# What does the cross-section tell about itself? An asset 

# pricing model with cross sectional moments 

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#### Abstract

This paper derives and tests a five factor asset pricing model based on cross-sectional moments, in addition to market return and the market earnings yield. The three cross-sectional factors are the cross-sectional variance of returns; the cross-sectional variance of the dividend-to-price ratio; and the cross-sectional covariance between returns and dividend yields. This model can be theoretically justified as a generalization from the Intertemporal CAPM, by relaxing the representative investor feature and assuming two types of investor heterogeneity - heterogeneous shocks in wealth, and heterogeneous "intertemporal risks". The model is denoted as the Generalized ICAPM (GICAPM). The empirical tests show that the Generalized ICAPM is able to price reasonably well the Fama and French (1993) portfolios, and compares favorably with the Fama and French (1993) model. These results are robust to additional classes of portfolios and different estimation methodologies. Moreover, the GICAPM explains the value premium anomaly.

Keywords: Asset pricing; Cross-section of stock returns; Cross-sectional moments; Intertemporal CAPM; Idiosyncratic risk; Heterogeneous investors; Value premium JEL classification: G11;G12; G14; E44


## 1 Introduction

Since the pioneer work of Constantinides and Duffie (1996) and Heaton and Lucas (1996) several studies have been developing asset pricing models where the representative consumer feature is relaxed. Common to some of these studies is the assumption that the idiosyncratic shocks in wealth/consumption faced by investors are not fully insurable by financial markets, that is, markets are incomplete, e.g., Constantinides and Duffie (1996), Heaton and Lucas (1996), Brav, Constantinidies and Geczy (2002), Cogley (2002), and Jacobs and Wang (2004). Moreover, the focus of this literature has been on trying to explain both the equity premium and risk free rate puzzles. More recently, by using data on individual consumption, Jacobs and Wang (2004) have employed a linear factor model that relies on the first two cross-sectional moments of consumption growth, to explain the cross-section of stock returns, and in particular the Fama and French (1993) portfolios. Nevertheless, two criticisms often applied to the tests of consumption based models with individual consumption, are the measurement error associated with that data, and the limited time-series data available that restricts the statistical power associated with the tests.

On another line of research in asset pricing, the Merton (1973) Intertemporal CAPM (ICAPM) postulates that state variables, which predict market returns, should act as risk factors that price the cross-section of average returns. Among the papers that implemented empirically testable versions of the original ICAPM, are Campbell $(1993,1996)$, and more recently Chen (2003), Brennan, Wang and Xia (2004), Campbell and Vuolteenaho (2004), and Maio (2005a,b). Common to these papers in the derivation of the pricing equations analyzed in the cross section of returns, is the assumption of a representative investor.

In alternative, this paper derives a theoretical asset pricing model that represents a generalization from the ICAPM, by relaxing the representative investor assumption. In particular, the model allows for two types of investor heterogeneity. First, there are idiosyncratic shocks in wealth, which are not fully insurable by financial markets. Second, each investor is assumed to have different "intertemporal risks", i.e., they have different reference portfolios and different state variables that proxy for changes in future portfolio returns. The result is a five factor model, whose factors
are the change in aggregate wealth; aggregate intertemporal risk; dispersion on investor's wealth; dispersion in investor's intertemporal risk; and the comovement across investors between shocks in wealth and in intertemporal risk. The cross-sectional factors are the novelty relative to the standard ICAPM, and arise from the existence of investor heterogeneity. The model is denoted as the Generalized ICAPM (GICAPM).

In the empirical implementation of the model, due to measurement issues, the cross-sectional variance of returns is used as a proxy for the dispersion in wealth across investors. Furthermore, the cross-sectional variance for the dividend-to-price ratio is used as proxy for the dispersion in intertemporal risk. Finally the market return is used instead of changes in market wealth, and the market earnings yield is the proxy used for aggregate intertemporal risk.

The empirical test of the model shows that the Generalized ICAPM is able to price reasonably well the Fama and French (1993) portfolios, and compares favorably with the Fama and French (1993) model. These results are robust to tests made with additional classes of portfolios - industry portfolios and alternative characteristic portfolios sorted on the cash flow-to-price, earnings-to-price and dividend-to-price ratios. In addition, the results are robust to different estimation methodologies, the two-stage GMM procedure with equally weighted pricing errors in the first stage, and in alternative, the two-stage method with the Hansen and Jagannathan (1997) weighting matrix, in the first stage. Moreover the Generalized ICAPM is able to price at least as well as the Fama and French (1993) model, the extreme growth and value portfolios, that is, the GICAPM explains the value premium anomaly.

The rest of the paper is organized as follows. Section 2 presents the model and discusses the issues involved in its empirical implementation, and Section 3 presents the estimation and evaluation results for the test in the cross-section of portfolio returns. Finally, Section 4 concludes.

## 2 A Generalized Intertemporal CAPM (GICAPM)

This section presents the theoretical model, and then builds on some analysis that enable the model's empirical implementation.

### 2.1 The model

In this sub-section, I derive the theoretical asset pricing model that represents a generalization from the Merton (1973) Intertemporal CAPM (ICAPM). To save space, only the main steps in the model derivation are presented here, while the full derivation is provided in Appendix (A).

The standard ICAPM derives from the consumption/portfolio choice problem of a representative investor, and hence there is no room for investor heterogeneity. In the following model, I will allow for investor heterogeneity in two different ways. First, there are idiosyncratic shocks in wealth, which are not fully insurable by financial markets, similarly to Constantinides and Duffie (1996), Heaton and Lucas (1996), Brav, Constantinidies and Geczy (2002), Cogley (2002), Jacobs and Wang (2004), among others. This market incompleteness causes the individual consumption and individual portfolio choice decisions to differ across investors. Second, each investor is assumed to have different "intertemporal risks", i.e., they have different reference portfolios and different state variables that proxy for changes in future individual portfolio returns. While the first type of heterogeneity is also present in consumption-based asset pricing models with idiosyncratic shocks, the heterogeneity in "intertemporal risk" is restricted to the ICAPM case.

Consider an economy with $I$ investors and $N$ financial assets. The consumer/portfolio choice problem for investor $i$ can be represented as

$$
\begin{align*}
& J_{t}\left(W_{t}^{i}, z_{t}^{i}\right) \equiv \max _{\left\{C_{t+j}^{i}\right\}_{j=0}^{\infty}\left\{\omega_{n, t+j}^{i}\right\}_{j=0}^{\infty}} \mathrm{E}_{t}\left[\sum_{j=0}^{\infty} \delta^{j} U\left(C_{t+j}^{i}\right)\right] \\
& \text { s.t. }\left\{\begin{array}{c}
W_{t+1}^{i}=R_{p, t+1}^{i}\left(W_{t}^{i}-C_{t}^{i}\right) \\
R_{p, t+1}^{i}=g\left(z_{t}^{i}\right)
\end{array}\right. \text {, } \tag{1}
\end{align*}
$$

where

- $J_{t}($.$) represents the value function for investor i$, in period $t$;
- $C_{t}^{i}$ denotes the consumption of investor $i$, in period $t$, which drives utility $U\left(C_{t}^{i}\right)$;
- $\omega_{n, t}^{i}$ is the weight for asset $n$ in investor $i$ 's portfolio, at period $t$;
- $W_{t}^{i}$ is the total wealth for investor $i$, in period $t$;
- $R_{p, t+1}^{i}$ represents the portfolio gross return (realized at period $t+1$ ) for investor $i$;
- $z_{t}^{i}$ is the state variable that helps to forecast $R_{p, t+1}^{i}$;
- $\delta$ denotes a time-discount factor, which is assumed to be constant across investors.

Investors are assumed to have homogenous preferences and no private information, that is they share the public information set available at time $t$.

As shown in Appendix (A), investor $i$ 's Euler equation is given by

$$
\begin{gather*}
1=\mathrm{E}_{t}\left[M_{t+1}^{i} R_{n, t+1}\right]  \tag{2a}\\
M_{t+1}^{i}=\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right)}{J_{W, t}\left(W_{t}^{i}, z_{t}^{i}\right)} \equiv h\left(W_{t+1}^{i}, z_{t+1}^{i}\right) \tag{2b}
\end{gather*}
$$

where

- $R_{n, t+1}$ is the gross return on the $n$th asset $(n=1, \ldots, N)$;
- $M_{t+1}^{i}$ stands for the stochastic discount factor (SDF) or pricing kernel associated with investor $i$, at period $t+1$;
- $J_{W, t}($.$) represents the marginal value of wealth, in period t$.

The pricing equation (2a) is only valid for investor $i$, and hence, in order to have pricing implications for the whole economy, we need to aggregate the Euler equations across the $I$ individuals.

If the SDF for investor $i$ is a valid SDF, then the average pricing kernel in the economy will also price assets. By equally averaging across $I$ investors (who participate in the stock market), the pricing equation for the economy is represented by

$$
\begin{gather*}
1=\mathrm{E}_{t}\left[M_{t+1} R_{n, t+1}\right]  \tag{3a}\\
M_{t+1}=\frac{1}{I} \sum_{i=1}^{I} M_{t+1}^{i}=\frac{1}{I} \sum_{i=1}^{I} \frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right)}{J_{W, t}\left(W_{t}^{i}, z_{t}^{i}\right)}=\frac{1}{I} \sum_{i=1}^{I} h\left(W_{t+1}^{i}, z_{t+1}^{i}\right), \tag{3b}
\end{gather*}
$$

where $M_{t+1}$ represents the average $\operatorname{SDF}$ in the economy, at period $t+1$.
The function $h\left(W_{t+1}^{i}, z_{t+1}^{i}\right)$ in Equation (3b) can be approximated by a second order Taylor equation around the cross-sectional averages for wealth and the state variable,

$$
\begin{gather*}
h\left(W_{t+1}^{i}, z_{t+1}^{i}\right)=h\left(W_{t+1}, z_{t+1}\right)+h_{W}\left(W_{t+1}, z_{t+1}\right)\left(W_{t+1}^{i}-W_{t+1}\right)+h_{z}\left(W_{t+1}, z_{t+1}\right)\left(z_{t+1}^{i}-z_{t+1}\right)+ \\
\frac{1}{2} h_{W W}\left(W_{t+1}, z_{t+1}\right)\left(W_{t+1}^{i}-W_{t+1}\right)^{2}+\frac{1}{2} h_{z z}\left(W_{t+1}, z_{t+1}\right)\left(z_{t+1}^{i}-z_{t+1}\right)^{2}+ \\
h_{W z}\left(W_{t+1}, z_{t+1}\right)\left(W_{t+1}^{i}-W_{t+1}\right)\left(z_{t+1}^{i}-z_{t+1}\right) \\
W_{t+1}=\frac{1}{I} \sum_{i=1}^{I} W_{t+1}^{i}, z_{t+1}=\frac{1}{I} \sum_{i=1}^{I} z_{t+1}^{i} \tag{4}
\end{gather*}
$$

where

- $W_{t+1}$ represents the cross sectional average for wealth;
- $z_{t+1}$ denotes the cross sectional average for the state variable;
- $h_{W}, h_{z}, h_{W W}, h_{z z}, h_{W z}$ denote partial derivatives of $h($.$) with respect to either W_{t+1}$ or $z_{t+1}$.

By taking the average of (4) across the $I$ investors, it follows that the economy's SDF is given
by

$$
\begin{gather*}
M_{t+1} \equiv \frac{1}{I} \sum_{i=1}^{I} h\left(W_{t+1}^{i}, z_{t+1}^{i}\right)= \\
h\left(W_{t+1}, z_{t+1}\right)+\frac{1}{2} h_{W W}\left(W_{t+1}, z_{t+1}\right) V W_{t+1}+\frac{1}{2} h_{z z}\left(W_{t+1}, z_{t+1}\right) V Z_{t+1}+h_{W z}\left(W_{t+1}, z_{t+1}\right) C W Z_{t+1},  \tag{5}\\
V W_{t+1} \equiv \frac{1}{I} \sum_{i=1}^{I}\left(W_{t+1}^{i}-W_{t+1}\right)^{2}  \tag{6}\\
V Z_{t+1} \equiv \frac{1}{I} \sum_{i=1}^{I}\left(z_{t+1}^{i}-z_{t+1}\right)^{2}  \tag{7}\\
C W Z_{t+1} \equiv \frac{1}{I} \sum_{i=1}^{I}\left(W_{t+1}^{i}-W_{t+1}\right)\left(z_{t+1}^{i}-z_{t+1}\right) \tag{8}
\end{gather*}
$$

where the first order cross-sectional moments have cancelled out. In the above expression for the aggregate SDF, the cross-sectional moments are as follows

- $V W_{t+1}$ represents the cross sectional variance for wealth;
- $V Z_{t+1}$ is the cross sectional variance associated with the state variable;
- $C W Z_{t+1}$ denotes the cross sectional covariance between wealth and the state variable.

The asset pricing model in equation (3a) can be represented in expected return-covariance form as

$$
\begin{equation*}
\mathrm{E}_{t}\left(R_{n, t+1}\right)-R_{f, t+1}=-\frac{\operatorname{Cov}_{t}\left(R_{n, t+1}, M_{t+1}\right)}{\mathrm{E}_{t}\left(M_{t+1}\right)} \tag{9}
\end{equation*}
$$

By taking a first-order Taylor approximation to $\operatorname{Cov}_{t}\left(R_{n, t+1}, M_{t+1}\right)$ and substituting in (9), I show in Appendix (A) that the ICAPM with heterogeneous investors is represented as

$$
\begin{align*}
\mathrm{E}_{t}\left(R_{n, t+1}\right)-R_{f, t+1} & =\lambda_{M t} \operatorname{Cov}_{t}\left(R_{n, t+1}, \frac{W_{t+1}}{W_{t}}\right)+\lambda_{z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, z_{t+1}\right)+\lambda_{V W t} \operatorname{Cov}_{t}\left(R_{n, t+1}, V W_{t+1}\right) \\
& +\lambda_{V Z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, V Z_{t+1}\right)+\lambda_{C W Z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, C W Z_{t+1}\right) \tag{10}
\end{align*}
$$

where

- $\mathrm{E}_{t}\left(R_{n, t+1}\right)-R_{f, t+1}$ is the conditional expected return (at time $t$ ) for asset $n$, in excess of the risk free rate;
- $\lambda_{M t} \operatorname{Cov}_{t}\left(R_{n, t+1}, \frac{W_{t+1}}{W_{t}}\right)$ measures the risk associated with changes in the average or market wealth;
- $\lambda_{z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, z_{t+1}\right)$ denotes the "intertemporal risk";
- $\lambda_{V W t} \operatorname{Cov}_{t}\left(R_{n, t+1}, V W_{t+1}\right)$ is the risk associated with dispersion in wealth, among investors;
- $\lambda_{V Z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, V Z_{t+1}\right)$ represents the risk for dispersion in investors' intertemporal risk;
- $\lambda_{C W Z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, C W Z_{t+1}\right)$ measures the risk associated with comovement between investors' wealth and "hedging" risk.

The risk prices associated with the market factor; the state variable factor; the cross sectional variance of wealth; the cross sectional variance for the state variable; and the cross sectional covariance between wealth and the state variable; are provided in Appendix (A).

The innovation in the asset pricing model (22) relative to the standard ICAPM, is the inclusion of the last three factors that measure the risks associated with dispersion (among investors) in individual wealth; dispersion in individual intertemporal risk; and the comovement between changes in individual wealth and changes in future individual portfolio returns. I will denote model (22) as the Generalized ICAPM (GICAPM).

In the case of an homogenous investor that does not face idiosyncratic shocks in both wealth and intertemporal risk, then we have

$$
V W_{t+1}=V Z_{t+1}=C W Z_{t+1}=0
$$

and the Merton's ICAPM arises as a special case of the Generalized ICAPM,

$$
\mathrm{E}_{t}\left(R_{n, t+1}\right)-R_{f, t+1}=\lambda_{M t} \operatorname{Cov}_{t}\left(R_{n, t+1}, \frac{W_{t+1}}{W_{t}}\right)+\lambda_{z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, z_{t+1}\right)
$$

If we assume constant risk prices and apply unconditional expectations to Equation (10), we obtain the Generalized ICAPM in unconditional form,

$$
\begin{align*}
\mathrm{E}\left(R_{n, t+1}-R_{f, t+1}\right) & =\lambda_{M} \operatorname{Cov}\left(R_{n, t+1}, \frac{W_{t+1}}{W_{t}}\right)+\lambda_{z} \operatorname{Cov}\left(R_{n, t+1}, z_{t+1}\right)+\lambda_{V W} \operatorname{Cov}\left(R_{n, t+1}, V W_{t+1}\right) \\
& +\lambda_{V Z} \operatorname{Cov}\left(R_{n, t+1}, V Z_{t+1}\right)+\lambda_{C W Z} \operatorname{Cov}\left(R_{n, t+1}, C W Z_{t+1}\right) \tag{11}
\end{align*}
$$

Notice that in the above model the shocks in wealth are assumed not to be completely idiosyncratic, in the sense that they are partially correlated across investors. This assumption seems economically plausible, since for example in a recession, while some investors will be more strongly affected than others, it is likely that most of them will suffer negative shocks in their respective incomes. Furthermore, the factors $V W_{t+1}, V Z_{t+1}$ and $C W Z_{t+1}$, although being related with investor heterogeneity, they are not by any means idiosyncratic. Instead, they reflect dispersion in wealth and intertemporal risk (and the comovement between those two), which is not diversified away.

