Hedging Collateralized Debt Obligations

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Abstract

In an empirical study, we investigate the ability of the Gaussian One-Factor Model to describe adequate hedging schedule for the iTraxx CDO. The model is able to provide some insurance against changes in several input spreads, however, it is completely fails when hedging second order effects, and correlation risk. Closer investigation reveals that the model performs very well during regular market times, reducing mean absolute hedge errors and root mean squared errors by more than 50% compared to an unhedged portfolio. A very large proportion of the hedge error, however, is caused by the largest 2.5% of observations. In these cases, the model sometimes even underperforms unhedged positions. We break down the hedge errors into first and second order effects, and in changes in correlation and further study and analyze possible extensions.

Keywords: Credit risk, correlation risk, collateralized Debt Obligation, CDO, large homogeneous portfolio approximation, LHP, Structured form model

JEL Classification Numbers: G13, G33

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1 Introduction

In recent years, the literature on the pricing of CDOs has multiplied. The debate on which model to use for the pricing of CDOs is still ongoing. However, it has become market practice to use a Gaussian One-Factor Model to back out different values of model correlation, the free parameter of the model, from the different securities' quotes. The so constructed *implied correlation* then takes a similar role to implied volatilities of equity options and is used to price related securities. In the presence of reliable prices, the model takes a role similar to the Black-Scholes Model in the pricing of equity derivatives. The use of implied correlation eliminates the problem of simultaneously pricing different instruments, a major drawback of practically all one-factor models. Consequently, the model cannot be judged by its ability to reproduce prices and hedging can be seen as the practically most relevant model application and test (Finger (2005), St. Pierre, Rousseau, Zavattero, van Eyeren, and Aurora (2004)).
To our knowledge, empirical validation of CDO pricing models in the literature is sparse, Bystrom (2006) is the only related empirical study we found. There exist a number of non-publicly available studies by banks. However, non of these studies we know analyzed a sample which was longer than a couple of month. Hence, this is the first article to investigate the Gaussian One-Factor Model's ability to prescribe adequate hedging schedules over a longer period of time.
The data for our study is based on the the iTraxx Index, which comprises the 125 most liquid Credit Default Swap contracts on European corporate entities. Based on this Index, a standardized CDO is issued half-yearly. Our dataset contains reliable, tradeable quotes of the Index, the CDO, and all Index constituents from June 22, 2004 until September 19, 2007. The time to maturity for all investigated contracts is 5 years. We calibrate a implied correlation structure for each day in the sample period in several ways and calculate hedge ratios against changes in an input spread. We find that a simple hedge against linear changes reduces

the mean average hedge error (MAHE) by about 50% compared to an unhedged position. The hedge error variance, measured by the root mean squared error (RMSE) is also substantially reduced, typically by 20-30%. Further study reveals that most of the hedge errors is caused by a few very large value changes. The highest 2.5% of observations account for between 25% and 60% of the MAHE and for between 78 and 98% of the RMSE. We scrutinize the serial dependence and the clustering of extreme value changes. However, we are not able to capitalize it to reduce the hedge errors. Furthermore, we study the models ability to hedge against second order effects and changes in correlation and try to correct for a possible dependence between implied correlations and spread changes. However, neither of these attempts is successful. Whereas adjusting the hedge ratios seems hardly to affect the hedge errors, hedging against second order effects and correlation changes dramatically increases both, MAHE and RMSE.

The articles contribution to the literature is the first empirical study of the market benchmark model for the valuation of CDOs. Our results confirm that the use of implied correlations is feasible and helpful in regular, low volatility times. In times of extreme movements of input quantities, however, the model fails. While these results are not surprising, they have not been documented in the literature yet. An implication of our research is that an extension of the Gaussian model should rather include jumps in spreads and dynamic correlations than changed factor distributions.

The remainder of this article is structured as follows: the model studied is introduced in Section 2. Section 3 presents the investigated dataset and discusses two remarkable periods, the auto crisis in May 2005 and the subprime crisis in Summer 2007. The main part of this article, the hedging study can found in Section 4. In Section 5 we present attempts to improve the hedge performance by adjusting hedge ratios. Section 6 concludes.

2 The Large Homogeneous Portfolio Gaussian Copula Model

In this section, we present the Gaussian Copula Model, the Large Homogeneous Portfolio Approximation (LHP), and two ways of calibrating the model to market data, compound and base correlations. In the empirical part of this paper, we calibrate the model to market prices. Hence, all presented equations hold under the pricing measure.

