Choice of financing by independent or bank-affiliated venture capital firm

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Abstract

This article studies to what extend the affiliation of the venture capital firms has an influence on the supply and the quality of the financing of holders of innovative projects. In particular, this research has for objective to understand the relative merits of the financing by independent venture capital firms or by firms affiliated to a banking network.

This article develops a model in which an entrepreneur is looking for a source of financing. He may be financed from an independent venture capital firm or from a bank-affiliated VC fund. I suppose that in case of financing by an independent fund, the cost of effort is lower thanks to more effective support. On the other hand, financing by a bank-affiliated VC fund eliminates asymmetry of information for the later investments. In this context, I study the terms of the optimal contracts offered to the entrepreneur. The entrepreneur determines his choice on the two contracts by comparing the gains of a better support with an independent, with the gains created with an affiliated firm which never takes ineffective continuation decisions unlike the independent VC firm. The affiliated firms of venture capital enhance the effort of the entrepreneur by tending to liquidate more but at the same time their slightest expertise reduce it. The model allows to suggest certain number of empirical predictions. I extend this analysis to the international differences between industries of venture capital dominated by independent or affiliated firms.

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1 Introduction

The expansion of the venture capital industry for these last decades has made possible the emergence of high-growth firms. This industry has for main specificity to increase the profitability of projects, thanks to a knowledge, an evaluation, an assistance offered to the entrepreneur. The literature in Finance has much studied the degree of implication of venture capital firms in the management of financed firms[1]. It highlighted the conflicts of interest between the entrepreneur and the venture capitalist, particularly by studying the financial contracts and securities most capable to solve these conflicts[2], which enable the venture capital firms to exit in the most advantageous way[3]. More recently, the literature studied the internal organization of venture capital firms and its influence on profitability. Kandel and al. (2006) study the conflicts of interests which can exist between the limited partners and the general partner in the VC fund structure. Dessi (2005) shows in her monitoring model that the entrepreneur and the general partner may collude at the expense of outside investors. To alleviate this problem, convertibles may be used to signal good projects, which joins the certification role of VC. Zarutskie (2006) highlights the fact that the past experience of a venture capitalist in this job or as an entrepreneur does influence the performance of the funds he manages. Nevertheless, many questions are still pending concerning the diversity of actors in the VC industry. Several types of structures of VC firms are observed. There exist independent firms, sometimes very specialized. This type of actor is often organized as a fund with a rather small implication of the limited partners. In parallel, there exist affiliated VC firms, for example to a large company, or to a large bank (e.g. *Crédit Lyonnais Venture Capital* in France). More extensively, there exist networks of VC firms affiliated to banks, organized as specialized funds. This question of the sources of financing and the affiliations of the companies of venture capital is important, because it has a direct impact on their operation. This article studies to what extend the affiliation of VC firms has an influence on the supply and the quality of the financing of the holders of innovative projects. In particular, this research has the objective to understand the relative merits of the financing by independent firms of venture capital or by firms affiliated to a banking network. Entrepreneurs wish to be financed from the investor which can provide them the most important added-value. The principal interest is thus to compare the effectiveness of these two types of structures: the independent firm, and the firm affiliated to a banking network.

This analysis is based on several important aspects. First of all, VC firms do not only provide funds. They also monitor the funded companies, support in the development of the strategy, help recruiting key personal, etc. An essential assumption devel-

1. See in particular Gorman and Sahlman (1989), Sahlman (1990), Hellmann and Puri (2002), Kaplan and Strömberg (2003) on the involvement of VC firms in the management of ventures.

2. See in this perspective advising models with double moral hazard-problem: Biais and Casamatta (1999), Casamatta (2003), Schmidt (2003), Repullo et Suarez (2004) ; monitoring models close to the approach develop in this paper: Holmstrom and Tirole (1997) and Dessi (2005).

3. See Bayar and Chemmanur (2006), Cumming (2002) on the problem of investment exits.

oped in this paper is that the VC firm affiliated to a banking network is less effective than its independent counterpart. This assumption is supported by Hellmann, Lindsey and Puri (2004), who made an empirical study on a sample of VC investments in the United States, differentiating if they are financed by firms affiliated to banks or by others not-affiliated. They show that the affiliated VC companies operate in the least risky stages and generally in collaboration with independent companies. The firms linked to bank do not have specific skills in the evaluation of the projects of investment compared to traditional VC firms. In fact they are more interested by the fact of finding complementarity with the other types of activities of their parent firm (e.g. traditional lending) than the profitability of VC activities; that tends to confirm the least expertise on the banks in this field, in spite of their skills in credit screening[4] which could not directly be transfered in VC activity. Secondly, the venture capitalists tend to stage their investments in order to control for the hazards[5]. In my model, two investments are made at two successive period by two different VC investors. These investors may be independent or affiliated to a banking network. At the second period, the first investor obtains a private information on the state of the project and may have to exit venture in an anticipated way. It is supposed that when a venture is financed by a firm affiliated to a banking network, the information acquired on the firm financed during the intermediate period is shared with the new investor because this one always belongs to the same network. It is not the case when the venture is financed by an independent VC firm because he has no link with the new investor.

I study a model comprising three periods: starting of the activity, development during the intermediate period and maturity of the financed project which could then be sold in case of success. The investment necessary to the full development of the project is staged during the first two periods. During the starting of the project, the entrepreneur makes an effort which cost is influenced by the nature of the investor that he chooses; thus the productivity of its effort is better if he chooses an independent investor. At the intermediate period, the state of nature is favorable or unfavourable according to the provided effort, and the VC firm may be hit by a liquidity shock which obliges him to exit from the venture precipitately, either by asking liquidation, or by reselling its shares. Then a second investor makes the new investment, and offers to repurchase the shares of the first investor at the conditions established in the initial contract. In the case of an affiliated VC firm, it will prefer to refinance the firm from a company belonging to the same network. It is supposed that at the intermediate period the first investor receives a signal on the state of the project. This signal is shared with the new investor only if it belongs to the same network as the first.

The results show that in the case of an independent VC firm, when the liquidity shock occurs the VC will always prefer to resell, thus means that the project is never liquidated, even in the unfavourable states of nature, where it would be socially optimal to.

4. Best and Shang (1993) study the skills of banks to acquire information while granting loans.

5. Highlighted by Sahlman (1990), Kaplan and Strömberg (2004) in their empirical study. For a theoretical point of view, see Gompers (1995), Cornelli and Yosha (2003) and Admati and Pfleiderer (1994).

There is thus inefficient continuation if the entrepreneur chooses an independent firm. On the contrary, in the case of a financing by an affiliated VC firm, the project is always liquidated in the unfavourable states of nature and continued in the favorable states. But when the liquidity shock does not occur, the independent VC firm may be incited to act as first best and liquidates when it is socially optimal to do so. The entrepreneur finally decides between two possible contracts which differ on the efficiency of support and on the way the investor behaves in bad states of nature. These two factors directly influence the NPV of the project obtained under the two financings and all actors take into account these inefficiencies. The independent VC firm which reinvests is asking less shares in particular when the probability of the shock is low and support is really more efficient. The entrepreneur chooses the contract which maximizes its payoff which is equal to the NPV of the project. He determines his choice by comparing the effects of a better support with an independent VC firm, with those of an absence of inefficient continuations with an affiliated company. The less productive is the support by the affiliated company compared to the independent VC firm, the less it is attractive for the entrepreneur. It is the same thing if the opportunity cost of continuation of project is high, in other words if the liquidation value of the assets which could be obtained during the intermediate period is low.

Comparing the parameters of the two optimal contract leads to several predictions. The independent VC firms should finance more projects with high and uncertain cash flows, those whose liquidation value is low (as in the case of projects involving a high human capital) and those requiring a more important support because of the sophistication of the project (e.g. scientific innovations), even those more distant geographically. The VC firms affiliated to a banking networks should finance the least sophisticated projects, with a high potential liquidation value at interim period. They thus have a broader financial basis that enables them to face the possible hazards. It counterbalances their least expertise, while having a restricted field of intervention. Such results may empirically tested by more frequent liquidations from VC firms affiliated to banks, or from VC firms which face low risk of liquidity shock (i.e. with a longer term).

This paper is related to the analysis by Hellmann (2002) who models the choice of an entrepreneur between a traditional investor, similar to the independent investor developed here, and a strategic investor, which is a VC firm affiliated to a large company. However in this analysis, by belonging to a network the strategic investor has an advantage compared to the independent VC firm: an externality, positive (synergy) or negative, on the assets of the parent firm according to the project which is financed. This externality if positive or very negative may allow the strategic VC to ask less shares of the project and thus be more competitive. In my analysis, the affiliated VC is linked to a bank that enables him to have access to large funds within the network. Another difference with the investor affiliated to a banking network is that the affiliation with a large company makes sometimes the strategic investor more effective than the independent investor thanks to the expertise than this affiliation provides. In my model, the bank-affiliated VC firm has less expertise. Moreover, Hellmann (2002) considers that the investor supports the cost of the effort, and the investor affiliated to a firm has a lower unit cost of effort. In my model, the entrepreneur is supporting the cost of effort. Lastly, contrary to my model, this one is not a dynamic model.

In addition, my results extend the analysis by Ueda (2004) and Winton and Yerramilli (2007), who study the differences between the financing by traditional bank lending or by venture capital and models the choice of financing of the entrepreneur between these two sources. As in our study, the theoretical model developed in Ueda (2004) supposes that the venture capitalist has an expertise enabling him to better apprehend the projects which are submitted to him (given the absence of asymmetry of information). However, he also has the possibility to develop a project without the entrepreneur who originally had the idea of it, thanks to a mechanism of expropriation. These results are consistent with mines in the best apprehension of large sophisticated projects by independent venture capitalists. They highlight the behavior of the venture capitalists affiliated to a banking network closely related to the behavior of the banks themselves in their activity of traditional lending, as Hellmann, Lindsey and Puri (2004) empirically showed. However this model deals neither with the effort nor with the asymmetry of information which can exist between the entrepreneur and the venture capitalist. Winton and Yerramilli (2007) show that venture capital should be chosen if the project to be financed has great disparity between expected cash flows from a safe and a risky strategy. A bad strategic choice may result from the desire of the entrepreneur to keep control of its firm and thus obtain private benefits. By providing monitoring, the venture capitalist is able to impose a safe strategy. However, they consider that banks are more subjects to liquidity shocks, because they do not impose liquidity restrictions to their investors contrary to venture capitalists. The liquidity shock studied here for the independent VC firm results from the fund being at the end of its life ; and the bank-affiliated VC firm is supposed to belong to banking networks with large financial resources, as in continental Europe for instance.

The question studied here, namely comparing the two optimal contracts offered by the independent company of venture capital and its counterpart affiliated to a banking network, raise many practical implications. From the point of view of the entrepreneur, we will predict which type of structure is most capable to create value added for him (assistance while hiring, advisements, etc), by weighing up the expertise and financial base, according to the type of project he has. In addition, it makes it possible to explain international differences in the structure of the VC industry; there are indeed markets of the venture capital dominated by the strong presence of VC firms affiliated to a banking network (for example in France) and markets dominated by independent VC firms (for example in the United States). Thus, Black and Gilson (1998) show in their empirical study that in 1994, 40% of the venture capital in France were provided by firms controlled by commercial banks, whereas in the United States, pension funds are the main contributors of the VC firms. According to the French ministry of Economy, in 2001 VC firms in France raised funds to 40% from banks, 11% from insurance companies and only 7% from pension funds. My study, focused on the differences of behavior of these two types of VC firms, makes it possible to evaluate the impact on the dynamism of this activity, crucial for innovating and creating wealth. The presence of firms linked to banks in the VC market appears to be an imperfect substitute to independent firms, particularly because the former are less effective when they monitor more sophisticated projects. But at the same time, since their potential financial base is very important within the network, they take better decisions on continuations of projects. My model suggests then that the best way of financing is an independent VC firm with a low probability to face a liquidity shock. It suggests that if the fund is able to stay for a long time in the venture, he guarantees to new financiers that the venture is robust. This result joins Admati and Pfleiderer (1994) who show in their model that the presence of an investor with constant shares over time is optimal.

This study also completes the banking literature, such as Rajan (1992) or Sharpe (1990). In these models the banks obtain information on their borrowers which is not observed by the external investors. Rajan (1992) models the choice of the project financing by an entrepreneur from an informed bank or from an external investor without information. In this model, the entrepreneur can liquidate the firm before the end of the project during the intermediate period before the realization of the final cash-flow, which is close to my model, however I integrate the effort of the entrepreneur and the support of the lender.

The model developed in this article is also based on the Finance literature which studies the links between the security liquidity and incentives of monitors, as in Aghion, Bolton and Tirole (2004) and Faure-Grimaud and Gromb (2004). The latter analyzes the relation between the actions of a large shareholder on the market and the quotation of a public firm, which is influenced by the exchanges between liquidity traders, a speculator and a market-maker. In their model, the large shareholder has to make an effort to increase the firm value. Besides, he may be hit before the final period by a liquidity shock which obliges him to sell in an anticipated way. The mechanism of the shock undergone by the majority shareholder is similar to a liquidity shock undergone by a venture capitalist obliged to close its fund, even if unlike my model the agent which is facing this risk is also exerting effort. It is in particular the case if the funds of venture capital arrives in end of lifetime and must be closed, whereas the financed project did not arrive yet at its end[6].

This paper is organized as follows: the second section is a general presentation of the model and characteristics of the game. In the third section I study and compare the parameters of the two optimal contracts offered by the independent VC firm and the affiliated VC firm, and then make empirical predictions. The fourth section presents possible extensions of the model. The fifth section concludes. The evidence are provided in appendix, as well as numerical examples.

6. See Kandel and al. (2006) and Sahlman (1990). Lerner and Schoar (2003) show that the managers of funds organized as partnerships use the weak liquidity of the financial securities of the funds in order to dissimulate their worse performance, the exit of the funds by limited partners constituting a negative signal on the performance of the general partner.

2 The model

An entrepreneur is endowed with an innovating project. This project lasts three periods numbered from t=0 to t=2 and needs financing at the first two periods (i.e. t=0 and t=1). The first investment is noted I_0. The final cash-flow, which comes at date 2, is risky and can be either positive and equal to R, either equal to 0. The probability to obtain a high cash-flow R depends on the state of nature determined at date 1. I suppose two states of nature. If the state of nature is favorable, then the project generates the cash-flow R with probability 1. If the state of nature is unfavorable, then it generates R with probability q, 0 with probability $1-q$. At date 1, the venture can be liquidated in which case it generates an immediate cash-flow L. If the project is continued, a second investment, noted I_1, will be necessary.

