Optimal Debt, Asset Substitution and Coupon Rating-Trigger Covenants

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Abstract

The current paper analyses an optimal debt contract that incorporates a rating trigger covenant so as to impose a coupon rate increase when the credit rating of the issuing firm is downgraded. It is shown that the presence of this particular kind of covenant albeit reducing the value of the leveraged firm when there is no operational flexibility, may assume an important role in terms of asset substitution when such flexibility is present. Such conclusion contradicts the conclusion of Bhanot and Mello (2006) regarding this type of covenants. The optimal coupon increase associated to the covenant is determined and a distinction regarding the nature of the covenant in terms of its effects (permanent or not) is also made.

JEL classification: G13, G33

1 Introduction

The widespread use of rating-based trigger covenants has been documented by some surveys conducted by rating agencies. According to Moody's (2002a)[25], out of 711 US corporate issuers rated Ba1 or higher, 87,5%, reported a total of 2819 rating triggers. From these, 21,1% were of the type "adjustment in interest or coupons", where the initial interest rate or coupon is revised in the event of a rating change. In Moody's (2002b)[26], it is reported that 59% out of 345 European corporate debt issuers used rating triggers, and a similar result (50% among more than 1000 US and European investment-grade debt issuers) was reported by Standard and Poor's (2002)[27]. As mentioned by Gonzalez et al (2004)[10], the main aimof such type of covenants, is: "(...) to protect lenders against credit deterioration and asymmetric information problems (...)". Indeed Bhanot and Mello (2006)[3] showed that rating trigger covenants that dictate an early partial amortization of the principal induce equity holders to pursue low risk strategies. However, in parallel these authors concluded that coupon based triggers "(...) do not inhibit asset substitution (...)". Our results contradict this statement and this is one of the main points of the current paper.

The purpose of the present work consists in analysing the optimal debt contract that includes coupon rating triggers. Specifically we intend to provide answers to the following questions:

- In what way the use of a coupon rating triggers covenant may have an effect on the optimal values of debt and equity?
- In what circumstances the use of this type of covenants may effectively prevent the substitution of assets?
- Is there an "optimal" coupon increase associated with the rating trigger covenant?

- In what circumstances is it optimal to issue debt with this kind of covenant instead of straight debt?
- What is the relevance of the nature (permanent or not) of the coupon's adjustment?

As in Bhanot and Mello (2006)[3], our starting point is Leland's (1994)[17] framework. Leland's model is one of the so called structural credit risk models class where the default of the issuing firm occurs whenever the value of its assets[1] (modeled by a diffusion process) reaches a lower threshold - the default barrier. Within this class of models we can further distinguish those in which this default barrier is set exogenously (e.g. Black and Cox (1976)[4], Kim, Ramaswamy and Sundaresan (1993)[16], Longstaff and Schwartz (1995)[20], Briys and de Varenne (1997)[5], Ericsson and Reneby (1998)[8], Schobel (1999)[24], Hsu, Sa-Requejo and Santa-Clara (2003)[13], Hui, Lo and Tsang (2003)[14], Taurén (1999)[28], Collin-Dufresne and Goldstein (2001)[6], Ju and Ou-Yang (2004)[15], Huang et al. (2003)) from the others wherein it is set endogenously. In this latter case, default occurs when equity holders find it optimal to stop financing the debt service (e.g. Black and Cox (1976)[4], Leland (1994)[17], Leland and Toft (1996)[19], Goldstein, Ju and Leland (2001)[12], Ericsson and Reneby (2003)[9]) or to strategically default in the coupon payments (e.g. Anderson and Sundaresan (1996)[1], Anderson, Sundaresan and Tychon (1996) [2], Mella-Barral and Peraudin (1997)[22], Mella-Barral (1999)[21], Fan and Sundaresan (2000)[11]). In parallel, the structural approach literature also includes a few articles that formally address the asset substitution problem (e.g. Mello and Parsons (1992)[23], Leland (1998)[18], Ericsson (2000)[7]).

Assuming a endogenous default and considering a perpetuity bond, Leland (1994)[17] analyses the optimal debt level in the presence of bankruptcy costs and tax benefits associated to the coupon payments. Since our aim is to study the inclusion in the debt contract of a covenant attached directly to the firm's credit rating, another credit event besides bankruptcy must be considered, namely the credit rating downgrade. This is achieeved, as in Bhanot and Mello (2006)[3], by establishing a second threshold - in the sense that if the assets value decreases enough to reach that new critical level, the firm's credit rating is downgraded and the coupon increase embodied in the covenant is triggered. It will be assumed that this threshold is greater than the default barrier, which implies that the possible occurrence of bankruptcy is always preceded by a rating change.

Within this framework, we will show that, when equity holders have operational flexibility to alter the firm's risk, the presence of a coupon type rating trigger covenant in the debt contract can effectively prevent the substitution of assets, by forcing them to pursue low risk strategies. The attainment of such result and the inherent benefits in terms of agency costs will, in turn, depend on the level of the minimum risk strategy available and on the spectrum of possible risk profiles. The lower the minimum risk level (which must be below some critical level) and the higher the risk spectrum, the greater are the benefits associated with the covenant.

The paper is organized as follows: in section 2 the valuation framework is established and the expressions for the value of debt, equity, bankruptcy costs, tax benefits and the leveraged firm are derived. Section 3 addresses the optimal debt level. Section 4 focus on the prevention of the substitution of assets. Section 5 investigates the case of non permanent effects of the rating trigger covenant and section 6 concludes the paper.

2 The Valuation Framework

2.1 Base model

The following assumptions are assumed:

H1. The risk free interest rate, r, is constant;

[1]Or other state variable such as the firm's cash-flow.

H2. The value of the firm's assets, V_t, is described by the following continuous diffusion process, under the risk neutral probability:

$$dV_t/V_t = (r - \alpha)dt + \sigma dW_t$$

Where r is the risk free interest rate, α is the cash payout rate, σ volatility of assets return and dW_t an increment of a standard Brownian motion.

H3. Debt comprises a bond sold at par value with infinite maturity, that promises the payment of a continuous coupon flow of Cdt until the firm's rating is downgraded. The bond indenture includes a rating based covenant that dictates an increase in the coupon rate, from the moment when the credit rating of the firm is downgraded. We name this new coupon by $C\Delta$, with $\Delta > 1$. Thus, $(\Delta - 1)100\%$ corresponds to the percentage increase in the coupon rate. It is also assumed that this new coupon, once effective, is permanent and irreversible[2] irrespective of the future behaviour of the value of the assets, unless the firm enters in bankruptcy.

H4. Definition of the rating change: the rating downgrade is modelled through the specification of an exogenous constant barrier. The credit event is triggered when the value of the assets reaches this pre-specified level. Labelling this constant threshold as V_{B1}, we define the time of the rating change as:

$$\tau_1 = \inf\{t \geq 0 : V_t \leq V_{B1}\}$$

That is, the first time the value of the assets passes though the barrier level V_{B1}.

H5. Definition of bankruptcy (liquidation of the firm): Bankruptcy occurs when the value of the company's assets reaches a second threshold V_{B2} (the second barrier). Although this barrier is initially set exogenously, later on it will be determined endogenously in order to maximize the equity value. Thus, the firm enters in bankruptcy and is liquidated when the equity holders are not willing to carry on with the debt service. Defining τ_2 as the liquidation time, we have:

$$\tau_2 = \inf\{t \geq \tau_1 : V_t \leq V_{B2}\}$$

Note that $\tau_2 \geq \tau_1$, implying that the bankruptcy event is always preceded by the firm's rating downgraded ($V_{B1} \geq V_{B2}$).

H6. Bondholders recovery value: When the firm is liquidated, the bondholders get a fraction ρ of the value of the company's assets, ρV_{B2}, with $0 < \rho \leq 1$, and $(1 - \rho)V_{B2}$ the bankruptcy costs.

H7. We consider the existence of a corporate taxe rate defined by ι, thus, at each instant in time an amount of ιCdt (before the rating downgrade) or $\iota \Delta Cdt$ (after the rating downgrade) is tax deductible.

Given the above description Figure 1 ilustrastes possible paths for the value of firm's assets in different scenarios.

2.2 Bond value

The cash flow provided by the bond to its holder will be a function of financial health of the issuing firm, embodied by its credit rating. Thus, while the value of the firm's assets remains above the threshold V_{B1}, $t < \tau_1$, the bondholders receive a continuous coupon flow of Cdt. After the firm's rating has been downgraded - which occurs when V_{B1} is crossed for the first time - the coupon

[2] In section 5 this assumption is relaxed

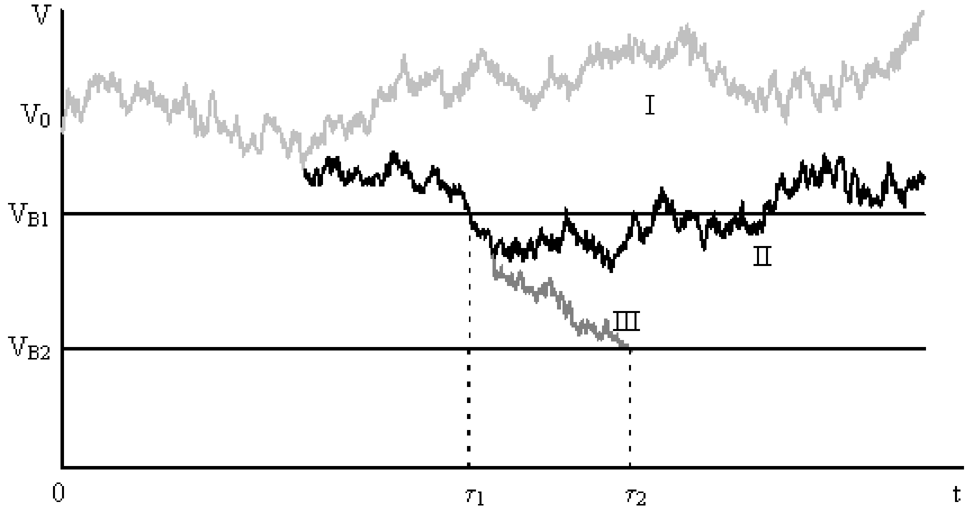

Figure 1: Three sample path for the assets values associated to the three possible scenario. Assets value remains above V_{B1} - I; assets value crosses V_{B1} but remains above V_{B2}, - II and finally the assets value reaches V_{B2} - III.

