Optimal Gradual Annuitization: Quantifying the Costs of Switching to Annuities

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Abstract

We compute the optimal dynamic asset allocation policy for a retiree with Epstein-Zin utility. The retiree can decide how much he consumes and how much he invests in stocks, bonds, and annuities. Pricing the annuities we account for asymmetric mortality beliefs and administration expenses. We show that the retiree does not purchase annuities only once but rather several times during retirement (gradual annuitization). We analyze the case in which the retiree is restricted to buy annuities only once and has to perform a (complete or partial) switching strategy. This restriction reduces both the utility and the demand for annuities.

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1 Introduction

Two major trends are responsible for the increasing public awareness of longevity risk: first, as nations move from public pay-as-you-go to privately funded pension systems, the retiree himself becomes responsible for managing longevity risk over his remaining lifetime. Second, employers are shifting from defined benefit (DB) plans to either hybrid or defined contribution plans (DC). The constant payout life-annuity is a bond-based investment with longevity insurance protecting the retiree from outliving his resources (Mitchell et. al. 1999). Thus, life-annuities are almost identical to public pensions with respect to their payout structure. Longevity insurance is possible because the insurer (insurance company or government) absorbs the longevity risk by pooling many annuitants. Usually, annuity contracts guarantee constant life-long payments to the annuitant. No other form of retirement withdrawal plan can offer fixed payments as long as the individual is alive without exposing the individual to longevity risk. However, there are pitfalls related to longevity insurance too. Once the money is spent on the annuity premium, the retiree loses control over his funds and gives up flexibility. Neither can the premium can be transferred to his heirs nor can he stay financially flexible during retirement to pay health expenditures or finance a motor yacht.

Political regulations of subsidized privately funded pensions do not agree on the importance of longevity insurance. Some governments have mandated that tax-qualified retirement saving plans include a mandatory annuity which begins after a certain period. In the US, of course, annuitization is not compulsory for 401(k) plans; as a result, most retirees roll them over to an Individual Retirement Account and manage the funds themselves, subject to the tax laws requiring minimum distributions to begin at age $70\frac{1}{2}$. Other governments introduced tax sheltered and subsidized savings plans as well while making annuitization compulsory at a certain age. In the UK, accumulated assets had to be annuitized by age 75 (this rule expired in April 2006). In Germany, "Riester" plans offer a tax inducement if life-annuity payments start from age 85 on and the withdrawn amounts are either constant or increasing prior to age 85. Therefore, it seems to be that governments want to have simple and standardized rules for annuitization applied to a large, heterogeneous group of retirees.

Theoretically, complete annuitization is only optimal for restrictive assumptions. Yaari (1965) finds that all assets should be annuitized - given a single riskless asset, actuarially fair annuity premiums, and no bequest motive.[1] Davidoff et al. (2005)

[1] Richard (1975) was the first to include the uncertainty of the time of death in a continuous life-cycle framework and to extend Merton's (1971) model to include instantaneous term life insurance. However, this framework lacks the realism of an actual insurance market because Richard (1975) models instantaneous life insurance and annuity demand symmetrically.

are more specific about the conditions for complete annuitization. As long as the insurance market is complete and the return on the annuity is above the reference asset, a retiree without a bequest motive completely annuitizes his entire wealth. If the assumption of complete markets is relaxed or if there is a bequest motive then partial annuitization becomes optimal.

A certain literature string on this topic has just compared the pros and cons of alternative phased withdrawal plans versus life-annuities paying constant benefits. In this context, some studies compute the probability of running out of money before the retirees uncertain date of death. Follow-on work by Dus, Maurer, and Mitchell (2005) extended the previous research by quantifying risk and return profiles of fixed versus variable withdrawal strategies using a shortfall framework. A natural extension is whether retirees might benefit from following a mixed strategy, where the portfolio could possibly involve both a life-annuity and a withdrawal plan. Devolder and Hainaut (2005) consider an initial annuitization strategy that mixes withdrawal strategies and life-annuities contemporaneously, but do not allow for deferring annuitization into future periods. Therefore, part of the literature comes up with the search for the optimal time to switch from withdrawal plans to life-annuities and hence allows for mixing withdrawal plan and life-annuities not contemporaneously but inter-temporally. In this context, complete switching strategies entirely use the remaining funds of a withdrawal plan to purchase a life-annuity. The recommendation of these studies is to switch to annuities at a certain point during retirement, usually if the mortality credit exceeds the equity premium. Milevsky and Young (2002) and Kingston and Thorp (2005), for instance, also try to explain the annuity puzzle describing empirically low levels of annuitization by introducing the real option to delay annuitization. In fact, the authors calculate the optimal deterministic time to switch completely to an annuity while following optimal investment and consumption policies before the actual switching time. Blake et al. (2003) estimate the optimal deterministic and stochastic switching times to completely shift from investments in bonds and stocks to constant real life-annuities, while Stabile (2003) exclusively focuses on stochastic switching times (a.k.a. stopping times to switch to annuities). Optimal switching times to annuities are also investigated in a shortfall framework when the retiree self-annuitizes his portfolio prior to switching to an annuity (see Milevsky, Moore, and Young, 2006).[2]

Contrary to the previous switching literature, Kapur and Orszag (1999) and Milevsky and Young (2003), and Horneff, Maurer, and Stamos (2006) investigate gradual annuitization strategies.[3] Gradual annuitization refers to the strategy whereby

[2]Blake et al. (2003) solve the combined optimal control and stopping time problem in discrete time by relying on numerical methods. Milevsky et al. (2006) and Stabile (2003) solved this problem in a continuous time setting by restating it as a variational inequality.

[3]Kapur and Orszag (1999) and Milevsky and Young (2003) assume time-additive CRRA pref-

a retiree can purchase annuities several times during retirement. The retiree should gradually annuitize his wealth meaning that he should purchase annuities with a certain part of his wealth several times during retirement in order to have the optimal tradeoff between the inflexibility of annuities and the longevity insurance they offer.

Recent research referring to annuitization has not yet considered the impact of regulatory restrictions vis-à-vis the theoretically optimal annuitization strategy. Our goal is to examine the utility losses and the effects on the demand for annuities caused by imposing switching restrictions. Besides complete switching we also want to introduce as a novelty partial switching strategies which allow for mixing withdrawal plans and life-annuities contemporaneously. In this framework we allow annuities to be purchased only once too. We show how a retiree optimally consumes, (dis-)saves, and purchases annuities in the following three cases: complete switching, partial switching, and gradual annuitization. We consider the following liquid assets: risky stocks and riskless bonds. The inflexibility or illiquidity of annuities is modeled by imposing the restriction that the retiree cannot sell previously purchased annuities. We assume that the retiree's utility function is of the Epstein/Zin (1989) form, he possibly has a bequest motive, and faces borrowing restrictions. By resorting to numerical backward optimization, we derive the optimal policies.

