of extracting concessions from creditors by threatening costly liquidation. More general renegotiation games played between holders of debt and equity were considered by Mella-Barral (1999), extended to the case of diffusely held debt by Hege and Mella-Barral (2005), where they look at distressed exchanges. Here, a different set of issues will be addressed, and consequently renegotiation will be disallowed and equity holders seen as essentially passive. In the spirit of Bolton and Scharfstein (1996), the focus of attention will be on the inefficiencies in reorganisation that might be caused by dispersed creditors, and their likely effect, here on the values of equity, debt, and the firm.

In the next section (section 2), the coordinated benchmark model is discussed: Debt and equity are priced (subsection 2.1), and optimal liquidation boundaries are derived (subsection 2.2). The coordination game and its influence on liquidation is discussed in section 3. The results are discussed in section 4, including how they might be modified by the presence of bankruptcy codes (subsection 4.1). Section 5 concludes.

2 The coordinated benchmark model[1]

Suppose that the projects that the firm is engaged in produce a (pre-tax) net cash flow that will be divided between a holder of debt or creditor, who will receive a coupon, and the holder of equity or the owner, who receives the residual cash flows. The owner has no wealth other than her equity stake in the firm. Cash flows are observable and verifiable in the sense of Grossman and Hart (1986), so that contracts can be written that prevent strategic debt service (Anderson and Sundaresan, 1996; Mella-Barral and Perraudin, 1997) or the diversion of cash flows.

Furthermore assume that asset sales are not allowed. Assume that all forms of negative dividends are not allowed[2], such that coupon payments can only be met out of cash flows. These assumptions are similar to the ones made by Anderson and Sundaresan (1996) or Broadie, Chernov, and Sundaresan (forthcoming), but in contrast to e.g. the assumptions made by Leland (1994) and others, where typically the owner will make payments to the firm to ensure that coupons can be paid. Taken together, the assumptions imply that there will be situations in which the cash flow is insufficient to pay the coupon, and the coupon will not be paid in full. It is likely that this happens in practice, at least in some situations[3].

[1] Note that mathematically, the setup presented here is similar to the one presented by Naqvi (2003)

[2] This means e.g. that dilution is not possible.

[3] Varma and Cantor (2005) report that of about slightly more than 1,000 "initial default events"

Assume that in situations in which interest payments are missed, the creditor can initiate legal proceedings to liquidate the firm. Typically, a creditor can sue to obtain a judgement lien, which can be attached to an asset of the debtor if debt is not fully serviced, and perfected, i.e. the creditor can, after incurring legal expenses, recover part of the money owed by liquidation.

This setup will produce incentives for the owner to gamble for resurrection, as will be explained in greater detail in section 2.2.

Mathematically, suppose pre-tax cash flows (net of costs) have the following dynamics under the pricing measure $\mathbb{Q}$ defined by the money market account as the numeraire:

$$dx = \mu x dt + \sigma x dW \tag{1}$$

Assume that the interest rate is constant at r, and that $\mu < r$. All cash flows net of costs that the firm generates are subject to a constant corporate tax rate τ.

The value of receiving the pre-tax cash flow forever is

$$\frac{x_t}{r - \mu}. \tag{2}$$

The value of receiving the pre- or post-tax cash flow are simply scaled versions of the original geometric Brownian motion describing the cash flows, and can be interpreted as the pre- and post-tax going-concern value of the firm respectively[4].

Assume that that the going-concern value of the firm can at any point in time be swapped irreversibly for a constant liquidation payment K.

2.1 Introducing debt and equity

We can now introduce equity and debt, which are claims that receive different parts of the cash flow. Suppose that the creditor receives a coupon c and the holder of equity (the owner) receives the residual post-tax cash flow, $(1-\tau)(x-c)$, as long as cash flows exceed the promised coupon. When cash flows fall below the promised coupon, the owner does inject cash or sells assets to make up the shortfall: Assume

recorded by Moody's for the period between 1983 to 2003, more than 55% consist of missed interest payments and "grace period defaults", which are missed interest payments that are eventually repaid (the remaining cases are direct chapter 11 filings, distressed exchanges, missed principals, direct chapter 7 filings and chapter 11 prepacks). Emery and Cantor (2003) report that between 23% - 25% of firms that default on payments are not in formal bankruptcy. Furthermore, Altman and Bana (2003) indicate that for a sample of 339 bankruptcies that they studied, 62% of the firms default on payments at the date of their bankruptcy filing, but the average time between defaulting on a payment until declaring bankruptcy still is 2.7 months. This is because for some companies in there sample (all of which eventually end up in bankruptcy, of course), the time between missing payments and going into bankruptcy is very long, and can sometimes be over one year. It is also likely that there are other ways in which companies can effectively avoid making payments to creditors, e.g. by delaying the payments due to trade creditors etc., which do not even appear in the data discussed in these studies.

[4]A structural model of the value of corporate debt and equity can either be based on cash flows or on the going-concern value of the firm. In the second case, liquidation is typically assumed to produce a payoff that is a fraction of the going concern value of the firm (e.g. Leland, 1994). Of course, with this type of specification, liquidation is never optimal for an unlevered firm. Although a cash-flow based formulation is equivalent, it serves to illustrate that an alternative specification of liquidation payoffs is plausible.

that if the promised coupon exceeds the cash available to pay the coupon, the owner receives nothing, and the creditor receives whatever the firm can pay. Later, we will argue that if the coupons are not paid in full, creditors have the right to liquidate the firm. Some mechanism determines a point $\bar{x}$, and the firm is liquidated when the cash flow hits this barrier. The liquidation proceeds K are then paid to the creditor, and the owner receives nothing. This is plausible if K is less than the "principal", and we do not allow deviations from absolute priority. Since the debt is perpetual, following Mella-Barral and Perraudin (1997), I take c/r to be the "principal", so that $K < c/r$, i.e. in the event of liquidation, the creditor receives a payment that is worth less than the value of perpetual risk-free debt paying a coupon c. This makes debt risky.

To summarize, the payoffs to debt and equity are as follows:

$$\text{payoff to equity at } t = \begin{cases} (1-\tau)\max(x_t - c, 0) & \text{before liquidation} \\ 0 & \text{at liquidation} \end{cases} \tag{3}$$

and

$$\text{payoff to debt at } t = \begin{cases} c - \max(c - x_t, 0) & \text{before liquidation} \\ K & \text{at liquidation} \end{cases} \tag{4}$$

Note that this setup ignores that defaults have to be cured (i.e. interest payments that have been missed have to be paid at a later date if a sufficient amount of money becomes available), and is therefore an approximation. Although a proper treatment would be desirable, there is a trade-off in terms of complexity versus obtaining closed form solutions, since introducing cures would introduce path dependency[5].

Requiring that the discounted gains from holding these assets are martingales under $\mathbb{Q}$ produces two pricing ODEs, one for the region where the promised coupon is paid in full, and one for the region where the coupon payments equal the (insufficient) cash flows, each with two constants of integration. To solve for the price of a claim, four boundary conditions are required.

For the case of debt, denoting D_1 as the price of debt when the coupon is paid in full and D_2 as the price of debt when the coupon is not paid in full, we can impose the following boundary conditions:

$$\lim_{x\to\infty} D_1(x) = \frac{c}{r} \tag{5}$$

$$D_1(c) = D_2(c) \tag{6}$$

$$D_1'(c) = D_2'(c) \tag{7}$$

$$D_2(\bar{x}) = K \tag{8}$$

[5] states that as the cash flow becomes very large, debt essentially becomes riskless. [6] is a value-matching condition that states that at the point where the dynamics

[5] A possibly more complete and accurate model in this sense is the one proposed by Broadie, Chernov, and Sundaresan (forthcoming), for which no closed form solutions are derived. Also, Broadie, Chernov, and Sundaresan (forthcoming) treat any missed interest payment as being equivalent to the firm being in bankruptcy reorganization, which is different from the treatment here (see section 4.1 for a discussion of bankruptcy).

of the discounted gains process change (when the cash flow is just equal to the coupon), the value of the solution to both differential equations nevertheless has to be the same (since there is only one claim). [7] is required to rule out arbitrage as the cash flow falls below the coupon (see e.g. Dixit, 1993), and [8] is another value-matching condition that states that when the firm is liquidated, the value of debt is equal to the liquidation value.

These boundary conditions produce 4 linear equations in 4 unknowns (the constants of integration), which can be easily solved (although some of the expressions turn out to be large). It is shown in the appendix that this produces the following equation for the price of debt:

$$D(x,\bar{x}) = \begin{cases} D_1(x,\bar{x}) & \text{when } x \geq c \\ D_2(x;\bar{x}) & \text{when } x \leq c \end{cases} \tag{9}$$

where

$$D_1(x,\bar{x}) = \frac{c}{r} + \left(D_2(c,\bar{x}) - \frac{c}{r}\right)\left(\frac{x}{c}\right)^{-\gamma} \tag{10}$$

and

$$D_2(x;\bar{x}) = \frac{x}{r-\mu} - Z\left(\frac{x}{c}\right)^{\delta} + \left(K - \frac{\bar{x}}{r-\mu} + Z\left(\frac{\bar{x}}{c}\right)^{\delta}\right)\left(\frac{x}{\bar{x}}\right)^{-\gamma} \tag{11}$$

where in turn

$$Z = \frac{\gamma}{\delta+\gamma}\left(\frac{1+\gamma}{\gamma}\frac{c}{r-\mu} - \frac{c}{r}\right). \tag{12}$$

D_1 is the value of receiving the coupon until c is hit, in which case the value of receiving the coupon is swapped for D_2. D_2 this is the value of receiving the cash flow, minus the value of an upper barrier claim that swaps the cashflow for the coupon (barrier payoff Z), plus the value of a lower barrier claim that represents the liquidation boundary; liquidation means losing the cash flows and the barrier claim with barrier payoff Z and receiving the liquidation payment K. Here δ is the positive root of the fundamental quadratic equation of geometric Brownian motion.

