The Coskewness Puzzle

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Comments welcomed

In testing the 3M-CAPM, Dittmar (2002) assumes that the representative investor's utility is concave in wealth. This assumption is, however, overly restrictive and, as a consequence, his test is not conclusive. We propose instead a novel test based on a positivity constraint on the estimated stochastic discount factor (SDF) and, more importantly, an upper bound on its volatility. The former restriction rules out arbitrage opportunities, while the later rules out unduly large Sharpe ratios, based on a sensible upper bound on investors' risk aversion. Together, these restrictions reduce the risk of spurious estimates of the 3M-CAPM parameters without the need to impose overly-restrictive assumptions about the shape of investor's utility. We find that the 3M-CAPM is empirically admissible in the cross-section of excess returns on industry-sorted portfolios but, crucially, it is rejected when the set of test asset payoffs is augmented to include portfolios managed on the basis of conditioning information, even under a very loose SDF volatility upper bound. In light of the considerable explanatory power of coskewness in the cross-section of stock returns, this gives rise to a coskewness puzzle.

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In testing the 3M-CAPM, Dittmar (2002) assumes that the representative investor's utility is concave in wealth. This assumption is, however, overly restrictive and, as a consequence, his test is not conclusive. We propose instead a novel test based on a positivity constraint on the estimated stochastic discount factor (SDF) and, more importantly, an upper bound on its volatility. The former restriction rules out arbitrage opportunities, while the later rules out unduly large Sharpe ratios, based on a sensible upper bound on investors' risk aversion. Together, these restrictions reduce the risk of spurious estimates of the 3M-CAPM parameters without the need to impose overly-restrictive assumptions about the shape of investor's utility. We find that the 3M-CAPM is empirically admissible in the cross-section of excess returns on industry-sorted portfolios but, crucially, it is rejected when the set of test asset payoffs is augmented to include portfolios managed on the basis of conditioning information, even under a very loose SDF volatility upper bound. In light of the considerable explanatory power of coskewness in the cross-section of stock returns, this gives rise to a coskewness puzzle.

1. Introduction

There is evidence that, controlling for covariance with popular market portfolio proxies, assets that display higher coskewness with the latter offer lower average returns. For example, Harvey and Siddique (2000) find that coskewness is important and commands on average a risk premium of 3.6 percent per annum. Kraus and Litzenberger (1976), Friend and Westerfield (1980), Harvey and Siddique (2000), among others, explain this empirical regularity on the basis of a three-moment extension of the Capital Asset Pricing Model (henceforth, 3M-CAPM). Two crucial 3M-CAPM predictions are, first, that the inter-temporal marginal rate of substitution (henceforth, IMRS) of a rational, expected utility maximizing representative investors is a valid stochastic discount factor (henceforth, SDF) that prices all assets and, second, that the SDF can be approximated as a quadratic polynomial in the market return. A crucial corollary of these two predictions is that the market portfolio is efficient. Consistently with the 3M-CAPM, Dittmar (2002) finds that a SDF quadratic in market wealth provides a much better fit to the observed cross-section of stock returns than a linear model. Both Dittmar (2002) and Post, Levy and van Vliet (2005), however, find that the superior performance of the 3M-CAPM is greatly reduced when the SDF is restricted to be decreasing in wealth and thus when the representative investor's utility function is restricted to display risk aversion over all values of sample wealth.

It is not uncommon, in the extant literature, to find experimental evidence consistent with non-concave utility and local risk-seeking and specifications of investors' preferences that admit this type of behaviour. Active stock traders appear to play negative-sum games and their behaviour can sometimes be interpreted as 'gambling' (see Statman (2002)). Psychologists, led by Kahneman and Tversky (1979), find experimental evidence for local risk seeking behavior. Friedman and Savage (1948) and Markowitz (1952) argue that the willingness to purchase both insurance and lottery tickets implies that marginal utility is increasing over a range. See Hartley and Farrell (2001) and Post and Levy (2002) for a discussion. Post, Levy and Van Vliet (2005), however, argue that non-concave utility is problematic from the 3M-CAPM point of view. In essence, they point out that, if the representative investor's utility function is not concave, the market portfolio is not guaranteed to maximize her expected utility function and, therefore, the 3M-CAPM does not necessarily hold even if the SDF is quadratic in such portfolio.

The problematic shape of the representative investor's utility function, implied by unrestricted estimates of the 3M-CAPM, represents a puzzling conundrum. On the one hand, Dittmar's (2002) results and the critique put forth Levy and Van Vliet (2005) suggest that estimating coskewness premia under no restriction on the shape of the SDF might lead to spurious estimates of the parameters of the 3M-CAPM. On the other hand, in tests of the 3M-CAPM, concavity of utility is a sufficient but not necessary condition for the market portfolio to maximize expected utility. This implies that it is formally impossible to make any conclusive inference on the empirical validity of the 3M-CAPM if the latter is rejected when this condition is imposed in estimation, as in the tests performed by Dittmar (2002) and Post, Levy and Van Vliet (2005). Such tests, in fact, amount to tests of the joint hypothesis that the 3M-CAPM holds and utility is concave.

To avoid this problem, we test the 3M-CAPM under weaker assumptions about the representative investor's preferences. We only impose a positivity restriction on the estimated SDF, to rule out arbitrage opportunities, and an upper bound on its volatility, to rule out unduly high Sharpe ratios (henceforth, SR). The SDF volatility upper bound applies under a fairly broad family of utility functions and mild assumptions about the distribution of returns and it is a generalization of the bound derived by Ross (2005). Consistently with the 3M-CAPM, these restrictions are based on the assumption that the representative investor's preferences display non-satiation, satisfy a reasonable upper bound on relative risk aversion and display non-increasing absolute risk aversion.

We find that, while a SDF quadratic in a market return proxy performs much better than its linear counterpart, even under a reasonable upper bound on SDF volatility, it is badly rejected when the set of test asset payoffs is augmented to include portfolio managed on the basis of conditioning information. Unlike in Dittmar's (2002) study, based on the overly-restrictive assumption about the concavity of the representative investor's utility function, this finding allows us to formally reject the 3M-CAPM. An additional finding is that the extent to which coskewness drives away the explanatory power of the loadings on the Fama and French (1993) size and book-to-market factor mimicking portfolios heavily depends on the amount of allowed SDF volatility.

In the next Section, we present some background analytical results on stochastic discount factor pricing and we discuss the problem of bounding from above SDF volatility. In Section 3, we outline the estimation methodology that underlies our tests. In Section 4, we present our dataset. In Section 5, we present our main empirical results. In Section 6, we discuss the implications of our findings for the 3M-CAPM and the extent to which they give rise to a coskewness puzzle. In the final Section, we restate our main findings and present our conclusions.