We estimate the utility loss of switching strategies for a retiree with and without a bequest motive. Thereby, we also consider a scenario in which annuity markets do not exist (pure withdrawal plan) and a scenario in which the retiree is forced to purchase annuities with all of his wealth initially (initial annuitization). While utility losses for the partial and the complete switching strategies are still high with respect to gradual annuitization, the losses of those strategies are much lower compared to the losses related to the pure withdrawal plan or the initial annuitization.

The remainder of this paper is organized as follows. Section 2 introduces the discrete time model, the individual's preferences and explains the different annuitization strategies. Section 3 shows the optimal asset allocation policy for the bequest and no-bequest case. Section 4 reports the results from Monte Carlo simulations and from a comparative welfare analysis before section 5 concludes.

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erences without bequest motives and labor/pension income. Kapur and Orszag's (1999) model does only account for tontines while Milevsky and Young's (2003) also accounts for inflexible life-annuities. Therefore, the first one resembles a standard stochastic control problem, whereas Milevsky and Young's (2003) is of the barrier control type. Horneff, Maurer, and Stamos (2006) have analyzed the optimal life-annuity demand and the welfare gains from annuities taking into account the entire life-cycle of an individual with Epstein/Zin preferences, bequest motives, and uninsurable labor income.

2 The Model

In this section, we introduce the model we apply to the problems identified in the previous paragraph. First, we define the individual's preferences and then we will introduce the three possible annuitization strategies: gradual annuitization, partial switching, and complete switching.

We consider a retiree turning 65 in $t = 0$ who has a constant retirement income Y and initial retirement-savings of S_0. We truncate the retiree's maximum age to 100 in $T = 36$. Hence, we have $t \in \{0, ..., T+1\}$ because the retiree leaves estate to his heirs in $T + 1$. The retiree has a subjective probability p_t^s that he survives until $t + 1$ given that he is alive in t. Furthermore, the individual is characterized through Epstein-Zin utility defined over a single non-durable consumption good. Let C_t be the consumption level and B_t be the bequest at time t. Then Epstein-Zin preferences as in Epstein and Zin (1989) are described by

$$V_t = \left\{ (1 - \beta p_t^s) C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[p_t^s V_{t+1}^{1-\rho} + (1 - p_t^s) k \frac{(B_{t+1}/k)^{1-\rho}}{1-\rho} \right]^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}}, \quad (1)$$

where ρ is the level of relative risk aversion (RRA), ψ is the elasticity of intertemporal substitution (EIS), β is the discount factor and k the strength of the bequest motive. Since $p_T^s = 0$ equation (1) reduces in T to

$$V_t = \left\{ C_t^{1-1/\psi} + \beta \mathbb{E}_t \left[k \frac{(B_{t+1}/k)^{1-\rho}}{1-\rho} \right]^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}},$$

which gives us the terminal condition for V_T.

2.1 Gradual Annuitization Strategy

The gradual annuitization strategy is the most general case we consider in our analysis. It refers to the intertemporal asset allocation problem among equity, bonds, and life-annuities as well as the consumption choice of a finite horizon long-lived agent in a setting in which the annuities purchased to date provide constant payments for the individual's remaining lifetime, but additional annuities can be purchased over time. Each year t the retiree can use his wealth on hand W_t to consume C_t, to buy stocks S_t and bonds M_t, and to purchase life-annuities PR_t. Therefore, the budget constraint is

$$W_t = C_t + S_t + M_t + PR_t. \quad (2)$$

The next year's wealth on hand W_{t+1} comprises the public pension income Y, the new value of the last-period stock investment S_tR_{t+1} and the bond investments M_tR_f and the payouts of all previously purchased life-annuities L_{t+1}:

$$W_{t+1} = Y + S_tR_{t+1} + M_tR_F + L_{t+1}, \tag{3}$$

where R_t denotes the real risky stock return and R_f the real bond gross return. The risky stock return is assumed to be i.i.d. lognormal distributed with the expected return μ and volatility σ. The purchase of the annuity with the amount PR_t delivers constant payouts P_t

$$P_t = PR_t/a_t,$$

where the annuity factor a_t is given by

$$a_t = (1+\delta)\sum_{s=1}^{\infty}\left(\prod_{u=t}^{t+s} p_u^a\right) R_f^{-s}.$$

p_u^a are the survival probabilities used by the life-annuity provider and δ is the expense factor. Since these probabilities can be different from the retiree's ones, we are then able to model asymmetric mortality beliefs. The constant payout life-annuity is an asset class with a distinctive return profile, as payments are conditional on the annuitants survival. The capital of those who die is allocated across surviving members of the cohort. Accordingly, a survivors one-period total return from an annuity is a function of his capital return on the assets plus a mortality credit. Other things equal, the older the individual, the higher is the mortality credit, i.e. the higher is the compensation for the inflexibility of the life-annuity.

In $t+1$ the sum of all payouts from previously purchased annuities is

$$L_{t+1} = \sum_{i=0}^{t} P_i. \tag{4}$$

At this point we want to highlight our assumption that the investor is not restricted to use annuity payouts for consumption purposes only, as in Blake et al. (2003), Milevsky and Young (2002), Kingston and Thorp (2005), and Stabile (2003). The investor has the full flexibility to decide on how to spend the annuity payouts. They can be used to consume, to purchase bonds or stocks or even to purchase additional annuities. Furthermore, we impose borrowing constraints:

$$M_t, S_t, PR_t \geq 0, \tag{5}$$

since we do not allow the investor to borrow against future pension income and to sell life-annuities. Hence, from the individual's perspective, the premium paid initially cannot be recovered. If the retiree dies, bequest B_t will be given by the remaining financial wealth $B_{t+1} = M_t R_f + S_t R_{t+1}$ since annuity payouts cannot be transferred to his heirs.

2.2 Partial Switching Strategy

Partial switching limits the freedom of choice given in the gradual annuitization strategy. The partial switching restriction urges the retiree to purchase annuities only once but gives him the freedom to decide how much wealth he shifts to annuities. Thus, let τ denote the stochastic stopping time at which the switching takes place. Then, we can add the following restriction to the model. If switching took place in an earlier period, no further annuity purchases would be allowed for periods to come:

$$PR_t = 0 \quad \forall t > \tau.$$

Then the budget restrictions (2) and (3) can be restated as

$$W_t = \begin{cases} C_t + S_t + M_t + PR_t & ,t \leq \tau \\ C_t + S_t + M_t & ,t > \tau \end{cases} \tag{6}$$

$$W_{t+1} = Y + S_t R_{t+1} + M_t R_f + \begin{cases} 0 & ,t \leq \tau \\ P_\tau & ,t > \tau \end{cases}. \tag{7}$$

Again, annuity payouts can be used not only to consume but also to purchase bonds and stocks.