The value of equity is derived in a similar manner. In the appendix, it is shown that for a similar set of boundary conditions, we obtain

$$E_1(x;\bar{x}) = (1-\tau)\left\{\frac{x}{r-\mu} - \frac{c}{r}\right\} + \left(E_2(c;\bar{x}) - (1-\tau)\left\{\frac{c}{r-\mu} - \frac{c}{r}\right\}\right)\left(\frac{x}{c}\right)^{-\gamma} \tag{13}$$

$$E_2(x;\bar{x}) = (1-\tau)\left(Z\left(\frac{x}{c}\right)^{\delta} - Z\left(\frac{\bar{x}}{c}\right)^{\delta}\left(\frac{x}{\bar{x}}\right)^{-\gamma}\right). \tag{14}$$

In the region where the coupon is paid, the value of equity (E_1) is the value of receiving the cash flow, minus the value of paying the coupons. When cash flows become insufficient to pay coupons, the value of equity (E_2) becomes equal to the value of the barrier claim that pays the cash flow minus coupons once cash flows exceed coupons again, minus the value of losing this barrier claim when the firm is liquidated.

The market value of the levered firm is the sum of the market values of equity (E) and debt (D):

$$L_1(x;\bar{x}) = E_1(x;\bar{x}) + D_2(x;\bar{x}) = \\ (1-\tau)\frac{x}{r-\mu} + \tau\frac{c}{r} + \left(L_2(c;\bar{x}) - \left((1-\tau)\frac{c}{r-\mu} + \tau\frac{c}{r}\right)\right)\left(\frac{x}{c}\right)^{-\gamma}, \quad (15)$$

i.e. in the region where the coupon is paid, the firm is the value of the post-tax cash flows plus the value of the tax shield, plus the value of swapping this for L_2 when the cash flow falls below c. This in turn is given by

$$L_2(x;\bar{x}) = E_2(x;\bar{x}) + D_2(x;\bar{x}) = \\ \frac{x}{r-\mu} - \tau Z\left(\frac{x}{c}\right)^{\delta} + \left(K - \left(\frac{\bar{x}}{r-\mu} - \tau Z\left(\frac{\bar{x}}{c}\right)^{\delta}\right)\right)\left(\frac{x}{\bar{x}}\right)^{-\gamma}, \quad (16)$$

i.e. the value of the now untaxed cash flow, minus the value of a barrier claim that represents having to pay taxes on cash flows net of interest once the cash flows exceed interest payments, plus the value of the lower barrier claim that swaps all this for the liquidation payment.

2.2 Optimal liquidation

Very generally, letting L denote the value of the levered firm, taking derivatives of

$$L = E + D \quad (17)$$

w.r.t. a boundary $\bar{x}$ at which the firm will be liquidated (by a mechanism yet to be determined), allows comparing the choices of $\bar{x}$ that would be optimal for the different parties:

$$\frac{\partial L}{\partial \bar{x}} = \frac{\partial E}{\partial \bar{x}} + \frac{\partial D}{\partial \bar{x}} \quad (18)$$

The socially optimal liquidation point $\bar{x}_L$ is given by the first order condition which sets the LHS of [18] equal to zero. Unless this point is also optimal for holders of equity (i.e. at $\bar{x}_L$, $\partial E/\partial \bar{x} = 0$), which implies that it must also be optimal for the holders of debt (i.e. at $\bar{x}_L$, $\partial D/\partial \bar{x} = 0$), the points at which holders of equity and holders of debt respectively would like to liquidate the firm will be on opposite sides of $\bar{x}_L$. This immediately follows from the fact that if the LHS of [18] is zero, and the two terms on the RHS individually are not equal to zero, they must be of different sign for [18] to be satisfied. We therefore have either

$$\bar{x}_D < \bar{x}_L < \bar{x}_E \quad (19)$$

or

$$\bar{x}_E < \bar{x}_L < \bar{x}_D. \quad (20)$$

Intuitively, this makes sense: The social optimum must be a compromise (lie between) what the of holders of debt and holders of equity want.

This kind of insight forms the basis of the discussion by Mella-Barral (1999), who looks at the case where holders of equity determine $\bar{x}$, and goes on to produce a version of the argument of Haugen and Senbet (1978), that costless Coasian bargaining between holders of debt and equity can achieve the socially optimal outcome, with the surplus being divided among holders of debt and equity.

In our case, $\bar{x}_E < \bar{x}_L < \bar{x}_D$, since holders of equity never benefit, and always lose from liquidation - they have incentives to gamble for resurrection. Conversely, there are situations where the holders of debt gain from liquidation.

This is good, since the alternative case, $\bar{x}_D < \bar{x}_L < \bar{x}_E$, is not a plausible setup for talking about the optimal liquidation decisions of creditors. If holders of equity are the first to want to shut down a firm, it is hard to argue for anything but holders of equity determining the liquidation point, due to the limited liability nature of equity. In essence, holders of equity can always simply walk away from a firm.

Note that the assumption that there cannot be negative dividends to equity is very important in this context. For example, in the model of Leland (1994), equity holders inject cash (e.g. via dilution), to pay the coupon, as long as it is still optimal for them to do so. In this kind of model, it is natural to think of bankruptcy as equity holders walking away (i.e. refusing to inject more cash into the firm).

As a consequence, in this type of model, holders of debt always receive their coupon prior to bankruptcy. Note that this means that holders of debt *never* have an incentive to liquidate, *unless* they receive something worth more than receiving the coupon forever in liquidation. Interestingly, this would imply that this type of risky debt would be worth more than risk-free debt paying the same coupon, which is somewhat implausible. It is therefore hard to argue for situations in which $\bar{x}_E < \bar{x}_L < \bar{x}_D$ in this type of model, i.e. for situations in which holders of equity have incentives to gamble for resurrection.

Here, giving holders of debt incentives to liquidate is achieved producing situations in which the actual debt service payments (which are *less* than the coupon) are so low that it is better to liquidate.

In the appendix, it is shown that the optimal boundary that holders of debt would choose to maximize the value of debt $\bar{x}_D$, is lower than the level of the coupon, and higher than the socially optimal liquidation point:

$$\bar{x}_L < \bar{x}_D < c. \qquad \text{(Propositions 1 and 2)}$$

Intuitively, $\bar{x}_D$ has to be higher than the socially optimal point, because unlike the holders of the unlevered firm, creditors do not benefit from the full upside potential of the cash flow since the payments they receive are always capped at the level of the coupon. Hence when weighing the liquidation payment against their lower continuation value, they will decide to liquidate earlier.

A real-world example of this would be the commonly expressed view that floating charge holders in the UK liquidated too early, as for example reported by Woolridge (1987): "floating charge holders in the UK apply themselves ruthlessly to the realization of assets to satisfy the charge [...] in some cases with scant regard for the future of the company".

Also, since the liquidation payoff is less than the value of just receiving the coupon forever, creditors will not want to liquidate immediately when they can, i.e. when the firm just begins to have problems paying the coupon. If the liquidation value was higher than the value of receiving the coupon, liquidation would of course be a positive thing, and creditors would want to liquidate as soon as possible. We have already ruled out cases like these, however, to ensure that debt is risky, in the sense that it will be worth *less* than the equivalent risk-free debt.

Note that as a consequence of $\bar{x}_D$ being larger than $\bar{x}_L$, we know that the optimal liquidation point from the point of view of holders of equity must be below the social optimum. In fact, since holders of equity gain nothing from liquidation in the model presented here, whereas they always lose the possibility of receiving a positive cash flow in the future, they will never want to liquidate. Holders of equity prefer to gamble for resurrection, because the costs of this behaviour would be borne by holders of debt.

The fact that $\bar{x}_D > \bar{x}_L$ can be interpreted as a filtering failure in the sense of White (1994): Viable firms are liquidated when it is not social optimal to do so (although here, so far, no mention has been made of bankruptcy. See section 4.1.), where socially optimal actions are defined as those actions that maximize the value of the levered firm.

Having established how coordinated creditors would behave in this setup, it is now time to look at uncoordinated creditors.

3 The coordination game

The setup developed here is similar to the one of Morris and Shin (2004), with the crucial difference that the game here needs to be adapted to take into account valuation in continuous time. The equilibrium in the game will now dictate the point at which the firm is liquidated. Since $\bar{x}_D$ was chosen optimally, it is easy to see that changing the liquidation boundary will lead to a lower debt value.

