Hedging Portfolios of Financial Guarantees *

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January 15, 2007

*We acknowledge the financial support from the Institut de Finance Mathématique de Montreal (IFM2). All errors are the authors' sole responsibility.

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Abstract

We propose a framework *a la* Davis et al. (1993) and Whalley and Wilmott (1997) to study dynamic hedging strategies on portfolios of financial guarantees in the presence of transaction costs. We contrast four dynamic hedging strategies including a utility-based dynamic hedging strategy, in conjunction with using an asset-based index, with the strategy of no hedging. For the proposed utility-based strategy, the portfolio rebalancing is triggered by the tradeoff between transaction costs and utility gains. Overall, using a Froot and Stein (1998) and Perold (2005) type of risk-adjusted performance measurement metric, we find the utility-based strategy to be a good compromise between the delta hedging strategy and the passive stance of doing nothing. This results is even stronger with higher transaction costs. However, if the insured firms assets are not traded, the guarantor can use an index-based security as hedging instrument, especially in a high transaction costs environment.

Keywords: Financial guarantee, Credit insurance, Dynamic hedging.

1 Introduction

Enterprise risk management is nowadays a must for all institutions especially financial institutions. In this article, we use a risk management framework *a la* Merton and Perold (1993) and Froot and Stein (1998) to study hedging strategies by financial guarantee providers who hold invariably portfolios composed of several financial guarantee contracts. For instance, firms in the financial services industry can diversify away the systematic risk and/or insure (reinsure), hedge, retain (e.g., Bodie and Merton (1999)), and undertake alternative risk transfer (e.g., Banks (2004)). However, these risk management strategies cannot be implemented at no cost and perfectly. Further, in the domain of managing financial guarantees, which is the focus of our study, it is widely recognized that the portfolio credit risk cannot be completely diversified away. Therefore, financial guarantee providers need to find strategies to enhance their risk-adjusted returns.

Following Leland (1985), there is a significant literature on hedging derivatives with the same underlying asset and with transaction costs. Davis et al. (1993) propose a utility-based hedging model with negative exponential utility function. Later, Whalley and Wilmott (1997) use an asymptotic approach to hedge a call option. Alternatively, Edirisinghe et al. (1993) propose a dynamic programming approach with binomial trees to hedge an option with transaction costs. To overcome huge transaction costs associated with dynamic hedging, Derman et al. (1994) and Carr et al. (1998) propose static hedging models. Unfortunately, to our knowledge, there is no studies on hedging portfolios of financial guarantees with several underlying assets in the presence of transaction costs.[1]

There are limitations with the existing models. Indeed, in many real life situations, the portfolio to be hedged contains several underlying securities, hence many sources of risk to hedge. Moreover, the portfolio manager has to rebalance the portfolio repeatedly. On the one hand, the dynamic hedging methodology of Hodges and Neuberger (1989), Davis et al. (1993) and Whalley and Wilmott (1997) is less appropriate since the introduction of other sources of risk makes the problem more complex and no analytical

[1] There is a substantial literature on dynamic hedging and replication of derivatives under transaction costs both in discrete and continuous time, e.g. Avellaneda and Paras (1994), Boyle and Vorst (1992), Clewlow and Hodges (1997), Zakamouline (2005) among others.

solution can be derived. The approach of Edirisinghe et al. (1993) becomes cumbersome since the calculation time evolves exponentially with the number of rebalancing points and multiple risks. One the other hand, the static hedging approach of Derman et al. (1994) and Carr et al. (1998) which requires many replicating instruments with specific characteristics that may not be available in the market makes the hedging much less efficient.

In this paper, we study relatively simple hedging strategies that allow for multiple sources of risk, many rebalancing dates and non-zero transaction costs. We consider five strategies: (i) the doing nothing strategy, (ii) the dynamic delta hedging and (iii) utility-based hedging strategies using the underlying assets, (iv) the dynamic delta hedging and (v) utility-based hedging strategies using a security-based index as alternative hedging instrument.

In the spirit of Merton and Perold (1993), Froot and Stein (1998), and Perold (2005), to compare the performance of our five strategies, we use the relatively modern performance metric, the so-called risk-adjusted performance measure or RAPM, which is defined as the ratio of the portfolio expected return over its value at risk (VAR).[2] To better apprehend the impact of the parameters on our hedging strategies, we focus our numerical exercises on a portfolio composed of two financial guarantees. Overall, based on our parameters values, we found the utility-based hedging with the underlying assets to be a better compromise between the delta hedging strategy and the passive stance of doing nothing. This result remains stronger even with higher transaction costs. However, if the insured firms assets are not traded, the guarantor can use an index-based security as hedging instrument, especially in a high transaction costs environment.

Institutionally, managing and hedging portfolios of financial guarantees require the guarantor to set reserves and economic capital. Setting a risk-based capital or capital at risk allows us to capture the changes in the capital allocation associated with the hedging decisions. By doing so, we capture the portfolio diversification feature and price the risk

[2]Rather than benchmarking as done in the portfolio performance measurement literature which requires the construction of a proper benchmark portfolio, here we simply compare different strategies using the risk-adjusted return metric. Moreover, we recognize the critics associated with the use of VAR, however, since the objective of this paper is not the calculation of VAR per se, we believe that our main messages will hold using other improved measures of VAR.

associated with the tails of the distribution inherent to credit risk.[3] Unlike Smith and Stulz (1985) and Morellec and Smith (2006), the focus of our paper is to study hedging strategies of portfolios of financial guarantees. However, the implications of our study are consistent with their assertion that hedging can increase firm value.

The rest of the paper is structured as follows. In section 2, we present the model. In section 3, we discuss the dynamic hedging strategies. In section 4, we present the simulation parameters and discuss the results. Section 5 concludes.

2 General model

Before presenting the general model of the mutiple-risk sources case, we present a one risky asset portfolio case to capture the essence of the hedging problem.

2.1 The single underlying asset portfolio case

To gain the insight of our paper, we first start by providing the model with only one underlying asset. We present a utility-based dynamic hedging model to replicate a single option with one underlying asset.