2.3 Complete Switching Strategy

If complete switching is imposed, further restrictions have to be added. If the retiree decides to switch, no investments into stocks and bonds will be allowed any longer. Therefore, we have to add to the above restriction the following ones:

$$S_t, M_t = 0 \quad \forall t \geq \tau \tag{8}$$

Thus, once the retiree decides to switch he has to shift all his savings into annuities and he has to use all annuity payouts for consumption purposes only. From then on his consumption stream is deterministic:

$$C_t = P_\tau + Y \quad \forall t \geq \tau.$$

Plugging the restriction (8) into (6) and (7) leads to

$$W_t = C_t + \begin{cases} S_t + M_t & ,t < \tau \\ PR_T & ,t = \tau \\ 0 & ,t > \tau \end{cases} \quad W_{t+1} = Y + \begin{cases} S_t R_{t+1} + M_t R_f & ,t < \tau \\ P_\tau & ,t \geq \tau \end{cases} .$$

2.4 Numerical Solution

Each year the retiree must choose how much he consumes, saves in stocks and bonds, and to what extent he buys life- annuities. Thereby, he maximizes his life-time utility under consideration of the corresponding budget restrictions as well as the short-selling restrictions. We use the public pension income to normalize the state and policy variables. The normalized variables are denoted as lower case letters. The optimal policy depends on three state variables: normalized cash on hand w_t , normalized annuity payouts from previously purchased annuities l_t and age t. Since an analytic solution to this type of problem does not exist to our knowledge, we use dynamic programming techniques to maximize the value function by backward induction.

We solve the problems in a three-dimensional state space by backward induction. For solving the gradual annuitization and partial switching problem the continuous state variables wealth on hand w_t and annuity payouts l_t have to be discretized and the only discrete state variable is age t. For each grid point we calculate the optimal policy and the value of the value function. Thereby, the expectation operator in (1) is computed by resorting to Gaussian quadrature integration and the optimization is done by numerical constrained minimization. We derive the policy functions for gradual annuitization (i=GA) and partial switching (i=PS), $s_i(w, l, t)$, $m_i(w, l, t)$, $pr_i(w, l, t)$, $c_i(w, l, t)$ and the value function $v_i(w, l, t)$ by cubic-splines interpolation.

For solving the complete switching problem we can omit the state annuity payouts l but have to introduce an indicator variable I which is 1 if the retiree decides to switch and 0 otherwise. For each combination of wealth and age in the grid we compute the optimal utility for the case that the retiree switches and that he does not switch. The policy delivering a higher utility is then the optimal one. In the case of switching, utility is trivial to compute since c_t, s_t, m_t, and b_t are constant from that time on. In the case of no switching the value function is computed by using cubic splines interpolation. The policy function is then given by $s_i(w, I, t)$, $m_i(w, I, t)$, $pr_i(w, I, t)$, and $c_i(w, I, t)$.[4]

[4]The numerical optimization in the 3 dimensional grid for the gradual annuitization and partial switching case is done in the matter of hours and in the complete switching case with only 2 dimensional grid it is done in a matter of minutes on a standard personal computer with Pentium IV processor and 2,400 Mhz using Matlab.

3 Optimal Annuitization and Asset Allocation Policies

3.1 Without Bequest Motives

This section shows the optimal policy for each annuitization strategy and displays the annuity purchases as well as stock and bond investments for the case without a bequest motive. We choose the following preference parameters: coefficient of relative risk aversion $\rho = 3$, elasticity of intertemporal substitution $\psi = 0.2$, discount factor $\beta = 0.96$, and bequest weight $k = 0$. We set the real interest rate R_f to 2 percent, the equity premium $\mu - R_f$ to 4 percent and stock volatility σ to 18 percent, which is in line with the recent life-cycle literature. The expense factor δ is set to 7.3 percent for male annuitants.[5] The retiree's and the annuitant's survival probabilities are taken from the 2000 Population Basic mortality table and the 1996 US Annuity 2000 Aggregate Basic respectively.

The optimal policies for annuity purchases are depicted in figure 1. All four graphs show the optimal annuity purchases as a function of current age and normalized wealth.

The upper left graph reflects the complete switching case. The barrier at which the investor completely annuitizes separates the age-wealth space into two regions. The higher the wealth the earlier the retiree wants to shift his accumulated wealth to life-annuities. If the wealth remains low, annuitization will never become optimal. Since the retiree can only choose between a liquid stock/bond portfolio and illiquid life-annuities exclusively, he switches to annuities rather late. Annuitization is postponed because the decision is reduced to a mutually exclusive investment decision.

In the no-annuitization region the need for staying flexible and the desire to gain the equity premium is predominant for the retiree. On the flip side, in the annuitization region, the retiree wants to avoid the risk of not consuming his wealth entirely and of leaving bequest behind in case he dies. At the same time he wants to hedge himself against longevity risk. Due to the restriction he is not able to optimally exploit the advantages of annuities while maintaining a partially liquid portfolio.

If we allow for partial switching the pattern of annuitization is very similar to the case of complete switching as the upper right hand graph shows. However, the retiree starts switching earlier but with less wealth. If normalized wealth is sufficiently high, he will even switch at the beginning of his retirement phase (age 65). Overall, the

[5] This factor is taken from the 1995 annuity value per premium dollar computed on an after tax basis by Mitchell et al. (1999). We refer the interested reader to this article for a greater discussion of the explicit and implicit costs related to annuities.

