BUNDLED FINANCIAL CLAIMS - A MODEL OF HYBRID CAPITAL

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Abstract. There is a large class of infinite horizon financial instruments which incorporates elements of both debt and equity, collectively denoted "hybrid capital". The Bank for International Settlements (BIS) has devised the fundamental requirements for how hybrid capital may qualify as a part of core ("Tier 1") regulatory capital for banks. We present valuation models for hybrid capital in the set-up of Black and Cox (1976) and Leland (1994) and derive new valuation formulas incorporating these special features. In particular, we take into account the issuer's right to omit hybrid coupon payments and to call the hybrid capital at par value starting from a given date. In doing so, we build on formulas developed in Mjøs and Persson (2005). We show that hybrid capital actually carry risk and clarify interesting links between their valuation and overall corporate capital structure as guidance both for market participants and regulators alike.

1. Introduction

Increasingly complex structured securities are a consequence of the sophistication of financial markets. The commercial drive behind this development is - as always - to seek any remaining arbitrage opportunities either in terms of risks or rewards. Complexity is a function of issuer and investor preferences, investment banks' commercial creativity and finally regulatory, legal or tax frameworks. In the case of hybrid capital[1] which encompass elements of both debt and equity, the

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[1]*Hybrid capital* is a class of infinite horizon coupon-paying securities for which the value is bounded above similarly as for debt, but which carry almost the same downside risk as equity. These securities have many names but the generic structures are very similar on a global basis due to commonality in the regulation of the

regulatory and tax-considerations define the most unique characteristics. Typical 'hybrid capital' is a perpetual coupon paying security, senior only to common equity, includes an issuer option to call the security as from 10 years after the date of issue and the right of the issuer to forego coupon-payments without it constituting a default. In sum, the commercial "raison d'être" for hybrid capital is that it is a qualifying form of risk-carrying capital which in most cases have tax-deductible dividends or coupons. Our focus is to better understand the valuation of and capital structure impacts from hybrid capital generally and in particular the specific structures usually issued by banks. Standard asset pricing models value each separate element included in a complex security and conduct a "sum-of-the-parts" valuation. This approach disregards any endogenous effects on the securities and the issuer caused by a combined set of fixed and conditional claims. Our paper models, values and contributes to a better understanding of hybrid capital and comments on its effect on optimal shareholder bankruptcy behavior for a given capital structure. The complexity makes the valuation a particularly challenging research task, even under strict assumptions regarding to market efficiency and symmetric information. Mjøs and Persson (2005) developed fundamental valuation formulas using a barrier-options-approach, whilst we in this paper analyze a completely specified version of hybrid capital as found in the capital markets.

1.1. **The market for hybrid capital.** As of mid 2005, the (global) stock of outstanding hybrid capital was estimated at \$376 billion[2] Amongst the issuers were 57 % banks, 8 % insurance companies, 15 % utilities and 12 % industrial companies. This dominance of regulated industries reflects both that the securities are tailor-made to specific regulations and that the infinite horizon is well suited to sectors where regulations and supervision reduce the expected default risk. An international comparison also shows that this dominance is even clearer in markets outside the US.

All banks in the developed part of the world are under strict regulations compared to other sectors, in particular regarding risk exposure, risk management and required capital. The foundation for the global regime for risk capital in banks is the 1998 Basle Accord on Capital Standards, see Committee on Banking Supervision (1988). This document laid out the general principles for calculating the prudent capital requirements for banks and the criteria for what constituted acceptable forms of risk capital, as well as certain deductions from this capital. We introduce these regulations in more detail below. In addition to

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issuers, primarily banks and insurers. Preferred stock, Trust Preferred Stock and Capital Securities are terms used on variants of hybrid capital, in particular in the US.

[2]Source: Lehman Brothers "The Capital Securities Market. Composition and Trends".

strengthening the global regulatory focus on risk capital in financial institutions, the Accord represented a standard which secured a large degree of commonality between capital regulations between jurisdictions. An element of this was a more standardized global structure for hybrid capital. National variations include tax-deductability and contingent rights in case of financial distress. This high degree of standardization of hybrid capital issued by banks and to a large degree also insurance companies leads us to focus our examples on hybrid capital for financial institutions. The valuation formulas and related analysis are generic and may be usefully applied in any setting, not just for banks or insurance companies.

Although a research subject in itself, we conduct our analysis under the assumption that an issuer's main motivation for issuing hybrid capital is to optimize between regulatory requirements, after-tax cost-of-capital and capital-structure considerations. A bank may issue hybrid capital instead of raising new common equity and thus both potentially save tax and avoid the direct and indirect costs related to seasoned equity offerings. Benston, Irvine, Rosenfeld, and Sinkey (2003) analyze 105 issues of hybrid capital by US bank holding companies in the years 1995-1997 with regards to what characterizes issuers vs. non-issuers. They find distinct differences in that issuers are larger, have higher tax-rates, more uninsured funding and lower equity ratios. Their event studies also find significant abnormal returns on the common stock of issuers who filed after the Federal Reserve's new favorable tax-treatment of hybrid capital[3] was announcement on 21 October 1996. The latter is taken as a confirmation of the shareholder value enhancement from such issues.

1.2. **Regulatory framework.** Banks and insurance companies are subject to extensive regulations. Researchers have not yet reached a consensus as to what market failures that justify such regulations, but in his review, Santos (2000), points to two common issues: Firstly, the risk of a systemic crisis when the banks as liquidity providers experience a "run" from depositors. Secondly, fragmented depositors' limited incentives and ability to properly monitor banks and thus the implicit risk of moral hazard in banks. These risks are met by deposit insurance, rules and supervision regulating the creation and operation of banks, and finally specific capital requirements to capture a sufficient part of the remaining risk exposure of both society and depositors.

[3]Trust Preferred Securities(TPS)

Under the current regulations[4] the assets of a bank are weighted primarily according to credit risk and the capital requirement is 8 % of this basis. This requirement may be met by capital of different priority and risk-exposure. Tier I includes common equity and hybrid capital, Tier II constitutes subordinated debt and is split between a perpetual "upper-level" and a dated "lower-level" category. Tier III is short term subordinated debt which may only cover market price risks.

The basic requirements defined by BIS for hybrid capital to qualify as part of Tier I capital are[5]:

"Hybrid (debt/equity) capital instruments. This heading includes a range of instruments which combine the characteristics of equity capital and of debt. Their precise specifications differ from country to country, but they should meet the following requirements:

- they are unsecured, subordinated and fully paid-up,
- they are not redeemable at the initiative of the holder without the prior consent of the supervisory authority,
- they are available to participate in losses without the bank being obliged to cease trading (unlike conventional subordinated debt),
- although the capital instrument may carry an obligation to pay interest that cannot permanently be reduced or waived (unlike dividends on ordinary shareholders' equity), it should allow service obligations to be deferred (as with cumulative preference shares) where the profitability of the bank would not be supported.

Cumulative preference shares, having these characteristics would be eligible for inclusion in this category."

National implementations of these principles are necessarily more practically phrased and are usually very similar as to the following (Find ref!!):

- Hybrid capital is only senior to equity capital in case of distress and liquidation.
- Hybrid capital have to be fully paid-in, any authorized payments are not included.
- The issuer have the right to call the securities at coupon-payment[6] dates after 10 years from the date of issue. The execution of this

[4]BIS is well under way with a new set of capital requirements popularly labelled "Basel II" which includes a broader range of risks evaluated when setting the capital requirements. Basel II also represents a significant improvement in how individual institutions may calculate their exact capital requirements. (ref) These regulations are expected to be implemented around xx.

[5]Committee on Banking Supervision (1988) Annex 1, D, (d), page 18-19.