2. Asset Pricing and SDF Volatility

The SDF is the random variable m_{t+1} that satisfies the following condition for all payoffs x_{t+1} and payoff prices p_t:

$$p_t = E_t\left(m_{t+1}x_{t+1}\right) \tag{1}$$

Here, the expectation is taken conditional on the available information set. A well known theorem, credited to Harrison and Kreps (1979), says that, given free portfolio formation and the law of one price, such a variable exists and, under the additional assumption of no arbitrage, it is positive. Factor models specify m_{t+1} as a linear function of a set of factors f_{t+1}:

$$m_{t+1} = a_t + b_t' f_{t+1} \tag{2}$$

Letting $f'_{t+1} = \left[R_{m,t+1} \quad R^2_{m,t+1}\right]$, we model the SDF imposing on (2) a second order polynomial structure defined over the market return, $R_{m,t+1}$:

$$m_{t+1} = 1 + b_{1,t} R_{m,t+1} + b_{2,t} R^2_{m,t+1} \tag{3}$$

We call this specification the quadratic market model (QMFM). While linear in polynomials of the market return, it implies that m_{t+1} is a non linear function of the latter. Based on (3), the variance $\sigma^2_t(m_{t+1}) = b_t' Var_t(f_{t+1}) b_t$ of the SDF is:

$$\sigma^2_t(m_{t+1}) = b^2_{1,t}\sigma^2_t(R_{m,t+1}) + b^2_{2,t}\sigma^2_t(R^2_{m,t+1}) + 2b_{1,t}b_{2,t}Cov_t(R_{m,t+1}, R^2_{m,t+1}) \tag{4}$$

Here, the terms $\sigma^2_t(R_{m,t+1})$, $Cov_t(R_{m,t+1}, R^2_{m,t+1})$ and $\sigma^2_t(R^2_{m,t+1})$ are related to the market variance, skewness and kurtosis. Taking unconditional expectations of both sides of (4) and assuming, for the time being, that its parameters are not time-varying yield:

$$\sigma^2(m_{t+1}) = b^2_1\sigma^2(R_{m,t+1}) + b^2_2\sigma^2(R^2_{m,t+1}) + 2b_1 b_2 Cov\ (R_{m,t+1}, R^2_{m,t+1}) \tag{5}$$

When pricing excess-returns under the 3M-CAPM, as briefly explained in Appendix A, (3) can be seen as the marginal utility (henceforth, MU) of a representative investor with preferences that can be approximated using a third order Taylor expansion of a generic non-polynomial standardized utility function. Therefore, the relation in (5) links the volatility of the SDF to the parameters that capture the marginal investor's preference for portfolio volatility and skewness. Its usefulness is that, if we find a meaningful way of bounding these parameters, i.e. if we bound the elements of b, we then have a bound on the volatility of the SDF that prices the assets.

To do this, following Ross (2005), we place an upper bound on the relative risk aversion (henceforth RRA) of the marginal investor, i.e. we let $RRA \leq RRA_V$. Since U is, by assumption, a standardized utility function, the representative investor's RRA around the point of expansion is $U'' = -(b_1 + 2b_2 R_{m,t+1})$. Imposing the upper bound RRA_V on RRA, this implies

$$-(b_1 + 2b_2 R_{m,t+1}) \leq RRA_V \quad (6)$$

or,

$$b_1 \geq -(RRA_V + 2b_2 R_{m,t+1}) \quad (7)$$

Assume further non increasing absolute risk-aversion (NIARA) and thus $b_2 \geq 0$. A necessary condition for (7) is that $b_1 \geq -RRA_V + Max(-2b_2 R_{m,t+1})$. Under $b_2 \geq 0$, this implies $b_1 \geq -RRA_V - 2b_2 R_{m,t+1}^{\min}$, where $R_{m,t+1}^{\min}$ denotes the minimum value of the range over which the representative investor's market return probability density function is defined. Making the reasonable assumption that the minimum of the market return is a negative number, a necessary condition for this inequality to hold is that:

$$b_1 \geq -RRA_V \quad (8)$$

Local risk aversion, a milder assumption than concave utility (i.e., global risk aversion), implies $b_1 < 0$. For the latter to hold, (7) implies that:

$$RRA_V + 2b_2 R_{m,t+1} > 0 \tag{9}$$

Similarly, for (9) to hold over the entire range of the possible negative values of the return on market wealth (i.e., the values to which the representative investor assigns positive probability), it must be that:

$$b_2 < -\frac{\frac{1}{2} RRA_V}{R_{m,t+1}} \qquad R_{m,t+1} < 0 \tag{10}$$

For this inequality to hold over the entire range of the possible market returns, it must be that:

$$b_2 < -\frac{\frac{1}{2} RRA_V}{R_{m,t+1}^{\min}} \tag{11}$$

Combining (5), (8) and (11), we obtain an upper bound on the volatility of the SDF:

$$\sigma^2(m_{t+1}) \leq b_{V,1}^2 \sigma^2(R_{m,t+1}) + b_{V,2}^2 \sigma^2(R_{m,t+1}^2) + 2b_{V,1,t} b_{V,2} Cov\ (R_{m,t+1}, R_{m,t+1}^2) \tag{12}$$

Here, based on (8), $b_{V,1} = -RRA_V$ and, based on (11), $b_{V,2} = -\frac{\frac{1}{2} RRA_V}{R_{m,t+1}^{\min}}$. This upper bound on the volatility of the SDF depends on quantities related to the volatility, skewness and kurtosis of the portfolio held by the marginal investor and on the preferences for portfolio volatility and skewness embodied by a monotone, concave transformation of his utility function (i.e., the transformation with *RRA* corresponding to the upper bound RRA_V). It applies under an arbitrary non-polynomial utility function at least three times continuously differentiable, as long as its expectation is quasi-concave (to ensure efficiency of the market portfolio), and under any distribution of returns for which moments of at least the first four orders exist (and are finite). Thus, it must hold also under the 3M-CAPM. It is therefore a generalization of a similar result derived by Ross (2005) that instead applies only when $U(W_{t+1})$ is quadratic or returns are normally distributed. All we need, in order to compute the bounds in (8) and (11), and thus the bound in (12), is an upper bound on the relative risk aversion of the marginal investor and the assumption about NIARA. The bound also applies when the SDF parameters are conditionally time-varying, as long as (8) and (11) hold.

To identify a suitable value for the RRA bound, we follow Ross' (2005) advice and experimental evidence on investors' RRA provided by the extant literature. Ross (2005) suggests imposing an upper bound of 5 on the relative risk aversion of the marginal investor, i.e. $RRA_V = 5$. Among the motivations advanced by Ross (2005) to do so, the one that most easily applies to a world with possibly non-normally distributed returns and non quadratic utility is the simple observation that a relative risk aversion higher than 5 would imply that the marginal investor is willing to pay more than 10 percent per annum to avoid a 20 percent volatility of his wealth (i.e., about the unconditional volatility of the S&P from 1926), which seems a rather large amount. A study by Meyer and Meyer (2005) has recently provided a comprehensive re-evaluation of the hitherto scattered empirical evidence on investors' risk aversion. They show that relative risk aversion estimates reported by the extant literature are less heterogeneous and extreme if one takes into account measurement issues and the outcome variable with respect to which each study defines risk aversion. Using returns on stock investments as the outcome variable, calculations by Meyer and Meyer (2005) show that the RRA coefficient in the classical Friend and Blume's (1975) study of household asset allocation choices ranges between 6.4 and 2.0, and decreases in investors' wealth. Using returns on the investors' overall wealth, including real estate and a measure of human capital, the RRA estimate ranges between 3.0 and 2.4. The same calculations show[1] that the RRA implied by Barsky et al. (1997) study ranges between 0.8 and 1.6.