I assume that it is optimal to liquidate the project when the state is unfavorable and to continue the activity with a new investment when the state is favorable. That means taking the following assumptions:

$$R - I_1 > L$$

$$q R - I_1 < L$$

Moreover, by choosing to liquidate the first investor do not recover all its investment and consequently $-I_0 + L < 0$. It is also supposed that $R > I_0 + I_1$.

The probability s that the state of the project is good is influenced by the effort e provided by the entrepreneur. To simplify, I suppose $s = e$. Thus, we have $e \in [0; 1]$. The entrepreneur chooses his effort during period 0. Effort is costly. We note the cost of effort in the following way:

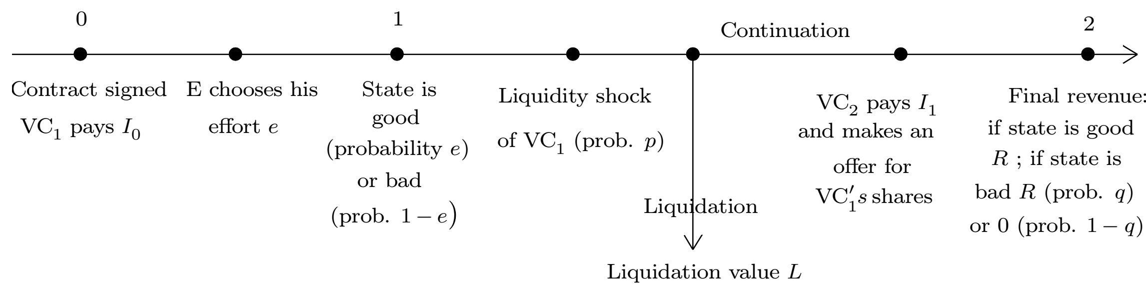
$$c(e) = c\frac{e^2}{2}$$

It is immediately noticed that the weaker the parameter c is, the less expensive the effort is.

The entrepreneur does not have any initial wealth and must obtain financing for his project from a venture capital firm. I consider an industry with two types of venture capitalists: independent firms, and bank-affiliated VC firms. At date 0, choosing a type of investor for I_0 impacts several factors. It is supposed that an independent venture capitalist brings a better support to the entrepreneur, which makes its effort less expensive. In the formula of the cost of the effort, I note c_I and c_B respectively the parameter c with an independent or an affiliated financing. One thus have $c_I < c_B$. In addition, at date 1, the VC firm which invested I_0 is privately informed of the state of the project and decides whether to continue or give up the project. If he decides not to liquidate, he has to find a new VC firm which will have to immediately invest I_1 and may have to repurchase the shares of the first investor if the latter is hit by a liquidity shock. I suppose that the liquidity shock of VC_1 is observed by all agents. If he is bank-affiliated, he will choose another firm belonging to his network.

Lastly, all the agents are supposed to be risk-neutral and the riskless rate is 0. All revenues are verifiable.

To sum up, I represent the events in the following timeline:



VC_1 is the first venture capitalist investor.

VC_2 is the second venture capitalist investor

E is the entrepreneur

In the following section I study the optimal contracts relative to the game.

3 Optimal contract with uncertain exit of the first venture capitalist

As previously supposed, the first VC firm is unable to refinance the venture at period 1. He may be hit at the same time by an uncertain liquidity shock which obliges him to resell his shares (i.e. a total exit of the project) . This probability is p such that $p \in [0, 1[$.

The entrepreneur can be financed from an independent firm, or from a firm affiliated to a banking network. A contract is signed with this first investor who offers to finance I_0 in exchange of a fraction of the liquidation value L of the venture. I note L_I or L_B according to the type of the investor these rights, the remainder L_E being allocated to the entrepreneur, so we have $L = L_I + L_E$.

At period 1, a second VC firm is offered to reinvest I_1, which is the new investment required for the venture to develop. As he knows that he may be hit by a liquidity shock at this period, the first investor negotiates with the entrepreneur at date 0 a repurchase value of his investment noted P_I or P_B. It is supposed that this repurchase value, proposed to the second VC firm at date 1, is paid at the same time as this VC firm pays I_1 : if this condition were not true, the new investor could liquidate the firm at the same time while being informed of the state of the project. Furthermore, it is supposed the liquidation value L is lower than continuation cash-flow $q\,R$ when the state of the project is bad. Lastly, in return of his investment, this new investor is allocated in the initial contract a share of the capital noted δ which gives him rights on the final cash-flow if the project is bring to its completion.

In the case the liquidity shock does not occur, the first investor remains in the venture but is still unable to refinance it. A new VC investor is called upon in order to refinance and pay I_1. In return of their investment, the first and second investors are allocated rights on the final cash flow respectively noted δ_1 and δ_2 if the project is not liquidated and then continued at period 1.

According to his signal on the state of the project, the initial investor decides either to continue the project, in which case the second VC firm is solicited in order to accept the terms of the initial contract, or to liquidate. I suppose that in the case of a first I_0 financing from an VC firm affiliated to a banking network, this firm calls upon another firm belonging to the same network. In this case, the signal observed by the first investor (i.e. the state of the project) is shared with the second. This last assumption is coherent with the information sharing between the VC firms affiliated to the same banking network. On the other hand, if the first investor is independent, another firm of the same nature will have to reinvest and possibly repurchase its shares, following the conditions established in the initial contract.

I finally suppose that the liquidity shock is supposed to be perfectly observable by all agents, whatever their nature.

To sum up, the contract is made up of the following parameters : L_I or L_B and L_E that defines the liquidation rights of the first investor and the entrepreneur ; δ which are the rights on the final cash-flow of the second investor when the first investor is hit by a liquidity shock and P_I or P_B which are the price paid by the second investor to the first one for his shares ; and if not δ_1 and δ_2 which are the rights on the final cash flow of the first and the second investor when both are present at the end of the game.

I successively study the contract corresponding to first-best solution, the optimal contract signed with an independent investor, and the one signed with an investor affiliated to a banking network.

3.1 First-best solution

The net present value of the project is worth:

$$-I_0+e\,(R-I_1)+(1-e)\,L-c\frac{e^2}{2}.$$

The social optimum corresponds to the situation which maximizes the surplus. Considering the characteristics of the investors, it can be reached only by calling upon an independent VC firm which induces in fact the weakest unit cost of effort. The optimal level of effort is such that:

$$\max_e\; -I_0+e\,(R-I_1)+(1-e)\,L-c_I\frac{e^2}{2}$$

Let us note e^{FB} the solution to this equation. It is necessary to fix a level of effort such as $e^{\mathrm{FB}}=\frac{R-I_1-L}{c_I}$ to maximize the surplus[7]. For consistency, I suppose so that $e^{\mathrm{FB}}<1$:

Assumption 1. $R-I_1-L<c_I$

7. It is supposed that $R-I_1-L>0$ and thus $e^{\mathrm{FB}}>0$.

3.2 Financing by independent venture capitalist

The contract specifies the respective parts of the entrepreneur and of the venture capitalist on the liquidation value L noted L_E and L_I in case of liquidation ; P_I is the price paid by the second investor to repurchase the share of the first when he is obliged to exit the firm and δ is the share of the final income given to the second investor, the remainder is allocated to the entrepreneur ; if the liquidity shock does not occur δ_1 and δ_2 are the respective shares of the final income being allocated to the first and the second investor, $1-\delta_1-\delta_2$ being the share allocated to the entrepreneur. This contract is signed at date 0.

At date 1, the first investor may be hit by a liquidity shock with probability p. It is supposed that this shock is perfectly observable by all agents. It implies that the P_I parameter may be contingent on this event: I note respectively P_{IS} and $P_{\overline{\mathrm{IS}}}$ the price to be paid for the shares of the first investor in case of shock or not. Besides, the first investor is at the same time privately informed of the state of the project (i.e. good or bad) ; the second investor cannot directly learn this information.

The first investor has full control on the firm's future. He has the possibility to close the business in an anticipated way or not. If he is hit by a liquidity shock, he chooses between selling or liquidating, because he has to recover his stake: the VC fund is at the end of its life and has to be closed. If it is not the case, he chooses between selling, liquidating or remaining present in the venture. The second investor can only accept the conditions of the initial contract negotiated at date 0 or not. The entrepreneur will want to maximize his utility, choosing the parameters of the contract so as to. The following table sumps up the possible strategies of the first investor:

	Liquidity shock	No liquidity shock
Good state	Sell / Liquidate	Stay / Sell / Liquidate
Bad state	Sell / Liquidate	Stay / Sell / Liquidate

Given all these assumptions, a perfect bayesian equilibrium may be founded in this game. I show that only two equilibriums are possible. One pooling equilibrium, in which the strategy of the first investor is to sell whatever happens ; and one separate equilibrium in which he sells his stakes all the time except in good state with no liquidity shock, in this case he will prefer to stay.

3.2.1 Pooling equilibrium strategy

A pooling equilibrium is reached if the first investor always chooses the same action. It is the case if he always liquidate or sell the firm. He cannot choose to always liquidate. Suppose the latter is true. He earns a constant payoff which is a share of the liquidation value of the firm L, in return of his first investment I_0. As it is supposed that $L-I_0<0$, he makes losses. The pooling equilibrium in which he always decides to sell the firm is possible if this condition is true[8]:

$$P_I - I_0 \geqslant 0$$

Suppose it is true. Then, the entrepreneur want to maximize his profits solving:

$$\max_{\delta, e, P_I} e\,(1-\delta)\,R + (1-e)\,(1-\delta)\,q\,R - c_I\,\frac{e^2}{2}$$

The second investor cannot directly infer the state of nature by observing the decisions of the first investor. He computes his payoffs by estimating the probability that the state of nature is good. The probability has been previously defined as equal to the the effort of the entrepreneur, i.e. e. He then only invests iff:

$$e\,\delta\,R + (1-e)\,\delta\,q\,R - I_1 - P_I \geqslant 0$$

In this case, the maximization program is the following:

$$\max_{\delta, e, P_I, L_I} e\,(1-\delta)\,R + (1-e)\,(1-\delta)\,q\,R - c_I\,\frac{e^2}{2}$$

$$s.t.\; e \in \operatorname{argmax} e\,(1-\delta)\,R + (1-e)\,(1-\delta)\,q\,R - c_I\,\frac{e^2}{2}$$

$$s.t.\; e\,\delta\,R + (1-e)\;\delta\;q\;R - I_1 - P_I \geqslant 0$$

$$s.t. - I_0 + P_I \geqslant 0$$

The following proposition characterizes the parameters of the optimal contract in the case of a pooling equilibrium strategy, chosen by the first investor. For convenience, I then note contract$_{\mathrm{PE}}$ the contract corresponding to this situation. It is possible for certain amounts of income, which results in adopting the following hypothesis:

Assumption 2. $\left[\frac{R^2}{c_I}\,(1-q)^2 + q\,R\right]^2 \geqslant 4\,\frac{R^2(1-q)^2}{c_I}\,(I_0 + I_1)$

Proposition 1. *When the entrepreneur chooses an independent VC firm, he can sign a first contract whose parameters are such that:*

$$P_I^* = I_0$$

$$\delta^* = \frac{1}{2} + \frac{c_I\,(q\,R - \sqrt{\Delta_i})}{2\,R^2\,p\,(1-q)^2}$$

$$\text{with } \Delta_i = \left[\frac{R^2}{c_I}\,(1-q)^2 + q\,R\right]^2 - 4\,\frac{R^2(1-q)^2}{c_I}\,(I_0 + I_1)$$

If all these conditions relative to the pooling equilibrium are respected, a contract is feasible. This contract is equivalent to the only possible contract when liquidity shock is certain (i.e. with $p = 1$). In this case, the firm is never liquidated which results in inefficient continuations. The first investor always leaves the firm, even if he has the possibility to stay, and earns a constant payoff equal to his initial investment.

3.2.2 Separate equilibrium strategy

A separate equilibrium is reached if the first investor follow a different strategy given the two different states of nature. In this case, the entrepreneur and the second investor updates their beliefs by observing the first investor.

8. It is no longer necessary to make a distinction on the P_I parameter whether liquidity shock occurs or not, since if leads to the same conclusion.

Suppose first that the first investor is hit by a liquidity shock. He chooses between liquidating or selling the venture to the second investor, which give him the respective payoffs L_I and P_I. Suppose that $P_{\mathrm{IS}} < L_{\mathrm{IS}}$. In this case, the first investor always liquidate the firm. But given that $L_{\mathrm{IS}} \leqslant L$ and that $-I_0 + L < 0$, he makes losses. Consequently, he always want to sell the venture to the second investor if the following condition is respected:

$$-I_0 + P_{\mathrm{IS}} \geqslant 0$$

I suppose it true.

Suppose now that the first investor if not hit by a liquidity shock. He then chooses between liquidating, selling the venture to the second investor, or staying. If he chooses not to liquidate, the second investor always has to invest I_1. Suppose that the best strategy for the first investor given the parameters of the contract is to stay when the state of nature is favorable and to sell or liquidate if not. That means taking the following assumptions:

$$\delta_1 R \geqslant \max(P_{I\bar{S}}, L_{I\bar{S}})$$

$$\delta_1 q R \leqslant \min(P_{I\bar{S}}, L_{I\bar{S}})$$

Suppose that $P_{I\bar{S}} > L_{I\bar{S}}$. In this case when the state of nature is unfavorable, the second investor repurchases the shares of the first investor. Then, the second investor only accept to invest if: $\delta_{\bar{S}} q R - I_1 - P_{I\bar{S}} \geqslant 0$. If we suppose that $L_{I\bar{S}} = L$ (to be checked afterwards), it cannot be true given that it is supposed that $q R - I_1 - L \leqslant 0$. In this case the first investor always liquidate when the state of nature is unfavorable since the second investor would not accept to invest. Consequently, $P_{I\bar{S}} < L_{I\bar{S}}$ and the second assumption can be rewritten as $\delta_1 q R \leqslant L_{I\bar{S}}$.

