The Valuation of Deposit Insurance in an Arbitrage-free Basel II Consistent Framework

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Working Paper

preliminary

July 2006

Keywords: deposit insurance, loan portfolio, maturity gap, structural model

(JEL G13, G21, G22, G28)

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* Parts of this research were done while Oliver Entrop was visiting the School of Banking and Finance, University of New South Wales. He thanks Terry Walter and the academic and administrative staff for their hospitality and support.

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Abstract

This paper analyzes the joint influence of the quality of a bank's loan portfolio, the bank's maturity gap and its deposit rate policy on the value of deposit insurance in an arbitrage-free Basel II consistent framework. We develop and apply a two-stage structural model of a bank where deposit insurance is a European put option on the loan portfolio, the default of each loan is driven by the borrower's asset value and interest rates are stochastic. Modeling the firms' asset values by conditional independent three-factor geometric Brownian motions and applying forward measure techniques, we obtain semi-analytical solutions of arbitrage-free deposit insurance premiums for sufficiently large and homogenous loan portfolios. We show for realistic parameter combinations that the correlation within the loan portfolio and the bank's maturity gap can have a material impact on fair deposit insurance premiums. Furthermore, we find that fair deposit insurance premiums can depend negatively on the borrowers' default risk when the bank faces a maturity gap. The fair deposit insurance premium for a bank holding defaultable loans can even be lower than for a bank investing in default-free bonds.

(JEL G13, G21, G22, G28)

1 Introduction

Since the seminal work by Merton (1974), analyzing and valuing credit risk by structural models and option pricing theory in both theoretical and empirical work has had a long tradition. Structural models where the default event is driven by the firm's asset value have been developed in order to better understand the pricing of defaultable bonds and derivatives on corporate bonds (e.g., Longstaff and Schwartz, 1995, Briys and de Varenne, 1997) and of vulnerable derivatives (e.g., Johnson and Stulz, 1987, Klein, 1996). Another branch of literature develops and applies structural models in order to analyze topics of theoretical corporate finance such as financing decisions of companies and optimal default policies (e.g., Leland, 1994, Uhrig-Homburg, 2005).

In conjunction with banks, structural models have widely been applied to analyzing and valuing deposit insurance contracts. Following Merton (1977), a variety of papers such as Marcus and Shaked (1984), Ronn and Verma (1986), Pennacchi (1987), Crouhy and Galai (1991), Allen and Saunders (1993), Duan, Moreau and Sealey (1995), Anderson and Cakici (1999), Falkenheim and Pennacchi (2003) and others, model and value deposit insurance in an option-theoretical framework. They mainly differ in assumptions related to the deposit insurance agencies' behavior, the particular type of option representing the deposit insurance, and the statistical methods and data sets when empirical analyses are carried out. With the exception of Crouhy and Galai (1991) and Anderson and Cakici (1999), models coincide in the assumption of banks' asset value to follow geometric Brownian motion under stochastic or non-stochastic interest rates.

The economic intuition behind using geometric Brownian motion to model the asset values of banks is straightforward: It does not solely reflect the value of a bank's total balance assets but also the value of all its future income.[1] This is a natural approach if equity, as a call option on the asset value, is to be used to infer major bank risk parameters such as asset volatility from market data in empirical analyses. However, it is not obvious how the implied distributional assumptions and the related “abstract” risk parameters are related to the major sources of risk for banks that accounted for the majority of historical bank defaults and for the financial distress of banks over the last decades: the credit risk of the (real) assets the bank holds, especially of loans and bonds, and the maturity gap, i.e. the difference between the maturity or duration of the loans and/or bonds and the bank’s debt.

First steps to analyze the effect of (real) assets the banks holds and of the maturity gap, respectively, on the valuation of deposit insurance in structural models were taken by Crouhy and Galai (1991) and Anderson and Cakici (1999). Crouhy and Galai (1991) assume that the assets of the bank consist of a geometric Brownian motion and a default-free short-term bond. This setup allows them to revert to the option pricing formulas by Rubinstein (1983). Anderson and Cakici (1999) analyze a model with stochastic interest rates according to Cox, Ingersoll and Ross (1985), where the bank holds a single default-free or defaultable bond whose maturity is assumed to be larger than the maturity of the deposits, i.e. when the bank faces a positive maturity gap. For their numerical results they rely on numerical solutions of partial differential equations.

Our approach contributes to recent literature in several ways: We develop a two-stage structural model for banks under stochastic interest rates with which we can jointly analyze a) the influence of the quality of a bank’s loan portfolio, b) the bank’s maturity gap, and c) its deposit rate policy on the fair value of deposit insurance. Each single loan the bank holds is a derivative on the borrower’s assets that we

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1 Of course, another reason for using geometric Brownian motion is related to its nicely tractable mathematical attributes.

assume to follow conditional independent 3-factor geometric Brownian motions. This approach is consistent with conditional independent factor models widely applied to model loan portfolio distributions under the real-world measure (e.g., Schönbucher, 2000) and also serve, in the 1-factor case, as a theoretical foundation of Basel II-risk weight functions (e.g., Wilde, 2001). Applying the theory of forward martingale measures (Geman, El Karoui and Rochet, 1995) and large-sample approximation techniques[2] (Vasicek, 1991, Gordy, 2003) we obtain semi-analytical solutions of arbitrage-free deposit insurance premiums when the bank holds a sufficiently large and homogenous loan portfolio and deposit insurance is modeled as a European put option on the loan portfolio.

This setup allows us to analyze the effect of a) default-free and defaultable bonds, b) positive and zero maturity gaps, c) different correlations within the loan portfolio, d) different correlations between the asset values of the borrowing firms and interest rates, and e) the effect of idiosyncratic risk in the borrower's asset value on the value of deposit insurance. Additionally, we can analyze f) the influence of the deposit rate policy.

The paper is organized as follows: Section 2 describes the model. We first provide the general setup and the stochastic processes we use. Then we derive pricing formulas for the loans and the general pricing formula for deposit insurance. Finally, we consider two special cases: a bank holding default-free loans and a bank with a maturity gap of zero, respectively. Section 3 provides numerical examples for fair deposit insurance premiums with realistic parameter combinations and analyzes the economic behavior of our model. This section also reports on several robustness checks we carried out to assess whether our major results depend on the concrete

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2 A related approach under the real-world measure has recently been introduced by Grundke (2004). However, his main focus is the effect of jointly modeling credit and interest rate risk on a bond portfolio's value at risk.

modeling of the borrowing firms' default mechanism, of the recovery rate in case of default, or of the interest rate dynamics. Finally, Section 4 discusses policy implications for deposit insurance agencies and regulators, and concludes.

2 Model

2.1 Technical basics

We assume an underlying probability space (Ω, *F*, *P*) equipped with a filtration $(F_t)_t$ that is generated by the processes defined later and that fulfills the usual conditions. Trading in securities takes place continuously with a finite time horizon [0, *T'*]. We assume the economy to be arbitrage-free and complete. Under mild regularity conditions, this ensures the existence of a unique equivalent spot martingale measure *Q* (Harrison and Pliska, 1981, Heath, Jarrow and Morton, 1992). Under *Q*, non-dividend-paying security prices discounted by the money market account are martingales. Let *P*(*t*, *T*), $0 \leq t \leq T \leq T'$, denote the price at time *t* of a default-free zero bond maturing in *T* with face value 1. *P*(*t*, *T*) can be represented by

$$P(t, T) = E_t^Q\left(\exp\left(-\int_t^T r(s)\, ds\right)\right), \tag{1}$$

where *r*(*s*) is the continuously compounded short rate in *s* and $E_t^Q(\cdot)$ denotes F_t-conditional expectation with respect to *Q*. Let *R*(*t*, *T*) denote the continuously compounded spot rate in *t* for *T*, i.e. the continuously compounded yield of *P*(*t*, *T*), and $R_{ac}(t, T)$ its annually compounded equivalent. The instantaneous forward rate in *t* for *T* is defined by $f(t, T) = -\frac{\partial \ln P(t, T)}{\partial T} = R(t, T) + \frac{\partial R(t, T)}{\partial T} T$.

When dealing with stochastic interest rates, as we will do in this paper, it is often more convenient to apply (risk-adjusted) forward martingale measures instead of the spot martingale measure for the valuation of derivatives. Let Q^T denote the equivalent T-forward measure (Geman, El Karoui and Rochet, 1995) and $Q_t^T(\cdot)$ and $E_t^T(\cdot)$ F_t-conditional probability and expectation with respect to Q^T, respectively. Under Q^T, T-year forward prices of non-dividend-paying securities, i.e. security prices discounted by the default-free zero bond $P(t, T)$ that matures in T, are martingales.[3] If C_T denotes the payoff of a European derivative in T, its value in $t \leq T$ is given by

$$C_t = P(t, T)\, E_t^T(C_T). \tag{2}$$

Hence, the value equals the discounted expected payoff under the T-forward measure. Since the discount factor is the value of a default-free zero bond the representation (2) later allows us to analyze the maturity gap effect and the credit risk effect separately.

2.2 General setup

Throughout the paper, we consider a bank (or a depository institution) with today's total asset value $A(0)$ that is financed through equity and deposits with face value FV_{DP} and maturity $S = 1$ year. A maturity of 1 year is the common assumption in structural models dealing with deposit insurance. It is motivated by the monitoring period of deposit insurance agencies.

The bank promises its depositors an annually compounded interest rate of R_{DP} that is paid at the maturity date of the deposits. Since we aim to analyze fair values of deposit insurance, we assume that the total promised payment

$$DP(S) = DP(1) = FV_{DP}\,(1 + R_{DP}) \tag{3}$$

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3 If interest rates are non-stochastic, the forward measures and the spot martingale measure coincide.

is guaranteed by some default-free[4] deposit insurance agency. Therefore, today's value of deposits $DP(0)$ is given by the promised payment discounted by the annually compounded ($S = 1$)-year default-free spot rate $R_{ac}(0, S)$:

$$DP(0) = \frac{FV_{DP}\,(1 + R_{DP})}{1 + R_{ac}(0,\,1)} \tag{4}$$

In order to avoid arbitrage opportunities for private investors we assume $R_{DP} \leq R_{ac}(0, 1)$. In contrast, we allow for arbitrage opportunities for banks in the context of deposits, i.e. $R_{DP} < R_{ac}(0, S)$ is possible, which is consistent with the market segmentation hypothesis by Jarrow and van Deventer (1998) in the context of deposits when markets are arbitrage-free in all other respects. The difference between the face value of deposits and the value $DP(0)$ is usually referred to as deposit premium. For simplicity, we assume a linear relationship between the default-free interest rate and the deposit rate:[5]

$$R_{DP} = b_1 + b_2\, R_{ac}(0, S), \tag{5}$$

where the constants b_1 and b_2 denote the basic deposit rate and the so-called deposit rate elasticity[6], respectively. Together, b_1 and b_2 display the bank's deposit rate policy that relates the deposit rates to the default-free interest rates.

Deposit insurance in our model can be identified as a European put option on the bank's asset value with strike price $DP(S)$ and expiry date $S = 1$. Hence, the payoff of the deposit insurance in S equals

$$DI(S) = \max(DP(S) - A(S);\, 0). \tag{6}$$

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4 This assumption allows us to ignore counterparty risk of the deposit insurance contract.

5 In an equilibrium framework, Hutchison and Pennacchi (1996) derive a (for a bank) optimal relationship between interest rates and deposits rate that is approximately linear.

6 Note, that this "elasticity" is rather a sensitivity than an elasticity. However, we use this expression since it is common practice.

The key to the analysis is the modeling of the bank's asset A. We assume that the bank holds n loans. The loans are zero bonds that all mature in $T \geq S$. Hence, for $T > S$ the bank faces a maturity gap of $T - S > 0$ years, whereas for $T = S$ the bank has a maturity gap of zero. The total face value of the loan portfolio equals FV_{LP}, each loan has a face value of $FV_{LP}\, n^{-1}$. Let the value of loan i in $t \leq T$ be given by $P_{d,i}(t, T)$ per one unit of face value, i.e. its total value equals $FV_{LP}\, n^{-1}\, P_{d,i}(t, T)$. The value of the loan portfolio is then given by:

$$LP(t, T) = FV_{LP}\, n^{-1} \sum_{i=1}^{n} P_{d,i}(t, T). \tag{7}$$

Hence, the payoff of the deposit insurance in S equals:

$$\begin{aligned} DI(S) &= \max(DP(S) - LP(S, T); 0) \\ &= \max(FV_{DP}\,(1 + R_{DP}) - FV_{LP}\, n^{-1} \sum_{i=1}^{n} P_{d,i}(S, T); 0). \end{aligned} \tag{8}$$

Applying the S-forward measure, we find today's value of deposit insurance (see (2)):

$$DI(0) = P(0, S)\, E_0^S(D(S)). \tag{9}$$

The next step is to model the loans. As already pointed out in Section 1, we value each loan by applying a separate structural model. We assume that the default of a borrowing firm is driven by its total asset value. In order to get (semi-)analytical, tractable solutions in our later analysis we rely on a comparatively simple default mechanism of the firms. More precisely, firm i defaults if and only if its total asset value $V_i(t)$ falls below some default point D_i at the maturity date T of the loan, i.e. if $V_i(T) < D_i$. In case of default, the bank receives a fraction δ_i of the notional amount.[7]

[7] An exogenous recovery rate is, for example, also considered by Longstaff and Schwartz (1995).

Hence, the payoff of the loan in T per one unit of face value and of the loan portfolio, respectively, is given by:

$$P_{d,i}(T, T) = 1 + (\delta_i - 1)\, 1_{\{V_i(T) < D_i\}} = \delta_i + (1 - \delta_i)\, 1_{\{V_i(T) \geq D_i\}} \tag{10}$$

$$LP(T, T) = FV_{LP}\, n^{-1} \sum_{i=1}^{n} P_{d,i}(T, T) = FV_{LP}\, n^{-1} \sum_{i=1}^{n} \left(1 + (\delta_i - 1)\, 1_{\{V_i(T) < D\}}\right) \tag{11}$$

Note that each loan per one unit face value can be interpreted as a portfolio of a default-free zero bond with face value δ_i and a defaultable zero bond with face value $(1 - \delta_i)$ and recovery rate 0. Taking expectations with respect to the T-forward measure, we have at any time $t < T$:

$$\begin{aligned} P_{d,i}(t, T) &= P(t, T)\, E_t^T(1 + (\delta_i - 1)\, 1_{\{V_i(T) < D_i\}}) \\ &= P(t, T)\, ((1 + (\delta_i - 1)\, Q_t^T(V_i(T) < D_i)) \end{aligned} \tag{12}$$

and

$$\begin{aligned} LP(t, T) &= FV_{LP}\, P(t, T)\, n^{-1} \sum_{i=1}^{n} \left(1 + (\delta_i - 1)\, E_t^T\, (1_{\{V_i(T) < D_i\}})\right) \\ &= FV_{LP}\, P(t, T)\, n^{-1} \sum_{i=1}^{n} \left(1 + (\delta_i - 1)\, Q_t^T(V_i(T) < D_i)\right) \end{aligned} \tag{13}$$

By substituting into (7) and (8), we have the following representation of today's value of deposit insurance:

$$\begin{aligned} DI(0) = P(0, S)\, E_0^S(\max(&FV_{DP}\, (1 + R_{DP}) \\ &- FV_{LP}\, P(S, T)\, n^{-1} \sum_{i=1}^{n} (1 + (\delta_i - 1)\, Q_S^T(V_i(T) < D); 0)) \end{aligned} \tag{14}$$

2.3 Stochastic processes

We need to specify the stochastic processes for the interest rate term structure and the asset values of the firms. First, we specify the processes under the spot martingale measure. Next, we provide the dynamics of the processes under the forward measures that we need to evaluate (12) to (14).

