The Stochastic Volatility Model, Regime Switching and VaR in International Equity Markets

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Abstract

In this paper, we model international equity markets according to two stochastic volatility models: the log-normal SV model which is estimated by quasi-maximum likelihood with the kalman filter, and the two-regime switching model which is estimated by maximum likelihood with the Hamilton filter. Then based on the one-day-ahead forecast conditional volatility from both models, we evaluate the Value-at-Risk (VaR) in each market. We find that the VaR estimates, in general, are higher for the SV model than those for the regime-switching model for all markets and over all horizons. The exception is Nikkei225, where in both cases, the SV model generate lower VaR values than those of the regime switching model. Comparing the VaRs calculated directly from the two models and the unconditional return distribution, there appears to be a tendency for the two conditional models to generate smaller VaRs, except for the Japanse market, where the SV model produces smaller VaRs than those obtained from the returns. Considering how the VaRs increase with horizon, generally, according to the regime switching model, VaRs increase more slowly with horizon than the SV approach.

Keywords: Regime Switching; Stochastic Volatility Model;Value at Risk
JEL classification: G1; G12; G14; G15

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1. Introduction

Volatility is a key ingredient for derivative pricing, portfolio optimization and value-at-risk analysis. Hence, accurate estimates and good modelling of stock price volatility are of central interest in financial applications. The valuation of financial instruments is complicated by two characteristics of the volalitily process. First, It is generally acknowledged that the volatility of many financial return series is not constant over time and exhibit prolonged periods of high and low volatility, often referred to as volatility clustering (Mandelbrot, 1963; Engle 1982: among many others). Second, volatility is not directly observable[1]. Two models have been developed which capture this time-varying autocorrelated volatility process: the GARCH and the Stochastic Volatility (SV) model. GARCH models define the time-varying variance as a deterministic function of past squared innovations and lagged conditional variances whereas the variance in the SV model is modelled as an unobserved component that follows some stochastic process. Stochastic volatility models are also attractive because they are close to the models often used in Financial Theory to represent the behavior of financial prices. Furthermore, their statistical properties are easy to derive using well-known results on long-normal distributions. Finally, compared with the more popular GARCH models, they capture in a more appropriate way the main empirical properties often observed in daily series of financial returns (see, for example, Carnero *et al.*, 2003). For surveys on the extensive GARCH literature we refer to Bollerslev *et al.* (1992), Bera and Higgins (1993) and Bollerslev *et al.* (1994). SV models are reviewed in, for example, Taylor (1994), Ghysels *et al.* (1996) Shephard (1996), and Broto and Ruiz (2004)). Both models are defined by their first and second moments. The Stochastic Volatility model introduced by Taylor (1986) provides an alternative to GARCH models in accounting for the time-varying and persistent volatility as well as for the leptokurtosis in financial return series. It arises from the mixture-of-distributions hypothesis which assumes that the volatility process is driven by the unobservable flow of price-relevant information. These models present two main advantages over ARCH models. The first one is their solid theoretical background, as they can be interpreted as discretized versions of stochastic volatility continuous-time models put forward by modern finance theory (see Hull and White (1987)). The second is their ability to generalize from

[1] For a comprehensive review of volatility measures and their properties see Andersen, Bollersleve and Diebold (2002) and for forecasting financial volatility see the survey by Poon and Granger (2003).

univariate to multivariate series in a more natural way, as far as their estimation and interpretation are concerned. On the other hand, SV models are more difficult to estimate than ARCH models, due to the fact that it is not easy to derive their exact likelihood function. For this reason, a number of econometric methods have been proposed to solve the problem of estimation of SV models.

The stochastic volatility model defines volatility as a logarithmic first-order autoregressive process, which is a discrete-time approximation of the continuous-time Ornstein-Uhlenbeck diffusion process used in the option-pricing literature (See Hull and White (1987). It is an alternative to the GARCH models which have relied on simultaneous modelling of the first and second moment. For certain financial time series such as stock index return, which have been shown to display high positive first-order autocorrelations, this constitutes an improvement in terms of efficiency; see Campell *et al.* (1997, Chapter 2). The volatility of daily stock index returns has been estimated with SV models but usually results have relied on extensive pre-modelling of these series, thus avoiding the problem of simultaneous estimation of the mean and variance. Koopman and Hol Uspensky (2002) proposed the Stochastic Volatility in Mean model (SVM) that incorporates volatility as one of the determinants of the mean. The fact that we are able to estimate an SV model that includes volatility as one of the determinants of the mean makes the model suitable for empirical applications between the mean and variance of returns. The SVM model can be viewed as the SV counterpart of the ARCH-M model of Engle *et al.* (1987) with the main difference between the two models is that the ARCH-M model intends to estimate the relationship between expected returns and expected volatility, whereas the aim of the SVM model is to simultaneously estimate the *ex ante* relation between returns and volatility and the volatility feedback effect.

Another way of modelling economic time series is to define different states of the world or regimes, and to allow for the possibility tha the dynamic behavior of economic variables depends on the regime that occurs at any given point in time. By "state-dependent dynamic behavior" of a time series, it is meant that certain properties of the time series, such as its mean, variance and /or autocorrelation, are different in different regimes. Regime switching models were first introduced in the econometrics literature by Goldfeld and Quandt (1973) to provide a simple way to model endogenously determined structural breaks or regime shifts in parameters. Hamilton (1989) generalizes this setting by allowing the mixing probability to be time-varying function of the history of the data. To illustrate the importance of stochastic regime switching for financial time series, for example, LeBaron (1992) shows that the autocorrelations of stock returns are related to the level of volatility of these returns. In particular, autocorrelations tend to be larger during periods of low volatility and smaller during periods of high volatility[2]. The periods of low and high volatility can be interpreted as distinct regime - or, put differently, the level of volatility can be regarded as the regime-determining process. In this setup, the level of volatility is not known with certainty and what we can do is to make a sensible forecast of this level, and hence, of the regimes that will occur in the future, by assigning probabilities to the occurrence of the different regimes.