flow rises to $\Delta C dt$ and remains at this level[3] until the firm enters in bankruptcy - when V_{B2} is reached - and is liquidated, generating a cash flow of ρV_{B2} to the bondholders. Notice that, in terms of payoffs, debt can be understood as the sum of two assets: a straight bond (without any covenant) paying a continuous coupon of Cdt and an asset that pays a stream of $(\Delta-1)Cdt$ (which corresponds to the additional coupon triggered by the covenant) from the moment in which the credit rating is downgraded until the occurrence of bankruptcy (between τ_1 and τ_2). Keeping this in mind, to calculate the value of the bond it is only necessary to determine the expected value (under the risk neutral measure) of all corresponding cashflows discounted at the risk free rate. Defining the bond value before the rating downgrade (for $t \leq \tau_1$) by $B^I(V_t, C, \Delta)$ and after the rating downgrade (for $t > \tau_1$) by $B^{II}(V_t, C, \Delta)$ we have:

$$\begin{aligned} B^I(V_t, C, \Delta) &= \mathbb{E}\left[\int_t^{\tau_2} e^{-r(s-t)} C ds \middle| \mathcal{F}_t\right] + \mathbb{E}\left[e^{-r(\tau_2-t)}\rho V_{B2} \middle| \mathcal{F}_t\right] \\ &+ \mathbb{E}\left[e^{-r(\tau_1-t)}\int_{\tau_1}^{\tau_2} e^{-r(s-\tau_1)}(\Delta-1) C ds \middle| \mathcal{F}_t\right] \end{aligned} \tag{1}$$

$$B^{II}(V_t, C, \Delta) = \mathbb{E}\left[\int_t^{\tau_2} e^{-r(s-t)} \Delta C ds \middle| \mathcal{F}_t\right] + \mathbb{E}\left[e^{-r(\tau_2-t)}\rho V_{B2} \middle| \mathcal{F}_t\right] \tag{2}$$

Whose corresponding values are:

$$B^I(V_t, C, \Delta) = \frac{C}{r} + \left(\rho V_{B2} - \frac{C}{r}\right)\left(\frac{V_t}{V_{B2}}\right)^{-X} + \frac{(\Delta-1)C}{r}\left[\left(\frac{V_t}{V_{B1}}\right)^{-X} - \left(\frac{V_t}{V_{B2}}\right)^{-X}\right] \tag{3}$$

$$B^{II}(V_t, C, \Delta) = \frac{\Delta C}{r} + \left(\rho V_{B2} - \frac{\Delta C}{r}\right)\left(\frac{V_t}{V_{B2}}\right)^{-X} \tag{4}$$

[3] Remember that, for the moment, we are assuming a permanent and irreversible effect of the covenant.

Where:

- $X = \dfrac{\sqrt{\left(r-\alpha-\frac{\sigma^2}{2}\right)^2 \, + \, 2r\sigma^2} + \left(r-\alpha-\frac{\sigma^2}{2}\right)}{\sigma^2} > 0$

- $\left(\dfrac{V_t}{V_{Bi}}\right)^{-X}$ is the probability-weighted discount factor of the assets value reaching V_{Bi} from V_t.

Before the rating downgrade (expression (3)), the debt value corresponds to a perpetuity of Cdt (first term in the right hand side) corrected by the loss that will take place upon bankruptcy (second term) and adjusted by the increase in the coupon flow associated to the impact of the rating trigger covenant in case of a rating downgrade (third term). Notice that the first two terms refer to a straight bond ($\Delta = 1$) as in Leland (1994)[17].

2.3 Equity value

The value of equity will be given by the expected present value of the continuous dividend stream received by equity holders - defined as the cash payout of the firm ($\alpha V_t dt$) minus the coupon payment to bond holders adjusted by the tax shields, ($(1-\iota)Cdt$ before the rating downgrade and $(1-\iota)\Delta Cdt$ after). For the valuation of the adjusted coupon payments it is possible to apply an approach similar that used for bond valuation: the global coupon stream might be subdivided in a continuous payment of $(1-\iota)Cdt$ until τ_2 and an additional continuous payment of $(1-\iota)(\Delta-1)Cdt$ between τ_1 and τ_2. Thus, defining equity value by $E^I(V_t, C, \Delta)$, before the rating downgrade and $E^{II}(V_t, C, \Delta)$ after, we have:

$$\begin{aligned} E^I(V_t, C, \Delta) &= \mathbb{E}\left[\int_t^{\tau_2} e^{-r(s-t)} \left[\alpha V_s - (1-\iota)C\right] ds \middle| \mathcal{F}_t\right] + \\ &+ \mathbb{E}\left[e^{-r(\tau_1 - t)} \int_{\tau_1}^{\tau_2} e^{-r(s-\tau_1)} \left[-(1-\iota)(\Delta-1)C\right] ds \middle| \mathcal{F}_t\right] \end{aligned} \tag{5}$$

$$E^{II}(V_t, C, \Delta) = \mathbb{E}\left[\int_t^{\tau_2} e^{-r(s-t)} \left[\alpha V_s - (1-\iota)\Delta C\right] ds \middle| \mathcal{F}_t\right] \tag{6}$$

Whose corresponding value expressions are:

$$\begin{aligned} E^I(V_t, C, \Delta) &= V_t \; - \; \frac{(1-\iota)\,C}{r} \; + \; \left[\frac{(1-\iota)\,C}{r} - V_{B2}\right]\left(\frac{V_t}{V_{B2}}\right)^{-X} \; - \\ &- \; \frac{(1-\iota)\,(\Delta-1)\,C}{r}\left[\left(\frac{V_t}{V_{B1}}\right)^{-X} - \left(\frac{V_t}{V_{B2}}\right)^{-X}\right] \end{aligned} \tag{7}$$

The first line represents the value of equity if straight debt is issued ($\Delta = 1$) - in accordance with Leland's model results. As for the second line, the correction factor in the dividend flow associated with the rating trigger covenant - that is, the value of the extra stream of coupon payments realized between τ_1 and τ_2. Notice that these accrued payments affect negatively the value of equity, since the expression in the vertical brackets has always a positive sign. In fact, this expression represents the diference between the probability-weighted discount factors for the rating downgrade and for bankruptcy, where the former is greater than the latter since $V_{B1} > V_{B2}$.

After the rating downgrade, equity value would be given by:

$$E^{II}(V_t, C, \Delta) = V_t \; - \; \frac{(1-\iota)\,\Delta C}{r} \; + \; \left[\frac{(1-\iota)\,\Delta C}{r} - V_{B2}\right]\left(\frac{V_t}{V_{B2}}\right)^{-X} \tag{8}$$

2.4 Value of tax shield

The value of the tax shield is given by the flow of $\iota C dt$ until bankruptcy (at τ_2) plus the flow associated to the coupon increase between the rating downgrade and default (between τ_1 and τ_2). Thus we have:

$$TB^{I}(V_t, C, \Delta) = \mathbb{E}\left[\int_t^{\tau_2} e^{-r(s-t)} \iota C ds \middle| \mathcal{F}_t\right] \; + \; \mathbb{E}\left[e^{-r(\tau_1 - t)} \int_{\tau_1}^{\tau_2} e^{-r(s-\tau_1)} \iota\,(\Delta - 1) C ds \middle| \mathcal{F}_t\right] \tag{9}$$

$$TB^{II}(V_t, C, \Delta) = \mathbb{E}\left[\int_t^{\tau_2} e^{-r(s-t)} \iota \Delta C ds \middle| \mathcal{F}_t\right] \tag{10}$$

Yielding:

$$TB^{I}(V_t, C, \Delta) = \frac{\iota C}{r}\left[1 - \left(\frac{V_t}{V_{B2}}\right)^{-X}\right] \; + \; \frac{\iota(\Delta - 1)C}{r}\left[\left(\frac{V_t}{V_{B1}}\right)^{-X} - \left(\frac{V_t}{V_{B2}}\right)^{-X}\right] \tag{11}$$

$$TB^{II}(V_t, C, \Delta) = \frac{\iota \Delta C}{r}\left[1 - \left(\frac{V_t}{V_{B2}}\right)^{-X}\right] \tag{12}$$

2.5 Value of bankruptcy costs

Since the bankruptcy costs considered in the present model only refer to the loss of a fraction $(1-\rho)$ of the firm's assets upon liquidation (at τ_2) there is no need to differentiate the corresponding value before and after the rating downgrade. Thus we have:

$$BC(V_t, C, \Delta) = \mathbb{E}\left[e^{-r(\tau_2 - t)}\,(1-\rho)\,V_{B2} \middle| \mathcal{F}_t\right] \tag{13}$$

Whose correspondent solution is:

$$BC(V_t, C, \Delta) = (1-\rho)\,V_{B2}\left(\frac{V_t}{V_{B2}}\right)^{-X} \tag{14}$$

2.6 The value of the leveraged firm

The value of the leveraged firm can be calculated either from the bond ((3) or (4)) and equity ((7) or (8)) values, or using the values of the assets, tax shields ((11) or (12)) and bankruptcy costs ((14)):

$$v^{i}(V_t, C, \Delta) = B^{i}(V_t, C, \Delta) \; + \; E^{i}(V_t, C, \Delta) \; = \; V_t \; - \; BC(V_t, C, \Delta) \; + \; TB^{i}(V_t, C, \Delta) \tag{15}$$

With i = I or II, for $t \leq \tau_1$ or $t > \tau_1$ respectively.

2.7 The endogenous bankruptcy trigger

As previously noted, we are assuming that V_{B1} is greater than V_{B2}, so the bankruptcy event can only occur after τ_1. Thus, to obtain the barrier level that maximizes equity value, we must use the corresponding equity formula valid for $t > \tau_1$, namely expression (8):

$$E^{II}(V_t, C, \Delta) = V_t \; - \; \frac{(1-\iota)\,\Delta C}{r} \; + \; \left[\frac{(1-\iota)\,\Delta C}{r} - V_{B2}\right]\left(\frac{V_t}{V_{B2}}\right)^{-X}$$

Formally, the endogenous bankruptcy trigger is the value of V_{B2} that solves the following equation:

$$\left.\frac{\partial E^{II}(\cdot)}{\partial V_t}\right|_{V_t = V_{B2}} = 0$$

Such value will be given by:

$$V_{B2} = \frac{\Delta C\,(1-\iota)}{r}\frac{X}{(1+X)} \tag{16}$$

Such endogenous threshold is the same as in Leland (1994), since ΔC is the coupon value after the rating downgrade[4].

Incorporating the endogenous barrier V_{B2} in the expressions derived above yields:

$$B^{I}(V_t, C, \Delta) = \frac{C}{r}\left[1 + (\Delta - 1)\left(\frac{V_t}{V_{B1}}\right)^{-X} - \Delta k\left(\frac{V_t}{\Delta C}\right)^{-X}\right] \tag{17}$$

$$E^{I}(V_t, C, \Delta) = V_t - \frac{(1-\iota)\,C}{r}\left[1 + (\Delta - 1)\left(\frac{V_t}{V_{B1}}\right)^{-X} - \Delta m\left(\frac{V_t}{\Delta C}\right)^{-X}\right] \tag{18}$$

$$v^{I}(V_t, C, \Delta) = V_t + \frac{\iota C}{r}\left[1 + (\Delta - 1)\left(\frac{V_t}{V_{B1}}\right)^{-X} - \Delta h\left(\frac{V_t}{\Delta C}\right)^{-X}\right] \tag{19}$$

$$TB^{I}(V_t, C, \Delta) = \frac{\iota C}{r}\left[1 + (\Delta - 1)\left(\frac{V_t}{V_{B1}}\right)^{-X} - \Delta(1+X)m\left(\frac{V_t}{\Delta C}\right)^{-X}\right] \tag{20}$$

$$BC(V_t, C, \Delta) = \frac{\iota C\Delta\,[h - (1+X)m]}{r}\left(\frac{V_t}{\Delta C}\right)^{-X} \tag{21}$$

[4]Besides the definition of X, since Leland (1994), assumes a zero cash payout ratio.