Given that we know that the first investor only sell his shares when liquidity shock occurs, it is no longer necessary to distinguish the parameters. The separate equilibrium needs then the following conditions:

$$\delta_1 R \geqslant L_I$$

$$\delta_1 q R \leqslant L_I$$

The entrepreneur takes into account when the venture is liquidated and when it is continued and calculates his profit in the following way:

$$e(p(1-\delta)R + (1-p)(1-\delta_1-\delta_2)R) + (1-e)(p(1-\delta)qR + (1-p)(L-L_I)) - c_I\frac{e^2}{2}$$

that could be rewritten as:

$$p(1-\delta)R(e+(1-e)q) + (1-p)[e(1-\delta_1-\delta_2)R + (1-e)(L-L_I)] - c_I\frac{e^2}{2}$$

When the first investor liquidates, he makes losses since $L_I < I_0$. He thus has to recover these losses elsewhere to be incited to invest. Its participation constraint, according to the assumptions taken, is thus the following one:

$$p P_I + (1-p)(e\,\delta_1 R + (1-e)L_I) - I_0 \geqslant 0$$

The second investor will accept to invest in the venture if the following condition is true:

$$p(e\,\delta R + (1-e)\,\delta q R - P_I - I_1) + e(1-p)(\delta_2 R - I_1) \geqslant 0$$

While updating its beliefs, the second investor gets some information. He observes that i) when the liquidity shock occurs, the first investor always sell the firm, providing no information on the state of nature, and ii) if it does not occur, the first investor stay is state is good and liquidate if state is bad. The second investor will then ask:

$$e\,\delta\,R+(1-e)\,\delta\,q\,R-P_I-I_1\geqslant 0 \text{ if the shock occurs} \tag{1}$$

$$\delta_2\;R-I_1\geqslant 0 \text{ if not} \tag{2}$$

If these two constraints are satisfied, then the profitability constraint of the second investor is satisfied, it is thus redundant. If these conditions were not observed, he would prefer doing nothing and would oblige the first to liquidate with losses.

The maximization program is thus the following:

$$\max_{e,\delta,\delta_1,\delta_2,P_I,L_I}\; p\,(1-\delta)\,R\,(e+(1-e)\,q)+(1-p)\,[e\,(1-\delta_1-\delta_2)\,R+(1-e)\,(L-L_I)]-c_I\,\frac{e^2}{2}$$

$s.t. e\in\operatorname{argmax} p\,(1-\delta)\,R\,(e+(1-e)\,q)+(1-p)\,[e\,(1-\delta_1-\delta_2)\,R+(1-e)\,(L-L_I)]-c_I\,\frac{e^2}{2}$

$$s.t. P_I>L$$

$$s.t. \delta_1\,R\geqslant L_I$$

$$s.t. \delta_1\,q\,R\leqslant L_I$$

$$s.t. p\,P_I+(1-p)(e\,\delta_1\,R+\,(1-e)\,L_I)-I_0\geqslant 0$$

$$s.t. e\,\delta\,R+(1-e)\,\delta\,q\,R-P_I-I_1\geqslant 0$$

$$s.t. \delta_2\;R-I_1\geqslant 0$$

The following results characterize the parameters of the optimal contract. It is possible for certain amounts of income, which results in adopting the following hypothesis:

Assumption 3. $\forall p\in[0;1],\left[\frac{R\,p\,(1-q)^2+R\,(1-p)\,(1-q)\,(R-I_1-L)}{c_I}+q\right]^2\geqslant 4\,\frac{p\,(1-q)^2}{c_I}\left[\frac{I_0-L\,(1-p)}{p}+I_1\right]$

Proposition 2. *When the entrepreneur chooses an independent VC firm, he may sign a contract (noted* contract$_{\mathrm{SE}}$*) whose parameters are such that:*

$$\delta_1^*=\frac{L}{R}$$

$$\delta_2^*=\frac{I_1}{R}$$

$$P_I^*=\frac{I_0-L\,(1-p)}{p}$$

$$\delta^*=\frac{1}{2}+\frac{(1-p)(R-I_1-L)}{2\,R\,p\,(1-q)}+\frac{c_I\,(q\,R-\sqrt{\Delta_i})}{2\,R^2\,p\,(1-q)^2}$$

with $\Delta_i=\left[\frac{R^2\,p\,(1-q)^2+R\,(1-p)\,(1-q)\,(R-I_1-L)}{c_I}+q\;\;R\right]^2+4\,\frac{R^2\,p\,(1-q)^2}{c_I}\left[\frac{L\,(1-p)-I_0}{p}-I_1\right]$

$$L_I^*=L$$

In this case, the first investor takes better continuation decision. When he is hit by a liquidity shock, he always decide to sell the firm whatever the state of nature is, as in the previous case. But unlike the pooling equilibrium case, when liquidity shock does not happen, he liquidates when the state of nature is bad.

3.2.3 Comparison of the two cases

The next proposition study how the entrepreneur decides between the two contracts, corresponding to the pooling or to the separate equilibrium cases.

Proposition 3. *The entrepreneur will prefer* contract$_{\mathrm{SE}}$ *if one of the two following conditions is respected:*

I. $\varphi(e_{\mathrm{PE}}) \leqslant 0$

II. $\varphi(e_{\mathrm{PE}}) \geqslant 0$ *and* $\Delta_f \geqslant 0$ *and* $e_{\mathrm{SE}} \geqslant \frac{R - L - I_1 + p(-qR + L + I_1) - \sqrt{\Delta_f}}{c_I}$

with $\Delta_f = [R - L - I_1 + p(-qR + L + I_1)]^2 - 2c_I\, \varphi(e_{\mathrm{PE}})$

and $\varphi(e_{\mathrm{PE}}) = e_{\mathrm{PE}}\, R + (1 - e_{\mathrm{PE}})\, q\, R - c_I \frac{e_{\mathrm{PE}}^2}{2} - (1 - p)(L + I_1) - p\,(q\,R)$

The entrepreneur chooses between these two possible contracts. Their only difference is when the first investor is not hit by a liquidity shock and when the state of nature is bad: under contract$_{\mathrm{SE}}$, the venture is liquidated with profit L whereas in the contract$_{\mathrm{PE}}$ the project is continued with a lower profit $q\ R - I_1$. The entrepreneur gains nothing when the venture is liquidated under contract$_{\mathrm{SE}}$, and a part of uncertain profits under contract$_{\mathrm{PE}}$. Thus he may provide more effort under the first one because it is more attractive for him to work. Under contract$_{\mathrm{SE}}$ in this case the continuation decision follows first-best, contrary to the other contract in which it is impossible to incite the first investor to liquidate, generating inefficiency costs. More generally, the less frequently the first investor is hit by a liquidity shock (i.e. p is low), the more the first investor's continuation decisions follows first-best with contract$_{\mathrm{SE}}$. This contract is equivalent to first-best when $p = 0$. With contract$_{\mathrm{PE}}$, the situation is equivalent to the case in which the liquidity shock is certain, because the first investor always leaves the firm and the second investor cannot infer the state of nature from his information on the liquidity shock. In this case there are always inefficient continuations when state of nature is bad at interim period.

The proposition sums up how the entrepreneur chooses between the two contract in order to maximize its own profit. The first criteria is given by the function φ. It is negative if the the maximum profits in the contract$_{\mathrm{PE}}$ (i.e. with all shares) minus the cost of effort are lower than the efficiency gains of contract$_{\mathrm{SE}}$: $q\ R$ with probability p, and $L + I_1$ (liquidation value plus the opportunity gain of non-continuation decision) with probability $1 - p$. In this case, the contract$_{\mathrm{SE}}$ is always the best solution: the efficiency gains of this contract always compensate on average. Suppose now this function φ is positive. Then two conditions are required so that the contract$_{\mathrm{SE}}$ is the best choice. It is likely to be the best contract for the entrepreneur if φ or the unit cost of effort c_I are low. The level of effort in the contract$_{\mathrm{SE}}$ should be higher than a threshold depending on these parameters. The next corollary compares the two levels of effort obtained.

Corollary 4. *The effort with* contract$_{\mathrm{SE}}$ *is higher iff:* $(q\,R - I_1 - L)\,(1-p) + R\,(1-q)\,(-p\,\delta_{\mathrm{SE}} + \delta_{\mathrm{PE}}) \geqslant 0$.

Under the two contracts, the entrepreneur recovers the whole NPV created. This corollary is useful because if the level of effort provided under contract$_{\mathrm{SE}}$ is under first-best and superior or equal to the effort under contract$_{\mathrm{PE}}$, the entrepreneur will prefer the former because under the former the NPV obtained is then superior. Under this contract, when the first investor is not hit by a liquidity shock and the state of nature is bad the social payoff is equal to the liquidation value. With contract$_{\mathrm{PE}}$, the payoff is worth $q\,R - I_1$. As by hypothesis $q\,R - I_1 < L$, the entrepreneur chooses the first contract.

The corollary describes the condition so that the effort under contract$_{\mathrm{SE}}$ is better. To do so, it is necessary that $-p\,\delta_{\mathrm{SE}} + \delta_{\mathrm{PE}}$ may be as high as possible. It is the case is the second investor is asking more shares with liquidity shock of the first investor in a significant way under contract$_{\mathrm{PE}}$ and if liquidity shocks do not occur too frequently.

3.2.4 Comparison with first-best

The next corollary compares the level of effort when financing by independent VC firm is chosen and when first-best occurs. For convenience, I consider the case of contract$_{\mathrm{SE}}$ is chosen. To compare with contract$_{\mathrm{PE}}$, one has to set $p = 1$.

Corollary 5. *The entrepreneur provides the following level of effort :*

$$e^* = \frac{p\,(1-q)\,R}{c_I}\left(\frac{1}{2} - \frac{(1-p)(R - I_1 - L)}{2\,R\,p\,(1-q)} - \frac{c_I\,(q\,R - \sqrt{\Delta_i})}{2\,R^2\,p\,(1-q)^2}\right) + \frac{(1-p)\,(R - I_1 - L)}{c_I}$$

This effort is lower than first-best if:

$$I_1 + L < q\,R + \delta^*\,(1-q)\,R$$

As previously explained inefficient continuations of project occur when state is unfavourable if the first investor decides not to liquidate: in this case the total output $q\,R - I_1$ obtained is lower than the one that could be obtained if the first investor had decided to liquidate (i.e. L), as supposed. Given that the first investor may be better off by selling its shares than liquidating, he may prefer to continue the project and sell. The inefficiencies are the more frequent under contract$_{\mathrm{PE}}$ which is equivalent to the situation in which $p = 1$. In this case, the first investor never liquidate. He thus takes always inefficient continuation decisions when the state of nature is bad. and liquidity shock happens when contract$_{\mathrm{SE}}$ is chosen.

The situation differs when the entrepreneur chooses contract$_{\mathrm{SE}}$. When the first investor remains in the firm, he always take optimal continuation decisions and the entrepreneur provides the first-best effort corresponding to this particular case. If the probability of liquidity shock was equal to zero, this financing would join first-best. This contract is consequently more efficient than contract$_{\mathrm{PE}}$ because unlike it when the shock does not occur the first investor always takes efficient continuation decisions.

Depending on the gain obtained by more frequent optimal continuations, compared to the gains in the second contract, the entrepreneur chooses between these two contracts. The entrepreneur chooses contract$_{\mathrm{SE}}$ if the gains it permits by more efficient liquidation decisions are sufficiently higher than what he would gain at best in the contract$_{\mathrm{PE}}$.

The corollary study the conditions of providing more effort than first-best. Given that sometimes inefficient continuations occur, the entrepreneur could be incited to provide too much effort, since he obtains rights in unfavourable states of nature with liquidity shock on an expected value $q\,R$ lower than in first-best effort (liquidation value). However, the rights he obtains on the final cash flow are not always sufficient to incite him to provide too much effort. Indeed, we known that $q\;R\;-\;I_1\;<\;L$. For the entrepreneur not being led to provide too much effort, it is firstly necessary that he obtains a sufficiently weak share of the final cash-flow. In addition, it is impossible if the amount necessary to reinvest or the liquidation value is too important, because that increases the share of the income final required by the new investor, and thus reduces the entrepreneur's potential payoff in the cases of favorable state. Then, if the probability of success while the project seems to badly starts q is sufficiently weak, it also reduces the risk of providing too much effort.

3.3 Financing by a venture capitalist affiliated to a banking network

The final cash-flow is also verifiable here. As previously, the contract specifies the value of the liquidation rights held by the first investor here noted as L_B, in case of a decision of anticipated liquidation. It also fixes the share of the final cash-flow being allocated to the second investor noted δ and the price of repurchase of the initial investment noted P_B, in case of the first investor exits during interim period. In case he does not exit, the contract specifies the shares of the final cash-flow allocated to the first and second investor noted δ_1 and δ_2.

During the interim period, one state of nature is reached. Besides, the first investor may be hit by a liquidity shock with probability p. If he chooses not to liquidate, he always has to find another firm to make the second investment I_1 in the venture. As previously supposed, when the shock happens he has to early exit the firm, by selling its shares to a new investor. But unlike the independent VC firm, the affiliated firm is supposed to always asks to another firm within its network to finance the new investment and repurchase its shares if necessary. It implies that here the second investor perfectly observes the state of nature while repurchasing, thanks to information sharing. He also knows if the first firm is really the victim of a liquidity shock.

There are thus four possible cases, since there are two states of nature, and that the first investor could be hit by a liquidity shock or not.

Let us suppose firstly that in $t=1$, the state of the project is favorable.

Suppose the first investor is hit by a liquidity shock[9]. He then chooses between selling or liquidating. If he liquidated, he would make losses, because $I_0 > L$. He will prefer to continue the project if he wins more than when he liquidates. It is thus necessary that:

$$P_B \geqslant L_B$$

The second investor is then encouraged to invest only if :

$$\delta R - P_B - I_1 \geqslant 0$$

So we have $P_B \geqslant L_B$ and $\delta \geqslant \frac{P_B + I_1}{R}$.

If the first investor is not hit by a liquidity shock, he remains in the firm only if it is more profitable than to liquidate or to sell, that means:

$$\delta_1 R - I_0 \geqslant P_B - I_0$$

and

$$\delta_2 R - I_1 \geqslant 0$$

so that the second investor is incited to invest. It involves that $\delta_1 \geqslant \frac{P_B}{R}$ and $\delta_2 \geqslant \frac{I_1}{R}$.

Suppose now that in $t=1$, the state of the project is unfavorable.

If the liquidity shock occurs, as previously, the second will refuse to invest, because he should pay $P_I \geqslant L_I$ and would gain $\delta q R - P_I - I_1 < \delta\, q R - L - I_1 < 0$ if we suppose that $L_B = L$, to be checked afterwards in appendix.