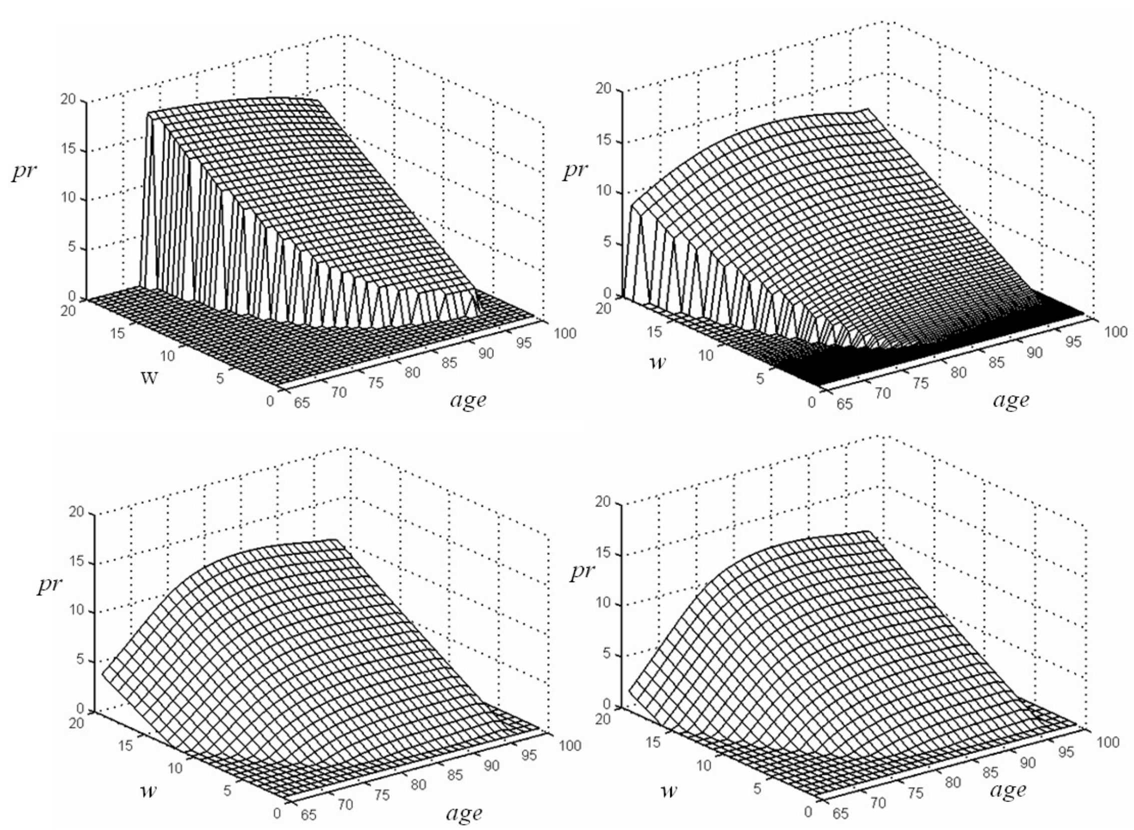

Figure 1: Optimal Annuity Purchases for Males with $RRA = 3$, $k = 0$, and $EIS = 0.2$. Upper left graph: complete switching case. Upper right graph: partial switching case. Lower left graph: gradual annuitization case ($l = 0$). Lower right graph: gradual annuitization case with previously purchased annuities ($l = 1/3$).

annuitization region in the age-wealth space is much larger compared to the complete switching case.

If we allow for gradual annuitization, the first time the retiree purchases annuities will be slightly before the initial purchase in the case of partial switching. He starts annuitizing low amounts of wealth compared to the more restrictive cases because he has the opportunity to purchase annuities later in life. The fourth graph depicts the case in which he has previously bought annuities paying one third of his yearly pension income. One can infer that the retiree still has demand for annuities. The demand is slightly lower than in the case in which he has not purchased annuities previously.

The optimal asset allocation policies for stocks and bonds are displayed in figure 2. The structural brakes in the optimal policies for stocks and bonds in the switching cases reflect the stopping times when the retiree switches to annuities. The optimal mix between bonds, stocks, and annuities consists actually mainly of stocks and annuities in all cases. The optimal stock exposure shrinks with age which is in line with popular investment recommendations promoted by many policy makers and

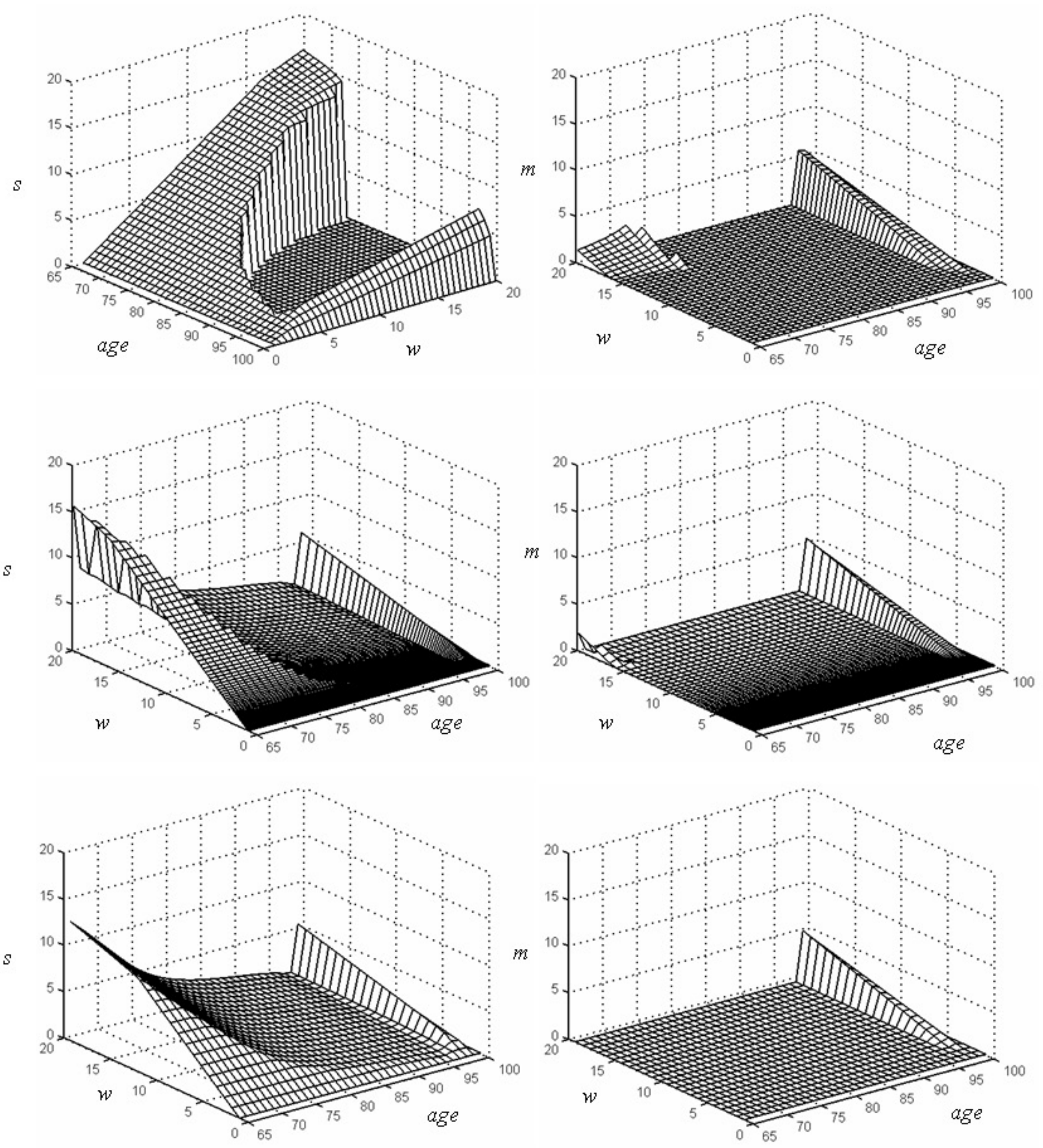

Figure 2: Optimal Allocation to Stocks (left column) and Bonds (right column) for Males with $RRA = 3$, $k = 0$, and $EIS = 0.2$. The upper, middle, and lower graphs correspond to the complete switching, partial, and gradual annuitization case, respectively.