[6]We choose to denote the amounts paid to service the securities "coupons" even though some versions of hybrid capital are categorized as preference shares which receive dividend payments and not coupons.

option requires explicit approval from the supervisory authority in charge of the bank to secure that a repayment of the hybrid capital does not make the issuer too weak in capital terms.

- To absorb risk in financially distressed situations, non-payment of coupons should not cause default.
- That hybrid capital should not exceed [15 %] of total Tier I capital.

The main variations between different countries' regulations regards whether unpaid coupons are permanently omitted or may be accumulated up to a maximum period (e.g., 5 years) but then constitutes a default, the position of the capital relative to equity in the case of refinancing a distressed bank, the maximum step-up of the coupon-rate if the call option is not exercised at the first possible date, and the tax-treatment of coupons for both issuer and investor. Some countries allow direct hybrid issuance where the coupons are fully treated as debt-coupons, these include the Scandinavian countries, Spain++. In the USA the most common form used by bank holding companies (BHCs) has been Trust Preferred Capital which has been issued by a special purpose vehicle which then re-lend the funds to the bank holding company as Upper Tier II subordinated debt. This structure allows for Tier I treatment of the funds in the consolidated bank-group accounts and tax-treatment as for regular debt, but no Tier I effect on the bank holding company separately.(Extend, refine and find references!)

In our analysis, we model hybrid capital as infinite horizon subordinated debt with a finite, embedded call-option, a step-up in the coupon-rate and an exogenously given risk of non-payment of coupons. These combined features characterize most issues of hybrid capital.

1.3. **Literature overview.** Hybrid capital has been studied from various angles, primarily related trying to explain issuance (e.g., Benston et al. (2003) as discussed above), its role in a regulatory perspective (e.g., Santos (2000)) and some event-studies of the effect of issuance on common stock values (see Krishnan and Laux (2005) as commented below).

The most direct precedence of our work is the paper by Emanuel (1983) which develops a valuation of preferred stock, equivalent to our term hybrid capital, based on the option-methodology of Black and Scholes. Emanuel applies a geometric Brownian motion for the development of the total value of the firm and develops partial differential equations for the value of equity, debt and preferred stock. The paper gives a comprehensive motivation for what criteria that drives management's dividend payment decisions and argues that preferential dividend rights and voting rights conditional on passed dividends are equivalent. The basic model assumes that as soon as firm value exceeds initial value, current and any accumulated preferred dividends will be paid. The derivative of preferred stock with respect to firm value is subject to current firm value and the size of any arrearage of preferred dividends, exemplifying the mix of debt and equity features. Preferred stock with non-cumulative dividends are also modelled by excluding the arrearage-term. Our paper extends Emanuel in some aspects. We provide an alternative way of modelling potential omission of hybrid coupons. Common for both approaches is that hybrid coupons are stopped when the asset value process hits an exogenous, arbitrary lower boundary (omission level). Our model includes debt class with priority before hybrid capital and thus contains a second boundary (bankruptcy level) where the company is in default. The choice of the first boundary, the omission level, affects the optimal bankruptcy level in our model. Another contribution compared to Emanuel's paper is the analysis of the issuer's embedded call option commonly found in practice. We develop closed form formulas for this option and also show how the existence of this option affects the optimal bankruptcy level and thereby the initial pricing of other contingent claims on the firm, in particular debt.

Regarding the existence and impact of hybrid capital per se, some specific papers are worth mentioning:

Engel, Erikson, and Maydew (1999) analyze the issuance of trust preferred stock using a sample of 158 issuances for the period 1993-1996. They find that issuers are willing to incur on average \$ 10 million in direct and \$ 43 million in indirect costs to reduce the debt-to-asset ratio by 12.8 % and also that the Net Present Value of the tax-benefits in replacing traditional preferred stock with trust preferred stock are around 28 %. Finally, they also find that the tax benefits are partially transferred to the investors (implicit taxes) through a higher pre-tax yield on tax-advantaged vs. regular preferred stock.

Krishnan and Laux (2005) study the impact of issuance of trust preferred stock on the return of common stock. They find significant positive abnormal return when the issuer gains specific financial benefits from the issue.

Beatty (2005) analyze whether changes in accounting treatment of hybrid capital impacts banks' propensity to issue hybrid capital when capital regulations are unchanged. The paper shows that market discipline is important in that bank behavior actually changed following the accounting changes.

Engel et al. (1999), Benston et al. (2003) and Krishnan and Laux (2005) all include extensive descriptions of the market for and structure of hybrid capital, in particular as seen in a US perspective.

1.4. **Focus of the paper.** Lack of contingent control when coupon-payments are defaulted, subordination after all debt, perpetual maturity and non-cumulative coupons are all equity-like features of hybrid capital. On the other hand, seniority before common equity, tax-deductible coupons, issuer call-features and no share in any profits are the main debt-like features. Our focus is on the valuation of these instruments in a given capital-structure setting and its impact on optimal shareholder bankruptcy behavior. Interesting topics like agency-problems and issuance dynamics are left out.

Our model is based on the Merton (1974)/Black and Cox (1976) models as extended by Leland (1994) and Goldstein, Ju, and Leland (2001) although we do not introduce frictions like tax or bankruptcy costs. Our model can not be used to explicitly analyze optimal capital structure. Throughout the paper we maximize the market value of equity by calibrating the coupon-rates of various debt and hybrid contracts.

Our paper is organized as follows: Section 2 presents our specifications of hybrid capital. Section 3 reproduces the standard results of Black and Cox (1976) with one class of debt. Section 4 explains our main results. Section 5 presents numerical examples. Section 6 discusses various specific features of hybrid capital in more detail. Section 7 contains numerical sensitivities and graphical illustrations and section 8 concludes. Some calculations are left for appendices.

2. Our Specification of Hybrid Capital

We include the following properties of hybrid capital:

(1) Hybrid capital has priority after senior debt, but before equity. In terms of priority it is similar to regular junior or subordinated debt. To incorporate this property hybrid capital is modelled as Black and Cox (1976) junior debt. In cases with several classes of debt with different priorities, hybrid capital will always have the lowest rank.
(2) Issuer has the right to call (repay) the hybrid capital at par at coupon-dates (usually quarterly) after a fixed period (usually 10 years) from the date of issue. For simplicity we assume that this option is *European* and only exercisable at the first possible date. Our assumed exercise strategy is in correspondence with observed market practice, see Mjøs and Persson (2005), who also develop valuation formulas capturing this aspect of hybrid capital.
(3) Unpaid coupons represent an irrevocable loss for the investor. They do not trigger bankruptcy, nor are they accumulated as

additional debt[7]. To explicitly model the possibility to omit payment of hybrid capital coupons we introduce an exogenous asset threshold level under which coupon payments are not paid.

(4) If the call option is not exercised at the first possible date, the annual coupon-rate is increased by a contractually agreed *step-up* (typically 75 - 150 bp).

Regular perpetual junior debt may contain a call option and coupon rate step-up. The crucial difference between junior debt and hybrid capital is thus the issuer's right to omit coupons without causing an event of default. BIS as well as leading rating agencies consider this feature as critical to accept hybrid capital as the highest ranking (Tier I) risk capital and not just as junior(Tier II) debt for financial institutions.

3. The valuation model and basic results

We consider the standard Black-Scholes-Merton economy and impose the usual perfect market assumptions:

- All assets are infinitely separable and continuously tradeable.
- No taxes, transaction cost, bankruptcy costs, agency costs or short-sale restrictions.
- There exists a continuously compounded constant riskless rate of return r.