### 2.2 Measuring the cross-sectional risks

In order to be able to empirically test the model, we need to measure the covariance terms include in the GICAPM in equation (11), and more specifically to obtain proxies for the factors $\frac{W_{t+1}}{W_{t}}, z_{t+1}$, $V W_{t+1}, V Z_{t+1}$ and $C W Z_{t+1}$. In the following empirical analysis, the market return, $R_{m, t+1}$, is used as a proxy for changes in the average or aggregate wealth, $\frac{W_{t+1}}{W_{t}}$. To measure the cross-sectional variance in wealth, $V W_{t+1}$, I use the cross-sectional variance (standard deviation) associated with individual asset returns,

$$
\begin{gathered}
\widehat{V R}_{t+1} \equiv \sqrt{\frac{1}{N} \sum_{n=1}^{N}\left(R_{n, t+1}-\overline{R_{n, t+1}}\right)^{2}} \\
\overline{R_{n, t+1}}=\frac{1}{N} \sum_{n=1}^{N} R_{n, t+1}
\end{gathered}
$$

where $\overline{R_{n, t+1}}$ denotes the cross-sectional average return. This measure seems economically intuitive, since with heterogeneous shocks in wealth, the net demand for a given stock will diverge across investors, thus affecting the prices and returns of that same stock. In addition, if we assume that the net demands will diverge within stocks, this creates dispersion within the cross-section of stock returns. For example, in response to the arising of credit constraints and negative income shocks in economic downturns, (which affect some investors more than others), it is likely that investors will increase their demand for certain categories of stocks (e.g., large/growth stocks) and decrease their demand for other categories (e.g., small/value stocks). It is important to assume that shocks in wealth are not perfectly negatively correlated across investors, otherwise they would have no effect on the total demand and hence on stock prices, i.e., the net individual demands would be zero, leaving prices unchanged). ${ }^{1}$ In addition, some investors will be more keen to hedge certain types of risks (e.g., recession risk), and thus will demand different classes of assets, than the other investors. Therefore, the dispersion in wealth shocks will have an impact on actual prices and returns for the whole cross-section of stocks, which translates into a higher dispersion among individual stock returns. On the other hand, Goyal and Santa-Clara (2003) find that a measure close to the cross-sectional variance of returns, is able to forecast future market returns. Thus, in an ICAPM context, such a variable should be included as an additional factor that prices the cross-section of returns.

Instead of using the whole cross-section of stocks, I rather compute $V R$ based on portfolio returns. This procedure has the advantage that one mitigates the estimation error arising from the noise effect associated with illiquid and small stocks, very much like the usual convention that asset pricing models are tested based on a group of portfolios rather than the complete cross-section of individual stocks. Nevertheless, the cross-sectional variance based on portfolio returns is related with the same measure based on individual stocks.. I use two classes of portfolios to evaluate $V R$. The first class are the 25 portfolios sorted on both size and book-to-market (BM) (hereafter,

[^1]SBM25) from Fama and French (1993), leading to

$$
\begin{gather*}
V R_{t+1} \equiv \sqrt{\frac{1}{25} \sum_{n=1}^{25}\left(R_{n, t+1}-\overline{R_{n, t+1}}\right)^{2}}  \tag{12}\\
\overline{R_{n, t+1}} \equiv \frac{1}{25} \sum_{n=1}^{25} R_{n, t+1}
\end{gather*}
$$

where $R_{n, t+1}$ denotes the return for the $n$th portfolio, $n=1, \ldots, 25$. As a robustness check, and to increase the number of returns used in the calculation of the cross-sectional return variance, I use 100 portfolios also sorted on both size and BM (SBM100, hereafter), available on Kenneth French's website, and which represent the intersection of 10 portfolios sorted on size and 10 portfolios sorted on BM. In this case, the measure for dispersion in returns becomes

$$
\begin{gather*}
V R_{t+1}^{*} \equiv \sqrt{\frac{1}{95} \sum_{n=1}^{95}\left(R_{n, t+1}-\overline{R_{n, t+1}^{*}}\right)^{2}}  \tag{13}\\
R_{n, t+1}^{*} \equiv \frac{1}{95} \sum_{n=1}^{95} R_{n, t+1}
\end{gather*}
$$

Due to missing observations during the period in analysis (January 1963 to December 2003), the portfolios $S B M_{1,3}, S B M_{7,10}, S B M_{10,8}, S B M_{10,9}, S B M_{10,10}$ - where the first number indexates the size quintile, and the second number refers to the BM quintile - are excluded from the sample, leading to a total of 95 portfolios.

In order to measure the investor dispersion associated with intertemporal risk, I will focus on a single state variable that constitutes a proxy for intertemporal risk - the dividend yield. The aggregate dividend yield (or similar financial ratios like the market earnings yield or the aggregate book-to-market ratio) represents the most widely used (and most important variable) to predict market returns, in the predictability of returns/asset pricing literature (for a non-exaustive list, see Fama and French $(1988,1989)$, Campbell and Shiller (1988a), Hodrick (1992), Campbell and Vuolteenaho (2004), Maio (2005a,b)). The predictive role of the market dividend yield can be rationalized in the context of the following dynamic accounting identity developed by Campbell
and Shiller (1988a),

$$
\begin{equation*}
d_{t}-p_{t}=\text { const. }+\mathrm{E}_{t} \sum_{j=0}^{\infty} \rho^{j}\left[r_{t+1+j}-\Delta d_{t+1+j}\right], \tag{14}
\end{equation*}
$$

where

- $d_{t}$ denotes $\log$ market dividend in period $t$;
- $p_{t}$ is the log market price index in period $t$;
- $r_{t+1}$ stands for the $\log$ market return realized at period $t$;
- $\rho$ is a linearization parameter related with the average dividend-to-price ratio.

The identity (14) is derived from the definition of market returns and by imposing a non-bubble condition that prices can not rise forever,

$$
\lim _{j \rightarrow \infty} \rho^{j} p_{t+j}=0,
$$

and the main message is that, conditional on future expected aggregate dividend (or cash flow) growth, a higher aggregate dividend yield today must be followed by higher expected market returns in the future, due to the mean-reversion in most stock prices. Naturally, the identity (14) is not only valid for the market as a whole, but also for each individual stock,

$$
\begin{gather*}
d_{n, t}-p_{n, t}=\text { const. }+\mathrm{E}_{t} \sum_{j=0}^{\infty} \rho_{n}^{j}\left[r_{n, t+1+j}-\Delta d_{n, t+1+j}\right],  \tag{15}\\
n=1, \ldots, N,
\end{gather*}
$$

where

- $d_{n, t}$ is the $\log$ dividend for asset $n$, in period $t$;
- $p_{n, t}$ is the $\log$ price for asset $n$, in period $t$;
- $r_{n, t+1}$ stands for the log return on asset $n$ realized at period $t$;
- $\rho_{n}$ is the linearization parameter for asset $n$.

From (15), it is clear that individual stocks's dividend-to-price ratios should help to forecast expected individual returns (conditional on expected dividend growth associated with the asset at hand) - given the mean reversion in individual stock prices - similarly to the predictive role played by the aggregate dividend yield over market returns. From this, it follows that when one wants to forecast individual returns, the forecasting power of asset dividend yields should be greater than the one associated with the market dividend yield. To assess this argument, I conduct the following predictive regression for each of the $S B M 25$ portfolios,

$$
\begin{gather*}
r_{n, t+1, t+k}=a_{k}^{n}+b_{k}^{n} D Y_{n, t}+u_{n, t+1, t+k},  \tag{16}\\
n=1, \ldots, 25,
\end{gather*}
$$

where

- $r_{n, t+1, t+k}$ is the continuously compounded excess return over $k$ periods, for portfolio $n$;
- $D Y_{n, t}$ represents the dividend yield associated with portfolio $n$, measured at time $t$;
- $u_{n, t+1, t+k}$ is the $k$-periods ahead forecasting error for portfolio $n$.

To allow the comparison with the predictive ability associated with the market dividend yield, I also compute the following regression for each portfolio,

$$
\begin{gather*}
r_{n, t+1, t+k}=a_{k}^{n}+b_{k}^{n} D Y_{t}+u_{n, t+1, t+k}  \tag{17}\\
n=1, \ldots, 25
\end{gather*}
$$

where $D Y_{t}$ denotes the market dividend yield. I use forecasting horizons of $1,3,12$ and 24 months ahead. To obtain the portfolio dividend yield data, I subtract the return data excluding dividends associated with $S B M 25$ (also available on Kenneth French's website), from the respective total
return data,

$$
\begin{gathered}
D Y_{n, t} \equiv \frac{D_{n, t}}{P_{n, t-1}}=R_{n, t}-\hat{R}_{n, t} \\
n=1, \ldots, 25
\end{gathered}
$$

where $\hat{R}_{n, t}$ denotes the ex-dividend return for portfolio $n$. Figure 1 plots the Newey and West (1987) asymptotic t-statistics (calculated with 5 lags) associated with $b_{k}^{n}$ in the above two regressions, for the $S B M 25$ portfolios. Panels A, B, C and D show the results for forecasting horizons of 1,3 , 12 and 24 months ahead, respectively. The main conclusion from Figure 1 is that in the cases of portfolios where the forecasting variables are statistically significant (at the $5 \%$ level), the tstatistics associated with the portfolio's dividend yield are in general greater than the corresponding ones for the market dividend yield. This is especially true for the lowest size quintiles (those in which predictability is stronger), and it is robust across forecasting horizons. ${ }^{2}$

In the Generalized ICAPM model above, the investor heterogeneity in intertemporal risk, is linked with the fact that each investor will have different portfolios (different weights assigned to the $N$ available assets), combined with the dispersion across assets in the predictability of returns. Therefore, in accordance with $V R$, the cross-section variance associated with "intertemporal risk" consists of the dispersion of portfolio dividend yield across the classes of portfolios SBM25 and SBM100. More specifically,

$$
\begin{gather*}
V I R_{t+1} \equiv \sqrt{\frac{1}{25} \sum_{n=1}^{25}\left(D Y_{n, t+1}-\overline{D Y_{t+1}}\right)^{2}}  \tag{18}\\
\overline{D Y_{t+1}} \equiv \frac{1}{25} \sum_{n=1}^{25} D Y_{n, t+1}
\end{gather*}
$$

[^2]and
\[

$$
\begin{gather*}
V I R_{t+1}^{*} \equiv \sqrt{\frac{1}{95} \sum_{n=1}^{95}\left(D Y_{n, t+1}-\overline{D Y_{t+1}^{*}}\right)^{2}},  \tag{19}\\
\overline{D Y_{t+1}^{*}} \equiv \frac{1}{95} \sum_{n=1}^{95} D Y_{n, t+1},
\end{gather*}
$$
\]

where $\overline{D Y_{t+1}}$ and $\overline{D Y_{t+1}^{*}}$ denote the cross-sectional average dividend yield for $S B M 25$ and $S B M 100$, respectively.

Given the measures (12-13) and (18-19), it follows that the factor associated with the investor comovement between changes in wealth and "intertemporal risk", $C W Z$, can be now approximated as

$$
\begin{align*}
& C I R R_{t+1} \equiv \frac{1}{25} \sum_{n=1}^{25}\left(R_{n, t+1}-\overline{R_{n, t+1}}\right)\left(D Y_{n, t+1}-\overline{D Y_{t+1}}\right),  \tag{20}\\
& C I R R_{t+1}^{*} \equiv \frac{1}{95} \sum_{n=1}^{95}\left(R_{n, t+1}-\overline{R_{n, t+1}^{*}}\right)\left(D Y_{n, t+1}-\overline{D Y_{t+1}^{*}}\right), \tag{21}
\end{align*}
$$

respectively for $S B M 25$ and $S B M 100$.
Finally, the aggregate intertemporal risk is measured by the smoothed log earnings yield associated with the S\&P composite index, $E Y 10$, which is based on a 10 year moving average of aggregate earnings. ${ }^{3}$

Figures 2-4 present the time-series for the cross-sectional risk factors, $V R, V R^{*}, V I R, V I R^{*}$, $C I R R, C I R R^{*}$, while Table 1 presents descriptive statistics for this group of variables in addition to the excess (value-weighted) market return, $R M R F$, and the smoothed log market earnings yield, $E Y 10$. We can see in Figure 2 that the peaks in both $V R$ and $V R^{*}$ are in general associated with periods coincident (or near) to NBER economic recessions. This is especially relevant in late 90 's and the early 2000 's, when there is a large increase in stock price dispersion, which might be related to the economic downturn occurred in 2001; the NASDAQ bubble and increased uncertainty

[^3]concerning both the state of the economy and future cash flows. Figure 3 shows that both $V I R$ and $V I R^{*}$ also tend to increase in recessions, although the biggest increases occur in late 80's (before the recession in early 90 's). In addition, the comovement between portfolio returns and dividend yields registers large swings (of either sign) around recessions. These findings are partially confirmed by the following regressions containing the NBER business cycle dummy variable (CYCLE, 1 for expansions, 0 for economic expansions), with OLS t-statistics in parenthesis, ${ }^{4}$
\[

$$
\begin{aligned}
V R_{t}= & 0.026-0.004 C Y C L E_{t}, \text { Adj. } R^{2}=0.010 \\
& (16.290)(-2.471) \\
V I R_{t}= & 0.002-0.0005 C Y C L E_{t}, \text { Adj. } R^{2}=0.066 \\
& (22.147)(-5.990) \\
C I R R_{t}= & 0.000-0.00001 C Y C L E_{t}, A d j \cdot R^{2}=0.005 \\
& (2.263)(-1.878) .
\end{aligned}
$$
\]

The descriptive statistics in Table 1 show that the cross-sectional factors are not highly autocorrelated, and also not strongly contemporaneously correlated among themselves. The biggest correlations are between $R M R F$ and $C I R R / C I R R^{*}$ (-0.398/-0.408), and between EY10 and $V I R / V I R^{*}(0.612 / 0.493)$

Given the cross-sectional moments in Equations (12-13), (18-19), and (20-21), the Generalized

[^4]ICAPM in Equation (11) is now given by

$$
\begin{align*}
\mathrm{E}\left(R_{n, t+1}-R_{f, t+1}\right) & =\lambda_{M} \operatorname{Cov}\left(R_{n, t+1}, R M R F_{t+1}\right)+\lambda_{E Y} \operatorname{Cov}\left(R_{n, t+1}, E Y 10_{t+1}\right)+\lambda_{V R} \operatorname{Cov}\left(R_{n, t+1}, V R_{t+1}\right) \\
& +\lambda_{V I R} \operatorname{Cov}\left(R_{n, t+1}, V I R_{t+1}\right)+\lambda_{C I R R} \operatorname{Cov}\left(R_{n, t+1}, C I R R_{t+1}\right),  \tag{22}\\
\mathrm{E}\left(R_{n, t+1}-R_{f, t+1}\right) & =\lambda_{M} \operatorname{Cov}\left(R_{n, t+1}, R M R F_{t+1}\right)+\lambda_{E Y} \operatorname{Cov}\left(R_{n, t+1}, E Y 10_{t+1}\right)+\lambda_{V R} \operatorname{Cov}\left(R_{n, t+1}, V R_{t+1}^{*}\right) \\
& +\lambda_{V I R} \operatorname{Cov}\left(R_{n, t+1}, V I R_{t+1}^{*}\right)+\lambda_{C I R R} \operatorname{Cov}\left(R_{n, t+1}, C I R R_{t+1}^{*}\right) . \tag{23}
\end{align*}
$$