We consider an guaranteed risky firm which asset, S_t, process is described as follows:

$$dS_t = \mu S_t dt + \sigma_t S_t dW_t, \tag{1}$$

where W_t is a standard Brownian motion. We also consider an riskless bond, B_t, with process

$$dB_t = r_t B_r dt, \tag{2}$$

where r_t is the risk-free rate at time t.

We define y_t the quantity of risky asset and B_t the amount of risk-free asset held by the guarantor. The transaction costs are assumed to be proportional to the value of the asset. We use θ to designate the proportion of transaction costs. The function L represents the value of the risky investment:

$$L(y_t, S_t) = \begin{cases} (1+\theta) y_t S_t, & \text{if } y_t < 0 \\ (1-\theta) y_t S_t, & \text{if } y_t > 0. \end{cases} \tag{3}$$

[3]See "Moody's Portfolio Risk Model for Financial Guarantors: Special Comment", Moody's Investors Services, Global Credit Research, July 2000, by R. Cantor, J. Dorer, L. Levenstein and S. Qian.

We want to maximize the utility function of the guarantor with respect to the cash-flows he will receive or pay at the maturity T of the guarantee. Let's assume that the guarantor has underwrite a guarantee contract on the firm's total debt K (equivalent to a short put on the firm asset). Thus, we can define the net wealth function of the guarantor as follows:

$$\Phi(T, B_T, y_T, S_T) = B_T + 1_{\{S_T < K\}}[L(y_T + 1, S_T) - K] + 1_{\{S_T \geq K\}} L(y_T, S_T). \tag{4}$$

The indirect utility function is defined as the maximum expected utility of the guarantor with respect to the hedging strategies

$$V(B) = \sup_{\psi \in \Omega(B)} \mathbb{E}[U(\Phi(T, B_T, y_T, S_T))], \tag{5}$$

where $\Omega(B)$ represents the set of possible strategies for a guarantor endowed with B amount and ψ is a strategy.

The optimal hedging strategy is obtained by solving a dynamic programming problem with the following indirect utility function

$$\begin{aligned} V(t, y_t, B_t, S_t) = \max_m \{ \quad & \max V(t, y_t + m\delta, B_t - (1+\theta) m\delta S_t, S_t), \\ & \max V(t, y_t - m\delta, B_t + (1-\theta) m\delta S_t, S_t), \\ & \quad \mathbb{E}[V(t + \Delta t, B_t \exp(r\Delta t), S_t \epsilon)] \} \end{aligned} \tag{6}$$

where the maximum is done with respect to m with values in $\{0, 1, 2, \ldots, \infty\}$, ϵ is the movement coefficient of the stock price and δ represents the portion of the asset that can be traded. The two terms of the utility function V are

$$\begin{aligned} V(t, y_t + m\delta, B_t - (1+\theta) m\delta S_t, S_t) \quad = & \\ \mathbb{E}\left\{V(t + \Delta t, y_t + m\delta, (B_t - (1+\theta) m\delta S_t) \exp(r\Delta t), S_t \epsilon)\right\} & \end{aligned} \tag{7}$$

and

$$\begin{aligned} V(t, y_t - m\delta, B_t + (1-\theta) m\delta S_t, S_t) \quad = & \\ \mathbb{E}\left\{V(t + \Delta t, y_t - m\delta, (B_t + (1-\theta) m\delta S_t) \exp(r\Delta t), S_t \epsilon)\right\}. & \end{aligned} \tag{8}$$

As in Edirisinghe et al. (1993), using a Markov chain decomposition, we need to compute several expectations. For each y, we need to compute the maximum value of the utility function with respect to all the strategies available to the guarantor.

For illustrative purpose, Figure 2.1 presents three trade regions for the replication of a put option. This figure has been obtained using a similar model as in Whalley and Wilmott (1997) who solve analytically the problem for a European call option. They provide asymptotic approximations of the Bellman-Jacobi equation of Davis et al. (1993) by characterizing the optimal hedge and the trade regions. In the top region, it is optimal to sell the underlying asset. In the bottom region, it is optimal to buy the underlying asset. And in the middle region, it is optimal to not trade because of the transaction costs.

2.2 The multiple-underlying assets portfolio case

Although, guarantors manage portfolios of more than two underlying assets, to address the main focus of the paper, we study a portfolio with two underlying securities. The simulation results obtained with two underlying assets convey the main message of the paper without loss of insight.

We consider a riskless asset B_t with process

$$dB_t = rB_t dt, \tag{9}$$

where r is the risk-free interest rate, and two risky securities $S_{1,t}$ and $S_{2,t}$ representing the assets of two clients firms. The processes of the two firms assets are

$$dS_{i,t} = \mu_i S_{i,t} dt + \sigma_i S_{i,t} dW_{i,t}, \quad i = 1, 2, \tag{10}$$

where the constants μ_i and σ_i are the instantaneous returns and returns' volatilities of the firms assets.

The guarantor underwrites separate guarantee contract with each client firm. Thus, each firm holds a put option written by the guarantor with exercise price the face value of its debt K_i. One special feature of the guarantee business is that guarantors usually hold portfolios composed of insured firms operating in the same industrial sector or having some common characteristics. Therefore we assume the existence an index-based security I which can be the index of the industry, and its process is given by

$$dI_t \quad = \quad \mu_I I_t dt + \sigma_I I_t dW_{I,t}, \tag{11}$$

where the constants μ_I and σ_I are the instantaneous return and returns' volatility of the industry index. We also consider a market index M with dynamics given by

$$dM_t \quad = \quad \mu_M M_t dt + \sigma_M M_t dW_{M,t}, \tag{12}$$

where the constants μ_M and σ_M are the instantaneous return and returns' volatility of the market index.

The above securities returns are correlated through their Brownian motions dW_i as follows: $\rho_{i,j} = corr(dW_i, dW_j)$, where i and j designated securities i and j.