Consequently, the first investor liquidates and makes losses because it gains $L_I - I_0 \leqslant 0$, which he will want to compensate for elsewhere and the second investor does not do anything.

If the shock of liquidity does not occur, the second invests only if $\delta_2\, q R - I_1 \geqslant 0$, or $\delta_2 \geqslant \frac{I_1}{q R}$. The first investor will want to continue the project only if it is more profitable for him than to liquidate, i.e. $\delta_1\, q R - I_0 > L$ (since $L_B = L$), which involves that $\delta_1 \geqslant \frac{L + I_0}{q R}$. However it involves $\delta_1 + \delta_2 \geqslant \frac{L + I_0 + I_1}{q R}$. However we have $\frac{L + I_0 + I_1}{q R} > 1$ given that $q R - I_1 - L < 0$. Thus as $\delta_1 + \delta_2 \leqslant 1$ it is not possible. Consequently the first investor always liquidates. He thus make losses because he gains $I_0 - L \leqslant 0$ and will seek to compensate for it elsewhere. That means he will ask:

$$e\,(p\, P_B + (1-p)\, \delta_1 R) + (1-e)\, L_B - I_0 \geqslant 0$$

In conclusion, when the state is favorable the first investor never liquidates, and when the state is unfavorable he always liquidates.

The entrepreneur takes into account how the first investor behaves. He thus calculates his profit in the following way:

$$e\,(p\,(1-\delta)\, R + (1-p)\,(1-\delta_1-\delta_2)\, R) + (1-e)\,(L - L_B) - c_B\, \frac{e^2}{2}$$

9. It is no longer necessary here to make contingent the parameters of the optimal contract on the realization of the liquidity shock, because *in fine* such parameters would not be different because of the equilibrium strategy.

Lastly, so that the first investor recover his investment, he will invest only if:

$$e\,(p\,P_B + (1-p)\,\delta_1\,R) + (1-e)\,L_B - I_0 \geqslant 0$$

The maximization program is thus the following:

$$\max_{\delta, e, L_B, P_B, \delta_1, \delta_2} e\,(p\,(1-\delta)\,R + (1-p)\,(1-\delta_1-\delta_2)\,R) + (1-e)\,(L-L_B) - c_B\,\frac{e^2}{2}$$

$$s.t.\, e \in \operatorname{argmax} e\,(p\,(1-\delta)\,R + (1-p)\,(1-\delta_1-\delta_2)\,R) + (1-e)\,(L-L_B) - c_B\,\frac{e^2}{2}$$

$$s.t.\, P_B \geqslant L_B$$

$$s.t.\, \delta\,R - P_B - I_1 \geqslant 0$$

$$s.t.\, \delta_1\,R \geqslant P_B$$

$$s.t.\, \delta_2\,R - I_1 \geqslant 0$$

$$s.t.\, e\,(p\,P_B + (1-p)\,\delta_1\,R) + (1-e)\,L_B - I_0 \geqslant 0$$

As previously, the contract is possible only if the final revenue is sufficiently high:

Assumption 4. $R \geqslant 2\sqrt{c_B\,(I_0 - L)} + I_1 + L$

Proposition 6. *When the entrepreneur chooses an affiliated VC firm, he signs a contract such that:*

$$\delta_1^* = \frac{P_B^*}{R}$$

$$\delta_2^* = \frac{I_1}{R}$$

$$\delta^* = \delta_1^* + \delta_2^* = \frac{R + L + I_1 - \sqrt{(R - I_1 - L)^2 + 4\,c_B\,(L - I_0)}}{2\,R}$$

$$P_B^* = \frac{R - I_1 + L - \sqrt{(R - I_1 - L)^2 + 4\,c_B\,(L - I_0)}}{2}$$

$$L_B^* = L$$

Corollary 7. *The entrepreneur provides an effort which is worth* $e^* = \frac{R - I_1 - L + \sqrt{(R - I_1 - L)^2 + 4\,c_B\,(L - I_0)}}{2\,c_B}$. *It is lower than first-best.*

Contrary to the financing by independent venture capitalist, the essential advantage is that the venture is systematically liquidated when the state of the project is unfavourable as in first-best. The only difference compared to the first-best situation holds in the larger cost of the effort because $c_B > c_I$, consequently the effort of the entrepreneur is limited by the lower productivity of the support provided by the bank-affiliated investor. Consequently, the effort provided is systematically under first-best. Besides, the value of repurchase required here should then be lower than the one with a financing by independent venture capitalist: an affiliated firm obtains all the liquidation value of the venture if the state is unfavourable, whether liquidity shock happens or not.

By sharing information within the banking network, bank-affiliated VC firms make it possible to take more efficient decisions, which compensates for their least expertise.

3.4 Comparison of the two financings

In this part, I compare successively the levels of effort, the shares held by investors and the NPV created by the two types of financing, in order to determine wich system is more effective and then be able to make empirical predictions. In appendix, numerical examples are provided with the two cases of a financing more attractive with an independent company and with an affiliated company.
For simplicity, I only consider the case of an independant VC financing with contract$_{\mathrm{SE}}$, wich is more general. To compare with contract$_{\mathrm{PE}}$ with independant VC firm, one has to set $p = 1$. I distinguish parameters relative to an independant or a bank-affiliated financing by respectively indicing them as I or B.

3.4.1 Effort

Proposition 8. *The entrepreneur provides more effort with the financing of an affiliated firm if and only if:*

$$\delta_I > 1 + \frac{(1-p)(R - I_1 - L)}{p\,(1-q)R} - \frac{(1-\delta_B)\,c_I}{c_B\, p\,(1-q)} \tag{3}$$

While financing with an independant VC firm, the entrepreneur provides an effort which is boosted by a more efficient support from the investor that reduces its cost. However, if the entrepreneur chooses an affiliated firm, his effort is encouraged by the fact that he obtains nothing in the bad states of nature which makes shirking less attractive, contrary to the independant financing when liquidity shock happens.
The equation (3) sums up how the entrepreneur arbitrages between these two types of financing. The intuition of the result is the following: the more important is the probability of liquidity shock, the more attractive if the financing by affiliated VC firm, because it makes too costly the inefficiencies of continuation decisions with an independant VC firm. Besides, the more efficient is the support provided by the affiliated investor compared to the independant, the more the effort provided joins the effort with an independant investor. At the optimum, the ratio $\frac{c_I}{c_B}$ mesuring the comparative efficiency of the two support should be then as high as possible, which is only possible if c_B mesuring the unit cost of effort with an affiliated investor is as close as possible from c_I. Lastly, the more important are the shares required δ_I required with early exit of the first investor, the more important would be the effort provided with the affiliated investor.

3.4.2 Shares of capital held by investors

Proposition 9. *When the first investor is not hit by a liquidity shock, his shares are higher when he is affiliated whereas the shares held by the second investor are equal under the two financings.*

When the first investor is hit by a liquidity shock, the second independant investor hold more shares than the second affiliated if:

$$e_I < \frac{(1-p)\, e^{\mathrm{FB}}\, R + e_B\, c_B\, p\, (1-q)}{c_I} \tag{4}$$

If the entrepreneur chooses an independant VC firm, inefficient continuations of the project occur when the first investor is hit by a liquidity show (with probability p) and when the state of nature is bad. With the other financing, the affiliated firm always takes efficient continuation decisions but offers a more costly support. It implies that if both investors were offering the same level of support efficiency, the shares held by the second independant investor when the first investor exits should always be more important so that he breakevens and compensates for his losses in bad states of nature.
When the first investor is not hit by a liquidity shock, his rights on the final cash flow are higher when he is affiliated because in this case there is no information asymetry relative to the quality of the project. These rights are equal to the opportunity gain of taking the decision to continue. With an independant firm, it is worth the liquidation value of the firm (in the separate equilibrium case, contrary to the pooling equilibrium case) whereas with an affiliated firm it is the potential repurchase price of its investment. Lastly, the second investor earns its investment value under both financings.
When the first investor is hit by a liquidity shock, things are more complicated. Several factors explain why the independant firm will ask more shares of the capital. Firstly, if the effort provided by the entrepreneur e_I is too low compared to the effort provided with an affiliated firm e_B. It indeed reduces the probability that good states of nature and high revenus occur. Secondly, if the probability that liquidity shock is too high: given that the last investor make losses in this case when the entrepreneur shirks, he will want to recover these losses thanks to the revenus generated by good states of nature. When the liquidity shock does not happen (with probability $1-p$), he breakevens and the entrepreneur provides first-best effort (i.e. e^{FB}). Furthermore, if the support provided by the independant investor is not sufficiently higher than the one provided by the affiliated investor (i.e. the ratio $\frac{c_B}{c_I}$ in the formula should be as low as possible). It is also the case if the probability that the project generates high revenues when the entrepreneur shirks q is low because it makes all the more important the expected losses when the second investor remains as sole investor at the end of life of the venture.

Corollary 10. *When the first investor is hit by a liquidity shock, the reinvesting independant firm always holds less shares than the reinvesting affiliated firm if* $c_I < \frac{(L+I_1)\,(1-q)^2}{q} - \frac{(1-p)(1-q)\,(R-I_1-L)}{p\,q}$.

There is a threshold of unit cost of effort c_I under which the independent VC firm is always asking less shares. This shreshold depends on several factors. Firstly, c_I should be small relatively to the amount to reinvest and the liquidation value: this sum $L+I_1$ which appears is the cost of the continuation, including the opportunity cost of continu-

ation L. At the same time, the NPV of the decision to continue the venture $R - I_1 - L$ when it is optimal to do so should not be too high. Then, the probability of success in bad states of nature q should also not be too high, because it makes too attractive for the entrepreneur the bad states, reducing its effort ; it is then less likely that the independant VC firm would ask less shares. Then, the probability p that the firm investor is hit by a liquidity shock may be high relatively to this unit cost of effort c_I: the effect of the good support prevails over the drawbacks of bad continuation decisions.
These differences of shares could be empirically checked from the comparison of implicit evaluation of investment projects by VC firms: the lower is the estimated value of the venture, the more shares the investor obtains.

3.4.3 NPV

Proposition 11. *The NPV created under a financing by an affiliated VC firm is more more important than the one created under an independant VC financing if the two following conditions are both respected:*

I. $R - L - I_1 > \sqrt{2\, c_B\, \gamma(e_I)}$

II. $e_B \geqslant \frac{R - I_1 - L - \sqrt{(R - I_1 - L)^2 - 2\, c_B\, \gamma(e_I)}}{c_B}$

with $\gamma(e_I) = p\,(e_I\, R + (1 - e_I)\, q\, R - I_1 - L) + (1 - p)\, e_I\, (R - I_1 - L) - c_I\, \frac{e_I^2}{2}$

This result is important since given that all participation constraints are binding at the optimum, the whole NPV is given back back to the entrepreneur. Comparing the NPV of two financing means then forecasting the choice of financing by the entrepreneur. He will choose the venture capitalist that offers him the maximum of NPV. It was previously showed that in the case of an independant VC financing, some inefficient continuation decisions are taken whereas with an affiliated VC financing, optimal continuation decisions are always taken but effort is more costly.
This proposition is made of several elements. First of all the function $\gamma(e_I)$; it is composed by i) the expected NPV at period 1 created by the systematic decision not to liquidate the project with an independant VC financing and liquidity shock occurs ii) the expected NPV in good states of nature of the decision to continue the project and liquidity shock does not occur iii) minus the cost of corresponding effort for the entrepreneur. It is the expected gains when continuation decision is taken minus the cost of effort of the entrepreneur. The lower this expected NPV is, the more attractive is the affiliated VC financing.
Two conditions must be satisfied so that the entrepreneur is better off by choosing an affiliated VC firm. The first condition requires that $R - I_1 - L$, i.e. the whole NPV of the continuation decision in good states of nature, must be high enough compared to the unit cost of effort with affiliated VC financing, and $\gamma(e_I)$. The second condition requires

that the level of effort provided e_B must be higher than a threshold. For a given level of effort e_B, the lower $\varphi(e_I)$ is, the more attractive is the affiliated VC financing.

An important thing may be lastly noted. Suppose that $p=0$, i.e. the probability of liquidity shock of the independant firm is zero. Then it is impossible that the affiliated VC financing is the best solution, because with the independant VC financing we join first-best. Finally, the higher p is, the more likely the affiliated financing is the best solution. The essential tradeoff of the model between the two financing is highlited here, mainly when the cost of non-liquidation decision is compensated by a less productive effort or not. My model shows that the main criteria of decision is the level of the NPV of the decision to continue the venture minus the cost of effort with independant financing. The lower it is, the more attractive is the affiliated VC financing.

3.5 Empirical predictions and discussion

Several tests may be realized in order to check the results of the model. Firstly, ventures financed from affiliated firms, or independant firms with long term objectives (i.e. organized as firm) should more often be liquidated. Besides, the shorter the fund life is, the less often it should liquidate. Secondly, the affiliated or long-term VC firms should prefer using convertibles or similar covenant, because it allows them to force liquidation and recover the whole liquidation cash-flow. Then, the riskier projects may be more valued by independent VC firms who will ask less shares. Lastly, high cash-flow projects should more be financed from independent firms, and the affiliated firms more concentrated on the projects with low or less sophisticated cash-flows, the projects whose monitoring requires less technical skills, or less distant geographically. Independant firms should also be more present in the financing of firms whose liquidation value of assets is low: firms endowed with specific assets, service activities, or those with strong human capital. Such results may be checked from two samples of ventures: firms financed from affiliated or long term VC firms on one hand, and firms financed from VC funds with short term objectives on the other hand.

Futhermore, some results in the litterature are close to mines. Kaplan and Schoar (2005) study the funds organized as partnership which prevail in the United States in private equity industry. They show that the performance of the manager of these funds tends to persist in time, which is unfavourable to new entrants. Hsu (2004) show that the best funds obtain weaker valorizations, which supports the fact that the monitoring performances are integrated in the evaluation of VC offers by the entrepreneur. He accepts to reduce his share to obtain a better support. Our model highlights this fact, however it does not suggest that the investor may extract rents, the entrepreneur recovering the whole NPV. My model suggests then that the best way of financing is an independant VC firm with a low probability to face a liquidity shock. It suggests that if the fund is able to stay for a long time in the venture, he garantees to new financiers that the venture is robust. This result joins Admati and Pfleiderer (1994) who show in their model that the presence of an investor with constant shares over time is optimal. But it depens on the possibility for new financiers to observe the liquidity shock.