Where:

$$
\begin{aligned}
m &= \left[\frac{(1-\iota)\,X}{r\,(1+X)}\right]^{X}\frac{1}{1+X} \\
h &= \left[1+X+\frac{(1-\rho)(1-\iota)\,X}{\iota}\right]m \\
k &= \left[1+X-\rho\,(1-\iota)X\right]m
\end{aligned}
$$

2.8 The effect of the rating trigger covenant

The coupon increase ($\Delta > 1$) related to the rating trigger covenant has two different direct effects. An higher Δ, will lead to:

1. An higher coupon after the rating downgrade;
2. An higher value for the endogenous bankruptcy threshold, and thus an higher probability of bankruptcy.

Such effects will produce different impacts in the values of the bond, equity, leveraged firm, tax shields and bankruptcy costs. Starting with debt, the first effect has a positive impact on its value whereas the second effect influences it negatively since an higher of probability of bankruptcy unambiguously means higher bankruptcy costs. The final outcome will depend on the initial coupon value, C. For low coupon values, the first effect dominates. The opposite occurs for high coupon values. The same happens regarding the tax benefits, albeit in a more pronounced way. Nevertheless the effect of an higher Δ on equity value is unambiguously negative. Finally, in general, higher values of Δ will lead to a decrease in leveraged firm values, although for lower coupons, the positive effect on the value of the bond may slightly overcome the negative impact on equity. Figure 2 ilustrates what we have said. It depicts the debt, equity and leverage firm value (values taken from Bhanot and Mello (2006)[3]).

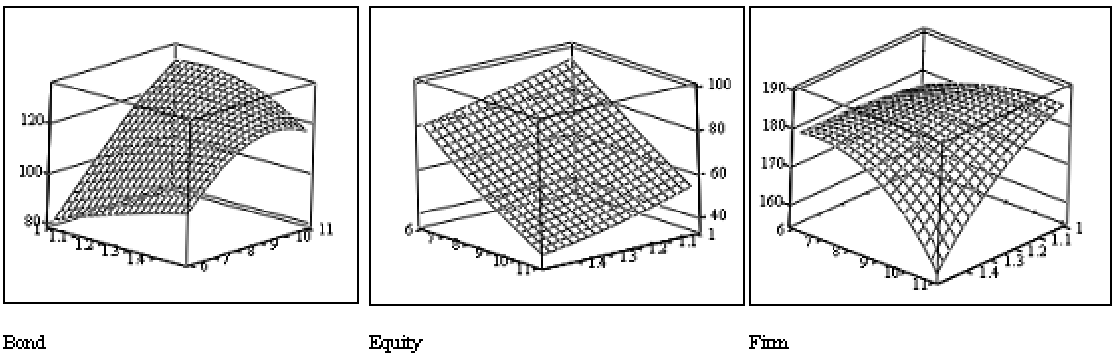

Figure 2: Bond value, equity value and leverage firm value as a function of coupon (ranging from 6 to 11) and Δ (ranging from 1 to 1,5), considering $V = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\sigma = 0,25$; $\iota = 0,35$ and $\rho = 0,4$.

3 The optimal debt level

3.1 The maximum coupon

As in Leland (1994)[17], each firm will face a maximum capacity to raise debt. Specifically, for a given value of Δ, there will be a value for the initial coupon (C) that maximizes the debt value. Formally we obtain this particular value solving the following equation:

$$\frac{\partial B^I(V_0,\cdot)}{\partial C} = 0$$

Obtaining:

$$C_M = \frac{V_0}{\Delta}\left[\frac{1+(\Delta-1)\left(\frac{V_0}{V_{B1}}\right)^{-X}}{\Delta k\,(1+X)}\right]^{1/X}$$

Once again with $\Delta = 1$, Leland's (1994) result is obtained. Notice that $\frac{\partial C_M}{\partial \Delta} < 0$, meaning that the use of such type of covenants reduces the firm's indebtedness capacity. Considering this result, it is worthwhile to point that the approach used by Bhanot and Mello (2006)[3] is in some way restrictive. The analyses conducted by those authors consisted in calculating the optimal level of straight debt (that is without any covenants) incurred by the firm and subsequently, for a given Δ - considering the existence of the covenant - in obtaining the particular coupon value that would generate that same amount of debt. Besides the fact that, in the firm's perspective, this debt level is far from being optimal (as we will see in the next subsection), it may be unattainable for some values of Δ, that is, higher than the correspondent maximum debt capacity. Illustrating considering a numerical example in which the debt level is 127,86[5] (value used by those authors considering $V = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\sigma = 0,25$; $\iota = 0,35$ and $\rho = 0,4$), the maximum possible value for the increase in the coupon embodied in the rating triggers covenant is 32% ($\Delta = 1,32$). For instance, for $\Delta = 1,4$, the maximum coupon is 10,388 and the correspondent debt value 124,64. For an asset value of 160, Bhanot and Mello (2006) framework would consider a debt level of 136,39. This amount would only be reachable for values of Δ lower than 1,24. Figure 3 below illustrates the influence of the coupon increase on the maximum coupon and the respective debt level considering two values for the initial assets value ($V_0 = 150$ and $V_0 = 160$).

3.2 The optimal coupon

In order to obtain the optimal debt level at the moment in which the securities are issued, we calculate the coupon that maximizes the leveraged firm value, ($v^I(V_0, C, \cdot)$). The corresponding first order condition implies that the first derivative of the leveraged firm value in respect to the coupon needs to zero:

$$\frac{\partial v^I(V_0, C,\cdot)}{\partial C} = 0$$

Resolving the above equation for C, yields:

$$C^* = \frac{V_0}{\Delta}\left[\frac{1+(\Delta-1)\left(\frac{V_0}{V_{B1}}\right)^{-X}}{\Delta h\,(1+X)}\right]^{1/X} \tag{22}$$

[5] Which corresponds to the optimal straight debt level when $V = 150$; $r = 0,07$; $\alpha = 0,01$; $\sigma = 0,25$; $\iota = 0,35$ and $\rho = 0,4$.

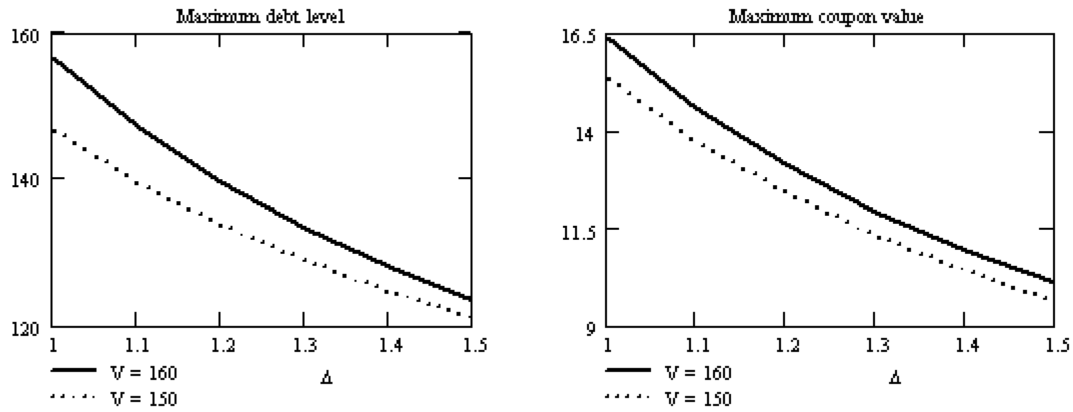

Figure 3: Left panel: maximum debt level at the emission date as a function of Δ; right panel: maximum coupon value as a function of Δ; considering: $V_0 = 150$ $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\sigma = 0,25$; $\iota = 0,35$ and $\rho = 0,4$.

From the above expression it can be verified that $\frac{\partial C^*}{\partial \Delta} < 0$, which means that the presence of this type of rating trigger covenants leads to lower optimal coupon values. Moreover, note that a decrease in the (optimal) coupon, generated by an higher Δ, more than compensates its own increase ($\frac{\partial \Delta C^*}{\partial \Delta} < 0$)[6] - even after the credit rating downgrade tackes place, the increased coupon value triggered by the covenant (ΔC^*) is less than the corresponding optimal coupon, if straight debt were issued instead[7]. Given that, it is not surprising that Δ has an unambiguously negative effect on the optimal debt level, as depicted in figure 4.

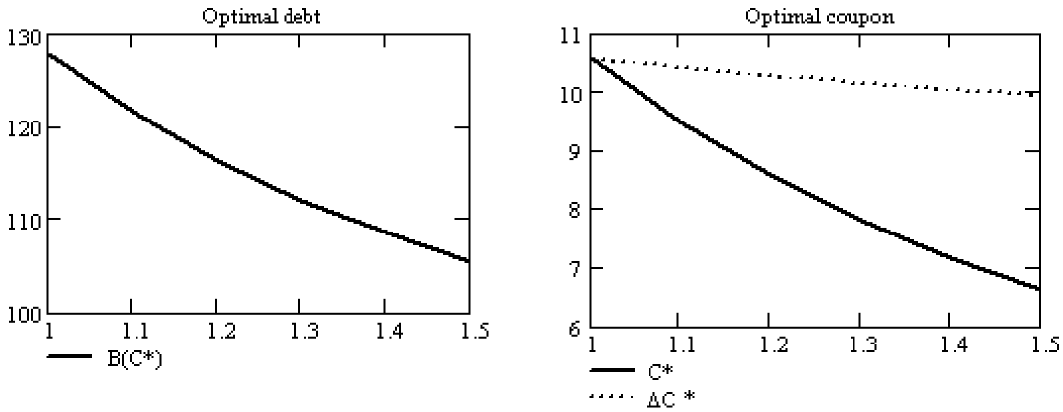

Figure 4: left panel: optimal debt level at the emission date as a function of Δ ; right panel: optimal coupon at the issuance date (C^*) and coupon value after the rating downgrade (ΔC^*) as a function of Δ. Both graphics considers $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\sigma = 0,25$; $\iota = 0,35$ and $\rho = 0,4$.

[6]Note that the relation of the endogenous bankruptcy threshold V_{B2} with Δ is reversed (being now negative) since it depends on ΔC. Incorporating the optimal coupon (expression (22)) in V_{B2} (expression(16)) yields $V_{B2}^* = V_0 \left\{ \frac{m}{\Delta h} \left[1 + (\Delta - 1)(V_0/V_{B1})^{-X} \right] \right\}^{1/X}$, and $\partial V_{B2}^* / \partial \Delta < 0$. Thus, higher values of Δ are associated with lower probabilities of bankruptcy.

[7]In fact, it can be showed that in the limit (when Δ tends to infinity) ΔC^* tends to the optimal coupon that would arise in the straight debt case when the asset value is V_{B1}.

The rationale of this result can be formulated as follows. Since the rating trigger ($\Delta > 1$) imposes an increase in the coupon rate, this higher coupon can be seen as a "forced" debt level rebalancing by the firm. But this forced rebalancing is on the opposite direction[8] of what would be the case if equity holders had the ability to freely restructure the capital structure, given the smaller value of assets (V_{B1}). In face of that, a lesser amount of debt is issued and a lower firm value is obtained (the decrease in the bankruptcy costs that arises from higher values of Δ is not enough to compensate for the decrease in the tax shield).

