Mutual versus Stock Insurers: Fair Premium, Capital, and Solvency*

Christian Laux† — Goethe University Frankfurt and CFS

Alexander Muermann‡ — University of Pennsylvania

October 2006

Abstract

Mutual insurance companies and stock insurance companies are different forms of organized risk sharing: policyholders and owners are two distinct groups in a stock insurer, while they are one and the same in a mutual. This distinction is relevant to raising capital, selling policies, and sharing risk in the presence of financial distress. Up-front capital is necessary for a stock insurer to offer insurance at a fair premium, but not for a mutual. In the presence of an owner-manager conflict, holding capital is costly. Free-rider and commitment problems limit the degree of capitalization that a stock insurer can obtain. The mutual form, by tying sales of policies to the provision of capital, can overcome these problems at the potential cost of less diversified owners.

Key Words ownership structure, insurance, owner-manager conflict, capital, default

JEL Classification G22, G32

*The authors wish to thank Neil Doherty, Scott Harrington, Paul Kofman, Harris Schlesinger, and Stephen Shore for their valuable comments.

†Goethe University Frankfurt, Mertonstr. 17, 60325 Frankfurt, Germany, email: laux@finance.uni-frankfurt.de

‡The Wharton School of the University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104-6302, USA, email: muermann@wharton.upenn.edu

1 Introduction

An insurance corporation "organizes" risk sharing between individuals, and in principle, there are two different ways to do this. First, risks can be shared within the pool of policyholders. A prominent example is a mutual insurer where policyholders are also the owners of the insurance corporation. In this case, policyholders have participating contracts as all participate in the insurer's surplus. Second, risks can be transferred from policyholders to another group of individuals (investors). Stock insurers, for example, transfer risks from policyholders to shareholders, i.e. the capital market. This transfer is achieved through the separation of rights to profits and rights to indemnity claims, thereby decoupling owners and customers (policyholders).

A large body of the insurance literature assumes that the level of aggregate future claims payment is certain. In this case, an actuarially fair premium equals the expected indemnity payment, and no capital beyond premiums is needed. Moreover, there is no insolvency and, with full insurance, the actuarially fair premium equals the expected loss. This implies that there is no difference between a mutual insurer and a stock insurer with respect to the required capital, the fair premium, and the distribution of risk. In this context, it is irrelevant whether rights to profits and rights to indemnity claims or, equivalently, owners and customers, are separated or not.

A different picture emerges if the level of total claims payment is uncertain. Then, the average policyholder's claim can be higher or lower than the insurer's total capital, which includes risk capital (equity) and premiums. If the policyholders' claims are higher than the total capital, the company is insolvent and total capital is distributed among policyholders. If claims are lower, the excess funds accrue to the owners of the insurer. Risk capital is important to reallocate funds from states where total claims are lower than total premiums to states where the reverse holds. The way that risk capital is raised within these two organizational forms is markedly different. A stock insurer first raises risk capital from investors (shareholders) and then sells insurance policies; while a mutual insurer raises risk capital through premiums (raising risk capital is tied to selling insurance contracts).[1] We show that these different forms of raising capital have different consequences in the presence of financial distress, in particular with respect to the actuarially fair premium, the degree of capitalization, and managerial incentives to sell insurance policies.

An individual who buys an insurance policy from a mutual insurer also receives an ownership right in the insurer. The premium net of the expected value of the ownership right equals the expected claims payment. Hence, the insurance premium is always actuarially fair, independent of the premium level paid by policyholders.[2] However, a stock insurer needs risk capital provided by shareholders in order to offer insurance at an actuarially fair premium. To illustrate this point, imagine that no risk capital is available and that policyholders pay some positive premium. If claims are lower than premiums, the stock insurer is solvent and excess funds accrue to shareholders. If claims are higher than premiums, the insurer is insolvent and the total payment to policyholders equals the collected premiums. Hence, given the limited liability of shareholders and zero capital, the expected payment to policyholders is lower than the premium, which is therefore not actuarially fair.[3]

By providing risk capital, shareholders pay for the expected surplus they will make when claims are lower than collected premiums. There is a unique actuarially fair premium for each level of capital, which is below a policyholder's expected loss and increases in the amount of capital that shareholders provide. For a low (fixed) level of capital, it can be optimal for risk-averse policyholders if the policy exhibits loading for all policyholders. The reason is that by collectively providing additional funds, additional risk sharing is created when the insurer is insolvent. This is costly to policyholders as these funds accrue to shareholders when the insurer is solvent. Therefore, there is a potential free-rider problem, as it is individually rational not to provide additional capital. Loading

[1] We do not consider subordinate debt. As there is a positive probability of default, subordinate debt cannot replace equity and as both organizational forms can use subordinate debt at the same terms, this simplifying assumption does not affect our qualitative results.

[2] We define an actuarially fair premium to be the rate that is fair to policyholders, i.e. it is equal to the expected claims payment. If there are no frictional costs of capital then this premium also provides a fair return to owners. If there are frictional costs of capital, e.g. stemming from the owner-manager conflict considered in this paper, the premium providing a fair return to owners might exceed the actuarially fair premium.

[3] In this case, shareholders earn a strictly positive expected return at the expense of policyholders.

is a way to tie the raising of additional capital to selling insurance policies and thus overcome this free-rider problem. A mutual can then be viewed as an organizational form that avoids the problem of "additional funds" accruing to external investors.

This leads to our main contribution, which is to show how the mutual and the stock forms differ in their ability to raise capital. We assume that frictional costs of capital stem from a conflict between the manager and owners, and this conflict exists for both stock and mutual insurers. The manager is able to expropriate a fraction of the surplus, and the firm returns only the remaining surplus to owners (shareholders or policyholders). Thus, it costs the same to provide a given amount of capital either through issuing shares or through premiums. The main difference between the stock and the mutual forms is that it is possible under the mutual form to restrict sales of policies to those who also provide capital. Since competition in the insurance market restricts the return to shareholders, it imposes an upper bound on the level of reimbursement for the (sunk) frictional cost of capital. The higher the frictional cost of capital, the lower is the amount of capital that can be raised and the degree of risk sharing provided by the insurer. For policyholders it would be optimal to collectively provide additional capital, despite the frictional cost, to improve risk sharing. Thus, the increase in utility from improved risk sharing compensates policyholders for a negative (unfair) return on the invested capital. However, if buying insurance policies and providing capital are separated, each policyholder has an incentive to free-ride on the capital provided by the others and will not provide capital. For a mutual, the sales of policies and provision of capital are linked, and the free-rider problem is overcome.

If the manager can expropriate a fraction of the insurer's surplus, there is another interesting difference between the two organizational forms with respect to the manager's incentives to increase the number of policyholders. We show that the incentives are generally higher for the mutual form than for the stock form. The ex ante available capital in a stock insurer provides a cushion for the manager, a fraction of which he is able to consume. The manager of a mutual has to "earn" this cushion by selling insurance contracts.

Our paper contributes to an understanding of the differences between stock and mutual insurers.

The existing literature discusses two main differences: differences in risk bearing (participating versus non-participating contracts) and differences in governance (reducing the owner-customer conflict versus reducing the owner-manager conflict). Smith and Stutzer (1990, 1995) focus on the different contractual structures of insurance contracts offered by mutual insurers (participating contract) and stock insurers (indemnity payment with a flat fee). They argue that undiversifiable risk drives participating contracts and that these contracts reduce problems of adverse selection and moral hazard. Mayers and Smith (1981, 2005) focus on governance issues and argue that different organizational forms have different advantages in dealing with different types of agency problems. The stock form is better suited to reducing the owner-manager conflict through the market for corporate control, whereas the mutual form internalizes the owner-customer problem at the expense of a higher owner-manager conflict. In our model, both organizational forms face an identical owner-manager conflict: the manager can extract a fraction of the surplus. The distinction that is central to our discussion is how the separation and non-separation of owners and customers differentially impacts the raising of capital and selling of policies in the presence of this agency problem. Thus, while the focus of Mayers and Smith is on how different organizational forms can influence (increase or decrease) different types of agency problems, we take the agency problem as given for both forms and analyze the effects that it has on raising capital and providing insurance. Assuming that the owner-manager problem is lower for a stock insurer than for a mutual insurer, as suggested by Mayers and Smith, would not affect our qualitative results.

Doherty and Dionne (1993) argue that when external capital is costly, consumers will substitute by bearing risk themselves. Zanjani (2004) combines the arguments of Mayers and Smith (1981) and Doherty and Dionne (1993): when external capital is costly, the level of external capital is low so that the owner-manager conflict becomes less important and the mutual form is chosen to reduce the owner-customer conflict. In our setting, the cost of capital includes agency costs that apply equally to both capital provided by shareholders and premiums paid by policyholders. Therefore, external capital is not more costly than internal capital. Nevertheless, it may be optimal for policyholders to provide the capital and thus bear the risk themselves. The benefit of capital provided by policyholders is that the free-rider and commitment problems can be overcome when raising capital is tied to selling policies. A direct consequence of tying ownership rights to policies is that policyholders, instead of investors, have to bear the insurer's surplus risk.

Smith and Stutzer (1995) and Zanjani (2004 and forthcoming) find that mutual insurers are used more often in times of financial crises. Zanjani argues that one reason for this phenomenon is that new stock insurers are more capital-intensive than mutual firms and that the use of mutuals in times of capital market distress may represent a move away from external capital in production. This finding is consistent with our model, as external (risk) capital is required to reallocate funds between policyholders and owners. These funds are not required for mutual firms, as policyholders are also owners. Moreover, Viswanathan and Cummins (2003) find that recent demutualizations were motivated by access to capital. Given a capital market where there are many investors, a stock insurer can have a competitive advantage over a mutual through improved risk sharing.