Markov switching models have been found to provide a flexible framework to handle many features of asset returns (see Hamilton 1989 and 1994). In particular, they allow for nonlinearities arising from persistent jumps in the model parameters. These modesl have several appealing features. First, they provide a convenient framework to endogenously identify regime shifts that are commonplace in financial data. Regimes are treated as latent processes which are not observable by the econometrician, but can be inferred from the estimation algorithm using observable data, such as the history of the asset's returns. Second, as Markov switching modesl belong to the mixture-of-distributions class of stochastic processes, they are as versatile as mixture models in capturing salient features of financial data such as time-varying volatilities, skewness, and letptorkurtosis. A detailed study of the statistical properties of Markov switching models by Timmerman (2000) shows the Markov switching models can indeed approximate general classes of density functions with a wide range of conditional moments. Ang and Bekaert (2001) show that Markov switching models with state-dependent means and variances can match exceedance correlations better than do standard GARCH models or bivariate jump diffusion processes.

This paper is organized as follows. In section 2, we introduce the two competing models (i.e, the regime switching model and the stochastic volatility model. In section 3, we describe the available data and characterize the stylized facts of the corresponding realized volatility. In section 4, we use the two models to compute the Value at Risk and assess their performance. Section 5 concludes.

[2]Returns on international equity markets were characterized by jumps, and these jumps tend to occur at the same time across countries, implying that conditional correlations between international equity returns tend to be higher in periods of high market volatility or following large downside moves. Evidence on jumps is provided by Jorion (1988), Akgiray and Booth (1988), Bates (1996), and Bekaert *et al.* (1998). And for evidence on changing conditional correlations see, for instance Ang and Chen (2002), Longin and Solnik (1995), Karolyi and Stulz (1996), and Chakrabarti and Roll (2000).

2. Models of volatility

The empirical regularities of asset returns (i.e., volatility clustering; squared returns exhibit prolonged serial correlation; and heavy tails and persistence of volatility) suggest that the behavior of financial time series can be captured by a model which recognizes the time-varying nature of return volatility as follows:

$$y_t = \mu_t + \sigma_t \varepsilon_t \tag{1}$$

$$\mu_t = a + \sum_{i=1}^{k} b_i x_{i,t} \tag{2}$$

with $\varepsilon_t \sim NID(0,1)$. μ_t represents the mean and depends on a constant a and regression coefficients $b_1, ..., b_k$. The explanatory variables $x_{1,t}, ...x_{k,t}$ may also contain lagged exogenous and dependent variables. The disturbance term ε_t is IID with zero mean and unit variance and a usual assumption of a normal distribution.

Following Shephard (1996), models of changing volatility can be usefully partitioned into observation-driven and parameter-driven models and both can be expressed using a parametric framework as: $y_t/z_t \sim N(\mu_t\ \sigma_t^2)$. In the first class, the autoregressive heteroskedasticity (ARCH) models introduced by Engle (1982) are the most representative example. In the second class, z_t is a function of an unobserved or latent component. The log-normal stochastic volatility model created by Taylor (1986) is the simplest and best known example:

$$y_t/h_t \sim N(0_t \exp(h_t))\ ,\ h_t = \alpha + \beta h_{t-1} + \eta_t\ ,\ \eta_t \sim NID(0, \sigma_\eta^2) \tag{3}$$

where h_t represents the log-volatility, which is unobserved but can be estimated using the observations. One interpretation for the latent h_t is to represent the random and unveven flow of new information, which is difficult to model directly, into financial markets. The most popular model from Taylor (1986), puts:

$$y_t = \varepsilon_t \exp(h_t/2) \tag{4}$$

and

$$h_t = \alpha + \beta h_{t-1} + \eta_t \tag{5}$$

where ε_t and η_t are two independent Gaussian white noises, with variances 1 and σ_η^2, respectively. Due to the Gaussianity of η_t, this model is called a log-normal SV model[3].

Another possible intepretation for h_t is to characterize the regime in which financial markets are operating and then it could be described by a discrete valued variable. The most popular approach to modelling changes in regime is the class of Markov switching models introduced by Hamilton (1989). In that case the model is:

$$y_t = \varepsilon_t \exp(h_t/2) \tag{6}$$

and

$$h_t = \alpha + \beta s_t \tag{7}$$

where s_t is a two state first-order Markov chain which can take values 0,1 and is independent of ε_t. The value of the time series s_t, for all t, depends only on the last value $s_{t-1,}$ for $i, j = 0, 1$:

$$P(s_t = j | s_{t-1} = i, s_{t-2} = i, ...) = P(s_t = j | s_{t-1} = i) = p_{ij} \tag{8}$$