## 3 Asset Pricing Tests

### 3.1 Model estimation and evaluation: Two-stage GMM

In this sub-section, the Generalized ICAPM from Equations (22-23) is estimated and evaluated, by using the two-stage GMM framework (Hansen (1982)), where the weighting matrix used in the first-stage is the identity matrix, and in the second stage the weighting matrix is the inverse of the moments (spectral density) matrix. Therefore, the first-stage GMM with equally weighted pricing errors is equivalent to an OLS cross-sectional regression of average excess returns on asset covariances, whereas the efficient GMM - which assigns more weight to pricing errors with lower variance - is analogous to the corresponding GLS cross-sectional regression (Cochrane (2001), Chapter 13). The $N$ sample moments correspond to the pricing errors for each of the $N$ test assets, i.e., the sample counterpart of $(22),{ }^{5}$

$$
g_{T}(\boldsymbol{\lambda}) \equiv \frac{1}{T} \sum_{t=0}^{T}\left\{\begin{array}{c}
\left(R_{n, t+1}-R_{f, t+1}\right)-\lambda_{M} R_{n, t+1} R M R F_{t+1}-\lambda_{E Y} R_{n, t+1} E Y 10_{t+1} \\
-\lambda_{V R} R_{n, t+1} V R_{t+1}-\lambda_{V I R} R_{n, t+1} V I R_{t+1}-\lambda_{C I R R} R_{n, t+1} C I R R_{t+1} \tag{24}
\end{array}\right\}=\mathbf{0},
$$

and similarly for model (23). The asymptotic standard errors associated with system (24) account for measurement error in the covariances. As a robustness check, I compute standard errors that

[^5]don't correct for estimation error in covariances, (i.e., threat the covariances as fixed regressors as opposed to generated regressors), arising from the following GMM system,
$g_{T}(\boldsymbol{\lambda}) \equiv \frac{1}{T} \sum_{t=0}^{T}\left\{\left(R_{n, t+1}-R_{f, t+1}\right)-\lambda_{M} \sigma_{n, M}-\lambda_{E Y} \sigma_{n, E Y}-\lambda_{V R} \sigma_{n, V R}-\lambda_{V I R} \sigma_{n, V I R}-\lambda_{C I R R} \sigma_{n, C I R R}\right\}=\mathbf{0}$,
\[

$$
\begin{equation*}
n=1, \ldots, N, \tag{25}
\end{equation*}
$$

\]

where $\sigma_{n, M} \equiv \operatorname{Cov}\left(R_{n, t+1}, R M R F_{t+1}\right), \sigma_{n, E Y} \equiv \operatorname{Cov}\left(R_{n, t+1}, E Y 10_{t+1}\right), \sigma_{n, V R} \equiv \operatorname{Cov}\left(R_{n, t+1}, V R_{t+1}\right)$, $\sigma_{n, V I R} \equiv \operatorname{Cov}\left(R_{n, t+1}, V I R_{t+1}\right), \sigma_{n, \text { CIRR }} \equiv \operatorname{Cov}\left(R_{n, t+1}, C I R R_{t+1}\right)$ denote the (previously) estimated covariances with the factors. ${ }^{6}$

The standard errors for the parameter estimates $\hat{\boldsymbol{\lambda}}$, associated with both first and second stage GMM, are respectively given by

$$
\begin{align*}
\operatorname{Var}(\hat{\boldsymbol{\lambda}})= & \frac{1}{T}\left(\mathbf{d}^{\prime} \mathbf{I}_{N} \mathbf{d}\right)^{-1} \mathbf{d}^{\prime} \mathbf{I}_{N} \hat{\mathbf{S}} \mathbf{I}_{N} \mathbf{d}\left(\mathbf{d}^{\prime} \mathbf{I}_{N} \mathbf{d}\right)^{-1}  \tag{26}\\
& \operatorname{Var}(\hat{\boldsymbol{\lambda}})=\frac{1}{T}\left(\mathbf{d}^{\prime} \hat{\mathbf{S}}^{-1} \mathbf{d}\right)^{-1} \tag{27}
\end{align*}
$$

where $\mathbf{I}_{N}$ is a $N$ order Identity matrix, $\mathbf{d} \equiv \frac{\partial g_{T}(\hat{\boldsymbol{\lambda}})}{\partial \hat{\boldsymbol{\lambda}}^{\prime}}$ represents the matrix of moments' sensitivities to the parameters, and $\hat{\mathbf{S}}$ is a estimator for the spectral density matrix $\mathbf{S}$. The variance-covariance matrix for the pricing errors, $\hat{\boldsymbol{\alpha}} \equiv g_{T}(\hat{\boldsymbol{\lambda}})$, is represented by

$$
\begin{gather*}
\left.\left.\operatorname{Var}(\hat{\boldsymbol{\alpha}})=\frac{1}{T}\left(\mathbf{I}_{N}-\mathbf{d}\left(\mathbf{d}^{\prime} \mathbf{I}_{N} \mathbf{d}\right)^{-1}\right) \mathbf{d}^{\prime} \mathbf{I}_{N}\right) \hat{\mathbf{S}}\left(\mathbf{I}_{N}-\mathbf{I}_{N} \mathbf{d}\left(\mathbf{d}^{\prime} \mathbf{I}_{N} \mathbf{d}\right)^{-1}\right) \mathbf{d}^{\prime}\right)  \tag{28}\\
\left.\left.\operatorname{Var}(\hat{\boldsymbol{\alpha}})=\frac{1}{T}\left(\mathbf{I}_{N}-\mathbf{d}\left(\mathbf{d}^{\prime} \hat{\mathbf{S}}^{-1} \mathbf{d}\right)^{-1}\right) \mathbf{d}^{\prime} \hat{\mathbf{S}}^{-1}\right) \hat{\mathbf{S}}\left(\mathbf{I}_{N}-\hat{\mathbf{S}}^{-1} \mathbf{d}\left(\mathbf{d}^{\prime} \hat{\mathbf{S}}^{-1} \mathbf{d}\right)^{-1}\right) \mathbf{d}^{\prime}\right), \tag{29}
\end{gather*}
$$

for first-stage and second-stage GMM, respectively ${ }^{7}$. The asymptotic test that the pricing errors are jointly zero (test of overidentifying conditions or J-test) is represented by

$$
\begin{equation*}
T \hat{\boldsymbol{\alpha}}^{\prime} \hat{\mathbf{S}}^{-1} \hat{\boldsymbol{\alpha}} \sim \chi^{2}(N-K) \tag{30}
\end{equation*}
$$

[^6]with $K$ being the number of factors used in the model ( $K=5$, in the benchmark model). The asymptotic statistic (30) enables us to formally accept or reject a given model. In alternative, we can compute two goodness-of-fit measures to evaluate the overall pricing ability of the model - the average pricing error (root mean square error, $R M S E$ ) and the cross-sectional OLS $R^{2}$. RMSE is represented by
\[

$$
\begin{equation*}
R M S E=\sqrt{\frac{1}{N} \sum_{n=1}^{N} \hat{\alpha}_{n}^{2}}, \tag{31}
\end{equation*}
$$

\]

and the cross-sectional OLS $R^{2}$ is

$$
\begin{gather*}
R_{O L S}^{2}=1-\frac{\sum_{n=1}^{N} \hat{\alpha}_{n}^{2}}{\sum_{n=1}^{N} \bar{R}_{n}^{2}},  \tag{32}\\
\bar{R}_{n}=\frac{1}{T} \sum_{t=0}^{T}\left(R_{n, t+1}-R_{f, t+1}\right)-\frac{1}{N} \sum_{n=1}^{N}\left\{\frac{1}{T} \sum_{t=0}^{T}\left(R_{n, t+1}-R_{f, t+1}\right)\right\}, \\
A d j . R_{O L S}^{2}=1-\left(1-R^{2}\right)\left(\frac{N-1}{N-K}\right) .
\end{gather*}
$$

$R_{O L S}^{2}$ measures the proportion of cross-sectional variance in excess returns not explained by the model, and $A d j . R_{O L S}^{2}$ stands for the adjusted cross-sectional $R^{2}$, which corrects for degrees of freedom in the model (number of factors). Both (31) and (32) represent intuitive measures, since they give equal weight to all pricing errors (arising from first-stage GMM). The corresponding GLS cross-sectional $R^{2}$ is given by

$$
\begin{equation*}
R_{G L S}^{2}=1-\frac{\hat{\boldsymbol{\alpha}}^{\prime} \boldsymbol{\Omega}^{-1} \hat{\boldsymbol{\alpha}}}{\overline{\mathbf{R}}^{\prime} \boldsymbol{\Omega}^{-1} \overline{\mathbf{R}}}, \tag{33}
\end{equation*}
$$

where $\overline{\mathbf{R}}$ is the vector containing the (cross-sectional) demeaned average returns, and $\boldsymbol{\Omega}$ is a diagonal matrix containing the elements from the main diagonal of $\hat{\mathbf{S}} .{ }^{8}$ The pricing errors, $\hat{\boldsymbol{\alpha}}$, are from the second-stage GMM estimation. In (33) the pricing errors with higher variance are given less weight, in accordance with the efficient estimation inherent to the second stage GMM.

Table 2 presents the estimation/evaluation results from first-stage GMM for both models (22) and (23), with the test assets being the SBM25 portfolios. Given the correlations among

[^7]factors documented in Table 1 (Panels B and C), it is convenient to orthogonalize the factors ( $V R, V I R, C I R R, E Y 10$ ) relative to $R M R F .{ }^{9}$ Table 2 also presents the results for two alternative asset pricing models, the traditional CAPM (from Sharpe (1964) and Lintner (1965)), and the Fama and French (1993) three factor model (FF3 hereafter), which has been empirically successful in explaining the cross-section of stock returns. The pricing equation for the FF3 model can be represented as
\[

$$
\begin{align*}
\mathrm{E}\left(R_{n, t+1}-R_{f, t+1}\right)= & \lambda_{M} \operatorname{Cov}\left(R_{n, t+1}, R M R F_{t+1}\right)+\lambda_{S M B} \operatorname{Cov}\left(R_{n, t+1}, S M B_{t+1}\right)  \tag{34}\\
& +\lambda_{H M L} \operatorname{Cov}\left(R_{n, t+1}, H M L_{t+1}\right),
\end{align*}
$$
\]

where $S M B$ and $H M L$ represent the size premium and value premium factors, respectively. The CAPM arises as a special case of (34) by imposing $\lambda_{S M B}=\lambda_{H M L}=0$. The results presented in first row confirm many previous findings (starting in Fama and French (1992)) that the CAPM performs poorly in pricing the $S B M 25$ portfolios, with a monthly average pricing error of $0.311 \%$, and a negative cross-sectional $R^{2}(-0.598)$. The FF3 model (in row 4) clearly improves relative to the CAPM, with a $R M S E$ of $0.139 \%$ and an adjusted $R^{2}$ of 0.654 . The results for the GICAPM (row 2) are not too different from those associated with FF3, with a RMSE only marginally higher ( $0.147 \%$ ) , and a cross-sectional $R^{2}$ of 0.575 . The GICAPM* in row 3 (whose cross-sectional factors are based on the SBM100 portfolios) provides the best overall results, with an average error of $0.122 \%$ per month and with $A d j . R_{O L S}^{2}$ being 0.704 (notice that $A d j . R_{O L S}^{2}$ corrects for the fact that the GICAPM /GICAPM* have two additional factors relative to FF3). Regarding the J-test, all four models are rejected, although the levels for $G I C A P M / G I C A P M^{*}$ are clearly lower in relation to both FF3 and CAPM. In terms of the covariance risk prices associated with the cross-sectional

[^8]$$
\hat{f}_{t}=\hat{\eta}_{0}+\hat{e}_{t}
$$
where $\hat{\eta}_{0}$ is the intercept, and $\hat{e}_{t}$ represents the residual from the following regression,
$$
f_{t}=\eta_{0}+\eta_{1} R M R F_{t}+e_{t}
$$
factors, both $\lambda_{V R}$ and $\lambda_{C I R R}$ are positive, while $\lambda_{V I R}$ assumes negative estimates. Furthermore, all three risk prices are strongly statistically significant.

While the previous results represent some evidence in favor of the GICAPM, it is important to orthogonalize the factors among themselves (and not only relative to $R M R F$ ) in order to better interpret the individual contribution from each factor. Following Campbell (1996), Patelis (1997), and Petkova (2006), I employ a first-order VAR,

$$
\begin{gather*}
\mathbf{x}_{t+1}=\mathbf{A} \mathbf{x}_{t}+\boldsymbol{\varepsilon}_{t+1}  \tag{35}\\
\mathbf{x}_{t} \equiv\left[R M R F_{t}, V R_{t}, V I R_{t}, C I R R_{t}, E Y 10_{t}\right]^{\prime}
\end{gather*}
$$

with A representing the coefficient VAR matrix, and $R M R F$ being positioned first in the VAR. The orthogonalized VAR residuals are then given by

$$
\begin{gathered}
\mathbf{w}_{t+1}=\mathbf{P}^{-1} \varepsilon_{t+1} \\
\mathbf{w}_{t} \equiv\left[w_{M, t}, w_{E Y, t}, w_{V R, t}, w_{V I R, t}, w_{C I R R, t}\right]^{\prime}
\end{gathered}
$$

with $\mathbf{P}$ representing a Choleski matrix. The Generalized ICAPM of Equation (22) is now given by

$$
\begin{align*}
\mathrm{E}\left(R_{n, t+1}-R_{f, t+1}\right) & =\lambda_{M} \operatorname{Cov}\left(R_{n, t+1}, R M R F_{t+1}\right)+\lambda_{E Y} \operatorname{Cov}\left(R_{n, t+1}, w_{E Y, t+1}\right)+\lambda_{V R} \operatorname{Cov}\left(R_{n, t+1}, w_{V R, t+1}\right) \\
& +\lambda_{V I R} \operatorname{Cov}\left(R_{n, t+1}, w_{V I R, t+1}\right)+\lambda_{C I R R} \operatorname{Cov}\left(R_{n, t+1}, w_{C I R R, t+1}\right) \tag{36}
\end{align*}
$$

and similarly for $G I C A P M^{*}$.
The VAR estimation results are provided in Table 3. In Panel A the factors are those associated with the GICAPM, and in Panel B, the factors are from the GICAPM*. We can summarize the estimation results as follows: $V R / V R^{*}$ are explained by their respective own lagged values, but also by $E Y 10 ; V I R\left(V I R^{*}\right)$ has a negative (positive) autocorrelation coefficient and they are both correlated with lagged $E Y 10 ; C I R R / C I R R^{*}$ are both negatively correlated with lagged market returns; $E Y 10$ has a very persistent autocorrelation coefficient, and it is also forecasted
by lagged market returns, which is related with some short-term momentum in stock prices; and finally market returns are mostly negatively forecasted by $C I R R / C I R R^{*}$.

The results for model (36) with $S B M 25$ as test portfolios, are presented in Table 4. Panel A reports the results from first stage GMM, and the efficient GMM estimation is reported in Panel $\mathrm{B}^{10}$. We can see that both GICAPM and GICAPM* provide similar results, with a lower average pricing error than in Table $2(0.104 \%)$. The $A d j . R_{O L S}^{2}$ are 0.781 and 0.783 , for GICAPM and GICAPM*, respectively. Regarding the risk price estimates, the asymptotic standard errors that correct for measurement error in covariances are large, although $\lambda_{E Y}, \lambda_{C I R R}$ and $\lambda_{M}$ are statistically significant, based on the non-corrected standard errors. The magnitudes of the risk prices are obviously different to their counterparts in Table 2, given the different proxies used (orthogonalized VAR innovations instead of the raw factors). In the second stage estimation, the risk price estimates have higher precision relative to the first stage counterparts, as indicated by the respective t-statistics, with both $\lambda_{C I R R}$ and $\lambda_{E Y}$ being statistically significant in either GICAPM or $G I C A P M^{*}$, while the market risk price is no longer significant. While one can not compare across models, the second stage $R M S E$ (based on second stage pricing errors) is significantly lower for the GICAPM in comparison with FF3. The GLS adjusted $R^{2}$, although being lower than the corresponding OLS estimates, nevertheless assumes reasonable values for both GICAPM and GICAPM* ( 0.685 and 0.314 , respectively), whereas in the FF3 model it has a residual magnitude (0.091). More relevant is the fact that both GICAPM and GICAPM* are not rejected by the J-test (p-values of 0.729 and 0.234 , respectively), whereas the FF3 model is strongly rejected. Therefore, these results reinforce the results in Table 2 that the Generalized ICAPM compares relatively well to the FF3 model in pricing the $S B M 25$ portfolios.