Among the vast variety of arbitrage-free or equilibrium interest rate models we rely on the class of affine term structure models (e.g., Dai and Singleton, 2000). More precisely, we apply the arbitrage-free Hull and White (1990) extension of the 1-factor equilibrium model by Vasicek (1977) with constant volatility. Vasicek (1977) and its (multi-factor) extensions are still widely applied as reference models when other term structure models are analyzed. Furthermore, for the aim of our analysis, its distributional assumptions (i.e. short rates are normal and default-free zero bond prices are lognormal)[8] later allow us to easily correlate the interest rate term structure with the asset values of the firms and hence to achieve a high degree of analytical tractability. We prefer the Hull and White model to the Vasicek model since, in our later numerical analysis, the Hull and White model allows us to change interest rate risk parameters such as volatility without affecting the implied term structure.

The Hull and White (1990) model assumes the short rate to evolve under the spot martingale measure Q according to:

$$dr(t) = (\vartheta(t) - a\, r(t))\, dt + \sigma\, dW_r(t), \tag{15}$$

where $W_r(t)$ denotes standard Brownian motion. The positive constants $\sigma > 0$ and $a > 0$ denote volatility and mean reversion speed, respectively. The time-dependent expression $\vartheta(t)$ is given by:

$$\vartheta(t) = \frac{\partial f(0, t)}{\partial t} + a\, f(0, t) + \frac{\sigma^2}{2\, a}(1 - \exp(-2\, a\, t)). \tag{16}$$

This specification ensures that the model can be fitted to any term structure of today's spot rates by calculating the instantaneous forward rates $f(0, t)$ from today's term structure (Hull and White, 1990). Default-free zero bond prices are given by

8 One of the shortcomings of Vasicek (1977) and its extension is the positive probability of negative short rates. However, this probability is negligible if realistic parameter combinations are considered.

$$P(t, T) = A(t, T) \exp(-B(t, T)\, r(t)), \tag{17}$$

$$\text{where } B(t, T) = \frac{1 - \exp(-a\,(T - t))}{a}$$

$$\text{and } A(t, T) = \frac{P(0, T)}{P(0, t)} \exp\left(B(t, T)\, f(0, t) - \frac{\sigma^2}{4\,a}(1 - \exp(-2\,a\,t))\, B(t, T)^2\right).$$

Next, we consider the dynamics of the asset values of the borrowing firms. We follow the majority of structural models (e.g., Merton, 1974, Longstaff and Schwartz, 1995, Briys and de Varenne, 1997) and assume the asset value of firm i to follow geometric Brownian motion with constant volatility η_i. Hence, under Q we have the dynamics:

$$dV_i(t)\,/\,V_i(t) = r(t)\,dt + \eta_i\, dW_i^*(t), \tag{18}$$

where $W_i^*(t)$ denotes standard Brownian motion. Since we are dealing with several risk drivers we have to specify correlation effects. For this reason, we break down $W_i^*(t)$ into three orthogonal factors. More precisely we assume a conditional independent 3-factor model for each asset value:

$$W_i^*(t) = \rho_i\left(\theta_i\, W_r(t) + \sqrt{1 - \theta_i^2}\, W_o(t)\right) + \sqrt{1 - \rho_i^2}\, W_i(t). \tag{19}$$

$W_r(t)$ is the risk driver of the short rate (see (15)). $W_o(t)$ and $W_i(t)$, $i = 1, 2, \ldots$, are mutually independent standard Brownian motions, also orthogonal to $W_r(t)$, and $0 \leq \rho_i \leq 1$ and $-1 \leq \theta_i \leq 1$ are constants. Substituting (19) into (18) yields:

$$\begin{aligned} dV_i(t)\,/\,V_i(t) &= r(t)\,dt + \eta_i\left(\rho_i\left(\theta_i\, dW_r(t) + \sqrt{1 - \theta_i^2}\, dW_o(t)\right) + \sqrt{1 - \rho_i^2}\, dW_i(t)\right) \\ &= r(t)\,dt + \eta_i\,\rho_i\,\theta_i\, dW_r(t) + \eta_i\,\rho_i\sqrt{1 - \theta_i^2}\, dW_o(t) + \eta_i\sqrt{1 - \rho_i^2}\, dW_i(t), \end{aligned} \tag{20}$$

The economic intuition behind (19) and (20) is as follows: instantaneous unexpected changes in the asset value of firm i are driven by a systematic factor $\left(\theta_i\, W_r(t) + \sqrt{1 - \theta_i^2}\, W_o(t)\right)$ that affects all asset values simultaneously, and by a firm-specific idiosyncratic factor $W_i(t)$ that solely affects firm i. The asset value loads to the systematic factor with factor loading ρ_i and to the idiosyncratic factor with the systematic factor with factor loading ρ_i and to the idiosyncratic factor with loading $\sqrt{1-\rho_i^2}$. Hence, unexpected changes in the asset values of firms i and j are instantaneously correlated by $\rho_i \rho_j$. The systematic factor consists of two other factors. It loads to the factor $W_r(t)$ driving the short rate with θ_i and with $\sqrt{1-\theta_i^2}$ to a factor $W_o(t)$, which represents that part of systematic risk that is not interest rate risk. Consequently, the instantaneous correlation of the asset value of firm i and the short rate is given by $\rho_i \theta_i$.

For valuing each loan (see (12)) we require the distribution of $V_i(T)$ under the T-forward measure, since the F_t-conditional probability of default has to be calculated. For valuing the deposit insurance (see (14)) we additionally have to use the joint distribution of $V_i(S)$ and $r(S)$ under the S-forward measure.[9] The joint dynamics of $r(t)$ and $V_i(t)$ under the T-forward measure Q^T can be calculated by taking the Radon-Nikodym derivative of Q^T with respect to Q and applying Girsanov's theorem:[10]

$$dr(t) = (\vartheta(t) - \sigma^2 / a\,(1 - e^{-a\,(T-t)}) - a\, r(t))\, dt + \sigma\, dW_r(t) \tag{21}$$

$$\begin{aligned} dV_i(t) / V_i(t) &= (r(t) - \eta_i\, \rho_i\, \theta_i\, \sigma / a\,(1 - e^{-a\,(T-t)}))\, dt \\ &+ \eta_i\, \rho_i\, \theta_i\, dW_r(t) + \eta_i\, \rho_i \sqrt{1-\theta_i^2}\, dW_o(t) + \eta_i \sqrt{1-\rho_i^2}\, dW_i(t) \end{aligned} \tag{22}$$

This means that $r(t)$ and $\ln(V_i(t))$ are bivariate normal under each measure. Let us first determine the F_t-conditional expectation and variance of $\ln(V_i(T))$ under Q^T. We have

$$E_t^T(\ln(V_i(T)) = \ln(V_i(t) / P(t, T)) - ½\, \Sigma_i(t, T)^2 \tag{23}$$

$$\text{and } Var_t^T(\ln(V_i(T)) = \Sigma_i(t, T)^2 \tag{24}$$

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9 $V_i(S)$ is required since the S-conditional probability $Q_S^T(V_i(T) < D)$ depends on $V_i(S)$.

10 The proof is straightforward along the lines of Brigo and Mercurio (2001), pp. 453-458. Technically, the standard Brownian motions are different under different measures. For convenience, we forbear from adding measure-specific subscripts.

with $\Sigma_i(t, T) = \sqrt{V(t, T) + \eta_i^2(T - t) + \frac{2\,\rho_i\,\theta_i\,\eta_i\,\sigma}{a}\left(T - t - \frac{1}{a}(1 - e^{-a(T-t)})\right)}$

and $V(t, T) = \frac{\sigma^2}{a^2}\left(T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right)$.

The three summands in the variance $\Sigma_i(t, T)^2$ can be interpreted quite well: $\eta_i^2(T - t)$ is that part of the variance that is caused by the asset volatility. If interest rates were non-stochastic (e.g., if $\sigma = 0$) $\Sigma(t, T)^2$ would equal $\eta_i^2(T - t)$. The first summand $V(t, T)$ is solely determined by the parameters that describe the risk structure of short rate dynamics. It appears in the variance expression because the short rate enters the drift of the asset value. The third summand adjusts for the instantaneous correlation (respectively the covariance) between unexpected changes in the short rate and the asset value. Note further, that the expectation $E_t^T(\ln(V_i(T))$ depends on both the asset value $V_i(t)$ and the short $r(t)$. The later holds because $P(t, T)$ is a function of $r(t)$ (see (17)).

The next step is to determine the joint distribution of $r(S)$ and $\ln(V_i(S))$, conditional on today. According to (23) and (24), the expected value and the variance of $\ln(V_i(S))$ equals $\ln(V_i(0) / P(0, S)) - ½\,\Sigma(0, S)^2$ and $\Sigma(0, S)^2$, respectively. The expected value and variance of $r(S)$ is given by the instantaneous forward rate $f(0, S)$ and $\sigma^2 (1 - e^{-2aS}) / (2\,a)$, respectively. The covariance between $r(S)$ and $\ln(V_i(S))$ equals:

$$CV = CoVar_0^S(r(S), \ln(V_i(S)) = \sigma\left(\frac{\sigma}{a} + \eta_i\,\rho_i\,\theta_i\right)\frac{1 - e^{-aS}}{a} - \frac{\sigma^2}{a}\frac{1 - e^{-2aS}}{2a}. \tag{25}$$

Note, that even in the case of instantaneously uncorrelated short rates and asset values, i.e. $\rho_i\,\theta_i = 0$, the covariance of both variables $r(S)$ and $\ln(V_i(S)$ under the S-forward measure need not be zero, since the short rate enters the drift of the asset values.

The above considerations allow us to give the following representation of the joint distribution of $r(S)$ and $\ln(V_i(S))$. Under the S-forward measure we have, conditional on today, i.e. $t = 0$, in distribution:

$$r(S) \sim f(0, S) + \sigma_1\, \varepsilon_1 \tag{26}$$

$$\ln(V_i(S)) \sim \ln(V_i(0) / P(0,S)) - \tfrac{1}{2}\, \Sigma_i(0, S)^2 + x_{1,i}\, \varepsilon_1 + x_{2,i}\, \varepsilon_2 + x_{3,i}\, \varepsilon_{3,i} \tag{27}$$

where ε_1, ε_2, $\varepsilon_{3,1}$, $\varepsilon_{3,2}$, ... are iid standard normal and

$$\sigma_1 = \sigma\sqrt{\frac{1}{2a}\,(1 - e^{-2aS})}, \tag{28}$$

$$x_{1,i} = CV\,/\,\sigma_1, \quad x_{2,i} = \sqrt{\Sigma_i(0, S)^2 - (1 - \rho_i^2)\, \eta_i^2\, S - x_{1,i}^2}, \quad x_{3,i} = \eta_i \sqrt{1 - \rho_i^2}\sqrt{S}. \tag{29}$$

The different components of (26) and (27) can be interpreted analogously to the components of (15) and (20): ε_1 represents the (systematic) “interest rate risk” that affects the short rate in S and all asset values simultaneously. ε_2 represents that part of total systematic risk of the asset values that is not related to interest rate risk and $\varepsilon_{3,i}$ the firm-specific idiosyncratic risk. Note that the loading $x_{3,i}$ is zero if and only if the asset values are instantaneously perfectly correlated. Hence, the effect of idiosyncratic $\varepsilon_{3,i}$ vanishes in this case.

2.4 Valuing loans and deposit insurance

According to (12), we need to calculate the F_t-conditional probability of default, i.e. of the event $\{V_i(T) < D_i\}$ under the T-forward measure, in order to calculate the value of a loan in t. We have $\{V_i(T) < D_i\} = \{\ln(V_i(T)) < \ln(D_i)\}$. Since $\ln(V_i(T))$ is normally distributed with parameters given by (23) and (24) we have via normalization of $\ln(V_i(T))$:

$$Q_t^T(V_i(T) < D_i)) = N\left(\frac{\ln\dfrac{D_i\, P(t, T)}{V_i(t)} + \tfrac{1}{2}\, \Sigma_i(t, T)^2}{\Sigma_i(t, T)}\right), \tag{30}$$

where $N(\cdot)$ denotes the cumulative standard normal distribution function. Substituting into (12) yields a closed-form solution for the value of a loan with face value 1 in t:

$$P_{d,i}(t, T) = P(t, T)\left(1 + (\delta_i - 1)\, N\left(\frac{\ln\frac{D_i\, P(t, T)}{V_i(t)} + ½\, \Sigma_i(t, T)^2}{\Sigma_i(t, T)}\right)\right) \tag{31}$$

Naturally, the value of the defaultable loan equals the value of a default-free bond if the recovery rate δ_i is set to 1. If the default point of firm i goes to zero, i.e. for $D_i \downarrow 0$, the value of the defaultable loan converges to the value of a default-free bond as well.

By substituting (31) into (14) the value of deposit insurance is given by

$$DI(0) = P(0, S)\, E_0^S(\max(FV_{DP}\,(1 + R_{DP}) - FV_{LP}\, P(S, T)\, n^{-1} \sum_{i=1}^{n} \left(1 + (\delta_i - 1)\, N\left(\frac{\ln\frac{D_i\, P(S, T)}{V_i(S)} + ½\, \Sigma_i(S, T)^2}{\Sigma_i(S, T)}\right)\right); 0)) \tag{32}$$

Since each loan is a derivative on the firm's assets, deposit insurance is a derivate on a portfolio of derivatives. Even in the case of a simple compound equity option, it is well-known that closed-form solutions do not generally exist when stochastic interest rates are taken into account (Frey and Sommer, 1998). Hence, we cannot hope to find a closed-form solution for the value of deposit insurance (32) in our far more complex case.

However, we can find a semi-analytical solution, i.e. an integral representation, of the value of deposit insurance (32), if we make the simplifying assumption that the borrowing firms are homogenous in their major risk parameters and if the number of loans in the portfolio is sufficiently large. Hence we assume that the firms' asset value and asset volatility, their default points and the factor loadings, ρ_i and θ_i, coincide. Thus we can drop the subscripts and have $V(0) = V_i(0) = V_j(0)$, $\eta = \eta_i = \eta_j$, $D = D_i = D_j$, $\rho = \rho_i = \rho_j$, $\theta = \theta_i = \theta_j$, $\Sigma(t, T) = \Sigma_i(t, T) = \Sigma_j(t, T)$. Note that the asset correlation equals ρ^2 with this notation.

Although we have assumed that today's asset values $V_i(0)$ are equal, the asset values $V_i(S)$ in S will differ due to the idiosyncratic factor $W_i(t)$ (see (20) and (22)). However, for large n, i.e. a large number of loans, the idiosyncratic factors $W_i(t)$ diversify in such a way that the limit distribution of the loan portfolio value in S is independent of stochastic components that are related to $W_i(t)$. Therefore, only the systematic factor(s) will matter for future loan portfolio values. This allows us to obtain a semi-analytical representation, i.e. an integral representation, of the value of deposit insurance (32).