We also contribute to the literature that analyzes the role of financial distress in insurance markets. Doherty and Schlesinger (1990) examine the demand for insurance under financial distress, and Mahul and Wright (2004a, 2004b) explore the optimal structure of insurance contracts for a mutual insurer with limited capital. This literature does not analyze the relation among capital, premiums, and financial distress. Instead, the authors fix the level of pre-paid premiums either directly or indirectly by fixing the probability of financial distress. Cagle and Harrington (1995) and Cummins and Danzon (1997) analyze insurance supply with capacity constraints and endogenous insolvency risk. Their approach is quite different from ours, as the authors focus on the effect of loss shocks on capitalization and premiums in insurance markets. Shareholders supply insurance based on maximizing the expected value of net cash flows under the assumption that capital is costly. The demand side is given by some exogenously specified demand curve, which is assumed to be negatively related to the price of insurance. In contrast to this literature, our paper focuses on the trade-off between price and quality of insurance, which is crucial for the distinction between a mutual and stock insurer in terms of the relation among capital, premiums, and risk sharing.

The pricing of insurance policies in the presence of insolvency risk is discussed by Doherty and

Garven (1986) and Gründl and Schmeiser (2002). Doherty and Garven were the first to propose a contingent-claims approach to deal with insolvency risk in pricing insurance contracts. They focus on price regulation in property-liability insurance. Gründl and Schmeiser compare different approaches for pricing double-trigger reinsurance contracts that are subject to default and also discuss the relation between risk capital and financial distress.

The paper is structured as follows. We present the model in Section 2 and examine the role of capital and actuarially fair premiums under the two different organizational forms in Section 3. In Section 4, we discuss the role of the corporate form in the presence of governance problems and managerial incentives to expand. We discuss the empirical predictions of our model in Section 5 and conclude in Section 6.

2 The Model

There are n identical, risk-averse individuals who maximize expected utility with respect to an increasing, concave utility function $u(\cdot)$. Each individual is endowed with initial wealth w_0 and faces a loss of size X_i, $i = 1, ..., n$. We assume that losses are independent and identically distributed according to a continuous distribution function F^1 with $F^1(0) = 0$ and density function f^1. The aggregate loss in the economy, $\sum_{i=1}^{n} X_i$, is then distributed according to the n-fold convolution $F^n = \left(F^1\right)^{*(n)}$ with density function f^n.

Risk sharing is organized through an insurance company, which can either be a stock insurer or a mutual insurer. The insurer is run by a risk-neutral manager.

Stock insurer We consider the following three stages of setting up a stock insurance company. At the first stage, the company raises risk capital C from shareholders. At the second stage, the manager sells insurance policies offering full coverage to the n individuals at a premium P per policy. At the third stage, losses are realized, and the total capital, $nP + C$, is distributed to policyholders and shareholders.

Mutual insurer A mutual insurance company is owned by its policyholders, who own the right to the insurer's surplus. The company does not raise capital from shareholders, but through selling policies at the second stage, i.e. the mutual has no capital other than the collected insurance premiums. Suppose that each policyholder pays a premium P^m for full coverage. At the third stage, losses are realized, and the company distributes the total capital, nP^m, amongst policyholders.

Policyholders' claims Let TC be the insurer's total capital, i.e. $nP + C$ in the case of a stock insurer and nP^m in the case of a mutual. The insurer is solvent if $\sum_{i=1}^{n} X_i \leq TC$, and insolvent otherwise. If the insurer is solvent, then policyholders are fully indemnified. If the insurer is insolvent, then the company declares bankruptcy and the total capital is split amongst policyholders according to some pre-specified bankruptcy rule, $I_i(X_1, ..., X_n)$, with $\sum_{i=1}^{n} I_i(X_1, ..., X_n) = TC$ and $E[I_i(X_1, ..., X_n)] = TC/n$ for all $i = 1, ..., n$. A pro-rata rule would be defined by[4]

$$I_i(X_1, ..., X_n) = \frac{X_i}{\sum_{i=1}^{n} X_i} \cdot TC.$$

Owners' claims The owners have a claim to the excess funds, i.e. total capital net of total claims payments, $TC - \sum_{i=1}^{n} X_i$, if the insurer is solvent. If the insurer is insolvent, the owners are protected by limited liability. We refer to the excess funds as surplus and introduce an owner-manager conflict by assuming that only the fraction $(1 - \alpha)$ of the surplus is returned to owners, where $\alpha \in [0, 1]$. That is, owners receive $(1 - \alpha)(TC - \sum_{i=1}^{n} X_i)$.

α can be interpreted as a measure of the severeness of the owner-manager conflict. For example, managers may be able to expropriate a fraction of the firm's surplus in the form of outright fraud, consumption of perks, or investment of some of the funds in projects with a negative net present value. This frictional cost of capital does not represent an exogenous bias for or against providing capital through either corporate form. While differences in the governance mechanisms certainly

[4]Such a sharing rule, where policyholders receive a share of the insurer's assets that is proportional to their claim, is assumed in much of the literature (see, e.g., Cummins and Danzon, 1997, who confirm that "this liquidation rule is consistent with the way insurance bankruptcies are handled in practice," footnote 22).

can result in shifts of the relative importance of different types of agency conflicts, the purpose of our paper is to highlight differences in organizational forms that are above and beyond direct differences in the relative importance of agency problems.

Another potentially important distinction is how well diversified owners are under each corporate form. Shareholders bear the surplus risk of a stock corporation while the pool of policyholders bears the surplus risk of a mutual. Depending on which group is larger and better diversified, risk sharing may be better for a mutual or for a stock insurer. For example, in the presence of high frictional costs in the capital market, a relatively small number of shareholders might hold a large fraction of the shares in an insurer. To the extent that shareholders are then not well diversified, they demand a risk premium for bearing surplus risk. We assume that shareholders in a stock insurer are well diversified and that the surplus risk is unsystematic risk. Shareholders are therefore risk-neutral with respect to the surplus risk. To the extent that this is also true for the policyholders in the mutual, they are also risk-neutral with respect to the wealth generated from the surplus.

The distinctions that are central to our discussion are the separation versus non-separation of owners and customers as well as the sequential versus simultaneous way of raising capital and selling policies given positive probability of financial distress.[5]

3 Actuarially Fair Premium and Financial Distress

In this section, we examine the role of capital provided by owners under the two organizational forms focusing on the companies' ability to offer an actuarially fair premium. We show that it can be optimal for policyholders in a stock insurer to raise additional funds through a loading in order to improve risk sharing. To highlight this effect, in this section we assume that there is no frictional cost of capital under either corporate form, i.e. $\alpha = 0$, and that the amount of capital provided by shareholders is exogenously fixed. We thus concentrate on the functions of selling policies and

[5] Customers may also be shareholders, but they are usually only a small subgroup in the case of stock insurers. In our analysis, we assume a distinct separation between owners and customers in a stock corporation in order to clarify the difference to a mutual organization.

distributing total capital to claimants and owners.

The actuarially fair premium, P_{fair}, equals the expected indemnity payment to the policyholder and is therefore lower than the policyholder's expected loss if there is a positive probability that the company can become insolvent. The actuarially fair premium therefore depends on the company's insolvency probability, which, in turn, depends on the level of the premium, as collected premiums are available for claim payments. Another important determinant of the insolvency risk and level of the actuarially fair premium is the amount of shareholders' capital in the company, C.

To clarify the interconnection between the actuarially fair premium and insolvency risk of a stock corporation, we examine the two extreme scenarios: zero capital and unlimited financial capital held by the company. With unlimited capital, i.e. $C = +\infty$, the company never goes bankrupt and policyholders are always fully indemnified. This implies that the actuarially fair premium equals the expected loss to the insured. If the stock corporation has no capital, i.e. $C = 0$, there is still a strictly positive probability that the company remains solvent and that the collected premiums exceed the aggregate claims by policyholders. In this case, the remaining funds are paid out to shareholders, and the expected payout to policyholders is lower than the premium, which contradicts the definition of an actuarially fair premium. Without capital the only actuarially fair premium is therefore zero. Capital provided by shareholders is needed for a stock insurer to provide insurance at an actuarially fair premium.

Under a mutual organization, policyholders are also the owners of the firm and all premiums collected are redistributed. Each policyholder receives $X_i + P^m - \frac{1}{n}\sum_{i=1}^{n} X_i$ in case of solvency and $I_i\left(X_1, ..., X_n\right)$ in case of insolvency. The premium therefore comprises the expected indemnity payment and the value of the ownership right. This implies that any premium P^m provided by the policyholder of a mutual insurer is actuarially fair.

In the following proposition, we formalize these arguments and show that for stock insurers the actuarially fair premium is increasing in the amount of capital provided by shareholders.

Proposition 1 *For a stock insurer, there exists a unique actuarially fair premium for each fixed* *level of capital provided by shareholders. Furthermore, the actuarially fair premium is strictly increasing in the amount of capital, from zero without capital to the level of the expected loss with unlimited capital. For a mutual insurer, any premium provided by policyholders is actuarially fair.*

Proof. See Appendix A.1. ∎

Proposition 1 highlights an important distinction between stock and mutual insurers. A sufficiently high level of capital C is required to offer a substantial amount of insurance at an actuarially fair premium in the case of a stock insurer. This capital has to be provided by shareholders who benefit from states in which total premiums exceed total claims. The role of capital is thus to reallocate funds from states where shareholders make a profit to states where total premiums are lower than the policyholder's losses.