In the situation described, there are economic incentives for a single creditor to buy out all others, in order to achieve the benefits associated with optimal (from the point of view of the creditor) liquidation, as well as improving the liquidation payoffs. The undesirable concentration of credit risk produced by concentrating the holdings of debt of a large firm in the hands of a single agent and/ or costs to bargaining are likely to counteract the economic incentives that would work towards the concentration of debt. Here, it is simply taken as a starting point that creditors are uncoordinated, and the question of whether or not the benefits of concentrating holdings of the debt outweigh the costs is not addressed.

Also, there is a question as to whether or not an institutional framework, in particular, a bankruptcy procedure, can prevent coordination failures. This discussion is delayed until section 4.1.

3.1 The actions of agents

Assume that in every period in which the aggregate coupon is not paid in full, the coupons due to different creditors are reduced by an equal proportion, and that subsequently any individual creditors can attempt to seize assets: Since the firm is not fulfilling its debt obligations, creditors can send out their lawyers to attempt to e.g. attempt to obtain a judicial lien on assets of the company. This action has a certain cost associated with it (paying a lawyer). Assume that the ability of the firm to defend itself against legal attack depends on the current cash flow, out of which the defense is financed. If enough creditors (relative to current cash flow) hire lawyers to seize assets, the firm is forced to liquidate. If the fraction of creditors attempting to seize assets is too low, the firm will not be forced to liquidate (it has sufficient cash to defend itself). Lawyers will have been paid already, though, if a creditor has attempted to seize assets.

3.1.1 Payoffs

Forced liquidation will take place immediately before a time t only when the fraction of creditors who decide to seize assets l is larger than or equal to $\frac{x_t}{c}$. This formulation ensures that it will be impossible for the firm to forcibly liquidated when $x_t > c$. As x_t falls, it implies that less creditors are required to initiate forced liquidation.

Attempting to seize assets produces an immediate cost s, which one can think of the cost of the lawyers, or "sharks" . If the firm is pushed into liquidation, an agent that has seized assets receives her share of the liquidation value K, whereas agents that have not participated receive 0. If the firm is not liquidated, agents that attempted to seize assets still incur the costs but both types of agents still hold their share of the debt.

Table 1 illustrates the instantaneous 'per unit of principal' payoffs that creditors need to take into account when making the decision whether to attempt to seize assets. These payoffs will generate (Bernoulli) utility. If the firm is *not* reorganised, creditors furthermore receive the (expected) continuation value (omitted in this table) of holding aggregate debt (not explicit in this table).

	liquidation	no liquidation
seize assets	$(1-s)\,K$	$-sK$
do not seize assets	0	0

Table 1: Payoffs to creditors in the discrete time game.

Note that these payoffs exhibit full strategic complementarities. Potentially more complicated and possibly more realistic payoffs, especially those which depend on the actual fraction of creditors attempting to seize assets are possible. It would for example be natural to argue that in the event of liquidation, assets will first be shared first among those agents that attempted to seize assets, and all remaining assets will be shared between those that did not attempt to seize assets. This kind of setup with *one-sided strategic complementarities* (Goldstein and Pauzner, 2005) is also feasible, although in this case information structures, and the way limits will be taken, would be more complicated. Note that in the limit derived below, the payoff does not depend on the fraction of creditors that attempt to seize in any case, so this modification would not change the continuous-time version of the argument.

3.1.2 Information content of prices

A necessary ingredient for coordination failure to arise is uncertainty about the actions of other agents. Without common knowledge of the fundamentals (the cash flow in our case) of an issuer, agents will not be completely sure of how other agents will act. Suppose there is private as well as public information, then provided that private information is sufficiently precise in relation to public information, i.e. there is sufficient uncertainty about the actions of others, this will create coordination failure.

In a comment on the paper by Morris and Shin (2000), Atkeson (2000) doubts that the coordination failure idea is applicable to pricing debt. He argues that if agents can see prices, there will be no coordination failure, because all information will be revealed in the prices - there is no role for private information, and hence uncertainty about the information of other agents. In the model presented here, there is no trade at the time private signals are received and agents act, hence the private information is not revealed through trading. To what extent people can act on private information before it is revealed in markets is an interesting issue in its own right - it is, however, simply taken as given in the present context.

Suppose that agents have to make a decision as to whether or not to seize assets after they have received a signal, but before the signals are revealed to all. Subsequently, signals are revealed to all, then trading occurs and information is integrated into prices. Then there might still be coordination failure, because the private information has not been made public at the time when the agents need to act.

3.1.3 Timing

Time increments are of size Δ. At time t, identical agents (the creditors) know the cash flow of this period, x_t. Agents are uncountably infinite and have mass 1. Relative changes in the cash flow are normally distributed (to obtain geometric Brownian motion in the limit). Aggregate debt trades at a price D_t which incorporates the information x_t. Let q denote a time increment that is smaller than Δ ($0 < q < \Delta$). At $t + q$, agents receive a signal ξ_i about the increase in the cash flow - subscript i indexes the different agents, where the time subscript is omitted to simplify notation. They form a posterior given their information. Given their posterior, they make a decision as to whether or not to attempt to seize assets.

After it has been determined that the firm will not fail in this period, we proceed to the next period: Signals are revealed, the cash flow value is revealed and the price $D_{t+\Delta}$ incorporating all the information $x_{t+\Delta}$ is formed. As a consequence of these timing assumptions, only public information will be incorporated into prices. This is important as it allows valuation by standard martingale techniques.

t	$t+q$	$t+\Delta$
signals revealed	no trading	signals revealed
x_t revealed	agents receive new signals	$x_{t+\Delta}$ revealed
trading	agents form posterior	trading
D_t formed	agents act	$x_{t+\Delta}$ formed
	firm is possibly liquidated	

Figure 1: Timing assumptions

3.1.4 Dynamics of cash flow and signals

The relative increase in the cash flow is normally distributed around a drift.

$$x_{t+\Delta} - x_t = \mu x_t \Delta + x_t \eta_t, \quad \eta_t \sim NID\left(0, \frac{1}{\alpha}\right) \tag{21}$$

At $t+q$, agents receive a signal ξ_i (subscript i indexes the different agents) about the impending change in x, with the distribution of the signal, conditional on the cash flow x_t given by

$$\xi_i = x_{t+\Delta} + x_t \varepsilon_i, \quad \varepsilon_i \sim NID\left(0, \frac{1}{\beta}\right), \tag{22}$$

where $\text{Cov}(\eta_t, \varepsilon_i) = 0$, i.e. the noise is orthogonal to the innovations in the cash flow.

From the signal ξ_i and the public information x_t, agents form a posterior about the cash flow of the firm in period $t+\Delta$, $x_{t+\Delta}$ (which is also normally distributed).

3.2 The solution

3.2.1 Basic procedure

The discrete time model is solved using the procedure as in Morris and Shin (2004). Suppose that agents follow a switching strategy around a certain posterior belief. Given the posterior belief around which agents switch, we can work out how many of them will seize assets, given the cash flow in the next period (posterior beliefs will be centred around this cash flow in the next period). The critical next-period cash flow for which the firm will be forcibly liquidated (given the belief in this period around which agents switch) can therefore be determined. This is the trigger point.

3.2.2 The discrete time trigger point

In appendix B, the following solution is derived (equation 118):

$$x^*_{t+\Delta} = c\Phi\left\{\frac{\alpha}{\sqrt{\beta}}\left(\frac{x^*_{t+\Delta}}{x_t} - 1 - \mu\Delta\right) + \frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}\{\theta\}\right\} \tag{23}$$

where θ is a ratio of utilities that reflects the comparison of seizing assets versus not seizing assets:

$$\theta = \frac{u((1-s)K) - u(0)}{u((1-s)K) - u(-sK)} \tag{24}$$

For risk averse agents, $\theta < 1 - s$, it attains its upper limit $1 - s$ when agents are risk-neutral. Note that θ is decreasing in s, and risk-aversion.

The trigger point x^*_t is unique if:

$$c\frac{1}{\sqrt{2\pi}}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t} < 1 \tag{25}$$

(see also appendix B.8, proposition 3, condition I). This is then the only equilibrium which survives iterated deletion of dominated strategies.

3.2.3 Continuous time limit

Now take the continuous time limit. If we want the cash flow process to tend to a geometric Brownian motion, we need (loosely speaking)

$$\lim_{\Delta \to dt} \frac{1}{\alpha} = \sigma^2 dt, \tag{26}$$

i.e. the variance of public information about the innovation in the cash flow to be proportional to time. So the variance of the innovation is $O(\Delta)$, or the precision is $O(\frac{1}{\Delta})$.

Now a sufficient condition for the uniqueness of the equilibrium described in equation (118) in continuous time, *regardless of the cash flow and the coupon c*, is that

$$\frac{1}{\beta} = o\left(\Delta^2\right), \tag{27}$$

i.e. that private information becomes more precise at a rate faster than Δ^2, because this ensures that condition (I) (s.a.) is always satisfied. This is just to say that we need the quality of private information to be sufficiently high in relation to the quality of public information in order for agents to be sufficiently uncertain about the actions of others to obtain coordination failure. As $\Delta \to dt$, $\Delta^2 \to 0$, and hence β grows at a faster rate than α. Consequently, $\frac{\alpha}{\sqrt{\beta}}$ tends to zero, so condition (I) will be satisfied for any permissible $x_{t+\Delta}$. Also, $\frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}} \to 1$. The resulting trigger point equation then reduces to

$$x^* = \theta c \le (1-s)c. \tag{28}$$

This means that the trigger point is a fraction of the coupon, where this fraction reflects the utility of seizing assets versus the utility of not seizing assets in situations in which the firm is liquidated and in situations where it is not liquidated. x^* attains an upper limit of $1-s$ when agents are risk neutral: For more risk averse agents and/or higher legal costs s one would expect a lower trigger point x^*.

