The Impact of the Interaction of Managers and Clients on Market Prices

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ABSTRACT: Traditional agent-based models are inhabited by boundedly-rational learning agents who switch between different trading strategies. We develop a model of a speculative market where the *clientele* (principals) allocate endowments across heterogeneous managers (agents). Each manager utilises different investment styles: fundamental, momentum and index. Without any learning, modelled by clients' reallocation of these investment funds, the market displays destabilised behaviour. This is a common finding in the literature if the market is dominated by momentum trading. We examine the market impact when clients actively seek to maximise their wealth by periodically reallocating their funds towards better performing managers. Intertemporal dynamics show that the fundamental manager initially benefits from this reallocation (learning) process at the expense of the momentum manager, but eventually loses out to the index manager. The fund outflows from the momentum manager reduces the level of market mispricing, albeit slowly. The correction is especially slow with serially correlated information signals. Even then we suggest that our model is likely to overstate the speed at which this destabilisation dissipates, and that such behaviour could characterise markets over extremely extended periods.

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Agent-based models have become a popular tool to examine the pricing implications of the interaction of investors pursuing different investment styles. In particular, they provide an alternative approach to behavioural finance for explaining the anomalous pricing that has been highlighted in numerous empirical studies. Bird et al. [2007] have demonstrated that markets composed of managers implementing various different styles display the under- and subsequent over- reaction to information that has been found in the empirical evidence. They highlighted that it is the momentum investor that is most disruptive to the price formation process, but also that the over-extrapolating fundamental investor has a destabilising influence, especially in trending markets.

The question that this raises is whether there is anything inherent in the way that markets operate that will limit the sustainability of the mispricing. The answer lies in the behaviour of the end investor and, in particular, the clients of the investment managers. Indeed, each client would be expected to pursue investment styles with the aim to maximize her wealth outcome irrespective of the pricing implications of the resource allocations in the selected styles (Barberis and Shleifer [2003]). In this paper we extend the model by Bird et al. in order to include such clients who switch between managers in the pursuit of the superior risk-adjusted return[1].

Our findings suggest that over time, there will be a tendency for clients to switch away from the more destablising managers, and, as they do, the extent of mispricings in markets will reduce. However, the speed at which this adjustment takes place will very much depend on the initial allocations of funds across the managers using differing investment styles, the extent to which information flows in markets are random rather than serially correlated, and the rules that clients adopt in determining the money flows. Overall, the evidence suggests that the mispricings, resulting from the destabilising impact on markets from investment styles such as momentum, will continue over an extended period of time if not forever. Our findings also shed some light on a number of other interesting issues, such as the impact of the interaction between managers pursuing different styles on the returns realised and the risks faced by each of the managers considered and also the implications of various switching rules followed by clients on their realised risks and returns. Interestingly, we obtained in many circumstances that switching would erode the clients' after-fee performance, questioning the investment incentives for clients to contribute, through wealth reallocation, to the learning process formulated in this paper.

The paper is organised as follows. In Section 1 we provide an overview of the literature and highlight the motivation for this study. We proceed in Section 2 to develop the model that we subsequently use to create a speculative market. In Section 3, we report the major findings of our model and discuss their significance. Finally, in Section 4 we provide an overall summary and discuss the implications for future research.

[1] This contrasts with other papers where it is considered to be the managers/traders who learn with respect to the success of their styles and so switch between styles (see, for instance, Lux [1995;1998], and Lux & Marchesi [1999;2000]). Such a learning strategy is unrealistic as managers have strong incentives not to vary their style, including being bound by contracts that preclude it.

Section 1: Background

Traditional economics assumes the existence of representative rational agents who act to maximise their utility. Milton Friedman [1953] was one of the first and most prominent advocates of rational agent models, envisaging the long-term situation where all the non-rational agents have been driven out by the evolutionary forces in the market leaving only the rational informed traders. Friedman's hypothesis is based on the full rationality of the traders, which incorporates the somewhat extreme informational assumption that all traders should know "…with certainty what their own payoffs would be in each period of time" (Copeland et al. [2005], p.363), with the only uncertainty being the asset clearing price.

Grossman [1976, 1977] and Grossman and Stiglitz [1980] stated that if in a competitive equilibrium arbitrage profits are eliminated by fully informative prices, it is impossible for the economy to be in equilibrium in the first place, consequently opening a breach in the efficient market hypothesis (Fama, [1970]). Figlewski [1978] was the first to extend the Grossman and Stiglitz's model to the case where the accumulated wealth of agents was a key factor in weighting each agent's demand, though he assumed that each agent knew the wealth level of the other(s). In this case the market does become informationally efficient if the distribution of wealth converges to a Pareto-optimal allocation.

The paradox identified by Grossman and Stiglitz provided crucial stimulus for the sunsequent development of many theoretical models of financial markets. DeLong et al. [1990], for instance, created a market where irrational noise traders through the destabilisation of market prices are able to earn higher expected returns. The explanation for this to happen is that rational arbitrageurs, because of the risk induced by the noise traders, cannot aggressively eliminate the mispricing existing in the market.

Sargent [1993], Arthur [1995] and Hommes [2001] questioned the unrealistic assumption of strong rationality, and tested whether Friedman initial hypothesis could still hold in a market where traders are only boundedly rational. Hong and Stein [1999] developed a framework where the interaction between informed traders (newswatchers), and uninformed (momentum) traders, both with constant absolute risk aversion (CARA) and bounded rationality, are able to explain the existence of an initial underreaction to private information and a subsequent overreaction before prices finally correct. This stream of research stimulated the development in recent years of a literature in heterogeneous agent-based models (HAM), comprising different interacting non-fully-rational agents populating financial markets (see also Brock & Hommes [1997;1998], Lux [1995;1998], Brock and LeBaron [1996], Farmer and Joshi [2002], and Iori [2002]). The main objective of these models has been to evaluate the impact of different beliefs about the future evolution of prices, relying in some cases (see Levy et al. [2000], and He & Li [2007]) on computational methods to overcome the obvious limitations of solving analytically the complex relations existing between agents. These developments provide a more realistic approach to the analysis of financial markets, where traders differ with respect to access to information, ability to process information, and/or attitudes toward risk, although they do suffer from an analysis that is subject to many degrees of freedom (see Hommes [2005]).

Almost all of the computational agent-based models rely on a particular learning process, based on the relation between market prices and available information, in order to regulate the traders' interactions. The evolutionary learning dynamics has been implemented using various tools, such as genetic algorithms based on fitness functions (Lettau [1997]), variation-imitation-decision (VID) models that help agents to augment their behaviour (see Brenner [1996]), neural networks structures where traders learn on lagged prices (see Beltratti & Margarita [1992]), and adaptive belief models based on discrete choice probability (Brock & Hommes [1997]).

Regardless of the 'learning toolbox' applied in the literature to impound the evolutionary forces into the price formation process, all the models relate the learning mechanism to the agents, who are able in this way to learn from their previous "mistakes" generated by their past investment strategies, and consequently adapt their believes and act by switching to other supposedly more sophisticated strategies (see, for instance, Lux [1995;1998] and Lux and Marchesi [1999;2000]). The agent role played in capital markets is universally performed by investment managers who market themselves on the basis of an investment style and commit themselves to this style through contracts they enter into with their clients. Despite the proliferating literature on the performance of professional managers, little attention has been devoted to the rules regulating the contract between managers and clients. Almazan et al. [2003] have been the first to analyse the implications of contractual restrictions, and found that they can be particularly strict and constrain the operations of mutual fund managers in the attempt to limit the costs arising in the agency problems between managers and clients (see also Smith & Warner [1973]).

The perception that the competition in (financial) markets represents some sort of selection environment is one of the fundamental ideas that permeate theories of evolutionary economics. Analysis of search and selection were already evident in a stylised evolutionary system of Schumpeter [1934], where a carrot and a stick motivate agents to reach better production methods or product (services). A crucial factor in most theorising by economists about market selection environments is the existence of a clear separation between agents (e.g. *money managers*) on the one hand, and principals (e.g. *institutional clientele*) on the other. The principal's evaluation of the agents' performance represents the dictate that should drive the resource allocation in the market (see Nelson & Winter [1982]).