The probabilities $(p_{ij})_{i,j=0,1}$ are called transition probabilities of moving from one state to the other. These transition probabilities are collected in the transition matrix P:

$$\begin{bmatrix} p_{00} & 1 - p_{11} \\ 1 - p_{00} & p_{11} \end{bmatrix} \tag{9}$$

which fully describes the Markov chain and also we get: $p_{00}+p_{01} = p_{10}+p_{11} = 1$. A two-state Markov chain can be represented by a simple AR(1) process as follows:

$$s_t = (1 - p_{00}) + (-1 + p_{00} + p_{11})s_{t-1} + \upsilon_t \tag{10}$$

where $\upsilon_t = s_t - E(s_t | s_{t-1}, s_{t-2}, ...)$ and the volatility equation can be written the following way:

$$h_t = \alpha + \beta s_t = \alpha + \beta[(1 - p_{00}) + (-1 + p_{00} + p_{11})s_{t-1} + \upsilon_t] \tag{11}$$

or

[3] Although the assumption of Gaussianity of η_t can seem *ad hoc* at first sight, Andersen *et al.* (2001, 2003) show that the log-volatility process can be well approximated by a Normal distribution.

$$h_t = (2-p_{00}-p_{11})+\beta(1-p_{00})+(-1+p_{00}+p_{11})h_{t-1}+\beta v_t = a+bh_{t-1}+\omega_t \quad (12)$$

which implies the same structure of the stochastic volatility model but with a noise that can take only a finite set of values.

2.1. Estimation of the models

A variety of estimation procedures has been proposed for the SV models, including for example the Generalized Method of Moments (GMM) used by Melino and Turnbull (1990), the Quasi Maximum Likelihood (QML) approach followed by Harvey *et al.* (1994) and Ruiz (1994), the Efficient Method of Moments (EMM) applied by Gallant *et al.* (1997), and Markov-Chain Monte Carlo (MCMC) procedures used by Jacquier *et al.* (1994) and Kim *et al.* (1998). Although each of these methods is reported to work well under certain conditions, it is difficult to assess their overall perforances across data sets. In this paper, the parameters of the SV model are estimated by exact maximum likelihood methods using Monte Carlo importance sampling techniques. We refer the reader to Koopman and Hol Uspensky (2002) for more explanations. The likelihood function for the SV model can be constructed using simulation methods developed by Shephard and Pitt (1997) and Durbin and Koopman (1997). We start by considering the standard SV model of the equation of the volatility process (10). The non-linear relation between log-volatility h_t and the observation equation of y_t does not allow the computation of the likelihood by linear methods such as the Kalman filter. For the SV model we can express the likelihood function as:

$$L(\psi) = p(y/\psi) = \int p(y,\theta/\psi)d\theta = \int p(y/\theta,\psi)p(\theta/\psi)d\theta \quad (11)$$

where $\psi = (\phi, \sigma_\eta, \sigma_\varepsilon)'$, $\theta = (h_1, ...h_T)'$. An efficient way of evaluating such expressions is by using importance sampling; see Ripley (1987, Chapter 5). A simulation devices is required to sample from an importance density $\widehat{p}(y/\theta,\psi)$ which is preferred to be as close as possible to the true density $p(y/\theta,\psi)$. A choice for the importance density is the conditional Gaussian density since in this case it is relatively straightforward to sample from $\widehat{p}(y/\theta,\psi) = g(y/\theta,\psi)$ using simulation smoothers such as the ones developed by de Jong and Shephard (1995) and Durbin and Koopman (2002). Guidelines for the construction of an importance model and the likelihood function for the SV model using this model are given by Hol and Koopman (2000). The SV in mean models were estimated using programs written in the Ox language of Doornik (1998) using SsfPack by

Koopman, Shephard and Doornik (1999). The Ox programs were downloaded from www.econ.vu.nl/koppman/sv/.

In this section we estimate the two models, i.e., the log-normal SV model which is estimated by quasi-maximum likelihood with the kalman filter, and the two-regime switching model which is estimated by maximum likelihood with the Hamilton filter.

The first model is:

$$y_t = \varepsilon_t \exp(h_t/2) \tag{12}$$

and

$$h_t = \alpha + \beta h_{t-1} + \eta_t \tag{13}$$

with ε_t and η_t independent Gaussian white noises. Their variances are 1 and σ_η^2, respectively. The volatility equation is characterized by the constant parameter α, the autoregressive parameter β and the variance σ_η^2 of the volatility noise. The mean is either imposed equal to zero or estimated with the empirical mean of the series. Since the specification of the conditional volatility is an autoregressive process of order one, the stationarity condition is $|\beta| < 1$. Moreover, the volatility σ_η must be strictly positive. In the estimation procedure the following logistic and logarithm reparameterizations:

$$\beta = 2\left(\frac{\exp(b)}{1+\exp(b)}\right) - 1 \text{ and } \sigma_\eta = \exp(s_\eta) \tag*{((14))}$$

have been considered in order to satisfy these conditions.

The second model is a particular specification of the regime switching model introduced by Hamitlon. Precisely the distribution of the returns is described by two regimes with the same mean but different variances and by a constant transition matrix:

$$y_t = \begin{cases} \mu + \sigma_0 \varepsilon_t \text{ if } s_t = 0 \\ \mu + \sigma_1 \varepsilon_t \text{ if } s_t = 1 \end{cases} \tag*{((15))}$$

and

$$\begin{bmatrix} p_{00} & 1 - p_{11} \\ 1 - p_{00} & p_{11} \end{bmatrix} \tag{16}$$

where s_t is a two-state Markov chain independent of ε_t, which is a Gaussian white noise with unit variance. The parameters of this model are the mean μ, the low and high standard deviation σ_0, σ_1 and the transition probabilities

p_{00}, p_{11}(also called regime transformations probabilities). As for the log-normal SV model, the logarithm and the logistic transformations ensure the positiveness of the volatilities and constrain the transition probabilities to assume values in the (0,1) interval. Further, for the log-normal SV model the returns are modified as follows:

$$y_t^* = \log(y_t - \overline{y}_t) + 1.27 \tag{17}$$

where $\overline{y}_t$ is the empirical mean. Thus, for the log-normal SV model the mean is not estimated but is simply set equal to the empirical mean. For the estimation, the starting values of the parameters are calculated considering the time series analyzed. For example, the sample mean is used as an approximation of the mean of the switching regime model and the empirical variance multiplied by appropriate factors is used for the high and low variance. However, for the log-normal SV model, a range of possible values of the parameters were fixed and a value is randomly extracted. This method proved to be useful for us, since we do not have an idea about the possible value of the parameters but want to better investigate the parametric space.