Despite the fact that the SBM25 portfolios have been the most challenging group of assets for the CAPM, several authors have raised some concerns about asset pricing tests that rely only on the size/BM portfolios. Lo and Mackinlay (1990) and Daniel and Titman (1997) advert for the

[^9]problems inherent with using portfolios sorted on stock characteristics. More recently, Lewellen, Nagel and Shanken (2006) stress that SBM25 exhibit a strong factor structure (i.e., the timeseries variation in returns is almost explained by only two factors, $S M B$ and $H M L$ ), and hence they argue that asset pricing models containing factors correlated with either $S M B$ or $H M L$ will artificially price the $S B M 25$ portfolios, and therefore, one should include as test assets additional portfolios, which are not so strongly correlated with either $S M B$ or $H M L$. In response to these concerns, I use 38 industry portfolios (IND38) (Fama and French (1997)), 10 portfolios sorted on the earnings-to-price ratio $(E / P) ; 10$ portfolios sorted on the cash flow-to-price ratio $(C F / P)$ and 10 portfolios sorted on the dividend-to-price ratio $(D / P)$ (Fama and French (1996)), as additional groups of test assets. The results associated with SBM25 in combination with the industry portfolios (SBM25 + IND38) are reported in Table 5. The average pricing errors are higher than the corresponding estimates in Table 4 - reflecting the biggest hurdle of simultaneously pricing $S B M 25$ and the industry portfolios - nevertheless, both GICAPM and GICAPM* still have lower $R M S E$ than FF3 $(0.154 \% / 0.153 \%$ versus $0.188 \%)$. The analysis for the cross-sectional $R^{2}$ confirms these findings, with GICAPM/GICAPM* having Adj. $R_{O L S}^{2}$ of $0.388 / 0.390$ compared to 0.127 for FF3. Furthermore, the $A d j \cdot R_{G L S}^{2}$ are similar to the OLS counterparts, in the case of GICAPM /GICAPM* (0.406/0.395), whereas FF3 has a large negative estimate (-1.374). All three models are rejected by the asymptotic test (30), which might be related with the large number of portfolios used in test, and the inherent problems in inverting the spectral density matrix. Regarding the individual significance of the risk prices, with the sole exception of $\lambda_{V I R}$ (which is not significant at the $10 \%$ level), all the risk prices in the GICAPM/GICAPM* are statistically significant at the $1 \%$ level, in both first and second stage estimation.

The results associated with all portfolios (i.e., by including the 30 additional characteristic portfolios, $S B M 25+C F / P+E / P+D / P+I N D 38$ ) are presented in Table 6. Essentially, the results confirm the findings from Table 5: (i) GICAPM/GICAPM* have a lower average pricing error than FF3; (ii) GICAPM/GICAPM* have higher $A d j . R_{O L S}^{2}$ than FF3; (iii) Both $A d j . R_{O L S}^{2}$ and $\operatorname{Adj} . R_{G L S}^{2}$ present similar values in the cases of GICAPM/GICAPM*; and (iv) finally FF3
has a negative $A d j . R_{G L S}^{2}$ estimate. Therefore, the outperformance of the Generalized ICAPM relative to the FF3 model is maintained when using additional classes of portfolios.

As an additional comparison between the Generalized ICAPM and FF3, I conduct the following asymptotic difference test (Cochrane (1996), Cochrane (2001)),

$$
\begin{equation*}
T \hat{\boldsymbol{\alpha}}_{r}^{\prime} \hat{\mathbf{S}}^{-1} \hat{\boldsymbol{\alpha}}_{r}-T \hat{\boldsymbol{\alpha}}_{\mathbf{u}}^{\prime} \hat{\mathbf{S}}^{-1} \hat{\boldsymbol{\alpha}}_{\mathbf{u}} \sim \chi^{2}\left(K^{*}\right) \tag{37}
\end{equation*}
$$

which compares the restricted model ( $\hat{\boldsymbol{\alpha}}_{r}$, excluding the factors we want to test) against an unrestricted model ( $\hat{\boldsymbol{\alpha}}_{\mathbf{u}}^{\prime}$ ), with $\hat{\mathbf{S}}$ being associated with the unrestricted model. $K^{*}$ denotes the number of restrictions (equal to $K_{u}-K_{r}$, where $K_{u}, K_{r}$ denote the number of factors associated with the unrestricted and restricted models, respectively). This test is equivalent to a likelihood ratio test and enables to evaluate whether the excluding factors are important to price assets. In our case, the unrestricted model will be the GICAPM in combination with the $S M B$ and $H M L$ factors,

$$
\begin{gather*}
\mathrm{E}\left(R_{n, t+1}-R_{f, t+1}\right)=\lambda_{M} \operatorname{Cov}\left(R_{n, t+1}, R M R F_{t+1}\right)+\lambda_{E Y} \operatorname{Cov}\left(R_{n, t+1}, w_{E Y, t+1}\right)+\lambda_{V R} \operatorname{Cov}\left(R_{n, t+1}, w_{V R, t+1}\right) \\
+\lambda_{V I R} \operatorname{Cov}\left(R_{n, t+1}, w_{V I R, t+1}\right)+ \\
\lambda_{C I R R} \operatorname{Cov}\left(R_{n, t+1}, w_{C I R R, t+1}\right)+\lambda_{S M B} \operatorname{Cov}\left(R_{n, t+1}, S M B_{t+1}\right)  \tag{38}\\
+
\end{gather*}
$$

Thus, both GICAPM/GICAPM* and FF3 represent special cases of (38). Table 7 reports the RMSE and cross-sectional $R^{2}$ associated with the unrestricted model (38), which is denoted by $G I C A P M+F F 3\left(G I C A P M^{*}+F F 3\right)$. For convenience, I replicate the corresponding estimates for GICAPM/GICAPM* and FF3 from Tables 4-6. The results associated with SBM25, SBM25+ $I N D 38$, and $S B M 25+C F / P+E / P+D / P+I N D 38$ are reported in Panels $\mathrm{A}, \mathrm{B}$ and C , respectively. We can see that the $R M S E$ estimates associated with both the unrestricted model and the Generalized ICAPM are very similar, and this pattern is robust across the three classes of test portfolios. On the other hand, the $A d j . R_{O L S}^{2}$ magnitudes associated with the general model (38) are slightly lower than the corresponding estimates for GICAPM/GICAPM*. This pattern is more accentuated in the case of $A d j \cdot R_{G L S}^{2}$, given that the FF3 model has either negligible
(Panel A, SBM25) or negative estimates (Panels B and C, augmented portfolios) of Adj. $R_{G L S}^{2}$. Therefore, these facts suggest that both $S M B$ and $H M L$ don't add explanatory power over the cross-section, in the presence of the factors contained in the Generalized ICAPM. The levels and respective p-values for the Difference test (37) - reported in the last two columns of Table 7 confirm these findings. In Panel A, we accept the null that neither group of excluded factors is important, although the p-values associated with GICAPM/GICAPM* (as restricted models) are significantly higher when compared to FF3 ( 0.866 for $G I C A P M^{*}$ versus 0.205 for FF3). In both Panels B and C, the test accepts the exclusion of both $S M B$ and $H M L$, i.e., both the GICAPM and GICAPM* are note rejected as restricted models. On the other hand, the test strongly rejects the exclusion of the four factors contained in the Generalized ICAPM ( $V R / V R^{*}$, $V I R / V I R^{*}, C I R R / C I R R^{*}, E Y 10_{t+1}$ ), with p-values significantly lower than $5 \%$. Therefore, the FF3 model as a special case of the general model, is rejected.

The weak performance of the FF3 model in pricing alternative classes of portfolios (e.g, industry portfolios) is partially related with the fact that both $S M B$ and $H M L$ were designed to price the SBM25 portfolios. In response to that, I reestimate both the Generalized ICAPM and FF3, by assigning a bigger weight to the SBM25 in comparison to the other portfolios, in the joint first stage GMM estimation. Thus, the first stage weighting matrix assigns a weight of 2 to each of the 25 size/BM portfolios, and a weight of 1 (as previously) to the remaining portfolios. The estimation is conducted for $S B M 25+I N D 38$ and $S B M 25+C F / P+E / P+D / P+I N D 38$. The "reduced" RMSE,

$$
\begin{equation*}
R M S E^{*}=\sqrt{\frac{1}{25} \sum_{n=1}^{25} \hat{\alpha}_{n}^{2}}, \tag{39}
\end{equation*}
$$

measures the average pricing error, within the SBM25 portfolios. In addition, the "weighted" cross-sectional $R^{2}$,

$$
\begin{equation*}
R^{* 2}=1-\frac{\hat{\boldsymbol{\alpha}}^{\prime} \mathbf{W}^{*} \hat{\boldsymbol{\alpha}}}{\overline{\mathbf{R}}^{\prime} \mathbf{W}^{*} \overline{\mathbf{R}}} \tag{40}
\end{equation*}
$$

is a proxy for the model's overall explanatory power, with the pricing errors weighted properly
according to $\mathbf{W}^{*} .{ }^{11}$ The results in Table 8 show that the estimates of $R M S E^{*}$ associated with GICAPM/GICAPM* are lower than the counterparts associated with the FF3 model, for both classes of portfolios. These results are confirmed by the adjusted weighted $R^{2}$, which assumes higher values for both GICAPM/GICAPM* in comparison with FF3 ( 0.531 for GICAPM* versus 0.316 for FF3 in Panel A, and 0.570 versus 0.404 in the estimation with all test portfolios). Therefore, by assigning more importance to the SBM25 portfolios, the Generalized ICAPM continues to do well relative to the FF3 model.

In order to have an assessment on the individual pricing errors, Figure 5 plots the estimated excess returns (vertical axis) against the realized excess returns associated with the SBM25 portfolios. Panels A, B, and C are associated with $G I C A P M, G I C A P M^{*}$ and FF3 models, respectively. The estimated returns are from the first stage estimation, in order to be able to compare across models. We can confirm the better fit around the $45^{\circ}$ line, in the case of both GICAPM /GICAPM* compared to FF3. The biggest outlier is the extreme small-growth portfolio (southern point) which is difficult to price for all three models. Figure 7, Panel A, presents the first stage pricing errors for $S B M 25$. We can see that for most portfolios, the pricing errors arising from GICAPM/GICAPM* compare favorably with those associated with FF3. In particular, FF3 has significantly higher pricing errors for portfolios $S B M_{21}, S B M_{45}, S B M_{51}, S B M_{55}$, where the first index denotes the size quintile and the second index measures the BM quintile. Figure 6 (Panels A, B and C) plots the estimated/realized excess returns for the industry portfolios. The graphs show that there is also significant less dispersion around the $45^{\circ}$ line, for the Generalized ICAPM in comparison with FF3. The graph for the industry pricing errors (Figure 8, Panel A), shows that for most industries, FF3 has larger pricing errors than GICAPM/GICAPM*. The difference in error magnitudes across models is greater for Food (FO, in the graph); Tobacco (SM); Wood (WO); Chemicals (CH); Metal (ME) and Transportation (TR).

[^10]
### 3.2 Model estimation and evaluation: The Hansen and Jagannathan (1997) distance

In alternative to the traditional first-stage estimation with equally pricing errors, several authors have used the weighting matrix associated with the Hansen and Jagannathan (1997) distance, in the first stage estimation (e.g., Jagannathan and Wang (1996), Hodrick and Zhang (2001), Jacobs and Wang (2004), among others). The HJ weighting matrix is given by

$$
\begin{equation*}
\mathbf{W}_{H J}=\left(\frac{1}{T} \sum_{t=1}^{T} \mathbf{R}_{t} \mathbf{R}_{t}^{\prime}\right)^{-1} \tag{41}
\end{equation*}
$$

where $\mathbf{R}_{t}$ is the vector containing asset returns at time $t$, thus assets with a larger second moment in returns are given less weight in the estimation. The HJ distance, which is equal to

$$
\begin{equation*}
H J=\left(\hat{\boldsymbol{\alpha}}^{\prime} \mathbf{W}_{H J} \hat{\boldsymbol{\alpha}}\right)^{\frac{1}{2}} \tag{42}
\end{equation*}
$$

can be interpreted as the minimum distance between a given candidate SDF an the set of all true SDF's. The HJ method shares with the first stage estimation (with equally weighted errors) the attractive feature that one can compare the results across different models, since they don't rely on the estimation of $\mathbf{S}$ (as it is the case with efficient GMM). Nevertheless, the results associated with $\mathbf{W}_{H J}$ are more difficult to interpret than the estimation with equally weighted errors. Moreover, often the second moments matrix of returns is near singular which causes difficulties in the inversion (Cochrane (1996)).

The results associated with the HJ method are presented in Table 9, for the case of the SBM25 portfolios. Panel A presents the first stage estimates, and Panel B reports the corresponding efficient estimates. The average pricing error is similar across the three models, although RMSE is not a convenient measure in this context, since the GMM estimation does not weight portfolios equally. More relevant is the fact that the HJ distance is lower for both GICAPM/GICAPM* relative to FF3 ( $0.341 / 0.331$ versus 0.367 ). Moreover, we can not reject the null that $H J$ is zero, for
both GICAPM /GICAPM* at the $5 \%$ significance level (p-values of 0.087 and 0.063 , respectively). On the other, we reject the null $H J=0$, in the case of $\mathrm{FF} 3^{12}$. By looking at $A d j \cdot R_{G L S}^{2}$ it happens that FF3 has higher values than in Table 4, but still with lower estimates in comparison with both GICAPM/GICAPM* (0.196 versus $0.432 / 0.285$ ). Compared to the corresponding t-statistics in Table 4, the first stage risk prices associated with $V R\left(V R^{*}\right)$ and $C I R R\left(C I R R^{*}\right)$ are now statistically significant. On the other hand, the three models are rejected by the J-test at the $5 \%$ level, although the test level associated with FF3 is the double of the corresponding levels for GICAPM/GICAPM*.

The estimation results for the augmented portfolios are reported in Tables 10 (SBM25 + $I N D 38$ ) and 11 (all portfolios). We can see that in both cases, the estimates associated with HJ are similar across the three models. Nevertheless, the FF3 model still has very low estimates for $\operatorname{Adj} \cdot R_{G L S}^{2}(-0.643$ in the case of $S B M 25+I N D 38$, and 0.001 in the case of all portfolios).

Figure 5 (Panels D, E and F) present the plot for the estimated/average returns associated with the $S B M 25$ portfolios, arising from the HJ estimation. We can see that both GICAPM/GICAPM* have a better fit relative to FF3, although the difference is not as relevant as in the estimation with equally-weighted errors (Panels A to C). The plot of the corresponding average pricing errors in Figure 7 (Panel B), confirms this evidence, with the portfolio pricing errors being more similar across models, (compared to Panel A in the same figure), with the greatest gap being with portfolio $S B M_{51}$. In the cases of the industry portfolios, both Figures 6 (Panels D, E and F) and 8 (Panel B) show that the pricing ability for the industry portfolios associated with FF3 is closest to both GICAPM/GICAPM*, in comparison with the case of equally weighted errors.

### 3.3 The Value Premium

The value premium refers to the CAPM anomaly in which growth stocks have low average returns and large negative pricing errors (associated with the CAPM) and value stocks have large average returns and positive pricing errors (Fama and French (1992, 1993)). We already saw in the

[^11]previous sub-sections that the GICAPM/GICAPM* model is able to do reasonably well - and even outperform the FF3 model - in pricing the SBM25 portfolios. This sub-section goes one step further and seeks to assess how GICAPM/GICAPM* perform in explaining the extreme growth and value portfolios in comparison to FF3, by assigning a larger weight to those portfolios in the first stage estimation. The analysis done above for the $S B M 25$ portfolios is replicated for the extreme BM quintiles within each size quintile: $S B M_{11}, S B M_{15}, S B M_{21}, S B M_{25}, S B M_{31}, S B M_{35}$, $S B M_{41}, S B M_{45}, S B M_{51}$, and $S B M_{55}$. The GMM weighting matrix assigns a weight of 2 for each of these 10 portfolios, and a weight of 1 for all remaining portfolios. In this case, the estimation is done with the three classes of portfolios, SBM25 (Panel A), SBM25 + IND38 (Panel B), and all portfolios (Panel C). The "reduced" RMSE is now given by
\[

$$
\begin{equation*}
R M S E^{*}=\sqrt{\frac{1}{10} \sum_{n=1}^{10} \hat{\alpha}_{n}^{2}} \tag{43}
\end{equation*}
$$

\]

where the $\hat{\alpha}_{n}$ 's denote the pricing errors associated with the 10 portfolios described above. The "weighted" cross-sectional $R^{2}$ is similar to that in (40) with $\mathbf{W}^{*}$ accounting for the new weights. The results presented in Table 12 indicate that the estimates of $R M S E^{*}$ associated with GICAPM/GICAP compare favorably with the corresponding estimates for FF3: In the case of the SBM25 portfolios, RMSE* achieves $0.114 \% / 0.134 \%$ for GICAPM and GICAPM*, respectively, compared to $0.182 \%$ for FF3. This pattern is maintained for the other two groups of portfolios in Panels B and C. On the other hand, the "weighted" cross-sectional $R^{2}$ assumes higher values for both GICAPM/GICAPM* in comparison to FF3 (0.799/0.779 versus 0.646 , in the case of SBM25).