We study a limited liability company with a capital structure consisting of three claims, infinite horizon debt, hybrid capital, and common equity. In line with Goldstein et al. (2001), we assume that the the company generates an EBIT (earnings before interest rates and taxes) cashflow denoted by δ_t given by

$$d\delta_t = \mu\delta_t dt + \sigma\delta_t dW_t, \tag{1}$$

where μ and σ are constants representing the drift and volatility respectively, and δ_0 is given. Here W_t is a standard Brownian motion under the equivalent martingale measure. The total market value A_t of the assumed perpetual EBIT stream from the company equals

$$\begin{aligned} A_t &= E_t^Q\left[\int_t^\infty e^{-r(s-t)}\delta_s ds\right] \\ &= \frac{\delta_t}{r-\mu}. \end{aligned} \tag{2}$$

[7] In the USA and some other jurisdictions, also hybrid capital for financial institutions with either cumulative interest or certain rights to the investors following a defined number (e.g. 20) of consecutive missed coupon payments qualifies as core/Tier 1 capital whilst most European and other supervisory authorities require no cumulation of missed coupons. We choose to focus on the non-cumulative setting. In Norway, issuers may omit coupons provided no dividends are paid on common stock. (Extend, specify, motivate!!)

This quantity is elsewhere in the literature referred to as *the unlevered value of the firm's assets.* This market value follows the process

$$dA_t = (rA_t - \delta_t)dt + \sigma A_t dW_t = \mu A_t dt + \sigma A_t dW_t \tag{3}$$

Observe that the volatility parameter of this market value is identical to the volatility of the cashflow process. We use the notation $A = A_0 = \frac{\delta_0}{r-\mu}$.

A general claim f on the assets under these assumptions satisfies the partial differential equation, see, e.g., Merton (1974),

$$\frac{1}{2}\sigma^2 A_t^2 f_{AA} + \mu A_t f_A - rf + f_t + C(A_t, t) = 0, \tag{4}$$

where f_t denotes the partial derivative of f with respect to (elapsed or calendar) time, f_A the partial derivative of f with respect to A_t, and f_{AA} the partial derivative of f_A with respect to A_t. Here $C(A_t, t)$ represents the time t continuous net coupon rate received by the owner of the claim f.

3.1. **A recollection of the Black and Cox (1976) results.** We review the results of Black and Cox (1976) and Leland (1994) for the simple case with one class of regular infinite horizon debt. Observe that we assume an underlying EBIT-process following Goldstein et al. (2001) and that equity-holders receive cashflows in excess of debt coupons as dividends. This assumption leads to a drift parameter μ different from the risk free rate r in the asset price process.

We denote the face value of the debt by D and assume that contractual debt payments per unit of time are given by cD, where the coupon rate c is assumed constant.

Let $\bar{A}$ denote the lower boundary of A_t where debt payments are stopped and the shareholders transfer the assets of the firm to the debtholders. We refer to $\bar{A}$ as the *bankruptcy level.*

The *time t market value* of one monetary unit paid upon bankruptcy, i.e., when the process A_t hits the boundary $\bar{A}$ is

$$F_t = F_t(A_t, \bar{A}) = (\frac{A_t}{\bar{A}})^{-\beta}, \tag{5}$$

where[8]

$$\beta = \frac{\mu - \frac{1}{2}\sigma^2 + \sqrt{(\mu - \frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} > 0. \tag{6}$$

[8]It can actually be shown that $\beta > \frac{2\mu}{\sigma^2}$.

3.2. **Valuation of debt and equity.** The time 0 market value of infinite horizon debt with continuous constant coupon payment is

$$D(A) = \frac{cD}{r} - J_D F_0(A, \bar{A}), \tag{7}$$

where

$$J_D = \frac{cD}{r} - \bar{A}.$$

Expression (7) for the market value of debt carries a nice intuition. $\frac{cD}{r}$ is the time 0 market value of infinite horizon default-free debt receiving a coupon payment rate of cD. In case of risky debt ($\bar{A} < D$), the debtholder upon bankruptcy looses a stream of infinite coupon payments which at the time of bankruptcy has market value $\frac{cD}{r}$. This loss has a time 0 market value of $F_0 \frac{cD}{r}$. The time 0 market value of debt when $\bar{A} = 0$ is then the difference between the market values of these two coupon streams. In a more realistic setting, $\bar{A} > 0$ and this is also the liquidation payoff to debt in case of bankruptcy. We can therefore interpret $J_D = (\frac{cD}{r} - \bar{A})$ as the debtholder's *net loss* upon bankruptcy. The time 0 market value of this net loss $J_D F_0$ therefore represents the reduction of the time 0 market value of riskfree debt due to default risk.

Since we have not included any market inefficiencies, e.g., taxes, liquidation costs, or strategic debt service opportunities at this stage, the time 0 value of equity is the residual of the market value of assets less the market value of debt, or

$$E(A) = A - D(A) = A - \frac{cD}{r} + J_D F_0(A, \bar{A}), \tag{8}$$

using equation (7).

3.3. **The optimal bankruptcy level.** Given c and D, we assume that the equityholders choose $\bar{A}$ so that the value of equity is maximized. By maximizing expression (8) with respect to $\bar{A}$ we determine the optimal $\bar{A}^*$ as

$$\bar{A}^* = \frac{\beta}{(\beta + 1)} \frac{cD}{r}. \tag{9}$$

4. Main results

We model hybrid capital including the properties described in section 2 and assume a more realistic capital structure including both infinite horizon senior debt, hybrid capital and equity. Our valuation results for a given capital structure facilitate the calibration of optimal bankruptcy levels and coupon rates.

We develop our valuation formulas by first determining payoffs and market values of all claims at time T, the time of expiry of the hybrid capital embedded option. We subsequently determine the time zero value of all claims, partially by applying barrier option formulas with the time T market values as underlying assets. Finally, we discuss the default risk of senior debt and hybrid capital respectively.

Our analysis is based on the assumption that the bankruptcy asset level before the expiration of the option is constant, but possibly different from the long term bankruptcy asset level. This assumption ignores any time dependencies caused by decreasing time to maturity of the hybrid capital embedded option. A companion paper, Mjøs and Persson (2005), demonstrates that this assumption has negligible influence on the resulting calibrated coupon rates for reasonable parameter values.

We define the stopping time τ as

$$\tau = \inf\{t \geq 0; A_t = B\}. \tag{10}$$

Here τ can be interpreted as the time of bankruptcy (if $\tau \leq T$).

4.1. **Time T valuation of the claims.** The market values of senior debt, hybrid capital and equity at time T represent the first step in the valuation of the financial claims at time zero.

4.1.1. *Valuation of senior debt at time T.*

Proposition 1. *The time T market value of senior debt with face value D and coupon rate c_S for a given bankruptcy asset level $\bar{A}$ is*

$$D_T^*(A_T) = \frac{c_S D}{r} - J_D^* F_T(A_T, \bar{A}), \tag{11}$$

where

$$J_D^* = \frac{c_S D}{r} - \min(\bar{A}, D),$$

and F_T is defined in expression (5).

Proof. This is the standard result for regular Black and Cox (1976) debt from expression (7) where the bankruptcy payoff term is replaced with $\min(\bar{A}, D)$ to explicitly include the case where senior debt has no default risk, i.e., $\bar{A} > D$. □

4.1.2. *Valuation of hybrid capital at time T.* The payoff to hybrid capital at time T is

$$H_T^* = \begin{cases} H_T - \max(H_T - H, 0) & \text{for } \tau > T \\ 0 & \text{otherwise,} \end{cases} \tag{12}$$

where H_T is given by expression (7) for standard Black and Cox (1976) debt. Note, however, the following differences: The face value of debt D is replaced by the face value of hybrid capital H. The coupon rate c is replaced by the stepped up coupon-rate $c_H + k$. Here c_H represents the coupon rate of hybrid capital before time T and k represents the increase *(step-up)* in this rate at time T provided that the option to call the hybrid capital has not been exercised.

Thus, the total time T payoff from hybrid capital H_T^* equals infinite horizon debt less the payoff from a *European call*-option on the hybrid capital exercisable at time T with exercise price H, provided the company is not bankrupt.