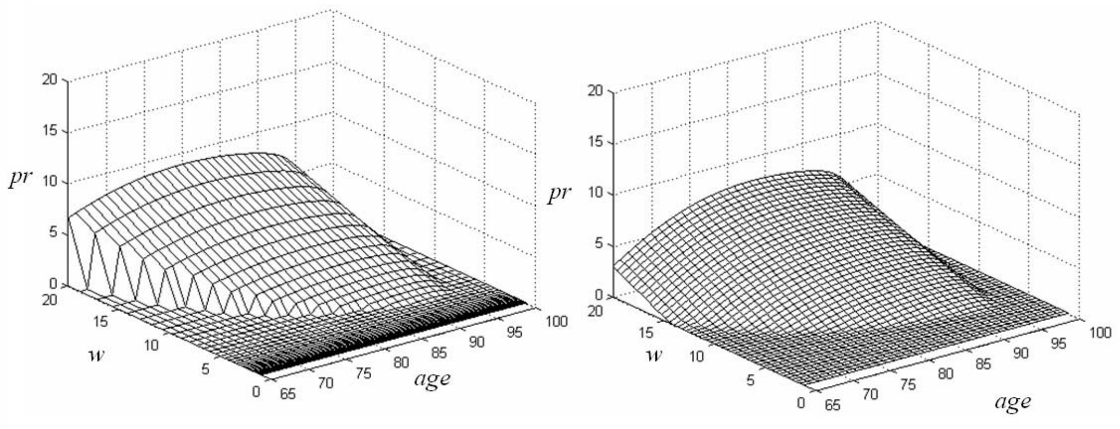

Figure 3: Optimal Annuity Purchases for Males with $RRA = 3$, $k = 1$, and $EIS = 0.2$. Left graph: partial switching case. Right graph: gradual annuitization case ($l = 0$).

financial planners. The reasons for the high equity fractions are manifold. First, we consider an investor with a relatively low risk aversion. Second, the mortality credit of annuities makes the bond yield appear less attractive. Hence, annuities replace bonds over time. Third, during the retirement period human capital represents the present value of the riskless pension income. Then, human capital is an implicit annuity holding because it perfectly resembles its payout structure and replaces bond demand. Fourth, in the case of complete switching the investor has to fully switch to annuities. Therefore, the retiree allocates his accumulated wealth mainly in stocks while anticipating the complete switch in the riskless annuity. At the same time he can also satisfy his risk appetite. In general, the stock exposure does not rise proportionally with wealth because annuities are purchased whenever a certain wealth level is reached.

3.2 With Bequest Motives

Introducing bequest motives has substantial effects on both the retiree's asset allocation and consumption strategy. This is because the retiree wants to have sufficient liquid financial wealth which he can bequeath in case he dies. Thus, he will align his policy in so far as he will never exhaust his entire savings. So he can always transfer liquid wealth if he dies. The more formal reason is that the marginal utility from leaving bequest becomes infinite if bequest converges to 0. The bequest motive mitigates the demand for annuities since annuity payment claims are not transferable to one's heirs (see Bernheim, 1991).

In a complete switching setting, a retiree with a bequest motive opts for a withdrawal plan exclusively. Our results are in line with the recent recommendations as far as the complete switching case is concerned. The retiree basically ignores