The previous analysis is further replicated for the separate cases of growth and value portfolios. The growth portfolios which are given a higher weight are $S B M_{11}, S B M_{21}, S B M_{31}, S B M_{41}$, $S B M_{51}$, whereas the extreme value portfolios are $S B M_{15}, S B M_{25}, S B M_{35}, S B M_{45}, S B M_{55}$. The results (not shown) largely confirm the findings in Table 12, with $R M S E^{*}$ achieving lower values in both GICAPM/GICAPM* relative to FF3 and with the "weighted" cross-sectional $R^{2}$ assuming higher values for both GICAPM/GICAPM* against FF3.

Within the class of $S B M 25$ portfolios, the most difficult portfolio to price is the extreme small-growth portfolio ( $S B M_{11}$ ), as shown in Figures 5 and 7. The pricing error estimates for this portfolio ( $\hat{\alpha}_{11}$ ) are $-0.324 \%,-0.357 \%$ and $-0.353 \%$ for GICAPM, GICAPM ${ }^{*}$ and FF3, respectively. When the first stage GMM estimation assigns a weight of 2 to $S B M_{11}$ and 1 to the remaining 24 portfolios in SBM25-i.e., more importance is given to the small-growth portfolio - then the estimated errors are $-0.235 \%,-0.279 \%$ and $-0.288 \%$ for GICAPM, GICAPM* and FF3, respectively.

What are the factors that drive the ability of the Generalized ICAPM to price the value premium? To address this issue, I calculate the risk premium (covariance times risk price) for each factor, and across each BM quintile. Table 13 reports the factor risk premiums and average pricing error across book-to-market quintiles, for both the GICAPM, GICAPM* and FF3 models ${ }^{13}$. We can see that in both GICAPM and GICAPM* the risk premium attached to CIRR is negative for growth stocks (denoted by $B M 1$ ) and positive for value stocks ( $B M 5$ ), producing a gap of $0.475 \% /-0.432 \%$ across extreme quintiles, for GICAPM and GICAPM*, respectively. In addition, the risk premium associated with $E Y 10$, is almost zero for growth stocks and largely positive for value stocks, with a resulting gap of $-0.338 \% /-0.315 \%$ respectively for GICAPM and GICAPM*. Thus both $C I R R$ and $E Y 10$ mimic the role played by $H M L$ in the FF3 model, into explaining the value premium. By comparing across models the average pricing errors per BM quintile, we can see that, with the sole exception of the middle BM quintile, both GICAPM/GICAPM* have lower average errors than the FF3 model.

In retrospect, the results of this subsection confirm that the Generalized ICAPM is able to price the value premium.

[^12]
## 4 Conclusion

This paper derives a theoretical asset pricing model that represents a generalization from the Intertemporal CAPM, by relaxing the representative investor assumption. There are two types of investor heterogeneity. First, there are idiosyncratic shocks in wealth, which are not fully insurable by financial markets. Second, each investor is assumed to have different "intertemporal risks", i.e., they face different state variables that proxy for changes in future portfolio returns. The result is a five factor model, whose factors are the change in aggregate wealth; aggregate intertemporal risk; dispersion on investor's wealth; dispersion in investor's intertemporal risk; and the comovement across investors between shocks in wealth and in intertemporal risk. The cross-sectional factors are the novelty relative to the standard ICAPM, and arise from the existence of investor heterogeneity. The model is denoted as the Generalized ICAPM (GICAPM).

In the empirical implementation of the model, due to measurement issues, the cross-sectional variance of returns is used as a proxy for the dispersion in wealth across investors. Furthermore, the cross-sectional variance for the dividend-to-price ratio is used as proxy for the dispersion in intertemporal risk. Finally the market return is used instead of changes in market wealth, and the market earnings yield is the proxy used for aggregate intertemporal risk.

The empirical test of the model shows that the Generalized ICAPM is able to price reasonably well the Fama and French (1993) portfolios, and compares favorably with the Fama and French (1993) model. These results are robust to tests made with additional classes of portfolios - industry portfolios and alternative characteristic portfolios sorted on the cash flow-to-price, earnings-to-price and dividend-to-price ratios. In addition, the results are robust to different estimation methodologies, the two-stage GMM procedure with equally weighted pricing errors in the first stage, and in alternative, the two-stage method with the Hansen and Jagannathan (1997) weighting matrix, in the first stage. Moreover, the Generalized ICAPM is able to price at least as well as the Fama and French (1993) model, the extreme growth and value portfolios, that is, the GICAPM explains the value premium anomaly.

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## A An ICAPM with cross-sectional risk

The problem for Investor $i$ is stated as

$$
\begin{aligned}
& J_{t}\left(W_{t}^{i}, z_{t}^{i}\right) \equiv \max _{\left\{C_{t+j}^{i}\right\}_{j=0}^{\infty},\left\{\omega_{n, t+j}^{i}\right\}_{j=0}^{\infty}} \mathrm{E}_{t}\left[\sum_{j=0}^{\infty} \delta^{j} U\left(C_{t+j}^{i}\right)\right] \\
& \text { s.t. }\left\{\begin{array}{c}
W_{t+1}^{i}=R_{p, t+1}^{i}\left(W_{t}^{i}-C_{t}^{i}\right) \\
R_{p, t+1}^{i}=g\left(z_{t}^{i}\right)
\end{array}\right.
\end{aligned}
$$

and can be represented in a dynamic programming framework, in the following form

$$
\begin{align*}
& J_{t}\left(W_{t}^{i}, z_{t}^{i}\right) \equiv \max _{C_{i}^{i}, \omega_{n, t}^{i}}\left\{U\left(C_{t}^{i}\right)+\delta \mathrm{E}_{t}\left[J_{t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right)\right]\right\} \\
& \text { s.t. }\left\{\begin{array}{c}
W_{t+1}^{i}=R_{p, t+1}^{i}\left(W_{t}^{i}-C_{t}^{i}\right) \\
R_{p, t+1}^{i}=g\left(z_{t}^{i}\right)
\end{array},\right. \tag{A.1}
\end{align*}
$$

where $J_{t}\left(W_{t}^{i}, z_{t}^{i}\right)$ denotes the value function for investor $i ; R_{p, t+1}^{i}$ is the gross return on investor $i$ 's reference portfolio; $z_{t}^{i}$ are the state variables that forecast $R_{p, t+1}^{i},{ }^{14}$ and $\omega_{n, t}^{i}$ is the weight for asset $n$ in the portfolio of investor $i$. The f.o.c. with respect to $C_{t}^{i}$ is equal to

$$
\begin{equation*}
U^{\prime}\left(C_{t}^{i}\right)=\delta \mathrm{E}_{t}\left[J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right) R_{p, t+1}^{i}\right], \tag{A.2}
\end{equation*}
$$

where $U^{\prime}\left(C_{t}^{i}\right)$ stands for the first partial derivative relative to $C_{t}^{i}$, and $J_{W, t+1}$ is the first partial derivative of $J_{t}($.$) w.r.t W_{t+1}^{i}$. By applying the envelope theorem to (A.1), $J_{W, t}($.$) can be repre-$ sented as
$J_{W, t}\left(W_{t}^{i}, z_{t}^{i}\right)=\frac{\partial C_{t}^{i}}{\partial W_{t}^{i}}\left\{U^{\prime}\left(C_{t}^{i}\right)-\delta \mathrm{E}_{t}\left[J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right) R_{p, t+1}^{i}\right]\right\}+\delta \mathrm{E}_{t}\left[J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right) R_{p, t+1}^{i}\right]$,
and by using Equation (A.2), Equation (A.3) simplifies to

$$
\begin{equation*}
J_{W, t}\left(W_{t}^{i}, z_{t}^{i}\right)=\delta \mathrm{E}_{t}\left[J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right) R_{p, t+1}^{i}\right] \tag{A.4}
\end{equation*}
$$

which can be rewritten, given (A.2), as follows

$$
\begin{equation*}
J_{W, t}\left(W_{t}^{i}, z_{t}^{i}\right)=U^{\prime}\left(C_{t}^{i}\right) . \tag{A.5}
\end{equation*}
$$

By updating (A.5), substituting the result in (A.2), and rearranging, we obtain the Euler equation for investor $i$,

$$
\begin{equation*}
1=\mathrm{E}_{t}\left[\frac{\delta U^{\prime}\left(C_{t+1}^{i}\right)}{U^{\prime}\left(C_{t}^{i}\right)} R_{p, t+1}^{i}\right]=\mathrm{E}_{t}\left[\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right)}{J_{W, t}\left(W_{t}^{i}, z_{t}^{i}\right)} R_{p, t+1}^{i}\right] . \tag{A.6}
\end{equation*}
$$

[^13]Given (A.6), the stochastic discount factor (SDF) for investor $i$ is equal to

$$
\begin{equation*}
M_{t+1}^{i}=\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right)}{J_{W, t}\left(W_{t}^{i}, z_{t}^{i}\right)} \equiv h\left(W_{t+1}^{i}, z_{t+1}^{i}\right) . \tag{A.7}
\end{equation*}
$$

Let the portfolio return for investor $i$ be represented as

$$
\begin{equation*}
R_{p, t+1}^{i}=\sum_{n=1}^{N-1} \omega_{n, t}^{i}\left(R_{n, t+1}-R_{f, t+1}\right)+R_{f, t+1}, \tag{A.8}
\end{equation*}
$$

with $R_{f, t+1}$ denoting a benchmark return (for example, the risk-free rate). ${ }^{15}$ Therefore, the f.o.c. with respect to $\omega_{n, t}^{i}$ is given by

$$
\mathrm{E}_{t}\left[J_{W, t+1}\left(W_{t+1}^{i}, z_{t}^{i}\right)\left(W_{t}^{i}-C_{t}^{i}\right)\left(R_{n, t+1}-R_{f, t+1}\right)\right]=0
$$

which can be rewritten as

$$
\begin{equation*}
\mathrm{E}_{t}\left[\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t}^{i}\right)}{J_{W, t}\left(W_{t+1}^{i}, z_{t}^{i}\right)} R_{n, t+1}\right]=\mathrm{E}_{t}\left[\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t}^{i}\right)}{J_{W, t}\left(W_{t+1}^{i}, z_{t}^{i}\right)} R_{f, t+1}\right] . \tag{A.9}
\end{equation*}
$$

By substituting (A.8) in (A.6), and rearranging, we obtain

$$
1=\sum_{n=1}^{N-1} \omega_{n, t}^{i} \mathrm{E}_{t}\left[\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t}^{i}\right)}{J_{W, t}\left(W_{t+1}^{i}, z_{t}^{i}\right)}\left(R_{n, t+1}-R_{f, t+1}\right)\right]+\mathrm{E}_{t}\left[\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t}^{i}\right)}{J_{W, t}\left(W_{t+1}^{i}, z_{t}^{i}\right)} R_{f, t+1}\right],
$$

and by using (A.9) leads to

$$
\begin{equation*}
1=\mathrm{E}_{t}\left[\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t}^{i}\right)}{J_{W, t}\left(W_{t+1}^{i}, z_{t}^{i}\right)} R_{f, t+1}\right]=\mathrm{E}_{t}\left[\frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t}^{i}\right)}{J_{W, t}\left(W_{t+1}^{i}, z_{t}^{i}\right)} R_{n, t+1}\right], \tag{A.10}
\end{equation*}
$$

for an arbitrary return $R_{n, t+1}$. By averaging across $I$ investors, the general pricing equation (A.10) can be rewritten as

$$
\begin{gather*}
1=\mathrm{E}_{t}\left[M_{t+1} R_{n, t+1}\right]  \tag{A.11}\\
M_{t+1}=\frac{1}{I} \sum_{i=1}^{I} M_{t+1}^{i}=\frac{1}{I} \sum_{i=1}^{I} \frac{\delta J_{W, t+1}\left(W_{t+1}^{i}, z_{t+1}^{i}\right)}{J_{W, t}\left(W_{t}^{i}, z_{t}^{i}\right)}=\frac{1}{I} \sum_{i=1}^{I} h\left(W_{t+1}^{i}, z_{t+1}^{i}\right), \tag{A.12}
\end{gather*}
$$

where $M_{t+1}$ represents the average SDF in the economy.
The function $h\left(W_{t+1}^{i}, z_{t+1}^{i}\right)$ in Equation (A.12) can be approximated by a second order Taylor

[^14]equation,
\[

$$
\begin{gather*}
h\left(W_{t+1}^{i}, z_{t+1}^{i}\right)=h\left(W_{t+1}, z_{t+1}\right)+h_{W}\left(W_{t+1}, z_{t+1}\right)\left(W_{t+1}^{i}-W_{t+1}\right)+h_{z}\left(W_{t+1}, z_{t+1}\right)\left(z_{t+1}^{i}-z_{t+1}\right)+ \\
\frac{1}{2} h_{W W}\left(W_{t+1}, z_{t+1}\right)\left(W_{t+1}^{i}-W_{t+1}\right)^{2}+\frac{1}{2} h_{z z}\left(W_{t+1}, z_{t+1}\right)\left(z_{t+1}^{i}-z_{t+1}\right)^{2}+ \\
h_{W z}\left(W_{t+1}, z_{t+1}\right)\left(W_{t+1}^{i}-W_{t+1}\right)\left(z_{t+1}^{i}-z_{t+1}\right) \\
W_{t+1}=\frac{1}{I} \sum_{i=1}^{I} W_{t+1}^{i}, z_{t+1}=\frac{1}{I} \sum_{i=1}^{I} z_{t+1}^{i} \tag{A.13}
\end{gather*}
$$
\]

with $W_{t+1}$ representing the cross sectional average or market wealth, and $z_{t+1}$ denoting the cross sectional average for the state variable. By taking the average across the $I$ investors, the average SDF in the economy is given by

$$
\begin{gather*}
M_{t+1} \equiv \frac{1}{I} \sum_{i=1}^{I} h\left(W_{t+1}^{i}, z_{t+1}^{i}\right)= \\
h\left(W_{t+1}, z_{t+1}\right)+\frac{1}{2} h_{W W}\left(W_{t+1}, z_{t+1}\right) V W_{t+1}+\frac{1}{2} h_{z z}\left(W_{t+1}, z_{t+1}\right) V Z_{t+1}+h_{W z}\left(W_{t+1}, z_{t+1}\right) C W Z_{t+1},  \tag{A.14}\\
V W_{t+1} \equiv \frac{1}{I} \sum_{i=1}^{I}\left(W_{t+1}^{i}-W_{t+1}\right)^{2}  \tag{A.15}\\
V Z_{t+1} \equiv \frac{1}{I} \sum_{i=1}^{I}\left(z_{t+1}^{i}-z_{t+1}\right)^{2}  \tag{A.16}\\
C W Z_{t+1} \equiv \frac{1}{I} \sum_{i=1}^{I}\left(W_{t+1}^{i}-W_{t+1}\right)\left(z_{t+1}^{i}-z_{t+1}\right) \tag{A.17}
\end{gather*}
$$

where $V W_{t+1}$ represents the cross sectional variance for wealth; $V Z_{t+1}$ is the cross sectional variance associated with the state variable; and $C W Z_{t+1}$ denotes the cross sectional covariance between wealth and the state variable.