Optimal Loading We now investigate the optimality of the actuarially fair premium based on policyholders' preferences. For a stock insurer, the actuarially fair premium, and thereby the likelihood that the company stays solvent, is increasing in the amount of capital C provided by shareholders. Policyholders' level of expected utility is therefore also increasing in C. If providing risk capital is costless, i.e. if capital markets are perfect, then it would be optimal to have unlimited risk capital available, i.e. $C = +\infty$. In this case, the insurer is always solvent, and full insurance is achieved at a fair premium, i.e. $P_{fair}(\infty) = E[X_1]$, as shown in Proposition 1. Policyholders are fully indemnified and thus not willing to pay a loading in excess of the fair premium.

Suppose now that a stock insurer's risk capital is limited by some level $\bar{C} < +\infty$, e.g., raising capital might be costless up to $\bar{C}$ but infinitely costly beyond $\bar{C}$. If the company is insolvent, then policyholders will not be fully indemnified. Their marginal utility is therefore higher in states in which the company is insolvent compared to states in which funds are sufficient to receive full coverage. As policyholders are risk-averse, they wish to transfer money from solvency-states to insolvency-states and in particular to those insolvency-states with relatively high claims. In the following proposition, we show that it is optimal for policyholders to pay a loading on top of the fair premium if capital provided by shareholders is limited.

Proposition 2 *For a stock insurer, if capital provided by shareholders is small (great), then it is optimal (not optimal) for policyholders to pay a loading in excess of the actuarially fair premium. Furthermore, if policyholders' preferences exhibit constant absolute risk aversion, then the optimal loading is decreasing in the level of capital provided by shareholders.*

Proof. See Appendix A.2. ■

This proposition implies that under constant absolute risk aversion there exists a critical threshold of capital such that it is optimal for policyholders to collectively pay a loading for all levels of capital below that threshold and not to pay a loading for all levels above that threshold.

By collectively paying a loading in excess of the actuarially fair premium, policyholders reduce their wealth in solvency-states to the benefit of shareholders. At the same time, more funds are available to be distributed to policyholders in insolvency-states. Reasonable bankruptcy rules may therefore create a form of coinsurance amongst policyholders if these additional funds accrue to those policyholders with relatively high claims.[6] Policyholders thus trade off higher premiums for additional insurance against the possibility that these funds are not used to pay claims and instead accrue to shareholders. In addition to this trade-off, the loading also reduces the probability of insolvency. On the one hand, this is beneficial to policyholders, as they are more likely to be fully indemnified. On the other hand, a reduction of the insolvency probability has a negative effect on the trade-off described above. It is now more likely that the additional funds accrue to shareholders. The proposition shows that creating this form of coinsurance in insolvency-states is particularly beneficial if there are relatively few funds available, i.e. if little capital is provided by shareholders.

If increasing shareholders' capital is not an option, paying a loading is akin to "back door" capital. In the extreme scenario, i.e. if $\bar{C} = 0$, we have shown in Proposition 1 that no insurance can be offered at an actuarially fair premium. By paying a loading, policyholders would in fact initiate risk sharing. Providing these additional funds, however, is costly for policyholders and generates strictly positive rents for existing shareholders. Alternatively, policyholders could form a

[6] Mahul and Wright (2004b) show that the Pareto-optimal mutual risk sharing contract with limited capital includes a deductible which is adjusted *ex-post* depending on realized losses to meet the capital constraint.

mutual in which they have a claim on the excess funds.

4 Governance Problems and the Role of the Corporate Form for Raising Capital, Selling Insurance, and Sharing Risk

4.1 Raising Capital

In this section, we endogenize the capital that can be raised from shareholders in the first stage of insurance company launch. As mentioned before, if capital markets are perfect, it is optimal for policyholders if the insurer raises infinite capital from shareholders and sells policies at the actuarially fair rate, that equals the expected value of the loss. If raising capital is costly, then there exists some optimal capital C^* and optimal premium P^* which maximize the policyholder's expected utility subject to the participation constraint of shareholders. We first examine a benchmark case: the optimal solution for a stock insurer if policyholders can commit to a premium P when capital is raised from shareholders. In general, P is determined by competition in the insurance market after the capital has been raised. We show that, for a high level of competition, the premium is not sufficient to cover the frictional cost of capital associated with the level of capital that would be optimal under commitment. The advantage of a mutual is that capital provision is tied to the sales of insurance. Therefore policyholders directly bear the frictional cost of capital. We show that the mutual form can implement the same outcome as a stock insurer with commitment.

Stock insurer with commitment Given a premium P, the capital that can be raised in the capital market is determined by the zero expected profit condition for shareholders

$$C = (1 - \alpha)\, E\left[\left(C + nP - \sum\nolimits_{i=1}^{n} X_i\right)^+\right]. \tag{1}$$

If policyholders can commit to a premium P after capital is raised from shareholders, the optimal capital C^* and premium P^* are determined by the following optimization problem

$$\left(C^*, P^*\right) \in \arg\max_{C,P} E\left[u\left(W_1\right)\right]$$

subject to (1) where each policyholder's final wealth is given by

$$W_1 = \begin{cases} w_S = w_0 - P & \text{if} \quad \Sigma_i X_i \leq C + nP \\ w_{IS} = w_0 - P - X_1 + \dfrac{X_1}{\sum_{i=1}^n X_i}\left(C + nP\right) & \quad\;\; \Sigma_i X_i > C + nP \end{cases}.$$

Lemma 1 *The zero expected profit condition for shareholders* (1) *provides a one-to-one, increasing mapping between capital and premium. Furthermore, the optimal premium* P^* *paid by policyholders includes a loading that compensates shareholders for the frictional cost of capital.*

Proof. See Appendix A.3. ■

For every level of premium, it is optimal for policyholders to raise as much risk capital as possible. However, the level of risk capital that can be raised for a given level of P is constrained by the governance problem imposed by $\alpha > 0$. For C^* and P^*, shareholders will make zero expected profit. In the proof to Lemma 1, we show that

$$n\left(P^* - P^*_{fair}\right) = \alpha E\left[\left(C^* + nP^* - \sum\nolimits_{i=1}^n X_i\right)^+\right],$$

with $P^*_{fair} = E\left[\left(X_i\right) \cdot 1_{\{\Sigma_i X_i \leq C^* + nP^*\}} + I_i\left(X_1, ..., X_n\right) \cdot 1_{\{\Sigma_i X_i > C^* + nP^*\}}\right]$. P^*_{fair} is defined as the expected indemnity payment to policyholders implied by C^* and P^*, and we refer to P^*_{fair} as the actuarially fair premium. Thus, nP^* equals the total expected payment to policyholders plus the expected frictional cost of capital. P^* is therefore the fair premium that assures that shareholders earn a fair return on their invested capital and exceeds the actuarially fair premium. Without a frictional cost of capital ($\alpha = 0$) the two levels of premium coincide, $P^*_{fair} = P^*$ (see footnote 2).

Stock insurer without commitment An important characteristic of the frictional cost of capital is that the cost is sunk once the capital has been raised. As an extreme example, consider the case where the insurer does not sell any policies. Shareholders are then only repaid $(1-\alpha)C$. For that reason, commitment to the premium P^* is crucial for raising C^*. Suppose that commitment is not possible and instead the premium is determined by competition in the insurance market after capital has been raised.[7] Shareholders will foresee this premium, and capital can only be raised to the extent that competition in the insurance market allows for (quasi) rents that accrue to shareholders. Thus, competition imposes an upper bound on the level of reimbursement for the (sunk) frictional cost of capital. We assume that the degree of competition is measured by proportional loading β, which it generates in the insurance market. A higher degree of competition will thus result in a lower loading and vice versa. Substituting $P = (1+\beta) P_{fair}$ into the zero expected profit condition for shareholders (1) yields

$$C = (1-\alpha) E\left[\left(C + n(1+\beta) P_{fair} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right]$$

which determines the capital that can be raised in the capital market.

Proposition 3 *Given a degree of competition in the insurance market as measured by the loading β, there exists a unique amount of capital $C(\beta)$ and an actuarially fair premium $P_{fair}(\beta)$, which satisfy the zero expected profit condition for shareholders. The loading compensates shareholders for the frictional cost of capital, i.e.*

$$n\beta P_{fair}(\beta) = \alpha E\left[\left(C(\beta) + n(1+\beta) P_{fair}(\beta) - \sum\nolimits_{i=1}^{n} X_i\right)^+\right].$$

Furthermore, the amount of capital $C(\beta)$ and overall premium $(1+\beta) P_{fair}(\beta)$ are both increasing in β and both decreasing in α.

[7] A commitment problem is also the reason why policies cannot be sold first. In this case, initial owners have an incentive to not raise additional capital. This problem is directly related to the debt-overhang problem discussed by Myers and Majluf (1984).

Proof. See Appendix A.4. ■

The two solutions with commitment and without commitment are identical if $P^* - P^*_{fair} = \beta P_{fair}(\beta)$. In this case, it also holds that $P^*_{fair} = P_{fair}(\beta)$. Thus, market competition resulting in a loading defined by $\beta^* = P^*/P^*_{fair} - 1$ implements the optimal solution with commitment. If the degree of competition is higher, that is $\beta < \beta^*$, then the company is undercapitalized relative to what would be optimal for policyholders with commitment. In addition, the higher the frictional cost of capital, α, the lower are the amount of capital that can be raised and the degree of risk sharing provided by the insurer. In the presence of this commitment problem, it would be optimal for policyholders to collectively provide additional capital to improve risk sharing despite the frictional cost. The increase in utility from improved risk sharing compensates policyholders for a negative (unfair) return of their stock. However, if buying insurance policies and buying stock are separated, each policyholder has an incentive to free-ride on the capital provided by the others and will not buy stock.