Therefore, the evolutionary process within markets is not realistically driven by the learning process of the agents (managers) but rather by the choices made by the principals (the manager's clients)[2]. In this paper, we develop a dynamic HAM, where the resource (wealth) allocation to the money manager is more naturally regulated by the decisions made by the ultimate investor (namely, their clients) as who will manage their funds. In our model this decision is based on a risk-adjusted evaluation of the managers' performance. Clients compete with each other to select skilled (rational or irrational) managers who can deliver superior performance. Therefore, superior past risk-adjusted performance is evidence of this skill, so clients move their funds from the worst to the best performing managers[3]. To our knowledge, this analysis constitutes the first attempt in the HAM literature to translate the allocation of money flows by the clients into an indirect learning process of the system. Moreover, although unrealistic, we will assume for simplicity and model tractability that clients would shift 100 per cent of their wealth from the worst to the best managers, regardless of the style-nature and risk connotation[4]. Moreover, our

[2] The managers not only stick to their style because of contractual restrictions but also because it has been shown that style drift results in a deterioration of their performance (see, Brown and Harlow, 2004).

[3] We do not assume that managers' superior performance has a decreasing returns to scale, cause this would imply an inverse relationship between clients' flows and consequent managers' performance.

[4] The alternative would be a more complex gradual shift of wealth from one manager to the others. Our expectation is that the only difference would be in terms of a longer time for the system to converge to the Pareto- optimal allocation.

bounded rational manager comprise the classic categories of fundamental, momentum (chartists) and index (passive) managers, in order not only to investigate the relation, if any, between the clients' money flows, time to market equilibrium, and degree of mispricing, but also the excess returns, volatility and final market allocations of wealth among the styles of money managers considered.

Section 2: The Model

Initial set up and the fundamental value

The approach that we take in this paper is to create a model of a speculative market with three heterogeneous managers (agents) who commence with different endowments as determined by their clients. The managers' interaction of information and wealth, which is composed of both a risk-free asset (cash) and a risky asset (shares), contribute to both the price formation process and the degree of market efficiency. The manager's investment decision relates to moving the wealth under their control between the risk free asset and the risky asset. The risk free asset is perfectly elastically supplied and pays a fixed gross rate of return of $R=1+r/K$, where r represents a constant quarterly risk-free rate[5], and K stands for the frequency, assumed to be weekly, of trading period per quarter (thus K is equal to 13 periods per quarter).

The sole source of information that affects prices is quarterly earnings announcements where the quarterly growth in earnings (g_T) is

- either a *random draw* (each quarter from a distribution with an assumed mean and standard deviation of 1 per cent),
- or *serially correlated*, oscillating around a long-term growth rate of 1% per quarter where the complete cycle takes three years (i.e. 12 quarters) each composed of a upward drift in earnings growth for six quarters followed by a downward drift in earnings growth (i.e. mean reversion) over the subsequent six quarters. Again there is a a random element (η_T) around this - otherwise regular - pattern over the three year cycle[6]. The equation for generating the serially correlated earnings growth numbers is given below:

$$g_T = g_0 + \delta \sin(\xi_T) + \eta_T \qquad \eta_T \approx N(0, 0.01) \tag{1}$$

where ξ_T represents the quarterly radians varying between 0 and 2π over the 12-quarter cycle, and δ (set equal to 1) represents the magnitude of the sin function.

The fundamental price is a very important part of our analysis as it serves as the benchmark against which to judge the efficiency of the prices actually established in the market. The fundamental price is calculated each quarter, after the release of the quarterly earnings, and it reflects our knowledge of both the current level of earnings and the process by which future earnings growth will be generated. For example where this process is random, then it is assumed

[5] This could also be modeled to follow some stochastic process.
[6] This pattern of earnings growth is representative of many of the findings in the literature (see DeBondt & Thaler [1985], Hong & Stein [1999], and Soffer & Walther [2000]).

that future earnings will grow at a constant g_T equal to 1 per cent per quarter[7]. The cost of equity capital (r) for the stock is assumed to be 2 per cent (fixed) per quarter.

The fundamental price (P_T^*) for the random earnings growth will be given by:

$$P_T^* = \frac{\lambda E_T(1+g_0)}{(r-g_0)} \tag{2}$$

where λ (equal to 0.8) is the fixed payout rate, and E_T (equal to \$1 in the first quarter) is the EPS in quarter T. Based on this information the initial share price will be \$80.80 as shown by the following calculations:

$$P_0^* = \frac{0.8(1)(1+0.01)}{(0.02-0.01)} = \$80.80$$

In order to establish whether and how different combinations of heterogeneous traders could affect the market dynamics, we test various possible scenarios of initial managers' endowments (or weights). These different initial *market fractions* are set out in Table 1. For each test case we run 400 Monte Carlo simulations in cases of either random or serially-correlated earnings growth over a period of 40 years (2,080 weeks). Moreover, for comparability purposes, we use the same Monte Carlo earnings paths for all the different market structures. This means that all the results we document are generated from the *same seeds* of random or serially correlated earnings paths. The model is calibrated with the objective that an equivalent information signal I_t will translate into a similar level of trading for each of the bounded rational managers.

Heterogeneous trading demands

As mentioned previously, the market is made up of a *fundamental manager*, who possesses superior information on the fundamental value of the risky asset, a *momentum manager*, who trades upon price movements over some prior period, and an *index manager*, who simply trades to replicate the market index. In the following discussion, we outline the nature of the demand function of each type of investor.

The *fundamental managers* believe that the market price is mean reverting to their perceived fundamental price. Therefore, give the common information set (I_t) formed at time t:{P_t, P_{t-1},…, D_t, D_{t-1},…}, they purchase (sell) the stocks when the current market price is below (above) the perceived fundamental price. The way they determine their perceived fundamental price is by applying a dividend discount approach in the same way as in the case of the calculation of fundamental price and so as a result they always "know" the fundamental price. Besides, the fundamental managers only trade when the difference between the current price and their perceived fundamental price is either above or below a no-trade zone, identified by a certain percentage, say $\overline{\alpha}_s \in [0, 1]$, of the current price level, in recognition of risks, and various trading and financing costs, associated with the investment. Because they are assumed to stabilise the market, their demand is proportional to the deviation of the market price from the fundamental price. Based on these assumptions, the demand of the fundamental investors $z_{s,t}$ at time t may be defined by the following piece-wise linear function:

[7] Similarly with serially correlated earnings growth, the fundamental price reflects the current level of earnings and the future pattern of earnings' growth ignoring the noise element.

$$z_{s,t} = \begin{cases} \alpha_s\left[P_{t,i}^{*} - (1+\overline{\alpha}_s)P_t\right] & if \quad P_{t,i}^{*} > (1+\overline{\alpha}_s)P_t \\ 0 & if \quad \left|P_{t,i}^{*} - P_t\right| < \overline{\alpha}_s P_t \\ \alpha_s\left[P_{t,i}^{*} - (1-\overline{\alpha}_s)P_t\right] & if \quad P_{t,i}^{*} < (1-\overline{\alpha}_s)P_t \end{cases} \tag{3}$$

where $P_{t,i}^{*}$ represents the perceived fundamental price, with i equal to the length of the extrapolation of the fundamental trader considered. The parameter $\overline{\alpha}_s > 0$ measures a required premium that incorporates both transactions costs and a compensation for the risks associated with the investment.