As stipulated earlier, the guarantor underwrites two put options to the client firms with initial values P_1 and P_2. Hence, its portfolio value is

$$P = P_1 + P_2. \tag{13}$$

Applying Ito's lemma to this expression yields

$$\begin{aligned} dP &= \left(\frac{\partial P_1}{\partial t} + \frac{\partial P_1}{\partial S_1} S_1 \mu_1 + \frac{1}{2}\frac{\partial^2 P_1}{\partial S_1^2}\sigma_1^2 S_1^2\right)dt + \frac{\partial P_1}{\partial S_1}\sigma_1 S_1 dW_1 \\ &\quad + \left(\frac{\partial P_2}{\partial t} + \frac{\partial P_2}{\partial S_2} S_2 \mu_2 + \frac{1}{2}\frac{\partial^2 P_2}{\partial S_2^2}\sigma_2^2 S_2^2\right)dt + \frac{\partial P_2}{\partial S_2}\sigma_2 S_2 dW_2 \\ &= \mu_P P dt + \frac{\partial P_1}{\partial S_1}\sigma_1 S_1 dW_1 + \frac{\partial P_2}{\partial S_2}\sigma_2 S_2 dW_2, \end{aligned} \tag{14}$$

where μ_P is the drift of the portfolio returns obtained by summing the terms before the dt and dividing the sum by P. This equation highlights explicitly the exposition of the returns to the underlying risk sources dW_1 and dW_2.

The delta hedging strategy of the portfolio consists of trading $\frac{\partial P_1}{\partial S_1}$ of S_1 and $\frac{\partial P_2}{\partial S_2}$ of S_2. Abstracting from the transaction costs, this means that we need to hold delta quantity of each asset in order to delta-hedge.[4]

[4]In incomplete markets, under stochastic volatility and/or stochastic risk-free interest rate, one may need to use the other Greeks of the option for hedging such as the gamma-hedging, which we left for further study.

In some cases, either the underlying assets are not available for trade in the market (for example non-publicly available firms where equities are not traded) or it can be too costly to trade all the required underlying. In these situations, one may prefer to use a sector-based index instrument for hedging. As a matter of fact, Ramaswami (1991) and Ramaswamy (2002, 2005) among others exploit the insight that when the put is the money, the put behaves as equity, then hedging the default risk of the bond is tantamount to hedging equity risk. Naturally, the guarantor gains by using an index instrument for hedging closely related to his activities or highly correlated to his portfolio. To see that, let's decompose the underlying risk sources as follows:

$$dW_i = \rho_{I,i} dW_I + \sqrt{1-\rho_{I,i}^2} dZ_i, \quad i = 1, 2, \tag{15}$$

where $\{dZ_i, \ i = 1, 2\}$ are two independent Brownian motions independent from dW_I. The exposure of the portfolio to the index risk dW_I is given by

$$\frac{\partial P_1}{\partial S_1} S_1 \sigma_1 \rho_{I,1} + \frac{\partial P_2}{\partial S_2} S_2 \sigma_2 \rho_{I,2}. \tag{16}$$

Comparing this expression with the dynamic of the index, the guarantor needs to trade the following amount of the index

$$\frac{\frac{\partial P_1}{\partial S_1} S_1 \sigma_1 \rho_{I,1} + \frac{\partial P_2}{\partial S_2} S_2 \sigma_2 \rho_{I,2}}{I \sigma_I}. \tag{17}$$

Doing so, he benefits from the correlation between the portfolio and the index.

The next section presents the hedging strategies used to manage the portfolio of guarantees.

3 Hedging strategies

We present below the replication strategies used to hedge the portfolio composed of two put options. Hereafter, we use interchangeably replication or hedging or rebalancing to designate the same action. We consider 24 rebalancing dates over the year, i.e. twice per month, and the time step is denoted by Δt. Let's denote by $EL_{i,t}$ the expected loss by firm i at time t, $UL_{i,t}$ its unexpected loss (set at 5% confidence level for purpose) corresponding to the value at risk, $VAR_{i,t}$.

At the signature of the two guarantee contracts, the guarantor charges the following premium to the client firm i

$$PREM_i = (1 + \varepsilon_i) \times EL_{i,0} + H_i \times UL_{i,0}, \tag{18}$$

where ε_i is a loading coefficient capturing all market imperfections, H_i represents the hurdle rate for the guarantee contract i, as suggested by Marrison (2002) who analyses project finance guarantee portfolios. The total premium raised by the guarantor from the firms is

$$\sum_i PREM_i = \sum_i ((1 + \varepsilon_i) \times EL_{i,0} + H_i \times UL_{i,0}), \tag{19}$$

We assume that $\sum_i \varepsilon_i \times EL_{i,0}$ is used to cover the current operating expenses and other fees related to the signature of the guarantee contracts. Thus, the ϵ_i are chosen in order to break even these fees.

Following the practice of capital at risk, in addition to the portfolio expected loss EL_0, the guarantor has to set aside economic capital equal to the total unexpected loss of the portfolio UL_0. Therefore, the guarantor's shareholders provide $UL_0 - \sum_i H_i \times UL_{i,0}$ to raise the economic capital level to UL_0. In sum, the guarantor collects the two premiums and manages its guarantee portfolio up to the maturity of the guarantees. The amount available for investment is then $EL_0 + UL_0$.

Our set-up assumes that the guarantor invests the portfolio total expected loss amount raised from the firm $EL_0 = \sum_i EL_{i,0}$ in a reserve account earning the risk-free interest rate r to cover future expected losses. The rest of the premium, the capital at risk UL_0, is invested at the cost of capital, r_G, given as follows

$$r_G dt = r dt + \beta_{GM}((\mu_M - r)dt + \sigma_M dW_M). \tag{20}$$

This is the ICAPM (Intertemporal Capital Asset Pricing Model) type cost of capital and asserts that the cost of capital is equal to the risk-free rate plus the guarantor's beta times the market excess return.

At each rebalancing date, the guarantor reevaluates the current value of the portfolio expected loss $EL_t = EL_{1,t} + EL_{2,t}$ and replenish or reduce its reserve account balance accordingly. Therefore, the guarantor's reserve account balance is set equal to the portfolio total expected losses and is assumed to be invested in the risk-free bond. The rest of its wealth is invested at the rate r_G. This purports to reflect sound risk management practice through the use of capital at risk to cover unexpected losses.