By applying the Steins's Lemma ${ }^{16}$ to $\operatorname{Cov}_{t}\left[R_{n, t+1}, M_{t+1}\right]$, we obtain

$$
\begin{gather*}
\operatorname{Cov}_{t}\left[R_{n, t+1}, M_{t+1}\right]= \\
W_{t}\left\{\mathrm{E}_{t}\left[h_{W}\left(W_{t+1}, z_{t+1}\right)\right]+\frac{1}{2} \mathrm{E}_{t}\left[h_{W W W}\left(W_{t+1}, z_{t+1}\right) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[h_{z z W}\left(W_{t+1}, z_{t+1}\right) V Z_{t+1}\right]+\right. \\
\left.\mathrm{E}_{t}\left[h_{W W z}\left(W_{t+1}, z_{t+1}\right) C W Z_{t+1}\right]\right\} \operatorname{Cov}_{t}\left(R_{n, t+1}, \frac{W_{t+1}}{W_{t}}\right)+ \\
\left\{\mathrm{E}_{t}\left[h_{z}\left(W_{t+1}, z_{t+1}\right)\right]+\frac{1}{2} \mathrm{E}_{t}\left[h_{W W z}\left(W_{t+1}, z_{t+1}\right) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[h_{z z z}\left(W_{t+1}, z_{t+1}\right) V Z_{t+1}\right]+\right. \\
\left.\mathrm{E}_{t}\left[h_{z z W}\left(W_{t+1}, z_{t+1}\right) C W Z_{t+1}\right]\right\} \operatorname{Cov}_{t}\left(R_{n, t+1}, z_{t+1}\right)+ \\
\frac{1}{2} \mathrm{E}_{t}\left[h_{W W}\left(W_{t+1}, z_{t+1}\right)\right] \operatorname{Cov}_{t}\left(R_{n, t+1}, V W_{t+1}\right)+\frac{1}{2} \mathrm{E}_{t}\left[h_{z z}\left(W_{t+1}, z_{t+1}\right)\right] \operatorname{Cov}_{t}\left(R_{n, t+1}, V Z_{t+1}\right)+ \\
\mathrm{E}_{t}\left[h_{W z}\left(W_{t+1}, z_{t+1}\right)\right] \operatorname{Cov}_{t}\left(R_{n, t+1}, C W Z_{t+1}\right) . \tag{A.18}
\end{gather*}
$$

By further noting that the expected conditional SDF is equal to

$$
\begin{align*}
\mathrm{E}_{t}\left(M_{t+1}\right)=\mathrm{E}_{t}\left[h\left(W_{t+1}, z_{t+1}\right)\right]+ & \frac{1}{2} \mathrm{E}_{t}\left[h_{W W}\left(W_{t+1}, z_{t+1}\right) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[h_{z z}\left(W_{t+1}, z_{t+1}\right) V Z_{t+1}\right]+ \\
& \mathrm{E}_{t}\left[h_{W z}\left(W_{t+1}, z_{t+1}\right) C W Z_{t+1}\right] \tag{A.19}
\end{align*}
$$

it follows that the general pricing equation,

$$
\mathrm{E}_{t}\left(R_{n, t+1}\right)-R_{f, t+1}=-\frac{\operatorname{Cov}_{t}\left(R_{n, t+1}, M_{t+1}\right)}{\mathrm{E}_{t}\left(M_{t+1}\right)}
$$

can be represented as,

$$
\begin{align*}
\mathrm{E}_{t}\left(R_{n, t+1}\right)-R_{f, t+1} & =\lambda_{M t} \operatorname{Cov}_{t}\left(R_{n, t+1}, \frac{W_{t+1}}{W_{t}}\right)+\lambda_{z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, z_{t+1}\right)+\lambda_{V W t} \operatorname{Cov}_{t}\left(R_{n, t+1}, V W_{t+1}\right) \\
& +\lambda_{V Z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, V Z_{t+1}\right)+\lambda_{C W Z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, C W Z_{t+1}\right) \tag{A.20}
\end{align*}
$$

In the above asset pricing model, the risk prices associated with the market factor, the state variable factor, the cross sectional variance of wealth, the cross sectional variance for the state variable, and the cross sectional covariance between wealth and state variable, are equal to the following expressions, ${ }^{17}$

$$
\begin{align*}
\lambda_{M t} & \equiv-W_{t} \frac{J_{W W}\left(W_{t}, z_{t}\right)+\frac{1}{2} \mathrm{E}_{t}\left[J_{W W W W}(.) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[J_{W W z z}(.) V Z_{t+1}\right]+\mathrm{E}_{t}\left[J_{W W W z}(.) C W Z_{t+1}\right]}{J_{W}\left(W_{t}, z_{t}\right)+\frac{1}{2} \mathrm{E}_{t}\left[J_{W W W}(.) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[J_{W z z}(.) V Z_{t+1}\right]+\mathrm{E}_{t}\left[J_{W W z}(.) C W Z_{t+1}\right]}, \\
\lambda_{z t} & \equiv-\frac{J_{W z}\left(W_{t}, z_{t}\right)+\frac{1}{2} \mathrm{E}_{t}\left[J_{W W W z}(.) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[J_{W z z z}(.) V Z_{t+1}\right]+\mathrm{E}_{t}\left[J_{z z W W}(.) C W Z_{t+1}\right]}{J_{W}\left(W_{t}, z_{t}\right)+\frac{1}{2} \mathrm{E}_{t}\left[J_{W W W}(.) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[J_{W z z}(.) V Z_{t+1}\right]+\mathrm{E}_{t}\left[J_{W W z}(.) C W Z_{t+1}\right]} \tag{A.21}
\end{align*}
$$

[^15]\[

$$
\begin{align*}
\lambda_{V W t} & \equiv-\frac{\frac{1}{2} J_{W W W}\left(W_{t}, z_{t}\right)}{J_{W}\left(W_{t}, z_{t}\right)+\frac{1}{2} \mathrm{E}_{t}\left[J_{W W W}(.) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[J_{W z z}(.) V Z_{t+1}\right]+\mathrm{E}_{t}\left[J_{W W z}(.) C W Z_{t+1}\right]}, \\
\lambda_{V Z t} & \equiv-\frac{\frac{1}{2} J_{W z z}\left(W_{t}, z_{t}\right)}{J_{W}\left(W_{t}, z_{t}\right)+\frac{1}{2} \mathrm{E}_{t}\left[J_{W W W}(.) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[J_{W z z}(.) V Z_{t+1}\right]+\mathrm{E}_{t}\left[J_{W W z}(.) C W Z_{t+1}\right]},  \tag{A.24}\\
\lambda_{C W Z t} & \equiv-\frac{\text { A.23) }}{J_{W}\left(W_{t}, z_{t}\right)+\frac{1}{2} \mathrm{E}_{t}\left[J_{W W W}(.) V W_{t+1}\right]+\frac{1}{2} \mathrm{E}_{t}\left[J_{W z z}(.) V Z_{t+1}\right]+\mathrm{E}_{t}\left[J_{W W z}(.) C W Z_{t+1}\right]} \tag{A.25}
\end{align*}
$$,
\]

In the above derivations for the factor risk prices I have used equation (A.7), and the following assumptions

$$
\left\{\begin{array}{rl}
\mathrm{E}_{t}\left[J_{W}\left(W_{t+1}, z_{t+1}\right)\right] & =J_{W}\left(W_{t}, z_{t}\right) \\
\mathrm{E}_{t}\left[J_{W W}\left(W_{t+1}, z_{t+1}\right)\right] & =J_{W W}\left(W_{t}, z_{t}\right) \\
\mathrm{E}_{t}\left[J_{W z}\left(W_{t+1}, z_{t+1}\right)\right] & =J_{W z}\left(W_{t}, z_{t}\right) \\
\mathrm{E}_{t}\left[J_{W W W}\left(W_{t+1}, z_{t+1}\right)\right] & =J_{W W W}\left(W_{t}, z_{t}\right) \\
\mathrm{E}_{t}\left[J_{W W z}\left(W_{t+1}, z_{t+1}\right)\right] & =J_{W W z}\left(W_{t}, z_{t}\right) \\
\mathrm{E}_{t}\left[J_{W z z}\left(W_{t+1}, z_{t+1}\right)\right] & =J_{W z z}\left(W_{t}, z_{t}\right)
\end{array} .\right.
$$

If there are neither idiosyncratic shocks attached to wealth and to intertemporal risk (i.e., there is no heterogeneity across investors), then we have

$$
V W_{t+1}=V Z_{t+1}=C W Z_{t+1}=0
$$

and the pricing equation (A.20) specializes to the Merton (1973) ICAPM,

$$
\begin{equation*}
\mathrm{E}_{t}\left(R_{n, t+1}\right)-R_{f, t+1}=\lambda_{M t} \operatorname{Cov}_{t}\left(R_{n, t+1}, \frac{W_{t+1}}{W_{t}}\right)+\lambda_{z t} \operatorname{Cov}_{t}\left(R_{n, t+1}, z_{t+1}\right) \tag{A.26}
\end{equation*}
$$

with risk prices being given by

$$
\begin{align*}
\lambda_{M t} & \equiv-W_{t} \frac{J_{W W}\left(W_{t}, z_{t}\right)}{J_{W}\left(W_{t}, z_{t}\right)},  \tag{A.27}\\
\lambda_{z t} & \equiv-\frac{J_{W z}\left(W_{t}, z_{t}\right)}{J_{W}\left(W_{t}, z_{t}\right)} . \tag{A.28}
\end{align*}
$$

By comparing (A.21-A.22) with the corresponding risk prices (A.27-A.28), it is clear that the risk prices associated with market wealth and "hedging opportunities" in the GICAPM, will not only depend on $J_{W}\left(W_{t}, z_{t}\right), J_{W W}\left(W_{t}, z_{t}\right)$ and $J_{W z}\left(W_{t}, z_{t}\right)$ - as in the standard ICAPM - but also on higher order derivatives of the value function, and hence in general, the market and intertemporal risk prices will be different in the two models.

Table 1: Descriptive statistics for factors
This table reports descriptive statistics for the factors used in the GICAPM model presented in Equations (22-23). The factors are the excess market return ( $R M R F$ ); the log smoothed market earnings yield ( $E Y 10$ ); the cross-sectional return variance ( $V R, V R^{*}$ ); the cross-sectional dividend yield variance ( $V I R, V I R^{*}$ ); and the cross-sectional covariance between returns and dividend yields ( $C I R R, C I R R^{*}$ ). $V R, V I R, C I R R$ are computed based on the $S B M 25$ portfolios, while $V R^{*}, V I R^{*}, C I R R^{*}$ are based on the $S B M 100$ portfolios. The sample is 1963:01-2003:12. $\rho$ designates the first order autocorrelation. The correlations between the factors are presented in Panels B and C. For further details refer to Section 2.

| Panel A |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Mean | St. Dev. | Min. | Max. | $\rho$ |
| RMRF | 0.005 | 0.045 | -0.231 | 0.161 | 0.055 |
| EY10 | -2.830 | 0.453 | -3.789 | -1.893 | 0.998 |
| $V R$ | 0.022 | 0.013 | 0.008 | 0.144 | 0.503 |
| $V I R$ | 0.001 | 0.001 | 0.0004 | 0.005 | 0.263 |
| CIRR | 0.000 | 0.000 | -0.0001 | 0.0001 | 0.104 |
| $V R^{*}$ | 0.030 | 0.013 | 0.015 | 0.154 | 0.584 |
| $V I R^{*}$ | 0.002 | 0.001 | 0.0006 | 0.009 | 0.383 |
| CIRR* | 0.000 | 0.000 | -0.0001 | 0.0001 | 0.115 |
| Panel B |  |  |  |  |  |
|  | RMRF | EY10 | $V R$ | $V I R$ | CIRR |
| RMRF | 1.000 | -0.012 | -0.018 | -0.001 | -0.398 |
| EY10 |  | 1.000 | -0.274 | 0.612 | 0.030 |
| $V R$ |  |  | 1.000 | -0.127 | -0.118 |
| $V I R$ |  |  |  | 1.000 | 0.041 |
| CIRR |  |  |  |  | 1.000 |
| Panel C |  |  |  |  |  |
|  | RMRF | EY10 | $V R^{*}$ | $V I R^{*}$ | CIRR* |
| RMRF | 1.000 | -0.012 | 0.026 | -0.039 | -0.408 |
| EY 10 |  | 1.000 | -0.313 | 0.493 | 0.033 |
| $V R^{*}$ |  |  | 1.000 | -0.173 | -0.139 |
| $V I R^{*}$ |  |  |  | 1.000 | 0.096 |
| CIRR* |  |  |  |  | 1.000 |

Table 2: Generalized ICAPM: Estimating factor risk-premia by first-stage GMM
This table reports the estimation and evaluation results for the GICAPM in Equations (22-23) and alternative factor models, applied to the $S B M 25$ portfolios. The estimation procedure is first stage GMM (equally weighted errors). Row 1 presents the results for the CAPM; Rows 2 and 3 show the results for the GICAPM and GICAPM* models, respectively; and Row 4 presents the results for the Fama and French (1993) model. The pricing factors in the GICAPM (GICAPM*) are the excess market return, RMRF; the $\log$ smoothed market earnings yield, $E Y 10$; the cross-sectional return variance, $V R\left(V R^{*}\right)$; the cross-sectional dividend yield variance, $V I R\left(V I R^{*}\right)$; and the cross-sectional covariance between returns and dividend yields, $C I R R\left(C I R R^{*}\right) . V R, V I R, C I R R$ are computed based on the SBM25 portfolios, whereas $V R^{*}, V I R^{*}, C I R R^{*}$ are based on the $S B M 100$ portfolios. The factors for the Fama and French (1993) model are $R M R F$; the size premium ( $S M B$ ); and the value premium ( $H M L$ ). The first line associated with each row presents the covariance risk price estimates, while the second line reports the asymptotic t-statistics. The third line shows the asymptotic t -statistics not corrected for estimation error in covariances. The column $\chi^{2}$ presents the levels (first line) and associated p-values (second line) for the asymptotic $\chi^{2}$ test. $R M S E$ refers to the square root of the average pricing error (in \%). The column $\operatorname{Adj} . R^{2} / R^{2}$ denotes the original (first line) and the adjusted (second line) cross-sectional OLS $R^{2}$, respectively. The sample is 1963:01-2003:12. Italic
and bold numbers denote statistical significance at the $10 \%$ and $5 \%$ levels, respectively. Further details are presented in Section 3.

| Row | $V R^{(*)}$ | $V I R^{(*)}$ | CIRR ${ }^{(*)}$ | EY 10 | RMRF | SMB | HML | $\chi^{2} / p\left(\chi^{2}\right)$ | RMSE(\%) | $R^{2} /$ Adj. $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  | 3.298 |  |  | 88.524 | 0.311 | -0.598 |
|  |  |  |  |  | (2.780) |  |  | (0.000) |  | -0.598 |
|  |  |  |  |  | (3.058) |  |  |  |  |  |
| 2 | 44.629 | -1591.411 | 15912.998 | 1.667 | 2.632 |  |  | 34.200 | 0.147 | 0.646 |
|  | (3.013) | (-3.296) | (3.262) | (1.856) | (2.005) |  |  | (0.025) |  | 0.575 |
|  | (3.616) | (-3.928) | (4.263) | (2.246) | (2.577) |  |  |  |  |  |
| 3 | 26.520 | -1356.220 | 15314.218 | 0.867 | 2.589 |  |  | 31.637 | 0.122 | 0.753 |
|  | (1.732) | $(-3.011)$ | (3.182) | (1.028) | (1.352) |  |  | (0.047) |  | 0.704 |
|  | (2.083) | (-5.023) | (3.919) | (1.322) | (2.533) |  |  |  |  |  |
| 4 |  |  |  |  | 2.146 | 3.132 | 9.086 | 63.855 | 0.139 | 0.683 |
|  |  |  |  |  | (1.824) | (1.976) | (5.312) | (0.000) |  | 0.654 |
|  |  |  |  |  | (2.102) | (2.062) | (5.193) |  |  |  |

Table 3: VAR coefficient estimates
This table presents the estimated coefficients (first line of each variable) and associated NeweyWest t-statistics (calculated with 5 lags, second line) for the first-order VAR presented in Equation (35). The VAR vector is given by $\left[R M R F_{t}, V R_{t}, V I R_{t}, C I R R_{t}, E Y 10_{t}\right]^{\prime}$ in Panel A, and [ $\left.R M R F_{t}, V R_{t}^{*}, V I R_{t}^{*}, C I R R_{t}^{*}, E Y 10_{t}\right]^{\prime}$ in Panel B, where $R M R F$ is the excess market return; $E Y 10$ is the $\log$ smoothed market earnings yield; $V R\left(V R^{*}\right)$ is the cross-sectional return variance; $\operatorname{VIR}\left(V I R^{*}\right)$ is the cross-sectional dividend yield variance; and $C I R R\left(C I R R^{*}\right)$ denotes the cross-sectional covariance between returns and dividend yields. The usable sample is 1963:02-2003:12. Italic (bold) t-statistics denote statistical significance at the $10 \%(5 \%)$ level. Adj. $R^{2}$ is the adjusted $R^{2}$. For further details refer to Section 3.