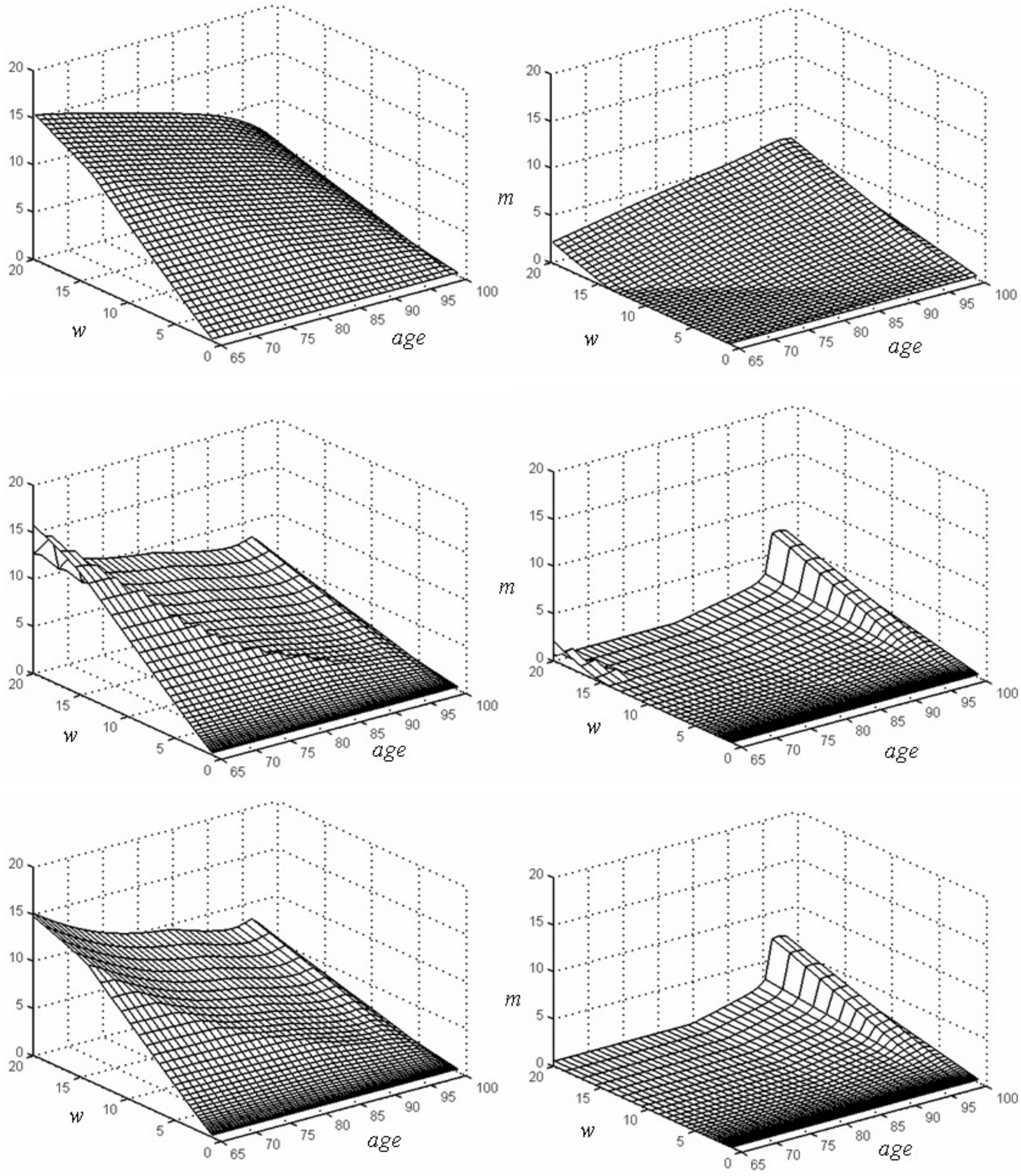

Figure 4: Optimal Allocation to Stocks (Left Column) and Bonds (Right Column) for Males with $RRA = 3$, $k = 1$, and $EIS = 0.2$. The upper, middle, and lower graphs correspond to the complete switching, partial, and gradual annuitization case, respectively.

the existence of annuity markets and chooses to invest only in stocks and bonds since he is only allowed to buy annuities with all his savings. This would reduce the bequest potential to zero. An exogenously complete switching restriction hence entirely prohibits the retiree from purchasing annuities and from gaining utility in the presence of annuity markets.

With the partial switching restriction (figure 3) the retiree buys annuities giving up a substantial fraction of his wealth. The remaining wealth is consumed and invested in liquid stocks and bonds. Holding liquid assets allows the retiree to transfer estate to his heirs. Contrary to the complete switching case, the retiree can gain utility from the existence of annuity markets although he has a bequest motive. In the gradual switching case he again purchases annuities several times during retirement. The demand for annuities in all the three cases is lower than in the case without a bequest motive but remains at a high level in the partial switching and gradual annuitization case.

The optimal asset allocation policy for all cases is given in figure 4. The demand for stocks shrinks with age similar to the no-bequest case due to the decrease of the human capital. Overall, the demand for stocks and bonds is higher than in the no-bequest case in order to be able to bequeath to his heirs. Especially, bonds enable the retiree to plan his bequest with a higher precision because the related bond return is certain. While life-annuities offer a higher return than bonds due to their increasing mortality credit, the retiree now prefers the lower return of the bonds to the life-annuities because he wants to keep liquid wealth instead.

4 Results from Monte Carlo Simulations

4.1 Expected Decumulation Profiles without Bequest

We carry out Monte Carlo simulations by generating 100,000 life-cycle trajectories for the base-line case. We assume that the retiree's savings are 10 times the pension income at the beginning of his decumulation phase. Hence, his entire accumulated wealth at age 65 is 11 times his pension income. He consumes and invests his wealth according to the optimal policy functions derived in the preceding section. Figure 5 shows the retiree's expected decumulation profiles for gradual annuitization, partial switching, and complete switching.

Starting from age 71 on annuity purchases occur rather late in the complete annuitization setting, whereas annuities are bought from age 66 on (see also figure 1) in other settings. These annuity purchases deliver additional payments allowing to consume even at very high ages (> 90) considerably more than the regular pension income. The retiree consumes more when he is young as he expects to gain more

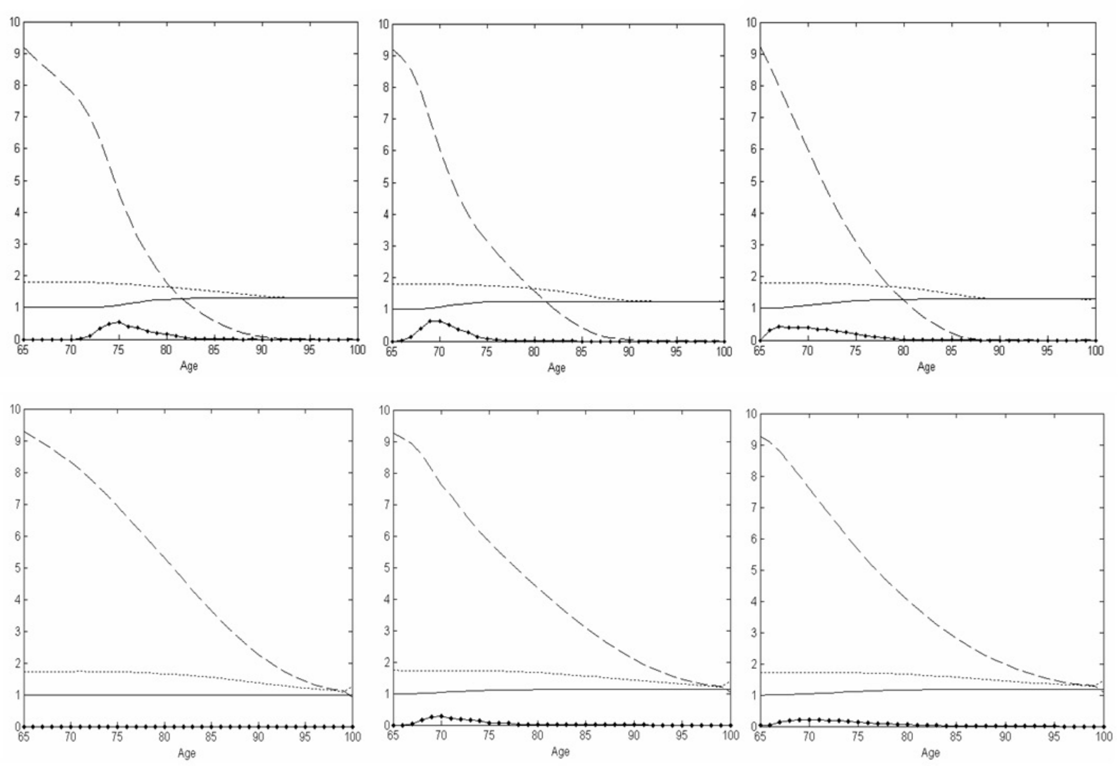

Figure 5: Decumulation Profiles for Males with $RRA = 3$ and $EIS = 0.2$. The upper panel depicts the case without bequest motive ($k = 0$) and the lower the case with one ($k = 1$). Left graph: complete switching. Middle graph: partial switching. Right graph: gradual annuitization. The dashed, dotted, solid and solid/asterisk lines reflect expected savings, consumption, pension income (life-annuity and public pension) and annuity purchases, respectively.

utility from early consumption since he weighs future utility from consumption with survival probabilities. At age 65 he consumes almost twice his pension income and in turn reduces wealth by 8 percent from 10 to 9.2. In expectation consumption is quite high because accumulated wealth is fully invested in stocks delivering high expected returns. Savings (dashed line) decline most rapidly in the case of gradual annuitization (right graph) becoming exhausted by age 87. In the other cases the exhaustion of savings occurs later at age 91 since the retiree is not allowed to buy annuities if he did it once before.