The price *momentum managers* (technical traders) believe that they can extrapolate the future price from the various patterns generated from the history of prices. Therefore, they purchase (sell) stocks that have changed in price over the previous f weeks (formation period) at an average growth rate greater (less) than g_0 equal to 1 per cent, and to hold the position created for h weeks before reversing the transaction. As in Bird et al. [2007], we consider two kinds of risk-averse momentum traders: a conservative short-term implementation, with a moving average rule (MA) based on a formation period of 13 weeks and a holding period of six weeks (i.e. f = 13, h =6), and an aggressive long-term implementation, with an MA based on formation period of 26-weeks and a holding period of 26-weeks (i.e. f = 26, h = 26)[8]. Moreover, as in the case of the fundamental managers, the momentum managers purchase (sell) stocks only when this difference is above (below) a non-trade zone identified by a certain percentage, say $\overline{\alpha}_m \in [0,1]$, which measures the required premium incorporating both transactions costs and a compensation for the risks associated with the investment. Because of their risk aversion, they increase their (long/short) positions initially when the trading signals generated are strong enough, though they are cautious when such signals become too strong. Based on the above assumptions, the demand function of the momentum investors $z_{m,t}$ at time t may thus be defined by the following piece-wise nonlinear increasing function of the trading signals (while the marginal demand is decreasing):

$$z_{m,t} = \begin{cases} \alpha_m \tanh\left(\frac{\beta_m}{\sigma^2}\left[P_t - (1+\overline{\alpha}_m)(1+g_0)P_{t-f}\right]\right) & if\ P_t > (1+\overline{\alpha}_m)(1+g_0)P_{t-f} \\ 0 & if\ \left|P_t - (1+g_0)P_{t-f}\right| < \alpha_m(1+g_0)\,P_{t-f} \\ \alpha_m \tanh\left(\frac{\beta_m}{\sigma^2}\left[P_t - (1-\overline{\alpha}_m)(1+g_0)P_{t-f}\right]\right) & if\ P_t < (1-\overline{\alpha}_m)(1+g_0)\,P_{t-f} \end{cases} \tag{4}$$

where $\alpha_m > 0$ measures the demand intensity, and $\overline{\alpha}_m > 0$ quantifies the trading costs incurred by the transactions.

The third typology of agents is constituted by the *index* (passive) *managers*. If we assume that the risky asset represents the benchmark, the index managers simply buy the current market price. Hence, their demand ($z_{i,t}$) is always equal to zero. The expectation is that the larger the

[8] Given the periodicity of the pricing cycle produced in our simulations, the shorter-term momentum investor is somewhat akin to the early-stage momentum investor identified in Lee and Swaminathan [2000] while the longer-term momentum trader is more akin to the late-stage momentum investor. The choice of the length of the formation and holding period of the two momentum traders does not affect the conclusions in terms of degree of market efficiency and stationarity of the system.

weight of the index traders in the market, the less the level of liquidity and, hence, the slower the mechanism of price adjustment to the fundamental price and the lower the level of volatility.

Clients' flows and wealth function

As proposed by Cootner [1967], the wealth redistribution from agents with inferior information to agents with superior information should permit the market, if initially not in equilibrium, to achieve and maintain informational efficiency in the long term. When convergence to efficiency is obtained, theoretically there should not be any difference between the final allocation of wealth among agents and their information quality. In the short run, though, if informed agents, say fundamental managers, have their forecast of the future dividends underweighted in the market, the price will not perform its function of aggregating information according to its quality, and it will not constitute the best estimate of the future price. We do not assume constant absolute risk aversion utility functions when we model the managers' demands because we want the latter to be dependent on managers' wealth (see also Levy et al. [2000], and He et al. [2007]). If agents possess superior information, they will progressively accumulate wealth over time, with their forecast being impounded more heavily in the market price. Consistent with this view, the contribution of this paper is to assume the presence of clients (principals) who carry out the task of allocating and redistributing their wealth between the managers (agents). In particular, the initial endowment of each manager as determined by their clients is constituted as follows: 5 per cent in cash position and 95 per cent in shares. To regulate the wealth allocation, we consider four possible types of clients who differ in terms of the rule that they follow for reallocating their funds between manager: one client who never redistributes irrespective of each manager's performance (C_0)[9], and the other three who redistribute the wealth at intervals of one, three and five years, according to some measure of performance calculated over the previous year (C_1), three years (C_3), and five years (C_5). The measure adopted by the clients to rank the managers according to their risk-adjusted performance is based on the relation between managers' excess return and its volatility (tracking error), both computed over the clients' different evaluation periods (one, three, and five years). The performance measure is expressed as follows: $\mu - \delta\omega$, where μ and ω represents, respectively, the manager's excess return and tracking error, whereas δ constitutes a measure of the risk-aversion (equal to 0.5[10]) of the clients.

After the initial allocation, if the current simulated week, say week 52 (or 1 year), does not match with a client switching strategy, say C_1, the cash ($Q_{p,t}$) and share ($N_{p,t}$) components of the wealth ($W_{c,p,t}$) of each client c in each manager p at time t, are simply carried over to the next period. The weekly calculation of the wealth, hence, can be expressed as follows:

$$W_{c,p,t} = w_{c,p,t}\left\{N_{p,t}P_t + \left\lfloor Q_{p,t}(1 + r_f/K) - \Delta N_{p,t}P_t \right\rfloor\right\} - F_{c,p}, \tag{5}$$

where $w_{c,p,t}$ denotes the proportion of wealth of client c allocated to manager p at time t, and $\Delta N_{p,t}$ indicates the time-t demand for manager p. We also introduce the presence of a

[9] In presence of clients with differing liquidity needs, high quality fund managers could impose stellar exit fees which obstruct withdrawals. Moreover, in addition to back-end loads, tax efficiency considerations, switching costs and other market frictions could impose disadvantages for clients who would like to mobilise their capital.

[10] We tested the sensitivity of the results to different values of the clients risk aversion and found that although the higher the aversion (e.g. $\delta>1$) the less the clients' flows to momentum traders, our conclusions in terms of time to stationarity and degree of market mispricing are largely unaffected.

shareholders' fixed-fees scheme $F_{c,p}$ to represent the costs incurred by managers in running their operations, such as for instance advisory fees, brokerage fees, and custodial, transfer agency, legal, and accountants fees. The fees, whose amount depends obviously on the type of manager considered, are automatically deducted each week from the cash position of each client *c*. The annual fees are assumed to be equal to 50 basis points for the fundamental manager, 30 basis points for momentum manager, and 10 basis points for the index trader (see also Sirri and Tufano [1998])[11].

The actual number of shares ($n_{p,t}$) to be purchased by manager *p* is determined by the product between their wealth, relative to the total wealth across all the managers in the market, and the level of their demand ($z_{p,t}$), as determined from their demand function. Based on the previous discussion, the excess demand of the risky asset $z_{e,t}$ at time *t* is then given by:

$$z_{e,t} \equiv \sum_{p=1}^{P} n_p z_{p,t}. \tag{6}$$

Market Maker and Market Clearing Price

In addition to the three types of managers, we also have a market maker whose role is to mediate transactions on the market out of equilibrium by providing liquidity. Hence the market maker will take a long position when $Z_{e,t}<0$ and a short position when $Z_{e,t}>0$. The change of the market makers' inventory G_t of the risky asset is equal to $Z_{e,t}$[12]. At the end of period *t* after the market maker has transacted, he adjusts the price for the next period in accordance with the experienced excess demand. Using μ to denote the corresponding speed of price adjustment of the market maker (or equivalently his aggregate risk tolerance) to the excess demand, then the classical price tâtonnement process at time *t+1* is given by:

$$P_{t+1} = P_t + \mu z_{e,t} \qquad \mu > 0. \tag{7}$$

The market maker behaviour in this model is highly stylised and similar to that proposed by Day and Huang [1990], where the market makers, in a market populated of fundamental and momentum traders, end up buying when the price is too high and selling when the price is too low. However, they could offset their losses by investing in their own account and by imposing fees for conducting the market.

The result of the calibration for all the simulations considered returned the following set of the parameters:

$$\mu = 0.5,\ \ \alpha_m = 5,\ \ \alpha_c = 2\mathrm{e}+03,\ \ \overline{\alpha}_s = \overline{\alpha}_c = \overline{\alpha}_m = 0.03/13,\ W_{c,p,0} = 6250$$

Section 3: The Findings

In a previous paper, Bird et al examined the impact on pricing of the interaction within markets of agents (managers) pursuing several different investment styles and found that combinations

[11] These fees are to be interpreted as management fees only. Therefore, we do not consider other fees usually charged by mutual funds in the market, such as the distribution fees (12b-1) and the expense-ratio fees. For a more detailed description and evidence on investment managers' fees, see Coates and Hubbard [2006] or refer to the documentation provided by the Investment Company Institute (ICI) at http://www.ici.org/.