Next, we present the passive strategy and four hedging strategies. The passive strategy of doing no hedging is the benchmark.

3.1 Strategy 0: The passive strategy of doing no hedging

The first strategy called the passive strategy consists of not hedging at all. However, both the reserve and risky accounts are reshuffled in order to maintain the reserve account to the level of the total expected loss.

3.2 Strategy 1: The dynamic delta hedging using the underlying assets

This strategy called delta hedging consists of performing the delta replication at the rebalancing dates using the insured firms assets. We denote by $\Delta_{i,t}$ the delta of stock i at time t. Here, we compute the delta of the portfolio and make the required trades on the underlying assets with transactions costs to obtain the hedged portfolio. We assume the transaction costs to be proportional to the trading amount, and the proportion coefficient θ is the same when buying or selling the securities. For example, at $t + \Delta t$, the transaction cost on trading stock i is

$$\theta \times |\Delta_{i,t+\Delta t} - \Delta_{i,t}| \times S_{i,t+\Delta t}. \tag{21}$$

3.3 Strategy 2: The utility-based dynamic hedging using the underlying assets

This strategy called utility-based hedging consists of using the utility maximization to determine the appropriate hedging dates and uses the underlying assets as hedging instruments. At each potential rebalancing date, in other words in this framework it is possible to have no replication at all at some dates. The guarantor must decide to rebalance or not it portfolio fully.

To rebalancing decision is based on the following indirect utility function

$$V_{t+\Delta t} = \max \mathbb{E}_{t+\Delta t}\Big[U(\Phi(T, B_{t+\Delta t}e^{r\tau}, \Delta_{1,t+\Delta t}, S_{1,T}, \Delta_{2,t+\Delta t}, S_{2,T}, B^G_{t+\Delta t}e^{r_G\tau}))\Big], \quad (22)$$

where T represents the same maturity of the two individual guarantee contracts, $\tau = T - t - \Delta t$ is the time to maturity and the function $\Phi(.)$ is a function of our underlying two state variables among others. $B_{t+\Delta t}$ is the reserve account balance at time $t + \Delta t$ multiplied by the compound factor to obtain its time T value, $\Delta_{i,t+\Delta t}$ is the number of stock i held, $S_{i,t+\Delta t}$ is the price of stock i, and $B^G_{t+\Delta t}$ represents the amount invested in the risky account (earning the rate of return r_G) multiplied by the corresponding compound factor.

Moreover, this is a self financing exercise because, at each rebalancing date, the total investment in the risk-free account and the risky account are equal to the previous time total investment value minus the total transaction costs. The reserve account at $t + \Delta t$ is

$$B_{t+\Delta t} = EL_{1,t+\Delta t} + EL_{2,t+\Delta t} \quad (23)$$

and the sum of the reserve account $B_{t+\Delta t}$ and the risky account $B^G_{t+\Delta t}$ is

$$\begin{aligned} B_{t+\Delta t} + B^G_{t+\Delta t} &= B_t e^{r\Delta t} + B^G_t e^{r_G \Delta t} \\ &\quad -(\Delta_{1,t+\Delta t} - \Delta_{1,t})S_{1,t+\Delta t} - |\Delta_{1,t+\Delta t} - \Delta_{1,t}|S_{1,t+\Delta t}\theta \\ &\quad -(\Delta_{2,t+\Delta t} - \Delta_{2,t})S_{2,t+\Delta t} - |\Delta_{2,t+\Delta t} - \Delta_{2,t}|S_{2,t+\Delta t}\theta. \end{aligned} \quad (24)$$

As stated above, the guarantor does not need to trade necessarily in the two stocks simultaneously, the decision to trade one or both underlying assets will be based on the indirect utility function.

Note that, in a single risky asset environment, as in Edirisinghe et al. (1993), the indirect utility function can be computed relatively easy using binomial trees. For two risky assets case where the assets correlation matters, it is important to look at all the possibilities, i.e. buying and selling portions of the two assets. The computation time of the dynamic programming approach in this case is too long and inefficient. This is why we follow the simulation approach.

In this utility-based hedging strategy, at each rebalancing date, the guarantor weighs the following four possible exclusive choices using the expected utility maximization:

Choice 1- Delta-replicate the portfolio using only stock 1, or

Choice 2- Delta-replicate the portfolio using only stock 2, or

Choice 3- Delta-replicate the portfolio using both stocks simultaneously, or

Choice 4- Do not hedge the portfolio.

For *Choice 1*, the decision function in equation (22) is simplified as follows

$$V_{t+\Delta t} = \max \mathbb{E}_{t+\Delta t} \left[U \left(\Phi(T, B_{t+\Delta t} e^{r\tau}, \Delta_{1,t+\Delta t}, S_{1,T}, \Delta_{2,t}, S_{2,T}, B^G_{t+\Delta t} e^{r_G \tau}) \right) \right], \qquad (25)$$

and the total investment in equation (24) becomes

$$\begin{aligned} B_{t+\Delta t} + B^G_{t+\Delta t} &= B_t e^{r\Delta t} + B^G_t e^{r_G \Delta t} \\ &\quad - (\Delta_{1,t+\Delta t} - \Delta_{1,t}) S_{1,t+\Delta t} - |\Delta_{1,t+\Delta t} - \Delta_{1,t}| S_{1,t+\Delta t} \theta. \end{aligned} \qquad (26)$$

For *Choice 2*, the equations are similar to the ones of *Choice 1*, except that stock 1 is replaced by stock 2. For *Choice 3*, the decision function and the sum of the investment accounts are given respectively by equations (22) and (24). Finally, for *Choice 4* when there is no trade, the decision function is

$$V_{t+\Delta t} = \max \mathbb{E}_{t+\Delta t} \left[U \left(\Phi(T, B_{t+\Delta t} e^{r\tau}, \Delta_{1,t}, S_{1,T}, \Delta_{2,t}, S_{2,T}, B^G_{t+\Delta t} e^{r_G \tau}) \right) \right], \qquad (27)$$

and the investment account value is

$$B_{t+\Delta t} + B^G_{t+\Delta t} = B_t e^{r\Delta t} + B^G_t e^{r_G \Delta t}. \qquad (28)$$

Comparing these four decision functions, the guarantor decides what transaction to undertake at time $t + \Delta t$ in order to maximize his expected utility with the portfolio held at that time.