Plugging the optimal coupon (expression (22)) into expressions (17), (18) and (19) we obtain respectively the reduced formulas for debt, equity and leveraged firm values, at the date the debt is issued:

$$B^*(V_0,\Delta) = V_0 \left[\frac{h(1+X)-k}{r}\right]\left[\frac{1+(\Delta-1)\left(\frac{V_0}{V_{B1}}\right)^{-X}}{\Delta h(1+X)}\right]^{\frac{1+X}{X}} \tag{23}$$

$$E^*(V_0,\Delta) = V_0 \left[1-\frac{(1-\iota)[h(1+X)-m]}{r}\left[\frac{1+(\Delta-1)\left(\frac{V_0}{V_{B1}}\right)^{-X}}{\Delta h(1+X)}\right]^{\frac{1+X}{X}}\right] \tag{24}$$

$$v^*(V_0,\Delta) = V_0 \left[1+\frac{\iota h(1+X)}{r}\left[\frac{1+(\Delta-1)\left(\frac{V_0}{V_{B1}}\right)^{-X}}{\Delta h(1+X)}\right]^{\frac{1+X}{X}}\right] \tag{25}$$

From the above expressions, it can be inferred that Δ affects negatively debt and firm values ($\partial B^*/\partial\Delta < 0$ and $\partial v^*/\partial\Delta < 0$), and positively equity value ($\partial E^*/\partial\Delta > 0$)(see figure 5).

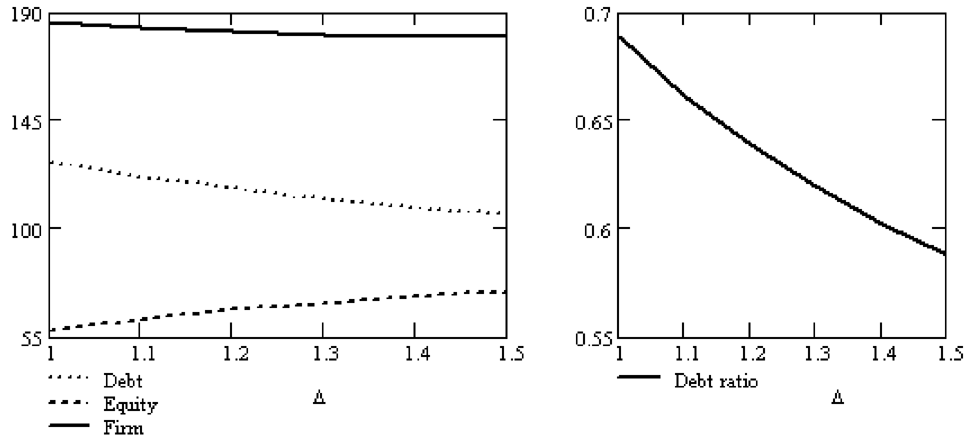

Figure 5: left panel: Leveraged firm, bond and equity values at the issuance date, considering the optimal coupon, as a function of Δ; right panel: Debt ratio, at the emission date, considering the optimal coupon (B^*/v^*), as a function of Δ. Both graphics considers $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\sigma = 0,25$; $\iota = 0,35$ and $\rho = 0,4$.

[8]Contrarily with what happens when the rating trigger covenant has attached a partial redemption of the debt principal instead of an increase in the coupon.

The positive relation in terms of equity value comes from the negative impact of Δ on the optimal coupon. Indeed, using expression (18), the total partial derivative of equity value in respect of Δ, considering the optimal coupon, is given by:

$$\frac{\partial E^{*}(V_0,\Delta)}{\partial \Delta} = \left.\frac{\S E^{I}(V_0,C,\Delta)}{\S \Delta}\right|_{C=C^{*}} = \left.\frac{\partial E^{I}(V_0,C,\Delta)}{\partial \Delta}\right|_{C=C^{*}} + \left.\frac{\partial E^{I}(V_0,C,\Delta)}{\partial C}\right|_{C=C^{*}} \frac{\partial C^{*}}{\partial \Delta}$$

Since, either $\partial E^{I}(\cdot)/\partial C$ and $\partial C^{*}/\partial \Delta$ have negative signs, their product yields a positive impact. Such indirect effect (trough the optimal coupon) more than compensates the negative direct influence of Δ on equity value ($\partial E^{I}(\cdot)/\partial \Delta < 0$).

As a final note, it must be pointed that the approach used by Bhanot and Mello (2006) (fixing the debt value at the optimal straight debt level ($\Delta = 1$)), can lead to misleading results. First of all, as previously referred (and shown in figure 5), the indebtedness of the firm would be clearly above the optimal level - in fact the relative excess in the coupon value regarding the correspondent optimal level could reach 40% (see figure 6). But the most importantly, is that it would give the misleading idea that a higher Δ negatively influences equity value, when indeed this relation is reversed at optimal levels. As we will see, this last result is central when the substitution of assets enters in the analysis.

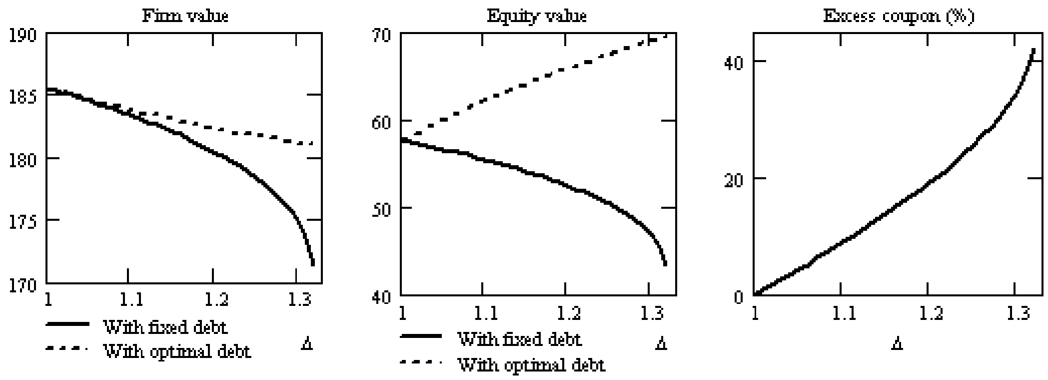

Figure 6: Leverage firm value (left panel) and equity value (middle panel) both as a function of Δ, considering optimal debt (dash line) and a fixed amount of debt (solid line); right panel: excess in the coupon value associated with a fixed amount of debt relative to optimal coupon value, in percentage, as a function of Δ. The graphics consider: $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\sigma = 0,25$; $\iota = 0,35$ and $\rho = 0,4$.

4 Asset substitution

One of the conclusions of the last section regarding the use of this type of rating trigger covenants in debt contracts was that the resulting value for the leverage firm was lower than the one associated with the use of straight debt. Given those inefficiency costs, in the perspective of the equity holders, this kind of covenants would never be a preferable option. However, as previously stated, one reason in favour of using this kind of debt relates to the asset substitution problem. Specifically, such covenants would have the ability to prevent equity holders from changing the firm's risk profile after debt is in place. Regarding this issue Bhanot and Mello (2006) concluded that: "In general, an increase in coupon level decreases firm value and does not inhibit (and may even stimulate) asset substitution"[9]. Such conclusion is different from ours. Indeed, we will show that not only coupon

[9] Remark 6 on page 91 from Bhanot and Mello (2006).

type rating trigger covenants can effectively induce equity holders to pursuit low risk strategies but also that the gains obtained in terms of agency costs can be greater than the inefficiency costs inherent to these covenants.

4.1 Operational flexibility and coupon rating triggers covenants

In the presence of operational flexibility, the substitution of assets occurs when, after debt is in place, equity holders have an incentive to rise the firm's risk level (the volatility parameter of the asset value diffusion process: σ), transferring value to equity in detriment of bondholders. As Leland (1994) showed and Bhanot and Mello (2006) have mentioned, this would happen if straight debt were used, since in that case the equity value would be a positive function of σ. Could the inclusion of coupon rating trigger covenant in the bond indenture alter this positive relation? The answer to this question is affirmative.

Recall that we can think of a bond promising a coupon payment of C until a rating downgrade occurs and ΔC (With $\Delta > 1$) thereafter, as the combination of two assets: a straight bond (without any covenant) that pays a coupon of C and an asset that generates an additional payment of $(\Delta - 1)C$ after the rating change. This distinction is useful to analyze and understand the effect of the asset volatility on the equity value when the rating trigger covenant is present. Let us remember the equity value formula obtained in section 2:

$$E^I(V_t, C, \Delta) = V_t \; - \; \frac{(1-\iota)\, C}{r} \; + \; \left[\frac{(1-\iota)\, C}{r} - V_{B2}\right]\left(\frac{V_t}{V_{B2}}\right)^{-X} \; -$$
$$- \; \frac{(1-\iota)\,(\Delta-1)\, C}{r}\left[\left(\frac{V_t}{V_{B1}}\right)^{-X} - \left(\frac{V_t}{V_{B2}}\right)^{-X}\right]$$

The first line of the above expression correspond to the equity value that would result from Leland's (1994) model, considering the issue of straight debt with a given coupon C. Such term is positively related to the volatility parameter. As for the second line, we have the adjustment on equity value associated to the rating trigger. Specifically, $\frac{(1-\iota)(\Delta-1)C}{r}\left[(V_t/V_{B1})^{-X} - (V_t/V_{B2})^{-X}\right]$ corresponds to the value of the additional payments (adjusted by the tax shields) made by equity holders to bondholders, as a result of a rating downgrade. This value depends on the probability-weighted discount factor of the value of assets reaching the barrier level V_{B1} $\left((V_t/V_{B1})^{-X}\right)$ and the bankruptcy threshold V_{B2} $\left((V_t/V_{B2})^{-X}\right)$. An higher volatility would increase both probabilities albeit, for some range of σ, the change in the former would be greater than the change in the latter - meaning that this adjustment value may also be a positive function of firm's risk. However, this component enters in the value expression with a negative sign (those payments reduces the dividend flow that accrues to equity holders), thus part of equity value could be negatively affected by an increase in σ. The higher the value of Δ the higher will be this negative effect be. The final impact of an increase in asset volatility on equity value will depend on the magnitude of these two opposing effects, which in turn will depend on the parameter values. The main point is that, for a given $\Delta > 1$, it may exist a range of firm risk levels where higher values of σ leads to lower values of equity values, contrarily to Bhanot and Mello (2006) conclusions.

The above analysis assumed the coupon as a given. What happens, when the optimal debt level (the optimal coupon) alongside with the endogenous bankruptcy trigger are considered? If we calculate the partial derivative of the optimal equity value (expression (24)) in respect to the volatility parameter ($\partial E^*/\partial\sigma$) we would conclude that it is almost always positive[10]. However such result is misleading in the sense that it incorporates a positive effect of volatility that comes from the optimal coupon ($\left.\frac{\partial E^I}{\partial C}\right|_{C=C^*} \frac{\partial C^*}{\partial \sigma} > 0$, because those two partial derivatives have negative

[10]For high values of Δ and in a range of lower values of σ, such partial derivative may turns out negative.

signs). Since the optimal coupon is only considered at the issuance date, we must distinguish the ex-ante from ex-post firm risk. The former corresponds to a "promised" risk level to bondholders, from which the optimal coupon is determined ($C^*(\sigma_{ex-ante})$) and the optimal amount of debt is issued. After debt is in place, this coupon does not change and what we previously stated holds[11] and concerns to the ex-post selection of firm's risk ($E^I(C^*, \sigma_{ex-post})$.