We present some graphical analysis of the returns and estimated volatility for both models. In the case of the log-normal SV model, the estimated volatility is obtained by using the Kalman smoother $\widehat{h}_{t/T} = E(h_t|Y^{*T})$, which is not very useful. In fact, we are interested in $E(\sigma_t|Y^T) = E(\exp(h_t/2)|Y^T)$, but $E(\exp(h_t/2)|Y^T) \neq \exp E(h_t/2)|Y^T))$. Thus, a first-order Taylor expansion of $\exp(h_t/2)$ around $\widehat{h}_{t/T}$ is considered and compute the conditional mean and estimated the volatility in the following way:

$$\widehat{\sigma}_{t/T} = E(\exp(\frac{h_t}{2})|Y^T) \cong \exp(\frac{\widehat{h}_{t/T}}{2}) + \frac{1}{8}\exp(\frac{\widehat{h}_{t/T}}{2})Q_{t/T} \tag{18}$$

In the case of the switching model, we present historical return series, the estimated volatility and the estimated switches between regimes. To estimated the volatility we consider the output of the Kim smoother. Since $\sigma_t = \exp(\alpha/2)(1 - s_t) + \exp((\alpha+\beta)/2)s_t = \sigma_0(1 - s_t) + \sigma_1 s_t$, we can compute:

$$\widehat{\sigma}_{t/T} = E(\sigma_t|Y^T) = \sigma_0 P(s_t = 0|Y^T) + \sigma_1 P(s_t = 1|Y^T) \tag{19}$$

where $P(s_t = 0|Y^T) = P(h_t = \alpha|Y^T)$ and $P(s_t = 1|Y^T) = P(h_t = \alpha + \beta|Y^T)$

The parameters of Markov switching model can be estimated by using maximum likelihood

2.2. Estimation Results

We examine the behavior of following equity markets. These are the S&P500 for USA, FTSE100 for United Kingdom, CAC40 for France, S&P/TSX for Canada, Nikkei225 for Japan, DAX for Germany, and Swiss Market for Switzerland. The price data was obtained from Datastream. Each of the price indices was transformed via first differencing of the log price data to create a series, which approximates the continuously compounded percentage return. The stock index prices are not adjusted for dividends following studies of French *et al.* (1987) and Poon and Taylor (1992) who found that inclusion of dividends affected estimation results only marginally. Returns are calculated on a continuously compounded basis and expressed in percentages, they are therefore calculated as $r_t = 100 * (\log(P_t/P_{t-1}))$, where P_t denotes the stock index in day t.

The summary statistics are presented in Table 1. We observe that the Swiss Market shows the highest mean returns followed by CAC40 and then the DAX. All the indices exchibit similar patterns of volatility represented by the standard deviation, with Nikkei225 having the highest variability and S&P/TSX having the lowest. We further observe that the returns are highly autocorrelated at lag 1, with S&P/TSX maintaining the highest autocorrelation. The high first-order autocorrelation reflects the effects of non-synchronous or thin trading, whereas highly correlated squared returns can bes seen as an indication of volatility clustering. The $Q(12)$ and $Q_s(12)$ test statistics, which is a joint test for the hypothesis that the first twelve autocorrelation coefficients on returns and squared returns are equal to zero, indicate that this hypothesis has to be rejected at the 1% significance level for all return series and squared return series[4]. Autocorrelation of squared returns is consistent with the presence of time-varying volatility such as GARCH effects. As pointed out by Lamoureux and Lastrapes (1990) and confirmed by Hamilton and Susmel (1994), regime shifts in the volatility process can also induce a spuriously high degree of volatility clustering.

Before estimating the models, we begin by testing whether there are indeed regime shifts in the stock markets. To do so, we apply Hansen's (1992) modified likelihood ratio test for regimes under the null hypothesis that returns are generated by a switching model. Detection of regime shifts requires non-standard tests, because the presence of nuisance parameters under the null of a single regime invalidates the use of standard likelihood ratio tests. Using empirical process theory, Hansen (1992) shows that a modified LR statistic can be applied

[4] A number of empirical studies has found similar results on market returns distributional characteristics. Kim and Kon (1994) showed similar results for 30 stocks in DJIA, S&P500, and CRSP indices. Campbell, Lo and Mackinlay (1997) concluded that daily US stock indexes show negatively skewed and positive excess kurtosis.

under non-standard conditions. We apply Hansen's test to evaluate the null hypothesis of a geometric random walk for daily stock prices against the alternative of a two-state Markov switching model.

The estimation results of the two models are reported in Tables 2 and 3. Table 2 presents the results of estimating the regime switching model in the different markets[5]. For this model, we can judge the persistence of the volatility from the values taken by the transition (or persistence) probabilities p_{00} and p_{11}, they are all high and higher than 0.90, confirming the high persistence of the volatility in all markets. The parameter which govern the mean process is also reported in the first column of Table 2 with the corresponding standard errors. The mean parameter is positive and statistically significant for all series, except being negative for Nikkei225. The estimation results of the log-normal SV model applied to international equity markets is reported in Table 3. All markets show strong persistence, since all the estimated autoregressive coefficients of the volatility equation (β) are higher than 0.90. Also all the volatility estimates are all highly significant and quite similar for all markets. In practice, for many financial time series this coefficient is often found to be bigger than 0.90. This near-unity volatility persistence for high-frequency data is consistent with findings from both the SV and the GARCH literature. Among all the markets, the Swiss market, FTSE100, Nikkei225 and Dax show the hightest variability in their volatility noise. For example, the standard deviation of the volatility noise in the FTSE100 is 0.1066, while that in the S&P500 is 0.071.