In the above description, we have introduced two dynamic hedging strategies (Strategy 1 and Strategy 2) using the portfolio underlying assets. However, sometimes it could be too costly and/or impractical (e.g., not traded, overly illiquid, institutional constraints, etc.) to replicate the portfolio using the underlying assets. One may then resort to use an security-based index hedging instrument such as the sector index I. As indicated earlier, in the financial guarantee business, it is common to see a guarantor specializing in particular industries. For that purpose, next, we introduce two additional strategies using an index hedging instrument.

3.4 Strategy 3: The dynamic delta hedging using an index instrument

This strategy consists of using the index to replicate the portfolio. It is similar in spirit to the hedging Strategy 1. At each rebalancing date, the industry risk is completely eliminated by the delta replication, but the residual (if any) firms idiosyncratic risks remain. Intuitively, this strategy can be attractive compare to Strategy 1 since less replication costs are required to delta hedged using the sector index hedge instrument. However, the guarantor portfolio risk may be higher since we do not hedge completely the total risk of the portfolio unless the portfolio is perfectly correlated with the index.

3.5 Strategy 4: The utility-based dynamic hedging using an index instrument

This strategy uses the utility-based hedging but with the index as hedging instrument. In this strategy, there is only two possible exclusive choices:
Choice 1- Delta-replicate the portfolio using the index, or
Choice 2- Do not hedge the portfolio.

We then have to compare only two decision functions. For *Choice 1*, the decision function is

$$V_{t+\Delta t} = \max \mathbb{E}_{t+\Delta t}\left[U\left(\Phi(T, B_{t+\Delta t}e^{r\tau}, \Delta_{I,t+\Delta t}, I_T, B^G_{t+\Delta t}e^{r_G\tau})\right)\right], \tag{29}$$

where $\Delta_{I,t+\Delta t}$ is the index, and the total investment is

$$B_{t+\Delta t} + B^G_{t+\Delta t} = B_t e^{r\Delta t} + B^G_t e^{r_G\tau} - (\Delta_{I,t+\Delta t} - \Delta_{I,t})I_{t+\Delta t} - |\Delta_{I,t+\Delta t} - \Delta_{I,t}|I_{t+\Delta t}\theta. \tag{30}$$

This represents the guarantor's investment in the reserve and the risky accounts minus the transaction costs. For *Choice 2*, the decision function is

$$V_{t+\Delta t} \quad = \quad \max \mathbb{E}_{t+\Delta t}\big[U(\Phi(T, B_{t+\Delta t}e^{r\tau}, \Delta_{I,t}, I_T, B^G_{t+\Delta}e^{r_G\tau}))\big]. \tag{31}$$

and the total investment is

$$B_{t+\Delta t} + B^G_{t+\Delta t} = B_t e^{r\Delta t} + B^G_t e^{r_G \tau}. \tag{32}$$

At each rebalancing date, the guarantor delta-replicates or not the portfolio based on the decision functions values.

3.6 Computing the RAPM

We now need to compute the risk-adjusted performance measurement (RAPM) of the guarantor. As mentioned in the introduction, rather than benchmarking as done in the portfolio performance measurement literature which requires the construction of a proper benchmark portfolio, here we simply compare different strategies using the RAPM metric. To do that, we proceed as follows. The proceeds of the guarantee net of underwriting fees is

$$\sum_i (EL_{i,0} + H_i \times UL_{i,0}). \tag{33}$$

Since the guarantor is short of two puts, we have the following payoff

$$-\sum_i P_i. \tag{34}$$

This payoff is equal to the sum of the two expected losses: $-\sum_i EL_{i,0}$. Combining equations (33) and (34) gives the net value

$$\sum_i H_i \times UL_{i,0}. \tag{35}$$

At the maturity of the guarantee contracts, the net gain to the guarantor is given by the total investment value (reserves and risky accounts) plus the value of the guarantee portfolio minus the realized guarantee payments made. Since the guarantor's shareholders initial capital contribution is $UL_0 - \sum_i H_i \times UL_{i,0}$, we can compute the return as follows

$$R = \frac{\text{Guarantee portfolio value} + \text{Investment value} - \text{ Realized guarantee payments}}{UL_0 - \sum_i H_i \times UL_{i,0}}. \tag{36}$$

To obtain the risk-adjusted performance measure, RAPM, we compute the value at risk, VAR, of the returns for each strategy and the RAPM is defined as follows:

$$RAPM = \frac{R}{VAR}. \tag{37}$$

As discussed earlier, the numerator is the excess return and the denominator is our chosen risk metric namely the value at risk rather the traditional standard deviation of returns. Note that, if we use as risk metrics the standard deviation and the semi variance, we would have obtained roughly the Sharpe ratio and the Sortino ratio.

In the next section, we run several simulations and report the returns obtained.

4 Simulations

As in Pellizzari (2005), we will run numerical simulations to obtain our results, however, we differs from this paper since its focus is on static hedging, while we conduct dynamic hedging.

4.1 Case 1: Positive correlations between securities

In this section, we implement the strategies described above. The baseline parameters values used for the simulation are: $\sigma_I = 0.3$, $\sigma_1 = 0.2$, $\sigma_2 = 0.4$, and $\sigma_M = 0.1$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.2$ the guarantor's beta with the market, $H_1 = H_2 = 0.2$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. We use the negative utility function for the guarantor

$$U(x) = -e^{-\lambda x}, \tag{38}$$

with risk aversion coefficient $\lambda = 1/100$. This utility function exhibits the feature of constant absolute risk aversion and is widely used for its simplicity.