Proposition 2. *The time T market value of hybrid capital with face value H and priority after senior debt with face value D, coupon rate c_H, time T coupon rate step-up k, for a given bankruptcy asset level $\bar{A}$, and given asset threshold value level U below which hybrid capital coupons are not paid, is*

$$H_T^*(A_T) = \frac{(c_H + k)H}{r} - J_H^* F_T(A_T, \bar{A}), \tag{13}$$

where

$$J_H^* = \frac{(c_H + k)H}{r} \frac{(\beta y^{-\alpha} + \alpha y^{\beta})}{(\alpha + \beta)} - (\bar{A} - D)^+, \tag{14}$$

$$\alpha = \beta + 1 - \frac{2\mu}{\sigma^2},$$

$$y = \frac{U}{\bar{A}} \geq 1, \tag{15}$$

and β is given in expression (6).

Proof. See appendix B. Expression (13) follows from equation (29), replacing c by $c_H + k$, and G by $(\bar{A} - D)^+$ to adjust the liquidation payoff due to the existence of senior debt. □

As opposed to J_D in the original debt expression (7) J_H^* represents the potential net loss to holders of hybrid capital both from bankruptcy and unpaid coupons.

The constant y represents the ratio between the coupon omission asset level U and the longterm bankruptcy asset level $\bar{A}$. In the case when $k = 0$ (no coupon rate step-up) and $y = 1$ (no risk of omitted coupons) this expression is simplified to the standard case of junior and senior debt. The term $(\bar{A} - D)^+$ in expression (14) reflects that hybrid capital has priority after senior debt in case of bankruptcy.

4.1.3. *Valuation of equity at time T.* The market value of equity at time T follows from the identity $E_T^*(A_T) = A_T - D_T^*(A_T) - H_T^*(A_T)$ and equations (13) and (11)

$$E_T^*(A_T) = A_T - \frac{c_S D + (c_H + k)H}{r} + (J_D^* + J_H^*) F_T(A_T, \bar{A}). \tag{16}$$

The optimal long term bankruptcy level at time T, denoted by $\bar{A}^*$, is found by differentiating expression (16) with respect to $\bar{A}$. The solution is

$$\bar{A}^* = \frac{\beta}{\beta + 1} \left(\frac{c_D D}{r} + \frac{(c_H + k)H}{r} y^{-\alpha}\right). \tag{17}$$

In the case where $k = 0$ and $y = 1$, $\bar{A}^*$ is identical to the similar quantity in the standard case with junior and senior debt.

4.2. **Time 0 valuation of the claims.** We now value the different claims at time 0 and include the effect of the embedded option in hybrid capital. The embedded call option has a fixed maturity, and its market value depends on remaining time to expiry. We assume a constant bankruptcy level B before time T. The long term bankruptcy level $\bar{A}^*$ is affected by this assumption through the coupon rates c_H and c_S.

4.2.1. *Valuation of senior debt at time* 0. The time 0 value of senior debt is equal to the time 0 market value of receiving D_T^* at time T if the company has not gone bankrupt, plus the time 0 market value of cashflows received before time T.

Proposition 3. *The time zero market value of senior debt is*

$$D_0^*(A) = \frac{c_S D}{r} - J_D^0 V_0(B) - J_D^* V_1(\bar{A}), \tag{18}$$

where

$$\begin{aligned} V_0(B) &= (\frac{A}{B})^{\alpha} N(d_1) + (\frac{A}{B})^{-\beta} N(d_2), \\ d_1 &= \frac{\ln(\frac{B}{A}) + (\mu - \frac{1}{2}\sigma^2 - \sigma^2\beta)T}{\sigma\sqrt{T}} \\ d_2 &= \frac{\ln(\frac{B}{A}) - (\mu - \frac{1}{2}\sigma^2 - \sigma^2\beta)T}{\sigma\sqrt{T}} \\ V_1(\bar{A}) &= (\frac{A}{\bar{A}})^{-\beta} N(-d_2) - (\frac{\bar{A}}{B})^{\beta} (\frac{A}{B})^{\alpha} N(d_1), \\ J_D^0 &= \frac{c_S D}{r} - \min(B, D), \end{aligned}$$

and β is given in expression (6) *and J_D^* in Proposition 1.*

Proof. The time 0 market value of debt consists of the market value of time T debt plus the market value of coupon payments and a potential bankruptcy payoff before time T, i.e.,

$$D_0^*(A) = \tag{19}$$

$$E^Q[D_T^* e^{-rT} 1_{\{\tau > T\}}] + E^Q[\int_0^{\tau \wedge T} c_S D e^{-rs} ds] + E^Q[\min(B, D) e^{-r\tau} 1_{\{\tau \leq T\}}].$$

Denote

$$V_0(B) = E^Q[e^{-r\tau} 1\{\tau \leq T\}],$$

where τ is defined in expression (10). From equation (11) we can write the first term of expression (19) as

$$E^Q[D_T^* e^{-rT} 1_{\{\tau > T\}}] = \frac{c_S D}{r} Q(\tau > T) e^{-rT} - J_D^* E^Q[e^{-rT} F_T(A_T, \bar{A}^*) 1_{\{\tau > T\}}].$$

Denote

$$V_1(\bar{A}) = E^Q[e^{-rT} F_T(A_T, \bar{A}) 1_{\{\tau > T\}}].$$

The integral in the second term of expression (19) can be written as

$$E^Q[\int_0^{\tau\wedge T} c_S D e^{-rs} ds] = \frac{c_S D}{r}(1 - Q(\tau > T)e^{-rT} - V_0(B))$$

The calculations of $V_0(B)$ and $V_1(\bar{A})$ are standard, see e.g. Mjøs and Persson (2005). The result follows by collecting terms.

□

4.2.2. *Valuation of hybrid capital at time* 0. The time 0 value of hybrid capital is equal to the time 0 market value of receiving H_T^* at time T plus the time 0 market value of cashflows received before time T. H_T^* is given in expression (12).

Proposition 4. *The time 0 market value of hybrid capital is*

$$H_0^*(A) = \frac{(c_H + k)H}{r} Q(\tau > T)e^{-rT} - J_H^* V_1(\bar{A}) - C + L + (B - D)^+ V_0(B), \tag{20}$$

where

$$C = C_0(A, H) - (\frac{B}{A})^{\beta-\alpha} C_0(\frac{B^2}{A}, H),$$

$$C_0(A, H) = (\frac{(c_H + k)H}{r} - H)e^{-rT} N(-d_2) - J_H^* (\frac{A}{\bar{A}})^{-\beta} N(-d_1),$$

$$d_1 = \frac{\ln(\frac{\bar{A}}{A}) - \frac{1}{\beta}(\ln(\frac{(c_H+k)H}{r} - H) - \ln(J_H^*)) + (\frac{r}{\beta} + \frac{1}{2}\sigma^2\beta)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\beta\sqrt{T},$$

$$Q(\tau > T) = N(d_3) - (\frac{A}{B})^{\alpha-\beta} N(-d_4),$$

$$d_3 = \frac{\ln(\frac{A}{B}) + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$d_4 = \frac{\ln(\frac{A}{B}) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$L = \frac{c_H H}{r}(1 - \kappa),$$

$$\begin{aligned}\kappa = {} & e^{-rT}[N(d_5) - (\frac{A}{B})^{\alpha-\beta} N(-d_6)] \\ & + \frac{\alpha}{\alpha+\beta}(\frac{B}{U})^{-\beta}[(\frac{A}{B})^{-\beta} N(-d_7) + (\frac{A}{B})^{\alpha} N(-d_8)] \\ & + \frac{\beta}{\alpha+\beta}(\frac{B}{U})^{\alpha}\left((\frac{A}{B})^{-\beta}[N(-d_9) + N(d_{10}) - N(d_{11})] - (\frac{A}{B})^{\alpha} N(-d_{12})\right),\end{aligned}$$

$$d_5 = \frac{\ln(\frac{A}{U}) + (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$d_6 = \frac{\ln(\frac{A}{B}) + \ln(\frac{U}{B}) - (\mu - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

$$d_7 = d_5 - \sigma\beta\sqrt{T},$$

$$d_8 = d_6 + \sigma\beta\sqrt{T},$$

$$d_9 = d_6 - \sigma\alpha\sqrt{T},$$

$$d_{10} = d_4 - \sigma\alpha\sqrt{T},$$

$$d_{11} = d_3 - \sigma\beta\sqrt{T}$$

$$d_{12} = d_4 + \sigma\beta\sqrt{T},$$

α and J_H^* are from Proposition 2, and β is given in expression (6)*.*

Proof. The time 0 market value of hybrid capital consists of the market value of time T hybrid capital including an embedded call option plus the market value of coupon payments and a potential bankruptcy payoff before time T, i.e.,