3. Value at Risk

VaR indicates the maximum potential loss at a given level of confidence (p) for a portfolio of financial assets over a specified time horizon (h). In practice, the value of a portfolio is expressed as a function of K risk factors such as interest rates, exchange rates or stock indexes. If their distribution is known, the VaR is a solution to the following problem:

$$p = \int_{-\infty}^{VaR(h,p)} f(x_{t+h})dx \tag{20}$$

with x being the value of the portfolio. Different methods have been proposed to calculate the VaR. One of them is the parametric model that can be used to forecast the portfolio return distribution, if this distribution is known in a closed

[5] The standard errors are calculated following Ruiz (1994) for the log-normal SV model and as the inverse of the information matrix for the switching model. In both cases the $z-$statistics asymptotically follow an $N(0,1)$ distribution.

form and the VaR simply being the quantile of this distribution. In the case of non-linearity we can use either Monte Carlo simulation or historical simulation approches. The advatange of the parametric approach is that the factors variance-covariance matrix can be updated using a general model of changing volatility. Having chosen the asset or portfolio distribution (usually the normal one), it is possible to use the forecasted volatility to characterise the future return distribution. Thus, $\widehat{\sigma}_{T+1/T}$ can be used to calculate the VaR over the next period. A different approach using the stochastic volatility (SV) model is to devolatize the observed return series and to revolatilize it with an appropriate forecasted value, obtained with a particular model of changing volatility. This approach is considered in several recent works (Barone-Adesi et al. (1998); Hull and White (1998)).

Consider a portfolio which perfectly replicates the composition of each stock marke index. Given the estimated volatility of the stochastic volatility model, the VaR of this portfolio can be obtained following the procedure proposed in Barone-Adesi et al. (1998). The historical portfolio returns are rescaled by the estimated volatility series to obtain the standardized residuals $u_t = y_t/\sigma_t$, $t = 1, ..., T$. This historical simulation can be performed by boostrapping the standardized returns to obtained the desired number of residuals $u_j^*, j = 1, ..., M$, where M can be arbitrarly large. To calculate the next period returns, it is sufficient to multiply the simulated residuals by the forecasted volatility $\widehat{\sigma}_{T+1/T} : y_j^* = u_j^* \widehat{\sigma}_{T+1/T}$, and then the VaR for the next day, at the desired level of confidence h, is then calculated as the Mth element of these returns sorted in ascending order.

To make the historical simulation consistent with empirical findings, the log-normal SV model and the regime switching model may be considered to describe the volatility behavior. Past returns are standardized by the estimated volatility to obtain standardized residuals. Statistical tests can confirm that these standardized residuals behave approximately as an iid series which exhibits heavy tails. Historical simulation can then be used. Finally, to adjust them to the current market conditions, the randomly selected standardized residuals are multiplied by the forecasted volatility obtained with the SV model.

The VaRs for the two models are presented together with the results obtained from unconditional returns in Tables 4 and 5. An examination of the results reveals that the VaR estimates, in general, are higher for the SV model than those for the regime-switching model for almost all markets and over all horizons. The exception is Nikkei225, where in both cases, whether using historical simulation or delta-normal approximation, the SV model generate lower VaR values than those of the regime switching model. The Comparing among the VaRs calculated directly from the two models and unconditional distribution of returns, there appears to be a tendency for the two conditional models to generate smaller VaRs, except again for the Japanse market, where the SV model produces smaller VaRs than those obtained from the returns. Considering how the VaRs increase with horizon, generally, according to the regime switching model, VaRs increase more slowly with horizon than the SV approach.

3.1. Back-testing the VaR models

The Value-at-Risk VaR_{t+1}^{p} measure promises that the actual return will only be worse than the VaR_{t+1}^{p}forecast p*100 of the time. Given a time series of past ex-ante VaR forecasts and past ex-post returns, we can define the "hit sequence" of VaR violations as:

$$I_{t+1} = \begin{cases} 1, \text{ if } R_{pf,t+1} < -VaR_{t+1}^{p} \\ 0, \text{ if } R_{pf,t+1} > -VaR_{t+1}^{p} \end{cases} \tag{21}$$

The hit sequence returns a 1 on day t+1 if the loss on that day was larger than the VaR number predicted in advance for that day. If the VaR was not violated, then the hit sequence returns a 0. When backtesting our models, we construct a sequence $\{I_{t+1}\}_{t+1}^{T}$ across T days indicating when the past violations occurred. We implement three test statistics derived from Christoffersen (1998), the unconditional, independenc, and conditional coverage[6]. Christoffersen (1998) idea is to separate out the particular predictions being tested, and then test each prediction separately. The first of these is that the model generates the "correct" frequency of exceedances, which is in this context is described as the prediction of correct unconditional coverage. The other prediction is that exceedances are independent of each other. This later prediction is important in so far as it suggests that exceedances should not be clusterd over time. to explain the Christoffersen approach, we briefly explain the three tests.