Mutual insurer The optimal capital P^m provided by each policyholder is determined by the optimization problem

$$\max_{P^m} E\left[u\left(W_1\right)\right]$$

where each policyholder's final wealth is given by

$$W_1 = \begin{cases} w_S = w_0 - P^m + \frac{1}{n}(1-\alpha)\left(nP^m - \sum_{i=1}^n X_i\right) & \text{if } \Sigma_i X_i \leq nP^m \\ w_{IS} = w_0 - P^m - X_1 + \dfrac{X_1}{\sum_{i=1}^n X_i} nP^m & \phantom{\text{if }} \Sigma_i X_i > nP^m \end{cases}.$$

There are several notable differences between the stock insurer and the mutual, which result from the policyholders also being the owners. To carve out the differences, we assume that the aggregate premiums, nP^m, can be decomposed into the payments for the insurance contracts, nP, and payments for the ownership rights, i.e. risk capital, C. Of course, any such decomposition is arbitrary, but to comparing the two, we define risk capital as the insurer's expected net surplus

$$C = (1-\alpha)\, E\left[\left(nP^m - \sum\nolimits_{i=1}^{n} X_i\right)^+\right]$$

and $nP = nP^m - C$. This definition does not influence our results if preferences exhibit constant absolute risk aversion (CARA).

Each policyholder's final wealth can then be divided into the sum of final wealths generated from the policy and the insurer's net surplus, i.e. $W_1 = W_1^I + W_1^S$ where

$$W_1^I = \begin{cases} w_S = w_0 - P & \text{if} \quad \Sigma_i X_i \leq nP^m \\ w_{IS} = w_0 - P - X_1 + \dfrac{X_1}{\sum_{i=1}^{n} X_i} nP^m & \phantom{\text{if}} \quad \Sigma_i X_i > nP^m \end{cases}$$

and

$$W_1^S = -\frac{C}{n} + \frac{1}{n}(1-\alpha)\left(nP^m - \sum\nolimits_{i=1}^{n} X_i\right)^+.$$

Suppose that the policyholder's utility function is additively separable in the two levels of final wealth, i.e.

$$u\left(W_1\right) = u_I\left(W_1^I\right) + u_S\left(W_1^S\right).$$

As mentioned in Section 2, policyholders may differ with respect to their risk aversion toward bearing the surplus risk compared to shareholders. A more subtle difference between the two organizational forms related to risk sharing results from the fact that the provision of capital is tied to the sales of insurance policies for a mutual. To focus on this effect, we assume that the owners of a mutual and the owners of the stock insurer are both risk-neutral with respect to surplus risk, i.e.

$$u\left(W_1\right) = u\left(W_1^I\right) + W_1^S.$$

The mutual firm's optimization problem that yields the optimal P^m, which maximizes policyholders' expected wealth, is then equivalent to the optimization problem of the stock corporation with commitment.

For a mutual, the sales of policies and provision of capital are linked, and the commitment and free-rider problems are overcome. Thus, by bundling insurance and capital, the mutual can raise the optimal amount of overall capital, nP^{m*}, which, for $\beta < \beta^*$, exceeds the overall capital that can be raised by a stock insurer in the presence of commitment problems, $nP(\beta) + C(\beta)$. Better risk sharing is therefore achieved in the states where the stock insurer would be insolvent: the mutual has a lower probability of insolvency and more funds to reallocate in the case of financial distress.

It is important to note that the benefit of linking the sales of policies and capital increases in α. If improved mechanisms of corporate governance reduce α, this advantage of a mutual decreases. Moreover, the benefit arises at the time when capital has to be raised. Thus, a mutual may dominate a stock insurer when capital has to be raised and corporate governance is weak.

4.2 Managerial Incentives to Expand

In this section, we show that the two different organizational forms can provide contrasting managerial incentives to expand the number of customers. To gain intuition for this difference, we first fix the level of capital C and focus on the effect of increasing the customer base on the probability of financial distress.

The number of policies underwritten is important for risk sharing as it affects the probability of financial distress and thereby the number of policyholders who have to bear the risk in case of financial distress. This is true for both organizational forms, a mutual and a stock insurer with limited capital. However, the effect of increasing the number of policyholders on risk sharing crucially depends on whether owners and customers are separated or whether they coincide. In fact, we show that increasing the number of policyholders has opposite effects for a mutual and a stock insurer with fixed capital C offering insurance at an actuarially fair rate.

Proposition 4 *Suppose that the number of policyholders is large such that the Central Limit Theorem can be applied. For a stock insurer with a given level of capital C and an implied actuarially* *fair premium $P_{fair}(n, C)$, the probability of financial distress is increasing in the number of policyholders. For a mutual insurer with premium $P^m > E[X_1]$, the probability of financial distress is decreasing in the number of policyholders.*

Proof. See Appendix A.5. ■

The difference between a stock insurer and a mutual insurer in this context is that the level of capital provided by shareholders is fixed in the case of a stock corporation. Increasing the customer base thus has a negative effect on shareholders' capital per policy, which outweighs the positive "diversification" effect and the effect on the fair premium. In contrast, since policyholders are also owners in the case of a mutual, selling actuarially fair insurance to policyholders is tied to ownership rights. Policyholders in a mutual provide capital and pay for the indemnity payment at the same time. Selling insurance policies at a fixed premium thus also raises capital, and the average total funds per policy remains constant.

The same intuition applies to differences in managerial incentives to expand the company. Suppose that a manager has to exert privately costly effort, $c(q)$, to sell q policies, which is increasing in q. Furthermore, the manager derives a private benefit of δ that is increasing in the surplus.[8] If q policies are sold, the manager's utility under the two organizational forms are

$$U^s(q) = \delta E\left[\left(C + qP - \sum_{i=1}^{q} X_i\right)^+\right] - c(q)$$

in case of a stock insurer and

$$U^m(q) = \delta E\left[\left(qP^m - \sum_{i=1}^{q} X_i\right)^+\right] - c(q)$$

in case of a mutual insurer. We assume that the objective is to sell n insurance contracts and analyze how the manager's utility changes when the number of policies increases from q to n under the two organizational forms.

[8] Note that we allow the benefit to the manager to differ from the cost to owners. Moreover, similar results obtain if the manager incurs a disutility from the company's insolvency.

Proposition 5 *Suppose that the number of policyholders is large such that the Central Limit Theorem can be applied. The managerial incentives to expand the number of customers are higher under the mutual form than under the stock form if* $P^{m*} > E[X_1]$.

Proof. See Appendix A.6. ■

The difference in incentives stems from the differences in how capital is raised and mirrors the intuition for Proposition 4. Stock insurers raise a fixed amount of capital first, then sell policies. Increasing the number of policies then reduces the average capital available for each policy. This effect dominates the benefit of a reduced variance of the average claim. Mutual insurers raise capital while selling insurance policies. Since the total premium and capital for ownership rights are constant, the benefit of reducing the variance of the average claim dominates. This implies that if the mutual and the stock insurer are equally capitalized, i.e. $\beta = \beta^*$, the managerial incentives to expand are higher under the mutual form. If the mutual company is better capitalized, i.e. $\beta < \beta^*$, we show in the proof of Proposition 5 that the managerial incentives to expand under a mutual form are increasing in the amount of capital, as long as $P^{m*} > E[X_1]$, and thereby are also higher than under the stock form.

The ex ante available capital in a stock insurer provides a cushion for the manager—the fraction δ of which he is even able to consume if the firm is solvent—while the manager of a mutual has to "earn" this cushion by selling insurance contracts.

5 Empirical Predictions and Evidence

In this section, we discuss the empirical predictions of our model for the relative advantage of one organizational form over the other. The advantage of a mutual insurer arises when capital has to be raised amidst large governance problems and when capital markets are underdeveloped. In this case, the stock form has a comparative disadvantage in raising capital due to the adverse consequences of the commitment problem, and a mutual company is able to offer a higher degree of risk sharing. This prediction is consistent with the finding by Smith and Stutzer (1995) and

Zanjani (2004 and forthcoming) that mutual insurers are used more often in times of financial crises. Zanjani argues that a reason is that new stock insurers are more capital-intensive than mutual firms and that the use of mutuals in times of distressed capital markets may substitute for (external) capital in production.

In our model, existing, well-capitalized stock insurers are not driven out of the market by the mutual form under normal circumstances. After a large shock to capital, however, e.g. due to a catastrophic event, the mutual form gains comparative advantage in raising capital and might replace stock insurers. This is consistent with empirical evidence that the mutual form dominates after large shocks. For example, the New York Fire of 1835 wiped out most stock insurers and stimulated the formation of mutual insurers (Smith and Stutzer, 1995). Arguably, governance problems were larger and stock markets less developed than today.

The benefit of a stock insurer arises in its ability to better spread risk when stock markets allow a high level of diversification through a large and dispersed group of investors. If governance problems are small, then the stock form can drive the mutual form out of the market. Alternatively, mutuals are forced to change their corporate form when the diversification advantage increases. This prediction is consistent with observed demutualization in countries with highly developed stock markets that are motivated by access to capital (Viswanathan and Cummins, 2003).