The main idea is that when coupon rating trigger covenant are present ($\Delta > 1$), after debt is in place, equity holders may have an incentive not to rise but to reduce the firm's risk profile since in doing so they are reducing the probability of the firm suffering a credit rating downgrade and consequently reducing the value of the extra payment attached to it. Such result will depend on the coupon increase embodied in the covenant (the specific value of Δ), on the promised firm's risk level ($\sigma_{ex-ante}$) and on the spectrum of possible risk levels available to equity holders. Figure 7 illustrates the above statement. The graph depicts the equity value as a function of $\sigma_{ex-post}$ and Δ assuming that optimal debt was issued with an ex-ante volatility of 0,25. As we can see, there exist a range of Δ values for which equity value increases with a sufficient reduction in firm's risk level.

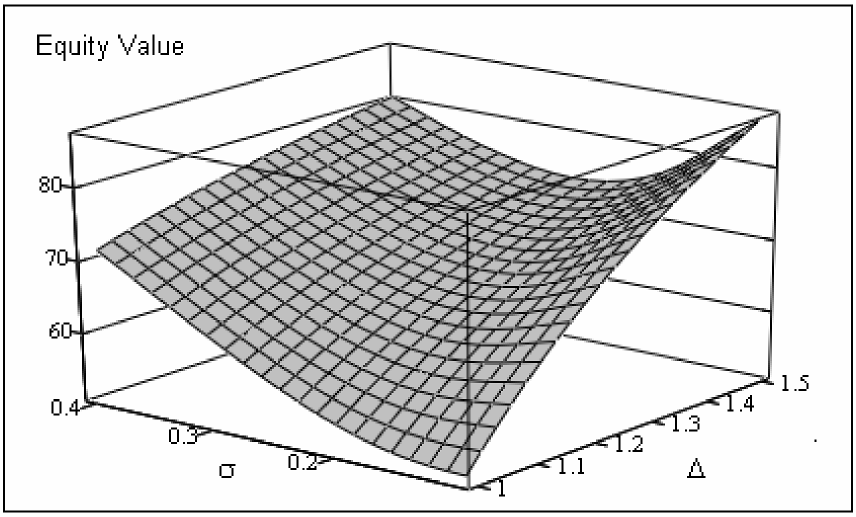

Figure 7: Equity value as a function of $\sigma_{ex-post}$ (ranging from 0,1 to 0,4) and of Δ (ranging from 1 to 1,5), after the emission of optimal debt assuming $\sigma_{ex-ante} = 0,25$; $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

4.2 A rational expectation framework and the equilibrium risk level

In a rational expectations setting[12], bondholders at the issue date, would only be willing to "accept" a "promised" risk level and thus to pay the corresponding debt value if it were a "credible" one, that is, if after the debt is in place, it would not be advantageous for equity holders to alter it. Thus, defining $\sigma^*_{ex-post}$ as the particular risk level, within the possible values associated with the operational flexibility, that maximizes the equity value after debt is in place, a rational expectations equilibrium occurs when $\sigma^*_{ex-post} = \sigma_{ex-ante}$.

Assume for now that the operational flexibility allows equity holders to choose between only two risk levels: σ_H and σ_L, where $\sigma_H > \sigma_L$. When straight debt is issued ($\Delta = 1$), given the positive relation between equity value and ex-post volatility, irrespective of $\sigma_{ex-ante}$, we will always

[11] In reality, the results are even reinforced.

[12] Such framework is used by Bhanot and Mello (2006) but they only applied it to partial amortization of principal rating trigger covenants since they concluded by the uselessness of coupon increase rating trigger covenants

have $\sigma^*_{ex-post} = \sigma_H$, and consequently, only the high risk level profile (σ_H) would constitute the rational expectation equilibrium. The numerical example in table 4 illustrates the case considering $\sigma_L = 0,10$ and $\sigma_H = 0,25$. When $\sigma_{ex-ante} = 0,10$, the corresponding optimal coupon value is 13,343, which would generate a cash inflow of 185,18. But, after debt is in place, equity holders would have an incentive to change the risk level to $\sigma_{ex-post} = 0,25$ since in doing so equity value would raise from 26,46 to 38,44 and the bond value would decrease to 143,19. Thus, anticipating this behaviour, bondholders wouldn't be wiling to finance the firm at that ex-ante risk level. If, instead the promised risk level were $\sigma_{ex-ante} = 0,25$, then equity holders wouldn't benefit from reducing it ex-post, since that would reduce the value of equity. Thus $\sigma_H = 0,25$ is the equilibrium level, with an associated firm value of 185,51. Note however that the firm value would be maximized at the low risk level (211,64). The difference between these two values derives from agency costs.

What happens when a coupon rating trigger is used? As we saw, the existence of such covenants alters the effect of ex-post volatility on equity values. In fact, depending on the particular value assumed by Δ, it is possible that $E^I(C^*(\Delta, \sigma_L), \Delta, \sigma_L) \geq E^I(C^*(\Delta, \sigma_L), \Delta, \sigma_H)$ be observed, resulting in a low risk profile equilibrium. Let us return to the numerical example of table 4 (lower panel). Consider a debt contract with a rating trigger covenant that imposes a 25% increase in the coupon value if a rating downgrade occurs ($\Delta = 1,25$). For a promised risk level of 0,10 ($\sigma_{ex-ante} = \sigma_L$), the optimal coupon would be 10,494, yielding a debt value of 148,07, an equity value of 51,22 and a firm value of 199,29. After debt is in place is there an incentive to pursuit higher risk strategies? No, since a higher ex-post volatility would mean a lower equity value (48,77). Equity holders would choose to maintain the low risk level $\sigma_{ex-post} = 0,10$, thus σ_L would correspond to an equilibrium. The same wouldn't happen if instead the ex-ante volatility were σ_H ($\sigma_{ex-ante} = 0,25$), since in that case the equity value would be maximized with a σ_L ex-post volatility ($\sigma_{ex-post} = 0,10$)[13].

$\Delta = 1$					
$\sigma_{ex-ante}$	Optimal coupon	$\sigma_{ex-post}$	Equity	Debt	Firm
0,25	10,609	0,25	57,65	127,86	185,51
		0,10	51,05	151,29	202,80
0,10	13,343	0,25	38,44	143,19	181,63
		0,10	26,46	185,18	211,64
$\Delta = 1,25$					
$\sigma_{ex-ante}$	Optimal coupon	$\sigma_{ex-post}$	Equity	Debt	Firm
0,25	8,177	0,25	67,50	114,23	181,73
		0,10	72,82	118,59	191,41
0,10	10,494	0,25	48,77	128,73	177,50
		0,10	51,22	148,07	199,29

Table 1: Equity, debt and firm value considering $\sigma_{ex-post}$ equal to 0,10 or 025, for different values of $\sigma_{ex-ante}$, assuming debt without covenant, $\Delta = 1$ (upper panel) and, debt with a rating trigger covenant that increases the coupon value in 25% after a rating downgrade, $\Delta = 1,25$ (lower panel); for $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

What debt contract should equity holders sold? Albeit the rating trigger covenant carries an inefficiency cost, as seen in the last section, it permits to avoid the agency costs present in straight debt. Thus the answer will depend on the magnitude of these two opposite effects. If the former is

[13] Note also that in that case, the reduction in ex-post volatility besides rising equity value would also rise the bond value. Despite this result, equity holders would always be better of promising a low risk ex-ante volatility since they obtain an higher value ($199,29 > 181,73 + (72,82 - 67,50)$).

greater then the latter, debt without covenant should be used, otherwise a rating trigger covenant is preferable. That is what happens in our numerical example. With the covenant, the equilibrium firm value is 199,29 which is greater than the corresponding value associated with straight debt equilibrium (185,51).

In order to conduct a generalization of the above analysis we will separate two aspects: the existence of Δ values for which a low a risk level equilibrium occurs; and the circumstances for which such equilibriums are optimal compared to the straight debt case.

4.3 Coupon rating trigger covenants and the low risk level equilibrium

We begin this section formulating the following statement:

Statement 1: When equity holders have operational flexibility to choose between two risk level profiles, σ_L and σ_H, with $\sigma_L < \sigma_H$, in principle it exists a bond with a coupon rating trigger covenant with an attached $\Delta = \Delta^*$ such that:

$$E^I(V_0, C^*(\Delta^*, \sigma_L), \Delta^*, \sigma_L) = E^I(V_0, C^*(\Delta^*, \sigma_L), \Delta^*, \sigma_H)$$

Moreover, for $\Delta > \Delta^*$, then:

$$E^I(V_0, C^*(\Delta, \sigma_L), \Delta, \sigma_L) > E^I(V_0, C^*(\Delta, \sigma_L), \Delta, \sigma_H)$$

$\square$

Implicit in the above statement is the idea that for reasonable values of potencial risk levels, we can always find a particular value of Δ (Δ^*) attached to a coupon rating trigger covenant that has the property of inhibiting equity holders from rising the ex-post risk level after debt is in place, an thus capable of giving credibility to an ex-ante low risk level.
The argument is as follows:

As we saw in section 3, when the optimal coupon is taken into account, the influence of Δ on equity value is always positive. Putting it differently, for a given volatility, an higher value of Δ leads to an increase in the equity value ($\partial E^*(V_0, \Delta, \sigma)/\partial\Delta = \partial E^I(V_0, C^*(\Delta, \sigma), \Delta, \sigma)/\partial\Delta > 0$). However, such increase is not σ-independent. Specifically, the lower the volatility , the greater the increase in equity value ($\frac{\partial(E^*(V_0,\Delta,\sigma)/\partial\Delta)}{\partial\sigma} < 0$)[14].

When the distinction between the ex-ante and ex-post volatility is made, the same result is obtained. That is, for a given $\sigma_{ex-ante}$, we have[15]:

$$\frac{\partial(\frac{\partial E^I(V_0, C^*(\Delta, \sigma_{ex-ante}), \Delta, \sigma_{ex-post})}{\partial\Delta})}{\partial\sigma_{ex-post}} < 0$$

What is the importance of this result? As we have seen, considering straight debt (with $\Delta = 1$), we always obtain[16]:

$$E^I(V_0, C^*(\Delta = 1, \sigma_L), \Delta = 1, \sigma_L) < E^I(V_0, C^*(\Delta = 1, \sigma_L), \Delta = 1, \sigma_H)$$

[14]Note that, $\frac{\partial E^*(\cdot)}{\partial\Delta} = \frac{(1-\iota)}{r}\left[1 - \left(\frac{V_0}{V_{B1}}\right)^{-X}\right] C^* \left(\frac{h(1+X)-m}{\Delta h X}\right)$, and C^*, X, m and h all depend on σ.