(21)

$$H_0^*(A) = E^Q[H_T^* e^{-rT} 1_{\{\tau > T\}}] - E^Q[(H_T^* - H)^+ e^{-rT} 1_{\{\tau > T\}}]$$

(22) $$+ E^Q[\int_0^{\tau \wedge T} c_H H 1\{A_s > U\} e^{-rs} ds] + E^Q[(B - D)^+ e^{-r\tau} 1_{\{\tau \leq T\}}].$$

Observe that equation (22) differs from equation (19) in two ways. First, it includes the hybrid capital embedded call option (the second term on the right hand side). Second, the coupon payments before time T only take place if $A_t > U$.

Denote

$$C = E^Q[(H_T^* - H)^+ e^{-rT} 1_{\{\tau > T\}}]$$

and

$$L = E^Q[\int_0^{\tau \wedge T} c_H H 1\{A_s > U\} e^{-rs} ds].$$

We calculate

$$E^Q[H_T^* e^{-rT} 1_{\{\tau > T\}}] =$$

$$\frac{(c_H + k)H}{r} Q(\tau > T) e^{-rT} - J_H^* E^Q[e^{-rT} F_T(A_T, \bar{A}) 1_{\{\tau > T\}}],$$

C and L are calculated in cite and cite, respectively. The result follows by collecting terms using the notation previously introduced. □

4.2.3. *Valuation of equity at time* 0. The market value of equity as the residual claim follows from equations (18) and (20)

(23) $$E_0^*(A) = A - D_0^*(A, 0) - H_0^*(A))$$

4.3. **Default risk of debt and hybrid capital.** To better understand the inherent default risk in our specified claims, we disregard the embedded call by analyzing the case when $t > T$.

In case of liquidation, $A = \bar{A}$ and the absolute priorities define the payoffs to the different claimants as

$$\begin{cases} D_L = \min(D, \bar{A}) \\ H_L = \min(H, (A - (\bar{A} - D))^+) \\ E_L = (\bar{A} - H_L - D_L)^+, \end{cases}$$

where D_L,H_L and E_L represent the liquidation payoffs to holders of debt, hybrid capital and equity, respectively. The analytically relevant cases in our setting are the following:

- *Case A:* $\bar{A} < D$, which yields no payoff to hybrid capital and makes senior debt risky in case of bankruptcy.
- *Case B:* $\bar{A} > D$, which yields positive liquidation payoff to hybrid capital but leaves senior debt risk-free in case of bankruptcy.

Hybrid capital in Case A is obviously more like equity, whilst it in Case B resembles junior debt. Our valuation formulas are generic and applicable in both situations.

In a 'going concern'-situation, senior debt carries the conventional conditional control rights in case of unpaid coupons, whilst hybrid capital by our definition does not. Seen from the issuer's side, however, hybrid capital coupons have to be paid to allow for paying dividends on common equity[9]. In our model, bankruptcy occurs if and when equityholders find it unattractive to continue servicing the company's debt and therefore declare bankruptcy. In case of senior debt, this is analytically uncomplicated. For hybrid capital, we may assume that even if coupon payments are stopped in certain distressed states, there also exists a lower bankruptcy asset level when the equityholders choose to permanently stop paying all coupons and let the company enter into bankruptcy. In a capital structure with both senior debt and hybrid capital, we have three levels of coupon payment:

$A > U$:	All coupons are paid.
$U > A > \bar{A}$:	Only senior debt coupons are paid.
$A \le \bar{A}$:	No coupons are paid and the company enters bankruptcy.

We assume that all coupons are treated equally in the first and the latter situations and thus that the bankruptcy asset level $\bar{A}$ is optimized with respect to all coupon payments. However, as equation (17) shows,

[9]This is a common requirement to discipline equityholders from immediately exploiting holders of hybrid capital, although such behavior would effectively terminate the market for hybrid capital.

the risk of unpaid coupons to hybrid capital impacts the calculation of the optimal bankruptcy level through $y^{-\alpha}$.

5. Numerical examples and comparisons

We illustrate our findings by showing the calibrated coupon-rates for senior debt and hybrid capital for a set of reasonable parameter values. All calibrations achieve 'issue-at-par' for the different claims.

Parameters	Values	Explanations
$\delta_0 c$	3	Initial EBIT
μ	2 %	Drift of EBIT
σ	0.2	Volatility of EBIT
r	5 %	Riskfree interest rate
A_0	100	Total asset value at time 0
D	60	Face value of senior debt
H	15	Face value of hybrid capital
E	25	Face value of equity
T	10	Expiration year of option

Table 1. Base case parameters.

		Barrier levels	
Alternative calibrated claims	Coupon rate	$\bar{A}$	U
Perpetual(Black&Cox)debt ($D = 70$)	5.718 %	49.04	n.r.
H^*: Fully specified hybrid (k=100 bp/y=1.05):			
$B = 0$:			
- Hybrid capital	10.440 %	57.31	60.18
- Senior debt	5.158 %	57.31	n.r.
B minimizing $D_0(A)^*$:			
- Hybrid capital	7.248 %	43.21	45.37
- Senior debt	5.419 %	43.21	n.r.

Table 2. Numerical calibrations

Our valuation formulas require a choice of pre-expiry bankruptcy level B which minimizes the value of the claims separately and in sum. This maximization of shareholder value is an economically credible contract element between shareholders and investors also *ex-post*. By inspecting how the time 0 market values of the two claims vary for different values of B, we selected two alternative approaches. Firstly, $B = 0$ which minimizes the values of both claims but may be seen as unreasonable. Secondly, the level of B that minimizes the value of senior debt $D_0(A)^*$ and thus varies by the choice of parameters. Neither choice are ideal but reflect the fact that the value of hybrid capital $H_0^*(A)$ is increasing monotonically in B whilst the value of senior debt has a minimum. The coupons are then calibrated to achieve issue at par for both claims. The comparison shows that hybrid capital featuring junior position, risk of lost coupons, embedded call option and coupon-rate step-up is indeed a risky claim demanding a higher coupon rate compared to straight perpetual debt.

6. Discussion of certain hybrid capital properties

In this section we further discuss the most important features of hybrid capital separately. We aim to focus on issues above and beyond those covered by Mjøs and Persson (2005).

6.1. **The risk of unpaid coupons.** The obligation to pay regular coupons is one of the defining characteristics of debt and motivates why hybrid capital typically have non-cumulative interest payments to qualify as Tier I risk capital for a financial institution. This is the property by which this capital absorb the required risk on a going-concern basis. The implication is that any missed payment is permanently lost and constitutes no future obligation for the issuer.An issuer will have to trade off the savings from missed coupon payments to any possible reputational damage potentially impacting future access to the capital markets. We do not explore these considerations further, but choose to treat the equityholders' decision to not pay coupons as exogenous to our model.