4.2 Expected Decumulation Profiles with Bequest

In order to analyze the implications of the bequest motive we run 100,000 Monte Carlo simulations in which we use the newly calculated optimal policies. We again assume that the retiree's savings are 10 times the pension income at the beginning of his decumulation phase. He consumes and invests his wealth according to the optimal policy functions derived in the preceding section. Figure 6 shows the re-

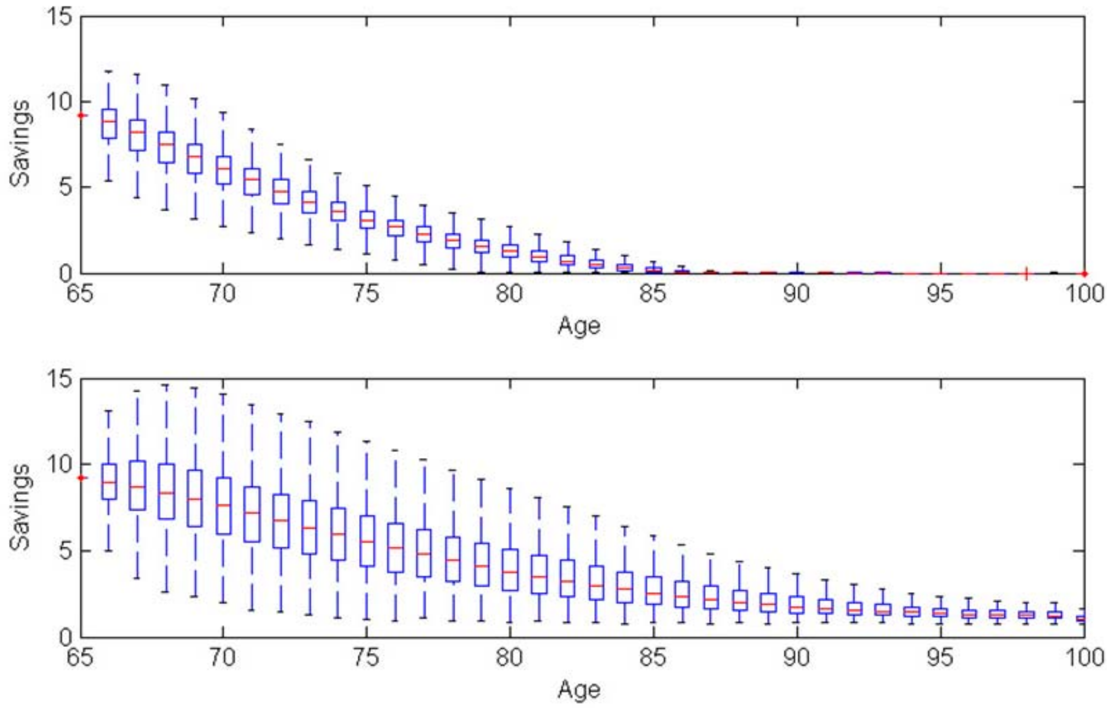

Figure 6: Box Plot of Savings for the Bequest and No-Bequest Case. The upper graph corresponds to the no-bequest case and the lower one to the bequest case.

tiree's expected decumulation profiles for gradual annuitization, partial switching, and complete switching.

The left graph corresponds to the complete switching case that is degenerated to a pure phased withdrawal plan as explained above. The middle (partial annuitization) and right graph (gradual annuitization) show that in expectation the annuity demand is substantially weaker than in the no-bequest case while the timing of annuity purchases remains similar. Again, the window for optimal annuity purchases is narrower in the partial switching than in the gradual switching case. Throughout the retirement period the individual keeps higher liquid savings than in the no-bequest case to be able to bequeath wealth in case he dies.

4.3 Distribution of Savings and Consumption

For the gradual annuitization case, the distribution of liquid savings over time is presented in figure 6 as a box plot. In the case with a bequest motive it has a higher level and larger variation than in the case without it. Dis-saving occurs slower in the bequest-case than in the no-bequest case. This can also lead to increasing savings for some random paths, e.g. until age 78 it is still likely that savings are higher than the initial savings in the bequest-case. Even in the 1 percent worst case the retiree can transfer substantial wealth, falling hardly below his yearly pension income, to

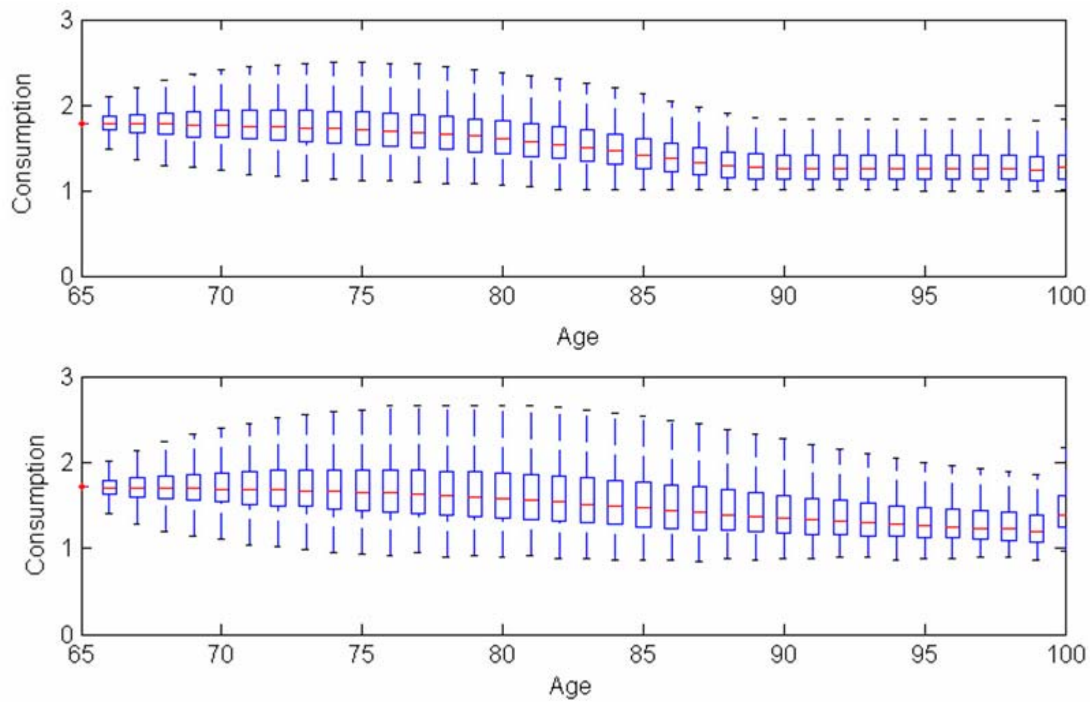

Figure 7: Box Plot of Consumption for the Bequest and No-Bequest Case. The upper graph corresponds to the no-bequest case and the lower one to the bequest case.

his heirs.