Note that the solution is constant. If we let the intermediate time period $(t+q)$ tend to the period immediately following it, the firm fails at t whenever $x(t)$ hits θc, i.e. when the cash flow is a fraction θ of the coupon. The boundary or trigger point is a decreasing function of the cost of attempting to seize assets. Agents are reluctant to seize assets if it is costly for them to do so. The optimal liquidation point for creditors is related to the liquidation value - the coordination failure point is related to the payoffs of the coordination failure game.

Due to the special assumptions about payoffs, this function turns out to be quite simple here - it is constant.

3.2.4 The actions of agents in the continuous time limit

Conditional on the cash flow in the next period, the probability that a creditor receives a signal which prompts it to seize assets is $\Phi\left\{\frac{1}{x_t}\sqrt{\beta}(\xi_t^* - x_{t+\Delta})\right\}$. As β tends to infinity, this probability tends either to 1 or to 0 for all non-marginal agents. What this means is that because all creditors essentially receive the same information (as the signal becomes infinitely precise), the agents will either all seize assets, or will all refrain from doing so. For any non-marginal creditor, the ex-ante probability of seizing assets when the other creditors do not do so tends to zero. Also, the probability of not seizing assets if all other creditors are seizing assets tends to zero. This is essentially because in the limit, agents receive the same signals, and there is no uncertainty about the cash flow. However, strategic uncertainty remains. This is a standard limiting result for this type of game for cases in which the precision of private information tends to infinity.

3.2.5 Pricing

Since the agents act in unison, the payoffs are the same for all agents: They receive the same payoffs as in the benchmark model prior to liquidation. The firm is liquidated once the cash flow hits the trigger point x^*, when all agents rush to seize assets. At this point, all agents receive a liquidation payoff $(1-s)K$. At times when trading occurs, all agents have the same information. Pricing is therefore standard, and the same formulas as before can be used, but with $\bar{x} = x^* = \theta * c$, and the liquidation payoff K replaced by $(1-s)K$.

4 Discussion

Vis-a-vis the coordinated case, the structure of the coordination failure game has two effects: Firstly, it changes the point at which the firm is liquidated. This can mean that the firm is liquidated at a point that is closer to the social optimum. This can raise firm value. Secondly, in the formulation presented so far, it lowers the liquidation payoff. This will lower firm value.

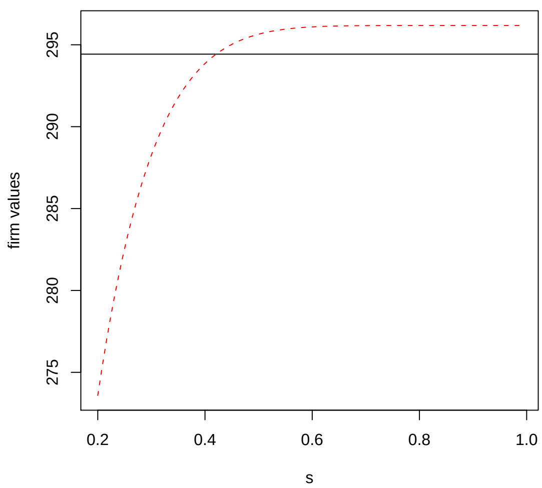

Figure 2: The red dashed line is the firm value under coordination failure, the solid black line is the firm value with coordinated creditors. The parameters are: $r = 0.07, \mu = 0.05, \sigma = 0.2, \tau = 0.2, K = 60, c = 6$, current cashflow $= 7$. Agents have power utility with coefficient of relative risk aversion of 2 and outside wealth of 60.

As it turns out, there are parameter values (in particular, typically high values for s and risk aversion that consequently lead to a low θ and hence a low x^*) for which the firm value in the coordination failure case can be higher than the firm value in the coordinated case, keeping everything else constant[6]. Figure 2 gives an example.

It can be argued that an outcome that is closer to the social optimum could also be achieved in the case of coordinated debt simply by lowering the liquidation payoff in that case as well. This is certainly the case, and in fact possibly provides an argument for the desirability of deviations from absolute priority.

It does raise the question, however, whether the effect generated here for the case of non-coordinated debt depends solely on the fact its liquidation payoff is assumed to be lower. To address this, we can compare firm values for situations in which the liquidation technology is the same in both cases, i.e. assume that the liquidation payoff is $(1-s)K$ in the case of coordinated and non-coordinated debt. As figure 3 shows, it is still possible to generate situations in which the firm value with non-coordinated debt exceeds the firm value with coordinated debt.

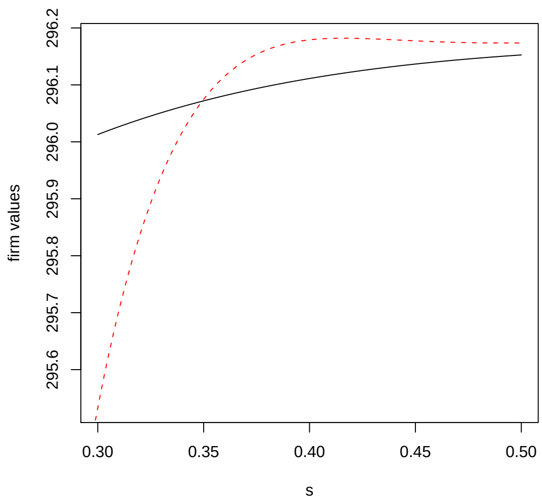

Figure 3: The red dashed line is the firm value under coordination failure, the solid black line is the firm value with coordinated creditors. The parameters are: $r = 0.07, \mu = 0.05, \sigma = 0.2, \tau = 0.2, K = 60, c = 6$, current cashflow $= 7$. Agents have power utility with coefficient of relative risk aversion of 2 and outside wealth of 30.

4.1 Bankruptcy codes

Bankruptcy codes, especially the feature of an automatic stay as well as preference law exist to promote equality of distribution among similarly situated creditors and deterring creditors from racing to dismember a financially distressed debtor,

[6]Due to the lack of a closed form solution for $\bar{x}_D$, it is difficult to *prove* for which parameter combinations this is the case. This will be addressed in future versions of this paper.

thereby helping the debtor work out its financial problems (see e.g. H.R. Rep. No. 595, 95th Cong., 1st Sess. 177-78 (1977) for the case of the US).

Although there are questions that could be raised as to whether bankruptcy procedures are effective in modifying payoffs to prevent coordination failures[7], it is probably reasonable to assume that in most cases they do.

Note that although creditors can sometimes initiate involuntary bankruptcy proceedings, it is not very common. In the US, for instance, it is much more typical for firms to apply for protection from its creditors. They do so when faced by creditors who attempt to individually seize assets (LoPucki, 1983).

In the context of the model presented here, we could assume that owners stand to lose from a race to seize assets (e.g. through reputational costs), and hence apply for protection from creditors just before the race happens.

Formally, we could structure this as a sequential move game in which owners have to decide whether or not to put the firm into bankruptcy, followed by creditors, who have to decide (simultaneously) whether to attempt to seize assets or not. If the payoff of the owners is such that she would prefer continuation over bankruptcy over disorderly liquidation, and she has perfect information about the cash flow in the next period, it is easy to see that she would apply for protection from creditors just before a run. The payoffs in the subsequent bankruptcy games do not have to reflect the payoffs in the coordination failure game, in particular, the total legal costs might be substantially lower. On top of this, if bankruptcy procedures take into account the option-value of shutting down the firm, this might produce a constrained-efficient outcome. In this setup, a bankruptcy procedure therefore seems very attractive. In essence, the only thing that coordination failure determines is the *point* at which the company enters bankruptcy.

It is, however, not so clear that the sequence of moves described above is realistic. In particular, taking into account the nature of preference law in the US (section 547 of title 11, United States Code), it might well be that the better way of thinking about the game would be to assume that creditors move first, followed by the owner. This is because if the creditors manage to seize assets, the owner can still put the firm into bankruptcy, and the payoffs in the coordination failure game can be modified retrospectively. Any preferential payoff that a creditor receives can be put back into the pot that will be shared across all creditors.

Of course, with this sequence, and if it is truly the case that no creditor receives more than any other creditor (of the same class of debt) in bankruptcy, this means that it is now a (weakly) dominant strategy *never* to attempt to seize assets. This would mean that bankruptcy codes are so effective at preventing runs that the firm is never put into bankruptcy.

It is obvious that the structure of the game matter a lot. It is likely that this is a fruitful area for further research.

[7] La Porta, Lopez-de Silanes, Shleifer, and Vishy (1998) for instance indicate that an automatic stay does not apply to secured debt in about 50% of all countries that they examine, which they take as evidence of strong creditors' rights.

5 Conclusion

Co-ordination failure between creditors affects how they liquidate a firm. This paper has argued that it is plausible that coordinated creditors have incentives to liquidate too early, and that uncoordinated creditors might have incentives to liquidate later, depending on the costs versus benefits of non-coordinated liquidation. It has been shown that it is possible that this increases firm value.