The asset threshold U represents the level where hybrid coupons are dropped. We refer to the latter level as the *omission level.* If asset values first drop below U and then again increase above U, hybrid coupon payments will remain unpaid only for this passed period.

When $U = \bar{A}$, $y = 1$ and the optimal bankruptcy equals the optimal bankruptcy in the case of regular debt, cf. equation (9). For large values of U and y, $y^{-\alpha}$ approaches zero. In the limit the optimal bankruptcy also goes towards zero since no hybrid coupon payments would ever be paid. For any other value of U, the optimal bankruptcy asset level will also be lower than with no risk of omitted coupons. The introduction of coupon risk via U into the model highlights the important connection between valuation and capital structure when evaluating hybrid capital. The level of coupon risk impacts both the value of hybrid capital as a claim on the company, but does also impact bankruptcy risk, coupon rates and values of any other claims on the issuing company. In case of financial institutions, this relationship illustrates how hybrid capital with coupon-risk may reduce the bankruptcy risk compared to e.g., regular subordinated debt and may therefore be a useful part of the risk-carrying capital. We explore this further in section 8.

The coupon rate for the hybrid capital given $\bar{A}$ and U is calculated from expression (20) and applying (15) as

$$c_h = \frac{r}{F_0 y^{\alpha-\beta} - 1}(F_0 \frac{\bar{A} y^{-\beta}}{H} - 1). \tag{24}$$

Equation (24) shows that c_h is falling in y since $(\alpha - \beta) > -\beta$, both exponentials being positive as discussed in Appendix B. The intuition behind this somewhat surprising result is that the reduction in the optimal long term bankruptcy level $\bar{A}$ for an increasing y contributes more to the market value of hybrid capital than the expected value of the omitted coupons. This is illustrated numerically below.

6.2. **Embedded issuer's call option and coupon rate step-up.** To increase the probability that the hybrid capital is called most issues include a 75-150 basispoints increase in the coupon rate as from the first possible expiration date of the option. This also increases the value of the option. The step-up reflects a natural difference in views between capital markets and regulators. The former prefers predictable, finite maturities for risky securities. The regulators, on the other hand, requires that risk-carrying capital have infinite maturities. Regulators solve this disagreement by setting an upper limit for the size of the step-up with the intention of supporting the permanence of the hybrid capital.

In our valuation-formula (20) the step-up rate, k, only directly impacts payoffs and values as from time T since no step-ups happen earlier.

A hybrid issuer facing worse refinancing terms at the expiry of the option will not exercise the call, leaving the investor with finite maturity only in the cases where the credit has improved and the investor generally would have preferred to remain invested. It is intuitively easy to see why this call feature is valuable to the issuer.

The lower the future time T market value of the firm's assets, A_T, is, the higher the required coupon-rate of newly issued hybrid capital would have to be. It is therefore only financially advantageous to let the option expire without exercise when the observed market *issue-at-par*-coupon rate at time T is higher than the existing contract coupon rate, including any agreed step-up. This situation will in our model occur for relatively low values of A_T, i.e., when the company is in a 'bad' state. The existence of the step-up, k, will further reduce this level and also make options called more frequently. This analysis depends on the refinancing assumptions applied and is clearly the case when the hybrid capital will be directly refinanced. If the hybrid capital instead is refinanced by regular infinite horizon debt, exercise is rational also for somewhat lower values of A_T since the issuer then gets rid of future payments of a higher coupon reflecting the premium of the now expired option as discussed in the introduction of Mjøs and Persson (2005).

7. Analysis of alternatives and sensitivities

In this section we apply the common set of base case assumptions as introduced in Section 5, and repeated and expanded in Table 3 below for the specific features. The extensions allow for testing the results on more realistic descriptions of the claims and the issuing company. This includes a revised capital structure where hybrid capital only represents 20% of non-equity capital and the remainder is senior debt, both being perpetual. In this section we maintain *issue-at-par* as the overriding objective i.e., all coupon-rates shown are calibrated to this end. The revised capital structure is applied unless explicitly motivated and analyzed.

The common set of corporate assumptions provides comparability between the various examples, but the relatively small size of the hybrid capital will naturally cause the variations in the observed coupon rates to be small.

Parameter	Value	Explanation
δ_0	3	Initial EBIT
μ	2 %	Drift of EBIT
σ	0.2	Volatility of EBIT
r	5 %	Riskfree interest rate
k	100 bp	Coupon rate step-up
y	1.05	Coupon loss/bankruptcy asset-levels
A_0	100	Total asset value at time 0
D	60	Face value of senior debt
H	15	Face value of hybrid capital
E	25	Face value of equity capital
T	10	Expiration year of hybrid option

Table 3. Extended base case parameters.

7.1. Sensitivities.

Figure 1 shows the calibrated coupon-rates for senior debt and hybrid capital for alternative assumed levels of annual EBIT-volatility. Both sets of rates are calculated when either holding $B = 0$ or choosing the B that minimizes the value of senior debt in each case. The graph shows that the the hybrid capital coupon rates are - as expected - very sensitive to firm volatility whilst senior debt rates are not. The hybrid capital coupons are also far more sensitive when the short term bankruptcy asset level is assumed to be 0 compared to optimized value. Intuitively this indicates that a larger B reduces the need for increased coupon rates for higher volatilities although this is an area for further research.

Table 4 shows the sensitivities of the calibrated coupon-rates for perpetual senior debt and hybrid capital for combinations of option

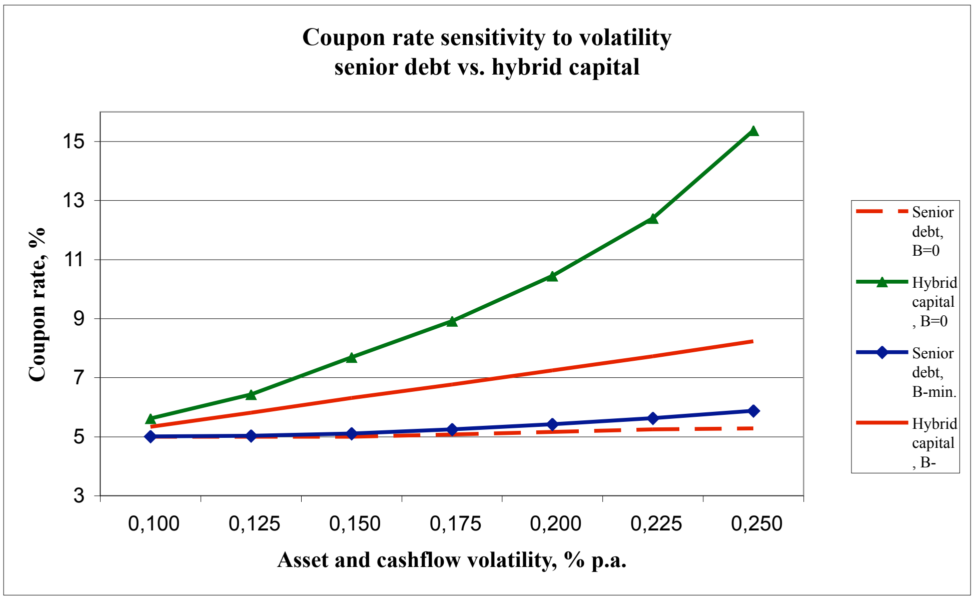

FIGURE 1. The figure shows the analytically calibrated issue-at-par coupon rates for hybrid capital and regular perpetual senior debt for assumed discrete levels of annual cashflow(EBIT)-volatility ranging from 0.10% to 0.25%. The coupon rates are calculated assuming short term bankruptcy level $B = 0$ and a B that minimizes the value of senior debt, respectively. See Table 3 for parameter values.

maturities and EBIT-volatilities and assuming a short term bankruptcy level $B = 0$. As expected, the coupon rates for hybrid capital grow in both volatility and maturity, reflecting increased risk exposure. Senior debt is close to risk free with coupon rates which grow in volatility but fall in option maturity.