In addition, the model explains the great international differences between local actors of the VC industry. Some markets of the venture capital are strongly dominated by bank-affiliated VC firms (for example in France) whereas others are dominated by independent VC firms (for example in the United States). Thus, Black and Gilson (1998) show in their empirical study that in 1994, 40% of the venture capital in France came from firms controlled by commercial banks, whereas in the United States the pension funds are the main contributors of the companies of venture capital. Pension funds give an independence to the general partner, by privileging the organization as funds and limited partnership. According to the French ministry of the economy, in 2001 VC firms in France raised funds to 40% from banks, 11% from insurance companies and only 7% from pension funds. A better understanding of these two types of VC firms makes it possible to assess the impact on the dynamism of this activity, crucial for innovation and growth. More generally, this study joins the general problem of financial intermediation: is it a handicap, or a substitute to the presence of very developed financial markets as in the United States? Black and Gilson (1998) underline that the low dynamism of special stock exchange segments for smaller firms in continental Europe is a handicap for VC investment exits: an exit by IPO is preferred by entrepreneurs, since it enables them to keep the control of the firm, contrary to an exit by selling to a bigger firm[10]. This can be related to the lower weight of VC investments relative to GDP in the European countries compared to the United States, except in United Kingdom, or to the fact that the European market regards LBO as part of the VC activity, unlike the US. However for Black and Gilson, it is not obvious that the American system is more powerful, the institutional differences making it possible to compensate for the difficulty to exit investments. Hellmann, Lindsey and Puri (2004) support that the VC markets dominated by the subsidiaries of bank can appear under-efficient and face the risk to hinder the diffusion of innovations, in addition to the greater difficulty to make IPO. Our study makes it possible to evaluate if the organisational differences are sources of inefficience. It shows that the presence of firms linked to banks in the VC market appears to be an imperfect substitute to independant firms, particularly because the former are less efficient while monitoring more sophisticated projects. But at the same time, since their potential financial base is very important within the network, they take better continuation decisions.

The results also supplement the literature studying the international differences in investments in private equity. Kaplan, Martel et Strömberg (2003) make an empirical study on VC investments in 32 countries, mainly european. They analyze the influence of the local legal status on the contractual clauses of VC investments. They show that the more experienced venture capitalists and those with highest return use contracts whose structure is inspired by American contracts, whatever the local legal status is. This type of contract thus appears optimal, in particular the use of the privileged convertible securities. Lerner and Schoar (2004) refutes this analysis: their study on the

10. Bayar and Chemmanur (2006) model the choice of VC investment exits between IPO or sale to another firm and highligh the conflict between the entrepreneur and the venture capitalist.

investments in private equity shows that the investors tend to adapt their contract to the local legal status (i.e system of the common law, French system, etc). They use a large variety of securities according to the countries; even the firms of American or British origin in their sample follow this behavior. The analysis highlights broader transactions with a better implicit valorization in the countries under the common law. However, the differences in behavior which are observed in these two studies do not take account the nature of shareholders of private equity groups, as Kaplan and al. (2003) recognize it. In this perspective, the study by Mayer, Schoors, and Yafeh (2001) supports that the nature of the investors (banks, insurance companies, firms, funds of pension) in the funds of venture capital involves differences between country. They study data from Germany, Israel, Japan and the United Kingdom. However, the data collected refer to the presence of a type of investor in the capital of funds and not to the amount invested in proportion of the capital of funds by them. Besides United Kingdom use more the system of funds and the financing of those by pension funds. This study shows that the VC firms financed by banks invest in later stages than individual and corporate-backed funds. The former tend also to invest more locally. This can be related with the model developed in this article: the farer the venture is, the more constraining is supporting and monitoring it. Their study could also be extended by taking account of the results of our model. The restricted scope of activity of the VC firms affiliated to banks may explain the international differences in the types of activities financed in venture capital, and a greater specialization of the United States in high-technology compared to Europe.

4 Conclusion

This article studies to what extend the affiliation of the venture capital firms has an influence on the supply and the quality of the choice of financing by entrepreneurs. Independent VC firms provide a better support, whereas bank-affiliated VC firms provide easier refinancing, by sharing information on the quality of projects with new investors found within the banking network.

In this paper, I design a model which highlights the advantages of each of these two ways of financing. In this model, the entrepreneur chooses to finance an early and a later investment from an affiliated or an independant VC firm. Then, the project generates a risky cash-flow at a last third period. At the interim period, the investor has to choose to close the project or not. In the case of continuation, he has to find another firm to reinvest and possibly repurchase its shares, if he is hit by a liquidity shock.

The results show that the bank-affiliated VC firm always acts as first-best in continuation decisions: it liquidates the project in the unfavourable states of nature and let it continue in the favorable states. On the contrary, the independent VC firm follows the same strategy only when the liquidity shock does not happen. That means that it prefers to resell while being hit by a liquidity shock, even in the unfavourable states of nature whereas it would be socially optimal to do so. The entrepreneur then arbitrages between two possible financings which offers on one hand a more efficient support with an independant firm and on the other hand more efficient continuation decisions with an affiliated firm.

Comparing the parameters of the two optimal contract leads to predic several interesting things. The entrepreneur chooses the contract which maximizes its payoff which is equal to the NPV of the project. He determines his choice by comparing the effects of a better support with an independent VC firm, with those of an absence of ineffective continuations with an affiliated company. The VC firms affiliated to a banking networks should finance the least sophisticated projects, with a high potential liquidation value at intermim period. They thus have a broader financial basis that enables them to face the possible hazards. It counterbalances their least expertise, while having a restricted field of intervention. On the contrary, the independant firms should finance more high tech projects, with low anticipated liquidation value. Such resultats may empirically tested by more frequent liquidations from VC firms affiliated to banks, or from VC firms which face low risk of liquidity shock (i.e. with a longer term), by studying a sample of firms financed according to these two types of VC firms.

This analysis may be extented to the international differences between industries of the venture capital dominated by independent or affiliated firms. The restricted sphere of activity of the bank-affiliated VC firms highlited in the model may explain the stronger emergence of high-tech firms in the countries dominated by funds and partnerships, as in the United States, compared to the financings of more traditional activities in France and Germany. It may also explain why LBO is a traditional activity of venture capitalists in these contries, whereas in the US it is not considered as venture capital.

This analysis offers several new perspectives of reaserch. From a theoretical point of view, the model may be extended by including the possibility of strategic exits. Faure-Grimaud and Gromb (2004) introduce into the extensions of their model the assumption that the majority shareholder may decide a strategic exit of the investment (i.e to leave the firm financed without shock of liquidity if it is profitable). This possibility could be included in my model. Lastly, the literature on partnerships in the United States showed that experience is an important skill in this industry and that in this type of organization the managers have little interest so that their partners imply themselves actively in management. If it the good managers are supposed to be a rare resource, it would be also intersting to study how managers are compensated in VC firms affiliated to banks and see if it is as attractive to good managers.

5 Appendix

Proof of proposition 1

In this case, the first investor always prefer to sell wether he is hit by a liquidity shock or not.

The maximization program is as follows:

$$\max_{\delta,e,L_B,P_I,\delta_1,\delta_2} e\,(1-\delta)\,R+(1-e)\,(1-\delta)\,q\,R-c_I\,\frac{e^2}{2}$$

$$s.t.\;e\in\operatorname{argmax} e\,(1-\delta)\,R+(1-e)\,(1-\delta)\,q\,R-c_I\,\frac{e^2}{2}$$

$$s.t.\,e\,(\delta\,R)+(1-e)\,\delta\,q\,R-I_1-P_I\geqslant 0$$

$$s.t.\,P_I-I_0\geqslant 0$$

It is easy to see that this maximization program is the same with $p=1$, i.e. when the probability of the liquidity shock is certain.
The level of effort maximizing the utility of the entrepreneur this function is $e^*=\frac{(1-\delta)\,(1-q)\,R}{c_I}$.
The unequation $P_I\geqslant I_0$ is in fact an equality. Let us suppose that $P_I>I_0$. It is possible to find $\varepsilon>0$ such that $P_I-\varepsilon>I_0$ and the function to be maximized increases: P_I^* should be as low as minimum to improve the entrepreneur's utility because it is negatively correled with the shares held by the second investor δ^*. Consequently we deduce that $P_I^*=I_0$.
We then have the simplified maximization program to solve:

$$\max_{\delta,e,P_I,L_I} e\,(1-\delta)\,R+(1-e)\,(1-\delta)\,q\,R-c_i\,(e^2/2)$$

$$s.t.\;e\in\operatorname{argmax} e\,(1-\delta)\,R+(1-e)\,(1-\delta)\,q\,R-c_i(e^2/2)$$

$$s.t.\;e\,\delta\,R+(1-e)\;\delta\;q\;R-I_1-I_0\geqslant 0$$

We suppose that $e^*<e^{\mathrm{FB}}$ where e^{FB} is the first-best level of effort. Consequently, in order to maximize her payoff, the entrepreneur has to enhance her effort.
The rationality constraint of the second investor is the following: $e\,(1-q)\,\delta\,R+\delta\,q\,R-I_1-I_0\geqslant 0$. At the optimum, it is an equality, because if it were not, one could reduce δ to increase e^* and the function to maximize.
We thus have $e\,(1-q)\,\delta\,R+\delta\,q\,R-I=0$, with $I=I_0+I_1$.
The maximization program may be reduced to:

$$\max_{\delta,e} e\,(1-\delta)\,(1-q)\,R+(1-\delta)\,q\,R-c_I\,(e^2/2)$$

$$s.t.\,e\,(1-q)\,\delta\,R+\delta\,q\,R-I=0$$

Let's replace e by e^* in this expression:

$$\max_{\delta,e}\frac{1}{2}\,\frac{[(1-\delta)\,(1-q)\,R]^2}{c_I}+(1-\delta)\,q\,R \tag{5}$$

$$s.t.\,\frac{[(1-q)\,R]^2}{c_I}\,\delta\,(1-\delta)+\delta\,q\,R-I=0 \tag{6}$$

I fix F as the function to maximize in (5). Let's compute the derivative with δ in the function F:

$$\forall\delta\in[0,1],\,\frac{\delta F}{\delta(\delta)}=-\,\frac{(1-\delta)\,(1-q)^2\,R^2}{c_I}-\mathrm{qR}<0.$$

The function to maximize is decreasing with δ. Consequently, the entrepreneur has to fix δ as low as possible and satisfying the equation (6) in order to maximize F.
I factor (6) by δ and find:

$$\delta^2\left[-\frac{R^2\,(1-q)^2}{c_I}\right]+\delta\left[\frac{R^2}{c_I}(1-q)^2+q\,R\right]-I=0 \tag{7}$$

This inequation describes a polynomial of the second degree of the form $a\,P_B^2+b\,P_B+c$, with $a<0, c<0$ and $b>0$. The discriminant is worth:

$$\Delta_i=\left[\frac{R^2}{c_I}\,(1-q)^2+q\,R\right]^2-4\,\frac{R^2(1-q)^2}{c_I}\,I$$

Let's firstly check that $\Delta_i\geqslant 0$. It is true if we suppose that:

$$\left[\frac{R^2}{c_I}\,(1-q)^2+q\,R\right]^2\geqslant 4\,\frac{R^2(1-q)^2}{c_I}\,I$$

I simplify this expression in the following way:

$$R\geqslant\frac{2}{1-q}\,\sqrt{c_I\,I}-\frac{q\,c_I}{(1-q)^2}$$

There exist two solutions to the equation (7): $\frac{-b-\sqrt{\Delta_i}}{2a}$ and $\frac{-b+\sqrt{\Delta_i}}{2a}$.
As $\sqrt{\Delta_i}\geqslant 0, -\sqrt{\Delta_i}\leqslant\sqrt{\Delta_i}\Longleftrightarrow -b-\sqrt{\Delta_i}\leqslant -b+\sqrt{\Delta_i}\Longleftrightarrow \frac{-b-\sqrt{\Delta_i}}{2\,a}\geqslant\frac{-b+\sqrt{\Delta_i}}{2\,a}$
To maximize his profits, the entrepreneur will want δ as small as possible, respecting $\delta\in[0;1]$.
As $a<0$, I check if $\sqrt{\Delta_i}-b\leqslant 0$. It is true because $-4\,a\,c\leqslant 0,\ b^2-4\,a\,c\leqslant b^2\Longrightarrow\sqrt{b^2-4\,a\,c}\leqslant b$.
The smallest root value is:

$$\frac{-\frac{R^2}{c_I}\,(1-q)^2-q\,R+\sqrt{\Delta_i}}{-2\,\frac{R^2\,(1-q)^2}{c_I}}=\frac{1}{2}+\frac{c_I\,(q\,R-\sqrt{\Delta_i})}{2\,R^2\,(1-q)^2}$$

It is inferior or equal to 1 if and only if:

$$\frac{c_I\,(q\,R-\sqrt{\Delta_i})}{2\,R^2\,(1-q)^2}\leqslant\frac{1}{2}$$

that I simplify in the following way:

$$-c_I\sqrt{\Delta_i}\leqslant R^2\,(1-q)^2-c_I\,q\,R$$

$$\Longleftrightarrow -\sqrt{\Delta_i}\leqslant R\left[\frac{R(1-q)^2}{c_I}-q\right] \tag{8}$$

The right-hand side of this inequation is positive if $\frac{R(1-q)^2}{c_I}-q\geqslant 0\Longleftrightarrow R\geqslant\frac{q\,c_I}{(1-q)^2}$.
Then, this inequaation is always true since its left-hand side is negative and its right-hand side is positive.
These conditions being checked, we thus have:

$$\delta^*=\frac{1}{2}+\frac{c_I\,(q\,R-\sqrt{\Delta_i})}{2\,R^2\,(1-q)^2}$$

Consequently, the parameters of the contract are the following:

$$P_I^* = I_0$$

$$\delta^* = \frac{1}{2} + \frac{c_I\,(q\,R - \sqrt{\Delta_i})}{2\,R^2\,p\,(1-q)^2}$$

$$\text{with } \Delta_i = \left[\frac{R^2}{c_I}\,(1-q)^2 + q\,R\right]^2 - 4\,\frac{R^2(1-q)^2}{c_I}\;I$$

$P_I > L_I$ since $I_0 > L$.

Proof of proposition 2

The optimal effort is calculated such as the derivative of the function to maximize is equal to zero.