| Panel A |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | RMRF | VR | VIR | CIRR | EY10 | Adj. ${ }^{2}$ |
| RMRF | $\begin{gathered} 0.011 \\ (0.233) \end{gathered}$ | $\begin{gathered} -0.194 \\ (-1.545) \end{gathered}$ | $\begin{gathered} -4.095 \\ (-1.261) \end{gathered}$ | $\begin{aligned} & -240.735 \\ & (-\mathbf{2 . 6 6 3}) \end{aligned}$ | $\begin{gathered} 0.007 \\ (1.144) \end{gathered}$ | 0.009 |
| $V R$ | $\begin{gathered} -0.014 \\ (-0.971) \end{gathered}$ | $\begin{gathered} 0.463 \\ (\mathbf{7 . 1 7 2}) \end{gathered}$ | $\begin{gathered} -0.562 \\ (-0.690) \end{gathered}$ | $\begin{gathered} 6.654 \\ (0.217) \end{gathered}$ | $\begin{gathered} -0.004 \\ (-\mathbf{2 . 0 0 8}) \end{gathered}$ | 0.270 |
| $V I R$ | $\begin{gathered} 0.000 \\ (0.059) \end{gathered}$ | $\begin{gathered} 0.007 \\ (4.011) \end{gathered}$ | $\begin{gathered} -0.185 \\ (-\mathbf{2 . 7 3 8}) \end{gathered}$ | $\begin{gathered} 1.594 \\ (1.277) \end{gathered}$ | $\begin{gathered} 0.001 \\ (\mathbf{1 4 . 6 0 2}) \end{gathered}$ | 0.399 |
| $C I R R$ | $\begin{gathered} -0.0001 \\ (-\mathbf{3 . 1 7 6}) \end{gathered}$ | $\begin{aligned} & 0.0001 \\ & (1.055) \end{aligned}$ | $\begin{gathered} 0.001 \\ (1.028) \end{gathered}$ | $\begin{gathered} 0.043 \\ (1.050) \end{gathered}$ | $\begin{aligned} & -0.000 \\ & (-0.138) \end{aligned}$ | 0.033 |
| EY 10 | $\begin{gathered} -0.453 \\ (-\mathbf{1 3 . 5 2 0}) \end{gathered}$ | $\begin{gathered} 0.172 \\ (1.621) \end{gathered}$ | $\begin{gathered} 0.766 \\ (0.322) \end{gathered}$ | $\begin{gathered} -8.777 \\ (-0.126) \end{gathered}$ | $\begin{gathered} 1.000 \\ (\mathbf{2 6 9 . 6 8 4}) \end{gathered}$ | 0.996 |
| Panel B |  |  |  |  |  |  |
|  | RMRF | $V R^{*}$ | $V I R^{*}$ | CIRR* | EY 10 | Adj. ${ }^{2}$ |
| RMRF | $\begin{gathered} 0.019 \\ (0.403) \end{gathered}$ | $\begin{gathered} -0.220 \\ (-1.631) \end{gathered}$ | $\begin{gathered} -1.834 \\ (-1.095) \end{gathered}$ | $\begin{aligned} & -207.158 \\ & (-\mathbf{2 . 1 0 0}) \end{aligned}$ | $\begin{gathered} 0.004 \\ (0.845) \end{gathered}$ | 0.005 |
| $V R^{*}$ | $\begin{gathered} -0.024 \\ (-1.826) \end{gathered}$ | $\begin{gathered} 0.540 \\ (8.359) \end{gathered}$ | $\begin{gathered} -0.254 \\ (-0.599) \end{gathered}$ | $\begin{gathered} 1.144 \\ (0.038) \end{gathered}$ | $\begin{gathered} -0.004 \\ (-\mathbf{2 . 7 1 9}) \end{gathered}$ | 0.361 |
| $V I R^{*}$ | $\begin{gathered} 0.000 \\ (0.034) \end{gathered}$ | $\begin{gathered} 0.003 \\ (1.090) \end{gathered}$ | $\begin{gathered} 0.186 \\ (\mathbf{2 . 4 8 0}) \end{gathered}$ | $\begin{gathered} 1.497 \\ (0.828) \end{gathered}$ | $\begin{gathered} 0.001 \\ (\mathbf{1 0 . 2 3 7}) \end{gathered}$ | 0.260 |
| $C I R R^{*}$ | $\begin{gathered} -0.000 \\ (-3.276) \end{gathered}$ | $\begin{gathered} 0.000 \\ (0.911) \end{gathered}$ | $\begin{gathered} 0.001 \\ (1.123) \end{gathered}$ | $\begin{gathered} 0.053 \\ (1.268) \end{gathered}$ | $\begin{gathered} 0.000 \\ (0.034) \end{gathered}$ | 0.031 |
| EY10 | $\begin{gathered} -0.462 \\ (-\mathbf{1 4 . 1 6 6}) \end{gathered}$ | $\begin{gathered} 0.197 \\ (1.758) \end{gathered}$ | $\begin{gathered} 0.476 \\ (0.390) \end{gathered}$ | $\begin{aligned} & -50.617 \\ & (-0.698) \end{aligned}$ | $\begin{gathered} 0.999 \\ (\mathbf{3 1 2 . 5 0 0}) \end{gathered}$ | 0.996 |

Table 4: Estimating factor risk-premia by two-stage GMM, SBM25
This table reports the estimation and evaluation results for the GICAPM/GICAPM* with orthogonal factors in Equation (36) and
 estimates (equally weighted errors) presented in Panel A, and second-stage estimates reported in Panel B. In each panel, Rows 1 and 2 show the results for the GICAPM and GICAPM* models, respectively; and Row 3 presents the results for the Fama and French (1993) model. The pricing factors in the $G I C A P M\left(G I C A P M^{*}\right)$ are the excess market return, $R M R F$; the log smoothed market earnings yield, $E Y 10$; the cross-sectional return variance, $V R\left(V R^{*}\right)$; the cross-sectional dividend yield variance, $V I R\left(V I R^{*}\right)$; and the cross-sectional covariance between returns and dividend yields, $C I R R\left(C I R R^{*}\right) . V R, V I R, C I R R$ are computed based on the $S B M 25$ portfolios, while $V R^{*}, V I R^{*}, C I R R^{*}$ are based on the $S B M 100$ portfolios. The factors for the Fama and French (1993) model are RMRF; the size premium $(S M B)$; and the value premium ( $H M L$ ). The first line associated with each row presents the covariance risk price estimates, while the second line reports the asymptotic t-statistics. The third line (only Panel A) shows the asymptotic t-statistics not corrected for estimation error in covariances. The column $\chi^{2}$ presents the levels (first line) and associated p-values (second line) for the asymptotic $\chi^{2}$ test. RMSE refers to the square root of the average pricing error (in \%). The column $A d j . R^{2} / R^{2}$ denotes the original (first line) and the adjusted (second line) cross-sectional $R^{2}$, respectively. Panel A reports OLS $R^{2}$ and Panel B presents GLS $R^{2}$. The sample is 1963:02-2003:12. Italic and bold numbers denote statistical significance at the $10 \%$ and $5 \%$ levels, respectively. Further details are presented in Section 3 .
Table 5: Estimating factor risk-premia by two-stage GMM, SBM25 + IND38
orthog-
in $\begin{array}{r}\text { com- } \\ \text { Table } \\ \hline\end{array}$.

Table 6: Estimating factor risk-premia by two-stage GMM, all portfolios
This table reports the estimation and evaluation results for the GICAPM/GICAPM* with orthogonal factors in Equation (36) and Fama and French (1993) model, applied to $S B M 25$ in combination with 38 industry portfolios (IND38); 10 portfolios sorted on cash flow-to-price ratio $(C F / P) ; 10$ portfolios sorted on earnings-to-price ratio $(E / P)$; and 10 portfolios sorted on dividend-to-price ratio $(D / P)$. In everything else, it is identical to Table 4.

| Row | $V R^{(*)}$ | $V I R^{(*)}$ | $C I R R^{(*)}$ | $E Y 10$ | $R M R F$ | $S M B$ | $H M L$ | $\chi^{2} / p\left(\chi^{2}\right)$ | $R M S E(\%)$ | $R^{2} / A d j . R^{2}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |


| Panel A (First Stage) |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.309 | -0.252 | 0.248 | -0.450 | $2.626$ |  |  |  | 0.133 | $\begin{aligned} & 0.505 \\ & 0.481 \end{aligned}$ |
|  | (2.603) | (-1.087) | (3.437) | (-2.705) | $(2.296)$ |  |  |  |  |  |
|  | (2.596) | (-1.197) | (3.549) | (-3.023) | (2.571) |  |  |  |  |  |
| 2 | 0.336 | -0.114 | 0.271 | -0.465 | 2.635 |  |  |  | 0.134 | 0.497 |
|  | (2.706) | (-0.475) | (3.323) | (-2.633) | (2.294) |  |  |  |  | 0.473 |
|  | (2.884) | (-0.563) | (3.746) | (-2.985) | (2.586) |  |  |  |  |  |
| 3 |  |  |  |  | 2.510 | 1.466 | 6.168 |  | 0.159 | $\begin{aligned} & 0.301 \\ & 0.284 \end{aligned}$ |
|  |  |  |  |  | (2.215) | (0.941) | (3.455) |  |  |  |
|  |  |  |  |  | (2.468) | (0.970) | (3.369) |  |  |  |
| Panel B (Second Stage) |  |  |  |  |  |  |  |  |  |  |
| 1 | 0.239 | -0.218 | 0.224 | -0.293 | 2.687 |  |  | 124.967 | 0.140 | 0.501 |


| Panel B (Second Stage) |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.239 | -0.218 | 0.224 | -0.293 | 2.687 | 124.967 | 0.140 | 0.501 |  |
|  | $(\mathbf{3 . 6 5 5})$ | $(-1.741)$ | $(\mathbf{4 . 1 0 4})$ | $(-\mathbf{2 . 9 0 5})$ | $(\mathbf{2 . 6 5 0})$ | $(0.002)$ |  | 0.477 |  |
| 2 | 0.227 | -0.166 | 0.229 | -0.280 | 2.876 |  | 120.331 | 0.144 | 0.468 |
|  | $(\mathbf{3 . 3 2 8})$ | $(-1.391)$ | $(\mathbf{4 . 0 2 9})$ | $(-\mathbf{2 . 7 0 3})$ | $(\mathbf{2 . 8 4 8})$ |  |  | 0.442 |  |
| 3 |  |  |  |  |  | 3.003 | 3.127 | 10.047 | 139.735 |
|  |  |  |  | $(\mathbf{2 . 8 3 9})$ | $(\mathbf{2 . 1 9 1})$ | $(\mathbf{6 . 5 4 0})$ | $(0.000)$ | 0.265 | -0.712 |

Table 7: Model comparison tests
This table reports evaluation measures for the GICAPM/GICAPM* in Equation (36) and the Fama and French (1993) model (FF3). Panels A, B, and C present the results for the estimation with SBM25; SBM $25+I N D 38$; and all portfolios, respectively. GICAPM $+F F 3$ and GICAPM* $+F F 3$ represent the unrestricted models. $R M S E$ refers to the square root of the average pricing error (in \%). $R_{O L S}^{2}$ and $A d j \cdot R_{O L S}^{2}$ denote the original and adjusted OLS $R^{2}$, respectively. $R_{G L S}^{2}$ and Adj. $R_{G L S}^{2}$ stand for the original and adjusted GLS $R^{2}$, respectively. Dtest and $p(D)$ denote respectively the level and associated p-values for the asymptotic difference $\chi^{2}$ test. The row labeled $F F 3^{*}$ presents the test values for the Fama and French (1993) model as a special case of GICAPM* $+F F 3$.

| Model | RMSE(\%) | $R_{O L S}^{2}$ | Adj. $R_{O L S}^{2}$ | $R_{G L S}^{2}$ | Adj. $R_{G L S}^{2}$ | $D$ test | $p(D)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A (SBM25) |  |  |  |  |  |  |  |
| GICAPM | 0.104 | 0.818 | 0.781 | 0.738 | 0.685 | 0.706 | 0.703 |
| FF3 | 0.139 | 0.683 | 0.654 | 0.167 | 0.091 | 3.224 | 0.521 |
| $G I C A P M+F F 3$ | 0.103 | 0.822 | 0.763 | 0.720 | 0.627 |  |  |
| GICAPM* | 0.104 | 0.819 | 0.783 | 0.429 | 0.314 | 0.288 | 0.866 |
| FF3* |  |  |  |  |  | 5.917 | 0.205 |
| GICAPM ${ }^{*}+F F 3$ | 0.103 | 0.823 | 0.764 | 0.496 | 0.327 |  |  |
| Panel B (SBM25+IND38) |  |  |  |  |  |  |  |
| GICAPM | 0.154 | 0.431 | 0.388 | 0.448 | 0.406 | 2.827 | 0.243 |
| FF3 | 0.188 | 0.157 | 0.127 | -1.291 | -1.374 | 12.548 | 0.014 |
| $G I C A P M+F F 3$ | 0.153 | 0.434 | 0.367 | 0.123 | 0.020 |  |  |
| GICAPM* | 0.153 | 0.433 | 0.390 | 0.437 | 0.395 | 2.389 | 0.303 |
| FF3* |  |  |  |  |  | 14.758 | 0.005 |
| GICAPM ${ }^{*}+F F 3$ | 0.153 | 0.435 | 0.368 | 0.291 | 0.207 |  |  |
| Panel C (SBM25+CF/P+E/P+D/P+IND38) |  |  |  |  |  |  |  |
| GICAPM | 0.133 | 0.505 | 0.481 | 0.501 | 0.477 | 1.656 | 0.437 |
| FF3 | 0.159 | 0.301 | 0.284 | -0.712 | -0.752 | 18.223 | 0.001 |
| $G I C A P M+F F 3$ | 0.133 | 0.506 | 0.469 | 0.534 | 0.500 |  |  |
| GICAPM* | 0.134 | 0.497 | 0.473 | 0.468 | 0.442 | 1.933 | 0.380 |
| FF3* |  |  |  |  |  | 17.941 | 0.001 |
| GICAPM ${ }^{*}+F F 3$ | 0.134 | 0.498 | 0.461 | 0.340 | 0.291 |  |  |

Table 8: Pricing the size/book-to-market portfolios
This table reports evaluation measures for the GICAPM/GICAPM* in Equation (36) and the Fama and French (1993) model (FF3). The SBM25 portfolios receive a bigger weight in the estimation, relative to the other portfolios. Panels A and B present the results for the estimation with $S B M 25+I N D 38$ and all portfolios, respectively. $R M S E^{*}$ refers to the square root of the average pricing error (in \%), associated with the SBM25 portfolios. $R^{2}$ and $\operatorname{Adj} . R^{2}$ denote the original and adjusted measures for the weighted cross-sectional $R^{2}$, respectively.

| Model | $R M S E^{*}(\%)$ | $R^{2}$ | Adj. $R^{2}$ |
| :---: | :---: | :---: | :---: |
| Panel A (SBM25+IND38) |  |  |  |
| GICAPM | 0.131 | 0.548 | 0.514 |
| GICAPM | 0.121 | 0.564 | 0.531 |
| FF3 | 0.154 | 0.340 | 0.316 |
| Panel B (SBM25+CF/P+E/P+D/P+IND38) |  |  |  |
| GICAPM | 0.133 | 0.583 | 0.563 |
| GICAPM | 0.124 | 0.590 | 0.570 |
| FF3 | 0.156 | 0.418 | 0.404 |

Table 9: Estimating factor risk-premia by the Hansen-Jagannathan method, SBM25
This table reports the estimation and evaluation results for the $G I C A P M / G I C A P M^{*}$ in Equation (36) and the Fama and French (1993) model, applied to the $S B M 25$ portfolios. The estimation procedure is the Hansen-Jagannathan (HJ) method, with first stage estimates (associated with the HJ weighting matrix) presented in Panel A, and second-stage estimates reported in Panel B. In each panel, Rows 1 and 2 show the results for the GICAPM and GICAPM* models, respectively; and Row 3 presents the results for the Fama and French (1993) model. The pricing factors in the GICAPM (GICAPM*) are the excess market return, RMRF; the $\log$ smoothed market earnings yield, $E Y 10$; the cross-sectional return variance, $V R\left(V R^{*}\right)$; the cross-sectional dividend yield variance, $\operatorname{VIR}\left(V I R^{*}\right)$; and the cross-sectional covariance between returns and dividend yields, $C I R R\left(C I R R^{*}\right) . V R, V I R, C I R R$ are computed based on the $S B M 25$ portfolios, while $V R^{*}, V I R^{*}, C I R R^{*}$ are based on the $S B M 100$ portfolios. The factors for the Fama and French (1993) model are $R M R F$; the size premium $(S M B)$; and the value premium ( $H M L$ ). The first line associated with each row presents the covariance risk price estimates, while the second line reports the asymptotic t-statistics. The column $\chi^{2}$ presents the levels (first line) and associated p -values (second line) for the asymptotic $\chi^{2}$ test. $R M S E$ refers to the square root of the average pricing error (in \%). The column $H J / p(H J)$ denotes the HJ-distance (first line) and respective p-values (second line). The column $\operatorname{Adj} . R^{2} / R^{2}$ denotes the original (first line) and the adjusted (second line) cross-sectional GLS $R^{2}$, respectively. The sample is 1963:02-2003:12. Italic and bold numbers denote statistical significance at the $10 \%$ and $5 \%$ levels, respectively. Further details are presented in Section 3.