Our model also sheds light on how capital and solvency regulation may affect the comparative advantage of the stock form relative to the mutual form. Initial minimum capital requirements imply that capital has to be raised before policies are sold. This puts the stock insurer at a relative disadvantage due to its commitment problem of raising funds through policies to reimburse shareholders for the high frictional cost of capital. If the required capital is very high for a stock insurer, the frictional cost of capital may be prohibitive.[9] In contrast, a regulatory requirement of

[9] Zanjani (forthcoming) analyzes the choice of organizational form by life insurance companies between 1900 and 1949. He finds that capital requirements are a major determinant of the choice of mutuals, which were formed in states with low initial capital requirements for mutuals and differentially high initial capital requirements for stock corporations. The economic rationale for this finding is not immediately clear. High capital improves risk sharing and may therefore be even beneficial. However, as we argue, it can be difficult to raise capital ex ante in the presence of frictional cost of capital.

maintaining a minimum level of capital to support insurance operations might reduce the relative disadvantage of stock insurers. Selling policies first and then being forced to raise some capital can reduce the problem that premiums may not cover the (sunk) frictional cost of capital if capital is raised first. Regulatory capital requirements can thus serve as a commitment device to the extent that the insurer can raise the required capital after policies have been sold.

6 Conclusion

In this paper, we emphasize the distinction between mutual and stock insurers in organizing risk sharing in the presence of governance problems that may exist under both corporate forms. In a stock corporation, the efficiency of risk sharing is inherently linked to the level of capital provided by shareholders and the degree to which shareholders are diversified. In a mutual corporation, risks are shared among policyholders only, and the efficiency of risk sharing therefore depends on the size of the pool of policyholders. In an efficient capital market without frictions and where shareholding is dispersed, risk sharing can optimally be organized through a stock insurer. However, in the presence of an owner-manager conflict where the manager can expropriate a fraction of the insurer's surplus, the insurance premium has to compensate shareholders for this expropriation of funds. When insurance policies are sold, shareholders already have exposed their capital, and competition in the insurance market may result in a premium that does not provide a sufficiently high (quasi) rent to cover the loss from expropriation. When governance problems are large, the level of capital and risk sharing in a stock insurer may be low. A mutual links the provision of capital and premium. Thus, policyholders directly bear the cost of providing capital. Moreover, policyholders cannot free-ride on others to provide capital at unfair terms.

A Appendix: Proofs

A.1 Proof of Proposition 1

For a stock insurer, the actuarially fair premium $P_{fair}(C)$ for a policyholder as a function of capital provided by shareholders is implicitly defined by

$$P_{fair}(C) = E\left[X_i \cdot 1_{\{\Sigma_i X_i \leq nP_{fair}(C)+C\}} + I_i(X_1, ..., X_n) \cdot 1_{\{\Sigma_i X_i > nP_{fair}(C)+C\}}\right].$$

Summing over all policies yields

$$\begin{aligned} nP_{fair}(C) &= E\left[\sum_{i=1}^{n} X_i \cdot 1_{\{\Sigma_i X_i \leq nP_{fair}(C)+C\}} + (nP_{fair}(C) + C) \cdot 1_{\{\Sigma_i X_i > nP_{fair}(C)+C\}}\right] \\ &= \int_0^{nP_{fair}(C)+C} x dF^n(x) + (nP_{fair}(C) + C)(1 - F^n(nP_{fair}(C) + C)), \end{aligned} \tag{2}$$

where F^n is the n-fold convolution of F^1 and thus the distribution function of the aggregate loss $\sum_{i=1}^{n} X_i$. For $C = 0$ we have

$$nP_{fair}(0) = \int_0^{nP_{fair}(0)} x dF^n(x) + nP_{fair}(0)(1 - F^n(nP_{fair}(0)))$$

which is satisfied for $P_{fair}(0) = 0$. For any $P_{fair}(0) > 0$ we deduce

$$\int_0^{nP_{fair}(0)} x dF^n(x) + nP_{fair}(0)(1 - F^n(nP_{fair}(0))) < nP_{fair}(0).$$

$P_{fair}(0) = 0$ is therefore the unique solution to (2).
For $C = +\infty$, the company is never insolvent and the actuarially fair premium is given by $P_{fair}(\infty) = E[X_1]$.
For any $0 < C < \infty$, define the expected loss for all policyholders from each paying a premium P as

$$f(P) = nP - \int_0^{nP+C} x dF^n(x) - (nP + C)(1 - F^n(nP + C)).$$

The actuarially fair premium $P_{fair}(C)$ is thus characterized by

$$f(P_{fair}(C)) = 0.$$

We have

$$f(0) = -\int_0^{C} x dF^n(x) - C(1 - F^n(C)) < 0$$

and

$$f(\infty) = \infty$$

for all $0 < C < \infty$. Furthermore

$$f'(P) = n - n(1 - F^n(nP + C)) = nF^n(nP + C) > 0.$$

As $f(\cdot)$ is a continuous function in P the intermediate value theorem implies that there exists a unique solution $P_{fair}(C) > 0$ for $f(P_{fair}(C)) = 0$.

Implicitly differentiating (2) with respect to C yields

$$nP'_{fair}(C) = \left(nP'_{fair}(C) + 1\right)\left(1 - F^n\left(nP_{fair}(C) + C\right)\right)$$

which implies

$$P'_{fair}(C) = \frac{1}{n}\frac{1 - F^n\left(nP_{fair}(C) + C\right)}{F^n\left(nP_{fair}(C) + C\right)} > 0$$

for all $C > 0$. The actuarially fair premium is thus strictly increasing in the amount of risk capital.

For a mutual insurer, suppose policyholders provide a premium P^m. This premium is actuarially fair if and only if

$$P^m = E\left[\left(X_i + P^m - \frac{1}{n}\sum\nolimits_{i=1}^{n} X_i\right)\cdot 1_{\{\Sigma_i X_i \leq nP^m\}} + I_i\left(X_1, ..., X_n\right)\cdot 1_{\{\Sigma_i X_i > nP^m\}}\right]$$

with $\sum_{i=1}^{n} I_i\left(X_1, ..., X_n\right) = nP^m$ and $E\left[I_i\left(X_1, ..., X_n\right)\right] = P^m$ for all $i = 1, ..., n$. Summing over all policies yields that total premiums provided, nP^m, are actuarially fair. Since all policies have the same expected value of payout, each single premium provided is actuarially fair.

A.2 Proof of Proposition 2

Suppose that the insolvency rule specifies a pro-rata rule, i.e.

$$I_i\left(X_1, ..., X_n\right) = \frac{X_i}{\sum_{i=1}^{n} X_i}\left(nP + \bar{C}\right),$$

and let Δ denote the loading in excess of the actuarially fair premium.[10] The final level of wealth of a policyholder, e.g. policyholder 1, is then given by

$$W_1(\Delta) = \begin{cases} w_S(\Delta) = w_0 - P - \Delta & \text{if } \sum_{i=1}^{n} X_i \leq n(P + \Delta) + \bar{C} \\ w_{IS}(\Delta) = w_0 - P - \Delta - X_1 + \frac{X_1}{\sum_{i=1}^{n} X_i}\left(n(P + \Delta) + \bar{C}\right) & \text{if } \sum_{i=1}^{n} X_i > n(P + \Delta) + \bar{C} \end{cases}.$$

where $w_S(\Delta)$ and $w_{IS}(\Delta)$ are the levels of final wealth in solvency- and insolvency-states respectively. The actuarially fair premium $P = P_{fair}\left(\bar{C}\right)$ is implicitly defined by (2). The policyholder's expected utility of final wealth is given by

$$\begin{aligned} & E\left[u\left(W_1(\Delta)\right)\right] \\ = \; & u\left(w_S(\Delta)\right)F^n\left(n(P + \Delta) + \bar{C}\right) + \int_0^\infty u\left(w_{IS}(\Delta)\right)dF^n\left(\sum\nolimits_{i=1}^{n} x_i\right) \\ = \; & u\left(w_S(\Delta)\right)F^n\left(n(P + \Delta) + \bar{C}\right) + \int_0^\infty \int_{n(P+\Delta)+\bar{C}-x_1}^\infty u\left(w_{IS}(\Delta)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right), \end{aligned}$$

[10] For expositional purposes, we focus on the pro-rata rule as bankruptcy rule. The results, however, are robust to any "reasonable" bankruptcy rule that allow to create the form of coinsurance decsribed above. More precisely, the bankruptcy rule must be such that the marginal benefit of an extra dollar under bankruptcy is increasing in the realized size of the loss.

where $x_{-1} = \sum_{i=2}^{n} x_i$. Differentiating expected utility with respect to Δ yields

$$\begin{aligned} \frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta} &= -u'\left(w_S\left(\Delta\right)\right)F^n\left(n\left(P+\Delta\right)+\bar{C}\right) \\ &\quad +\int_0^\infty \int_{n(P+\Delta)+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right)u'\left(w_{IS}\left(\Delta\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right). \end{aligned}$$