We compute the SDF volatility upper bounds, based on (8), (11) and (12), under two different upper bounds on RRA. The first bound is 5 and corresponds to the bound suggested by Ross (2005). The second bound is 6.4 and corresponds to the RRA coefficient of the most risk-averse cohort of investors in Friend and Blume (1975) study, as re-calculated by Meyer and Meyer (2005). The 5 and 6.4 upper bounds on RRA imply that the investor would be willing to pay no more than 10 and 12.8 percent per annum, respectively, to avoid a 20 percent volatility of her wealth. By introspection, these are arguably large amounts. The different assumptions used in the computation of the bounds, reported in Table 1, reflect sample moments estimated over the period 1927-2005 and portions thereof, and market lows over the same periods but they also apply in a non mean-variance world.

[1] Meyer and Meyer (2005) calculate somewhat higher values based on estimates provided by studies of the equity premium puzzle. Since these estimates are backed out parametrically from estimates of a particular asset pricing model, often based on a narrow definition of the market portfolio, they are of no interest for the purpose of computing the SDF volatility bound in (4e). Moreover, their use would imply a circular argument.

3. Estimating Factor Models under No-Good Deal Restrictions

When pricing excess-returns, the mean of the SDF can be set equal to an arbitrary value. This is because[2], as explained by Cochrane (2001), excess returns do not identify it. Thus, for convenience, we let the intercept in (2) equal one. Imposing the existence of a conditionally risk free rate, (3) is therefore equivalent to the following:

$$m_{t+1} = 1 + b_{1,t} r_{m,t+1} + b_{2,t} q_{t+1} \tag{13}$$

Here, $r_{m,t+1}$ and $q_{t+1} = R^2_{m,t+1} - R_{f,t}$ can be seen as a new set of factors. Restricting $b_{2,t}$ to equal zero yields a linear specification, that we label the linear market factor model (LMFM). Writing the SDF in (2) as a linear function of the size and book-to-market factor mimicking portfolios yields the the Fama and French (1993) 3-factor model (henceforth FFM). We also denote as QMFM-FFM the model that nests the QMFM and the FFM. Given a a set of n test asset payoffs x_{t+1}, a no arbitrage restriction and an upper bound on its volatity, the SDF parameters can be estimated solving the following problem:

$$\begin{aligned} &\min_{\{m\}} \;\; g_t' W_{n\times n} g_t \\ &s.t. \;\; m_{t+1} \geq 0, \;\; \sigma_t^2(m_{t+1}) \leq A_t \end{aligned} \tag{14}$$

with

$$g_t = E_t(m_{t+1} x_{t+1}) - p_t(x_{t+1}) \tag{15}$$

[2] This is strictly true only as long as the risk free rate is not unrealistically high.

The elements of the vector g_t correspond to the moment conditions implied by the factor pricing model in (1) and can be interpreted as pricing errors, while *W* is a suitable *n*x*n* weighting matrix[3]. The efficient choice is Hansen's (1982) optimal weighting matrix. This yields GMM estimates of the SDF parameters. Other choices are, however, admissible. An example is the identity matrix, which yields OLS. In (14), we might add the constraint $E_t(m_{t+1}f_{t+1}) = p_t(f_{t+1})$ to assign zero pricing error to the factors, whenever the latter are returns on traded assets. In estimation, we may use sample averages $E_T()$ instead of unconditional and we might expand the set of orthogonality conditions by imposing the pricing errors to be unpredictable using information carried by a vector of *k* instruments z_t:

$$\begin{aligned} &\min_{\{m\}} \; g_T' W_{n(1+k)\times n(1+k)} g_T \\ &s.t. \;\; m_{t+1} \geq 0, \;\; \sigma_T^2(m_{t+1}) \leq A \end{aligned} \tag{16}$$

$$g_T = E_T\left[(m_{t+1}x_{t+1} - p_t) \otimes z_t\right] \tag{17}$$

Under appropriate conditions, the minimization of the pricing error metric in (16) yields the same estimates as a classic 2-pass regression. Adding the constraint $E_T(m_{t+1}x_{EW\,t+1}) = E_T\left[p_t(x_{EW\,t+1})\right]$ forces the estimated model to assign zero pricing error to the equally weighted average payoff $x_{EW\,t+1}$ and thus, using the identity matrix as the

[3] This is similar to the approach followed by Cochrane and Saà-Requejo (2000) and Cochrane (2001) to extend, in incomplete markets, the pricing implications of the factor prices and of (1) to a non-redundant security. An important difference, however, is that Cochrane and Saà-Requejo (2000) and Cochrane (2001) motivate the volatility bound as a bound on the market SR, thus implicitly assuming quadratic utility3.

weighting matrix for the moment conditions, yields OLS second pass regression estimates. When working with excess returns, this constraint reduces to $E_T(m_{t+1} r_{EW\,t+1}) = E_T\left[p_t(r_{EW\,t+1})\right] = 0$, where $r_{EW\,t+1}$ is the excess return on the equally weighted portfolio of the test assets[4]. A zero intercept in second pass regressions is equivalent to no volatility bound in (16). On the contrary, an appropriate volatility bound gives point estimates corresponding to a second pass regression with intercept. This is because, by construction, all the cross-sectional variation in historical average excess returns captured by a second pass regression without intercept is explained by variation in factor loadings, leaving no portion of variation to be explained by the intercept, whereas imposing a volatility bound limits the explained portion of sample average excess returns.

4. Data

We use quarterly data, from 1952 to 2002, constructed sorting stocks of the Centre for Research on Security Prices (CRSP) database into an overall market portfolio, 17 and 30 industry portfolios[5]. We use quarterly data on 1 and 3 month T-Bill returns, on their spread and on Lettau and Ludvigson (2001) consumption-wealth ratio estimate. We use the quarterly returns on the 3-month US Government Treasury Bill as a proxy for the risk free rate.

[4] As another example, adding the constraint $E_T(m_{t+1} f_{t+1}) = E_T\left[p_t(f_{t+1})\right]$ and using the optimal weighting matrix yields GLS second pass regression estimates. This is because GLS assigns the largest weights to the moments estimated with most precision.

[5] We thank K. French for making this data publicly available for download.