Table 5 shows the alternative sensitivities of the calibrated coupon-rates for perpetual senior debt and hybrid capital for combinations of option maturities and EBIT-volatilities and assuming an optimized short term bankruptcy level B. As expected, both coupon rates grow in volatility. However, increased maturity decreases the rates for hybrid capital but increases them for senior debt. Optimal B falls in both volatility and maturity, but is not reported here.

Figure 2 shows how the coupon rates change for different assumed ratios y between the coupon omission level and the bankruptcy level. In line with the results shown in Figure 1, the senior debt coupon rate

Coupon-rates (%)	*Volatility*(σ)			
$B = 0$	0.10		0.20	
Maturity(T)	Senior	Hybrid	Senior	Hybrid
5 years	5.000	5.402	5.220	9.300
10 years	5.000	5.620	5.158	10.440
20 years	5.000	5.776	5.000	14.610

Table 4. Calibrated values of the coupon rates for senior perpetual debt and callable perpetual debt with embedded option, coupon rate step-up and exogenously defined risk of omitted hybrid coupons. The table shows combinations of number of years to expiry of the option (T) and annual EBIT(δ)-volatility (σ). The coupon rates are calculated assuming short term bankruptcy level $B = 0$. Remaining parameters are given in table 3

Coupon-rates (%)	*Volatility*(σ)					
Maturity(T)	0.10		0.20		0.30	
$Bmin(D_0)$	Senior	Hybrid	Senior	Hybrid	Senior	Hybrid
5 years	5.00	5.370	5.343	7.818	6.350	10.090
10 years	5.010	5.330	5.419	7.248	6.460	9.430
20 years	5.000	5.000	5.477	6.971	6.510	9.130

Table 5. Calibrated values of the coupon rates for senior perpetual debt and callable perpetual debt with embedded option, coupon rate step-up and exogenously defined risk of omitted hybrid coupons. The table shows combinations of number of years to expiry of the option (T) and annual EBIT(δ)-volatility (σ). The coupon rates are calculated assuming a short term bankruptcy level B that minimizes the value of senior debt. Remaining parameters are given in 3.

falls somewhat whilst the hybrid coupon rate grows exponentially for increased levels of y. The latter intuitively reflects the time 0 market value lost from unpaid coupons and how this increase in the risk of non-payments. In the case with an optimized B, this value seems to compensate the value of hybrid capital sufficiently to make the coupon rate almost insensitive to changes in volatility.

We have also analyzed the coupon rate sensitivities to changes in the hybrid coupon-rate step-up at time T if the option is not exercised. An increase in the step-up from 0 to 250 bp decreases the senior debt coupon by 27 bp whilst the hybrid capital coupon increase by 292 bp, assuming $B = 0$. With an optimized B, the changes are negligible.

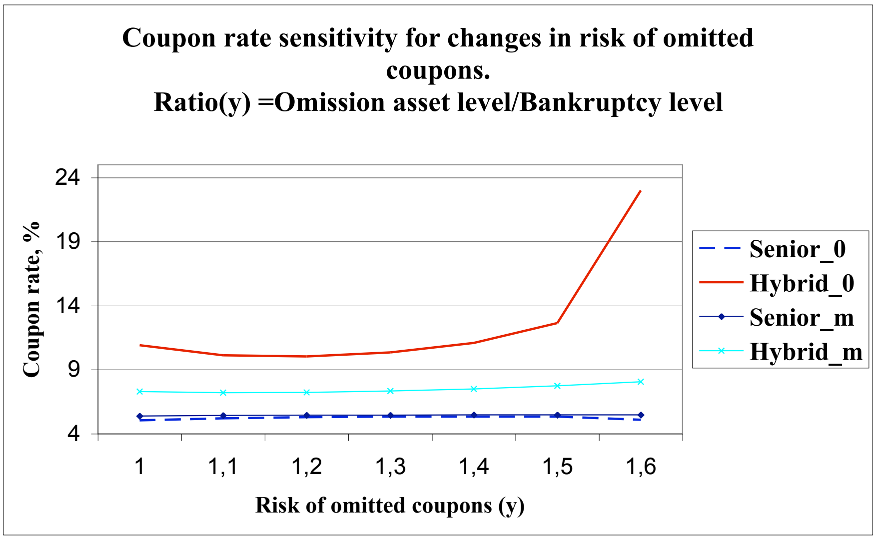

FIGURE 2. The figure shows the analytically calibrated issue-at-par coupon rates for senior debt and hybrid capital for increasing risk of omitted hybrid capital coupons. The risk of lost coupons is modelled as an exogenous asset level above the optimal bankruptcy level. If the asset process is between these two levels, no coupons are paid nor accumulated. The ratio between these levels is denoted y and represents the x-axis in the graph. The coupon rates are calculated assuming a short term bankruptcy level B that minimizes the value of senior debt. See Table 3 for parameter values.

The next sensitivity test varies the relative use of hybrid capital as share of non-equity financing of the company. Our base-case (See Table 3) includes hybrid capital as 20% of the sum of senior debt and hybrid capital. We have in addition modelled the coupon rates setting $H/(H + D) = 50\%$, leaving the sum of the claims unchanged. Figure 3 shows the resulting coupon-rates for senior debt and hybrid capital, respectively, given a range of assumed volatilities. The figure shows that the share of hybrid capital impacts the riskiness of both claims and in the 50%-case, senior debt is almost risk free. This result holds irrespective of the chosen assumption for B.

The regulatory rationale for accepting hybrid capital as part of the risk capital of financial institutions is its effect on the overall riskiness of the institution and exposure of the depositors. In our model, the calibrated coupon-rate of senior debt ('quasi-deposits') may be seen as a

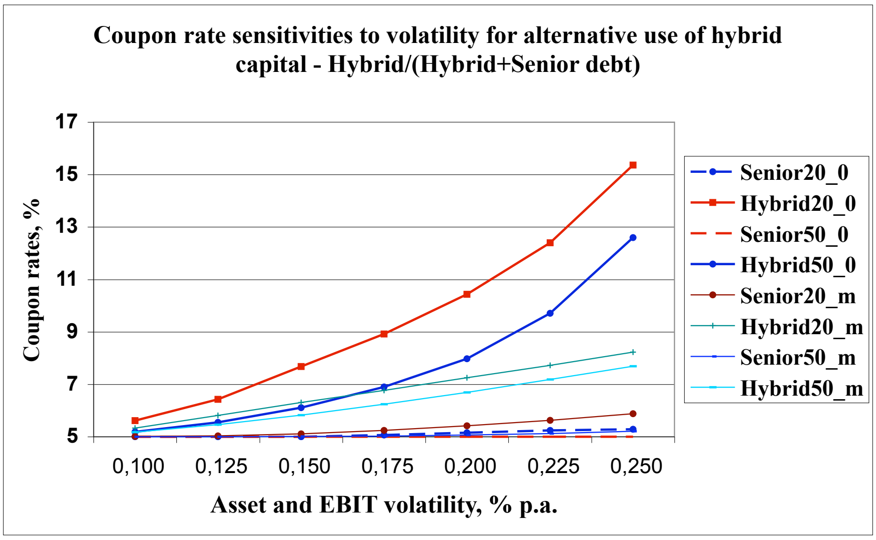

FIGURE 3. The figure shows the analytically calibrated issue-at-par coupon rates for senior debt and hybrid capital for alternative capital structures. The coupon rates are calculated assuming short term bankruptcy level $B = 0$ and a B that minimizes the value of senior debt, respectively. See Table 3 for parameter values.

reasonable proxy for the risk-exposure of depositors. We do a simplified test of the effects of hybrid capital by comparing the calibrate senior debt coupon rate for three different cases:

- *Case A:* A traditional bank with 10% equity capital and the remainder as senior deposits.
- *Case B:* A bank that raises 5% hybrid capital as additional risk-capital whilst retaining the equity ratio from Case 1.
- *Case C:* A more aggressive bank that replaces 1/3 of the equity by hybrid capital leaving the sum of risk-carrying capital constant at 10%.