Let us set F as the function to be maximized. It is worth:

$$F = p\,(1-\delta)\,R\,(e + (1-e)\,q) + (1-p)\,[e\,(1-\delta_1-\delta_2)\,R + (1-e)\,(L-L_I)] - c_I\,\frac{e^2}{2}$$

Let us compute its derivative with e:

$$\frac{\delta(F)}{\delta(e)} = p\,(1-\delta)\,\,(1-q)R + (1-p)\,[(1-\delta_1-\delta_2)\,R - L + L_I] - e\,c_I$$

The optimal effort e^* thus solves:

$$e^* = \frac{p\,(1-\delta)\,\,(1-q)R + (1-p)\,[(1-\delta_1-\delta_2)\,R - L + L_I]}{c_I}$$

The inequation (2) is equivalent to $\delta_2 \geqslant \frac{I_1}{R}$.

At the optimum, this inequality comes to an equality: let's suppose $\delta_2 > \frac{I_1}{R}$. One can find $\varepsilon > 0$ such that $\delta_2 - \varepsilon > \frac{I_1}{R}$ and the function to be maximized increases. In order to maximize her profits, the entrepreneur will want to fix δ_2 as small as possible; we then conclude that $\delta_2^* = \frac{I_1}{R}$.

From the unequation $\delta_1\,R \geqslant L_I$ we see that $\delta_1 \geqslant \frac{L_I}{R}$. In the same way it is possible to conclude that $\delta_1^* = \frac{L_I}{R}$.

We immediately deduce that:

$$e^* = \frac{p\,(1-\delta)\,\,(1-q)R + (1-p)\,(R - I_1 - L)}{c_I}$$

and

$$F = \frac{[p\,(1-\delta)\,\,(1-q)R + (1-p)\,(R - I_1 - L)]^2}{2\,c_I} + p\,(1-\delta)\,q\,R + (1-p)\,(L - L_I)$$

From the rationality constraint of the second investor (1) we see that $\delta \geqslant \frac{P_I + I_1}{R\,(e\,(1-q)+q\,)}$.

To maximize its utility the entrepreneur will want $\delta^* = \frac{P_I + I_1}{R\,(e\,(1-q)+q\,)}$.

From the rationality constraint of the first investor, we have $P_I \geqslant \frac{(\delta_1\,e\,p - \delta_1\,e)\,R + (1-e)\,L_I\,p + (e-1)\,L_I + I_0}{p}$ that may be easily simplified as $P_I \geqslant \frac{L_I\,(p-1) + I_0}{p}$. In order to maximize its profits, the entrepreneur will negiotiate a contract such that $P_I^* = \frac{L_I\,(p-1) + I_0}{p}$ (because P_I should be as small as possible to reduce δ^* and then the utility of the entrepreneur).

Let us now replace the level of effort by its optimal value in the expression of δ^*:

$$\delta\,(e\,R(1-q)+q\,R)-P_I-I_1\geqslant 0$$

$$\Longleftrightarrow\ \delta^2\left[-\frac{R^2\,p\,(1-q)^2}{c_I}\right]+\delta\left[\frac{R^2\,p\,(1-q)^2+R\,(1-p)\,(1-q)\,(R-I_1-L)}{c_I}+q\ R\right]+\left[\frac{L_I\,(1-p)-I_0}{p}-I_1\right]\geqslant 0$$

One will easily recognize a polynomial function $a\,\delta^2+b\,\delta+c$ whose characteristics are as follows:

$$a=-\frac{R^2\,p\,(1-q)^2}{c_I}<0$$

$$b=\frac{R^2\,p\,(1-q)^2+R\,(1-p)\,(1-q)\,(R-I_1-L)}{c_I}+q\,R>0$$

$$c=\frac{L_I\,(1-p)-I_0}{p}-I_1=-P_I^*-I_1<0$$

The discriminant is set as Δ_i and is worth:

$$\left[\frac{R^2\,p\,(1-q)^2+R\,(1-p)\,(1-q)\,(R-I_1-L)}{c_I}+q\,R\right]^2+4\,\frac{R^2\,p\,(1-q)^2}{c_I}\left[\frac{L_I\,(1-p)-I_0}{p}-I_1\right]=\Delta_i$$

For the contract to be possible it is necessary that the discriminant is positive (if not any solution would be negative), which is true if and only if $b\geqslant 2\sqrt{a\,c}$. I take this as assumption 1.

There are two possible solutions.

As $-\sqrt{\Delta}\leqslant\sqrt{\Delta}$, we have $\frac{-b-\sqrt{\Delta}}{2\,a}\geqslant\frac{-b+\sqrt{\Delta}}{2\,a}$.

The weakest root is:

$$\frac{-b+\sqrt{b^2-4\,a\,c}}{2\,a}\geqslant 0\,\text{since}\,c\leqslant 0$$

Thus:

$$\delta^*=\frac{1}{2}+\frac{(1-p)(R-I_1-L)}{2\,R\,p\,(1-q)}+\frac{c_I\,(q\,R-\sqrt{\Delta_i})}{2\,R^2\,p\,(1-q)^2}$$

We know that

$$F=\frac{[p\,(1-\delta)\ (1-q)R+(1-p)\,(R-I_1-L)]^2}{2\,c_I}+p\,(1-\delta)\,q\,R+(1-p)\,(L-L_I)$$

Let's compute the derivative of F with L_I:

$$\frac{\delta(F)}{\delta(L_I)}=\frac{-[p\,(1-q)\,R]\,[p\,(1-\delta)\ (1-q)R+(1-p)\,(R-I_1-L)]}{c_I}\,\frac{\delta(\delta^*)}{\delta(L_I)}-p\ q\ R\,\frac{\delta(\delta^*)}{\delta(L_I)}-(1-p)$$

The sign of the derivative is negative if if $\frac{\delta(\delta^*)}{\delta(L_I)}>0$.

If we compute that we obtain:

$$\frac{\delta(\delta^*)}{\delta(L_I)} = -\frac{1-p}{p\sqrt{\Delta_i}} < 0$$

$$\frac{\delta(F)}{\delta(L_I)} = \frac{[(1-q)(1-p)R]\,[p(1-\delta)\,(1-q)R + (1-p)(R-I_1-L)]}{c_I\sqrt{\Delta_i}} + \frac{q^2 R^2 (1-p)}{\sqrt{\Delta_i}} - (1-p)$$

A sufficient condition for this derivative to be positive is that $q^2 R^2 \geqslant \sqrt{\Delta_i}$. This last condition is true if R is high enough, that I take as assumption.
Given that this derivative is positive, we have $L_I^* = L$.
The parameters of the optimal contracts are the following:

$$\delta_1^* = \frac{L}{R}$$

$$\delta_2^* = \frac{I_1}{R}$$

$$P_I^* = \frac{L(p-1) + I_0}{p}$$

$$\delta^* = \frac{1}{2} + \frac{(1-p)(R-I_1-L)}{2Rp(1-q)} + \frac{c_I(qR - \sqrt{\Delta_i})}{2R^2 p(1-q)^2}$$

with $\Delta_i = \left[\frac{R^2 p(1-q)^2 + R(1-p)(1-q)(R-I_1-L)}{c_I} + q\;R\right]^2 + 4\frac{R^2 p(1-q)^2}{c_I}\left[\frac{L(1-p)-I_0}{p} - I_1\right]$

$$L_I^* = L$$

The first hypothesis on the first investor's behavior are respected, since:

$$P_I \geqslant L \Longleftrightarrow \frac{L(p-1)+I_0}{p} \geqslant L \Longleftrightarrow \frac{I_0 - L}{p} \geqslant 0$$

$$\delta_1 R \geqslant L_I \Longleftrightarrow L \geqslant L$$

$$L_I \geqslant \delta_1 q R \Longleftrightarrow L \geqslant q L$$

I finally compute the entrepreneur's payoff with this contract. It is worth:

$$F = \frac{[p(1-\delta)\,(1-q)R + (1-p)(R-I_1-L)]^2}{2c_I} + p(1-\delta)\,qR$$

This contract is prefered by the first investor to the contract in which he always sell its stake instead of liquidating, as I then show.

Proof of proposition 3

I study how the entrepreneur decides between the two contracts.
In contract$_{\mathrm{SE}}$, the NPV is worth:

$$p(eR + (1-e)\,qR) + (1-p)(eR + (1-e)(L+I_1)) - I_0 - I_1 - c_I\frac{e^2}{2}$$

that I simplify as:

$$\mathrm{NPV}_{\mathrm{SE}} = e_{\mathrm{SE}} R + (1 - e_{\mathrm{SE}}) \, (p \, q \, R + (1 - p)(L + I_1)) - I_0 - I_1 - c_I \, \frac{e_{\mathrm{SE}}^2}{2}$$

In the second contract, NPV is worth:

$$\mathrm{NPV}_{\mathrm{PE}} = e_{\mathrm{PE}} \, R + (1 - e_{\mathrm{PE}}) \, q \, R - I_0 - I_1 - c_I \, \frac{e_{\mathrm{PE}}^2}{2}$$

Let us compute the differential NPV between the two contracts:

$$\mathrm{NPV}_{\mathrm{SE}} - \mathrm{NPV}_{\mathrm{PE}} = e_{\mathrm{SE}}^2 \left(- \frac{c_I}{2} \right) + e_{\mathrm{SE}} \, (R - p \, q \, R - (1 - p) \, (L + I_1)) + (p \, q \, R + (1 - p)(L + I_1) - e_{\mathrm{PE}} \, R - (1 - e_{\mathrm{PE}}) \, q \, R + c_I \, \frac{e_{\mathrm{PE}}^2}{2})$$

That means searching when the entrepreneur will choose contract$_{\mathrm{SE}}$.
This function describes a polynomial $a \, e_{\mathrm{SE}}^2 + b \, e_{\mathrm{SE}} + c$ whose caracteristics are the following:

$$a = - \frac{c_I}{2} < 0$$

$$b = R - p \, q \, R - (1 - p) \, (L + I_1) = R - L - I_1 + p \, (- q \, R + L + I_1)$$

$$c = p \, q \, R + (1 - p)(L + I_1) - e_{\mathrm{PE}} \, R - (1 - e_{\mathrm{PE}}) \, q \, R + c_I \, \frac{e_{\mathrm{PE}}^2}{2}$$

The discriminant is worth $\Delta_f = b^2 - 4 \, a \, c$.
Suppose that the function described in c is positive. I then study the sign of the two roots of the polynomial.
The first root $\frac{- b - \sqrt{\Delta_f}}{2 \, a}$ is positive. The second root $\frac{- b + \sqrt{\Delta}_f}{2 \, a}$ is negative.
Consequently, as the level of effort are positive, the NPV in the first contract is greater if and only if:

$$e_{\mathrm{SE}} \leqslant \frac{b + \sqrt{\Delta}_f}{c_I}$$

Suppose now that the function described in c is negative.
If $\Delta \leqslant 0$, then it is impossible that the NPV in the contract$_{\mathrm{SE}}$ could be more important.
Suppose now that $\Delta_f \geqslant 0$.
The first root $\frac{- b - \sqrt{\Delta_f}}{2 \, a}$ is positive. The second root $\frac{- b + \sqrt{\Delta_f}}{2 \, a}$ is also positive.
Consequently, the NPV in the first contract is greater if and only if:

$$\frac{b - \sqrt{\Delta_f}}{c_I} \leqslant e_{\mathrm{SE}} \leqslant \frac{b + \sqrt{\Delta_f}}{c_I}$$

I then show that $e_{\mathrm{SE}} \leqslant \frac{b + \sqrt{\Delta_f}}{c_I}$ is always true, whatever is the sign of c.
It means checking if, by replacing e_{SE} by its value:

$$\frac{p \, (1 - \delta) \, (1 - q) R + (1 - p) \, (R - I_1 - L)}{c_I} \leqslant \frac{R - L - I_1 + p \, (- q \, R + L + I_1) + \sqrt{\Delta_f}}{c_I}$$

$$\Longleftrightarrow p \, (1 - \delta) \, (1 - q) R - p \, (R - I_1 - L) \leqslant p \, (- q \, R + L + I_1) + \sqrt{\Delta_f}$$

$$\Longleftrightarrow p \, (1 - \delta) \, (1 - q) R - p \, R \, (1 - q) \leqslant \sqrt{\Delta_f}$$

$$\Longleftrightarrow p \, (- \delta) \, (1 - q) R \leqslant \sqrt{\Delta_f}$$

In this unequality, the left-side member is always negative and the right-side member always positive. Then it is always true. Consequently, $e_{\mathrm{SE}} \leqslant \frac{b+\sqrt{\Delta_f}}{c_I}$ is always true.
In the proposition, I set $\varphi(e_{\mathrm{PE}}) = -c$.

Proof of corollary 4

Let us suppose that the level of effort is the same in the first and in the second contract. I then compare the two NPV in order to predict the entrepreneur's choice:

$$\begin{aligned}\mathrm{NPV}_{\mathrm{SE}}(e) - \mathrm{NPV}_{\mathrm{PE}}(e) &= e\,R + (1-e)\,(p\,q\,R + (1-p)\,(L + I_1)) - e\,R - (1-e)\,q\,R\\ &= (1-e)\,(1-p)\,(-q\,R + L + I_1) > 0 \text{ since } L + I_1 > q\,R\end{aligned}$$

If the effort in the first contract is better, then the function $\mathrm{NPV}_1(e_{\mathrm{SE}}) - \mathrm{NPV}_2(e_{\mathrm{PE}})$ remains positive if we suppose that the entrepreneur does not provide too much effort, implying that the NPV_1 is increasing with the level of its effort.
In the first contract effort is worth $e_{\mathrm{SE}} = \frac{p\,(1-\delta_1)\,(1-q)R + (1-p)\,(R - I_1 - L)}{c_I}$. In the second contract it is worth $e_{\mathrm{PE}} = \frac{(1-\delta_2)\,(1-q)\,R}{c_I}$.
I study when $e_{\mathrm{SE}} \geqslant e_{\mathrm{PE}}$:

$$\begin{aligned}&\Longleftrightarrow \frac{p\,(1-\delta_{\mathrm{SE}})\,(1-q)R + (1-p)\,(R - I_1 - L)}{c_I} \geqslant \frac{(1-\delta_{\mathrm{PE}})\,(1-q)\,R}{c_I}\\ &\Longleftrightarrow p\,(\delta_{\mathrm{PE}} - \delta_{\mathrm{SE}})\,(1-q)R + (1-p)\,(R\,\delta_{\mathrm{PE}}(1-q) + q\,R - I_1 - L) \geqslant 0\\ &\Longleftrightarrow p\,(-\delta_{\mathrm{SE}})\,(1-q)R + (1-p)\,(q\,R - I_1 - L) + R\,\delta_{\mathrm{PE}}(1-q) \geqslant 0\\ &\Longleftrightarrow -p\,[q\,R - I_1 - L + \delta_{\mathrm{SE}}\,(1-q)R] + q\,R - I_1 - L + R\,\delta_{\mathrm{PE}}(1-q) \geqslant 0\\ &\Longleftrightarrow (q\,R - I_1 - L)\,(1-p) + R\,(1-q)\,(-p\,\delta_{\mathrm{SE}} + \delta_{\mathrm{PE}}) \geqslant 0\end{aligned}$$

In this expression $q\,R - I_1 - L < 0$. A necessary condition is $\delta_{\mathrm{PE}} \geqslant p\,\delta_{\mathrm{SE}}$.