5. Empirical Results

We preliminarily estimate, by OLS and without volatility constraint, unconditional versions, i.e. with fixed b, of the QMFM, LMFM, FFM and QMFM+FFM using simple 2-pass regressions[6], both without and with intercept in second pass regressions. We estimate OLS standard errors corrected to take cross-sectional error correlation into account and, following Shanken (1992), for the fact that factor loadings are estimated. For each model, we also estimate the volatility $\sigma(m)$ of the corresponding SDF. This is done by taking the sample standard deviation of m_{t+1}, given the sample realizations of the factors and the point estimates of the model parameters.

The unconstrained estimates from 2-pass regressions, without and with intercept in the second pass regression, are reported in Table 2 and 3, respectively. The sample period is 1952-2002. The QMFM displays a much stronger explanatory power than the FFM. The sign of the market risk premium is positive, in accordance with the notion that the typical investor is averse to systematic stock market risk. The coefficient of the QMFM squared market return polynomial factor is negative, thus satisfying a necessary condition for NIARA and preference for skewness[7]. When the QMFM is estimated with no intercept in second pass regressions, its SDF takes negative values over a range of

[6] Since, as shown by Jagannathan and Wang (2002), the Beta-method and the SDF-method are equivalent in terms of consistency and asymptotic efficiency, we estimate the LMFM, QMFM and FFM following the former because of its greater simplicity. This way, we directly estimate the parameters of the beta-pricing representation of these models. In a 2-step procedure, we first regress the time series of the 30 industry portfolios and size and book-to-market sorted portfolios excess returns on the factors allowing for an intercept in the regression equations. This yields estimates of the factor loadings β. We then estimate the risk premia λ using a cross sectional regression of the average portfolio returns on the factor loadings estimated in the first step, without an intercept term. We then obtain the SDF representation of these estimates using (B6). This procedure yields exactly the same estimates of the SDF as (16) under the appropriate choice of the weighting matrix and no SDF positivity or volatility constraint (and without instruments apart from a constant).

the market return realizations, as shown in Figure 1, and thus violates the no-arbitrage requirement (i.e. it does not always assigns a positive price to strictly non-negative payoffs). The SDF is also very volatile, especially in the case of the QMFM and when the models are estimated with no intercept in second pass regressions. In particular, the QMFM displays a SDF volatility that is appreciably higher than the volatility bounds reported in Table 1.

Constrained Estimates

We then estimate the QMFM under positivity and volatility restrictions on its SDF. The (annualized) volatility constraint is set to 50, 75 and 100 percent, i.e. we set A in (16) equal to the corresponding quarterly SDF variance. The 50 percent bound is close to the SR of the market portfolio, yet it is somewhat higher than the latter to allow for the possibility that certain dynamic strategies may offer a reward for coskewness risk that the representative investor desires to shed. Based on (8), (11) and (12), it implies an upper bound of 2.5 on the relative risk aversion of the marginal investor. The other values of the volatility bound, i.e. 75 and 100 percent, broadly correspond to the values reported in Table 1.

The contrained estimates are reported in Table 4 and 5. In the former, we report the b parameter estimates while in Table 5 we report the corresponding the factor risk premia. The estimation is conducted without instruments, i.e. by setting z_t in (16) equal to 1. We

[7] Because there is considerable cross-sectional dispersion, industry returns are notoriously difficult to fit. Thus, relatively low coefficients of cross-sectional determination should not surprise and are in line with the estimates reported by Harvey and Siddique (2000).

use Hansen's (1982) optimal weighting matrix with zero lags [8] for the moment conditions. None of the models is rejected by a Chi-squared test based on Hansen's (1982) TJ_T statistics. The coskewness risk price is, however, substantially smaller than in the corresponding unconstrained estimates. For example, in the benchmark case, the QMFM estimated under the 75 percent annualized volatility bound, the coskewness risk price is almost cut in half relative to its unconstrained counterpart and the coskewness risk premium changes from -0.66 to -0.40 percent per quarter (from -2.64 to -1.6 percent per annum, i.e. about 1 percent less in absolute value). These are sizeable differences from a capital budgeting and valuation perspective.

To allow a comparison between the estimated models, we also report Hansen's (1982) TJ_T statistics re-calculated using a pre-specified weighting matrix, namely the optimal weighting matrix with zero lags of the QMFM+FFM model under a 100 percent SDF volatility bound. The difference between the QMFM and QMFM+FFM Hansen's (1982) TJ_T statistics is statistically insignificant. For example, in the case of the estimates under the 50 percent SDF volatility bound, this difference is 1.30, which is significant at the 0.522 level with two degrees of freedom. The difference between the FFM and QMFM+FFM Hansen's (1982) TJ_T statistics, however, is marginally significant. For example, in the case of the estimates under the 50 percent SDF volatility bound, this difference is 2.33, which is significant at the 0.127 level with one degree of freedom. Thus, while Hansen's (1982) TJ test marginally rejects the restriction that the FFM

[8] This specification of the optimal weighting matrix corresponds to a null hypothesis that the pricing errors are unpredictable. It neglects possible serial auto-correlation and cross-correlation of the pricing errors and thus it is sub-efficient. This simplification however is more robust to mispecification errors and, while it likely does not affect much the size of the test because returns are not very auto-correlated at this frequency, it increases its power. See Cochrane (2001) for a discussion.

model imposes on the model that combines QMFM and FFM, it does not reject the restriction imposed by the QMFM.

Conditional Estimates

Finally, and importantly, we estimate the QMFM and, for comparison, the FFM allowing for conditional variation in the parameters of the SDF and, following Dittmar (2002), augmenting the market portfolio by a proxy for the return on human capital, i.e. labour income Δy_t. We model time variation in b_t as a linear function of a set of conditioning variables. In a multi-period version of the 3M-CAPM, variation in the shape of the conditional SDF reflects changes in the investment opportunity set and thus variation in expected returns. We therefore seek conditioning variables that predict future market excess returns.

In Table 6, we report the correlations between the market excess return and various lags of candidate conditioning variables. The latter, following Dittmar (2002), include the market excess return itself, the 1 and 3 month T-Bill rate, the spread s_t between these two rates and, following Lettau and Ludvigson (2001), the consumption wealth ratio cay_t. We also include the first lag of q_t among the candidate conditioning variables. The variable that displays the largest correlation with future returns is cay_t and its explanatory power increases with the horizon, due its persistence. The first lag of q_t displays the second largest correlation (in absolute value) with one period ahead returns but its explanatory power fades away when longer-horizon returns are considered. This variable, moreover, displays a fairly high serial correlation, e.g. its autocorrelation is about 0.29. This implies that, since q_t is one of the factors, the inclusion of its lag as a conditioning variable would induce an identification problem. With the exception of s_t, all other variables display considerably less explanatory power. Using too many conditioning variables in modelling the dynamics of the SDF parameter might leave too few degrees of freedom in estimation. Parsimony is thus important and we consequently use only cay_t and s_t to capture the time-variation in risk prices.