The capital structure compares reasonably well to a simplified financial institution and the results are shown in Table **??**, assuming an optimized short term bankruptcy level B.

Our illustration shows that the required senior debt coupon rate is reduced if a stylized bank increases its risk-capital by raising hybrid capital in addition to its existing equity capital. We also see that replacing one third of equity by hybrid capital actually leads to a marginal reduction in the coupon rate required by senior claimants. This

Bank depositor's exposure	*Cases*(σ)		
	A	B	C
Capital structure:			
Senior debt	90	85	90
Hybrid capital	0	5	3.33
Equity capital	10	10	6.77
Senior debt coupon	6.471	6.198	6.467
Hybrid capital coupon	n.r.	9.569	9.794
B - short term bankruptcy	n.r.	70.54	75.388
A - long term bankruptcy	59.28	64.767	71.383

TABLE 6. Calibrated values for the coupon rate c_H for hybrid capital and c_d for senior debt for alternative capital structures. The coupon rates are calculated assuming a short term bankruptcy level B that minimizes the value of senior debt and is reported separately. Remaining parameters are given in 3.

result is preliminary also since the optimized short term bankruptcy level exceeds the long term level which is counterintuitive.

(TBC)

8. Conclusions and further research

Hybrid capital/preferred stock forms both a significant part of the capital of many companies, particularly financial institutions, and is a challenging research area. The valuation of this highly structured instrument also impacts the optimal capital structure of the issuer. We believe that our paper represents a first attempt to develop complete valuation models for hybrid capital including the common features found in the marketplace, notably issuer call-option, increased coupon-rate at the expiry date of the option and the right to omit coupon payments. The methodology is developed by using barrier call options to allow for bankruptcy risk before the exercise date. The exogenous decision by the company to omit coupons on hybrid capital impacts bankruptcy level and cost-of-capital across all claims issued. This feature, together with the junior rank in liquidation, are critical requirements when regulatory bodies acknowledges hybrid capital as part of the solvency capital for banks and insurance companies.

We have left many areas for future research, in particular the issue regarding when a company will choose to omit coupons. Insight from game theory or informational economics can probably be applied in this respect. The effects of taxes and various bankruptcy practises are also left for future research. There is obviously also a need for research into both the theory and the empirical evidence regarding the issuing decision. The valuation models also need to be empirically tested on actual values and yields both in the primary and secondary market of such securities.

This paper represents the first comprehensive valuation paper on hybrid capital including its many features. We provide closed form solutions for values and insight into what drives values and risk-elements.

Appendix A. Derivation of the market value for hybrid capital with risk of omitted coupons

In this case the hybrid coupons are only paid when $A_t > U$. The actual coupon-payment is thus dependent on A_t. We solve equation (4) separately for each region.

The general solution of equation (4) for the two regions is:

$$H(A) = \begin{cases} \frac{cH}{r} + K_1 A^{\alpha} + K_2 A^{-\beta} & \text{for } A \geq U \\ C_1 A^{\alpha} + C_2 A^{-\beta} & \text{for } \bar{A} \leq A \leq U. \end{cases} \tag{25}$$

where $\alpha, \beta > 0$, and are the positive solutions to the following equations:

$$\frac{1}{2}\sigma^2 \alpha(\alpha - 1) + \mu\alpha - r = 0$$
$$\frac{1}{2}\sigma^2 \beta(\beta + 1) - \mu\beta - r = 0$$

The solutions may be expressed as $\alpha = \lambda + \xi$ and $\beta = -\lambda + \xi$. Observe that $\alpha + \beta = 2\xi$, where

$$\lambda = \frac{\sigma^2 - 2\mu}{2\sigma^2},$$
$$\xi = \frac{1}{2\sigma^2}\sqrt{(\sigma^2 - 2\mu)^2 + 8\sigma^2 r}$$

The constants C_1 and K_1 in expression (25) can be found from the boundary conditions

$$\lim_{A \to \infty} H'(A) = 0 \Rightarrow K_1 = 0.$$

For $A = \bar{A}$, i.e. in bankruptcy, the boundary condition is $H(\bar{A}) = G$, where G represents the payoff to hybrid capital in case of bankruptcy. We obtain

$$C_1 \bar{A}^{\alpha} + C_2 \bar{A}^{-\beta} = G$$
$$C_1 = \bar{A}^{-\alpha}[G - C_2 \bar{A}^{-\beta}].$$

By inserting these expressions into the equation (25) we get

$$H(A) = \begin{cases} \frac{c_H H}{r} + K_2 A^{-\beta} & \text{for } A \geq U, \\ C_2(A^{-\beta} - \frac{A^{\alpha}}{\bar{A}^{\alpha+\beta}}) + G(\frac{A}{\bar{A}})^{\alpha} & \text{for } \bar{A} \leq A \leq U. \end{cases} \tag{26}$$

The first of the two additional equations necessary to determine K_2 and C_2 are found by assuming $H(A)$ is continuous for $A = U$, yielding

$$\frac{c_H H}{r} + K_2 U^{-\beta} = C_2 U^{-\beta} + (\frac{U}{\bar{A}})^{\alpha}[G - C_2 \bar{A}^{-\beta}] \tag{27}$$

$$K_2 = C_2(1 - (\frac{U}{\bar{A}})^{\alpha+\beta}) - \frac{c_H H}{r} U^{\beta} + G\frac{U^{\alpha+\beta}}{\bar{A}^{\alpha}}.$$

The second equation is found by the smooth pasting condition, i.e., assuming that the first derivatives of $H(A)$ are continuous at $A = U$,

$$-\beta K_2 U^{-(\beta+1)} = \alpha U^{(\alpha-1)} \bar{A}^{-\alpha}[\bar{A} - C_2 \bar{A}^{-\beta}] - \beta C_2 U^{-(\beta+1)},$$

$$K_2 = C_2(1 + \frac{\alpha}{\beta}(\frac{U}{\bar{A}})^{(\alpha+\beta)}) - G\frac{\alpha}{\beta}\frac{U^{\alpha+\beta}}{\bar{A}^{\alpha}} \tag{28}$$

The solution to equations (27) and (28) is found by equating them and solving for C_2:

$$C_2 = G\bar{A}^{\beta} - \frac{cH}{r}\frac{\beta}{(\alpha+\beta)}\frac{\bar{A}^{\alpha+\beta}}{U^{\alpha}}$$

The expression for C_2 may now be inserted into the expression for K_2:

$$K_2 = G\bar{A}^{\beta} - \frac{cH}{r}\frac{U^{\beta}}{(\alpha+\beta)}[\beta(\frac{\bar{A}}{U})^{\alpha+\beta} + \alpha].$$

and the complete solution based on equation (25) is

(29)

$$H(A) = \begin{cases} \frac{c_H H}{r} - [\frac{c_H H}{r}\frac{1}{(\alpha+\beta)}(\beta(\frac{\bar{A}}{U})^{\alpha} + \alpha(\frac{\bar{A}}{U})^{-\beta}) - G](\frac{A}{\bar{A}})^{-\beta} & \text{for } A \geq U, \\ \frac{c_H H}{r}\frac{\beta}{\alpha+\beta}(\frac{A}{U})^{\alpha} - [\frac{c_H H}{r}\frac{\beta}{\alpha+\beta}(\frac{A}{U})^{\alpha} - G](\frac{A}{\bar{A}})^{-\beta} & \text{for } \bar{A} \leq A \leq U. \end{cases}$$

This equation equals expression (7) when $U = \bar{A}$.

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