A private-information coordination failure game was introduced into a public-information martingale pricing model in a way that might indicate how other private information games might be included in continuous time models.

The analysis was done in the context of a model in which coupons are not paid in full when cash flows are insufficient to do so. This is different from models of the type proposed by e.g. Leland (1994), which cannot explain situations in which holders of equity want to gamble for resurrection, because in these cases creditors must want to foreclose early, which must imply payoffs that exceed those of risk-free debt.

It was speculated that filing for protection against creditors can seen as an attempt by owners of a firm to prevent a run of creditors to dismember a firm, and that therefore the payoffs to a coordination failure game might determine when a firm enters bankruptcy, even though in practice, these payoffs might never be observed. This issue will be addressed in more detail in future versions of this paper.

Also, there are issues of interest that this paper has not addressed so far, which are the shape that Coasian bargaining might take in the context presented here, if renegotiation costs were sufficiently low, as well as the question of the optimal capital structure and debt capacity. These will also be addressed in future versions of this paper.

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A The benchmark model

A.1 Values

Solutions to the valuation problem at hand are well known (see e.g. Dixit, 1993), but are summarized here for convenience.

It is well known that the value of a general dividends process $a + bx$ (i.e. a process consisting of a constant payoff and payoff proportional to cash flows) can be calculated by noting that the discounted gains process has to be a martingale under the risk-neutral measure $\mathbb{Q}$, which is the measure under which the dynamics of x have been specified. Requiring this gains process to be a martingale means that the drift has to be equal to zero. This leads to the following general pricing ODE (for the value F of the dividends $a + bx$)

$$\mu x F' + \frac{1}{2}\sigma^2 x^2 F''(x) + a + bx = rF, \tag{29}$$

where use has been made of the fact that x follows a geometric Brownian motion and that the value of perpetual claims on flows must be time-homogeneous (i.e. time derivatives drop out).

The solution to the homogeneous part of this ODE

$$\frac{1}{2}\sigma^2 x^2 F''(x) + \mu x F' - rF = 0 \tag{30}$$

will be of the form Cx^{β}. Inserting leads to the following quadratic equation for β, which Dixit (1993) calls the "fundamental quadratic equation of geometric Brownian motion":

$$\left(\mu\beta + \frac{1}{2}\sigma^2\beta(\beta - 1) - r\right) = 0 \tag{31}$$

This has two roots for β:

$$\delta = \frac{-\mu + \frac{1}{2}\sigma^2}{\sigma^2} + \frac{\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} \tag{32}$$

$$-\gamma = \frac{-\mu + \frac{1}{2}\sigma^2}{\sigma^2} - \frac{\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2 r}}{\sigma^2} \tag{33}$$

$$\tag{34}$$

where $\gamma > 0$ and $\delta > 1$.

Hence the solution for the homogeneous part (the complementary function) is of the type

$$F = Ax^{-\gamma} + Bx^{\delta}, \tag{35}$$

where A and B are constants of integration.

For the non-homogeneous part, it is necessary to assume $\mu < r$ for the integrals to converge. It is easy to derive a particular integral by positing that it is simply linear in x. The general solution is

$$F = Ax^{-\gamma} + Bx^{\delta} + \frac{a}{r} + \frac{bx}{r - \mu}, \tag{36}$$

where A and B will be determined by boundary conditions to be imposed.

A.1.1 The going-concern value

The going concern value of the firm can be derived by applying the appropriate boundary conditions to the general solution above, but also simply as follows:

$$E_t^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)} x_s ds\right] = \tag{37}$$

$$= \int_t^{\infty} e^{-r(s-t)} E_t^{\mathbb{Q}}\left[x_s\right] ds \tag{38}$$

$$= x_t \int_t^{\infty} e^{(\mu-r)(s-t)} ds = \frac{x_t}{r-\mu} \tag{39}$$

A.1.2 The value of debt

For debt, the pricing ODE in the region where the coupon is paid is

$$D_1'\mu x + \frac{1}{2}\sigma^2 x^2 D_1'' - rD_1 + c = 0 \tag{40}$$

This has a solution

$$D_1(x) = A_1 x^{-\gamma} + B_1 x^{\delta} + \frac{c}{r} \tag{41}$$

The pricing ODE in the region where the cash flow is paid is

$$D_2'\mu x + \frac{1}{2}\sigma^2 x^2 D_2'' - rD_2 + x = 0 \tag{42}$$

This has the solution

$$D_2(x) = A_2 x^{-\gamma} + B_2 x^{\delta} + \frac{x}{(r-\mu)} \tag{43}$$

The boundary conditions are as in the main text. The first condition (equation [5]) implies that $B_1 = 0$. The remaining three conditions provide a system of 3 linear equations in three unknowns (the constants of integration), which need to be determined:

$$c^{-\gamma}A_1 \quad -c^{-\gamma}A_2 \quad -c^{\delta}B_2 \quad = \frac{c}{r-\mu} - \frac{c}{r} \tag{44}$$

$$-\gamma c^{-\gamma}A_1 \quad +\gamma c^{-\gamma}A_2 \quad -\delta c^{\delta}B_2 \quad = \frac{c}{r-\mu} \tag{45}$$

$$\bar{x}^{-\gamma}A_2 \quad +\bar{x}^{\delta}B_2 \quad = K - \frac{\bar{x}}{r-\mu} \tag{46}$$

This system can easily be solved, although the expressions for A_1 is rather lengthy. The expression for B_2 is easiest to manage. Multiply [44] by γ and add to [45] to eliminate A_1 and A_2 and solve for B_2 to obtain:

$$B_2 = -c^{-\delta}Z \tag{47}$$

where

$$Z = \frac{\gamma}{\delta+\gamma}\left(\frac{1+\gamma}{\gamma}\frac{c}{r-\mu} - \frac{c}{r}\right). \tag{48}$$

We can now solve [46] for A_2 in terms of B_2 and insert to obtain

$$A_2 = \bar{x}^{\gamma} \left\{ K - \frac{\bar{x}}{r-\mu} + Z \left(\frac{\bar{x}}{c}\right)^{\delta} \right\} \tag{49}$$

Inserting [49] and [47] into [43] produces

$$D_2(x;\bar{x}) = \frac{x}{r-\mu} - Z\left(\frac{x}{c}\right)^{\delta} + \left(K - \frac{\bar{x}}{r-\mu} + Z\left(\frac{\bar{x}}{c}\right)^{\delta}\right)\left(\frac{x}{\bar{x}}\right)^{-\gamma} \tag{50}$$

D_1 is then best expressed in terms of $D_2(c;\bar{x})$. From the value-matching condition [6], we can deduce that

$$D_1(x;\bar{x}) = \frac{c}{r} + \left(D_2(c;\bar{x}) - \frac{c}{r}\right)\left(\frac{x}{c}\right)^{-\gamma} \tag{51}$$

where

$$D_2(c;\bar{x}) = \frac{c}{r-\mu} - Z + \left(\frac{c}{\bar{x}}\right)^{-\gamma}\left(K - \frac{\bar{x}}{r-\mu} + \left(\frac{\bar{x}}{c}\right)^{\delta} Z\right). \tag{52}$$

A.1.3 The value of equity

In the region where equity holders receive dividends, the pricing ODE is

$$E_1'\mu x + \frac{1}{2}\sigma^2 x^2 E_1'' - rE_1 + (1-\tau)(x-c) = 0. \tag{53}$$

In the region where equity holders receive nothing, the pricing ODE is

$$E_1'\mu x + \frac{1}{2}\sigma^2 x^2 E_1'' - rE_1 = 0. \tag{54}$$

We have

$$E_1(x) = A_1^E x^{-\gamma} + B_1^E x^{\delta} + (1-\tau)\left(\frac{x}{r-\mu} - \frac{c}{r}\right) \tag{55}$$

$$E_2(x) = A_2^E x^{-\gamma} + B_2^E x^{\delta} \tag{56}$$

with boundary conditions

$$\lim_{x\to\infty} E_1(x) - \frac{x}{r-\mu} = 0 \tag{57}$$

$$E_1(c) = E_2(c) \tag{58}$$

$$E_1'(c) = E_2'(c) \tag{59}$$

$$E_2(\bar{x}) = 0 \tag{60}$$

[57] implies that $B_1^E = 0$. The rest produce the following system of equations:

$$-c^{-\gamma}A_1^E + c^{-\gamma}A_2^E + c^{\delta}B_2^E = (1-\tau)\left(\frac{c}{r-\mu} - \frac{c}{r}\right) \tag{61}$$

$$\gamma c^{-\gamma}A_1^E - \gamma c^{-\gamma}A_2^E + \delta c^{\delta}B_2^E = (1-\tau)\frac{c}{r-\mu} \tag{62}$$

$$\bar{x}^{-\gamma}A_2^E + \bar{x}^{\delta}B_2^E = 0 \tag{63}$$

Multiplying [61] by γ and adding to [62], we can solve for B_2^E to obtain

$$B_2^E = c^{-\delta}(1-\tau)Z. \tag{64}$$

We can now solve [63] for A_2^E and insert [64] to obtain

$$A_2^E = -\bar{x}^{\gamma}\left(\frac{\bar{x}}{c}\right)^{\delta}(1-\tau)Z \tag{65}$$