[12] We also aim to extend our analysis to the case where the excess demand or supply by reducing or increasing the inventory of the market maker, leads the latter to vary the price according to $Z_{e,t}$ (see also Farmer and Joshi [2002]).

of styles typical of current markets had a very disruptive influence on the price formation process. In particular, they found that two very common types of managers in today's markets, momentum and over-extrapolating fundamental, cause both a wide oscillation of prices around their fundamental level and a significant increase in the risks faced by the other managers. The implication being that increased risk and inefficient pricing will remain a permanent feature of markets as long as these investment styles continue to be used by managers who have access to a large proportion of the funds invested by shareholders.

The probability of this remaining the case depends mostly on the willingness of clients to continue to allocate funds to managers pursuing such destabilizing strategies. This constitutes, at the same time, the motivation for us to consider in this paper the role of the clients, who initially allocate their funds to managers pursuing a range of investment styles, but who then become the engine for the redistribution of wealth in the market on the basis of a risk-adjusted evaluation of the performance of the same managers[13]. We report in this section our findings on the impact of the money flows on the price formation process within the system considered, and the returns realised and risks faced by both managers and their clients.

Test Cases

In order to evaluate the questions posed in this paper, we examine eight test cases as set out in Table 1, which differ in terms of the proportion of wealth that each style of manager constitutes of the total wealth as at the beginning of each simulation. We allow these proportions to change over time both because the managers will realise differing investment returns and because the clients periodically will move their funds from the worst to the best manager.

Mispricing

We track the price as well as the fundamental value of the risky asset at the end of each trading period (week), with our measure of mispricing being the standard deviation of the difference between these two levels. In Table 2 we document the degree of mispricing for the case where there is no learning (i.e. where the clients do not switch between managers) and for where there is learning, in the case where information signals are both random and serially correlated. We confirm the results from our previous study (without learning) that there is little mispricing where the market is composed on either the fundamental manager alone or the fundamental manager in combination with the index manager. However, mispricing greatly increases with the introduction of momentum managers and especially the long-term momentum manager whose implementation is less synchronised with the cycles in the market. This seems consistent with the analysis by Chiarella et al. [2006], where the extension of the lag length of a moving average rule for a momentum trader makes the rule "…smoother and more sluggish".

A question of particular importance to us is the extent to which the level of mispricing within markets is sensitive to the learning process. The good news is that in almost all test cases there is some, usually small, reduction in market mispricing where learning is allowed to occur. There are two cases where the mispricing measure reduces by around 25% where information signals are random, and they typically fall by a substantial, but lesser, amount where the information

[13] We outlined in Section 2 of this paper, the assumed rules followed by the clients when undertaking the reallocations of the wealth.

signals are serially correlated. The major reductions to mispricing are tied to those test cases involving a substantial allocation to managers who adopt a momentum strategy. This is a consequence of the general drift of funds away from these managers to fundamental and/or index managers, each of whose trading is less disruptive to the price formation process than is the case with momentum managers. In contrast to the other findings, learning leads to an increase in mispricing in test case 5 where we have a combination of fundamental and index managers. This finding is the direct consequence of the gentle drift of funds from the fundamental manager to the index manager which results in prices adjusting more slowly to the release of new information and, hence, gravitating slightly further away from the fair value. We will discuss these findings in more detail when we will concentrate, in the next sub-section, on the intertemporal pattern of wealth (funds) holdings experienced by the managers under the various test cases, both *with* and *without* learning.

Wealth holdings and mispricings

We report, in Table 3, the movement in the proportions of the relative wealth managed by each manager over the 40-year period covered by our simulations[14]. In those cases where there is no learning, this shift simply reflects the relative performance of the managers over the entire simulation period. Without learning, the fundamental manager almost always achieves the highest proportion of funds under management as a consequence of exploiting the mispricings created by the momentum managers, especially the long-term momentum implementation. The only case where the fundamental manager loses out in terms of funds under management is where his only competitor is the index manager (test case 5), because in this case there is little mispricing that the fundamental investor can exploit to offset his fee disadvantage.

When learning is introduced into the process via the clients switching between managers, there is a much larger movement in relative wealth towards the index manager than was the case without learning. This is to be expected, as the bulk of switches will be away from momentum managers and towards the index manager and, to a lesser extent, the fundamental manager. It is the long-term momentum, rather than the short-term momentum, manager who loses the most funds. This is best seen by looking at the wealth movements under test case six where the short-term momentum manager basically maintains his initial funds whereas there is a large transfer of wealth away from the long-term momentum manager. Finally, it should be noted that the switch away from the momentum managers to the other two managers is less where information signals are serially correlated reflecting that such markets produce longer trends which are more beneficial to the momentum style thus resulting in better performance of the momentum managers and so less loss of funds.

Moreover, it is also evident from Table 3 that there are test cases typically involving an index investor where the momentum managers seem to have lost almost all the funds that they had under management for their clients (C_1, C_3, and C_5), except for those managed on behalf of the client who never switches (C_0). This suggests the possibility that, under at least some of the test cases within the 40 years, a steady state is reached for the wealth allocations across the various managers. We get some insight into this issue from Figure 1, which illustrates the path of the average wealth managed by the managers under test cases 2 and 7 in the case of both no learning and learning in a market characterised by serially correlated information signals. A perusal of these subplots would suggest that a steady state of funds under the care of the various managers

[14] The wealth managed by the various managers is synonymous with their assets under management

is reached much faster where learning occurs, as a result of clients' reallocations, than in the case when no such flow of funds takes place.

In order to obtain more precise information as to when a steady state first sets in, we divided the 40 years of experience for each test case into four sub-periods of 10 years each, and used the *Augmented Dickey-Fuller* [1979] test (hereafter, ADF) to pinpoint the sub-period when (if ever) the equilibrium of the system is first reached. Our findings are reported in Table 4 – panel A. Consistent with our expectations, markets composed solely of fundamental and/or index managers (test cases 1 and 5) are always in a steady state both with and without learning and any mispricings are minimal. However, once momentum managers are introduced and in the absence of learning, there is only one instance of where the relative wealth allocations reach a steady state within the 40 years (namely, test case 7 where information signals follow a random walk). The introduction of learning results in a steady state being obtained in all test cases with the length of time required for this to occur being shorter (i) where information flows are random and (ii) where there is a long-term, rather than a short-term, implementation of the momentum strategy.

The results of the ADF lead also to the investigation of the extent of price correction perpetuated by the wealth redistribution. Indeed, an obvious question to ask is just how efficient do the markets become once a steady state has been reached in terms of the allocation of cash and share positions across the various types of managers? With this question in mind, we report in Table 4 – panel B, the calculated level of mispricing for the periods *before* the steady state is reached (*ex-ante*) and *after* the steady state set in (*ex-post*)[15]. In the case where information signals follow a random walk, there is a relatively large fall in all relevant test cases but to levels where mispricings, in most cases, is still observably higher than that exhibited by markets composed of only fundamental and/or index managers. The extent of the reductions in mispricing are slightly less in the case of markets characterised by serially correlated information flows reflecting the fact that the momentum managers retains a higher proportion of wealth under management in these markets and so continue to have a more disruptive impact on prices.

The conclusion that we draw from table 4 is that the learning process has a positive impact in reducing the level of mispricing that is largely attributable to the destabilising impact of the momentum managers. However, it is apparent that the initial mispricing is far from completely removed within the 40 years covered by our simulations of the market settings that we have considered. What is more, it is clear that in most test cases we have achieved a steady state in terms of wealth allocation across the various managers within the 40 years, suggesting the possibility that the learning process never leads to a situation of efficient pricing. We would only point out at this stage that the impact of the switching process on market mispricings has been overstated if anything. We will discuss this in more detail in Section 4 where we conclude that the switching process used by clients is unlikely to provide the sole solution for correcting the mispricing that we observe in markets.