The key to the analysis is the distribution of the value of the loan portfolio in S under the S-forward measure:

$$LP(S, T) = FV_{DP}\, P(S, T)\, n^{-1} \sum_{i=1}^{n} \left(1 + (\delta - 1)\, N\left(\frac{\ln \frac{D\, P(S, T)}{V_i(S)} + ½\, \Sigma(S, T)^2}{\Sigma(S, T)} \right) \right) \quad (33)$$

Conditional on today, the value of a default-free zero bond in S, $P(S, T)$, and the asset values $V_i(S)$ are random variables. $P(S, T)$ depends on the future short rate $r(S)$ (see (17)). For convenience, we shall write $P(S, T)(r(S))$ instead of $P(S, T)$. Hence:

$$LP(S, T) = FV_{DP}\, P(S, T)(r(S))\, n^{-1} \sum_{i=1}^{n} \left(1 + (\delta - 1)\, N\left(\frac{\ln \frac{D\, P(S, T)(r(S))}{V_i(S)} + ½\, \Sigma(S, T)^2}{\Sigma(S, T)} \right) \right) \quad (34)$$

The representation (26) and (27) of the joint distribution of $r(S)$ and $\ln(V_i(S))$ allows us to obtain the following representation of the value of the loan portfolio in S under Q^S:

$$LP(S, T) \sim FV_{DP}\, P(S, T)(f(0, S) + \sigma_1\, \varepsilon_1)$$
$$n^{-1} \sum_{i=1}^{n} \underbrace{\left(1 + (\delta - 1)\, N\left(\frac{\ln \frac{D\, P(0, S)\, P(S, T)(f(0, S) + \sigma_1\, \varepsilon_1)}{V_i(0)} + ½\, \Sigma(0, S)^2 - x_1\, \varepsilon_1 - x_2\, \varepsilon_2 - x_3\, \varepsilon_{3,i} + ½\, \Sigma(S, T)^2}{\Sigma(S, T)} \right) \right)}_{:= Z_i} \quad (35)$$

Note that, conditional on the "systematic" variables ε_1 and ε_2, the Z_i, defined in (35), are iid. Hence, conditional on $\varepsilon_1 = z_1$ and $\varepsilon_2 = z_2$, $n^{-1} \sum_{i=1}^{n} Z_i$ converges almost surely to

$E(Z_1 | \varepsilon_1 = z_1, \varepsilon_2 = z_2)$ according to the strong law of large numbers. Economically, this means that the influence of the idiosyncratic factors $\varepsilon_{3,i}$ that affect each loan value in S separately vanish for a large number of loans. Consequently, the future value of a sufficiently large loan portfolio is only driven by the systematic factor(s). This intuitive argument can be formalized by applying Lemma 1 from the Appendix with $\boldsymbol{X} = (\varepsilon_1, \varepsilon_2)$, $Y_i = \varepsilon_{3,i}$, and f given by the right hand side of (35). We find that $LP(S, T)$ converges in L^2 and, hence, in distribution to:

$$LP(S, T) \sim FV_{DP}\, P(S, T)(f(0, S) + \sigma_1 \varepsilon_1)$$

$$\int \left(1 + (\delta - 1)\, N\left(\frac{\ln \frac{D\, P(0, S)\, P(S, T)(f(0, S) + \sigma_1 \varepsilon_1)}{V_i(0)} + ½\, \Sigma(0, S)^2 - x_1 \varepsilon_1 - x_2 \varepsilon_2 - x_3 z_3 + ½\, \Sigma(S, T)^2}{\Sigma(S, T)} \right) \right) dN(z_3) \qquad (36)$$

Furthermore, since the payoff of a put option is a continuous bounded function of the underlying, we can conclude that the value of deposit insurance converges to

$$DI(0) = P(0, S) \int (\max(FV_{DP}\,(1 + R_{DP}) - FV_{LP}\, P(S, T)(f(0, S) + \sigma_1 z_1) \qquad (37)$$

$$\int \left(1 + (\delta - 1)\, N\left(\frac{\ln \frac{D\, P(0, S)\, P(S, T)(f(0, S) + \sigma_1 z_1)}{V_i(0)} + ½\, \Sigma(0, S)^2 - x_1 z_1 - x_2 z_2 - x_3 z_3 + ½\, \Sigma(S, T)^2}{\Sigma(S, T)} \right) \right) dN(z_3);\, 0))\, dN(z_1, z_2),$$

where $N(\cdot, \cdot)$ denotes the bivariate cumulative standard normal distribution function. From now on, we will work with the limit distribution (36) and the limit value of deposit insurance (37).

2.5 Special cases

In this section we analyze special cases of our general setup and hence special cases of our general pricing formula (37). Here we are dealing with the "extreme" cases: A) no credit risk, i.e. the loans are default-free, B) no maturity gap, i.e. $T = S$.

If there is no credit risk in the loan portfolio, i.e. the borrowing firms cannot default, the promised payment at maturity is safe, i.e. $P_d(T, T) = 1$. The loan portfolio thus behaves as if it consisted of a single default-free zero bond with face value FV_{LP}. Consequently, deposit insurance equals a put option on a default-free zero bond and we can apply the well-known formula for European puts with $T > S$ (Hull and White, 1990):

$$DI(0) = FV_{DP}\,(1 + R_{DP})\,P(0, S)\,N(-h + \sigma_P) - FV_{LP}\,P(0, T)\,N(-h), \tag{38}$$

$$\text{where } \sigma_P = \sigma\sqrt{\frac{1}{2a}\,(1 - e^{-2aS})}\,B(S, T) = \sigma_1\,B(S, T),$$

$$h = \frac{1}{\sigma_P}\ln\frac{P(0, T)\,FV_{LP}}{P(0, S)\,FV_{DP}\,(1 + R_{DP})} + \frac{\sigma_P}{2}.$$

Of course, this formula is a special case of our general formula (37) for $D \downarrow 0$, or $V_i(0) > D$ and $\eta \downarrow 0$. Note that for $T \downarrow S$ the value of deposit insurance converges to zero, since a bank with a maturity gap of zero cannot default in our framework if the loans are default-free.

In the case of no maturity gap i.e. $T = S$, the value of the loan portfolio in S is given by

$$\begin{aligned} LP(S, S) &= FV_{DP}\,n^{-1}\sum_{i=1}^{n}\left(1 + (\delta - 1)\,1_{\{V_i(S) < D\}}\right) \\ &= FV_{DP}\,n^{-1}\sum_{i=1}^{n}\left(1 + (\delta - 1)\,1_{\{\ln(V_i(S)) < \ln(D)\}}\right) \end{aligned} \tag{39}$$

We see that $r(S)$ does not enter (39). This is natural, since for $T = S$ the (expected) payoff of each loan need not be discounted from T to S. Hence it is only the distribution of $\ln(V_i(S))$ rather than the joint distribution of $\ln(V_i(S))$ and $r(S)$ that matters. Consequently, we can collect the terms $x_1\,\varepsilon_1$ and $x_2\,\varepsilon_2$ in (27) and have

$$\ln(V_i(S)) \sim \ln(V_i(0) / P(0, S)) - ½\,\Sigma(0, S)^2 + x_4\,\varepsilon_4 + x_3\,\varepsilon_{3,i} \tag{40}$$

under the S-forward measure, where ε_4 is standard normal and independent of $\varepsilon_{3,i}$, and x_4 is given by $x_4 = (x_1^2 + x_2^2)^{0.5}$. By substituting (40) into (39), taking expectation and applying analogous arguments as in the general case of Section 2.4 we find the limit distribution of the value of the loan portfolio in S:

$$LP(S,S) \sim FV_{DP}\,\left(1+(\delta-1)\,N\left(\frac{\ln\frac{D\,P(0,S)}{V_i(0)}+\frac{1}{2}\,\Sigma(0,S)^2-x_4\,\varepsilon_4}{x_3}\right)\right) \qquad \rho<1 \quad (41)$$

$$LP(S,S) \sim FV_{DP}\,\left(1+(\delta-1)\,1_{\{\varepsilon_4<(\ln(D\,P(0,S)/V_i(0))+\frac{1}{2}\Sigma(0,S)^2)/x_4\}}\right) \qquad \rho=1 \quad (42)$$

Of course, (41) and (42) are special cases of (36). $N\left(\frac{\ln\frac{D\,P(0,S)}{V_i(0)}+\frac{1}{2}\,\Sigma(0,S)^2-x_4\,\varepsilon_4}{x_3}\right)$ can be interpreted as the fraction of defaults in the loan portfolio under the S-forward measure. Observe that for $\rho = 1$, the value of the loan portfolio has a two-point distribution on $\{FV_{DP}, \delta FV_{DP}\}$. This is plausible, since in the case of $\rho = 1$, the loan portfolio behaves like a single defaultable zero bond. In contrast, in the case of instantaneously uncorrelated loans (see (41)), the value of the loan portfolio does not have a point distribution since x_4 is not zero. This is due to the fact that the short rate enters the drift of each asset value process. Hence the asset values in S are positively correlated, even if they are instantaneously uncorrelated. However, this effect vanishes if the short rate tends to zero or the mean reversion speed goes to infinity.

These considerations are illustrated by Figure 1 where the density of (41) for different asset correlations and short rate volatilities are shown. As in the later numerical analysis, we set the face value of the loan portfolio to that number that ensures a today's loan portfolio value of 100, according to (31). The total face value FV_{LP} of the loan portfolio thus equals 112.65, 112.81 and 113.27 for $\sigma = 0.005$, 0.02, and 0.04, respectively, but is, of course, independent of the asset correlation ρ^2 (see Table 2 for the other parameters). A large asset correlation favors large and small values of the loan portfolio, i.e. large and small portfolio values become more likely. Thus for increasing asset correlation, the distribution becomes broader. For $\rho^2 \uparrow 1$, the distribution converges to a two-point distribution, as already pointed out. For $\rho^2 \downarrow 0$, the distribution becomes tighter and converges to a one-point distribution if the asset volatility tends to zero.

3 Numerical results

3.1 General setting

To gain deeper insights into the economic behavior of our model, we analyze in this section the model outputs in dependence on the model input parameters. We first analyze the case of default-free loans (Section 3.2). We then consider a bank with a maturity gap of zero, but allow for defaultable loans (Section 3.3). Finally, we analyze the general case, i.e. allow for a positive maturity gap and defaultable loans (Section 3.4).

Throughout our analysis, we assume the bank to have a total asset value of $A(0) = 100$. Hence, the value of the loan portfolio is also 100 by definition. In our analysis we assume the loan portfolio to be priced fairly according to the general pricing formula (31) for each parameter combination we look at.[11] Thus for all parameter sets we adjust the face value of the loan portfolio to $FV_{LP} = A(0) / P_{d,i}(0, T)$ before calculating the value of deposit insurance.

Furthermore, we assume that the bank has issued deposits with face value $FV_{DP} = 95$.[12] For each parameter combination we calculate the value of deposit insurance $DI(0)$ according to the pricing formulas of Section 2 and report the fraction $DI(0) / FV_{DP}$, which is the value of deposit insurance per one unit of nominal deposits. We refer to this fraction as deposit insurance premium.

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[11] Hence, we are not running a comparative-static analysis in the sense of analyzing the effects of parameter changes to the value of deposit insurance when parameters of the loan portfolio, especially its face value, are exogenously given.

[12] We have also run our analysis with other face values of deposits. Since the put option's strike price is a linear function of the face value of deposits, a lower (higher) face value decreases (increases) the value of deposit insurance. Therefore, of course, the level of the deposit insurance value varies. However, we find that our qualitative results are substantially unaffected.

The Hull and White model requires the instantaneous forward rate curve and its derivation (see (16)), and hence a somewhat nicely behaved today's spot rate term structure. Since we also want to analyze effects of different forms of term structures, we parameterize today's annually-compounded spot interest rate term structure by the well-known Nelson and Siegel (1987) curve:

$$R_{ac}(0, T) = \beta_0 + (\beta_1 + \beta_2)\frac{1 - \exp(-T/\beta_3)}{T/\beta_3} - \beta_2 \exp(-T/\beta_3), \qquad (43)$$

where $\beta_0, ..., \beta_3$ ($\beta_3 > 0$) are constants. β_0 can be interpreted as the long-term spot rate, i.e. $\lim_{T\to\infty} R_{ac}(0, T) = \beta_0$, whereas $\lim_{T\downarrow 0} R_{ac}(0, T) = \beta_0 + \beta_1$ equals the annually compounded short rate. Thus we have $r(0) = \ln(1 + \beta_0 + \beta_1)$. If the difference between the long-term and short-term spot rate is interpreted as the slope, it is given by $-\beta_1$. If $\beta_2 = 0$, we see that a positive (negative) slope is equivalent to a concave (convex) curvature. The term structure is less curved if β_3 is higher. Based on (43), it is straightforward to calculate the instantaneous forward rate curve and its derivation.[13]

Throughout our analysis we will refer to 9 different term structures showing 3 different short-term levels, (l(ow), m(edium), h(igh)) and three different slopes, (d(own), f(lat), u(p)). These are specified by the Nelson and Siegel-parameters shown in Table 1 and are illustrated by Figure 2. Further, we assume a exemplary set of parameters that we refer to if the parameters are not explicitly specified otherwise. In this exemplary scenario we set the short-rate volatility $\sigma = 0.02$ and the mean reversion speed $a = 0.1$. These interest rate risk parameters are close to Hull and White (1993). Setting a deposit maturity of $S = 1$ year, we assume a maturity gap of 3 years, i.e. we set the maturity of the loans $T = 4$. If credit risk is taken into account, we set the borrowers' asset value and asset volatility to $V_0 = 10$ and $\eta = 0.1$, respectively, and the default point to $D = 8$. The chosen parameters determining the

[13] For simplicity, we omit the relevant formulas here.

borrowers' credit risk roughly imply a real-world default probability of 1%, assuming an average market price of risk for asset values of 0.15 (Huang and Huang, 2003) and then transforming risk-neutral default probabilities into real-world default probabilities along the lines of Crouhy, Galai and Mark (2000). According to Moody's (2000), a 4-year default probability of 1% is, on average, linked to the A rating category. In order to analyze the effect of "pure" credit risk we assume a zero recovery rate, i.e. $\delta = 0$, in the exemplary scenario. The loading to the systematic factor is set at $\rho = 0.447$, which means that the asset correlation ρ^2 equals 0.2, and is close to the Basel II assumptions. Further, we set $\theta = 0$. The basic deposit rate is $b_1 = 0$ and the deposit rate elasticity equals $b_2 = 1$. Hence in this scenario, the deposits are promised the 1-year default-free annually compounded spot rate, i.e. $R_{DP} = R_{ac}(0, 1)$. Table 2 summarizes the exemplary scenario.