The estimates of the conditional models are reported in Table 7. The conditional QMFM is denoted by CQMFM, while the conditional version of the FFM is denoted by C-FFM. The test asset payoffs are excess returns on the 17 industry sorted CRSP stock portfolios, augmented to include portfolios managed using conditional information, i.e. cross-products between the primitive excess-return payoffs and the conditioning variables. In estimation, we impose exact pricing of the stock market factor. Consistently with Dittmar's (2002) results, we find that the C-QMFM is superior, in terms of empirical fit, to the C-FFM. Under a positivity constraint on the SDF, the FFM is actually rejected while the C-QMFM is not.

Importantly, however, we find that the C-QMFM is overwhelmingly rejected even under very loose restrictions on SDF volatility, such as a 150 percent per annum upper bound (of course, as shown by un-tabulated results, the rejection is even more resounding under the lower bounds reported in Table 1). This result contrasts with Dittmar's (2002) study, as in the latter the human capital 3M-CAPM is not rejected when the SDF is restricted to be decreasing in wealth. A possible explanation for these contrasting results is that cay_t has a much larger predictive power than the conditioning variables used by Dittmar (2002). The maximal SR of the managed portfolios is thus higher and, therefore, it takes a more volatile SDF to price them.

5. The Coskewness Puzzle

As reported in Table 7, the QMFM is rejected under a SDF volatility upper bound as high as 150 percent per annum. This allows one to reject the 3M-CAPM. In fact, based on (5), (8) and (11), it would take a RRA of around 10 to explain such a high SDF volatility. The experimental and survey evidence summarized by Meyer and Meyer (2005) suggests that this is an un-plausibly high value for RRA. The evidence on the explanatory power of coskewness, coupled with the rejection of the QMFM under a reasonable SDF volatility upper bound, implies that, while coskewness is likely priced in the cross-section of stock returns, its price cannot be explained by the 3M-CAPM. Coskewness and thus, essentially, the tendency of stock returns to co-vary with market volatility are therefore stock characteristics that, just like covariance with firm and value factor mimicking portfolios, explain differences in average returns across stocks for reasons that we do not fully understand. This gives rise to yet another puzzle in empirical asset pricing, that we might label as the coskewness puzzle.

A possible explanation for this puzzle is that the quadratic market factor q_{t+1} proxies for other priced but omitted factors. As suggested by the relatively large correlation between the lagged quadratic market factor and the market excess return, reported in Table 6, one possibility is that coskewness proxies for exposure to time variation in expected returns. To gain some intuition, we therefore estimate the cross-sectional correlation between the QMFM factor loading on q_t, i.e. $\beta_{i,q}$, and the factor loading on $r_{m,t+1}z_t$, i.e. $\beta_{i,mz}$, of a conditional version of the LMFM. We estimate the factor loadings by running the usual time series regressions with intercepts of the industry portfolio excess returns on the factors and we estimate the time-series regressions in a maximum likelihood setting, by iterated system least squares. For the sake of robustness, we do not impose any constraint on the contemporaneous covariance of the residuals nor on their variance.

In Figure 2, we plot the cross-sectional 30-month rolling correlation between $\beta_{i,mz}$, and $\beta_{i,q}$ against 30-month rolling market returns. With the exception of the period 1959-1972, the plots of the cross-sectional correlation and of the market returns series seem to share the same 'trend' (with some sort of lag structure and a lot of noise) in the 3 (roughly) decades 1972-1982, 1982-1992, 1993-1999. This suggests that, while volatility exposure (captured by coskewness and thus by $\beta_{i,q}$) is almost the same as conditional beta exposure (i.e. the sensitivity of asset betas to the state of the economy captured by $\beta_{i,mz}$) at the peak of bull markets, they are almost unrelated at the bottom of a bear market.

Thus, it appears that, when valuations are high, coskewness is essentially generated by co-variation with market sensitivity to expected returns. This makes sense, as when valuations are high a small change in expected returns causes a large contemporaneous change in market valuation and thus a large (in absolute value) market return, i.e. the market return is more sensitive to expected returns changes when valuations are high. This produces more coskewness for any given level of asset sensitivity to the interaction between the market return and expected returns. This is not the case however when valuations are low. This simple mechanism explains, at least in part, the large explanatory power of coskewness, even in the presence of the reduced price of coskewness risk implied by QMFM estimates consistent with sensible SDF volatility upper bounds and thus with the 3M-CAPM. We leave a more formal investigation of this important issue for future research.

6. Main Findings and Conclusions

In this paper, we acknowledge the importance of Dittmar's (2002) findings and of the criticism of unrestricted 3M-CAPM tests put forth by Post, Levy and Van Vliet (2005), in that they highlight the danger of spurious estimates of the 3M-CAPM parameters. We emphasize, however, that a decreasing SDF, albeit sufficient, is not a necessary condition for the 3M-CAPM. In testing the latter, we thus impose alternative restrictions on the shape of the SDF, namely a positivity requirement and a volatility upper bound. These restrictions boil down to ruling out arbitrage opportunities and unduly high SR that to most investors would resemble obvious near arbitrage opportunities[9]. This way, we limit the risk of over-fitting the cross-section of asset returns without the need to resort to the overly-restrictive assumption that utility is concave.

Our results imply that the 3M-CAPM provides at best a partial explanation of the differences in average returns across stocks. In fact, while the QMFM fits well the cross-section of industry sorted portfolios, it badly fails to explain the cross-section of portfolios managed on the basis of conditioning information, even under relatively loose upper bounds on SDF volatility. The inability of the 3M-CAPM to account for the explanatory power of coskewness gives rise to a coskewness puzzle. The solution of the latter requires an explanation, different from the 3M-CAPM, for why the quadratic

[9] Beside, recognising that coskewness is an asset characteristic that explains a considerable portion of the cross-section of asset returns, such an approach is also consistent with a multi-factor, no-arbitrage perspective, along the lines of Ross' (1976) APT and especially Ross' (1978) linear pricing theory.

market factor is priced in the cross-section of stock returns. One possibility is that this factor proxies for priced but omitted factors. Because of the high cross-sectional correlation between coskewness and the loading on $cay_t r_{m,t+1}$ at particular times, q_t might capture the time-varying impact of changes in expected returns on stock market excess returns. Vanden's (2006) results suggest that it might proxy for omitted option-related factors. This possibility, while intriguing, requires however further scrutiny because Vanden's (2006) sample period is relatively short and, more importantly, it remains to be established whether his estimated SDF satisfies an appropriate volatility upper bound.

There is also the possibility that the managed portfolios correspond to unfeasible strategies, i.e. strategies with unfeasibly high SRs. Luttmer (1996), for example, shows how even modest proportional transaction costs, short sales restrictions and margin requirements considerably lower the mean-variance SDF frontier. We leave the investigation of these possible explanations of the coskewness puzzle for future research. Another fruitful avenue for future research is the application of sign and volatility constraints in tests of multi-factor models motivated by the Ross' (1976) APT.