[15]The partial derivative, $\frac{\partial E^I(V_0, C^*(\Delta, \sigma_{ex-ante}), \Delta, \sigma_{ex-post})}{\partial\Delta}$, is given by:

$$\frac{(1-\iota)C^*}{r}\left[m^p(1+X^p)\Theta - \Lambda + \left(\frac{1}{\Delta X}\right)\left[1 + (\Delta-1)\Lambda - \Delta m^p(1+X^p)\Theta\right]\left[X + \frac{1-\left(\frac{V_0}{V_{B1}}\right)^{-X}}{1+(\Delta-1)\left(\frac{V_0}{V_{B1}}\right)^{-X}}\right]\right],$$

Where $\Theta = \left(\frac{V_0}{\Delta C^*}\right)^{-X^p}$, $\Lambda = \left(\frac{V_0}{V_{B1}}\right)^{-X^p}$, C^*, X depends on $\sigma = \sigma_{ex-ante}$ and finally X^p, m^p are the same as X and m, but depends on $\sigma = \sigma_{ex-post}$.

[16]Remember that the volatility in $C^*(\cdot)$ is an ex-ante volatility wether in $E^I(\cdot)$ is an ex-post volatility.

What happens when the Δ value is increased? Both terms increase, but not by the same amount. The equity value with lower ex-post volatility (left hand term) will suffer a higher increase than the equity value with the higher ex-post volatility (right hand term), thus shrinking the difference between those two values. Eventually, for a sufficiently high value of Δ (Δ^*), the inequality turns to an equality, and for values higher than Δ^* the inequality is reversed. Let us give a numerical example (Table 2). For different values of Δ, considering $\sigma_{ex-ante} = \sigma_L = 0,10$, the optimal coupon is obtained. After debt is in place, we calculate the equity value for the two possible ex-post risk levels ($\sigma_L = 0,10$ and $\sigma_H = 0,25$). As table 2 shows, when straight debt is used ($\Delta = 1$), the low risk ex-post equity value is 26,46, while the high risk ex-post equity value is 38,44. If instead, a rating trigger covenant entailing a 10% increase in the coupon in case of a credit rating downgrade, were used ($\Delta = 1,1$), this would imply an equity value increase of 11,4 and 4,69 for the low and high ex-post volatility cases respectively. When the coupon increase is 19,85%, the equity values are the same for the two ex-post risk scenarios ($\Delta^* = 1,1985$). Thus, for coupon rating trigger covenants with $\Delta \geq 1,1985$, bondholders will be willing to accept the promised low risk level and pay the corresponding debt value since they know that after debt is in place they won't be "hurt" because equity holders don't have any incentive to alter the firm's risk. For $\Delta \geq 1,1985$, σ_L is a risk level equilibrium.

Δ	Optimal coupon ($\sigma_{ex-ant} = \sigma_L$)	Equity ($\sigma_{ex-post} = \sigma_L$)	Equity ($\sigma_{ex-post} = \sigma_H$)
1	13,343	26,46	38,44
1,1	12,041	37,76	43,13
1,1985	**10,98**	**46,99**	**46,99**
1,25	10,494	51,22	48,77

Table 2: Optimal coupon, assuming $\sigma_{ex-ante} = \sigma_L = 0,10$, and equity value for the two risk level scenarios ($\sigma_{ex-post} = \sigma_L = 0,10$ and $\sigma_{ex-post} = \sigma_H = 0,25$) considering different values of Δ; with $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

Moreover, fixing the low volatility level, the higher the upper volatility, the higher Δ^* will be. Conversely, fixing the high volatility level, the higher the lower level of volatility, the higher Δ^* will be (Figure 8).

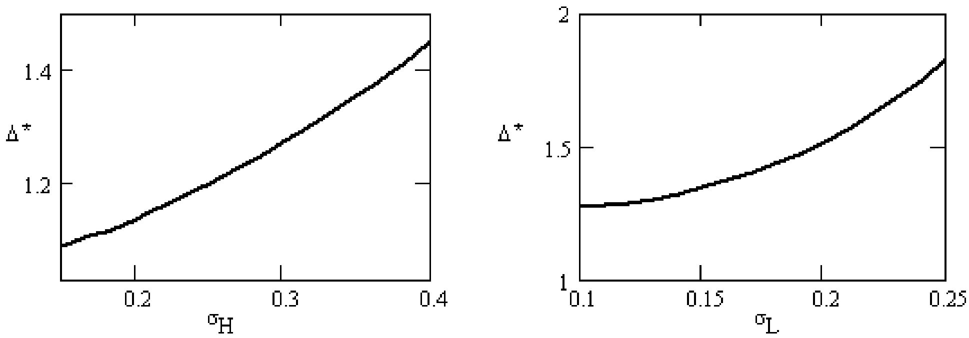

Figure 8: Left panel: Δ^* as a function of σ_H (ranging from 0,15 to 0,40), considering $\sigma_L = 0,10$; Right panel: Δ^* as a function of σ_L (ranging from 0,10 to 0,25), considering $\sigma_H = 0,30$; Both panel assume: $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

From the above analysis one question arises:

- What happens when the operational flexibility is not restricted to only two risk profiles (σ_L

and σ_H) but instead refers to an interval $[\sigma_L, \sigma_H]$?

There will be no change in the conclusions previously stated, since even when we were in the presence of an interval of possible risk levels, the analysis would always be centred between the two extremes values of that interval, namely σ_L and σ_H. The reason for that lies in the fact that equity value is a convex function of the ex-post volatility, irrespective of the particular value assumed by the ex-ante volatility. This means that, depending on the specific value taken by Δ, the ex-post volatility that maximizes the value of equity is always either σ_L or σ_H. Figure 9 illustrates this point. It depicts the equity value as a function of ex-ante and ex-post volatility, considering a Δ value of 1,25[17].

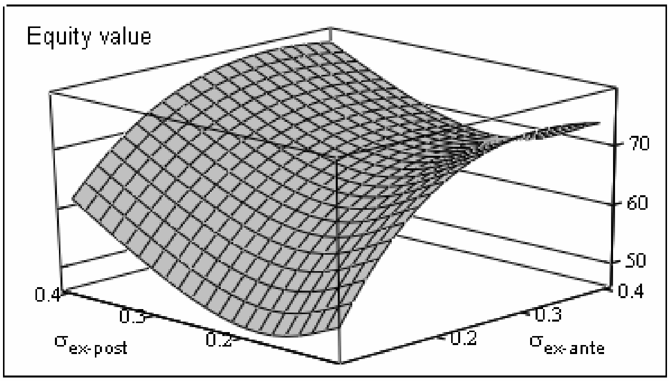

Figure 9: Equity value as a function of $\sigma_{ex-post}$ (ranging from 0,1 to 0,4) after the optimal debt is in place for $\Delta = 1,25$ and different values of $\sigma_{ex-ante}$ (ranging from 0,1 to 0,4), with $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

4.4 The optimal debt contract in the presence of operacional flexibility

After analyzing the existence of specific coupon increases ($\Delta \geq \Delta^*$), attached to a coupon a rating trigger covenant, capable of making a low risk level an equilibrium one, we turn to question about the optimality of such debt contracts when compared with straight debt.

Before beginning, let us state that in what concerns the coupon rating trigger covenant type of debt, only the particular Δ^* is relevant. Why? Because, as referred in section 3, the value of the leveraged firm is a decreasing function of Δ. Thus within all possible values of Δ (greater or equal than Δ^*), Δ^* is the one that maximizes the value of the firm.

Regarding the choice between the issuing straight debt or debt embodied with a Δ^* rating trigger covenant the following statement is formulated:

Statement 2: In a rational expectation framework, when equity holders have operational flexibility to choose a risk level within an interval $[\sigma_L, \sigma_H]$, for low values of σ_L there exist a σ^* such that $\sigma^* > \sigma_L$ for which:

$$v^*(V_0, \Delta^*, \sigma_L) = v^*(V_0, \Delta = 1, \sigma^*)$$

If $\sigma_H > \sigma^*$, then:

$$v^*(V_0, \Delta^*, \sigma_L) > v^*(V_0, \Delta = 1, \sigma_H)$$

and it is optimal to use a coupon rating trigger covenant in the debt contract.

Conversely, if $\sigma_H < \sigma^*$, then:

$$v^*(V_0, \Delta^*, \sigma_L) < v^*(V_0, \Delta = 1, \sigma_H)$$

[17]Notice that the ex-ante volatility, along with the Δ value, only determines the optimal coupon. After the optimal debt is in place, the equity value will depend on the ex-post volatility.

and it is optimal to issue straight debt.
For high values of σ_L, σ^* may not exist, and the optimal contract debt is always provided by straight debt.

□

Remember from section 3 that for a given volatility, the firm value that would result from the use of straight debt is always higher than the one associated with a coupon rating trigger covenant because of the inefficiency cost inherent to the latter. On the other hand, the lower the firm's risk is, the higher the corresponding firm value will be. Thus, in the presence of operational flexibility, the firm value would be maximized if straight debt were used and if the lower risk level were chosen ($v^*(V_0, \Delta = 1, \sigma_L)$). The problem arises after debt is in place, since, when straight debt is used, the equity holders would have an incentive to raise the firm's risk level (asset substitution problem). That is why, in a rational expectations framework, the equilibrium risk level, for straight debt, is always the higher risk profile, resulting in a firm value of $v^*(V_0, \Delta = 1, \sigma_H)$. The difference between those two values derives from the corresponding agency costs. The greater is the difference between the higher and the lower risk profiles (that is for low values of σ_L and high values of σ_H) the greater would those agency costs be. Let us return to the coupon rating trigger covenant debt. Albeit this kind of debt contract incorporates an inefficiency cost, when operational flexibility is present and for a specific coupon increase embodied in the covenant (Δ^*), it has the ability, in a rational expectations framework, of making the equilibrium risk level the lower risk profile (σ_L), resulting in a firm value of $v^*(V_0, \Delta^*, \sigma_L)$. Although this value is always lower than $v^*(V_0, \Delta = 1, \sigma_L)$ (being the difference, the inefficiency cost) , it can be higher than $v^*(V_0, \Delta = 1, \sigma_H)$ which constitutes the straight debt outcome. Thus, what underlies statement 2 is the simple idea that, when the agency cots of straight debt are greater than the inefficiency cots associated with the coupon rating trigger covenant debt, the latter should be used, since it maximizes the firm value. Otherwise, straight debt constitutes the optimal choice. The specific value of σ^*, is no more no less than the value of σ_H for which these costs are equal, yielding a same firm value. Figure 10 depicts the difference between the firm value when the coupon rating trigger covenant is used, and the firm value associated with straight debt ($v^*(V_0, \Delta^*, \sigma_L) - v^*(V_0, \Delta = 1, \sigma_H)$), as a function of σ_H, considering four values of σ_L.