3.2 Default-free loans

If we do not allow for default of the borrowing firms, the bank holds a single default-free bond. If we do not allow for a positive maturity gap of the bank, i.e. if we set $T = S$, the value of deposit insurance equals 0, since the bank cannot default by definition. In the following we assume $T > S$. According to (38), the value of deposit insurance is influenced by today's spot rate curve, its risk parameters σ and a, and b_1 and the deposit rate elasticity b_2. Let us first examine the influence of today's term structure on the value of deposit insurance when we set $b_1 = 0$ and $b_2 = 1$ as in our exemplary parameter set. Thus we have $FV_{DP} = 95\ (1 + R_{ac}(0, 1))$. Since $FV_{DP}\ (1 + R_{DP})\ P(0, S) = FV_{DP}$ and $FV_{LP}\ P(0, T) = A(0)$ hold, substituting into (38) yields:

$$DI(0) = FV_{DP}\ N(-h + \sigma_P) - A(0)\ N(-h), \tag{44}$$

$$\text{where } \sigma_P = \sigma\sqrt{\frac{1}{2a}(1 - e^{-2aS})}\ B(S, T) = \sigma_1\ B(S, T),$$

$$h = \frac{1}{\sigma_P}\ln\frac{A(0)}{FV_{DP}} + \frac{\sigma_P}{2}.$$

Hence today's value of deposit insurance is independent of today's term structure but, of course, it does depend on the two risk parameters, namely short rate volatility σ and mean reversion speed a, and on the time to maturity T.

Let us first consider the effect of T. Figure 3 shows the deposit insurance premium for different term structures and deposit rate elasticities in dependence on time to maturity. The "reference scenario" refers to a deposit rate elasticity of $b_2 = 1$. Clearly, the deposit insurance premium increases with increasing time to maturity. This is plausible since a higher time to maturity T yields a broader distribution of zero bond prices in $S = 1$ (see (17), (26)) under the S-forward measure and broader distributions cause higher put option values in general. Additionally, Figure 3 shows the results for deposit rate elasticities of 0.7, 0.3, and 0. We find that higher deposit rate elasticities lead to a higher deposit insurance premium. Of course, this effect is caused by the monotone-increasing relationship between the deposit rate elasticity and the strike price of the put option we look at (see (3), (5), (6)). As this relationship is independent of the distribution of the bank's assets, we omit the effect of the deposit rate elasticity in the subsequent sections.

Interestingly, Figure 3 illustrates that the deposit insurance premium is no longer unaffected by today's interest rate term structure if the deposit rate elasticity is smaller than one. The influence of the form of the term structure increases with lower deposit rate elasticity. Our numerical results indicate that both a higher level of the term structure and a higher slope cause the deposit insurance premium to decrease.

Figure 4 and Figure 5 show the effect of the short rate volatility and the mean reversion speed, respectively, on the deposit insurance premium for different time-to-maturities and different term structures. Except for the reference scenario, the deposit rate elasticity is set to $b_2 = 0.3$. Since, similar to the influence of T, a higher short rate volatility and a lower mean reversion speed cause a broader distribution of zero bond prices in $S = 1$, we find the deposit insurance premium to increase for higher σ and to decrease with higher a (see (17), (26)).

Summarizing the numerical results of this section, we found what was expected, especially in relation to the risk parameters of the term structure. However, it should be pointed out that the deposit rate policy of banks, measured by the deposit rate elasticity, can materially affect the value of deposit insurance as it defines the strike price of the put option.

3.3 No maturity gap

If the bank has a maturity gap of zero and holds only default-free loans it cannot default as already pointed out in Section 3.2. Hence, the deposit insurance premium is zero. Consequently, if the bank holds a portfolio of defaultable loans, the deposit insurance premium can never be lower than in the default-free case. From now on, we omit consideration of different today's interest rate term structures since the effects appear to be similar to those reported in Section 3.2. The same holds for the effects of short rate volatility and mean reversion speed in this section.

Figure 6 and Figure 7 illustrate the deposit insurance premium for different default points of the borrowing firms, i.e. different default risk of the firms, and different asset correlations with respect to the asset volatility (Figure 6) and the recovery rate (Figure 7). In all figures, we find that the deposit insurance premium increases with higher asset correlation. This is consistent with the analysis of Section 2.5 that a higher asset correlation yields a broader distribution of the value of the loan portfolio in S, which naturally leads to a higher value of put options. The economic intuition behind this is quite clear: The lower the asset correlation the better value fluctuations and defaults of individual loans can be diversified within the loan portfolio. Note that these fluctuations are solely caused by the asset values of the firms, i.e. the credit quality of the loan portfolio, since the value of the loan portfolio in S is not affected by the short rate in S, as we have already pointed out.

Furthermore, Figure 6 and Figure 7 imply that higher default risk, i.e. a higher default point D (all considered figures) or a higher asset volatility η of the borrowing firms (Figure 6), causes higher deposit insurance premiums. The influence of the asset correlation on the deposit insurance seems to be higher, the higher the default risk of the firms is. Naturally, a higher recovery rate causes a lower deposit insurance premium (Figure 7). To summarize this section, we should point out that the asset correlation of the borrowing firm, i.e. roughly speaking the correlation within the loan portfolio, materially affects the value of deposit insurance. Whereas for low asset correlations, results are similar to the default-free case, there is a large difference when the loan portfolio is not well diversified. However, an assumption of perfectly correlated loans, as implicitly made by Anderson and Cakici (1999), strongly overestimates the effect of credit risk in the loan portfolio on the value of deposit insurance.

3.4 General case

Let us next consider the general case where loans are defaultable with a maturity $T > S = 1$, i.e. the bank faces a positive maturity gap. In the case of a maturity gap of zero the asset correlation of the borrowing firms appeared to be of particular importance. We therefore first analyze how the asset correlation influences the value of deposit insurance. Figure 10 to Figure 13 show, for different asset correlations, the deposit insurance premium in dependence on the short rate volatility (Figure 10), the mean reversion speed (Figure 11), the default point (Figure 12) and the asset volatility (Figure 13). As in Section 3.3, we find the deposit insurance premium to increase with higher asset correlation. The economic intuition is analogous to the case of a zero maturity gap: lower asset correlation increases the ability to diversify the value fluctuations of individual loans in the portfolio that are caused by changing credit quality, thus yielding a tighter distribution of the portfolio value in *S* (under the *S*-forward measure) and decreasing the put option value. In the figures considered, the deposit insurance premium for a bank holding a default-free loan portfolio is also given, in addition to the deposit insurance premiums for the defaultable loan portfolios we have looked at so far. Interestingly, the value of deposit insurance in the default-free case need not necessarily be lower than the premium in the defaultable case. In fact, it can by far exceed the premium of the default-free case. This is an effect we did not find in the case of a zero maturity gap. By definition, the deposit insurance premium was zero for default-free loans. Hence, this must be caused by the positive maturity gap.

To clarify this in more detail, let us take a look at Figure 12. Here we find the deposit insurance premium for different asset correlations and maturity gaps in dependence on the default point, i.e. today's credit quality of the firms. For a low default point, the deposit insurance premiums for the default-free and the defaultable loan portfolios coincide or are close. For realistic default points like in our basis parameter set, the deposit insurance premium for the defaultable loan portfolio can be much lower than for the default-free loan portfolio, whereas for large default points we find the inverse effect.

The key to the analysis of this - at first glance contra-intuitive - behavior is the distribution of the value of the loan portfolio in $S = 1$ under the *S*-forward measure. Figure 8 shows the portfolio value for recovery rates δ of 0, 0.5, and 1, and a maturity gap of 3 years according to (36) where the asset correlation is set to the "extreme value" $\rho^2 = 0$ (see Table 2 for the other input parameters). On the axes, possible realizations of the systematic variables ε_1 and ε_2 are given. Remember that these variables are standard normal. Hence, they have an expected value of zero. Realizations below -3 and above 3 are very improbable. The lower graph of Figure 8 assumes a recovery rate of $\delta = 1$. Hence the loan portfolio behaves as if it were default-free. As a consequence, the value of the loan portfolio solely depends on ε_1

but is independent of ε_2. The upper graph shows the value of the loan portfolio for a recovery rate of zero. Naturally, the loan portfolio value depends on both ε_1 and ε_2. However, interestingly, the surface appears to be much flatter and hence less risky than in the default-free case. In fact, the default-free case implies a deposit insurance premium of 0.39%, whereas we have 0.12% for the defaultable case with $\delta = 0$. This means that in this example, a bank holding a 4-year default-free bond has to pay a higher deposit insurance premium than a bank holding a well-diversified portfolio of defaultable loans. This also implies that a higher recovery rate increases the fair deposit insurance premium. This effect can even appear when the bank holds a single defaultable bond, or alternatively if $\rho^2 = 1$ for certain parameter combinations.

This effect can be explained by the potential diversification between fluctuations of the value of default-free zero bonds in S, that also serve as discount factors from T to S under the S-forward measure, and fluctuation of the firms' asset values, i.e. fluctuations in the credit quality of the loan portfolio. The effect is more pronounced when we also have diversification between the credit quality of the borrowers in S, i.e. for low asset correlations.

The following formula rewrites the value of the loan portfolio (36), when it consists only of a single defaultable loan, i.e. for $\rho^2 = 1$, broken down into two factors, A and B:

$$FV_{LP} \underbrace{P(S,T)(f(0,S)+\sigma_1\varepsilon_1)}_{A} \underbrace{\left(1+(\delta-1)\,N\left(\frac{\ln\dfrac{D\,P(0,S)\,P(S,T)(f(0,S)+\sigma_1\varepsilon_1)}{V_i(0)}+\tfrac{1}{2}\Sigma(0,S)^2-x_1\varepsilon_1-x_2\varepsilon_2+\tfrac{1}{2}\Sigma(S,T)^2}{\Sigma(S,T)}\right)\right)}_{B} \tag{45}$$

A is the discount factor from T to S, i.e. the value in S of a default-free bond per one unit face value maturing in T, whereas B equals the expected loan portfolio payoff in S under the S-forward measure. In the default-free case, we have $B = 1$. Hence, the total risk of the loan portfolio value in S is solely related to fluctuations of A, that are caused by the randomness of ε_1. If we have a high (low) realization of ε_1, A is low (high). In the case of a defaultable loan, B is stochastic as well, depending on ε_1 and ε_2. If x_1 and x_2 in B were zero, B would behave conversely to A, i.e. B would be high (low) for high (low) realizations of ε_1. Roughly speaking, B would stabilize the fluctuations of the zero bond price A and vice versa. However, of course, in the case of a single defaultable bond, x_1 and x_2 are not zero, so the terms $-x_1 \varepsilon_1$ and $-x_2 \varepsilon_2$, that represent systematic risk of the asset value, in B can (over-)compensate the stabilizing effect.

In the case of a low correlated loan portfolio (see (36)), i.e. where ρ^2 is small, the stabilizing effect is much more likely (depending on the input parameters). For small ρ^2, the influence of the unsystematic risk on the loan portfolio value in S vanishes, as shown in Section 2.4 and the relevance of systematic risk is small, i.e. x_1 and x_2 are comparatively small.

Figure 8 also illustrates that for a higher recovery rate the loan portfolio distribution can become riskier. In view of the prior analysis this is plausible, since a higher recovery rate increases the default-free portion in the loan portfolio (see Section 2.2 and (10)).

Summarizing the major results of this section, we found that when a bank holds a defaultable loan portfolio and faces a positive maturity gap, the deposit insurance premium can depend strongly on the correlation within the loan portfolio, similar to the case of a zero maturity gap. In contrast, the deposit insurance premium can even be lower than for a bank holding default-free loans.

3.5 Robustness

We tested whether our results, especially the effect of correlation within the loan portfolio and the effect of the maturity gap, depend on the three main assumptions of our model: the simple default mechanism, i.e. default is only possible at the maturity date of the loans, the non-stochastic exogenous recovery rate, and the Hull and White

(1990) short rate model. Based on these assumptions we were able to derive the simple (semi-)analytical solution for deposit insurance premiums in section 2.4. For running several robustness checks, we modified our model in three ways: First, we allowed for early default of each borrower by including a default barrier similar to Longstaff and Schwartz (1995). Second, we analyzed the effect of a stochastic endogenous recovery rate that is proportional to the firms' asset values at default, similar to Klein (1996). Third, we replaced the Hull and White (1990) short rate model by the Cox, Ingersoll und Ross (1985) model.

For each of these modifications we calculated deposit insurance premiums for realistic parameter combinations and variations. Since closed-form or semi-analytical solutions are not available, we applied Monte Carlo simulation of the borrowers' asset values and the short rate under the spot martingale measure. As the payoff of deposit insurance in S depends on the values of the loans in S that cannot be determined in standard Monte Carlo procedures for a single run, we applied the least squares Monte Carlo simulation according to Longstaff and Schwartz (2001) to calculate these conditional expectations.

Our major results and the behavior of the model turned out to be robust against these modifications. The only exception is the default-free case when the Cox, Ingersoll and Ross (1985) model is applied. Clearly, the independence of the deposit insurance premium from today's term structure (Section 2.5) is a feature of the Hull and White (1990) model that cannot be expected to exist when other term structure models are applied. Additionally, the results of variants of the Cox, Ingersoll and Ross interest rate risk parameters (mean reversion speed and interest rate volatility) cannot directly be compared to the Hull and White results since a variation in the risk parameters always affects today's term structure in the Cox, Ingersoll and Ross model.

4 Conclusion and outlook

In this paper we provided a structural model in order to analyze the value of deposit insurance, depending simultaneously on credit risk, the maturity gap and the deposit rate policy. Under the real world measure, it is an extension of the well-known conditional-independent 1-factor model that serves as the theoretical foundation of the Basel II-risk weight functions. We have shown that the correlation within the loan portfolio and the bank's maturity gap can have a material impact on fair deposit insurance premiums. Furthermore, we find that fair deposit insurance premiums can depend negatively on the borrowers' default risk when the bank faces a maturity gap. The deposit insurance premium for a bank holding defaultable loans can even be lower than for a bank investing in default-free bonds. Consequently, in these cases a higher recovery rate or, alternatively, better collaterals causes higher deposit insurance premiums. The economic intuition behind these results is that in certain circumstances interest rate risk, i.e. a maturity gap, can diversify changes in the credit quality of the loan portfolio in a way that it is even less risky than a default-free bond. The influence of this effect increases with lower correlation within the loan portfolio.

[Discuss policy implications for deposit insurance agencies here]

[Discuss policy implications for banking supervision related to the joint treatment of credit risk, interest rate risk and other sources of bank risk such as commission risk, the future framework "Basel III" and Basel Committee on Banking Supervision (2004)]

For practical and empirical purposes, our approach can easily be extended to more realistic assumptions such as inhomogeneous loan portfolios, more advanced affine (e.g., Dai and Singleton, 2000) or essentially-affine (Duffee, 2002) term structure models and early-default structural models (e.g., Longstaff and Schwartz, 1995, Briys and de Varenne, 1997), respectively. Additionally, loans that are not priced fairly can be considered. The parameters can be chosen to be consistent with information of regulators or deposit insurance agencies about the bank's loan portfolio quality and maturity gap. However, (semi-)closed form solutions cannot be expected to exist in those settings.

Another challenging topic for future empirical research is the combination of our approach with the market-data-based empirical work of the papers mentioned in Section 1. This can be done by adding another geometric Brownian motion to the loan portfolio that reflects both other bank assets and the value of the remaining future income. Major risk parameters can then be inferred from both market data, i.e. equity prices, and accounting and regulatory data related to borrowers' default risk and the bank's maturity gap.

References

Allen, Linda; Saunders, Anthony (1993): Forbearance and valuation of deposit insurance as a callable put. Journal of Banking and Finance 17, 629-643.