Table 1
SDF Volatility Bounds

Panel A
(Volatility Bounds Calculations)

$R_{m,t+1}^{min}$	$R_{m,t+1}^{min}$ Period	RRA_V	b_1	b_2	$\sigma^2(m)$	$\sigma(m)$ annualized
		Quarterly 1927-2002:				
-33.5%	3Q 1931	5	-5.00	7.45	0.26	1.01
-29.2%	3Q 1929	5	-5.00	8.58	0.28	1.06
		Quarterly 1952-2002:				
-24.2%	3Q 1987	5	-5.00	10.31	0.17	0.78
-17.0%	3Q 2002	5	-5.00	14.71	0.18	0.78
		Monthly 1927-2005:				
-29.0%	Sep. 1931	5	-5.00	8.61	0.07	0.94
-20.1%	Oct. 1929	5	-5.00	12.43	0.08	0.97
		Monthly 1952-2005:				
-23.1%	Oct. 1987	5	-5.00	10.83	0.05	0.74
-16.1%	Aug. 1998	5	-5.00	15.52	0.05	0.76

Panel B
(Input Factor Variance-Covariance Matrices)

Quarterly Data

$$Var(f)_{1926-2002} = \begin{bmatrix} 1.31 & 0.35 \\ 0.35 & 0.34 \end{bmatrix} \qquad Var(f)_{1952-2002} = \begin{bmatrix} 0.67 & 0.03 \\ 0.03 & 0.01 \end{bmatrix}$$

Monthly Data:

$$Var(f)_{1926-2002} = \begin{bmatrix} 0.31 & 0.01 \\ 0.01 & 0.01 \end{bmatrix} \qquad Var(f)_{1952-2002} = \begin{bmatrix} 0.18 & 0.00 \\ 0.00 & 0.00 \end{bmatrix}$$

Notes. Panel A of this Table summarizes assumptions and computed values for the SDF volatility bound under different assumptions, corresponding to different sub-sample periods. Panel B reports the variance-covariance matrix estimates used in computing the bounds reported in Panel A. The factors are the market return and its square. All the variables are defined as in the text. The data frequency is either quarterly or monthly.

Table 2
Second Pass Regressions without Intercept (1952-2002)
Industry Portfolios

Model	r_{mt+1}	q_{t+1}	**SMB**	**HML**	R^2	**Adj. R^2**	**σ(m)**
			Panel A (Beta Pricing Representation)				
LMFM	1.71				3.7	3.7	
	(2.85)						
QMFM	1.90	-0.66			31.2	28.7	
	(3.12)	(-1.91)					
FFM	2.15		-0.34	-0.66	10.1	3.4	
	(3.64)		(-0.74)	(-1.22)			
QMFM+FFM	2.09	-0.54	-0.08	-0.71	35.6	28.1	
	(3.52)	(-1.70)	(-0.17)	(-1.19)			
			Panel B (SDF Pricing Representation)				
LMFM	-2.51						39.3
	(-2.83)						
QMFM	-4.08	47.51					125.2
	(-2.86)	(4.27)					
FFM	-4.15		3.86	0.20			64.5
	(-3.71)		(2.24)	(0.12)			
QMFM+FFM	-4.78	40.00	2.38	0.12			113.5
	(-3.12)	(3.47)	(1.20)	(0.473)			

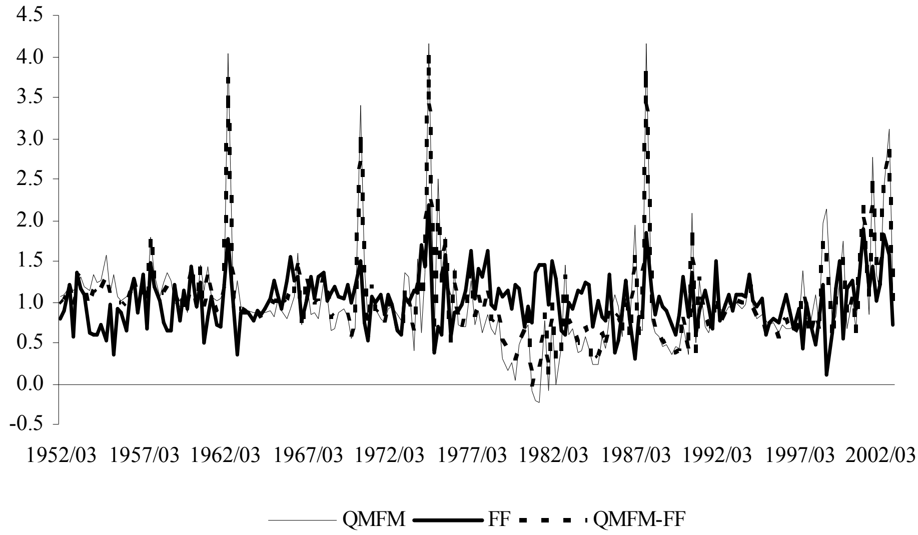
Notes. Panel A of this Table reports 2-step regression estimates of the beta-pricing representation of various factor models for the period 1952-2002. The second pass regressions are estimated without an intercept term. The top row indicates the factors included in each model. For each included factor, we report the risk premia point estimates in percentage and t-statistics in brackets. These are computed using OLS standard errors that account for correlated errors across test portfolios and using Shanken (1992) correction for the fact that the beta coefficients are estimated. The third and second last columns report the coefficient of determination R^2, both unadjusted and adjusted for the degrees of freedom, in percentage. Panel B reports the elements of the *b* vector, the negative of the risk prices, implied by the 2-pass regression estimates (without intercept in the second pass regressions) and, in brackets, associated t-statistics. These are computed using standard errors based on a specification of the *S* matrix that does not allow for serially correlated pricing errors. The last column reports the annualized volatility of the stochastic discount factor in percentage. The market Sharpe ratio is 40.4 percent. All the variables are defined as in the text. The data frequency is quarterly.

Table 3
Second Pass Regressions with Intercept (1952-2002)
Beta Pricing Representation Estimates

Model	r_{mt+1}	q_{mt+1}	SMB	HML	R^2	Adj. R^2	σ(m)
LMFM	0.44				3.8	0.4	10.3
	(0.89)						
QMFM	0.91	-0.51			36.1	31.3	96.7
	(1.51)	(-1.59)					
FFM	-0.20		0.16	-0.79	24.8	16.1	31.6
	(-0.34)		(0.34)	(-1.43)			
QMFM+FFM	0.32	-0.49	0.25	-0.77	46.9	38.5	85.5
	(0.54)	(-1.60)	(0.50)	(-1.34)			

Notes. This Table reports 2-step regression estimates of the beta-pricing representation of various factor models for the period 1952-2002. The second pass regressions are estimated with an intercept term. The top row indicates the factors included in each model. For each included factor, I report the risk premia point estimates in percentage and t-statistics in brackets. These are computed using OLS standard errors that account for correlated errors across test portfolios and using Shanken (1992) correction for the fact that the beta coefficients are estimated. The last three columns report the coefficient of determination R^2 (both unadjusted and adjusted for the degrees of freedom) and the annualized volatility of the stochastic discount factor in percentage. The market Sharpe ratio is 40.4 percent. All the variables are defined as in the text. The data frequency is quarterly.