We assume the following positive correlations between the securities returns: $\rho_{I,1} = 0.5$, $\rho_{I,2} = 0.7$, $\rho_{I,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = 0.5$. Since we are using the risk neutral probabilities, the drift of the securities returns will be equal to the risk free rate $r = 0.05$.

Since the market is imperfect because of the presence of transaction costs, in our framework, we assume the same transaction costs structure as in Leland (1985), therefore Black and Scholes (1973) formula hold provided we use the modified volatility, σ_i^*, of the hedging instruments derived by Leland (1985). For long call and put positions, the modified volatility is

$$\sigma_i^* = \sigma_i \left(1 - \sqrt{\left(\frac{8}{\pi \Delta t}\right) \frac{\theta}{\sigma_i}}\right)^{1/2}, \tag{39}$$

and for short call and put positions, it is

$$\sigma_i^* = \sigma_i \left(1 + \sqrt{\left(\frac{8}{\pi \Delta t}\right) \frac{\theta}{\sigma_i}}\right)^{1/2}. \tag{40}$$

We run 10 000 simulations (including 5000 antithetic variables) using the risk neutral probabilities to obtain the returns distributions of each of the following strategies described above:

Strategy 0- No hedging,

Strategy 1- Dynamic delta hedging with the underlying assets,

Strategy 2- Utility-based dynamic hedging with the underlying assets,

Strategy 3- Dynamic delta hedging with a security-based index hedging instrument,

Strategy 4- Utility-based dynamic hedging with a security-based index hedging instrument.

Table 1 presents the expected returns, the VAR and the RAPM of the five strategies. From Panel 1 of Table 1 with transaction costs proportion $\theta = 0.75\%$, comparing Strategies 0, 1, and 2, we observe that Strategies 1 and 2 perform better than Strategy 0. This means that the hedging strategies with the underlying assets are better than doing nothing. In the case of the use of a security-based index hedging instrument, we observe that the hedging strategies with the index (Strategies 3 and 4) are better than the no hedging strategy (Strategy 0). Moreover, the utility-based hedging strategy seems to be the better one.

From Panel 2 of Table 1, we observe changes in the strategies returns when the transaction costs proportion doubles. We observe that the delta hedging strategy produces on average lower returns with higher transaction costs. Compared to the hedging strategies using the underlying assets, delta-replicating with the index produces higher absolute returns, which is expected since less transaction costs incur when transacting with the index.

In this positive correlations scenario, the utility-based hedging strategy produces on average better results since positive correlations increase the future cash flows of the portfolio. Thus, even with high transaction costs, replication can result in increase utility relative to the passive strategy of doing nothing.

The graphs in Figure 2 show the distribution of the strategies returns. For all the strategies, we observe the skewness in the portfolio distribution, with more skewness in Strategy 0 relative to the hedging strategies. However, the skewness is less in the delta hedging strategies than in the utility-based strategies. Intuitively, the utility-based strategy produces the two simultaneous effects: the reduction of the portfolio risk from hedging and the gains in return from the skewness, hence a combination of the no hedging strategy and the delta-replication.

4.2 Case 2: Negative correlation between one of the firm and the other firm and the index

We use the same baseline parameters values except for the securities correlations. Here, we assume firm 1 to be negatively correlated with firm 2 and the security-based index hedging instrument: $\rho_{I,1} = -0.4$, $\rho_{I,2} = 0.5$, $\rho_{I,M} = 0.3$, $\rho_{1,2} = -0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$. This can happen for example if the firms do not belong to the same industry. As in the previous case, we run our simulation under risk neutral probabilities.

Table 2 presents the results for the five strategies. From Panel 1 of Table 2 with transaction costs proportion $\theta = 0.75\%$, we observe that the worst RAPM are obtained with the hedging strategies involving the index. The best strategy is the delta hedging with the underlying assets. Although, this strategy produces the higher RAPM, its return is the lowest. Indeed, the index being negatively correlated with the underlying asset 1 and positively correlated with the underlying asset 2, its correlation with the portfolio is poor, therefore the replication using the index is far less perfect.

From Panel 2 of Table 2 with transaction costs proportion $\theta = 1.50\%$, with regard to the distribution of returns, the same comments can be made as discussed in Table 1.

The strategies returns distributions are given by the graphs of Figure 3. We observe the same trend as the one discussed in the case of Figure 2.

5 Conclusion

In this paper we study dynamic hedging strategies for portfolios of financial guarantees using the utility-based dynamic hedging strategy with transaction costs. By considering multiple-risk sources within portfolios of financial guarantees, we extend previous works on dynamic hedging with transaction costs, e.g. Hodges and Neuberger (1989) and Whalley and Wilmott (1997).

We examine five hedging strategies: (i) the doing nothing strategy, (ii) the dynamic delta hedging and (iii) utility-based hedging strategies using the underlying assets, (iv) the dynamic delta hedging and (v) utility-based hedging strategies using a security-based index hedging instrumnent. In the spirit of Froot and Stein (1998), Merton and Perold (1993) and Perold (2005), we compare our strategies performance using the modern concept of risk-adjusted performance measure (RAPM) consisting of the ratio of the expected return over the value at risk of the portfolio. Consistent with the capital at risk practice, we use the expected losses as well as the unexpected losses or Value at Risk in order to capture the changes of capital allocation feature with the hedging strategies. A challenging avenue for future research will be to study the interactions between the capital structure, the capital requirements, the hedging strategies and the institution performance under the framework of portfolios of two guarantees and more.

To better apprehend the impact of the parameters on our hedging strategies, we focus our numerical exercises on a portfolio composed of two financial guarantees. Based on our parameters values, we found that the utility-based hedging strategy with the underlying assets is a better compromise between the delta hedging strategy and the passive stance of doing nothing. This result remains stronger even with higher transaction costs. However, if the insured firms assets are not trade, the guarantor can use an security-based index as hedging instrument, especially in a high transaction costs environment.

Our numerical exercise requires substantial amount of computation time, especially if one uses non risk neutralized probabilities when prices and deltas have to be computed numerically. One agenda for research will be finding more efficient simulation techniques.