Inserting [64] and [65] into [56] produces

$$E_2(x;\bar{x}) = (1-\tau)\left(Z\left(\frac{x}{c}\right)^{\delta} - Z\left(\frac{\bar{x}}{c}\right)^{\delta}\left(\frac{x}{\bar{x}}\right)^{-\gamma}\right). \tag{66}$$

It is now easiest to write E_1 in terms of E_2 and simply state that

$$E_1(x;\bar{x}) = (1-\tau)\left\{\frac{x}{r-\mu} - \frac{c}{r}\right\} + \left(E_2(c;\bar{x}) - (1-\tau)\left\{\frac{x}{r-\mu} + \frac{c}{r}\right\}\right)\left(\frac{x}{c}\right)^{-\gamma} \tag{67}$$

A.1.4 The value of the levered firm

We can now add the value of equity and the value of debt to obtain the value of the levered firm.

$$\begin{aligned} L_1(x;\bar{x}) &= E_1(x;\bar{x}) + D_2(x;\bar{x}) = \\ &(1-\tau)\frac{x}{r-\mu} + \tau\frac{c}{r} + \left(L_2(c;\bar{x}) - \left((1-\tau)\frac{c}{r-\mu} + \tau\frac{c}{r}\right)\right)\left(\frac{x}{c}\right)^{-\gamma}, \end{aligned} \tag{68}$$

where L_2 is given as

$$\begin{aligned} L_2(x;\bar{x}) &= E_2(x;\bar{x}) + D_2(x;\bar{x}) = \\ &\frac{x}{r-\mu} - \tau Z\left(\frac{x}{c}\right)^{\delta} + \left(K - \left(\frac{\bar{x}}{r-\mu} - \tau Z\left(\frac{\bar{x}}{c}\right)^{\delta}\right)\right)\left(\frac{x}{\bar{x}}\right)^{-\gamma}, \end{aligned} \tag{69}$$

A.2 Optimal liquidation boundaries

A.2.1 The optimal liquidation boundary for creditors

Note that D_1 depends on $\bar{x}$ only through the boundary payoff of D_2 at c, and that this dependence is positive. Hence maximizing D w.r.t. $\bar{x}$ is equivalent to maximizing D_2. Note furthermore that D_2 only depends on $\bar{x}$ via A_2. Since $x^{-\gamma} > 0$, choosing $\bar{x}$ to maximize D is equivalent to choosing $\bar{x}$ to maximize A_2. This will yield the optimal boundary from the point of view of holders of debt.

Note that

$$\frac{\partial A_2}{\partial \bar{x}} = \bar{x}^{\gamma-1}\left\{\gamma K - (1+\gamma)\frac{\bar{x}}{r-\mu} + (\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z\right\}. \tag{70}$$

Setting this partial derivative equal to zero will produce an equation that describes the optimal $\bar{x}$ from the point of view of the holders of debt.

In order to interpret this equation, it will be necessary to proceed via some intermediate steps.

Lemma 1.

$$r > -\gamma\mu. \tag{71}$$

Proof. Note that $\gamma > 0, r > 0$.

1. If $\mu \geq 0$, this is trivially satisfied.
2. If $\mu < 0$, it is not immediately obvious. Now $-\gamma$ is the negative root of the quadratic equation

$$Q(\beta) = -\frac{1}{2}\sigma^2\beta^2 - \left(\mu - \frac{1}{2}\sigma^2\right)\beta + r = 0 \tag{72}$$

We can obtain a lower bound on $-\gamma$ by noting that the line that is tangent to Q at $\beta = 0$ will intersect the β-axis at

$$\beta_0 = \frac{r}{\mu - \frac{1}{2}\sigma^2}, \tag{73}$$

which must be to the left of $-\gamma$ because Q is globally concave, hence $\beta_0 < -\gamma$, or, multiplying by the negative μ, $-\mu\gamma < \mu\beta_0$. Now if

$$\mu\beta_0 < r, \tag{74}$$

it will follow that [77] is satisfied. Inserting the value of β_0 produces (after rearranging)

$$\mu > \mu - \frac{1}{2}\sigma^2 \tag{75}$$

which is true since $\sigma^2 > 0$.

□

Lemma 2.

$$Z > 0 \tag{76}$$

Proof. An intuitive explanation is that the right to the cash flow that is capped at level c must be worth less than the value of the uncapped cash flow. Hence the difference between the values of the capped and the uncapped cashflow, which is equal to the value of swapping the cash flow for the coupon whenever the cash flow exceeds the coupon, must be negative.

Mathematically, note that we can arrange the inequality $Z > 0$ to yield

$$r > -\gamma\mu. \tag{77}$$

using $\gamma > 0, \delta > 0$, and $c > 0$. $Z > 0$ therefore follows from lemma 1. □

Proposition 1. *There exists an $\bar{x}_D$, such that*

$$\bar{x}_D < c \tag{78}$$

Proof. In order to characterize $\bar{x}_D$, which should be defined by

$$\left.\frac{\partial A_2}{\partial \bar{x}}\right|_{\bar{x}=\bar{x}_D} \equiv 0 \tag{79}$$

it will be necessary to discuss the properties of the implicit function

$$f(\bar{x}) = g(\bar{x}) + h(\bar{x}) = 0 \tag{80}$$

where

$$g(\bar{x}) = \gamma K - (1+\gamma)\frac{\bar{x}}{r-\mu}, \tag{81}$$

i.e. the linear part of $f(\cdot)$, and

$$h(\bar{x}) = (\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z, \tag{82}$$

i.e. the non-linear part. Note that the linear part has a positive intercept and is decreasing. Now it follows from $\delta > 1$ and lemma 2 that $\forall \bar{x} > 0$

$$h(\bar{x}) > 0, \tag{83}$$

$$h'(\bar{x}) > 0, \tag{84}$$

$$h''(\bar{x}) > 0. \tag{85}$$

This implies that for $\bar{x} > 0$, $f(\cdot)$ is a strictly convex function that always lies above $g(\cdot)$. If there is a solution to $f(\bar{x}) = 0$, this will lie to the right of the solution of $g(\bar{x}) = 0$.

There are either zero, one or two solutions to $f(\bar{x}) = 0$. It can be easily verified that $f'(c) = 0$, and that $f''(c) > 0$, so that $f(\cdot)$ is minimized at $\bar{x} = c$. Furthermore, it is easily seen that

$$f(c) = \gamma\left(K - \frac{c}{r}\right) \tag{86}$$

Due to our assumption that $K < \frac{c}{r}$, $f(c) < 0$. This implies that there are two solutions, the first representing a local maximum of A_2 and hence D, and the second representing a local minimum. Note that the local minimum must lie to the right of c, and is therefore of no consequence. It follows that D is maximized for a $\bar{x} =\in [0, c]$, where this is the first solution of [79]. We denote this as $\bar{x}_D$.

Since $f'(\bar{x}) < 0, \quad \forall \bar{x} \in [0, c]$ by virtue of $f(\cdot)$ being minimized at c, the sign of any derivatives of $\bar{x}_D$ w.r.t. any of the parameters entering $f(\cdot)$ is equal to the sign of the partial derivative of $f(\cdot)$ w.r.t. that parameter. For instance, we can see that

$$\frac{\partial \bar{x}_D}{K} > 0, \tag{87}$$

as the liquidation value rises, liquidation becomes more attractive and is optimal at an earlier stage.

Also

$$\frac{\partial \bar{x}_D}{c} < 0, \tag{88}$$

i.e. as the coupon rises, creditors want to liquidate the firm *later* (ceteris paribus). This is because for a given cash flow, creditors now receive a larger part of the potential upside of rising cash flows.

A.2.2 The socially optimal boundary for the levered firm

Defining

$$\frac{\partial A_2^L}{\partial \bar{x}} = \bar{x}^{\gamma-1}\left\{\gamma K - (1+\gamma)\frac{\bar{x}}{r-\mu} + \tau(\delta+\gamma)\left(\frac{\bar{x}}{c}\right)^{\delta} Z\right\}, \tag{89}$$

a similar argument to the one above will show that the socially optimal point, i.e. the one that maximizes firm value will be given by the first order condition

$$\left.\frac{\partial A_2^L}{\partial \bar{x}}\right|_{\bar{x}=\bar{x}_L} \equiv 0. \tag{90}$$

Similarly to the argument above, this implies that $\bar{x}_L$ is a solution to

$$f^L(\bar{x}) = g(\bar{x}) + \tau h(\bar{x}) = 0 \tag{91}$$

Proposition 2.

$$\bar{x}_L < \bar{x}_D \tag{92}$$

Proof. If $0 < \tau < 1$ (as is reasonable for a corporate tax rate), this implies that $\forall \bar{x}$, $\tau h(\bar{x}) < h(\bar{x})$. It is easily seen that this implies that $\bar{x}_L$ has the solution to $g(\bar{x}) = 0$ as a lower bound, and $\bar{x}_D$ as an upper bound. □

□

A.2.3 The optimal liquidation boundary for equity

A quick inspection of the value of equity indicates that E_1 only depends on $\bar{x}$ through E_2, and that this dependence is positive. Maximizing E_2 therefore maximizes E. Also, it can be seen that E_2 is is maximized for $\bar{x} \to 0$. Equity holders want to gamble for resurrection, and never want the firm to be liquidated, because they never gain and always lose from liquidation.