Anderson, Ronald W.; Cakici, Nusret (1999): The Value of Deposit Insurance in the Presence of Interest Rate and Credit Risk. Financial Markets, Institutions & Instruments 8(5), 45-62.

Basel Committee on Banking Supervision (2004): Principles for the Management and Supervision of Interest Rate Risk. Bank for International Settlement, Basel.

Brigo, Damiano; Mercurio, Fabio (2001): Interest rate models: theory and practice. Springer, Berlin.

Briys, Eric; de Varenne, François (1997): Valuing risky fixed rate debt: an extension. Journal of Financial and Quantitative Analysis 32, 239–248.

Cox, John C.; Ingersoll, Jonathan E.; Ross, Stephen A. (1985): A Theory of the Term Structure of Interest Rates. Econometrica, Vol. 53, 385-407.

Crouhy, Michel; Galai, Dan (1991): A contingent claim analysis of a regulated depository institution. Journal of Banking and Finance 15, 73-90.

Crouhy, Michel; Galai, Dan; Mark, Robert (2000): A comparative analysis of current credit risk models. Journal of Banking and Finance 24, 59-117.

Dai, Qiang; Singleton, Kenneth J. (2000): Specification of Affine Term Structure Models. Journal of Finance 55, 1943-1978.

Duan, Jin-Chuan; Moreau, Arthur F.; Sealey, C. W. (1995): Deposit insurance and bank interest rate risk: Pricing and regulatory implications. Journal of Banking and Finance 19, 1091-1108.

Duffee, Gregory R. (2002): Term Premia and Interest Rate Forecasts in Affine Models. Journal of Finance 57, 405-443.

Falkenheim, Michael; Pennacchi, George (2003): The Cost of Deposit Insurance for Privately Held Banks: A Market Comparable Approach. Journal of Financial Services Research 24, 121-148.

Frey, Rüdiger; Sommer, Daniel (1998): The Generalization of the Geske-Formula for Compound Options to Stochastic Interest Rates is not trivial – A Note. Journal of Applied Probability 35, 501-509.

Geman, Helyette; El Karoui, Nicole; Rochet, Jean-Charles (1995): Changes of Numéraire, Changes of Probability Measure and Option Pricing. Journal of Applied Probability 32, 443-458.

Gordy, Michael B. (2003): A risk-factor model foundation for ratings-based bank capital rules. Journal of Financial Intermediation 12, 199-232.

Grundke, Peter (2004): Integrating Interest Rate Risk in Credit Portfolio Models. Journal of Risk Finance, 5(2), 6-15.

Harrison, J. Michael; Pliska, Stanley, R. (1981): Martingales and Stochastic Integrals in the Theory of Continuous Trading. Stochastic Processes and Their Applications 11, 215-260.

Heath, David; Jarrow, Robert; Morton, Andrew (1992): Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claim Valuation. Econometrica 60, 77-105.

Huang, Jing-Zhi; Huang, Ming (2003): How much of the Corporate Credit Yield Spread is due to Credit Risk? Working Paper, Penn State University, New York University and Stanford University.

Hull, John; White, Alan (1990): Pricing Interest-Rate-Derivative Securities. Review of Financial Studies 3, 573-592.

Hull, John; White, Alan (1993): One-Factor Interest-Rate Models and the Valuation of Interest-Rate Derivative Securities. Journal of Financial and Quantitative Analysis 28, 235-254.

Hutchison; David E.; Pennacchi, George G. (1996): Measuring Rents and Interest Rate Risk in Imperfect Financial Markets: The Case of Retail Bank Deposits. Journal of Financial and Quantitative Analysis 31, 399-417.

Jarrow, Robert A.; van Deventer, Donald R. (1998): The arbitrage-free valuation and hedging of demand deposits and credit card loans. Journal of Banking and Finance 22, 249-272.

Johnson, Herb; Stulz, René (1987): The Pricing of Options with Default Risk. Journal of Finance 42, 267-280.

Klein, Peter (1996): Pricing Black-Scholes options with correlated credit risk. Journal of Banking and Finance 20, 1211-1229.

Longstaff, Francis A.; Schwartz, Eduardo S. (1995): A Simple Approach to Valuing Risky Fixed and Floating Debt. Journal of Finance 50, 789-819.

Longstaff, Francis A.; Schwartz, Eduardo S. (2001): Valuing American Options by Simulation: A Simple Least Squares Approach, Review of Financial Studies 14, 113-147.

Marcus, Alan J.; Shaked, Israel (1984): The Valuation of FDIC Deposit Insurance Using Option-pricing Estimates. Journal of Money, Credit and Banking 16, 446-460.

Merton, Robert C. (1974): On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. Journal of Finance 29, 449-470.

Merton, Robert C. (1977): An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees: An Application of Modern Option Pricing Theory. Journal of Banking and Finance 1, 3-11.

Nelson, Charles R.; Siegel, Andrew F. (1987): Parsimonious Modeling of Yield Curves. Journal of Business 60, 473-489.

Pennacchi, George (1987): A Reexamination of the Over- (or Under-) Pricing of Deposit Insurance. Journal of Money, Credit and Banking 19, 340-360.

Ronn, Ehud I.; Verma, Avinash K. (1986): Pricing Risk-Adjusted Deposit Insurance: An Option-Based Model. Journal of Finance 61, 871-895.

Rubinstein, Mark (1983): Displaced diffusion option pricing. Journal of Finance 38, 213-217.

Schönbucher, Philipp J. (2000): Factor models for portfolio credit risk. Working Paper, Bonn University.

Uhrig-Homburg, Marliese (2005): Cash-flow shortage as an endogenous bankruptcy reason. Journal of Banking and Finance 29, 1509-1534.

Vasicek, Oldrich (1977): An equilibrium characterization of the term structure. Journal of Financial Economics 5, 177-188.

Vasicek, Oldrich (1991): Limiting Loan Loss Probability Distribution. KMV Corporation, Document Number: 999-0000-046, Revision 1.0.0.

Wilde, Tom (2001): The IRB approach explained. Risk Magazine 14(5), 87-90.

Appendix

Lemma 1: Let (Ω, F, P) be a probability space, $\boldsymbol{X}$ an m-dimensional vector of random variables, and Y_i, $i = 1, 2, \ldots$ be iid random variables that are independent of $\boldsymbol{X}$. Let f be a real-valued function on R^{m+1} with $E(f(\boldsymbol{X}, Y_1)^2) = c < \infty$. Then we have

$$\lim_n n^{-1} \sum_{i=1}^{n} f(\boldsymbol{X}, Y_i) = E(f(\boldsymbol{X}, Y_1)|\boldsymbol{X}) \text{ in } L^2.$$

Proof The conditional expectations $E(f(\boldsymbol{X}, Y_i) \mid \boldsymbol{X})$ are iid. Hence, we have

$$E((n^{-1} \sum_{i=1}^{n} f(\boldsymbol{X}, Y_i) - E(f(\boldsymbol{X}, Y_1)|\boldsymbol{X}))^2)$$

$$= E(E((n^{-1} \sum_{i=1}^{n} f(\boldsymbol{X}, Y_i) - n^{-1} \sum_{i=1}^{n} E(f(\boldsymbol{X}, Y_i)|\boldsymbol{X}))^2|\boldsymbol{X}))$$

$$= E(n^{-2} E((\sum_{i=1}^{n} (f(\boldsymbol{X}, Y_i) - E(f(\boldsymbol{X}, Y_i)|\boldsymbol{X})))^2|\boldsymbol{X}))$$

$$= E(n^{-2} \mathrm{Var}(\sum_{i=1}^{n} f(\boldsymbol{X}, Y_i) |\boldsymbol{X})) = E(n^{-1}\mathrm{Var}(f(\boldsymbol{X}, Y_1) |\boldsymbol{X}))$$

$$\leq E(n^{-1}E(f(\boldsymbol{X}, Y_1)^2 |\boldsymbol{X})) = n^{-1} c$$

□

Tables

Table 1: Specification of spot rate curves

	Abbreviation	β_0	β_1	β_2	β_3
low down	ld	0.01	0.02	0	2
low flat	lf	0.03	0	0	2
low up	lu	0.05	-0.02	0	2
medium down	md	0.03	0.02	0	2
medium flat	mf	0.05	0	0	2
medium up	mu	0.07	-0.02	0	2
high down	hd	0.05	0.02	0	2
high flat	hf	0.07	0	0	2
high up	hu	0.09	-0.02	0	2

This table shows different parameterizations of today's annually compounded spot rate curve $R_{ac}(0, \cdot)$. They are illustrated by Figure 2. The beta-parameters refer to the Nelson and Siegel (1987)-parameterization: $R_{ac}(0, T) = \beta_0 + (\beta_1 + \beta_2)\,(1 - \exp(-T / \beta_3)) / (T / \beta_3) - \beta_2 \exp(-T / \beta_3)$.

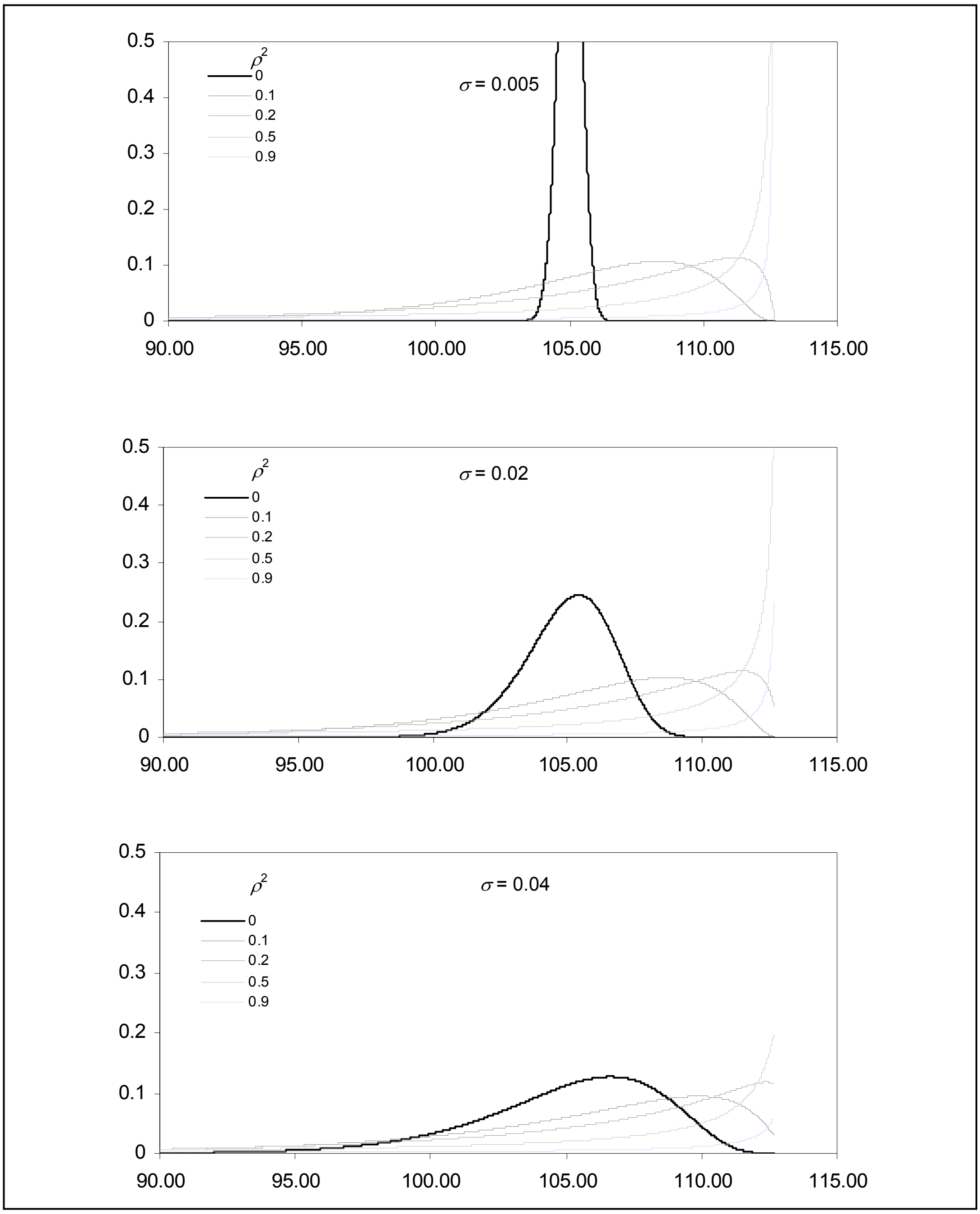
Table 2: Exemplary scenario

Interest Rate Term Structure				
Today's term structure (mf)	β_0	β_1	β_2	β_3
	0.05	0	0	2
Risk parameters				
Short rate volatility	σ	0.02		
Mean reversion speed	a	0.1		
Loans				
Maturity loan	T	4		
Recovery rate	δ	0		
Asset value borrowing firm(s)	$V(0)$	10		
Asset volatility borrowing firm(s)	η	0.1		
Default Point borrowing firm(s)	D	8		
Instantaneous asset correlation	ρ^2	0.2		
Instantaneous correlation asset value / short rate	θ	0		
Bank				
Value loan portfolio	$A(0)$	100		
Face value deposits	FV_{DP}	95		
Basic deposit rate	b_1	0		
Deposit rate elasticity	b_2	1		

This table reports the exemplary set of parameters for our analysis. If not otherwise specified, later numerical results refer to these parameters.