Figure 1
Second Pass Regressions (1952-2002)



Notes. This Figure reports the SDF time-series implied by the 2-step regression point estimates of the QMFM, FFM and QMFM+FFM. The data is quarterly data for the period 1952-2002.

Table 4
GMM Estimates with Volatility Bound – SDF Representation
Industry Sorted Portfolios

Model	r_{mt+1}	q_{mt+1}	SMB	HML	TJ_T	TJ_T*	σ(m)
FFM	-2.72		2.00	1.54	21.30	19.84	50.0
	(0.007)		(0.138)	(0.184)	(0.749)	(0.799)	
	-3.39		2.81	2.54	20.86	20.91	67.2
	(0.002)		(0.053)	(0.075)	(0.749)	(0.747)	
QMFM	-2.06	17.37			20.27	16.86	50.0
	(0.052)	(0.069)			(0.872)	(0.913)	
	-2.41	28.55			16.59	15.40	75.0
	(0.024)	(0.005)			(0.956)	(0.949)	
	-2.61	39.48			14.76	15.04	100.0
	(0.025)	(0.000)			(0.981)	(0.957)	
QMFM+FFM	-2.20	15.51	1.12	0.87	18.97	16.18	50.0
	(0.040)	(0.093)	(0.273)	(0.310)	(0.838)	(0.932)	
	-2.55	27.24	1.10	0.97	16.00	14.67	75.0
	(0.033)	(0.010)	(0.283)	(0.294)	(0.936)	(0.963)	
	-2.73	38.67	0.96	0.84	14.39	14.39	100.0
	(0.033)	(0.001)	(0.313)	(0.322)	(0.967)	(0.967)	

Notes. This Table reports GMM parameter estimates with positivity and volatility bound on the SDF of various factor models for the period 1952-2002. For each included factor, we report the corresponding b_k (the negative of the risk price) point estimate and its p-value in brackets. Three sets of TJ_T statistics with p-value in brackets are reported. The first are Hansen's (1982) TJ_T statistics. The second and third sets of TJ_T statistics are calculated using a common weighting matrix for all models, the S matrix of QMFM+FFM with 100 percent volatility bound, and Hansen and Jagannathan (1997) second moment matrix. The last column reports the annualized volatility of the stochastic discount factor in percentage. The market portfolio Sharpe ratio is 40.4 percent. All the variables are defined as in the text.

Table 5
GMM Estimates with Volatility Bound – Beta Pricing Representation
Industry Sorted Portfolios

Model	$\mathbf{r_{mt+1}}$	$\mathbf{q_{mt+1}}$	**SMB**	**HML**	$\mathbf{R^2}$	**Adj. $\mathbf{R^2}$**	**σ(m)**
FFM	1.77		-0.02	-0.90	14.5	4.6	50.0
	(3.04)		(-0.04)	(-1.96)			
	2.26	0.00	-0.09	-1.30	16.3	6.6	67.2
	(3.83)	(-0.02)	(-0.19)	(-2.30)			
QMFM	1.08	-0.23			21.8	16.0	50.0
	(1.86)	(-1.10)					
	1.11	-0.40			31.6	29.2	75.0
	(1.84)	(-1.26)					
	1.04	-0.56			36.0	33.7	100.0
	(1.71)	(-1.71)					
QMFM+FFM	1.16	-0.20	0.01	-0.59	33.0	22.3	50.0
	(2.00)	(-0.97)	(0.03)	(-1.28)			
	1.20	-0.38	0.01	-0.65	40.7	33.9	75.0
	(2.04)	(-1.26)	(0.02)	(-1.14)			
	1.12	-0.55	0.00	-0.62	42.2	35.6	100.0
	(1.88)	(-1.76)	(0.00)	(-1.06)			

Notes. This Table reports the percentage risk premia corresponding, in a beta-pricing representation, to the GMM estimates with positivity and volatility bound on the SDF. For each included factor, we report the corresponding λ_k point estimate and t-statistic in brackets. These are computed using GLS standard errors that account for correlated errors across test portfolios and Shanken's (1992) correction for the fact that the beta coefficients are estimated. The third and second last two columns report the percentage coefficient of determination R^2, both unadjusted and adjusted for the degrees of freedom. The last column reports the SDF volatility.

Table 6
Percentage Correlations
1952-2002

	$r_{m,t+1}$	$r_{m,t+1\rightarrow t+12}$	$r_{m,t+1\rightarrow t+24}$	$q_{,t}$	$r_{m,t}$	$r_{m,t-12}$	$r_{TB1M,t}$	$r_{TB3M,t}$	s_t	cay_t
$r_{m,t+1}$	1.00									
$q_{,t}$	0.25	-0.03	-0.02	1.00						
$r_{m,t}$	0.06	0.23	0.06	0.16	1.00					
$r_{m,t-12}$	0.04	0.28	-0.10	0.05	-0.06	1.00				
$r_{TB3M,t}$	-0.13	-0.04	0.05	-0.50	-0.07	-0.03	1.00			
$r_{TB1M,t}$	-0.05	-0.03	0.08	-0.36	-0.02	0.01	0.91	1.00		
s_t	0.16	0.04	0.05	0.07	0.07	0.07	0.20	0.28	1.00	
cay_t	0.29	0.45	0.24	0.05	0.18	0.08	-0.02	0.04	0.09	1.00

Notes. This Table reports percentage estimates of the correlation between factors and various predictive variables. The sample period is 1952-2002. The symbol $r_{m,t\rightarrow t+k}$ denotes the return from t to t+k (a multi-period return, with k that denotes the number of quarters), $r_{TB3M,t}$ and $r_{TB1M,t}$ are the yields on the 3 and 1 month T-Bill, respectively, and s_t is the spread between the former and the latter. All the other variables are defined as in the text.

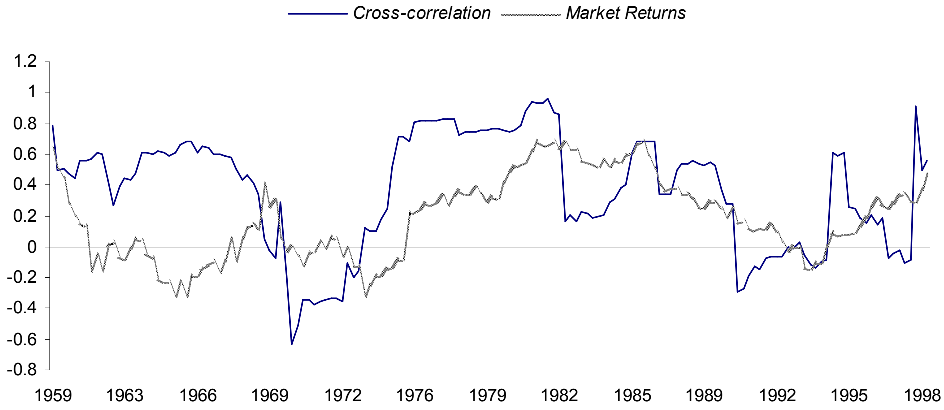
Table 7
GMM Estimates – Covariance-pricing representation
17 Industry Sorted Portfolios + Managed Portfolios