A.3 The optimal capital structure

We can find the level of c that maximizes the value of the firm L_1, taking into account that in general $\bar{x}$ will depend on c. Any solution c^* that we find must obey $c^* < x_0$, where x_0 is the initial cash flow.

(to be completed)

A.4 Debt capacity

We can also find the c that maximizes the value of debt to see whether there is an upper limit to the money that can be raised by issuing debt. Again, it is important to note that $\bar{x}$ will in general depend on c, and any solution c^* that we find must obey $c^* < x_0$, where x_0 is the initial cash flow.

(to be completed)

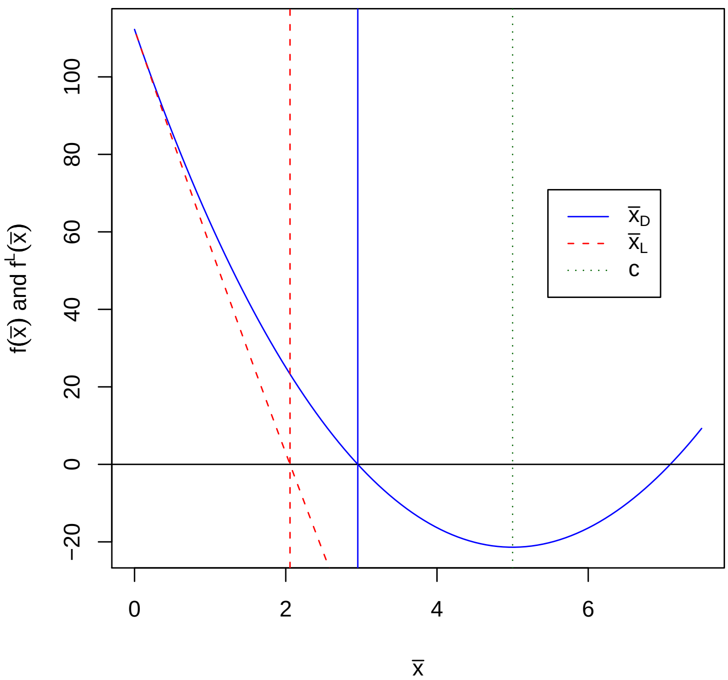

Figure 4: Optimal $\bar{x}$s. The parameters are: $r = 0.07, \mu = 0.02, \sigma = 0.2, \tau = 0.2, K = 60, c = 5$.

B Solving the discrete time game

B.1 Basic procedure

We follow the same procedure as Morris and Shin (2004) to solve the model. Suppose that agents follow a switching strategy around a certain posterior belief. Given the posterior belief around which agents switch, it is possible to derive the fraction of them that will attempt to grab assets, given the cash flow in the next period (posterior beliefs will be centred around this cash flow in the next period). We can therefore work out what the critical next-period cash flow is for which the firm will fail, given the belief in this period around which agents switch.

Also, we can use the fact that agents will switch if they believe that they will obtain a higher utility from doing so. Once we have defined utilities, this allows us to derive the critical posterior belief, given a critical next period cash flow for which the firm fails.

So we have two equations in two unknowns, which can then be solved for the critical cash flow for which the firm fails - the trigger point.

B.2 Information

For convenience, the assumptions about information are restated here. The relative increase in the cash flow is normally distributed around a drift.

$$x_{t+\Delta} - x_t = \mu x_t \Delta + x_t \eta_t, \quad \eta_t \sim NID\left(0, \frac{1}{\alpha}\right) \tag{93}$$

Agents receive a signal ξ_i (subscript i indexes the different agents) about this increase with a distribution conditional on the cash flow V_t given by

$$\xi_i = x_{t+\Delta} + x_t \varepsilon_i, \quad \varepsilon_i \sim NID\left(0, \frac{1}{\beta}\right), \tag{94}$$

with $\mathrm{Cov}(\eta_t, \varepsilon_i) = 0$, i.e. the noise is orthogonal to the innovations in the fundamental.

B.3 Posteriors

From the signal ξ_i and the public information x_t, agents form a posterior about the cash flow in period $t + \Delta$, $x_{t+\Delta}$ which is normal with mean and variance given by

$$\rho_i = E[x_{t+\Delta}|\xi_i] = \frac{\alpha(1 + \mu\Delta)x_t + \beta\xi_i}{\alpha + \beta} \tag{95}$$

and

$$\mathrm{Var}(x_{t+\Delta}|\xi_i) = \frac{(x_t)^2}{\alpha + \beta}. \tag{96}$$

B.4 Critical value of $x_{t+\Delta}$ for which the firm is liquidated

Given the posterior belief around which agents switch, we work out how many of them will grab assets, given the cash flow in the next period (posterior beliefs will be centred around this cash flow in the next period). We then work out what the critical next-period cash flow is for which the firm fails, given the belief in this period around which agents switch.

Suppose agents follow a switching strategy around ρ^*, i.e. agents grab assets when their posterior is below ρ^*. Then an agent will not grab assets if and only if the private signal is bigger than

$$\xi^* = \frac{\alpha+\beta}{\beta}\rho^* - \frac{\alpha}{\beta}(1+\mu\Delta)x_t. \tag{97}$$

Conditional on state $x_{t+\Delta}$, the distribution of ξ_i is normal with mean $x_{t+\Delta}$ and precision $\frac{\beta}{x_t^2}$. So the ex-ante probability for any agent of grabbing assets is equal to

$$\Phi\left\{\frac{1}{x_t}\sqrt{\beta}(\xi^* - x_{t+\Delta})\right\}. \tag{98}$$

where Φ is the cumulative standard normal density function.

As the number of agents tends to infinity, the fraction of agents that grab assets will be equal to this ex ante probability for any individual agent by the law of large numbers.

Since the firm fails if the fraction that grabs assets is

$$l \geq \frac{x_{t+\Delta}}{c}, \tag{99}$$

the critical value of $x_{t+\Delta}$ (denoted by $x^*_{t+\Delta}$) for which the firm fails at t is given by

$$x^*_{t+\Delta} = c\Phi\left\{\frac{1}{x_t}\sqrt{\beta}\left(\xi^* - x^*_{t+\Delta}\right)\right\} \tag{100}$$

or

$$x^*_{t+\Delta} = c\Phi\left\{\frac{1}{x_t}\left(\frac{\alpha}{\sqrt{\beta}}\left(\rho^* - (1+\mu_x\Delta)x_t\right) + \sqrt{\beta}\left(\rho^* - x^*_{t+\Delta}\right)\right)\right\}. \tag{101}$$

B.5 Utility

Utility is time-separable, with the Bernoulli utility $u(\cdot)$ being defined in terms of immediate payoffs. Immediate payoffs in any intermediate period $t+q$ are as described in the payoff matrix in the main text:

	liquidation (L)	no liquidation ($\neg L$)
grab assets (G)	$(1-s)K$	$-sK$
do not grab assets ($\neg G$)	0	0

In the following, denote the Bernoulli utility in the case the agent decides to grab assets and the firm is liquidated by

$$u(G,L) = u((1-s)K), \tag{102}$$

the Bernoulli utility in the case the agent decides to grab assets, but the firm is not liquidated by

$$u(G, \neg L) = u(-sK), \tag{103}$$

and the Bernoulli utility in the case the agent decides not to grab assets, whether or not the firm is liquidated by

$$u(\neg G, L) = u(\neg G, \neg L) = u(\neg G) = u(0). \tag{104}$$

Assume that in addition to the Bernoulli utility, the agent receives the (expected) continuation value of holding the debt in case the firm is not liquidated.

B.6 Critical value of ρ

We now use the fact that agents will switch if they believe that they will obtain a higher utility from doing so. We then derive the critical posterior belief, given a critical next period cash flow for which the firm is reorganised.

Now the marginal agent (one that is indifferent between grabbing assets or not) has a posterior over the cash flow which has its mean just at the switching point (i.e. ρ for this agent is equal to ρ^*). For her the expected utility of not grabbing assets should just equal the expected utility of grabbing assets. This *defines* the switching point. Use F to denote the posterior cumulative distribution (given the belief) over the cash flow $x_{t+\Delta}$. U denotes the continuation utility (which will depend on the belief over states of the cash flow).