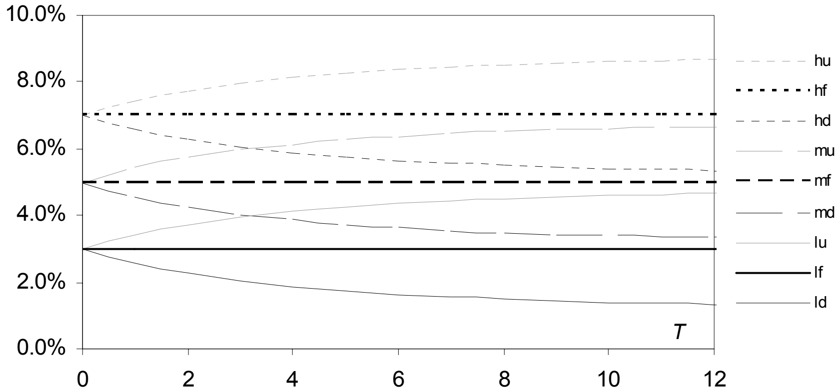
Figures

Figure 1: No maturity gap: densities of the value of the loan portfolio in $S = 1$ under the S-forward measure for different asset correlations and different short rate volatilities



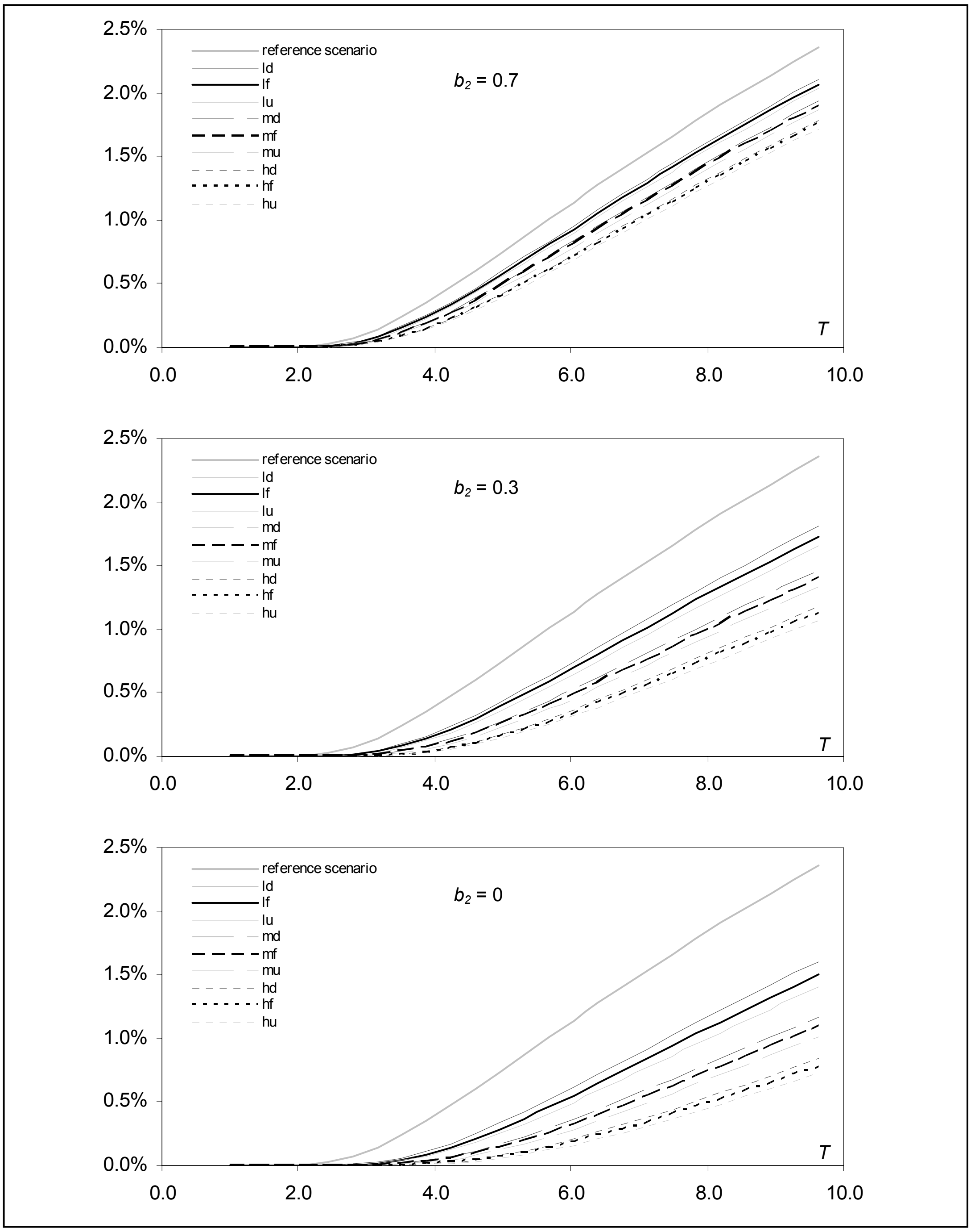
This figure shows densities of the value of the loan portfolio in $S = 1$ under the S-forward measure for different asset correlations ρ^2 and different short rate volatilities σ when the maturity gap is zero. The value of the loan portfolio is given by (41). The default point D of the firms is set to 9. For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 2: Spot rate curves



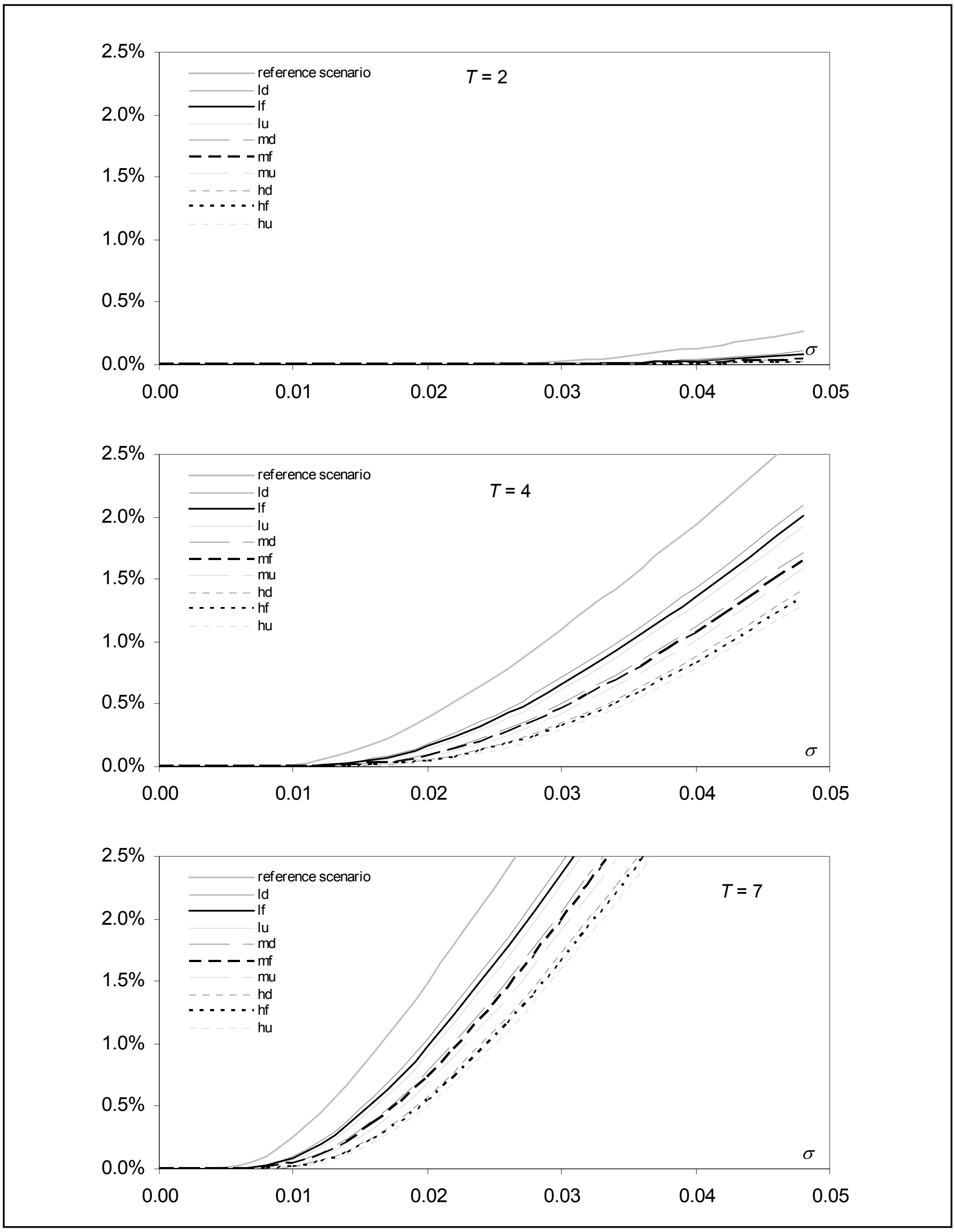
This figure illustrates the spot rates curves $R_{ac}(0, T)$ parameterized in Table 1.

Figure 3: Default-free loans: deposit insurance premium for different term structures and different deposit rate elasticities in dependence on the loans' maturity



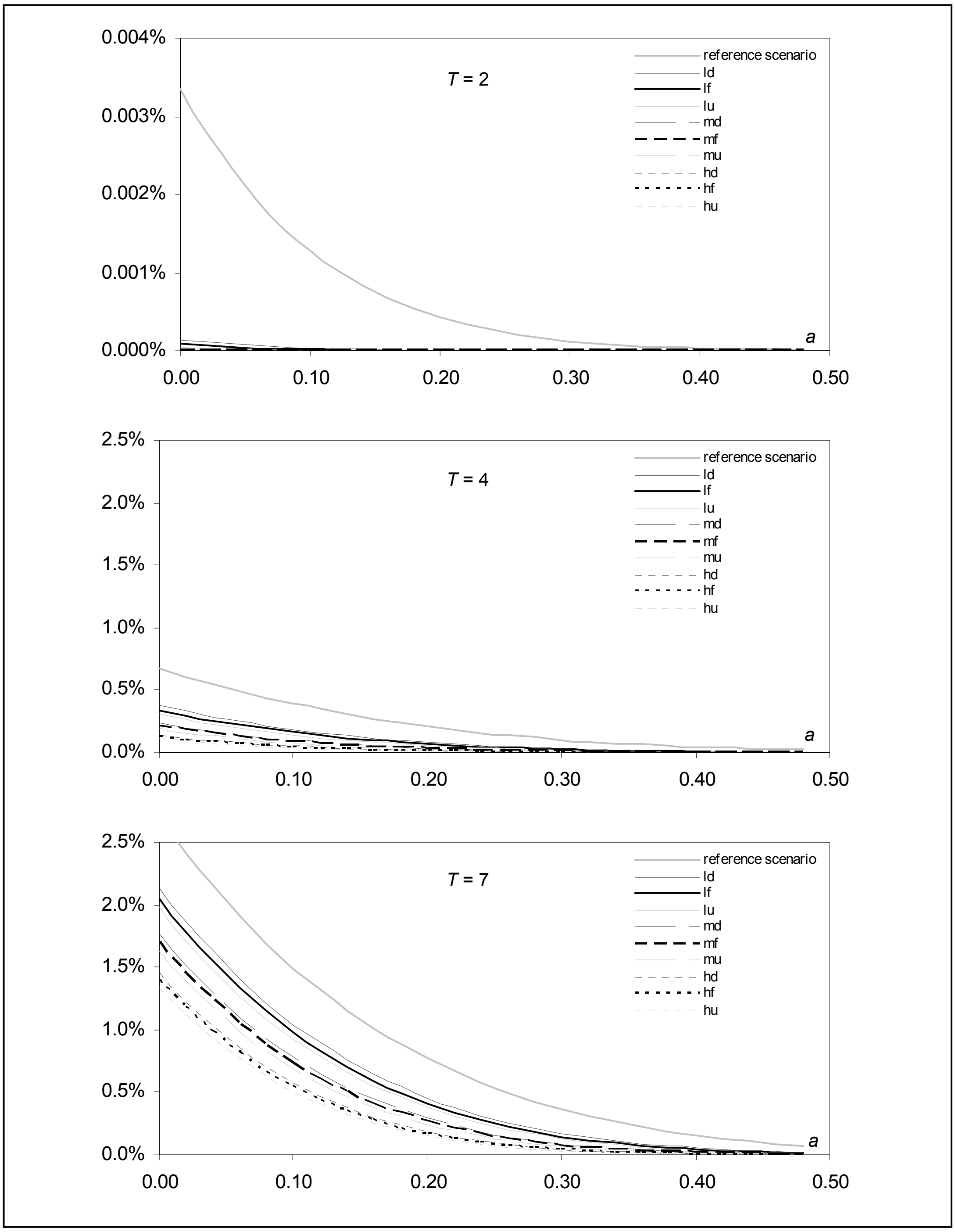
This figure shows the deposit insurance premium for different term structures and different deposit rate elasticities in dependence on the loans' maturity when loans are default-free. The reference scenario refers to a deposit rate elasticity of $b_2 = 1$. The abbreviations "ld" ... "hu" refer to different term structures specified by Table 1. The deposit insurance premium is calculated by (38) divided by the face value of deposits 95. For every parameter set the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 4: Default-free loans: deposit insurance premium for different term structures and different maturities of the loans in dependence on the short rate volatility



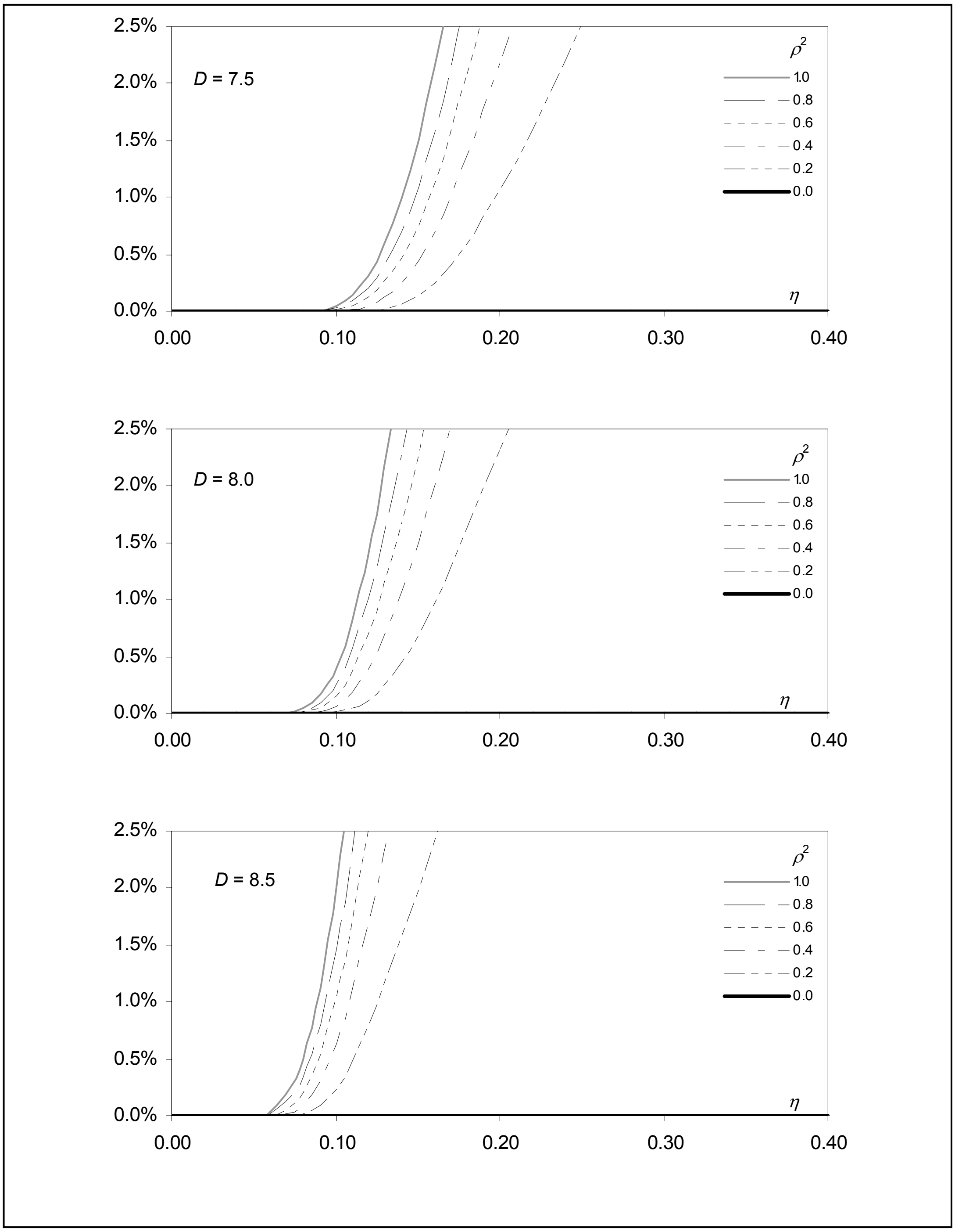
This figure shows the deposit insurance premium for different term structures and different maturities of the loans in dependence on the short rate volatility when loans are default-free. The reference scenario refers to a deposit rate elasticity of $b_2 = 1$. The abbreviations "ld" ... "hu" refer to different term structures specified by Table 1. When these term structures are applied, the deposit rate elasticity is set to $b_2 = 0.3$. The deposit insurance premium is calculated by (38) divided by the face value of deposits 95. For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 5: Default-free loans: deposit insurance premium for different term structures and different maturities of the loans in dependence on the mean reversion speed