The second derivative is given by

$$\begin{aligned} \frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta^2} &= u''\left(w_S\left(\Delta\right)\right)F^n\left(n\left(P+\Delta\right)+\bar{C}\right) - nu'\left(w_S\left(\Delta\right)\right)f^n\left(n\left(P+\Delta\right)+\bar{C}\right) \\ &\quad +\int_0^\infty \int_{n(P+\Delta)+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right)^2 u''\left(w_{IS}\left(\Delta\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right) \\ &\quad -nu'\left(w_S\left(\Delta\right)\right)\int_0^\infty \left(-1+\frac{nx_1}{n\left(P+\Delta\right)+\bar{C}}\right)f^{n-1}\left(n\left(P+\Delta\right)+\bar{C}-x_1\right)dF^1\left(x_1\right) \\ &= u''\left(w_S\left(\Delta\right)\right)F^n\left(n\left(P+\Delta\right)+\bar{C}\right) \\ &\quad -\frac{n^2}{n\left(P+\Delta\right)+\bar{C}}u'\left(w_S\left(\Delta\right)\right)\int_0^\infty x_1 f^{n-1}\left(n\left(P+\Delta\right)+\bar{C}-x_1\right)dF^1\left(x_1\right) \\ &\quad +\int_0^\infty \int_{n(P+\Delta)+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right)^2 u''\left(w_{IS}\left(\Delta\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right) \\ &< 0. \end{aligned}$$

Expected utility is thus globally concave in Δ and any inner solution $\Delta^*\left(\bar{C}\right)$ to the FOC

$$\frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta}|_{\Delta=\Delta^*(\bar{C})} = 0 \tag{3}$$

is therefore the unique global maximum. For $\bar{C} = \infty$, we have $P_{fair}\left(\infty\right) = E\left[X_1\right]$ and the first derivative is given by

$$\frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta}|_{\bar{C}=\infty} = -u'\left(w_0 - E\left[X_1\right] - \Delta\right) < 0.$$

As expected utility is decreasing in Δ, we get the corner solution $\Delta^*\left(\infty\right) = 0$.[11] As $\Delta^*\left(\bar{C}\right)$ is continuous in $\bar{C}$, $\Delta^*\left(\bar{C}\right) = 0$ for large values of $\bar{C}$. For $\bar{C} = 0$, we have $P_{fair}\left(0\right) = 0$ (Proposition A.1) and the first derivative is given by

$$\begin{aligned} &\frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta}|_{\bar{C}=0} \\ = &-u'\left(w_0-\Delta\right)F^n\left(n\Delta\right) \\ &+\int_0^\infty \int_{n\Delta-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right)u'\left(w_0-\Delta-x_1\left(1-\frac{1}{x_1+x_{-1}}\left(n\Delta\right)\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right). \end{aligned}$$

[11] The participation constraint for risk-neutral shareholders providing capital imposes $\Delta \geq 0$.

Evaluating this derivative at $\Delta = 0$ yields

$$\begin{aligned}\frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta}|_{\bar{C}=0,\Delta=0} &= \int_0^\infty \int_0^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) u'\left(w_0-x_1\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\ &= E\left[u'\left(w_0-X_1\right)\left(-1+\frac{nX_1}{X_1+X_{-1}}\right)\right] \\ &= Cov\left(u'\left(w_0-X_1\right), \frac{nX_1}{X_1+X_{-1}}\right) + E\left[u'\left(w_0-X_1\right)\right] E\left[-1+\frac{nX_1}{X_1+X_{-1}}\right].\end{aligned}$$

We have

$$\begin{aligned}E\left[-1+\frac{nX_1}{X_1+X_{-1}}\right] &= -1+nE\left[\frac{X_1}{\Sigma_{i=1}^n X_i}\right] \\ &= -1+\sum_{i=1}^n E\left[\frac{X_i}{\Sigma_{i=1}^n X_i}\right] \\ &= 0\end{aligned}$$

and therefore

$$\frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta}|_{\bar{C}=0,\Delta=0} = Cov\left(u'\left(w_0-X_1\right), \frac{nX_1}{X_1+X_{-1}}\right) > 0.$$

This implies $\Delta^*(0) > 0$. Again, as $\Delta^*\left(\bar{C}\right)$ is continuous in $\bar{C}$, $\Delta^*\left(\bar{C}\right) > 0$ for small values of $\bar{C}$. Total differentiation of the FOC (2) with respect to $\bar{C}$ and Δ implies

$$\frac{d\Delta^*\left(\bar{C}\right)}{d\bar{C}} = -\frac{\frac{\partial^2 E[u(W_1(\Delta))]}{\partial\Delta\partial\bar{C}}|_{\Delta=\Delta^*(\bar{C})}}{\frac{\partial^2 E[u(W_1(\Delta))]}{\partial^2\Delta}|_{\Delta=\Delta^*(\bar{C})}}.$$

As expected utility is globally concave in Δ we derive

$$sign\left(\frac{d\Delta^*\left(\bar{C}\right)}{d\bar{C}}\right) = sign\left(\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}}|_{\Delta=\Delta^*(\bar{C})}\right). \quad (4)$$

Recall that $P = P_{fair}\left(\bar{C}\right)$, i.e.

$$\begin{aligned}&\frac{\partial E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial \Delta} \\ = \; & -u'\left(w_0 - P_{fair}\left(\bar{C}\right) - \Delta\right) F^n\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}\right) \\ & + \int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\ & \quad \cdot u'\left(w_0 - P_{fair}\left(\bar{C}\right) - \Delta - x_1\left(1-\frac{1}{x_1+x_{-1}}\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}\right)\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).\end{aligned}$$

The cross-derivative is then given by

$$
\begin{aligned}
&\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}} \\
=\; & P'_{fair}\left(\bar{C}\right) u''\left(w_S\left(\Delta\right)\right) F^n\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}\right) \\
& -\left(nP'_{fair}\left(\bar{C}\right)+1\right) u'\left(w_S\left(\Delta\right)\right) f^n\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}\right) \\
& +\int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)\;+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\
& \quad \cdot\left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right) u''\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
& -\left(nP'_{fair}\left(\bar{C}\right)+1\right) u'\left(w_S\left(\Delta\right)\right) \\
& \quad \cdot\int_0^\infty \left(-1+\frac{nx_1}{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}}\right) f^{n-1}\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}-x_1\right) dF^1\left(x_1\right) \\
=\; & P'_{fair}\left(\bar{C}\right) u''\left(w_S\left(\Delta\right)\right) F^n\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}\right) \\
& +\int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)\;+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\
& \quad \cdot\left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right) u''\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right) \\
& -\frac{n\left(nP'_{fair}\left(\bar{C}\right)+1\right)}{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}} u'\left(w_S\left(\Delta\right)\right) \int_0^\infty x_1 f^{n-1}\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}-x_1\right) dF^1\left(x_1\right).
\end{aligned}
$$

In Proposition A.1, we have shown that $P'_{fair}\left(\bar{C}\right)>0$ which implies

$$
\begin{aligned}
\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}} \quad < \quad & \int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)\;+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\
& \cdot\left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right) u''\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).
\end{aligned}
$$

Introducing the constant coefficient of absolute risk aversion $R_a=-\frac{u''(w)}{u'(w)}$ yields

$$
\begin{aligned}
\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}} \quad < \quad & -R_a\int_0^\infty \int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}-x_1}^\infty \left(-1+\frac{nx_1}{x_1+x_{-1}}\right) \\
& \cdot\left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right) u'\left(w_{IS}\left(\Delta\right)\right) dF^{n-1}\left(x_{-1}\right) dF^1\left(x_1\right).
\end{aligned}
$$

For $-1+\frac{nx_1}{x_1+x_{-1}}>0$ we have

$$
-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)>\frac{1}{n}
$$

and thus

$$-\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)\left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right)<-\frac{1}{n}\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)$$

For $-1+\frac{nx_1}{x_1+x_{-1}}<0$ we have

$$-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)<\frac{1}{n}$$

and thus

$$-\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)\left(-P'_{fair}\left(\bar{C}\right)+\frac{x_1}{x_1+x_{-1}}\left(nP'_{fair}\left(\bar{C}\right)+1\right)\right)<-\frac{1}{n}\left(-1+\frac{nx_1}{x_1+x_{-1}}\right).$$

This implies

$$\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}}<-R_a\frac{1}{n}\int_0^\infty\int_{n\left(P_{fair}\left(\bar{C}\right)+\Delta\right)+\bar{C}-x_1}^\infty\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)u'\left(w_{IS}\left(\Delta\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right).$$

The FOC (3) for $\Delta^*\left(\bar{C}\right)$ implies

$$\begin{aligned}&\int_0^\infty\int_{n\left(P+\Delta^*\left(\bar{C}\right)\right)+\bar{C}-x_1}^\infty\left(-1+\frac{nx_1}{x_1+x_{-1}}\right)u'\left(w_{IS}\left(\Delta^*\left(\bar{C}\right)\right)\right)dF^{n-1}\left(x_{-1}\right)dF^1\left(x_1\right)\\ =\;&u'\left(w_S\left(\Delta^*\left(\bar{C}\right)\right)\right)F^n\left(n\left(P+\Delta^*\left(\bar{C}\right)\right)+\bar{C}\right)\end{aligned}$$

and therefore

$$\begin{aligned}\frac{\partial^2 E\left[u\left(W_1\left(\Delta\right)\right)\right]}{\partial\Delta\partial\bar{C}}\Big|_{\Delta=\Delta^*\left(\bar{C}\right)}&<-R_a\frac{1}{n}u'\left(w_S\left(\Delta^*\left(\bar{C}\right)\right)\right)F^n\left(n\left(P_{fair}\left(\bar{C}\right)+\Delta^*\left(\bar{C}\right)\right)+\bar{C}\right)\\&<0.\end{aligned}$$

Finally, (4) implies

$$\frac{d\Delta^*\left(\bar{C}\right)}{d\bar{C}}<0.$$

A.3 Proof of Lemma 1

We prove that the condition

$$C=(1-\alpha)E\left[\left(C+nP-\sum\nolimits_{i=1}^n X_i\right)^+\right]\tag{5}$$

is a one-to-one, increasing mapping between C and P. Let P be given and define the function f by

$$\begin{aligned}f\left(C\right)&=C-(1-\alpha)E\left[\left(C+nP-\sum\nolimits_{i=1}^n X_i\right)^+\right]\\&=C-(1-\alpha)\left((C+nP)F^n\left(C+nP\right)-\int_0^{C+nP}xdF^n\left(x\right)\right).\end{aligned}$$

We have

$$\begin{aligned} f(0) &= -(1-\alpha)\left(nPF^n(nP) - \int_0^{nP} x dF^n(x)\right) < 0 \\ f(\infty) &= \infty \end{aligned}$$

and

$$f'(C) = 1 - (1-\alpha) F^n(C+nP) > 0.$$

This shows that for all P there exists a unique $C = C(P)$ such that (5) is satisfied.