Model	r_{mt+1}	$r_{mt+1}cay_t$	$r_{mt+1} s_t$	Δy_{t+1}	$\Delta y_{t+1} cay_t$	$\Delta y_{t+1} s_t$	SMB_{t+1}	$SMB_{t+1}cay_t$	$SMB_{t+1} s_t$	HML_{t+1}	$HML_{t+1}cay_t$	$HML_{t+1}s_t$	THJ_T	σ(m)
C-FFM	-2.0	-14.8	-528.3	27.0	-884.9	-14368.8	0.2	-33.2	597.7	0.6	33.9	-216.4	173.0**	46.0
m > 0	(0.101)	(0.879)	(0.529)	(0.819)	(0.862)	(0.750)	(0.925)	(0.812)	(0.693)	(0.801)	(0.731)	(0.814)		
	6.6	0.0	0.0	-0.0	0.0	0.0	0.8	0.0	0.0	-1.9	0.01	-0.0		

Model	r_{mt+1}	$r_{mt+1}cay_t$	$r_{mt+1}s_t$	q_{mt+1}	$q_{mt+1}cay_t$	$q_{mt+1}s_t$	Δy_{t+1}	$\Delta y_{t+1}cay_t$	$\Delta y_{t+1}s_t$	q_{yt+1}	$q_{yt+1}cay_t$	$q_{yt+1}s_t$	THJ_T	σ(m)
C-QMFM	-5.4	-143.2	109.4	-2.9	1212.5	-5643.2	-92.7	-975.9	-44011.8	165.5	-465.6	-6272.8	103.8	224.0
	(0.002)	(0.150)	(0.893)	(0.870)	(0.205)	(0.354)	(0.409)	(0.836)	(0.311)	(0.000)	(0.580)	(0.157)		
	6.6	0.0	0.0	-1.6	-0.0	0.0	0.1	0.0	0.0	-2.6	-0.0	0.0		
C-QMFM	-3.5	-132.3	260.8	-3.5	1111.7	-3030.3	-17.2	-201.6	-41265.1	114.6	-303.8	-1560.0	109.9*	150.0
m > 0	(0.037)	(0.183)	(0.749)	(0.838)	(0.244)	(0.619)	(0.855)	(0.963)	(0.201)	(0.000)	(0.718)	(0.709)		
	3.1	0.0	0.0	-1.0	-0.0	0.0	0.0	0.0	0.0	-1.9	-0.0	0.0		

Notes. This Table reports GMM parameter estimates of various factor models for the period 1952-2002, under exact pricing of the stock market factor (see Appendix A for details on how this restriction is imposed). For each included factor, I report the corresponding b_k (the negative of the risk price) point estimate, its p-value in brackets, and the corresponding risk premium. I also report the Hansen and Jagannathan (1997) distance. I denote by ** and * significance at the 1 and 5 percent level. The last column reports the annualized volatility of the stochastic discount factor in percentage. The market portfolio Sharpe ratio is 40.4 percent. The conditioning variable cay is scaled by a factor of 100. All the variables are defined as in the text.

Figure 2
Coskewness vs. Market Return Sensitivity to Expected Returns



Notes. The darker line in this figure plots the 30-quarter (7 years and 6 months) rolling cross-sectional correlation between the conditional LMFM coefficients $\beta_{i,\mathrm{mz}}$ (multiplied by a factor 10 to facilitate visual comparison) and the conditional QMFM coefficients $\beta_{i,\mathrm{q}}$. The former are the sensitivities of industry excess returns to the product of the market excess-return and the conditioning variable (the lagged consumption-wealth ratio) whereas the latter are the coskewness coefficients. The jagged line represents the rolling time series of 30-quarter returns on the market portfolio.

Appendix A: Third Order Expansion of Utility

When pricing excess returns under the 3M-CAPM, we can let the SDF equal the representative investor's marginal utility (MU) in place of her IMRS. This is because an investor's IMRS is, essentially, her marginal utility growth and the price of excess returns is zero. Modelling the SDF as MU is a convenient simplification that we will adopt here, i.e. we let $m_{t+1} = U'(R_{m,t+1})$, where $U(W_{m,t+1})$ denotes the representative investor's utility function. Taking a third order Taylor expansion of $U(W_{m,t+1})$, we can write the representative investor's MU as follows,

$$U'(R_{m,t+1}) \cong 1 + b_1 R_{m,t+1} + b_2 R^2_{m,t+1}$$

Here, $R_{m,t+1} \equiv \frac{W_{m,t+1}}{W_{m,t}}$, $b_{V1} \equiv \frac{1}{2} V''(W_{m,t}) W_{m,t}$, $b_{V2} \equiv \frac{1}{6} V'''(W_{m,t}) W^2_{m,t}$, and $W_{m,t}$ is an initial wealth level around which $U(W_{m,t+1})$ is expanded in a Taylor series. Normalizing this level to one, $U(W_{m,t+1})$ is standardized in such a way that $U(W_{m,t}) = U(1) = 0$ and $U'(W_{m,t}) = U'(1) = 1$. This standardization is legitimate since utility functions are unique only up to a linear transformation.

Appendix B: Alternative Representations

The price of excess returns is by definition equal to zero. Thus, denoting by $r_{i,t+1}$ the excess return on the i-th asset, (1) can be rewritten as follows:

$$0 = E_t\left(m_{t+1}r_{i,t+1}\right) \tag{B1}$$

The cross-sectional implications that the model given by (2) and (B1) imposes on the cross-section of expected returns can be represented in a number of ways. We will mainly consider their covariance and the beta-pricing representations:

$$E_t(r_{i,t+1}) \cong -Cov_t\left(r_{i,t+1}, f_{t+1}\right)b_t \tag{B2}$$

$$E_t\left(r_{i,t+1}\right) = \beta'_{i,t}\lambda_t \tag{B3}$$

Where,

$$E_t\left(m_{t+1}\right) = a_t + b'_t E_t\left(f_{t+1}\right) \cong 1 \tag{B4}$$

$$\beta_{i,t} = Var_t\left(f_{t+1}\right)^{-1} Cov_t\left(f_{t+1}, r_{i,t+1}\right) \tag{B5}$$

$$\lambda_t \cong -Var_t\left(f_{t+1}\right)b_t \tag{B6}$$

Here, expectations and variances are denoted by familiar symbols and the time subscript indicates that they are conditional on the information set available at t. The elements of $-b_t$ can be seen as the factor risk prices, $\beta_{i,t}$ can be seen as a vector of coefficients from the regression of asset i on the factors and its elements are factor loadings. The elements of the λ_t vector are the factor risk premia.

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