The distribution of consumption over the retirement phase is presented in figure 7 as a box plot. The overall shape of the consumption distributions over the retirement period is similar. The overall level of consumption in both cases declines with age. The variability of consumption in the case with bequest is higher since the retiree never uses his entire savings mainly invested in stocks and reacts to different wealth states. Even though he retiree in the no-bequest case uses his liquid wealth up by age 87, his consumption remains constant above the pension income because he receives constant payouts from previously purchased annuities hedging longevity risk. The level of consumption from this age on is equal to his pension plus annuity income.

4.4 Welfare Analysis

The substantial demand for annuities suggests that considerable utility gains can be generated through the presence of annuity markets in general and hedging longevity risk is important for the retiree. Governments have an intrinsic motivation to promote longevity insurance: first, insurance products can avoid old-age poverty that might otherwise burden the social safety net; second, governments can also be interested in reducing intergenerational transfers to limit divergence in aggregate wealth distribution.

Regulation of privately funded pension systems can either leave the annuitization decision entirely up to the retiree or can establish mandatory annuitization guidelines. The first idea suggests that the individual is prudent enough to decide in a responsible manner about hedging longevity risk himself, while the second approach assumes that regulation is necessary to bring the advantages to the forefront. In this context, it is desirable to have simple and feasible annuitization rules that can be applied to a large, heterogeneous group of retirees.

While in the US annuitization is voluntary for tax sheltered retirement saving plans (401 K), some European governments introduced tax sheltered and subsidized plans making annuitization compulsory at a certain age. For instance, in the UK, accumulated assets had to be annuitized by age 75 prior to April 2006. In Germany, "Riester" plans offer a tax inducement if life-annuity payments start from age 85 on and the withdrawn amounts are either constant or increasing prior to age 85. Both examples involve switching savings to annuities at a particular age that is the same for all annuitants. In the optimal policy section, we showed that the switching age depends not only on the retiree's preference but also on the level of savings. In general, switching strategies seem to comprise the necessary simplicity needed for regulating privately funded pensions systems. The complete and partial switching strategies we analyzed give more freedom in the sense that retiree can choose the switching age.

In the previous asset allocation section, we have seen that annuity demand is weakened by exogenously imposed switching regulation and annuitization is postponed or even circumvented. Exogenously imposed restrictions could cause considerable utility losses making annuity investments look less attractive compared to the unrestricted case. Therefore, the switching strategy sets counterproductive incentives for annuitization.

In order to quantify the utility loss, we conduct a welfare analysis similar to Mitchell et al. (1999). To benchmark results we include pure withdrawal plans and initial annuitization into our analysis. We compute the equivalent losses in financial wealth for every age relative to gradual annuitization in order to measure the expected utility losses in monetary terms. Apparently, the expected utility is always higher for individuals who can voluntarily purchase annuities. The equivalent loss in financial wealth is defined as the reduction in savings implied by following a restricted policy. Therefore, we equate the expected utility values of retirees with and without restrictions as far as annuitization is concerned by lowering the individuals' financial wealth in the gradual annuitization case.

Table 1 displays the equivalent losses in savings for four suboptimal annuitization strategies. To calculate the equivalent losses, we use the optimal policies derived in the optimal policy section for both the bequest and no-bequest case. Overall, losses

	Partial Switching		Complete Switching		Withdrawal Plan		Annuity at Age 65	
Age	65	75	65	75	65	75	65	75
No Bequest	2.12	20.07	2.54	22.14	2.99	27.94	12.85	26.61
Bequest	3.64	38.57	5.97	42.76	5.97	42.76	n/a	n/a

Table 1: Equivalent Loss in Financial Wealth for Different Suboptimal Strategies Compared to Gradual Annuitization.

are of substantial magnitude, especially at age 75 after following the suboptimal strategies for 10 years. Losses are higher for the complete switching than for the partial switching case which is in line with adding more restrictions. If the retiree has a bequest motive, losses turn out to be almost twice as high as in the no-bequest case. The withdrawal plan performs worse than the switching strategies except for the bequest case in which the complete switching strategy is equal to the phased withdrawal plan. While the initial annuitization strategy appears to be the worst among all suboptimal strategies at age 65, the loss becomes smaller than the one of the withdrawal plan at age 75.

5 Conclusion

Initial switching, and gradual annuitization strategies have only been treated separately in the insurance literature so far. We compare all cases in terms of annuity demand, asset allocation and welfare. In addition, partial switching that has not been considered in the previous literature is introduced. Our paper also contributes to the literature by accounting for non-additive utility and bequest motives.

We find that the retiree seeks longevity insurance even if he has a bequest motive. Optimally, the retiree prefers purchasing annuities several times to switching to annuities only once. In the bequest case, the retiree always keeps a certain amount of liquid savings. Our analysis shows that the introduction of switching restrictions has substantial effects on both annuity demand and welfare. Switching restrictions cause the annuitization age to be postponed and the overall demand for annuities to be weaker. If the retiree has a bequest motive, the annuity demand will vanish completely for the complete switching case. The recent literature uses this result as a possible explanation for the empirically low annuity demand (annuity puzzle).

Our welfare analysis supports that the presence of life-annuities hedging longevity risk is valuable to the retiree. Switching restrictions produce welfare losses equivalent to a decrease in financial wealth of up to 5.97 percent at age 65 and up to 42.76 percent at age 75. The welfare analysis also shows that the simple initial, complete, and even our newly introduced partial switching strategies lead to severe utility losses especially if the individual gets old. Considering the utility losses is essential since most of the recent research on the accumulation period as well as the decumulation period has exogenously imposed one of the simplified annuitization strategies. Also from a policy viewpoint, it is important to account for the utility losses because some governments have already mandated that tax-qualified retirement saving plans include a mandatory annuitization. In light of our results, it seems to be somewhat questionable whether governments should impose restrictions on annuitization exogenously.

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