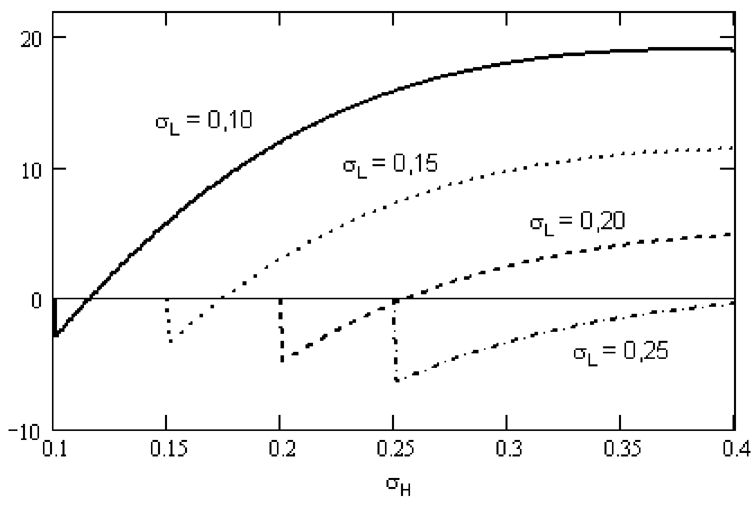

Figure 10: Difference between firm value when coupon rating trigger covenant are present and firm value when straight debt is used, ($v^*(V_0, \Delta^*, \sigma_L) - v^*(V_0, \Delta = 1, \sigma_H)$) as a function of σ_H (ranging from σ_L to 0,4), for different values of σ_L (0,10; 0,15; 0,20 and 0,25), with $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

Generally speaking, the higher the spectrum of possible risk level profiles, the higher the superiority of coupon rating trigger covenant type of debt relative to straight debt will be. Notice however that this difference does not depend solely on the gap between the two extreme risk levels, but fundamentally on the particular value assumed by the low risk level. Specifically, for a fixed gap, the lower the σ_L the higher will be the difference between the firm values. Put it differently, a low value of σ_L yields a low value of σ^* (which is the value of σ_H for which the two firm values are the same), as σ_L is increased, this will result also in an increase in σ^*, but in a more pronounced way, as figure 10 illustrates (σ^* corresponds to the intersection points of the curve lines with the horizontal axis) and table 3 shows. In fact, for high values of the low risk level (e.g. σ_L = 0,25), σ^* may even not exist.

σ_L	σ^*	Δ^*
0,10	0,116	1,062
0,15	0,174	1,119
0,20	0,255	1,359
0,25	-	-

Table 3: σ^* (the value of σ_H for which: $v^*(V_0, \Delta^*, \sigma_L) = v^*(V_0, \Delta = 1, \sigma_H)$) and the correspondent Δ^* value for different values of σ_L; with $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

This outcome is explained by the fact that, for a fixed gap, the lower the low volatility level is, the higher the agency costs of straight debt are and the lower the inefficiency cots of coupon rating trigger covenant debt will be (since a lower coupon increase (Δ^*) is needed), raising the difference between both firm values. Returning to table 3, if $\sigma_L = 0,10$, for values of σ_H greater than 0,116, it is optimal to use a coupon rating trigger covenant type of debt. In contrast, if $\sigma_L = 0,25$, is always optimal to use straight debt. Table **??**eports firm values associated with the two types of debt ($v^*(V_0, \Delta^*, \sigma_L)$ and $v^*(V_0, \Delta = 1, \sigma_H)$) along with the agency and inefficiency cots considering different values for low and high risk levels.

Risk levels		**Straight Debt**		**Coupon rating trigger**			**Difference**
σ_L	σ_H	Firm value	Agency costs	Δ^*	Firm value	Inefficiency costs	
0,10	0,15	200,85	10,79	1,087	206,67	4,97	+ 5,82
	0,25	185,51	26,13	1,198	201,40	10,25	+15,88
	0,40	173,68	37,96	1,448	192,64	19,0	+18,96
0,15	0,20	192,18	8,68	1,155	195,22	5,63	+ 3,05
	0,25	185,51	15,34	1,239	192,78	8,07	+ 7,27
	0,40	173,68	27,18	1,618	185,09	15,76	+ 11,42
0,20	0,25	185,51	6,66	1,344	185,18	7,00	- 0,34
	0,40	173,68	18,5	2,05	178,49	13,68	+ 4,82

Table 4: Firm value associated with straight debt ($v^*(V_0, \Delta = 1, \sigma_H)$) and the corresponding agency cost($v^*(V_0, \Delta = 1, \sigma_L) - v^*(V_0, \Delta = 1, \sigma_H)$); firm value associated with a coupon rating trigger covenant ($v^*(V_0, \Delta^*, \sigma_L)$), the corresponding coupon increase (Δ^*) and the inefficiency costs ($v^*(V_0, \Delta = 1, \sigma_L) - v^*(V_0, \Delta^*, \sigma_L)$); the difference between the two firm values ($v^*(V_0, \Delta^*, \sigma_L) - v^*(V_0, \Delta = 1, \sigma_H)$), for different risk levels profiles; with $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

5 Reversibility case

So far, one of the assumptions underlying the analysis was that after the rating downgrade (after the first threshold had been crossed) the bondholder would receive the new (increased) coupon forever or until the occurrence of bankruptcy, irrespective of further movements of the value of the assets. Specifically, after the occurrence of a rating downgrade, if the value of the assets crossed V_{B1} from below, which could be seen as a firm's credit rating upgrade, the coupon value would not revert to the initial value (as before the rating downgrade), since it was assumed that the effects of the covenant were permanent and irreversible. Notice that such assumption leads to an hysteresis phenomena regarding the firm's, equity and debt values. Concretely, the values of these variables, considering the same asset value (V), are different after and before the rating downgrade.

In the present section that assumption will be relaxed. Thus, we consider that whenever the value of the assets crosses V_{B1} from above, will generate a rating downgrade which in turn leads to an increase in the coupon payment to bondholders. However, we also consider that whenever the value of the assets crosses V_{B1} from below, the credit rating returns to the initial level and consequently the coupon is also re-established at the initial value. In other words, the increased coupon is only effective as long as the value of the assets remains below the credit change threshold. Thus the effects of the covenant are now temporary instead of permanent.

5.1 Bond value

In this new framework, for valuation purposes, the relevant distinction is no more before or after τ_1, but rather above or below V_{B1}. The diference between these two states lies on the coupon received by the bondholder (C in the first case and ΔC in the second). We will continue to define B^I (B^{II}) as the debt value when V_t is above (below) the credit change threshold (V_{B1}). Those value expressions will be obtained solving the following differencial equations:

- For $V_t > V_{B1}$:

$$\frac{\sigma^2}{2} V^2 B^I_{VV} + (r-\alpha) V B^I_V - rB^I + C = 0 \tag{26}$$

- For $V_t \leq V_{B1}$:

$$\frac{\sigma^2}{2} V^2 B^{II}_{VV} + (r-\alpha) V B^{II}_V - rB^{II} + \Delta C = 0 \tag{27}$$

Where B^i_V and B^i_{VV}, represents the first and second partial derivative of $B^i(V,\cdot)$, $(i = I, II)$ in relation to V respectively.

The solutions of (26) and (27) are respectively:

$$B^I(V,\cdot) = A_1 V^{-X} + A_2 V^{-X+2\theta} + \frac{C}{r} \tag{28}$$

$$B^{II}(V,\cdot) = A_3 V^{-X} + A_4 V^{-X+2\theta} + \frac{\Delta C}{r} \tag{29}$$

Where X is defined as previously, and $\theta = \frac{\sqrt{(r-\alpha-\sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2}$. Notice that $-X < 0$ and $-X + 2\theta > 1$.

The constants A_1, A_2, A_3 and A_4 are to be determined with the following boundaries conditions:

a_1) $\lim_{V \to \infty} B^I(V,\cdot) < \infty$

b_1) $B^I(V_{B1},\cdot) = B^{II}(V_{B1},\cdot)$

$c_1)$ $\quad B^{I}_{V}(V_{B1},\cdot) = B^{II}_{V}(V_{B1},\cdot)$

$d_1)$ $\quad B^{II}(V_{B2},\cdot) = \rho V_{B2}$

Condition a_1) states the nonexistence of speculative bubbles, implying that the debt value must be finite for high values of the firm's assets, leading to $A_2 = 0$. Conditions b_1) and c_1) relates the value matching and smooth pasting conditions of debt value when the state of the firm in terms of credit rating changes (downgrade vs upgrade). Finally, condition d_1) corresponds to the value matching condition that must be satisfied regarding the bankruptcy event. After some algebra the following expressions are obtained:

$$B^{I}(V_t, C, \Delta) = \frac{C}{r} + \left(\rho V_{B2} - \frac{C}{r}\right)\left(\frac{V_t}{V_{B2}}\right)^{-X} + \frac{(\Delta-1)\,C}{r}\left[\left(\frac{V_t}{V_{B1}}\right)^{-X} - \left(\frac{V_t}{V_{B2}}\right)^{-X}\right] -$$

$$- \frac{(\Delta-1)\,C}{r}\left(\frac{X}{2\theta}\right)\left(\frac{V_t}{V_{B1}}\right)^{-X}\left[1 - \left(\frac{V_{B1}}{V_{B2}}\right)^{-2\theta}\right] \tag{30}$$

$$B^{II}(V_t, C, \Delta) = \frac{\Delta C}{r} + \left(\rho V_{B2} - \frac{\Delta C}{r}\right)\left(\frac{V_t}{V_{B2}}\right)^{-X} -$$

$$- \frac{(\Delta-1)\,C}{r}\left(\frac{X}{2\theta}\right)\left(\frac{V_t}{V_{B1}}\right)^{(-X+2\theta)}\left[1 - \left(\frac{V_t}{V_{B2}}\right)^{-2\theta}\right] \tag{31}$$

Note that the resulting formulae are similar to those ones obtained in section 2 (expression (3) and (4)) except for the last term. Indeed, the bond value is equal to the previous one plus this adjustment term which is related to the reversibility of the covenants. This adjustment enters with a negative sign, lowering the debt value, since it captures the reduction on bondholders cash flow, resulting from a possible upgrade after the occurrence of downgrade.

5.2 Equity value

In terms of equity, as for the debt, the difference between the two states (above or below V_{B1}) lies on dividends received by the equity-holder ($\alpha V - (1-\iota)\,C$ in the first case and $\alpha V - (1-\iota)\,\Delta C$ in the second). Defining E^{I} and E^{II} as the equity value for the two possible states, these are governed by the following differential equations:

- For $V_t > V_{B1}$:

$$\frac{\sigma^2}{2}V^2 E^{I}_{VV} + (r-\alpha)\,V E^{I}_{V} - rE^{I} + \alpha V - (1-\iota)\,C = 0 \tag{32}$$

- For $V_t \leq V_{B1}$:

$$\frac{\sigma^2}{2}V^2 E^{II}_{VV} + (r-\alpha)\,V E^{II}_{V} - rE^{II} + \alpha V - (1-\iota)\,\Delta C = 0 \tag{33}$$

Whose correspondent solutions are, respectively:

$$E^{I}(V,\cdot) = A_1 V^{-X} + A_2 V^{-X+2\theta} + V - \frac{(1-\iota)\,C}{r} \tag{34}$$

$$E^{II}(V,\cdot) = A_3 V^{-X} \; + \; A_4 V^{-X+2\theta} \; + V \; - \frac{(1-\iota)\,\Delta C}{r} \tag{35}$$

The constants A_1, A_2, A_3 and A_4 are to be determined with the following boundaries conditions:

a_2) $\quad \lim_{V\to\infty} E^I_V(V,\cdot) \leq 1$

b_2) $\quad E^I(V_{B1},\cdot) = E^{II}(V_{B1},\cdot)$

c_2) $\quad E^I_V(V_{B1},\cdot) = E^{II}_V(V_{B1},\cdot)$

d_2) $\quad E^{II}(V_{B2},\cdot) = 0$

Once again, condition a_2) states the nonexistence of speculative bubbles, implying that for high values of the firm's assets, any additional increase in this variable is reflected in an increase, at most in of the same magnitude, in equity value, leading to $A_2 = 0$. Conditions b_2) and c_2) relates the value matching and smooth pasting condition of the equity value that must be verified when the firm's rating is downgraded (V_{B1} is crossed from above) or upgraded (V_{B1} is crossed from below). Finally, condition d_2) imposes that when the firm's assets are liquidated (in the event of bankruptcy) equity holders gets nothing.