Proof of corollary 5

Let us use the results on the parameters of optimal contract in order to determine e^* :

$$e^* = \frac{p\,(1-\delta^*)\,(1-q)R + (1-p)\,(R - I_1 - L)}{c_I}$$

It is easy to see that e^* is positive since $0 \leqslant \delta^* \leqslant 1$. The same argument leads us to notice:

$$\begin{gathered}p\,(1-\delta^*)\,(1-q)R + (1-p)\,(R - I_1 - L) \leqslant p\,(1-q)R + (1-p)\,(R - I_1 - L)\\ p\,(1-q)R + (1-p)\,(R - I_1 - L) = R\,(1 - p\,q) - I_1 - L \leqslant R - I_1 - L\end{gathered}$$

We know that $R - I_1 - L < c_I$. We thus have $\delta^* \leqslant 1$.

I then check if it is possible that the entrepreneur provides too much effort. It means checking the following inequality:

$$e^* < e^{\mathrm{FB}}$$

Let us express this inequality using the value of shares held by the investor. We thus have to solve:

$$\frac{p\,(1-\delta^*)\,(1-q)R+(1-p)\,(R-I_1-L)}{c_I}<\frac{R-I_1-L}{c_I}$$

$$\Longleftrightarrow p\left[\frac{(1-\delta^*)\,(1-q)R-(R-I_1-L)}{c_I}\right]<0$$

$$\Longleftrightarrow I_1+L<q\,R+\delta\,(1-q)\,R$$

$$\Longleftrightarrow \delta>\frac{I_1+L-q\,R}{(1-q)\,R}$$

This expression can also be solved using the value of δ function with the final revenue. The extended value of e is the following:

$$\frac{p\,(1-q)\,R}{c_I}\left(\frac{1}{2}-\frac{(1-p)(R-I_1-L)}{2\,R\,p\,(1-q)}-\frac{c_I\,(q\,R-\sqrt{\Delta_i})}{2\,R^2\,p\,(1-q)^2}\right)+\frac{(1-p)\,(R-I_1-L)}{c_I}$$

Proof of proposition 6

Let us transform some of the constraints. We have $\delta \geqslant \frac{P_B+I_1}{R}$.

At the optimum, this inequality comes to an equality: let us suppose $\delta > \frac{P_B+I_1}{R}$. One can find $\varepsilon>0$ such that $\delta-\varepsilon>\frac{P_B+I_1}{R}$ and the function to be maximized increases. We have consequently $\delta=\frac{P_B+I_1}{R}$

Applying this process to $\delta_1 \geqslant \frac{P_B}{R}$ and $\delta_2 \geqslant \frac{I_1}{R}$, we find $\delta_1=\frac{P_B}{R}$ and $\delta_2=\frac{I_1}{R}$, which implies that $\delta=\delta_1+\delta_2$.

We also have $e\,(p\,P_B+(1-p)\,\delta_1\,R)+(1-e)\,L_B-I_0\geqslant 0$. Since $p\,P_B+(1-p)\,\delta_1\,R=P_B$, this constraint becomes $e\,P_B+(1-e)\,L_B-I_0\geqslant 0$.

Finally, we can transform the maximization program :

$$\max_{\delta,e,L_e,P_B}\; e\,(1-\delta)\,R+(1-e)\,L_E-c_B\,(e^2/2) \tag{9}$$

$$s.t.\, e\in \operatorname{argmax} e\,(1-\delta)\,R+(1-e)\,L_E-c_B(e^2/2)$$

$$s.t.\, P_B\geqslant L_B \text{ avec } L_B=L-L_E$$

$$s.t.\, \delta\,R-I_1-P_B\geqslant 0 \tag{10}$$

$$s.t.\, P_B\geqslant L_B$$

$$s.t.\, e\,P_B+(1-e)\,L_B-I_0\geqslant 0 \tag{11}$$

It is easy to note that this program is the same as with $p=1$, i.e. when the probability for the first investor to exit during the intermediary period is certain.

The incentive constraint of the entrepreneur (9) leads him to an effort such that:

$$e^*=\frac{(1-\delta)\,R-L+L_B}{c_B}$$

We have supposed that $(1-\delta)\,R-L\geqslant 0$. Thus $e^*\geqslant 0$.

Let us suppose $e^*\leqslant e^{\mathrm{FB}}$.

Consequently, the entrepreneur, in order to maximize his profit, will increase his effort.

Let us replace e by its computed value in the constraint of participation of the first investor (11).

$$\frac{(1-\delta)\,R-L_E}{c_B}\,P_B+\left[1-\frac{(1-\delta)\,R-L_E}{c_B}\right]L_B-I_0\geqslant 0$$

Let F be the function to be maximized described by the incentive constraint of the entrepreneur.

Let us show that in order to maximize F, the entrepreneur had to fix δ as low as possible.

By replacing e with its optimal value e^* into (9), we find:

$$F=\frac{(1-\delta)\,R-L_E}{c_B}\,[(1-\delta)\,R-L_E]+L_E-\frac{[(1-\delta)\,R-L_E]^2}{2\,c_B}$$

$$\Longleftrightarrow F=\frac{[(1-\delta)\,R-L_E]^2}{2\,c_B}+L_E=\frac{[(1-\delta)\,R-L+L_B]^2}{2\,c_B}+L-L_B$$

The function to be maximized is decreasing with δ. It is thus necessary to fix δ as small as possible in order to maximize the expected value of the entrepreneur. The inequation (10) can be rewritten in the following way:

$$\delta\geqslant\frac{I_1+P_B}{R}$$

As it was shown that δ was to be as weak as possible to maximize F, we immediately deduce that $\delta=\frac{I_1+P_B}{R}$.

Thus $e^*=\frac{R-I_1-P_B-L+L_B}{c_B}$ and $F=\frac{(R-I_1-P_B-L+L_B)^2}{2\,c_B}+L-L_B$.

Let us rewrite the inequation (11) by replacing e by the computed value:

$$\frac{R-I_1-P_B-L+L_B}{c_B}\,(P_B-L_B)+L_B-I_0\geqslant 0$$

We now solve P_B.

$$-\frac{P_B^2}{c_B}+P_B\left[\frac{R-I_1+2\,L_B-L}{c_B}\right]-L_B\left[\frac{R-I_1-L+L_B}{c_B}-1\right]-I_0\geqslant 0$$

This inequation describes a polynomial of the second degree of the form $a\,P_B^2+b\,P_B+c$, with $a<0, b>0$ and $c<0$.

The function F is decreasing with δ, and as $\delta=\frac{I_1+P_B}{R}$, it is decreasing with P_B. It is thus necessary to find P_B as small as possible, such that $P_B\geqslant L_B$ is respected.

In the P_B expression previously obtained, the discriminant is such that:

$$\Delta_b=\left[\frac{R-I_1+2\,L_B-L}{c_B}\right]^2-\frac{4}{c_B}\left[L_B\left[\frac{R-I_1-L+L_B}{c_B}-1\right]+I_0\right]$$

$$\Longleftrightarrow \Delta_b=\left[\frac{R-I_1+L_B-L}{c_B}\right]^2+\frac{L_B^2}{c_B}+\frac{R-I_1-L+L_B}{c_B}\left[\frac{2\,L_B}{c_B}-\frac{4\,L_B}{c_B}\right]+\frac{4\,(L_B-I_0)}{c_B}$$

$$\Longleftrightarrow \Delta_b=\left[\frac{R-I_1-L}{c_B}\right]^2+\frac{4\,(L_B-I_0)}{c_B}$$

$$\Delta_b\geqslant 0 \text{ if and only if } (R-I_1-L)^2\geqslant -\,c_B\,(4\,(L_B-I_0))$$

The member of right-hand side is positive since $L_B \leqslant L < I_0$. One can then take the square root of each of the two members and obtains:

$$R - I_1 - L \geqslant 2\sqrt{c_B (I_0 - L)}$$

We take this constraint as an hypothesis such that the contract could be possible.
Then, the smallest solution root of the polynomial is $\frac{-b + \sqrt{\Delta_b}}{2a}$. It is positive if $b > \sqrt{\Delta} \Longleftrightarrow -4ac < 0$, which is true.
We thus have:

$$P_B = \frac{R - I_1 + 2L_B - L - \sqrt{(R - I_1 - L)^2 + 4c_B (L_B - I_0)}}{2}$$

Let us rewrite the function to be maximized:

$$F = \frac{(R - I_1 - P_B - L + L_B)^2}{2c_B} + L - L_B$$

$$\Longleftrightarrow F = \frac{\left(R - I_1 - L + \sqrt{(R - I_1 - L)^2 + 4c_B (L_B - I_0)}\right)^2}{8c_B} + L - L_B$$

Let us calculate the derivative with L_B :

$$\frac{\delta(F)}{\delta(L_B)} = \frac{\sqrt{(R - L - I_1)^2 + 4c_B (L_B - I_0)} + R - L - I_1}{2\sqrt{(R - L - I_1)^2 + 4c_B (L_B - I_0)}} - 1$$

$$\Longleftrightarrow \frac{\delta(F)}{\delta(L_B)} = -\frac{1}{2} + \frac{R - L - I_1}{2\sqrt{(R - L - I_1)^2 + 4c_B (L_B - I_0)}}$$

The derivative of F with L_B is thus positive if:

$$\frac{R - L - I_1}{2\sqrt{(R - L - I_1)^2 + 4c_B (L_B - I_0)}} > \frac{1}{2}$$

$$\Longleftrightarrow R - L - I_1 > \sqrt{(R - L - I_1)^2 + 4c_B (L_B - I_0)}$$

The two members of this inequality are positive, and while squaring we find:

$$(R - L - I_1)^2 > (R - L - I_1)^2 + 4c_B (L_B - I_0)$$

$$\Longleftrightarrow 4c_B (L_B - I_0) < 0$$

$$\Longleftrightarrow L_B - I_0 < 0$$

Since $L_B \leqslant L < I_0$, this constraint is always satisfied. Thus the derivative of F with L_B is positive. Consequently, in order to maximize its profit, the entrepreneur will negotiate a contract such that L_B is as high as possible. That thus means that $L_B = L$.
Thus,

$$P_B = \frac{R - I_1 + L - \sqrt{(R - I_1 - L)^2 + 4c_B (L - I_0)}}{2}$$

$$\delta = \frac{R + L + I_1 - \sqrt{(R - I_1 - L)^2 + 4c_B (L - I_0)}}{2R}$$

Let us check that $0 \leqslant \delta \leqslant 1$. This comes to check if:

$$-R-I_1-L \leqslant -\sqrt{(R-I_1-L)^2+4\,c_B\,(L-I_0)} \leqslant R-I_1-L$$

The right-hand side of the inequality is checked since its left term is negative and its right term is positive. Let us rewrite the left-hand side inequality by squaring:

$$(R+L+I_1)^2 \geqslant (R-I_1-L)^2+4\,c_B\,(L-I_0) \Longleftrightarrow R\,(L+I_1) \geqslant c_B\,(L-I_0)$$

This inequality is always true because its left member is positive and the right member is negative (because $L<I_0$).
For the contract to be possible it is necessary that the following assumption is respected:

$$R \geqslant 2\,\sqrt{c_B(I_0-L)}+I_1+L$$

Proof of corollary 7
I firstly calculate :

$$e_B^* = \frac{R-I_1-P_B-L+L_B}{c_B} = \frac{R-I_1-L+\sqrt{(R-I_1-L)^2+4\,c_B\,(L-I_0)}}{2\,c_B}$$

It is easy to see that this expression is positive.
As previously, I study:

$$e_B^* - e^{\mathrm{FB}} < 0$$

$$\Longleftrightarrow \frac{R-I_1-L+\sqrt{(R-I_1-L)^2+4\,c_B\,(L-I_0)}}{2\,c_B} - \frac{R-I_1-L}{c_I} < 0$$

$$\Longleftrightarrow (R-I_1-L)\left[\frac{c_I-2\,c_B}{2\,c_B\,c_I}\right] + \frac{c_I\sqrt{(R-I_1-L)^2+4\,c_B\,(L-I_0)}}{2\,c_B\,c_I} < 0$$

$$\Longleftrightarrow c_I\sqrt{(R-I_1-L)^2+4\,c_B\,(L-I_0)} < (R-I_1-L)\,(2\,c_B-c_I)$$

As both members of this last inequality are positive, one can square them and find, after simplification by c_I:

$$(R-I_1-L)^2+4\,c_B\,(L-I_0) < (R-I_1-L)^2\left[\frac{2\,c_B}{c_I}-1\right]^2$$

$$\Longleftrightarrow (R-I_1-L)^2\left(1-\left[\frac{2\,c_B}{c_I}-1\right]^2\right)+4\,c_B\,(L-I_0)<0$$

$$\Longleftrightarrow (R-I_1-L)^2\,\frac{2\,c_B}{c_I}\left[2-\frac{2\,c_B}{c_I}\right]+4\,c_B\,(L-I_0)<0$$

$$\Longleftrightarrow (R-I_1-L)^2 > -\frac{c_I^2\,(L-I_0)}{c_I-c_B}\ \text{car}\ c_I<c_B$$

The right-hand side member of this inequality is negative since $c_I<c_B$ and $L<I_0$.
Thus this expression is always true, and the entrepreneur never provides too much effort

with this contract. Futhermore, this result implies that $0 \leqslant e^* \leqslant 1$.