3.1.1. Unconditional Coverage Testing

According to this test, we are interested in testing if the fraction of violations obtained from our models, call it π, is significantly different from the promised fraction, p. We call this the unconditional coverage hypothesis. To test this, we write the likelihood of an i.i.d. Bernoulli (π) hit sequence:

[6] For other methods and elements in backtesting VaR models, see Christoffersen and Diebold (2000), Christoffersen and Pelletier (2003), McNeil and Frey (2000), Diebold, Gunther, and Tay (1998), and Diebold, Hahn, and Tay (1999).

$$L(\pi) = \prod_{t=1}^{T} (1-\pi)^{1-I_{t+1}} \pi^{I_{t+1}} = (1-\pi)^{T_0} \pi^{T_1} \tag{22}$$

where T_0 and T_1 are the number of 0s and 1s in the sample. π can be estimated from $\widehat{\pi} = T_1 / T$ - that is, the observed fraction of violations in the sequence. Plugging the estimate back into the likelihood function gives the optimized likelihood as: $L(\widehat{\pi}) = (1 - T_1 / T)^{T_0} (T_1 / T)^{T_1}$. Under the unconditional coverage null hypothesis that $\pi = p$, where p is the known VaR coverage rate, we have the likelihood: $L(p) = \prod_{t=1}^{T} (1-p)^{1-I_{t+1}} p^{I_{t+1}} = (1-p)^{T_0} p^{T_1}$. The unconditional coverage hypothesis using a likelihood ratio test can be checked as:

$$LR_{uc} = -2 \ln[L(p)/L(\widehat{\pi})] \tag{23}$$

Asymptotically, as T goes to infinity, the test will be distributed as a χ^2 with one degree of freedom. Substituting in the likelihood functions, we write:

$$LR_{uc} = -2 \ln[(1-p)^{T_0} p^{T_1} / \{(1 - T_1/T)^{T_0} (T_1/T)^{T_1}\}] \sim \chi^2 \tag{24}$$

The VaR model is rejected or accepted either using a specific critical value, or calculating the P-value associated with our test statistic.

3.1.2. Independence testing

According to this test, the hit sequence is assumed to be dependent over time and that it can be described as a so-called first-order Markov sequence with transition probability matrix:

$$\Pi_1 = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix} \tag{25}$$

These transition probabilities simply mean that conditional on today being a nonviolation (that is, $\mathrm{I}_t = 0$), then the probability of tomorrow being a violation (that is, $\mathrm{I}_{t+1} = 1$) is π_{01}. The probability of tomorrow being a violation given today is also a violation is: $\pi_{11} = \Pr(\mathrm{I}_t = 1 \text{ and } \mathrm{I}_{t+1} = 1)$. Accordingly, the two probabilities $\pi_{01}$and π_{11} describe the entire process. The probability of a nonviolation following a nonviolation is $1 - \pi_{01}$, and the probability of a nonviolation following a violation is $1 - \pi_{11}$. If we observe a sample of T observations, then the likelihood function of the first-order Markov process can be written as:

$$L(\Pi_1) = (1-\pi_{01})^{T_{00}} \pi_{01}^{T_{01}} (1-\pi_{11})^{T_{10}} \pi_{11}^{T_{11}} \tag{26}$$

where $T_{ij}, i, j = 0, 1$ is the number of observations with a j following an i. Taking first derivatives with respect to $\pi_{01}$and π_{11} and setting these derivatives to zero, we can solve for the maximum likelihood estimates: $\widehat{\pi}_{01} = ((T_{01}/(T_{00}+T_{01}))$ and $\widehat{\pi}_{11} = ((T_{11}/(T_{10} + T_{11}))$. Using the fact that the probabilities have to sum to one, we have: $\widehat{\pi}_{00} = 1-\widehat{\pi}_{01}$ and $\widehat{\pi}_{10} = 1-\widehat{\pi}_{11}$, which can be used to determine the matrix of the estimated transition proabilities.

In the case of the hits being independent over time, then the probability of a violation tomorrow does not depend on today being a violation or not, and we can write $\pi_{01} = \pi_{11} = \pi$, and the transition matrix can becomes: $\widehat{\Pi}_1 = \begin{bmatrix} 1-\widehat{\pi} & \widehat{\pi} \\ 1-\widehat{\pi} & \widehat{\pi} \end{bmatrix}$. Then in this case, we can test the independence hypothesis that $\pi_{01} = \pi_{11}$ using a likelihood ratio test:

$$LR_{ind} = -2\ln[L(\widehat{\pi})/L(\widehat{\Pi}_1)] \sim \chi^2_1 \tag{28}$$

where $L(\widehat{\pi})$ is the likelihood under the alternative hypothesis from the LR_{uc} test.

3.1.3. Conditional Coverage Testing

Ultimately, we care about simulatenously testing if the VaR violations are independent and the average number of violations is correct. We can test jointly for independence and correct coverage using the conditional coverage test:

$$LR_{cc} = -2\ln[L(p)/L(\widehat{\Pi}_1)] \sim \chi^2_2 \tag{29}$$

which corresponds to testing that $\pi_{01} = \pi_{11} = p$. Notice that the LR_{cc} test takes the likelihood from the null hypothesis in the LR_{uc} and combines it with the likelihood from the alternative hypothesis in the LR_{ind} test. Therefore, $LR_{cc} = -2\ln[L(p)/L(\widehat{\Pi}_1)] = -2\ln[\{L(p)/L(\widehat{\pi})\}\{L(\widehat{\pi})/L(\widehat{\Pi}_1)\}]$ or $LR_{cc} = -2\ln[L(p)/L(\widehat{\pi})] - 2\ln[L(\widehat{\pi})/L(\widehat{\Pi}_1)] = LR_{uc} + LR_{ind}$.