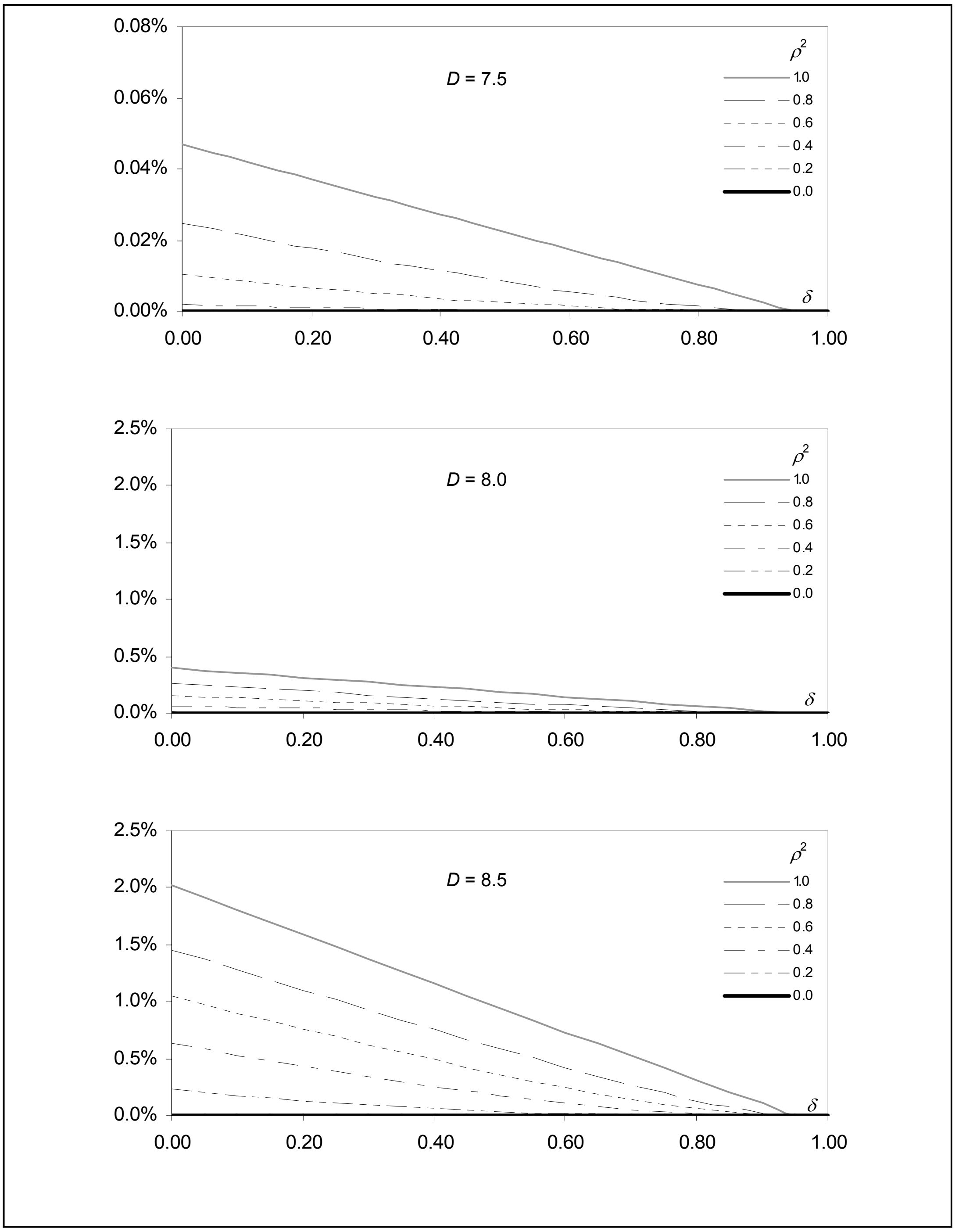
This figure shows the deposit insurance premium for different term structures and different maturities of the loans in dependence on the mean reversion speed when loans are default-free. The reference scenario refers to a deposit rate elasticity of $b_2 = 1$. The abbreviations “ld” ... “hu” refer to different term structures specified by Table 1. When these term structures are applied the deposit rate elasticity is set to $b_2 = 0.3$. The deposit insurance premium is calculated by (38) divided by the face value of deposits 95. For every parameter set, the face value of the loan portfolio is adjusted to yield a today’s loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 6: No maturity gap: deposit insurance premium for different asset correlations and different default points in dependence on the asset volatility



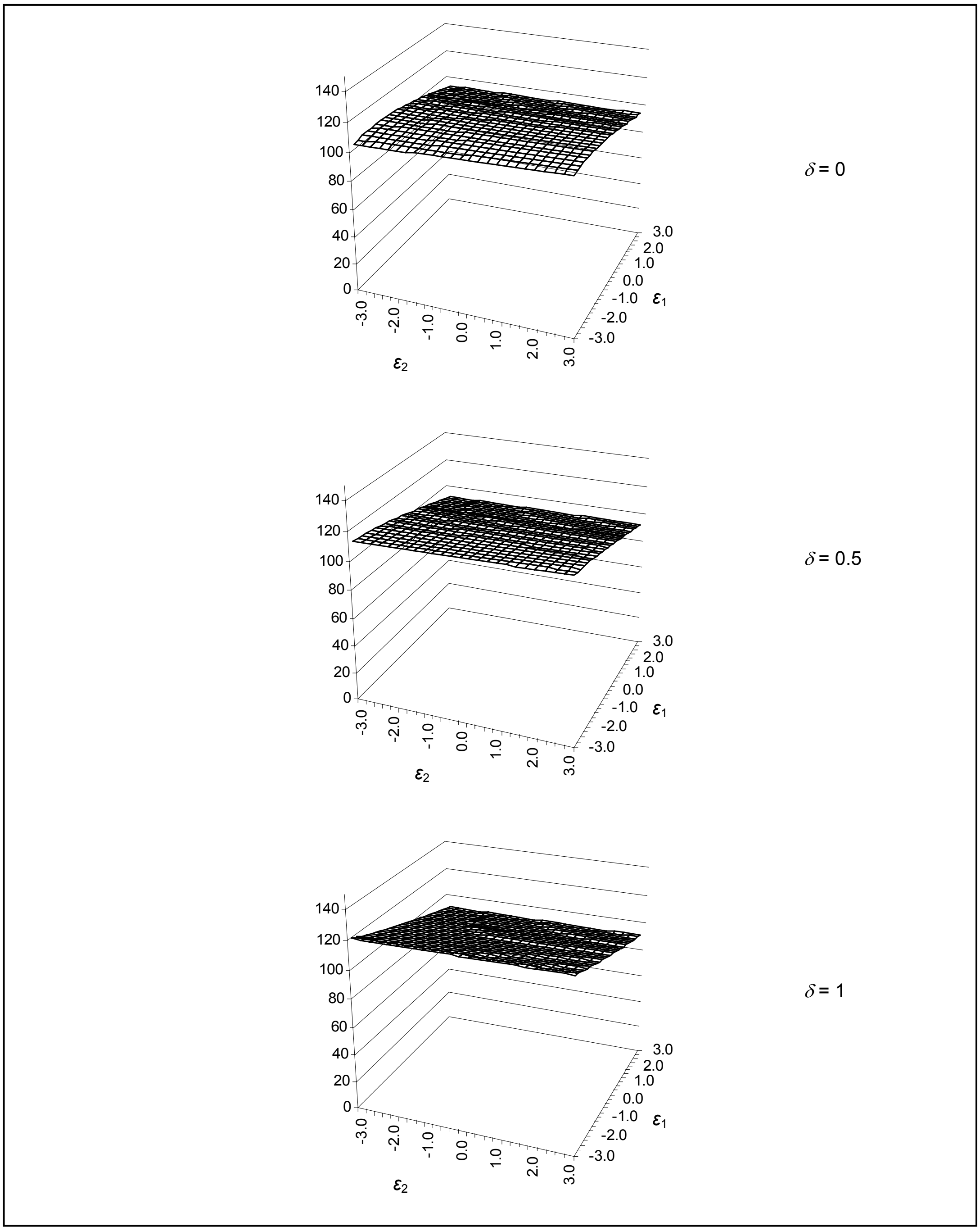
This figure shows the deposit insurance premium for different asset correlations and different default points in dependence on the asset volatility when the maturity gap is zero. The deposit insurance premium is calculated by (41) and (9) divided by the face value of deposits 95. For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 7: No maturity gap: deposit insurance premium for different asset correlations and different default points in dependence on the recovery rate



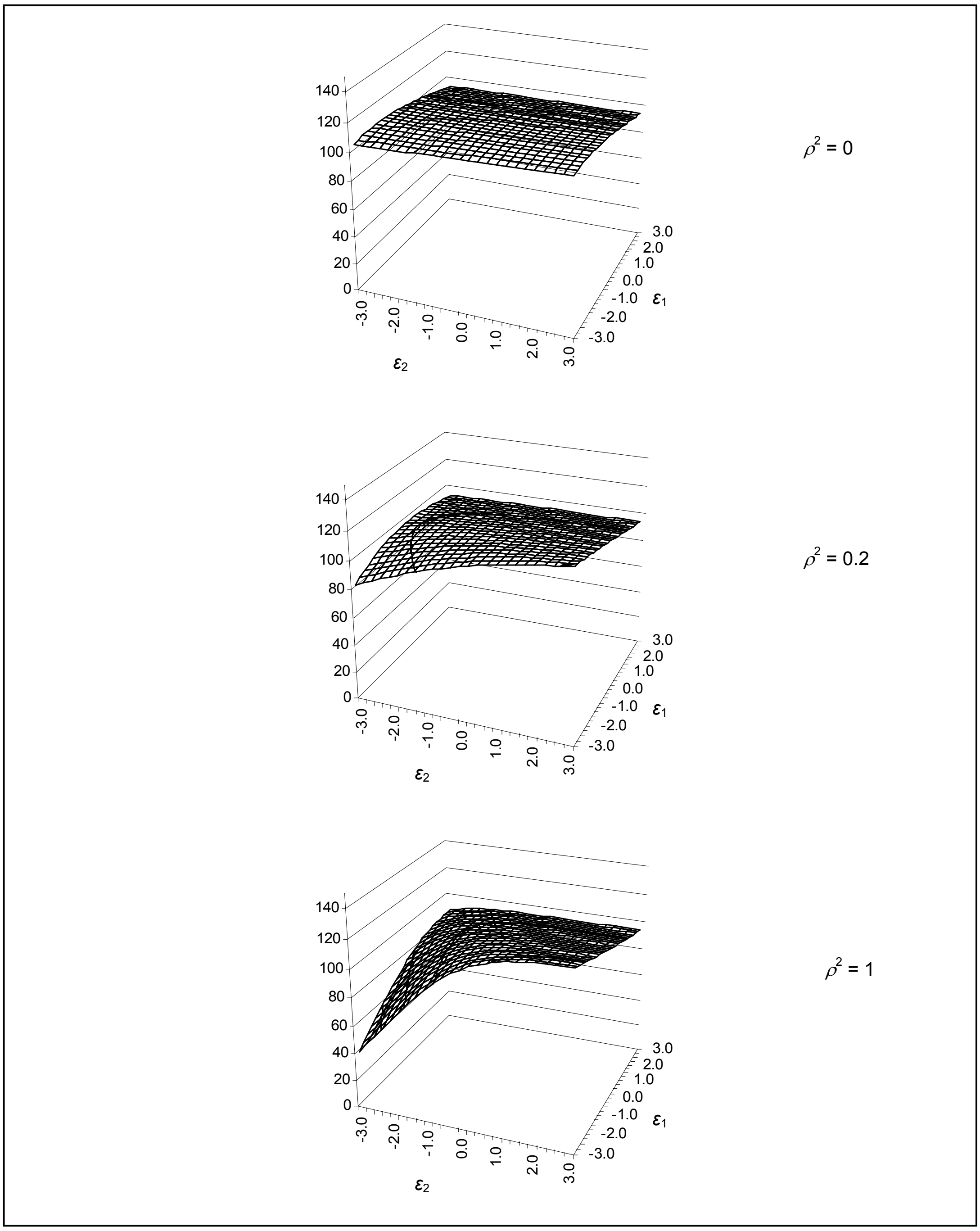
This figure shows the deposit insurance premium for different asset correlations and different default points in dependence on the recovery rate when the maturity gap is zero. The deposit insurance premium is calculated by (41) and (9) divided by the face value of deposits 95. For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 8: General case: value of the loan portfolio in S = 1 under the S-forward measure for different recovery rates in dependence on the realizations of the systematic variables ε_1 and ε_2



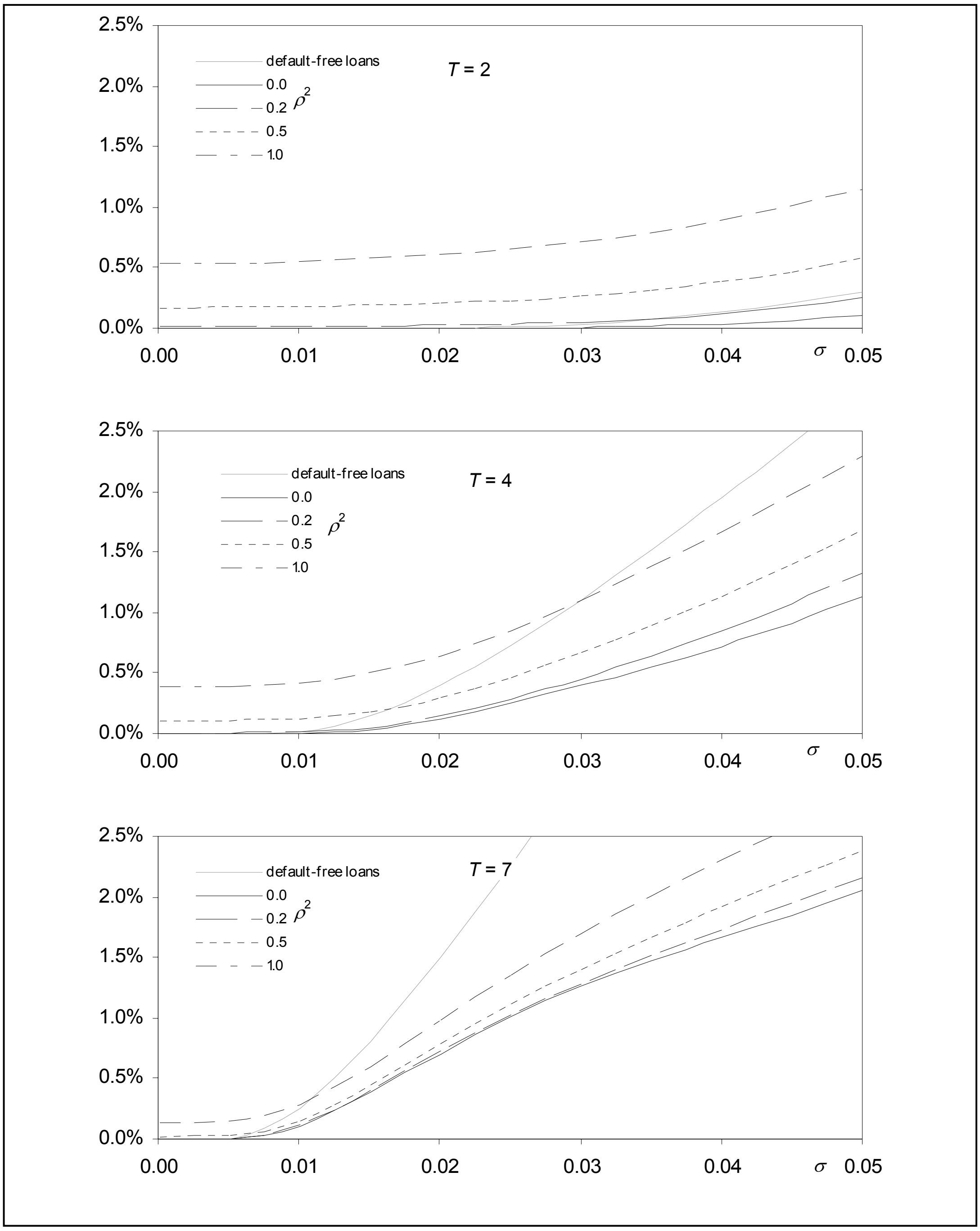
This figure shows the value of the loan portfolio in S = 1 under the S-forward measure for different recovery rates in dependence on the realizations of the systematic variables ε_1 and ε_2 in the general case according to (36). For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). The asset correlation is set to $\rho^2 = 0$. If not otherwise specified, input parameters are given by Table 2.