Proof of proposition 8

I determine under which conditions the effort provided by the entrepreneur is larger in the case of a financing by VC firm affiliated to a banking network. It is equivalent than testing the following inequality:

$$e_I^* - e_B^* < 0$$

Let us express this inequality according to the shares held by the second investor in case the first exits totally. We thus have to solve:

$$\frac{p\,(1-\delta_I)\;(1-q)R + (1-p)\,(R - I_1 - L)}{c_I} - \frac{(1-\delta_B)\,R}{c_B} < 0$$

$$\Longleftrightarrow \frac{p\,(\delta_I - 1)\;(1-q)R}{c_I} > \frac{(1-p)\,(R - I_1 - L)}{c_I} - \frac{(1-\delta_B)\,R}{c_B}$$

$$\Longleftrightarrow \delta_I - 1 > \frac{(1-p)\,(R - I_1 - L)}{p\;(1-q)R} - \frac{(1-\delta_B)\,c_I}{c_B\,p\;(1-q)}$$

$$\Longleftrightarrow \delta_I > 1 + \frac{(1-p)\,(R - I_1 - L)}{p\;(1-q)R} - \frac{(1-\delta_B)\,c_I}{c_B\,p\;(1-q)}$$

Proof of proposition 9

Let us firstly compare the shares held by investors in the case the first investor remains in the firm. The shares held by the second investor are the same and are worth $\frac{I_1}{R}$.
If the entrepreneur finances with an independant firm the first investor is allocated $\delta_{1,I} = \frac{L}{R}$, whereas with an affiliated firm $\delta_{1,B} = \frac{P_B}{R} = \frac{R - I_1 + L - \sqrt{(R - I_1 - L)^2 + 4\,c_B\,(L - I_0)}}{2\,R}$.

$$\delta_{1,B} - \delta_{1,I} = \frac{R - I_1 - L - \sqrt{(R - I_1 - L)^2 + 4\,c_B\,(L - I_0)}}{2\,R} > 0$$

because $R - I_1 - L > \sqrt{(R - I_1 - L)^2 + 4\,c_B\,(L - I_0)} \Longleftrightarrow 4\,c_B\,(L - I_0) < 0$ which is true.
Consequently the shares held by the second independant investor are less important than the shares held by the affiliated one when the first investor remains in the venture.
Now let us compare the shares held by the second investor when the first exits. I study the case under wich the shares held by the independant VC firm are more important. I

express this inequality according to the level of efforts in the two contracts:

$$\delta_I > \delta_B$$

$$\Longleftrightarrow 1 - \frac{e_I c_I}{p(1-q)R} + \frac{(1-p)(R - I_1 - L)}{p(1-q)c_I} > 1 - \frac{e_B c_B}{R}$$

$$\Longleftrightarrow - \frac{e_I c_I}{p(1-q)R} + \frac{(1-p)(R - I_1 - L)}{p(1-q)c_I} > - \frac{e_B c_B}{R}$$

$$\Longleftrightarrow e_I c_I < \frac{(1-p)(R - I_1 - L)R}{c_I} + e_B c_B p(1-q)$$

$$\Longleftrightarrow e_I < \frac{(1-p)e^{\text{FB}} R + e_B c_B p(1-q)}{c_I}$$

Proof of corollary 10

Let us write the first inequality in the previous proof as a function of the level of revenues:

$$\frac{1}{2} + \frac{(1-p)(R - I_1 - L)}{2Rp(1-q)} + \frac{c_I(qR - \sqrt{\Delta_i})}{2R^2 p(1-q)^2} - \frac{R + L + I_1 - \sqrt{(R - I_1 - L)^2 + 4c_B(L - I_0)}}{2R} > 0$$

$$\Longleftrightarrow \sqrt{(R - I_1 - L)^2 + 4c_B(L - I_0)} < R + \frac{c_I(qR - \sqrt{\Delta_i})}{R(1-q)^2} - (R + L + I_1) + \frac{(1-p)(R - I_1 - L)}{p(1-q)}$$

It is impossible if the right-hand side member is negative. This member is positive if $\frac{\sqrt{\Delta_i}}{R(1-q)^2} < R + \frac{c_I q}{(1-q)^2} - (R + L + I_1) + \frac{(1-p)(R - I_1 - L)}{p(1-q)}$.
In the same way, this inequality could not be satisfied if its right-hand side is negative, i.e. if $\frac{c_I q}{(1-q)^2} < L + I_1 - \frac{(1-p)(R - I_1 - L)}{p(1-q)} \Longleftrightarrow c_I < \frac{(L + I_1)(1-q)^2}{q} - \frac{(1-p)(1-q)(R - I_1 - L)}{pq}$. It would implie that $R + \frac{c_I(qR - \sqrt{\Delta_i})}{R(1-q)^2} - (R + L + I_1 < 0$, and consequently it is impossible to have $\delta_I^* > \delta_B^*$, thus the shares held by the affiliated firm will be more important.
For the other cases, one can rewrite these inequalities as a polynomial function and then check for the intervals of solution.

Proof of proposition 11

The NPV of the project with the financing by an independant VC firm is the following, with a separate equilibrium contract:

$$\text{NPV}_I = p(e_I R + (1 - e_I)qR) + (1-p)(e_I R + (1 - e_I)(L + I_1)) - I_0 - I_1 - c_I \frac{e_I^2}{2}$$

$$= p(e_I R + (1 - e_I)qR) + (1-p)(e_I(R - I_1 - L)) + L(1-p) + I_1(-p) - I_0 - c_I \frac{e_I^2}{2}$$

$$\text{since } (1-p)(e_I R + (1 - e_I)(L + I_1)) = (L + I_1)(1-p) + (1-p)(e_I(R - I_1 - L))$$

With a pooling equilibrium contract, the NPV is the same with $p=1$.
WIth the financing of an affiliated firm, it is worth:

$$\mathrm{NPV}_B = e_B\, R + (1-e_B)\, L - I_0 - e_B\, I_1 - c_B\, \frac{e_B^2}{2} = L - I_0 + e_B\, (R - L - I_1) - c_B\, \frac{e_B^2}{2}$$

Let us determine when the NPV created under a financing by an affiliated firm is more important. We thus have to solve:

$$\mathrm{NPV}_B - \mathrm{NPV}_I > 0$$

$\Longleftrightarrow L - I_0 + e_B\, (R - L - I_1) - c_B\, e_B^2/2 - p\, (e_I\, R + (1-e_I)\, q\, R) - (1-p)\, (e_I\, (R - I_1 - L)) - L\, (1-p) - I_1\, (-p) + I_0 + c_I\, \frac{e_I^2}{2} > 0$

$\Longleftrightarrow \frac{-c_B\, e_B^2}{2} + e_B\, (R - L - I_1) + p\, (I_1 + L) - p\, (e_I\, R + (1-e_I)\, q\, R) - (1-p)\, (e_I\, (R - I_1 - L)) + c_I\, \frac{e_I^2}{2} > 0$

In this unequality, it appears a polynomial function with e_B $a\, e_B^2 + b\, e_B + c$, whose characteristics are the following : $a<0$, $b>0$ and c of undetermined sign. It is thus positive only with values of e_B bounded by the roots of the polynomial function.
If the discriminant is negative, it is impossible to have a higher level of effort with a financing from an affiliated firm. It is worth:

$$\Delta_c = (R - L - I_1)^2 + 2\, c_B \left[p\, (I_1 + L) - p\, (e_I\, R + (1-e_I)\, q\, R) - (1-p)\, (e_I\, (R - I_1 - L)) + c_I\, \frac{e_I^2}{2} \right]$$

It is then only positive if $c>0$, or $(R - L - I_1)^2 > -\, 2\, c_B \left[p\, (I_1 + L) - p\, (e_I\, R + (1 - e_I)\, q\, R) - (1-p)\, (e_I\, (R - I_1 - L)) + c_I\, \frac{e_I^2}{2} \right]$ if not.
I set $\gamma(e_I) = -c = -\, p\, (I_1 + L) + p\, (e_I\, R + (1-e_I)\, q\, R) + (1-p)\, (e_I\, (R - I_1 - L)) - c_I\, \frac{e_I^2}{2}$.
Thus $\Delta_c = (R - L - I_1)^2 - 2\, c_B\, \gamma(e_I)$.
Suppose Δ_c is positive. It is only possible if $\gamma(e_I) < 0$, or $(R - L - I_1)^2 > 2\, c_B\ \gamma(e_I)$ if not. Thus $\Delta_c = (R - L - I_1)^2 - 2\, c_B\, \gamma(e_I)$.
It is easy to note that $\gamma(e_I) = p\, (e_I\, R + (1-e_I)\, q\, R - I_1 - L) + (1-p)\, e_I\, (R - I_1 - L) - c_I\, \frac{e_I^2}{2}$. It is positive because by hypothesis $\mathrm{NPV}_I = p\, (e_I\, R + (1-e_I)\, q\, R - I_0 - I_1) + (1-p)\, (e_I\, (R - I_0 - I_1) + (1-e_I)\, (L - I_0)) - c_I\, \frac{e_I^2}{2} \geqslant 0$ and $L < I_0$.
The two roots are $\frac{-b - \sqrt{\Delta_c}}{2\, a} > 0$ and $\frac{-b + \sqrt{\Delta_c}}{2\, a}$. The last root is negative if $c>0$ and positive if not. The first root is higher than the last root.
Since $a<0$ and $b>0$, the lower root $\frac{-b + \sqrt{\Delta_c}}{2\, a}$ is positive if $-b + \sqrt{\Delta_c} < 0 \Longleftrightarrow b^2 > b^2 - 4\, a\, c \Longleftrightarrow c < 0 \Longleftrightarrow \gamma(e_I) > 0$. This last condition being checked, then the root value is also under 1 because $R - L - I_1 < c_I$ and $c_I < c_B$.
The higher root $\frac{-b - \sqrt{\Delta_c}}{2\, a}$ is positive.
I show that the condition $e_B \leqslant \frac{-b - \sqrt{\Delta_c}}{2\, a}$ is always true.

Using that $e_B = \frac{R - I_1 - L + \sqrt{(R - I_1 - L)^2 + 4\, c_B\, (L - I_0)}}{2\, c_B}$, I simplify the previous unequality as:

$$\frac{R - I_1 - L + \sqrt{(R - I_1 - L)^2 + 4\, c_B\, (L - I_0)}}{2\, c_B} \leqslant \frac{R - I_1 - L + \sqrt{(R - I_1 - L)^2 - 2\, c_B\, \gamma(e_I)}}{c_B}$$

$$\Longleftrightarrow \sqrt{(R - I_1 - L)^2 + 4\, c_B\, (L - I_0)} \leqslant R - I_1 - L + 2\, \sqrt{(R - I_1 - L)^2 - 2\, c_B\, \gamma(e_I)}$$

We know that both members are positive. Then,

$\Longleftrightarrow 4\ c_B\ (L - I_0) \leqslant 4\ (R - I_1 - L)\ \sqrt{(R - I_1 - L)^2 - 2\, c_B\, \gamma(e_I)} + 4((R - I_1 - L)^2 - 2\, c_B\, \gamma(e_I))$

The left hand-side member is negative since $L < I_0$. The right hand side is positive because $(R - L - I_1)^2 - 2\, c_B\, \gamma(e_I) \geqslant 0$.
Consequently, e_B is always lower than the highest root of the polynomial function.

Numerical exemples

Let us study here two situations in which the entrepreneur will prefer the independant and the bank-affiliated financing. For simplicity, assume the liquidity shock is certain, i.e. $p = 1$.

First case

The exogen parameters are as follows:

R	215
I_0	30
I_1	30
q	0,2
L	15
c_I	200
c_B	209

Assumptions taken in the paper must be checked for:

- $L = 15 < I_0 = 30$, $R - I_1 - L = 170 > 0$ et $q\, R - I_1 = 13 < L = 15$; $R > I_0 + I_1$.
- Assumption 1 : $R - I_1 - L = 170 < c_I = 200$
- Assumption 2 (and 3) : $\left[\frac{R^2}{c_I}\ (1 - q)^2 + q\ R\right]^2 = 36450\ \geqslant 4\ \frac{R^2 (1 - q)^2}{c_I}\ (I_0 + I_1) = 33024$
- Assumption 4 : $2\, \sqrt{c_B\, (I_0 - L)} + I_1 + L = 126, 98 \leqslant R = 215$

Let us compute the rounded parameters of the contract obtained with the independent and the bank-affiliated VC firms:

Parameters	First-best	Independent VC firm	Bank-affiliated VC firm
P_I^*	/	30	36,04
δ^*	/	0,54	0,31
L_B^*	/	/	15
e^*	0,85	0,39	0,71
VAN	57,25	35,3	53,08

In this case, the NPV with bank-affiliated financing is greater. The entrepreneur will choose to finance from this VC firm because he recovers all the NPV under the two contracts. The shares obtained by the second investor are also lower because the firm is systematically liquidated in bad states of nature by the first bank-affiliated investor, and effort is higher. The differences of productivity of support between the two investors is not as sufficient so that the independent VC firm is the best choice.

Second case

The bank-affiliated VC firm is here less attractive, because c_B is worth twice as c_I (the independendant firm is twice more efficient while supporting venture, and q is higher. To meet assumptions, one has to increase L.

The new parameters are as follows :

R	215
I_0	30
I_1	30
q	0,25
L	25
c_I	200
c_B	400

As previously, assumptions must be checked.

- $L = 25$, $R - I_1 - L = 160 > 0$ et $q\,R - I_1 = 23,75 < L = 25$; $R > I_0 + I_1$.
- Assumption 1 : $R - I_1 - L = 160 < c_I = 200$
- Assumption 2 (and 3) : $\left[\frac{R^2}{c_I}\,(1-q)^2 + q\,R\right]^2 = 33767 \;\geqslant 4\,\frac{R^2(1-q)^2}{c_I}\;(I_0 + I_1) = 29025$
- Assumption 4 : $2\sqrt{c_B\,(I_0 - L)} + I_1 + L = 144,44 \leqslant R = 215$

The parameters obtained under the two financings, and relative NPV and effort are as follows:

Parameters	First-best	Independant VC firm	Bank-affiliated VC firm
P_I^*	/	30	38,67
δ^*	/	0,51	0,32
L_B^*	/	/	25
e^*	0,8	0,39	0,37
VAN	59	41,72	26,77

Financing by an independant VC firm is here more attractive because it makes it possible to obtain more NPV. The level of effort is also more important. The fact the the second investor obtain more shares due to inefficient continuation decisions is largely compensated by a more productive effort.

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