The Christoffersen approach enables use to test both coverage and independence hypotheses at the same time. Moreover, if the model fails a test of both hypotheses combined, his approach enable us to test each hypothesis separately, and so establish where the model failure arises.

4. Conclusion

Under the Bank of International Settelement Regulation, investment firms and banks are permitted to use their own internal risk management models to calculate the required capital to cover losses in their trading positions. Given that such models are now in widespread usage, it is crucial that a body of research is generated that compares between different approaches to computing value at risk. This paper proposes two different models, namel the log stochastic volatility model and regime switching model for calculating value at risk. Our approach has been applied to international equity markets, and was compared to the unconditional measures from the actual returns. We also examined the performance of VaRs calculated directly from the two models. It was observed that the two models generated smaller VaRs than the unconditional distributional method, except for those of Japan, where the SV model produces smaller VaRs than those obtained from the returns. Considering how the VaRs increase with horizon, generally, according to the regime switching model, VaRs increase more slowly with horizon than the SV approach.

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Table 1 : Summary Statistics of daily returns

	S&P500	FTSE100	NIKKEI225	DAX	S&P/TSX	CAC40	SM
Mean	-0.0089	0.017	-0.026	0.020	-0.007	0.029	0.037
S.D.	1.008	1.089	1.495	1.080	0.745	1.002	1.200
Skewness	-0.205	-0.069	0.123	-0.506	-0.632	0.358	-0.237
Kurtosis	7.538	5.684	5.051	8.248	9.174	6.448	7.486
J.B.	2731	950.28	561.6	3757	5225	1631	2678
ρ_1	0.052	0.020	-0.028	0.078	0.117	0.074	0.051
ρ_2	0.0051	-0.041	-0.050	-0.013	-0.012	0.040	-0.004
ρ_3	0.064	-0.085	0.018	-0.016	0.019	-0.024	-0.041
$Q(12)$	60.04	59.04	15.87	72.17	87.72	87.41	29.57
ρ_{s1}	0.182	0.214	0.099	0.107	0.129	0.165	0.232
ρ_{s2}	0.244	0.302	0.123	0.165	0.173	0.188	0.268
ρ_{s3}	0.191	0.255	0.153	0.160	0.110	0.141	0.212
$Q_s(12)$	1186	2173	373	796	687	1013	1691

Notes: J.B. is the Jarque-Bera normality test statistic with 2 degrees of freedom; ρ_k is the sample autocorrelation coefficient at lag k with asymptotic standard error $1/\sqrt{T}$ and $Q(k)$ is the Box-Ljung portmanteau statistic based on $k-$squared autocorrelations. ρ_{sk} are the sample autocorrelation coefficients at lag k for squared returns and $Q_s(12)$ is the Box-Ljung portmanteau statistic based on $12-$squared autocorrelations.

Table 2: Results of the regime switching model applied to international equity markets

Stock Index	μ	Low Persis.Pr.	High Persis.Pr.	Low V	High V	FV	LogL
S&P500	0.00055 0.000143	0.986 0.0037	0.981 0.0049	0.00618 0.000156	0.0144 0.00034	0.00725	-10278.7
FTSE100	0.00027 0.000156	0.993 0.0022	0.981 0.0057	0.00757 0.00015	0.0166 0.00049	0.00760	-10158.3
NIKKEI225	-0.00017 0.000232	0.980 0.0046	0.964 0.0085	0.108 0.00027	0.0204 0.00064	0.0204	-8974.4
DAX	0.00064 0.000199	0.990 0.0027	0.981 0.0052	0.0091 0.00022	0.0221 0.00059	0.0091	-9284.9
S&P/TSX	0.00054 0.000118	0.987 0.00323	0.976 0.0062	0.0052 0.00012	0.0137 0.00036	0.0053	-10907.2
CAC40	0.00035 0.000213	0.994 0.0018	0.977 0.0075	0.0108 0.00018	0.0229 0.00081	0.0108	-9260.85
SM	0.00076 0.000165	0.986 0.0030	0.959 0.0093	0.0079 0.00018	0.0197 0.00072	0.0181	-9942.67

Notes: A two-regimes switching model introduced by Hamilton is applied to equity markets and estimated by maximum likelihood with the Hamilton filter. In this model the returns are distributed with the same mean and different variances and a constant transition matrix. The standard errors are calculated following Ruiz (1994) as the inverse of the information matrix for the switching model and result in $z-$statistics asymptotically following an $N(0,1)$ distribution. μ is the mean value and LogL represents the loglikelihood.

Table 3: Results of estimating the log-normal SV model applied to international equity markets

	Constant	AR part	SD	Forecasted volatility	Loglik
S&P500	-0.0513 0.0261	0.996 0.00195	0.0711 0.0143	0.0013	-4252.87
FTSE100	-0.131 0.048	0.990 0.00352	0.1066 0.0171	0.00087	-4104.7
NIKKEI225	-0.240 0.0823	0.981 0.0062	0.122 0.0227	0.00140	-4232.73
DAX	-0.118 0.0426	0.990 0.0032	0.1228 0.0186	0.00172	-4159.45
S&P/TSX	-0.0522 0.0277	0.996 0.0019	0.074 0.0144	0.00064	-4128.85
CAC40	-0.054 0.0254	0.993 0.00288	0.067 0.0138	0.0113	-4193.88
SM	-0.235 0.0722	0.9825 0.0053	0.156 0.0235	0.00095	-4201.88

Notes: The log-normal SV model is applied to equity markets and estimated by quasi-maximum likelihood with the kalman filter. The volatility equation is characterized by the constant parameter α (constant) , the autoregressive parameter β (AR part) and the variance σ_η^2 of the volatility noise (SD). The standard errors are calculated following Ruiz (1994) for the log-normal SV model and result in $z-$statistics asymptotically following an $N(0,1)$ distribution.