We can write:

$$\begin{aligned}
& \int_{-\infty}^{x^*_{t+\Delta}} u(G, L) dF && + && \int_{x^*_{t+\Delta}}^{\infty} u(G, \neg L) dF && (105)\\
& && + && \delta \int_{-x^*_{t+\Delta}}^{\infty} U dF && (106)\\
= \quad & \int_{-\infty}^{x^*_{t+\Delta}} u(\neg G, L) dF && + && \int_{x^*_{t+\Delta}}^{\infty} u(\neg G, \neg L) dF && (107)\\
& && + && \delta \int_{-x^*_{t+\Delta}}^{\infty} U dF && (108)
\end{aligned}$$

If agents ignore the effect of their actions on the probability of the firm being liquidated (which is plausible if they are atomistic), we can write

$$\begin{aligned}
& u(G, L) \Pr\left(x_{t+\Delta} \le x^*_{t+\Delta}\right) + && u(G, \neg L) \Pr\left(x_{t+\Delta} > x^*_{t+\Delta}\right) && (109)\\
= \quad & u(\neg G, L) \Pr\left(x_{t+\Delta} \le x^*_{t+\Delta}\right) + && u(\neg G, \neg L) \Pr\left(x_{t+\Delta} > x^*_{t+\Delta}\right) && (110)
\end{aligned}$$

Noting that $\Pr\left(x_{t+\Delta} \le x^*_{t+\Delta}\right) = 1 - \Pr\left(x_{t+\Delta} > x^*_{t+\Delta}\right)$, we can rewrite this as

$$\Pr\left(x_{t+\Delta} > x^*_{t+\Delta}\right) = \frac{u(G, L) - u(\neg G, L)}{u(G, L) + u(\neg G, \neg L) - u(\neg G, L) - u(G, \neg L)} \tag{111}$$

Since it was assumed that $u(\neg G, L) = u(\neg G, \neg L)$, this is

$$\Pr\left(x_{t+\Delta} > x^*_{t+\Delta}\right) = \frac{u(G, L) - u(\neg G, L)}{u(G, L) - u(G, \neg L)} \tag{112}$$

or, with the Bernoulli utilities as defined above

$$\Pr\left(x_{t+\Delta} > x^*_{t+\Delta}\right) = \frac{u((1-s)K) - u(0)}{u\left((1-s)K\right) - u(-sK)}. \tag{113}$$

It will be useful to label this ratio of utilities as θ.

$$\theta = \frac{u((1-s)K) - u(0)}{u\left((1-s)K\right) - u(-sK)}. \tag{114}$$

Note that

$$0 \leq \theta \leq (1-s) \tag{115}$$

where the upper limit is attained when agents are risk neutral, and the lower limit is attained when $s = 1$. Also note that $\partial\theta/\partial s < 0$.

For the marginal agent (which has a normal posterior with conditional mean ρ^* and variance $(x_t)^2/(\alpha+\beta)$), the probability $\Pr(x_{t+\Delta} > x^*_{t+\Delta})$ is given by

$$\Pr\left(x_{t+\Delta} > x^*_{t+\Delta}\right) = \Phi\left\{\frac{\sqrt{\alpha+\beta}}{x_t}\left(\rho^* - x^*_{t+\Delta}\right)\right\} \tag{116}$$

where Φ denotes the cumulative normal density function.

We can equate [116] and [113] to obtain

$$x^*_{t+\Delta} - \rho^* = \frac{x_t}{\sqrt{\alpha-\beta}}\Phi^{-1}(\theta). \tag{117}$$

This equation together with (101) pins down the critical value of beliefs and the cash flow.

B.7 Equilibrium forced reorganisation

Combining equations (117) and (101) we can solve for the failure point at which the cash flow in the next period causes failure in this period:

$$x^*_{t+\Delta} = c\Phi\left\{\frac{\alpha}{\sqrt{\beta}}\left(\frac{x^*_{t+\Delta}}{x_t} - 1 - \mu\Delta\right) + \frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}(\theta)\right\} \tag{118}$$

Reorganisation at time $t+q$ will occur when x hits x^* at $t+\Delta$.

B.8 Uniqueness

To simplify notation, define

$$Y = \frac{\alpha}{\sqrt{\beta}}\left(\frac{x^*_{t+\Delta}}{x_t} - 1 - \mu\Delta\right) + \frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}}\Phi^{-1}(\theta) \tag{119}$$

and

Condition I. $c\frac{1}{\sqrt{2\pi}}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t} < 1$

Proposition 3. *The trigger point $x^*_{t+\Delta}$ is unique if condition (I) is satisfied.*

Proof. This is a version of the proof in Morris and Shin (2004). A sufficient condition for a unique solution is that the slope of

$$c\Phi\left\{Y\right\} \tag{120}$$

is less than one everywhere. This slope is equal to

$$c\varphi\left\{Y\right\}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t}. \tag{121}$$

It reaches a maximum where the argument of the normal density is 0, the maximum there will be $\frac{1}{\sqrt{2\pi}}$. Hence a sufficient condition for a unique solution is that

$$c\frac{1}{\sqrt{2\pi}}\frac{\alpha}{\sqrt{\beta}}\frac{1}{x_t} < 1. \tag{122}$$

□

B.9 Uncertainty in the limit

It can be shown that the marginal or pivotal agent views the fraction of creditors that attempt to grab assets as a random variable that is uniformly distributed in the continuous-time limit, and hence that strategic uncertainty remains. Note that these kind of results have been discussed at length elsewhere Morris and Shin (2002).

Proposition 4. *The distribution of the fraction that attempt to grab assets l given the belief ρ^* of the marginal agent is uniform in the limit.*

Proof. The proportion of creditors who receive a signal lower than ξ^* is

$$l = \Phi\left\{\frac{\sqrt{\beta}}{x_t}\left(\xi^* - x_{t+\Delta}\right)\right\}. \tag{123}$$

The question to ask is: What is the probability that a fraction less than z of the other bondholders receive a signal higher than that of the marginal agent, conditional on the marginal agent's belief, or what is

$$\Pr\left(\left(1-l\right) < z \mid \rho^*\right)? \tag{124}$$

Now the event

$$1 - l < z \tag{125}$$

is equivalent to

$$1 - \Phi\left\{\frac{\sqrt{\beta}}{x_t}\left(\xi^* - x_{t+\Delta}\right)\right\} < z \tag{126}$$

or (rearranging)

$$x_{t+\Delta} < \xi^* + \frac{x_t}{\sqrt{\beta}}\Phi^{-1}\left\{z\right\}. \tag{127}$$

So the probability we are looking for is

$$\Pr\left(x_{t+\Delta} < \xi^* + \frac{x_t}{\sqrt{\beta}}\Phi^{-1}\left\{1-z\right\} \mid \rho^*\right). \tag{128}$$

The posterior of the marginal agent over $x_{t+\Delta}$ has mean ρ^* and variance $\frac{x_t^2}{\alpha+\beta}$, hence this probability is

$$\Pr\left((1-l) < z \mid \rho^*\right) = \Phi\left\{\frac{\sqrt{\alpha+\beta}}{x_t}\left(\xi^* + \frac{x_t}{\sqrt{\beta}}\Phi^{-1}\left\{z\right\} - \rho^*\right)\right\}. \tag{129}$$

Now as we take limits, $\rho^* \to \xi^*$, since private information becomes infinitely more precise than public information (the agent attaches all weight to the signal and none to the mean of the prior), and $\frac{\sqrt{\alpha+\beta}}{\sqrt{\beta}} \to 1$. It follows that

$$\Pr\left((1-l) < z \mid \rho^*\right) = z, \tag{130}$$

so the cumulative distribution of $1-l$ is the identity function, which implies that the density of $1-l$, and hence also l, will be uniform. □

C Asset grabs and firm value

The cost of asset grabbing, or "sending out the sharks" s has an influence on the firm value via determining the liquidation boundary as well as the (disorderly) liquidation payoff. In general, this effect is not monotonic.

Letting L^C denote values of levered firms that have debt subject to coordination failure, with the corresponding liquidation boundary and payoff, the derivative of L_1^C w.r.t. s is

$$\frac{\partial L_1^C(x;x^*)}{\partial s} = \frac{\partial L_2^C(c;x^*)}{\partial s}\left(\frac{x}{c}\right)^{-\gamma} \tag{131}$$

where

$$\begin{aligned}\frac{\partial L_2^C(c;x^*)}{\partial s} &= \frac{\partial\left((1-s)K - \theta\frac{c}{r-\mu} + \theta^\delta\tau Z\right)\theta^\gamma}{\partial s} && (132)\\ &= -K\theta^\gamma + \frac{\partial\theta}{\partial s}\left\{\gamma L_2(c;x^*) - \theta\frac{c}{r-\mu} + \delta\tau Z\theta^\delta\right\}\theta^{\gamma-1} && (133)\end{aligned}$$

This can be positive as well as negative.

We can plot the difference in the values of the levered firm with coordination failure, and without coordination failure:

$$L_1^C - L_1 = \left(L_2^C(c;x^*) - L_2(c;\bar{x}_D)\right)\left(\frac{x}{c}\right)^{-\gamma} \tag{134}$$

If this difference is positive, the firm value is higher without coordination failure. The further away the firm is from being in a position where it is unable to pay the cash flow, the less matters what happens in liquidation.

The difference between L_2^C and L_2 can be seen to be

$$\begin{aligned}L_2^C(c;x^*) - L_2(c;\bar{x}_D) = &\left\{(1-s)K - \left(\frac{\theta c}{r-\mu} - \tau Z\theta^\delta\right)\right\}\theta^\gamma\\ &- \left\{K - \left(\frac{\bar{x}_D}{r-\mu} - \tau Z\left(\frac{\bar{x}_D}{c}\right)^\delta\right)\right\}\left(\frac{c}{\bar{x}_D}\right)^{-\gamma} \qquad (135)\end{aligned}$$

Raising s has the effect of lowering the liquidation value, and lowering the probability that it will be paid. The first effect makes the coordination failure case less attractive. The second effect is ambiguous and depends on the going-concern value of the firm at liquidation versus its liquidation value.