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Figure 1: **The trading regions when hedging a put option in the presence of transaction costs**

We use the following parameters values: $K = 100$, $T = 0.2$, $r = 0.05$, $\sigma = 0.2$, $\mu = 0.07$ and $\theta = 0.01$.

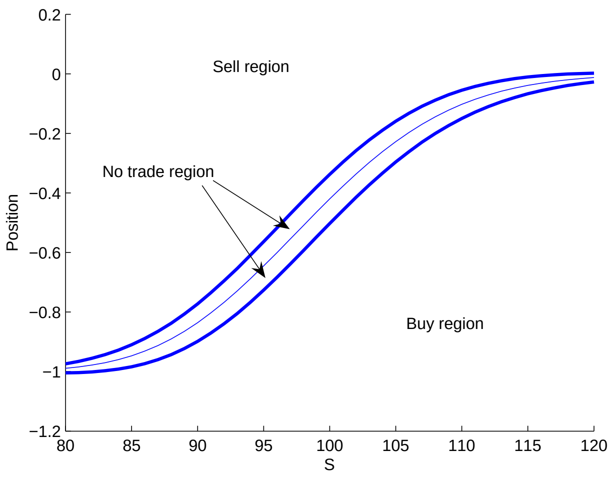


Table 1: **Strategies returns, VAR and RAPM for positive correlations between the securities returns**

These tables have been generated by simulations using the following baseline parameters values: $\sigma_I = 0.3$, $\sigma_1 = 0.2$, $\sigma_2 = 0.4$, and $\sigma_M = 0.1$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.2$ the guarantor's beta with the market, $H_1 = H_2 = 0.2$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. We use the negative utility function for the guarantor $U(x) = -e^{-\lambda x}$, with constant risk aversion coefficient $\lambda = 1/100$. The risk free rate $r = 0.05$. We assume the following positive correlations between the securities returns: $\rho_{I,1} = 0.5$, $\rho_{I,2} = 0.7$, $\rho_{I,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = 0.5$. The transaction costs proportion $\theta = 0.75\%$. In Panel 1, we use the transaction costs proportion $\theta = 0.75\%$ and in Panel b, $\theta = 1.50\%$.

Panel 1: The transaction costs portion $\theta = 0.75\%$

	Strategy 0	Strategy 1	Strategy 2	Strategy 3	Strategy 4
R	0.4060	0.2765	0.3684	0.3146	0.3588
VAR	1.5939	0.6993	0.9597	1.1857	1.2859
R/VAR	0.2547	0.3954	0.3839	0.2653	0.2790

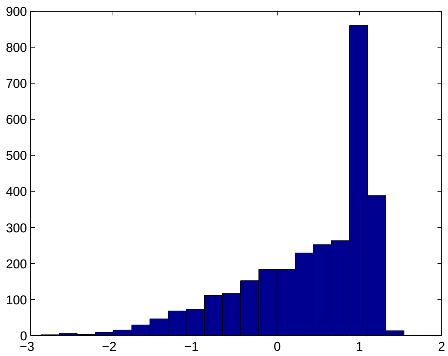
Panel 2: The transaction costs portion $\theta = 1.5\%$

	Strategy 0	Strategy 1	Strategy 2	Strategy 3	Strategy 4
R	0.4060	0.1676	0.3472	0.2318	0.3844
VAR	1.5939	0.7068	1.1047	1.2418	1.6076
R/VAR	0.2547	0.2371	0.3143	0.1867	0.2391

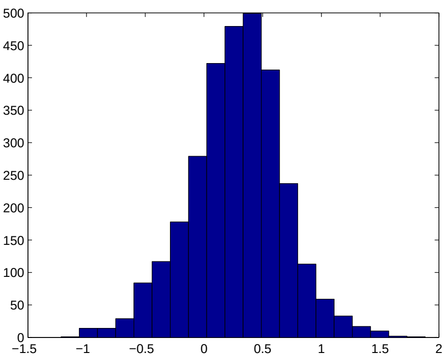
Figure 2: **Strategies returns distributions with positive correlations between securities**

These graphs have been generated by simulations using the following baseline parameters values: $\sigma_I = 0.3$, $\sigma_1 = 0.2$, $\sigma_2 = 0.4$, and $\sigma_M = 0.1$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.2$ the guarantor's beta with the market, $H_1 = H_2 = 0.2$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. We use the negative utility function for the guarantor $U(x) = -e^{-\lambda x}$, with constant risk aversion coefficient $\lambda = 1/100$. The risk free rate $r = 0.05$. We assume the following positive correlations between the securities returns: $\rho_{I,1} = 0.5$, $\rho_{I,2} = 0.7$, $\rho_{I,M} = 0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$, $\rho_{1,2} = 0.5$. The transaction costs proportion $\theta = 0.75\%$.

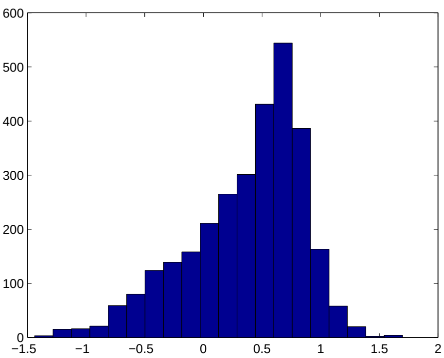
Strategy 0: No hedging



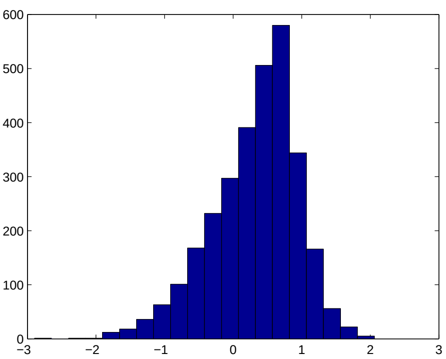
Strategy 1: Delta hedging with the assets