Figure 9: General case: value of the loan portfolio in S = 1 under the S-forward measure for different asset correlations in dependence on the realizations of the systematic variables ε_1 and ε_2



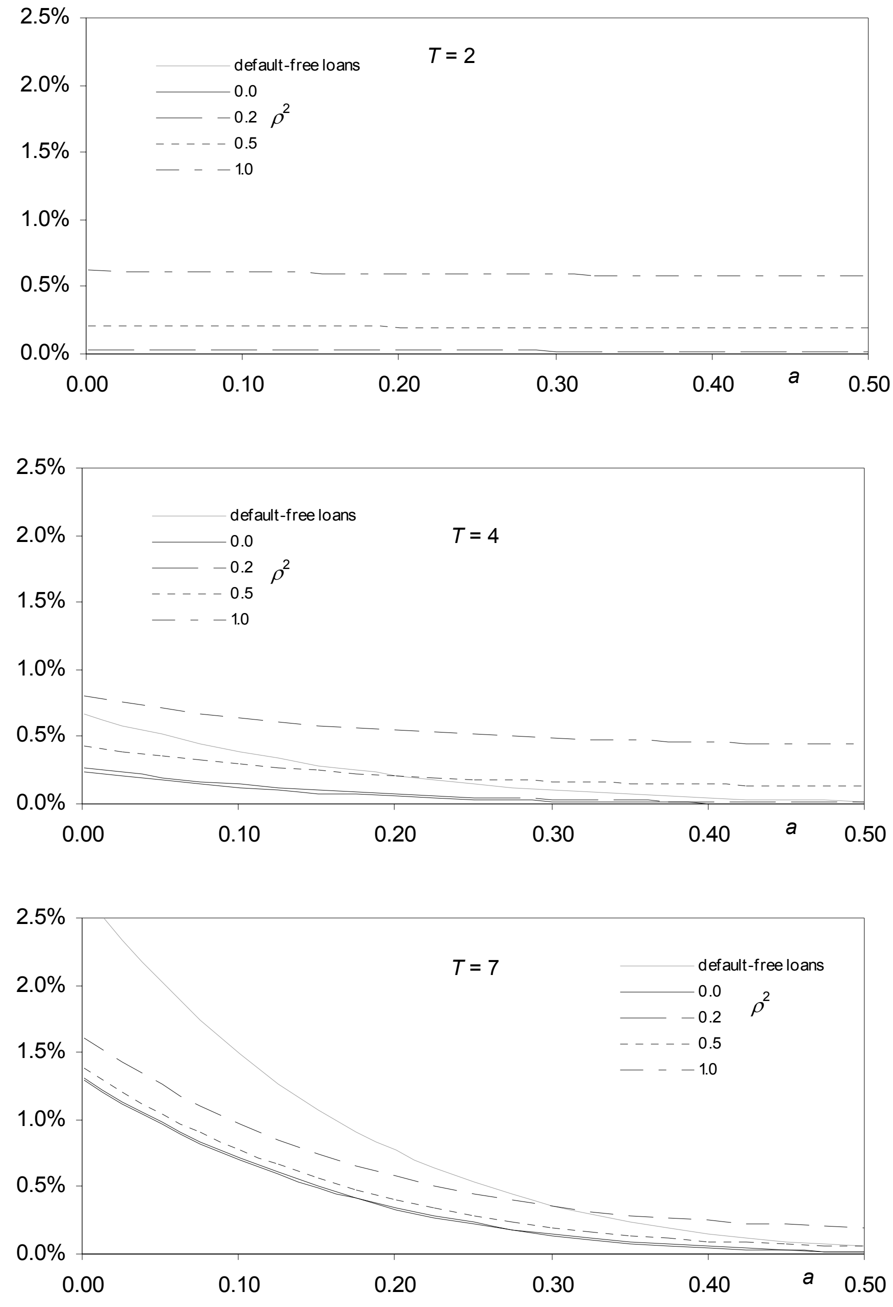
This figure shows the value of the loan portfolio in S = 1 under the S-forward measure for different asset correlations in dependence on the realizations of the systematic variables ε_1 and ε_2 in the general case according to (36). For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 10: General case: deposit insurance premium for different asset correlations and maturity gaps in dependence on the short rate volatility



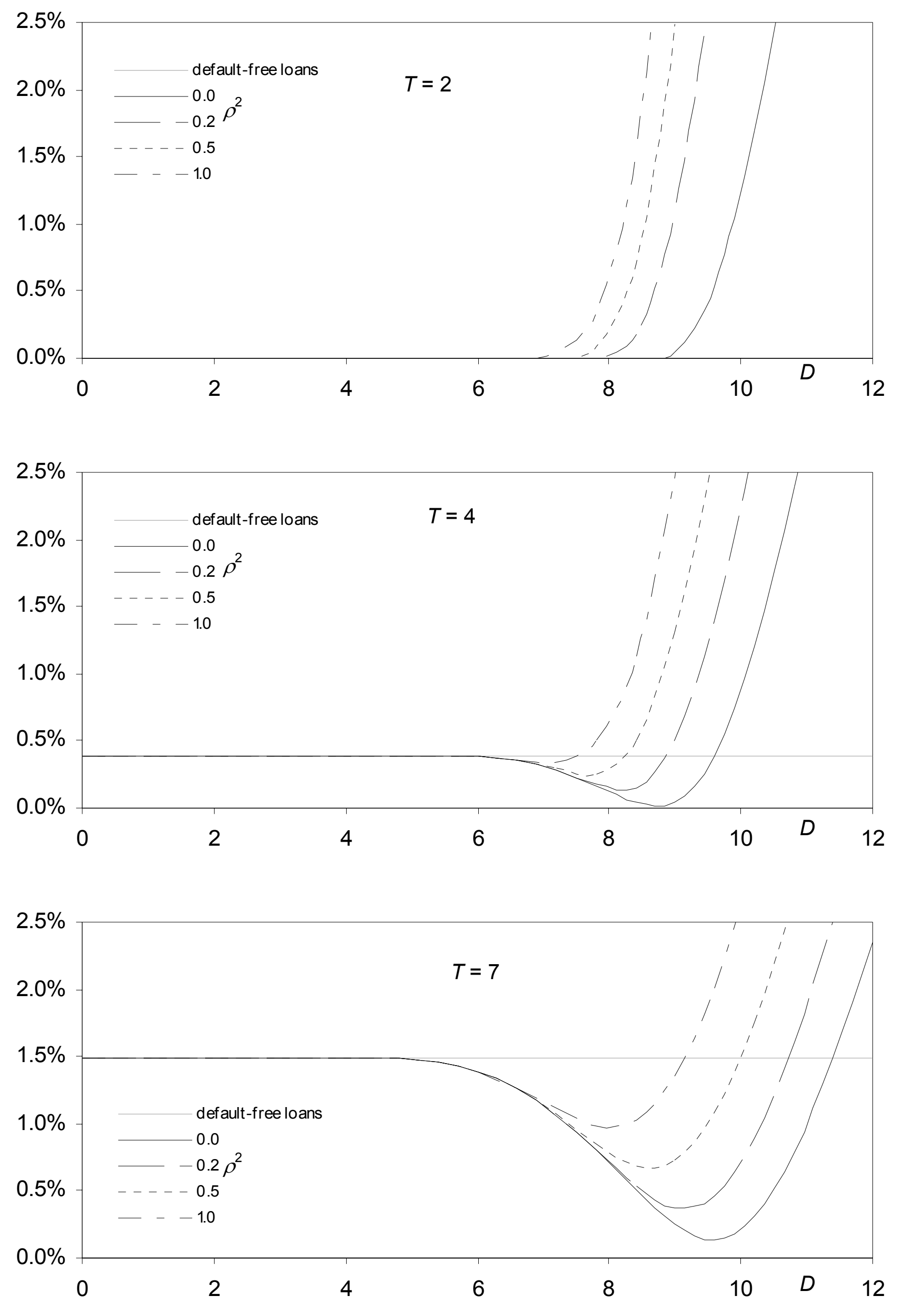
This figure shows the deposit insurance premium for different asset correlations and maturity gaps in dependence on the short rate volatility. Additionally, the respective value for default-free loans is shown. The deposit insurance premium is calculated by (37) and (38), respectively, divided by the face value of deposits 95. For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 11: General case: deposit insurance premium for different asset correlations and maturity gaps in dependence on the mean reversion speed



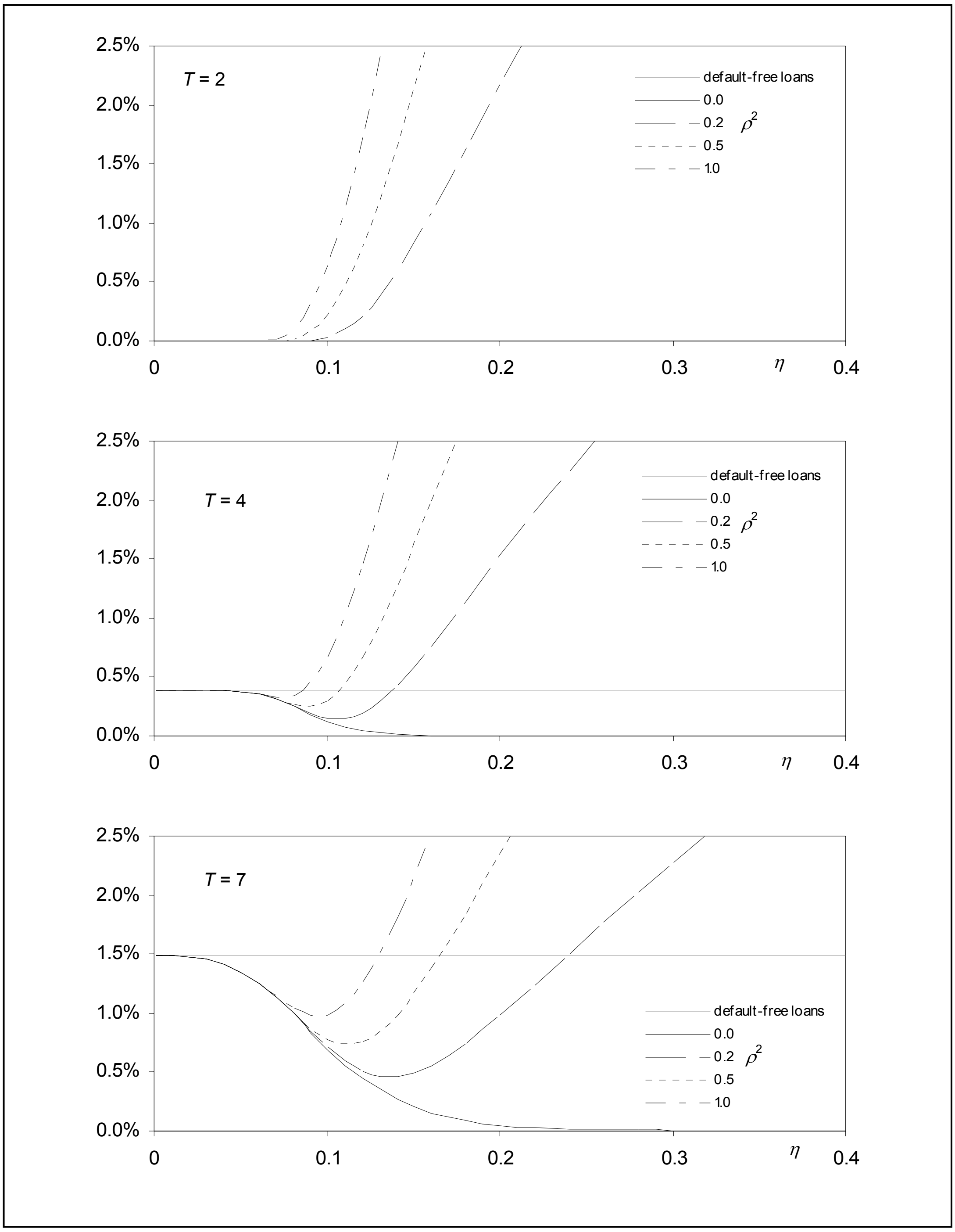
This figure shows the deposit insurance premium for different asset correlations and maturity gaps in dependence on the mean reversion speed. Additionally, the respective value for default-free loans is shown. The deposit insurance premium is calculated by (37) and (38), respectively, divided by the face value of deposits 95. For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 12: General case: deposit insurance premium for different asset correlations and maturity gaps in dependence on the default point



This figure shows the deposit insurance premium for different asset correlations and maturity gaps in dependence on the default point. Additionally, the respective value for default-free loans is shown. The deposit insurance premium is calculated by (37) and (38), respectively, divided by the face value of deposits 95. For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.

Figure 13: General case: deposit insurance premium for different asset correlations and maturity gaps in dependence on the asset volatility



This figure shows the deposit insurance premium for different asset correlations and maturity gaps in dependence on the asset volatility. Additionally, the respective value for default-free loans is shown. The deposit insurance premium is calculated by (37) and (38), respectively, divided by the face value of deposits 95. For every parameter set, the face value of the loan portfolio is adjusted to yield a today's loan portfolio value of 100, according to (31). If not otherwise specified, input parameters are given by Table 2.