[15] The figures calculated are based upon the presumption that the equilibrium persists from the beginning of the 10-year period in which it is identified. The more conservative assumption that stationarity persists from the end of this period, instead, would be based on the hypothesis that a level of relatively small mispricing is reached after a longer period of time. However, we did not find any significant different in the findings by adopting one or the other time cut-off.

Market behaviour

Even though not documented in the tables, the annual returns based on the fundamental and market prices are 4% (the difference between the rate on the risky asset and the rate on the risk-free asset), in the case of random information flows, and 4.5% in the case of serially correlated information flows[16]. The standard deviations of the market returns across the eight market structures are reported in Table 5. In the absence of any learning, it is evident that the momentum managers, and especially the short-term momentum manager, induce a large level of volatility in markets. When we move our attention to considering the effect of learning on the absolute volatility of the market returns, we notice that, in general, learning leads to significant reductions in the magnitude of the volatility in the market (similar to Hommes [2005] with slow learning). Indirectly, this is caused by the funds being directed away from the momentum managers with at least initially much of it going to the informed (fundamental) manager, whose actions ensure that information is more quickly impounded in price and, thus, causing the volatility of the market returns to converge towards the volatility of the returns based on the fundamental price.

Manager performance

In Table 6 Panel A, we report the after-fee excess returns and three risk measures (namely, volatility, tracking error and downside risk[17]) for the managers under each test case, *with* and *without* learning, where the information signals are random. We repeat the analysis when the information signals are serially correlated in the panel B of the table. The major findings with respect to returns can be summarised as follows:

- The fundamental manager experiences a positive excess return (after fees) in those cases where no learning is present and the long-term momentum manager is present. There is a sizable reduction in these excess returns when learning is introduced, especially in the random markets, reflecting the fact that the reduced presence of momentum managers marketing markets results in there being less opportunities for the fundamental manager to exploit. In the Black's phrasing, "[be]cause information traders trade with noise traders more than with other information traders, cutting back on noise trading also cuts back on information trading" (1986, p.533).
- In contrast, the biggest loser is the long-term momentum manager, who underperforms by in excess of 1.0% in those cases where there is no *Darwinian* selection by the clients. These losses reduce significantly when there is learning, reflecting the reduced mispricing that exists in such circumstances which translates into "better" performance for the long-term momentum manager and so less opportunities for the fundamental manager.
- As one would expect the index manager always generates a negative return just equal to the fees that they impose on their clients.

[16] Please note that fees do not impact on these returns.
[17] Our downside risk measure is the median of the minimum annual return earned over any 12-month period across all the Monte Carlo simulations.

In relation to the risks faced by each of the managers, while it is true that the three risk measures do not tell exactly the same story, it appears that the long-term momentum manager faces the greatest risk, followed, in order, by the fundamental manager, the short-term momentum manager and, finally, the index manager. The momentum managers lead to more absolute and relative volatility as a result of causing markets to overshoot their fundamental value in both directions and this causes the volatility experienced by the other active investors to also increase. The level of risk introduced by the momentum managers seems also to be slightly higher in markets with serially correlated signals than in those with random signals. More salient is the fact that the risk faced by each manager typically decreases with the introduction of learning, reflecting the increasing stationarity of the system and the greater efficiency of the price in incorporating all the available information. Therefore, learning in the market should provide a fillip to the fundamental managers in their attempt to drive the price back to fair valuation.

Client performance

As a final stage of this study, we analyse the risk and (net of fees) return performance of the clients. It is of interest to evaluate whether any of the particular rules assumed in the model proves optimal, *a posteriori*. As under each test case each client starts with an equal amount of wealth allocated in cash and shares to each manager, they obviously all realise the same return and experience the same risk where there is no reallocation (namely, no learning). Therefore, in this scenario the return that they received is the market return less the weighted average of the fees imposed by each kind of manager.

In Table 7, we provide evidence of the actual results for the clients of the reallocation of their funds, in terms of opportunities for differing risk/return outcomes across managers. An examination of the table provides some suggestion that switching results in slightly better outcomes for clients where information (and so market prices) are random and where there is the alternative to invest through index managers. This reflects the fact that in such test cases, there is a general drift towards greater index management which brings down the total fees being paid by clients. This finding changes when one considers serially correlated markets where the evidence seems to support the conclusion that switching results in worse outcomes with respect to both return and risk in most of test cases. Indeed, in these cases one could make a strong case to support that no switching would be the preferred strategy.

These findings with respect to the clients are of some interest in terms of their interpretation as to how much learning we might expect to see in the markets. In the random markets, where we did see the stronger evidence that switching leads to less market mispricing, there is some evidence to suggest that switching would be a good strategy for clients to pursue, provided there is an index manager. However in the serially-correlated markets, there is little evidence to suggest that switching is a wise policy for clients to adopt, which brings to question whether we would see much learning occurring in such markets, supporting the proposition that significant mispricing might continue in these markets over extremely extended periods of time.

Section 4: Conclusions

In this study, we developed a simple asset-pricing model with heterogeneous managers in a simulated market where a learning mechanism is dictated by the relationship between managers' risk-adjusted performance and clients' fund flows. This evolutionary force exerted by the clients is consistent with the literature on mutual fund flows. Our results confirm previous evidence (Beltratti & Margarita [1992], and Brenner [1996]) that learning processes do result in some improvements in pricing efficiency within speculative markets. However, this results seems to be dependent upon a number of assumptions: (i) the initial endowments (market fractions) allocated by the clients to the managers; (ii) the switching rules clients adopt in determining the reallocation of their wealth (cash plus share positions) across the managers; (iii) the hypothetical random or serially correlated path followed by the information signals (namely, earnings announcement) hitting the market.

Our findings on clients' learning highlight that both the speed of the convergence towards a steady state of wealth allocation across managers and the resulting reduction in market mispricing is a consequence of clients seeking to maximize their wealth by moving funds to the better performing managers[18]. The results show that the market compensates (by better returns) sophisticated fundamental managers who both identify and exploit mispricing, enabling them to attract clients flows from other managers. As a consequence, the more funds are moved to the fundamental trader, the smaller the deviation of the market price of the risky asset from its fundamental value (mispricing). Hence, the quality of information used by the informed managers is rewarded, making the information search a social benefit (in the short run). This mechanism contributes to the restoration of long term market equilibrium, where the initial price destabilisation introduced by the momentum traders is reduced. As envisaged by Black [1986], the fewer irrational (momentum) traders populate the system, the smaller the mispricing opportunities available to be exploited by the fundamental manager, and hence the inferior his performance going forward. The reason for this phenomenon being that the momentum traders are responsible for diverting the price away from fair valuation, and the extent to which this occurs is directly dependent on their actual presence in the market as measured by the proportion of total wealth under their management.

Naturally, the previous process is limited in the real world as a consequence of the existence of more noise in the market which blurs the relationship between past and future performance. Our expectation is that the convergence to a steady state is slower in real markets, resulting in prices being corrected even more gradually than suggested by our naïve model. Indeed, our simulations produce market cycles that are much shorter than those experienced in real markets which results in our fundamental manager not having to wait too long to benefit from their investment decisions as a result of the price of the risky asset reverting to its fundamental value. Consequently, fundamental managers consistently outperform the momentum managers to an extent that is not replicated in the real world and, so, there is a much more rapid drift of wealth away from the momentum managers to the fundamental manager in our simulations than that we could expect in real financial markets. Therefore, our findings are likely to overstate the speed of convergence to the steady state. One can get an insight in to this expectation from our results on the faster convergence in presence of random information signals (where market cycles are extremely short) as compared to that of a system with serially correlated information flows

[18] Our model is embedded with a fixed fee scheme on managers' performance.