Applying these boundary conditions yields the final expressions for equity:

$$\begin{aligned} E^I(V_t, C, \Delta) = V_t \; - \; \frac{(1-\iota)\,C}{r} + \left[\frac{(1-\iota)\,C}{r} - V_{B2}\right]\left(\frac{V_t}{V_{B2}}\right)^{-X} \; - \\ - \frac{(\Delta-1)\,(1-\iota)\,C}{r}\left[\left(\frac{V_t}{V_{B1}}\right)^{-X} - \left(\frac{V_t}{V_{B2}}\right)^{-X}\right] + \\ + \frac{(\Delta-1)\,(1-\iota)\,C}{r}\left(\frac{X}{2\theta}\right)\left(\frac{V_t}{V_{B1}}\right)^{-X}\left[1-\left(\frac{V_{B1}}{V_{B2}}\right)^{-2\theta}\right] \end{aligned} \tag{36}$$

$$\begin{aligned} E^{II}(V_t, C, \Delta) = V_t - \frac{(1-\iota)\,\Delta C}{r} + \left(\frac{(1-\iota)\,\Delta C}{r} - V_{B2}\right)\left(\frac{V_t}{V_{B2}}\right)^{-X} \; + \\ + \frac{(\Delta-1)\,(1-\iota)\,C}{r}\left(\frac{X}{2\theta}\right)\left(\frac{V_t}{V_{B1}}\right)^{(-X+2\theta)}\left[1-\left(\frac{V_t}{V_{B2}}\right)^{-2\theta}\right] \end{aligned} \tag{37}$$

As for the debt, we can verify that these expressions are similar to expressions (7) and (8) obtained in section 2, except for the last term. Thus, the equity values with temporary covenant effects are also equals to those with permanent effects plus a component associated to the reversibility of the coupon increase. However, such adjustment adds value to equity since the occurrence of an upgrade (after a downgrade had taken place), re-establishes the coupon payments to their initial values, (C instead of ΔC) and the continuous dividend stream rises in conformity.

To obtain the endogenous bankruptcy triggers we must consider the smooth pasting condition associated with the boundary condition d_2), namely:

$$\partial E^{II}(\cdot)/\partial V_t\big|_{V_t=V_{B2}} = 0$$

Thus, using expression (37) we get:

$$V_{B2}\frac{(1+X)\,r}{X\,(1-\iota)\,C} \;+\; (\Delta-1)\left(\frac{V_{B2}}{V_{B1}}\right)^{-X+2\theta} - \Delta = 0 \tag{38}$$

From the above equation we can see that the optimal threshold has no closed form solution so it can only be obtained numerically. Note also that for a same coupon (C) and coupon increase (Δ) the endogenous V_{B2} obtained from equation (38) is lower than the one associated with the previous case. Given the temporary nature of the coupon increase, the equity holders are willing to support higher negative dividends before entering in bankruptcy.

5.3 Optimal debt

Turning to the optimal debt, we determine the optimal coupon by maximizing the leverage firm value at the emission date: $v^I(V_0, C, \Delta) = E^I(V_0, C, \Delta) + B^I(V_0, C, \Delta)$. As usual the first condition for this optimization problem is that the first derivative of the leverage firm value in respect to the coupon be zero. Notice however that in calculating this derivative we must bear in mind that the optimal bankruptcy trigger (V_{B2}) also depends on the coupon. In fact, equation (38), implicitly defines V_{B2} as a function of C, besides the other parameters ($V_{B2}(C;\cdot)$). Thus $\partial v^I/\partial C = 0$ is given by the following equation[18]:

$$\frac{\partial E^I}{\partial C} \;+\; \frac{\partial E^I}{\partial V_{B2}}\frac{\partial V_{B2}}{\partial C} \;+\; \frac{\partial B^I}{\partial C} \;+\; \frac{\partial B^I}{\partial V_{B2}}\frac{\partial V_{B2}}{\partial C} = 0 \tag{39}$$

The endogenous bankruptcy threshold value of V_{B2} along with the optimal coupon (C^*) are thus obtained numerically, solving simultaneously equation (38) and (39).

Since, in the present case, the increase in the coupon is only temporary - it is only effective while the value of the assets remains below the threshold V_{B1} - one would expect that the corresponding optimal coupon, alongside with the optimal debt, would be higher than in the previous case - where, after the first passage time of the value of the assets thought V_{B1}, a permanent coupon increase occurs. This is illustrated in Figure 11. The negative relation of the optimal coupon (C^*) with Δ still holds[19] but the rate of decrease is now, in absolute terms, much lower ($|(\partial C^*/\partial\Delta)_{temporary}| < |(\partial C^*/\partial\Delta)_{permanent}|$) which, besides resulting in an higher value for C^*, has the particularity of turning the accrued coupon (ΔC^*) a positive function of Δ ($\partial\Delta C^*/\partial\Delta > 0$).

Comparing with the pure debt case, albeit an higher Δ means an higher coupon payment while the asset value is below the credit change threshold (V_{B1}), resulting in a lower dividend stream, it also means a lower coupon payment when the asset value is above it, which in turn rises the equity value. Since the latter effects overcomes the former, the equity value continues to be positively related to Δ[20].

[18] To obtain the partial derivative $\partial V_{B2}/\partial C$ we make use of the implicit function theorem. Specifically, define the left hand side of equation (38) as $F(V_{B2}, C)$, then:

$$\frac{\partial V_{B2}}{\partial C} = -\frac{\partial F(V_{B2},C)/\partial C}{\partial F(V_{B2},C)/\partial V_{B2}} = \frac{V_{B2}/C}{1+\frac{(\Delta-1)(1-\iota)CX(-X+2\theta)}{r(1+X)V_{B1}}\left(\frac{V_{B2}}{V_{B1}}\right)^{(-X+2\theta-1)}}$$

.

[19] Depending on the particular value assumed by V_{B1}, for very low volatility values this relation may turn positive with high values of Δ, or even be strictly positive .

[20] Related to what was refereed in the previous footnote, for very low volatility values we may obtain a negative relation between the equity value and Δ.

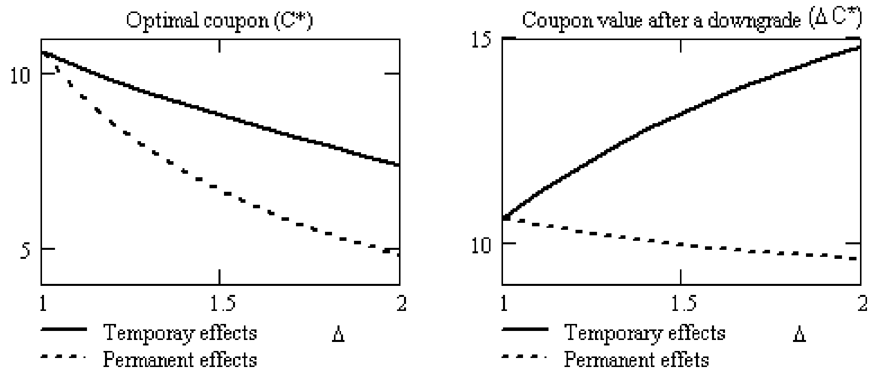

Figure 11: Right panel: optimal coupon as a function of Δ, when the coupon increase is temporary (solide line) or permanent (dashed line); left panel: coupon value after the occurrence of a downgrade (ΔC^*), when the coupon increase is temporary (solid line) or permanent (dashed line); with $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\sigma = 0,25$; $\iota = 0,35$ and $\rho = 0,4$.

5.4 Asset substitution

What happens to the Δ^* value? Remember from the previous sections that Δ^* was defined as the particular value of the coupon increase for which the low volatility level (σ_L) would constitute a rational expectation equilibrium, specifically, Δ^* would verified the following equation: $E^I(V_0, C^*(\sigma_L), \Delta^*, \sigma_{ex-post} = \sigma_L) = E^I(V_0, C^*(\sigma_L), \Delta^*, \sigma_{ex-post} = \sigma_H))$. For the present case, when Δ^* exists[21] it assumes a significantly much higher value than the one associated with the permanent effects case. In other words, given the temporary nature of the coupon increase, it is more "difficult" to force equity holders to pursuit low risk strategies. The consequence of that is a comparatively lower firm value albeit, in some cases, it could still be higher than the one associated with straight debt as table 5 shows.

Risk levels		**Temporary coupon rating trigger**			**Difference**	
σ_L	σ_H	C^*	Δ^*	Firm value	Permanent effects	Straight debt
0,15	0,20	10,97	2,144	191,24	- 3,98	- 0,94
	0,25	10,82	2,693	188,26	- 4,52	+ 2,75
0,20	0,25	8,80	1,999	181,53	- 3,65	- 3,98
	0,40	7,27	2,918	175,77	- 2,72	+ 2,09

Table 5: Optimal coupon, Δ^* and the correspondent firm value ($v^I(V_0, C^*(\sigma_L), \Delta^*, \sigma_L)$), considering a coupon rating trigger covenant with temporary effects ; the difference between that firm value and the ones associated with permanent effects and straight debt, for different risk levels profiles; with $V_0 = 150$; $V_{B1} = 120$; $r = 0,07$; $\alpha = 0,01$; $\iota = 0,35$ and $\rho = 0,4$.

6 Conclusion

We have determined the optimal values concerning debt, equity and the leverage firm value when the debt contract is embedded with a covenant which establishes a permanent increase in the coupon rate when the credit rating of the firm is downgraded. It has been shown that the use of these type of rating trigger covenant generates inefficiency costs reflected in a lower firm value when compared with the use of straight debt. Nevertheless, in the presence of operational flexibility,

[21]When Δ and the optimal coupon are positively related, Δ^* doesn't exists.

such covenants have the ability to "force" the equity holders to sustain a low risk level profile and thus preventing the occurrence of asset substitution. In this environment, the decision concerning what kind of debt should be issued will depend on the tradeoff between the agency costs of straight debt and the inefficiency costs attached to the rating trigger covenant type of debt, which in turn will depend on the available spectrum of risk profiles. Specifically, for low values of the lower volatility level, the covenant should be used, otherwise, the straight debt is a preferable option. When the increase in the coupon, triggered by the credit rating downgrade, is not permanent but instead is only effective when the assets value is above the credit rating threshold, the advantages associated to the covenant are reduced albeit in some cases can still generate better results than straight debt.

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