Table 10: Estimating factor risk-premia by the Hansen-Jagannathan method, SBM25+IND38
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Table 11: Estimating factor risk-premia by the Hansen-Jagannathan method, all portfolios
This table reports the estimation and evaluation results for the GICAPM/GICAPM* in Equation (36) and the Fama and French (1993) model, applied to SBM25 in combination with 38 industry portfolios (IND38); 10 portfolios sorted on cash flow-to-price ratio $(C F / P) ; 10$ portfolios sorted on earnings-to-price ratio $(E / P)$; and 10 portfolios sorted on dividend-to-price ratio $(D / P)$. The estimation procedure is the Hansen-Jagannathan (HJ) method. In everything else, it is identical to Table 9.

| Row | $V R^{(*)}$ | $V I R^{(*)}$ | $C I R R^{(*)}$ | EY10 | RM RF | $S M B$ | HML | $\chi^{2} / p\left(\chi^{2}\right)$ | RMSE(\%) | $H J / p(H J)$ | $R^{2} /$ Adj. $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Panel A (First Stage) |  |  |  |  |  |  |  |  |  |  |  |
| 1 | $\begin{gathered} 0.154 \\ (\mathbf{2 . 4 1 9}) \end{gathered}$ | $\begin{gathered} -0.109 \\ (-0.921) \end{gathered}$ | $\begin{gathered} 0.116 \\ (\mathbf{2 . 1 9 9}) \end{gathered}$ | $\begin{gathered} -0.168 \\ (-1.840) \end{gathered}$ | $\begin{gathered} 1.218 \\ (1.255) \end{gathered}$ |  |  |  | 0.372 | $\begin{gathered} 0.540 \\ (0.000) \end{gathered}$ |  |
| 2 | $\begin{gathered} 0.178 \\ (\mathbf{2 . 8 5 1}) \end{gathered}$ | $\begin{gathered} -0.112 \\ (-0.942) \end{gathered}$ | $\begin{gathered} 0.139 \\ (\mathbf{2 . 5 5 0}) \end{gathered}$ | $\begin{gathered} -0.174 \\ (-1.875) \end{gathered}$ | $\begin{gathered} 1.256 \\ (1.284) \end{gathered}$ |  |  |  | 0.358 | $\begin{gathered} 0.535 \\ (0.000) \end{gathered}$ |  |
| 3 |  |  |  |  | $\begin{gathered} 1.258 \\ (1.152) \\ \hline \end{gathered}$ | $\begin{gathered} 2.109 \\ (1.411) \end{gathered}$ | $\begin{gathered} 5.606 \\ (\mathbf{3 . 3 7 5}) \\ \hline \end{gathered}$ |  | 0.298 | $\begin{gathered} 0.539 \\ (0.000) \\ \hline \end{gathered}$ |  |
| Panel B (Second Stage) |  |  |  |  |  |  |  |  |  |  |  |
| 1 | $\begin{gathered} 0.237 \\ (\mathbf{4 . 0 3 5}) \end{gathered}$ | $\begin{gathered} -0.207 \\ (-1.816) \end{gathered}$ | $\begin{gathered} \hline 0.191 \\ (\mathbf{3 . 8 8 9}) \end{gathered}$ | $\begin{gathered} -0.260 \\ (-\mathbf{3 . 0 0 3}) \end{gathered}$ | $\begin{gathered} 2.280 \\ (\mathbf{2 . 4 5 6}) \end{gathered}$ |  |  | $\begin{aligned} & 147.691 \\ & (0.000) \end{aligned}$ | 0.173 |  | $\begin{aligned} & \hline 0.183 \\ & 0.144 \end{aligned}$ |
| 2 | $\begin{gathered} 0.255 \\ (\mathbf{4 . 4 9 8}) \end{gathered}$ | $\begin{gathered} -0.212 \\ (-1.854) \end{gathered}$ | $\begin{gathered} 0.212 \\ (\mathbf{4 . 1 5 9}) \end{gathered}$ | $\begin{gathered} -0.255 \\ (-\mathbf{2 . 9 0 8}) \end{gathered}$ | $\begin{gathered} 2.391 \\ (\mathbf{2 . 5 6 3}) \end{gathered}$ |  |  | $\begin{aligned} & 142.628 \\ & (0.000) \end{aligned}$ | 0.158 |  | $\begin{aligned} & 0.335 \\ & 0.303 \end{aligned}$ |
| 3 |  |  |  |  | $\begin{gathered} 2.406 \\ (\mathbf{2 . 3 0 9}) \end{gathered}$ | $\begin{gathered} 3.509 \\ (\mathbf{2 . 4 5 3}) \end{gathered}$ | $\begin{gathered} 10.054 \\ (\mathbf{6 . 5 3 7}) \end{gathered}$ | $\begin{gathered} 140.824 \\ (0.000) \end{gathered}$ | 0.206 |  | $\begin{aligned} & 0.024 \\ & 0.001 \end{aligned}$ |

Table 12: Pricing growth and value portfolios
This table reports evaluation measures for the GICAPM/GICAPM* in Equation (36) and the Fama and French (1993) model (FF3). The extreme growth and value portfolios receive a bigger weight in the estimation, relative to the other portfolios. Panels A, B and C present the results for the estimation with SBM25, SBM25 + IND38 and all portfolios, respectively. RMSE* refers to the square root of the average pricing error (in \%), associated with the extreme portfolios. $R^{2}$ and $\operatorname{Adj} . R^{2}$ denote the original and adjusted measures for the weighted cross-sectional $R^{2}$, respectively.

| Model | $R M S E^{*}(\%)$ | $R^{2}$ | Adj. $R^{2}$ |
| :---: | :---: | :---: | :---: |
| Panel A (SBM25) |  |  |  |
| GICAPM | 0.114 | 0.832 | 0.799 |
| GICAPM | 0.134 | 0.816 | 0.779 |
| $F F 3$ | 0.182 | 0.676 | 0.646 |
| Panel B (SBM25+IND38) |  |  |  |
| GICAPM | 0.173 | 0.506 | 0.468 |
| GICAPM | 0.162 | 0.517 | 0.481 |
| FF3 | 0.192 | 0.290 | 0.265 |
| Panel C (SBM25+CF/P+E/P+D/P+IND38) |  |  |  |
| GICAPM | 0.176 | 0.548 | 0.527 |
| GICAPM | 0.165 | 0.551 | 0.530 |
| FF3 | 0.191 | 0.381 | 0.367 |

Table 13: Average risk premia and pricing errors across BM quintiles This table reports the average risk premium (covariance times risk price) for each factor, across the book-to-market BM) quintiles. The models are the GICAPM (Panel A), GICAPM* (Panel B), and the Fama and French (1993) model (Panel C). $E(R)$ denotes the average excess return for each BM quintile, and $\bar{\alpha}$ represents the average pricing error per quintile, for each model. The risk price estimates are obtained from Table 4. For a description of the factors see Table 4. All the values are presented in percentage points. BM1 denotes the lowest BM quintile, and Dif. denotes the difference across extreme quintiles. The sample is 1963:02-2003:12. For further details, refer to Section 3.

| Panel A (GICAPM) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{E}(R)$ | RMRF | $V R$ | VIR | CIRR | EY 10 | $\bar{\alpha}$ |
| BM1 | 0.404 | 0.603 | 0.128 | 0.004 | -0.414 | 0.090 | $-0.007$ |
| BM2 | 0.631 | 0.511 | 0.078 | -0.020 | -0.172 | 0.247 | -0.012 |
| BM3 | 0.738 | 0.453 | 0.043 | -0.036 | -0.045 | 0.313 | 0.010 |
| BM4 | 0.875 | 0.422 | 0.048 | -0.040 | 0.040 | 0.394 | 0.011 |
| BM5 | 0.952 | 0.451 | 0.053 | -0.050 | 0.061 | 0.428 | 0.099 |
| Dif. | -0.548 | 0.152 | 0.075 | 0.055 | -0.475 | -0.338 | -0.016 |
| Panel B (GICAPM*) |  |  |  |  |  |  |  |
|  | $\mathrm{E}(R)$ | RMRF | $V R^{*}$ | $V I R^{*}$ | CIRR* | EY 10 | $\bar{\alpha}$ |
| BM1 | 0.404 | 0.620 | 0.145 | 0.005 | -0.368 | 0.001 | 0.002 |
| BM2 | 0.631 | 0.525 | 0.079 | 0.037 | -0.145 | 0.152 | -0.018 |
| BM3 | 0.738 | 0.466 | 0.042 | 0.042 | -0.026 | 0.219 | -0.004 |
| BM4 | 0.875 | 0.434 | 0.045 | 0.036 | 0.051 | 0.291 | 0.017 |
| BM5 | 0.952 | 0.464 | 0.049 | 0.048 | 0.064 | 0.316 | 0.011 |
| Dif. | -0.548 | 0.156 | 0.096 | -0.043 | -0.432 | -0.315 | -0.009 |
| Panel C (FF3) |  |  |  |  |  |  |  |
|  | $\mathrm{E}(R)$ | RMRF | $S M B$ | HML | $\bar{\alpha}$ |  |  |
| BM1 | 0.404 | 0.562 | 0.214 | -0.354 | -0.018 |  |  |
| BM2 | 0.631 | 0.476 | 0.155 | 0.037 | -0.037 |  |  |
| BM3 | 0.738 | 0.422 | 0.115 | 0.204 | -0.003 |  |  |
| BM4 | 0.875 | 0.394 | 0.100 | 0.331 | 0.050 |  |  |
| BM5 | 0.952 | 0.421 | 0.121 | 0.446 | -0.036 |  |  |
| Dif. | -0.548 | 0.141 | 0.093 | -0.800 | 0.018 |  |  |



Panel B (3 months)


Panel D (24 months)


Panel C (12 months)
Panel A (1 month)
Panel SBM25 portfolio returns, described in Section 2 . The t-statistics are based on the Newey and West (1987) standard errors computed with 5 lags. Panels A, B, C, and D present the results for forecasting horizons of 1, 3, 12, and 24 months, respectively. $i j$ denotes the portfolio associated with $i$ th size and $j$ th BM quintile. $D Y i$ and $D Y M$ represent the portfolio $i$ 's and market dividend yields, respectively. CV is the $5 \%$ critical value (1.96). The original sample is 1963:01-2003:12.


Panel A


Panel B

Figure 2: Cross-sectional variance in returns
This figure plots time-series of the cross-sectional variance in portfolio returns. In Panel A, the crosssectional return variance is based on the $S B M 25$ portfolios ( $V R$ ), and in Panel B it is based on the SBM100 portfolios $\left(V R^{*}\right)$. The sample is 1963:01-2003:12. Further details are presented in Section 2.


Panel A


Panel B

Figure 3: Cross-sectional variance in the dividend-to-price ratio
This figure plots time-series of the cross-sectional variance in portfolio dividend yields. In Panel A, the cross-sectional dividend yield variance is based on the $S B M 25$ portfolios (VIR), and in Panel B it is based on the SBM100 portfolios $\left(V I R^{*}\right)$. The sample is 1963:01-2003:12. Further details are presented in Section 2.


Panel A


Panel B

Figure 4: Cross-sectional covariance between returns and dividend-to-price ratios This figure plots time-series of the cross-sectional covariance between portfolio returns and dividend yields. In Panel A, the cross-sectional covariance is based on the SBM25 portfolios (CIRR), and in Panel B it is based on the SBM100 portfolios $\left(C I R R^{*}\right)$. The sample is 1963:01-2003:12. Further details are presented in Section 2.


Panel A (GICAPM)


Panel B (GICAPM*)


Panel C (FF3)


Panel D (GICAPM, HJ)


Panel E (GICAPM*, HJ)


Panel F (FF3, HJ)

Figure 5: Model fit: Size/BM portfolios
This Figure plots the estimated excess returns (vertical axis) versus the realized excess returns (horizontal axis), in the case of the SBM25 portfolios. The models are the GICAPM, GICAPM* and the Fama and French (1993) model. In Panels A, B, C, the estimates are associated with equally weighted estimation (Table 4), and in Panels D,



Panel A (GICAPM)


Panel B (GICAPM*)


Panel D (GICAPM, HJ)


Panel E (GICAPM*, HJ)


Figure 6: Model fit: Industry portfolios
This Figure plots the estimated excess returns (vertical axis) versus the realized excess returns (horizontal axis), in the case of the industry portfolios. The models are the GICAPM, GICAPM* and the Fama and French (1993) model. In Panels A, B, C, the estimates are associated with equally weighted estimation (Table 5), and in Panels D, E, F, the estimates are associated with the 5 Fansen-Jagannathan (HJ) procedure (Table 10).


Panel A


Panel B

Figure 7: Pricing errors: Size/BM portfolios
This figure plots the pricing errors associated with Figure 5. Panel A presents the pricing errors associated with equally weighted estimation (Table 4), and Panel B reports the estimates associated with the Hansen-Jagannathan (HJ) procedure (Table 9). $i j$ denotes the portfolio associated with $i$ th size and $j$ th $B M$ quintile.



Panel B

Figure 8: Pricing errors: Industry portfolios
This figure plots the pricing errors associated with Figure 6. Panel A presents the pricing errors associated with equally weighted estimation (Table 5), and Panel B reports the estimates associated with the Hansen-Jagannathan (HJ) procedure (Table 10).


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[^1]:    ${ }^{1}$ Notice that given the assumption of market incompleteness, the investors can not fully insure their shocks in wealth with the available assets.

[^2]:    ${ }^{2}$ For the last size quintiles, the t-statistics associated with the market and portfolio measures of the dividend yield, are more similar and in some cases reverse in relative magnitudes, which is not surprised given that the market dividend yield is associated with the value-weighted index and thus more tilted towards larger caps.

[^3]:    ${ }^{3}$ The smoothed earnings yield is available from Robert Shiller's website, and is employed by Campbell and Shiller (1998), Campbell and Vuolteenaho (2004), and Maio (2005a), among others.

[^4]:    ${ }^{4}$ In the case of $V R^{*}, V I R^{*}$ and $C I R R^{*}$, the regressions present similar results

    $$
    \begin{aligned}
    V R_{t}^{*}= & 0.033-0.004 C Y C L \mathrm{E}_{t}, A d j . R^{2}=0.009 \\
    & (20.866)(-2.362) \\
    V I R_{t}^{*}= & 0.002-0.0005 C Y C L \mathrm{E}_{t}, \text { Adj. } R^{2}=0.032 \\
    & (19.757)(-4.167) \\
    C I R R_{t}^{*}= & 0.000-0.00001 C Y C L \mathrm{E}_{t}, \text { Adj. } R^{2}=0.008 \\
    & (2.793)(-2.277)
    \end{aligned}
    $$

[^5]:    ${ }^{5}$ The factors are previously demeaned.

[^6]:    ${ }^{6}$ Jagannathan and Wang (1998) point out that under some circumstances, the precision associated with noncorrected standard errors is not necessarily overstated when compared to corrected standard errors.
    ${ }^{7}$ The second stage GMM estimation is associated with system 24.

[^7]:    ${ }^{8}$ The GLS $R^{2}$ is similar to the one employed in Ferson and Harvey (1999).

[^8]:    ${ }^{9}$ Given a factor $f_{t}$, the orthogonalized factor is computed as

[^9]:    ${ }^{10}$ In the estimation of $\mathbf{S}$, no lagged moments are considered, since the conditional implications of the asset pricing model force the moments to be serially uncorrelated. Nevertheless, the results associated with Newey and West (1987) standard errors (calculated with one lag), are similar to those associated with White (1980) standard errors.

[^10]:    ${ }^{11}$ In the calculation of the "weighted" cross-sectional $R^{2}$, all the pricing errors are included.

[^11]:    ${ }^{12}$ The p-values associated with the test $H J=0$, are calculated as in Jagannathan and Wang (1996) and Hodrick and Zhang (2001), with 10.000 simulations of a weighted sum of $\chi^{2}(1)$ distributions.

[^12]:    ${ }^{13}$ The average risk premium per BM quintile is equal to the risk price times the average covariance associated with that quintile.

[^13]:    ${ }^{14}$ For notational convenience let's assume that there is only one state variable, i.e., $z_{t}^{i}$ is a scalar.

[^14]:    ${ }^{15}$ The normalization that the benchmark return is the $N$ th asset does not play any role in the derivation.

[^15]:    ${ }^{16}$ See Cochrane (2001), Chapter 10.
    ${ }^{17}$ The functions with unspecified arguments (.), have arguments $\left(W_{t+1}, z_{t+1}\right)$.