(longer market cycles but still much shorter than those experienced in reality). The slower convergence in the case of serially-correlated information signals is a consequence of market phases which are more favourable to the momentum investor. This results in an increase in the number of measurement periods when the momentum managers produces after-fee returns greater than those achieved by the fundamental manager.

There are several other reasons why our model overstates the speed of the convergence to equilibrium, where more of the funds are managed by the fundamental and index managers and less by the momentum managers resulting in significant reductions in the level of mispricings in markets. First, our learning process is based on a 100 per cent switch to the most productive source of management performance. Though unrealistic, we could have modeled money flows by clients with a more gradual and style-concerned approach, but the most likely outcome of this, in the current settings, would have been to slow down the time-to-stationarity of the system. Second, as documented by Gruber [1996], Sirri and Tufano [1998], Berk and Green [2004] and Berk and Tonks [2007], the empirical evidence on equity mutual funds over the past two decades suggest an option-like nature of the relationship between past performance ranking of mutual funds and growth rate in the clients' flows, with a correct concentration of funds towards high performing managers but an incorrect rate of divestment from poor performing managers. Regardless of the many reasons put forward to explain this anomaly[19], what we can assert is that should the clients' behaviour in our model follow these patterns, we would witness a greater *inertia* in their activity, with a more pronounced magnitude of mispricing, particularly when it comes to shifting funds away from the disruptive influence of unsophisticated momentum managers. Finally, the clientele preference for managers may be influenced by factors other than wealth which will influence not only their initial choice of managers but also cause them to not base their reallocation decisions purely on manager performance. For instance, Gruber [1996] mentioned the possibility of clients being more influenced by managers' skills in marketing their products rather their investment performance. Another example is the preference of clients towards managers who hold a preponderance of recently well-performing growth stocks (i.e. momentum managers) as opposed to those who hold mostly out-of-favour value stocks (i.e. fundamental managers) resulting in them being hesitant to switch even in the case of better performance by the fundamental managers. Behaviours such as these can bias the decision process of the clients, and significantly extend the period until a steady state is realised (if ever).

Our analysis indicated that the mispricing within markets, attributable to the interaction of managers pursuing different investment styles, would be reduced, but far from be eliminated by the more realistic learning process that we introduced. Further, we highlighted that the switching process offered clients little in the way of improved performance. Therefore, it is the clients that might find that switching is not a worthwhile activity and, hence, effectively remove the learning from the process. This being the case, we might revert to the situation of no learning where our results clearly indicate that in the majority of the test cases examined, the markets will remain inefficient for extremely extended periods of time, raising the open question as whether other factors would cause clients to switch other than their economic interests (*shareholder activism*).

[19] Sensitivity to fees or change in the fees, low response rate of some irrational clientele to fire underperforming managers, and inability of the rational clientele to short sell inefficient managers.

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Table 1
The Initial Endowments of the Managers

In this table, we document the initial endowments of the managers for each test case (or market structure). These percentages then vary according to both the weekly proportion of the traders' wealth in relation to the total wealth in the market and the clients' flows. We considered three types of managers. The *fundamental* trader trades on the difference between a stock's current price and its perceived fair value. The *momentum* trader trades on the basis of the price movement over some previous defined formation period and holds the stocks for a fixed holding period. The momentum managers are separated into either a short implementation, with a formation period of 13 weeks and a holding period of 6 weeks, or a long implementation, with a formation period of 26 weeks and a holding period of 26 weeks. The *Index* trader, instead, simply replicates the market.

Test Cases	Fundamental	Momentum [Short-term]	Momentum [Long-term]	Index
1	100%			
2	25%	25%	25%	25%
3	50%	50%		
4	50%		50%	
5	50%			50%
6	34%	33%	33%	
7	34%		33%	33%
8	34%	33%		33%

Table 2
The Effect of Clients' Flows on the Degree of Market Mispricings

In this table, we document the impact of the clients' learning process (implemented *via* inter-managers money flows) on the average (across the Monte Carlo simulations) magnitude of mispricing (in dollar terms) in the market. The results are reported for both a random and a serially-correlated path in the earnings-per-share (EPS)across the eight different initial market structures as documented in Table 1. As evidenced, the mispricing drops, as expected, when a learning mechanism is introduced in the system. Only test case 1 and 5 do not seem to experience such reductions. For test case 1, the mispricing remains constant because the market structure comprises only fundamental managers. In test case 5, instead, the learning process determines a larger shift of wealth from fundamental to index managers, contributing, hence, to diluting the price corrections through time.

Degree of Mispricing ($)				
Test cases	Random Information Signals		Serially-Correlated Information Signals	
	No learning	*Learning*	*No Learning*	*Learning*
1	0.49	0.49	0.90	0.90
2	4.71	4.12	5.35	4.92
3	3.85	3.19	4.17	3.67
4	5.36	4.69	5.67	4.85
5	0.86	1.25	1.98	2.59
6	5.67	4.08	6.06	4.80
7	4.35	3.71	4.99	4.54
8	3.47	2.66	3.74	3.35

Table 3
Wealth Flows between the Managers under the Various Test Cases

In this table, we document the changes in the percentages of total wealth being managed by each of the managers from the starting allocation (Table 1) and that being managed at the end of the 40 years. These percentage changes are calculated for each of the 400 simulations and then averaged. We considered three types of managers. The *fundamental* trader (Fund) trades on the difference between a stock's current price and its perceived fair value. The *momentum* trader trades on the basis of the price movement over some previous defined formation period and holds the stocks for a fixed holding period. The momentum managers are separated into either a short implementation (Short M), with a formation period of 13 weeks and a holding period of 6 weeks, or a long implementation (Long M), with a formation period of 26 weeks and a holding period of 26 weeks. The *index* trader (Index), instead, simply replicates the market.

Wealth Flows																
Test Cases	Random Information Signals								Serially-Correlated Information Signals							
	No learning				*Learning*				*No learning*				*Learning*			
	Fund	Short M	Long M	Index	Fund	Short M	Long M	Index	Fund	Short M	Long M	Index	Fund	Short M	Long M	Index
1	0%	---	---	---	0%	---	---	---	0%	---	---	---	0%	---	---	---
2	7%	1%	-10%	2%	-16%	-14%	-24%	54%	10%	-1%	-9%	0%	-5%	-9%	-14%	29%
3	8%	-8%	---	---	9%	-9%	---	---	9%	-9%	---	---	14%	-14%	---	---
4	20%	---	-20%	---	20%	---	-20%	---	17%	---	-17%	---	16%	---	-16%	---
5	-5%	---	---	5%	-11%	---	---	11%	1%	---	---	-1%	0%	---	---	0%
6	11%	3%	-14%	---	12%	6%	-18%	---	13%	-2%	-11%	---	13%	-1%	-12%	---
7	10%	---	-13%	3%	-24%	---	-28%	52%	13%	---	-11%	-1%	2%	---	-17%	14%
8	2%	-4%	---	3%	-24%	-26%	---	50%	7%	-5%	---	-2%	-11%	-18%	---	29%

Table 4 - Panel A

Time to Steady State Allocation across Managers *With* and *Without* Learning

This table reports the time required for the system to reach equilibrium.. For this purpose, we decided to apply an Augmented Dickey-Fuller test (hereafter, ADF) to examine if the relative wealth managed by the various managers is ADF stationary and, if yes, how many periods were necessary to reach this condition. The ADF is computed on 15 lagged periods (weeks), as returned by the Akaike, Schwartz and Hannan-Quinn Information Criteria, and implemented over four 10-year periods. The results are reported for both a random and a serially-correlated path in the earnings-per-share (EPS), *with* and *without* learning, across the eight different *initial* market structures (test cases) documented in Table 1. The situations where the market (structure) starts and remains in equilibrium, as returned by the ADF test, are indicated as "*Always*". If the market only converges to equilibrium, as indicated by the ADF, we document, in square brackets, the period (in years) necessary for the system to reach the stationarity. Finally, the cases where the market (structure) does never converge to equilibrium are indicated as "*Never*".