Implicitly differentiating (5) with respect to P yields

$$C'(P) = \frac{(1-\alpha) nF^n(C(P)+nP)}{1-(1-\alpha)F^n(C(P)+nP)}.$$

Thus $C'(P) > 0$ for $0 \le \alpha < 1$ and $C'(P) = 0$ for $\alpha = 1$.

The solution P^* can be decomposed in the actuarially fair premium

$$P^*_{fair} = E\left[X_i \cdot 1_{\{\Sigma_i X_i \le C^* + nP^*\}} + \frac{X_i}{\sum_{i=1}^n X_i}(C^* + nP^*) \cdot 1_{\{\Sigma_i X_i > C^* + nP^*\}}\right]$$

and a premium loading $P^*_{load} = P^* - P^*_{fair}$. Summing over all policies yields

$$nP^*_{fair} = E\left[\min\left(\sum\nolimits_{i=1}^n X_i, C^* + nP^*\right)\right].$$

Combining this equation with (1) implies

$$nP^*_{load} = \frac{\alpha}{1-\alpha}C^* = \alpha E\left[\left(C^* + nP^* - \sum\nolimits_{i=1}^n X_i\right)^+\right].$$

A.4 Proof of Proposition 3

Given a loading β, we show that the two equations

$$C = (1-\alpha) E\left[\left(C + n(1+\beta) P_{fair} - \sum\nolimits_{i=1}^n X_i\right)^+\right] \tag{6}$$

and

$$nP_{fair} = E\left[\min\left(\sum\nolimits_{i=1}^n X_i, C + n(1+\beta) P_{fair}\right)\right] \tag{7}$$

uniquely determine $C(\beta)$ and $P_{fair}(\beta)$, and that $C(\beta)$ and $(1+\beta) P_{fair}(\beta)$ are increasing in β and decreasing in α.

Combining both equations (6) and (7) yields

$$\beta n P_{fair} = \frac{\alpha}{1-\alpha} C. \tag{8}$$

Note that this implies

$$\beta n P_{fair} = \alpha E\left[\left(C + n(1+\beta) P_{fair} - \sum\nolimits_{i=1}^n X_i\right)^+\right].$$

Substituting (8) into (6) yields

$$C = (1-\alpha) E \left[\left(\frac{\beta+\alpha}{\beta(1-\alpha)} C - \sum\nolimits_{i=1}^{n} X_i \right)^{+} \right]. \tag{9}$$

We define that function f by

$$\begin{aligned} f(C) &= C - (1-\alpha) E \left[\left(\frac{\beta+\alpha}{\beta(1-\alpha)} C - \sum\nolimits_{i=1}^{n} X_i \right)^{+} \right] \\ &= C - (1-\alpha) \int_0^{\frac{\beta+\alpha}{\beta(1-\alpha)}C} \left(\frac{\beta+\alpha}{\beta(1-\alpha)} C - x \right) dF^n(x) \end{aligned}$$

with derivatives

$$\begin{aligned} f'(C) &= 1 - \frac{\beta+\alpha}{\beta} F^n \left(\frac{\beta+\alpha}{\beta(1-\alpha)} C \right). \\ f''(C) &= -\frac{(\beta+\alpha)^2}{\beta^2(1-\alpha)} f^n \left(\frac{\beta+\alpha}{\beta(1-\alpha)} C \right) < 0. \end{aligned}$$

Note that $f(0) = 0$, $f(\infty) = -\infty$, and $f'(0) = 1 > 0$. f is therefore positive for small values of C, negative for large values of C, and concave in C. This implies that there exists a unique $C(\beta) > 0$ such that $f(C(\beta)) = 0$, i.e. such that (9) is satisfied. Defining $P_{fair}(\beta) > 0$ by (8), $P_{fair}(\beta) = \frac{\alpha}{(1-\alpha)n\beta} C(\beta)$, implies that $C(\beta)$ and $P_{fair}(\beta)$ are the unique strictly positive solutions to (6) and (7).

Differentiating (9)with respect to β yields

$$C'(\beta) = -\frac{\frac{\alpha}{\beta^2} F^n \left(\frac{\beta+\alpha}{\beta(1-\alpha)} C(\beta) \right)}{1 - \frac{\beta+\alpha}{\beta} F^n \left(\frac{\beta+\alpha}{\beta(1-\alpha)} C(\beta) \right)} \cdot C(\beta).$$

Equation (9) implies

$$1 - \frac{\beta+\alpha}{\beta} F^n \left(\frac{\beta+\alpha}{\beta(1-\alpha)} C(\beta) \right) = \frac{-(1-\alpha) \int_0^{\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)} x dF^n(x)}{C(\beta)} < 0 \tag{10}$$

and therefore $C'(\beta) > 0$. Differentiating the overall premium, $(1+\beta) P_{fair}(\beta)$, with respect to β and using equation (8) implies

$$\begin{aligned} \frac{\partial}{\partial\beta} \left((1+\beta) P_{fair}(\beta) \right) &= \frac{\partial}{\partial\beta} \left(\frac{\alpha(1+\beta)}{n(1-\alpha)\beta} C(\beta) \right) \\ &= -\frac{\alpha}{n(1-\alpha)\beta} \frac{1 - (1-\alpha) F^n \left(\frac{\beta+\alpha}{\beta(1-\alpha)} C(\beta) \right)}{\beta \left(1 - \frac{\beta+\alpha}{\beta} F^n \left(\frac{\beta+\alpha}{\beta(1-\alpha)} C(\beta) \right) \right)} C(\beta) > 0. \end{aligned}$$

Differentiating (9)with respect to α yields

$$C_\alpha(\beta) = \frac{-\int_0^{\frac{\beta+\alpha}{\beta(1-\alpha)}C(\alpha)}\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)-x\right)dF^n(x) + \frac{1+\beta}{\beta(1-\alpha)}F^n\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)\right)C(\beta)}{1-\frac{\beta+\alpha}{\beta}F^n\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)\right)}.$$

Substituting (10) into this equation implies

$$C_\alpha(\beta) = -\frac{1}{1-\alpha}\frac{1-\frac{1+\beta}{\beta}F^n\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)\right)}{1-\frac{\beta+\alpha}{\beta}F^n\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)\right)}\cdot C(\beta).$$

(10) and $\alpha \leq 1$ imply

$$1-\frac{1+\beta}{\beta}F^n\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)\right) < 1-\frac{\beta+\alpha}{\beta}F^n\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)\right) < 0$$

and thus $C_\alpha(\beta) < 0$. Differentiating (8) with respect to α implies

$$\begin{aligned}\partial P_{fair}(\beta)/\partial\alpha &= \frac{1}{\beta n}\frac{\alpha}{1-\alpha}C_\alpha(\beta) + \frac{1}{\beta n}\frac{1}{(1-\alpha)^2}C(\beta)\\ &= \frac{1}{\beta n(1-\alpha)}\frac{1-F^n\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)\right)}{1-\frac{\beta+\alpha}{\beta}F^n\left(\frac{\beta+\alpha}{\beta(1-\alpha)}C(\beta)\right)}\cdot C(\beta) < 0.\end{aligned}$$

A.5 Proof of Proposition 4

For a stock insurer with limited capital C and implied fair premium $P_{fair}(n, C)$, the probability of financial distress is given by

$$prob\left(\sum_{i=1}^n X_i > nP_{fair}(n,C) + C\right).$$

Applying the Central Limit Theorem (CLT) yields

$$prob\left(\sum_{i=1}^n X_i > nP_{fair}(n,C) + C\right) = prob\left(Z > z(n)\right)$$

where Z is standard normally distributed and

$$z(n) = \frac{\sqrt{n}}{\sigma(X_1)}\left(P_{fair}(n,C) - E[X_1]\right) + \frac{1}{\sqrt{n}\sigma(X_1)}C.$$

This implies that

$$sign\left(\frac{\partial}{\partial n}prob\left(\sum_{i=1}^n X_i > nP_{fair}(n,C)+C\right)\right) = -sign\left(z'(n)\right). \tag{11}$$

Recall equation (2) which defines $P_{fair}(n, C)$ through

$$nP_{fair}(n,C) = E\left[\sum_{i=1}^n X_i \cdot 1_{\{\Sigma_i X_i \leq nP_{fair}(n,C)+C\}} + (nP_{fair}(n,C)+C)\cdot 1_{\{\Sigma_i X_i > nP_{fair}(n,C)+C\}}\right].$$