Table 4: VaR measures obtained by using historical simulation method

Time Horizon	5	10	15	5	10	15	5	10	15
	unconditional distribution			conditional distribution			conditional distribution		
Stock Index	historical returns			Log-normal SV			Regime switching		
S&P500	11.28	15.95	19.54	9.78	13.84	16.95	5.68	8.04	9.84
FTSE100	10.88	15.39	18.84	5.52	7.81	9.56	4.96	7.02	8.60
NIKKEI225	13.32	18.83	23.07	1.44	2.04	2.50	13.33	18.08	23.09
DAX	14.42	20.39	24.97	12.80	18.08	22.17	6.11	8.64	10.59
S&P/TSX	13.93	19.70	24.12	13.04	9.02	11.05	5.37	7.59	9.30
CAC40	13.04	18.45	22.59	9.66	13.67	16.74	7.60	10.75	13.17
SM	12.78	18.08	22.15	7.13	10.08	12.35	11.78	16.67	20.41

Notes: The table reports the VaR estimates based on conditional and unconditional distribution of the returns and calculated by historical simulation method. The VaR are calculated for 5-,10- and 15-days holding period with the significance level is 1%. Unconditional distribution measures are based on historical returns, while conditional distribution are those obtained by weighting the standardized residuals by the forecasted volatility.

Table 5: VaR obtained by delta-normal approximation

Time Horizon	5	10	15	5	10	15	5	10	15
	unconditional distribution			conditional distribution			conditional distribution		
Stock Index	historical returns			Log-normal SV			Regime switching		
S&P500	5.52	7.81	9.56	4.82	6.81	8.34	3.66	5.18	6.35
FTSE100	5.66	8.01	9.81	3.68	5.21	6.38	3.86	5.46	6.69
NIKKEI225	7.77	11.00	13.47	0.84	1.19	1.46	10.29	14.52	17.78
DAX	7.76	10.98	13.44	7.27	10.28	12.59	4.59	6.49	7.95
S&P/TSX	4.74	6.71	8.22	2.99	4.24	5.19	2.67	3.77	4.62
CAC40	7.33	10.36	12.69	6.11	8.65	10.59	5.53	7.82	9.57
SM	6.24	8.83	10.82	4.07	5.75	7.05	0.07	12.83	15.71

Notes: The table reports the VaR estimates based on historical data. The significance level is 1% and VaR are calcuated based on 5-, 10- and 15-days time horizons. Unconditional distribution measures are based on historical returns, while conditional distribution are those obtained by weighting the standardized residuals by the forecasted volatility.

Table 6: Unconditional, Conditional and Independence Coverage Tests based on Log-normal Stochastic Volatility model

	Unconditional		Independence		Conditional	
	1% (LR_{uc})	5%(LR_{uc})	1%(LR_{ind})	5%(LR_{ind})	1%(LR_{cd})	5%LR_{cd})
S&P500	0.081 (NR)	0.224 (NR)	4.455 (R)	0.479 (NR)	4.535 (NR)	0.704 (NR)
FTSE100	3.701 (R)	1.371 (NR)	0.368 (NR)	0.0001(NR)	4.070 (NR)	1.371 (NR)
NIKKEI225	1.951 (NR)	0.003 (NR)	0.552 (NR)	0.904 (NR)	2.503 (NR)	0.907 (NR)
DAX	1.064 (NR)	2.296 (NR)	0.700 (NR)	0.776 (NR)	1.765 (NR)	3.072 (NR)
S&P/TSX	0.721 (R)	0.148 (NR)	12.843(R)	16.01(R)	13.56(R)	16.15(R)
CAC40	3.071 (R)	0.224 (NR)	10.521 (R)	0.479 (NR)	13.59(R)	0.704(NR)
SM	1.064 (NR)	2.043 (NR)	0.700 (NR)	0.052 (NR)	1.765 (NR)	2.095 (NR)

Notes: The table reports the unconditional, conditional and independence coverage tests based on the Log-Normal Stochastic Volatility model. R indicates rejection and NR indicates NO rejection of the VaR model -Significance 10%

Table 7: Unconditional, Conditional and Independence Coverage Tests (Christoferesen book) Results based on Regime Switching model

	Unconditional		Independence		Conditional	
	1%	5%	1%	5%	1%	5%
S&P500	1.479 (NR)	1.176 (NR)	0.623 (NR)	0.566 (NR)	2.102 (NR)	1.742 (NR)
FTSE100	5.171 (R)	2.843 (R)	2.252 (NR)	0.010 (NR)	7.424 (R)	2.854 (NR)
NIKKEI225	1.94 (NR)	0.021 (NR)	0.552 (NR)	2.523 (NR)	2.497 (NR)	2.545 (NR)
DAX	5.977 (R)	2.296 (NR)	2.102 (NR)	0.035 (R)	8.079 (R)	2.33 (NR)
S&P/TSX	2.475 (NR)	1.581 (NR)	0.486 (NR)	5.317 (R)	2.962 (NR)	6.898 (R)
CAC40	1.064 (NR)	0.829 (NR)	0.700 (NR)	0.074 (NR)	1.765 (NR)	0.904 (NR)
SM	3.071 (R)	0.995 (NR)	2.746 (R)	2.188 (NR)	5.818 (R)	3.184 (NR)

Notes: The table reports the unconditional, conditional and independence coverage tests based on the regime switching model. R indicates rejection and NR indicates NO rejection of the VaR model -Significance 10%