Test cases	Random Information Signals		Serially Correlated Information Signals	
	No learning	*Learning*	*No learning*	*Learning*
1	Always	Always	Always	Always
2	Never	[10-20]	Never	[30-40]
3	Never	[10-20]	Never	[20-30]
4	Always	Always	[30-40]	Always
5	Always	Always	Always	Always
6	Never	[10-20]	Never	[30-40]
7	[30-40]	[10-20]	Never	[10-20]
8	Never	[20-30]	Never	[30-40]

Table 4 - Panel B

Mispricing both Pre and Post Steady State *With* and *Without* Learning

This table reports the market mispricing, in dollar terms, over the periods both before and after the steady state is achieved. The results are reported for both a random and a serially-correlated path in the earnings-per-share (EPS), *with* and *without* learning, across the eight different *initial* market structures (test cases) as documented in Table 1. Where the market (structure) starts and remains in equilibrium (see also Tabel 4 - Panel A), as returned by the ADF test, we document the extent of the mispricing in the section "*After*". If the market only converges to equilibrium, as indicated by the ADF, we document the extent of mispricing in both sections "*Before*" and "*After*" (the system enter stationarity), in order to quantify the degree of price correction. Finally, in the cases where the market (structure) does never converge to equilibrium we report the extent of mispricing in the section "*Before*", but we obviously omit to report the mispricing for section "*After*".

Test Cases	Random Information Signals				Serially Correlated Information Signals			
	No learning		*Learning*		*No learning*		*Learning*	
	Before	After	Before	After	Before	After	Before	After
1	---	0.49	---	0.49	---	0.90	---	0.90
2	4.71	---	4.44	3.79	5.35	---	6.31	3.52
3	3.85	---	3.48	2.92	4.17	---	3.91	3.43
4	---	5.36	---	4.69	6.74	4.59	---	4.85
5	---	0.86	---	1.25	---	1.98	---	2.59
6	5.67	---	5.02	3.14	6.06	---	5.65	3.95
7	4.72	3.95	4.46	2.96	4.99	---	4.92	4.15
8	3.47	---	3.45	1.86	3.74	---	3.86	2.83

Table 5
Annual Volatility of Fundamental Value and Market Prices

In this table we document the impact of the clients' learning process (implemented *via* inter-managers money flows) on the (annual) average (across the Monte Carlo simulations) volatility in the market. The results are reported for both a random and a serially-correlated path in the earnings-per-share (EPS), each separated according to whether or not there is any money flows (learning), across the eight different initial market structures (test cases), documented in Table 1. For comparability purposes, we also illustrate the volatility of the fundamental (fair) price (first line).

	Standard Deviation (% pa)			
	Random Information Signals		Serially Correlated Information Signals	
	No learning	*Learning*	*No learning*	*Learning*
Fair Price (P*)	0.7%	0.7%	1.0%	1.0%
Testcase 1	1.6%	1.6%	2.4%	2.4%
Testcase 2	4.3%	1.6%	4.1%	2.7%
Testcase 3	5.9%	3.0%	6.0%	5.0%
Testcase 4	4.2%	2.9%	4.0%	3.5%
Testcase 5	1.1%	1.1%	1.8%	1.8%
Testcase 6	5.2%	3.5%	4.7%	4.0%
Testcase 7	3.9%	2.6%	3.3%	1.9%
Testcase 8	4.6%	1.9%	4.1%	2.5%

Table 6 - Panel A

After-fee Performance of the Managers when Earnings per Share follow a Random Walk

In this table we document the main descriptive statistics of the performance of the managers when earnings per share follow a random walk. We consider three types of managers. The *fundamental* trader trades on the difference between a stock's current price and its perceived fair value. The *momentum* trader trades on the basis of the price movement over some previous defined formation period, and holds the risky asset for a fixed holding period. The momentum managers are separated into either a short implementation, with a formation period of 13 weeks and a holding period of 6 weeks, or a long implementation, with a formation period of 26 weeks and a holding period of 26 weeks. The *index* trader, instead, simply replicates the market. The statistics include the managers' excess return, absolute volatility, standard deviation of the excess returns or tracking error (*T.E.*), and a measure of the downside risk. These results are calculated for the two opposite scenarios of *learning* and *no-learning* in the clients' flows.

Excess return and risk measures (% pa)																	
Random Path of the Earnings per Share																	
Testcases	Learning	*Excess Return*				*Volatility*				*T.E.*				*Downside Risk*			
		Fund	**Short M**	**Long M**	**Index**	**Fund**	**Short M**	**Long M**	**Index**	**Fund**	**Short M**	**Long M**	**Index**	**Fund**	**Short M**	**Long M**	**Index**
1	*No*	-0.53%	---	---	---	2.22%	---	---	---	0.58%	---	---	---	-2.12%	---	---	---
2	*No*	0.23%	-0.20%	-1.52%	-0.15%	4.56%	3.76%	4.94%	3.75%	1.15%	0.17%	1.24%	0.16%	-3.03%	-0.47%	-5.12%	-0.43%
3	*No*	-0.18%	-0.78%	---	---	6.52%	6.17%	---	---	1.32%	0.66%	---	---	-1.97%	-1.77%	---	---
4	*No*	0.39%	---	-1.51%	---	4.18%	---	5.41%	---	0.60%	---	1.51%	---	-1.03%	---	-5.77%	---
5	*No*	-0.58%	---	---	-0.09%	1.60%	---	---	0.98%	0.54%	---	---	0.05%	-3.38%	---	---	-0.29%
6	*No*	0.26%	-0.25%	-1.37%	---	4.93%	4.42%	5.86%	---	1.03%	0.23%	1.60%	---	-2.05%	-0.67%	-5.56%	---
7	*No*	0.27%	---	-1.38%	-0.10%	3.62%	---	4.08%	3.25%	0.74%	---	0.91%	0.20%	-2.21%	---	-5.18%	-0.36%
8	*No*	-0.35%	-0.51%	---	-0.10%	5.37%	4.34%	---	4.15%	1.63%	0.25%	---	0.23%	-4.31%	-1.38%	---	-0.38%
1	*Yes*	-0.53%	---	---	---	2.22%	---	---	---	0.58%	---	---	---	-2.12%	---	---	---
2	*Yes*	-0.41%	-0.27%	-0.65%	-0.07%	2.90%	2.45%	2.64%	2.42%	0.64%	0.11%	0.28%	0.12%	-3.62%	-0.59%	-1.85%	-0.51%
3	*Yes*	-0.49%	-0.42%	---	---	3.71%	3.39%	---	---	0.70%	0.22%	---	---	-2.23%	-1.06%	---	---
4	*Yes*	-0.04%	---	-1.13%	---	3.76%	---	3.73%	---	0.80%	---	0.81%	---	-2.37%	---	-5.04%	---
5	*Yes*	-0.63%	---	---	-0.12%	1.25%	---	---	1.13%	0.46%	---	---	0.21%	-4.07%	---	---	-0.85%
6	*Yes*	-0.35%	-0.40%	-0.70%	---	3.67%	3.46%	3.54%	---	0.63%	0.19%	0.44%	---	-2.17%	-0.96%	-2.63%	---
7	*Yes*	-0.31%	---	-0.66%	-0.06%	2.54%	---	2.16%	2.13%	0.66%	---	0.27%	0.14%	-6.69%	---	-3.71%	-1.25%
8	*Yes*	-0.53%	-0.32%	---	-0.08%	2.49%	2.14%	---	2.16%	0.58%	0.11%	---	0.16%	-3.57%	-0.60%	---	-0.31%

Table 6 - Panel B

After-fee Performance of the Managers when Earnings per Share are Serially Correlated

In this table we document the main descriptive statistics of the performance of the managers when earnings per share are serially correlated through time. We consider three types of managers. The *fundamental* trader trades on the difference between a stock's current price and its perceived fair value. The *momentum* trader trades on the basis of the price movement over some previous defined formation period, and holds the stocks for a fixed holding period. The momentum managers are separated into either a short implementation, with a formation period of 13 weeks and a holding period of 6 weeks, or a long implementation, with a formation period of 26 weeks and a holding period of 26 weeks. The *index* trader, instead, simply replicates the market. The statistics include the managers' excess return, absolute volatility, standard deviation of the excess returns or tracking error (T.E.), and a measure of the downside risk (D/S). These results are calculated for the two opposite scenarios of *learning* and *no-learning* in the clients' flows.