Strategy 2: Utility hedging with the assets



Strategy 3: Delta hedging with the index



Strategy 4: Utility hedging with the index

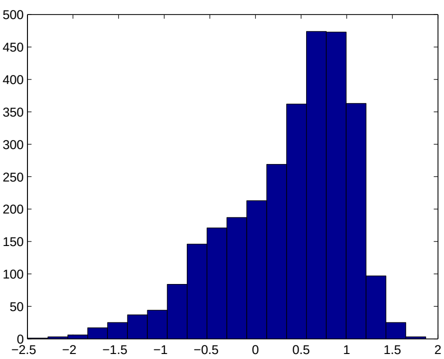


Table 2: **Strategies returns, VAR and RAPM when firm 1 is negatively correlated with firm 2 and the security-based hedging instrument**

These tables have been generated by simulations using the following baseline parameters values: $\sigma_I = 0.3$, $\sigma_1 = 0.2$, $\sigma_2 = 0.4$, and $\sigma_M = 0.1$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.2$ the guarantor's beta with the market, $H_1 = H_2 = 0.2$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. We use the negative utility function for the guarantor $U(x) = -e^{-\lambda x}$, with constant risk aversion coefficient $\lambda = 1/100$. The risk free rate $r = 0.05$. We assume firm 1 to be negatively correlated with firm 2 and the security-based index hedging instrument: $\rho_{I,1} = -0.4$, $\rho_{I,2} = 0.5$, $\rho_{I,M} = 0.3$, $\rho_{1,2} = -0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$. In Panel 1, we use the transaction costs proportion $\theta = 0.75\%$ and in Panel b, $\theta = 1.50\%$.

Panel 1: The transaction costs portion $\theta = 0.75\%$

	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5
R	0.3989	0.2673	0.3675	0.3632	0.3871
VAR	1.1485	0.6377	0.9930	1.1687	1.1719
R/VAR	0.3473	0.4192	0.3700	0.3108	0.3303

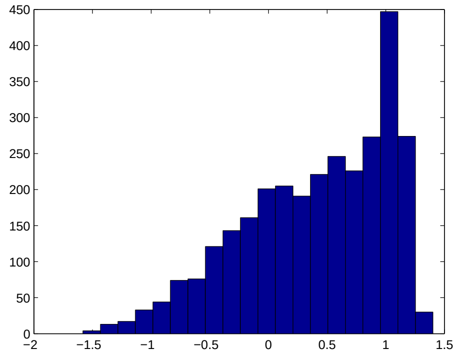
Panel 2: The transaction costs portion $\theta = 1.5\%$

	Strategy 1	Strategy 2	Strategy 3	Strategy 4	Strategy 5
R	0.3989	0.1741	0.3854	0.3080	0.3807
VAR	1.1485	0.6624	1.2016	1.1083	1.1828
R/VAR	0.3473	0.2628	0.3207	0.2779	0.3219

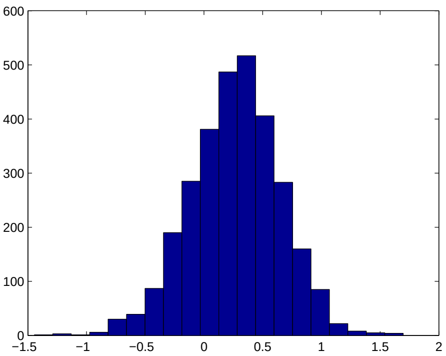
Figure 3: **Strategies returns distribution when firm 1 is negatively correlated with firm 2 and the security-based hedging instrument**

These graphs have been generated by simulations using the following baseline parameters values: $\sigma_I = 0.3$, $\sigma_1 = 0.2$, $\sigma_2 = 0.4$, and $\sigma_M = 0.1$ for the securities returns volatilities, $\mu_I = 0.10$, $\mu_1 = 0.08$, $\mu_2 = 0.12$ and $\mu_M = 0.10$ are the instantaneous mean returns of the securities, $S_{1,0} = S_{2,0} = 100$ the firms initial values, $I_0 = 100$ the index initial value, $M_0 = 100$ the market initial value, $\beta_{GM} = 1.2$ the guarantor's beta with the market, $H_1 = H_2 = 0.2$ the hurdle rates, $K_1 = K_2 = 100$ the firms debt face values. We use the negative utility function for the guarantor $U(x) = -e^{-\lambda x}$, with constant risk aversion coefficient $\lambda = 1/100$. The risk free rate $r = 0.05$. We assume firm 1 to be negatively correlated with firm 2 and the security-based index hedging instrument: $\rho_{I,1} = -0.4$, $\rho_{I,2} = 0.5$, $\rho_{I,M} = 0.3$, $\rho_{1,2} = -0.3$, $\rho_{1,M} = 0.35$, $\rho_{2,M} = 0.25$. The transaction costs proportion $\theta = 0.75\%$.

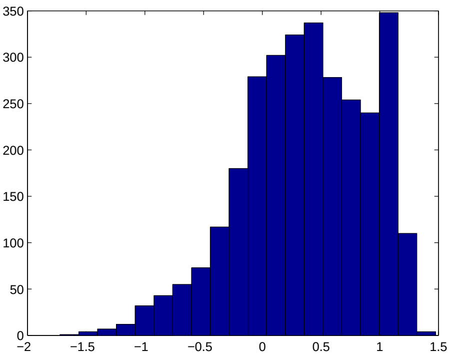
Strategy 0: No hedging



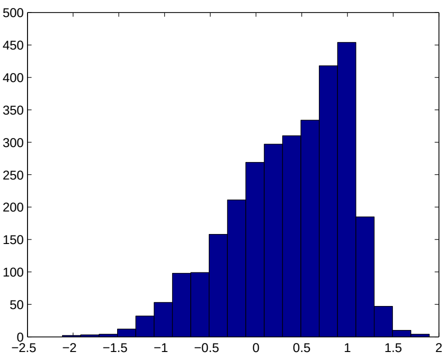
Strategy 1: Delta hedging with the assets



Strategy 2: Utility hedging with the assets



Strategy 3: Delta hedging with the index



Strategy 4: Utility hedging with the index