Applying the CLT to this equation yields

$$z(n) - \frac{1}{\sqrt{n}\sigma(X_1)}C = \int_{-\infty}^{z(n)} z dN(z) + z(n)(1 - N(z(n))),$$

where $N(\cdot)$ is the cumulative distribution function of the standard normal distribution. By implicitly differentiating with respect to n we deduce

$$z'(n) + \frac{1}{2n\sqrt{n}\sigma(X_1)}C = z'(n)(1 - F(z(n)))$$

which implies

$$z'(n) = -\frac{1}{2n\sqrt{n}\sigma(X_1)F(z(n))}C < 0.$$

Equation (11) then proves

$$\frac{\partial}{\partial n} prob\left(\sum_{i=1}^{n} X_i > nP_{fair}(n, C) + C\right) > 0.$$

For a mutual insurer with fixed premium $P^m > E[X_1]$, the probability of financial distress is given by

$$prob\left(\sum_{i=1}^{n} X_i > nP^m\right).$$

Applying the CLT yields

$$prob\left(\sum_{i=1}^{n} X_i > nP^m\right) = prob\left(Z > \frac{\sqrt{n}}{\sigma(X_1)}(P^m - E[X_1])\right),$$

which implies

$$\frac{\partial}{\partial n} prob\left(\sum_{i=1}^{n} X_i > nP^m\right) < 0.$$

A.6 Proof of Proposition 5

Incentives stemming from the effect on the utility are higher under the mutual form if

$$U^m(n) - U^m(q) > U^s(n) - U^s(q). \tag{12}$$

For $\beta = \beta^*$, $P^{m*} = P(\beta^*) + C(\beta^*)/n$ and $U^s(n) = U^m(n)$. The inequality holds if $U^s(q) > U^m(q)$. For $P^{m*} = P(\beta^*) + C(\beta^*)/n$, we obtain

$$\begin{aligned} U^s(q) &= \delta E\left[\left(C(\beta^*) + qP(\beta^*) - \sum_{i=1}^{q} X_i\right)^+\right] \\ &> \delta E\left[\left(\frac{q}{n}C(\beta^*) + qP(\beta^*) - \sum_{i=1}^{q} X_i\right)^+\right] \\ &= U^m(q) \end{aligned}$$

and condition (12) holds.

For $\beta < \beta^*$, the mutual insurer is more capitalized than the stock insurer, i.e. $P^{m*} > P(\beta) + C(\beta)/n$. We prove (12) by showing that the managerial incentives to expand a mutual insurer are increasing in the capital of the company. We thus have to show that

$$U^m(n|P^{m*}) - U^m(q|P^{m*}) > U^m(n|\gamma P^{m*}) - U^m(q|\gamma P^{m*})$$

for $0 \leq \gamma < 1$ and

$$U^m(q|P^{m*}) = \delta E\left[\left(qP^{m*} - \sum\nolimits_{i=1}^{q} X_i\right)^+\right] - c(q).$$

This inequality is equivalent to

$$\begin{aligned} & E\left[\left(nP^{m*} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] - E\left[\left(n\gamma P^{m*} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] \\ > \; & E\left[\left(qP^{m*} - \sum\nolimits_{i=1}^{q} X_i\right)^+\right] - E\left[\left(q\gamma P^{m*} - \sum\nolimits_{i=1}^{q} X_i\right)^+\right]. \end{aligned}$$

We will prove the inequality by using the Central Limit Theorem (CLT) to show that $E\left[\left(nP^{m*} - \sum_{i=1}^{n} X_i\right)^+\right] - E\left[\left(n\gamma P^{m*} - \sum_{i=1}^{n} X_i\right)^+\right]$ is increasing in n. The CLT implies

$$\begin{aligned} & E\left[\left(nP^{m*} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] - E\left[\left(n\gamma P^{m*} - \sum\nolimits_{i=1}^{n} X_i\right)^+\right] \\ = \; & \frac{1}{\sqrt{2\pi}}\frac{n\sqrt{n}}{\sigma(X_1)}\left(\int_0^{z_2(n)} (z_2(n) - z)\, e^{-\frac{1}{2}z^2} dz - \int_0^{z_1(n)} (z_1(n) - z)\, e^{-\frac{1}{2}z^2} dz\right) \end{aligned} \tag{13}$$

where $z_1(n) = \frac{\gamma P^{m*} - E[X_1]}{\sigma(X_1)/\sqrt{n}}$ and $z_2(n) = \frac{P^{m*} - E[X_1]}{\sigma(X_1)/\sqrt{n}}$. The first term of the product is increasing in n. Differentiating the second term with respect to n yields

$$\begin{aligned} & \frac{\partial}{\partial n}\left(\int_0^{z_2(n)} (z_2(n) - z)\, e^{-\frac{1}{2}z^2} dz - \int_0^{z_1(n)} (z_1(n) - z)\, e^{-\frac{1}{2}z^2} dz\right) \\ = \; & z_2'(n) \int_0^{z_2(n)} e^{-\frac{1}{2}z^2} dz - z_1'(n) \int_0^{z_1(n)} e^{-\frac{1}{2}z^2} dz \\ = \; & \frac{1}{2\sigma(X_1)\sqrt{n}}\left((P^{m*} - E[X_1]) \int_0^{z_2(n)} e^{-\frac{1}{2}z^2} dz - (\gamma P^{m*} - E[X_1]) \int_0^{z_1(n)} e^{-\frac{1}{2}z^2} dz\right) \end{aligned}$$

Define the function f by

$$f(\gamma) = (P^{m*} - E[X_1]) \int_0^{z_2(n)} e^{-\frac{1}{2}z^2} dz - (\gamma P^{m*} - E[X_1]) \int_0^{z_1(n)} e^{-\frac{1}{2}z^2} dz.$$

If $\gamma P^{m*} - E[X_1] \leq 0$ then $f(\gamma) > 0$ (as $P^{m*} > E[X_1]$) and (13) is increasing in n. Suppose $\gamma P^{m*} - E[X_1] > 0$. Then $f(1) = 0$ and

$$f'(\gamma) = -P^{m*} \int_0^{z_1(n)} e^{-\frac{1}{2}z^2} dz - (\gamma P^{m*} - E[X_1]) \frac{P^{m*}}{\sigma(X_1)/\sqrt{n}} e^{-\frac{1}{2}z_1(n)^2} < 0.$$

Since f is strictly decreasing and $f(1) = 0$, we derive that $f(\gamma) > 0$ for all $0 \leq \gamma < 1$. (13) is therefore increasing in n.

References

[1] Cagle, J.A.B. and S.E. Harrington (1995) "Insurance Supply with Capacity Constraints and Endogenous Insolvency Risk", *Journal of Risk and Uncertainty* 11, p. 219-232

[2] Cummins, J.D. and P.M. Danzon (1997) "Price, Financial Quality, and Capital Flows in Insurance Markets", *Journal of Financial Intermediation* 6, p. 3-38

[3] Doherty, N.A. and G. Dionne (1993) "Insurance with Undiversifiable Risk: Contract Structure and Organizational Form of Insurance Firms", *Journal of Risk and Uncertainty* 6, p. 187-203

[4] Doherty, N.A. and J.R. Garven (1986) "Price Regulation in Property-Liability Insurance: A Contingent-Claims Approach", *Journal of Finance* 41, p. 1031-1050

[5] Doherty, N.A. and H. Schlesinger (1990) "Rational Insurance Purchasing: Consideration of Contract Nonperformance", *Quarterly Journal of Economics* 55, p. 243-253

[6] Gruendl, H. and H. Schmeiser (2002) "Pricing Double-Trigger Reinsurance Contracts: Financial Versus Actuarial Approach", *Journal of Risk and Insurance* 69, 449-468

[7] Mahul, O. and B.D. Wright (2004a) "Implications of Incomplete Performance for Optimal Insurance", *Economica* 71, p. 661-670

[8] Mahul, O. and B.D. Wright (2004b) "Efficient Risk Sharing within a Catastrophe Insurance Pool", *working paper*

[9] Mayers, D. and C.W. Smith (2005) "Agency Problems and the Corporate Charter", *Journal of Law, Economics, & Organization* 21, p. 417-440

[10] Mayers, D. and C.W. Smith (1981) "Contractual Provisions, Organizational Structure, and Conflict Control in Insurance Markets", *Journal of Business* 54, p. 407-434

[11] Myers, S.C. and N.S. Majluf (1984) "Corporate Financing and Investment Decisions when Firms Have Information that Investors do not have", *Journal of Financial Economics* 13, p. 187-221

[12] Smith, B.D. and M. Stutzer (1995) "A Theory of Mutual Formation and Moral Hazard with Evidence from the History of the Insurance Industry", *Review of Financial Studies* 8, p. 545-577

[13] Smith, B.D. and M. Stutzer (1990) "Adverse Selection, Aggregate Uncertainty, and the Role for Mutual Insurance Contracts", *Journal of Business* 63, p. 493-510

[14] Viswanathan, K.S. and J.D. Cummins (2003) "Ownership Structure Changes in the Insurance Industry: An Analysis of Demutualization", *Journal of Risk and Insurance* 70, p. 401-437

[15] Zanjani, G. (2006) "Regulation, Capital, and the Evolution of Organizational Form in U.S. Life Insurance", *American Economic Review* (forthcoming)

[16] Zanjani, G. (2004) "Mutuality and the Underwriting Cycle: Theory with Evidence from the Pennsylvania Fire Insurance Market, 1873-1909", *working paper*