Serially-Correlated Earnings per Share																	
Testcases	Learning	*Excess Return*				*Volatility*				*T.E.*				*Downside Risk*			
		Fund	**Short M**	**Long M**	**Index**	**Fund**	**Short M**	**Long M**	**Index**	**Fund**	**Short M**	**Long M**	**Index**	**Fund**	**Short M**	**Long M**	**Index**
1	*No*	-0.38%	---	---	---	3.29%	---	---	---	0.97%	---	---	---	-6.16%	---	---	---
2	*No*	0.28%	-0.14%	-1.18%	-0.08%	4.90%	3.78%	5.36%	3.91%	1.69%	0.23%	1.79%	0.29%	-15.58%	-1.04%	-13.53%	-0.83%
3	*No*	0.07%	-0.63%	---	---	6.66%	6.53%	---	---	1.50%	0.67%	---	---	-9.04%	-4.52%	---	---
4	*No*	0.45%	---	-1.32%	---	4.48%	---	5.65%	---	1.07%	---	1.75%	---	-8.10%	---	-14.62%	---
5	*No*	-0.31%	---	---	-0.05%	2.45%	---	---	1.58%	1.09%	---	---	0.04%	-10.14%	---	---	-0.71%
6	*No*	0.39%	-0.30%	-1.26%	---	5.25%	4.68%	6.73%	---	1.47%	0.34%	2.38%	---	-11.89%	-1.07%	-15.93%	---
7	*No*	0.30%	---	-1.05%	-0.15%	4.02%	---	4.42%	3.39%	1.22%	---	1.13%	0.22%	-12.24%	---	-11.40%	-0.87%
8	*No*	-0.06%	-0.44%	---	-0.05%	5.46%	4.56%	---	4.23%	1.74%	0.42%	---	0.14%	-13.22%	-2.43%	---	-0.75%
1	*Yes*	-0.38%	---	---	---	3.29%	---	---	---	0.97%	---	---	---	-6.16%	---	---	---
2	*Yes*	-0.17%	-0.23%	-0.40%	-0.13%	3.78%	3.15%	3.40%	3.12%	1.36%	0.28%	0.54%	0.25%	-16.93%	-2.33%	-5.99%	-2.53%
3	*Yes*	-0.20%	-0.38%	---	---	4.75%	4.35%	---	---	0.97%	0.44%	---	---	-6.64%	-3.31%	---	---
4	*Yes*	0.31%	---	-0.68%	---	4.59%	---	4.62%	---	1.35%	---	1.14%	---	-8.75%	---	-12.14%	---
5	*Yes*	-0.40%	---	---	-0.08%	2.34%	---	---	1.66%	0.83%	---	---	0.32%	-8.83%	---	---	-3.63%
6	*Yes*	-0.08%	-0.26%	-0.49%	---	4.67%	4.10%	4.41%	---	1.32%	0.27%	0.62%	---	-7.46%	-3.42%	-7.20%	---
7	*Yes*	0.26%	---	-0.40%	-0.19%	3.61%	---	3.13%	2.87%	1.42%	---	0.51%	0.30%	-12.34%	---	-8.02%	-5.17%
8	*Yes*	-0.45%	-0.08%	---	0.01%	3.26%	2.54%	---	2.37%	1.07%	0.12%	---	0.19%	-12.93%	-1.15%	---	-2.07%

Table 7
Main Descriptive Statistics of the Performance of the Clients

In this table we document the main descriptive statistics of the performance of the clients when earnings per share either follow a random walk or are serially correlated. We consider four types of clients, distinguished according to the strategy employed in regulating the inter-managers money flows. The client C_0, never switches after allocating the initial endowments to each bounded rational manager considered in the particular test case. The client C_1 maximises his wealth by switching from the worst to the best manager according to a 1-year evaluation period of the risk-adjusted performance. The client C_3 maximises his wealth on the basis of a 3-year evaluation period of the risk-adjusted performance. Finally, client C_5 maximises his wealth according to a 5-year evaluation period of the risk-adjusted performance. The statistics include the clients' excess return and its standard deviation (or tracking error). These results are calculated for the two opposite scenarios of *learning* and *no-learning* process of the clients.

Excess returns and tracking errors (% pa)											
Testcases	Performance Measures	Random Information Signals					Serially Correlated Information Signals				
		No learning	*Learning*				*No learning*	*Learning*			
		All	C0 [no switch]	C1 [1-year]	C3 [3-year]	C5 [5-year]	All	C0 [no switch]	C1 [1-year]	C3 [3-year]	C5 [5-year]
1	*Excess Return*	-0.53%	-0.53%	-0.53%	-0.53%	-0.53%	-0.38%	-0.38%	-0.38%	-0.38%	-0.38%
	Tracking Error	0.58%	0.58%	0.58%	0.58%	0.58%	0.97%	0.97%	0.97%	0.97%	0.97%
2	*Excess Return*	-0.30%	-0.34%	-0.33%	-0.38%	-0.36%	-0.14%	-0.09%	-0.24%	-0.22%	-0.26%
	Tracking Error	0.52%	0.25%	0.36%	0.54%	0.41%	0.63%	0.40%	0.76%	0.70%	0.75%
3	*Excess Return*	-0.35%	-0.44%	-0.50%	-0.51%	-0.50%	-0.28%	-0.37%	-0.39%	-0.36%	-0.33%
	Tracking Error	0.95%	0.39%	0.63%	0.54%	0.58%	1.03%	0.69%	1.15%	1.04%	1.06%
4	*Excess Return*	-0.38%	-0.49%	-0.63%	-0.45%	-0.34%	-0.25%	-0.17%	-0.47%	-0.47%	-0.22%
	Tracking Error	0.61%	0.63%	0.91%	0.76%	0.85%	0.77%	1.02%	1.52%	1.39%	1.27%
5	*Excess Return*	-0.25%	-0.30%	-0.28%	-0.24%	-0.24%	-0.19%	-0.08%	-0.27%	-0.13%	-0.13%
	Tracking Error	0.25%	0.36%	0.14%	0.12%	0.15%	0.52%	0.83%	0.50%	0.49%	0.50%
6	*Excess Return*	-0.33%	-0.44%	-0.47%	-0.46%	-0.47%	-0.13%	-0.17%	-0.46%	-0.28%	-0.26%
	Tracking Error	0.72%	0.32%	0.59%	0.64%	0.57%	0.77%	0.58%	1.35%	1.08%	0.91%
7	*Excess Return*	-0.27%	-0.35%	-0.29%	-0.33%	-0.33%	-0.18%	0.00%	-0.16%	-0.28%	-0.24%
	Tracking Error	0.44%	0.20%	1.08%	0.28%	0.19%	0.63%	0.58%	0.76%	0.86%	0.84%
8	*Excess Return*	-0.26%	-0.33%	-0.33%	-0.34%	-0.32%	-0.15%	-0.17%	-0.16%	-0.14%	-0.24%
	Tracking Error	0.61%	0.36%	0.43%	0.40%	0.41%	0.77%	0.56%	0.68%	0.63%	0.63%

Figure 1
Intertemporal Market Fractions *With* and *Without* a Learning Process of the Clients

The figure illustrates the change through time of the market weights of the several managers operating in two different market structures: test case 2 and test case 7. The information signals (EPS) are assumed to be serially correlated through time. Test case 2 (upper subplots) comprises all the possible heterogeneous agents considered (fundamental, short and long momentum, and index managers). Test case 7 (lower subplots), instead, refers to only three of such managers (fundamental, long momentum, and index managers). For each test case, we consider the intertemporal dynamics of the market fractions in the two opposite situations of absence and presence of a learning mechanism. The learning is implemented through the inter-manager money flows generated by the clients and based on a periodical risk-adjusted evaluation of the performance of the managers.