A CDO distributes the losses from default events occurring in a portfolio of credit risky instruments to a number of credit risky notes according to certain externally specified rules. The distribution of losses on these tranches is given by that of the total loss on the entire portfolio. Thus, each model for the credit portfolio loss also is a CDO pricing model. A large number of models for the losses of credit portfolios exist, see for example Bluhm, Overbeck, and Wagner (2003) for a survey. The most important input quantity for all these models are the portfolio components probabilities. These are influenced by a large number of micro-economic factors, such as asset and equity value, (implied) volatility, and leverage, and macro-economic factors, such as interest rates and the business cycle, as well as measures of information quality. The influence of these factors has been confirmed and quantified by numerous studies by, for example, Collin-Dufresne, Goldstein, and Martin (2001), Eom, Helwege, and Huang (2004), Ericsson, Reneby, and Wang (2006), or Yu (2005). Consequently, any model has to take into account a large number of factors. Moreover, since also the dependence between the factor matters, the number of model parameters grows exponentially with the number of portfolio components. While this is important for many applications such a managing loan portfolios, it can be undesirable or unneccessary for other applications where reliable market prices exist. In the market for equity derivatives for example, the Black-Scholes model is used to quote option prices as implied volatilities. In this case, the model serves to facilitate communication and hence, its simplicity is a value in itself. Similarly to the use of implied volatilities for equity options, it has become common to quote implied correlations in the market for synthetic CDOs. This is only meaningful if everyone agrees on the same model, which is the Large Homogeneous Gaussian Copula Model (LHGM). However, this does not imply that all market participants agree on the same model for pricing purposes.

It is generally understood and accepted that Gaussian models cannot describe the distribution of financial data accurately. This is normally attributed to the light tails and the symmetry of the normal distribution. In comparison to other models however, these models are well understood and easy to handle. Thus, they are very popular and widely used, frequently in modifications to adjust for their shortcomings. We refer to Embrechts, McNeil, and Straumann (2002) and Embrechts, Frey, and McNeil (2005) for an overview and discussion of Gaussian models; Andersen and Sidenius (2005a), and Burtschell, Gregory, and Laurent (2005) discuss possible extensions and give an overview of factor models for CDO pricing.

In a factor (copula) model, an entity i defaults, if the factor

$$X_i = \alpha_i M + \sqrt{1 - \alpha_i^2} W_i \tag{1}$$

falls below a certain specified level.[1] The factor X_i is the weighted sum of a systematic market component M, affecting all entities in the portfolio, and an idiosyncratic component W_i, which affects i only. The model is further determined by the assumption of specific distributions for M, W_i and α_i. In a Gaussian Copula Model M and W_i are independent standard normally distributed and the

[1] The factor is frequently perceived as the asset value of a firm, the barrier as the present value of its debt.

α_i are constant.[2] The correlations between different entities' factors are

$$\rho_{ij} = \alpha_i \alpha_j \tag{2}$$

and are referred to as the *default correlations*. In combination with a model which relates the entities default probabilities and the respective loss-given-default to their factors X_i, a distribution of the portfolio loss is completely specified. For convenience, the time until the default event is frequently assumed to be exponentially distributed with a constant default intensity. The loss given default is often assumed to be a constant fraction of the invested notional.

For a fixed time horizon t Vasicek (1977) derived the distribution of the relative loss on a large homogeneous portfolio of identical debtors. All debtors default with probability p, the loss in the case of default is he same and equal to 1, and the default correlation is equal to ρ between any two debtors. Conditional on the realization of the common factor M, the relative portfolio loss is binomially distributed. Increasing number of obligors in the portfolio to infinity and integrating over the distribution of the common factor, the unconditional distribution of the relative portfolio θ is:

$$F_G(\theta) = \boldsymbol{P}\left(\text{loss} \leq \theta\right) = \Phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}(\theta) - \Phi^{-1}(p)}{\alpha}\right), \tag{3}$$

which is the Large Homogeneous Portfolio Approximation (LHP). The existence of closed-form LHPs is commonly seen as desirable for any factor model since it allows for the fast calculation of prices. O'Kane and Schloegl (2005) published a note for the LHP for the t-distribution case. Hull and White (2005) further generalize the t-distribution case. Moosbrucker (2006) and Kalemanova, Schmid, and Werner (2007) give the LHP for the Variance Gamma and Normal Inverse Gaussian distributions, respectively. As the LHGM, none of these models can perfectly price the investigated CDO. Thus they would also need to be calibrated

[2] Under practically all factor specifications the X_i follow a tractable distribution F_i; thus, considering the transformed factors $F_i\left(X_i\right)$, the equivalence to copula models becomes obvious.

using a correlation structure. However, it is questionable whether the calibration of an implied correlation structure is feasible: correlation looses its meaning outside of the Gaussian setting. Moreover, all of these models will have more parameters that prices exist for the calibration. Since that, we found no reason to prefer any of these models in an empirical study and, hence, only present the results for the LHGM.

All the above equations are independent of time and describe the distributions for a given horizon of time. This is due to the transformation by the factors' distribution and, consequently, the copula is independent of time. In a model calibration, however, the default probability depends on time. Substituting the probability of default prior to time t, $p(t)$, for p in equation (3) introduces time dependence to the model.

2.1 Default Probabilities

To obtain an estimate for the default probabilities, we solve a simple Credit Default Swap (CDS) pricing equation. A CDS is quoted as the spread s which makes the expected value of the premiums on the contract equal to the expected value of its loss. In a CDS initiated at time $t_0 = 0$ with premium payments at $t_1, ..., t_T$, the premium receiver (seller) of the contract suffers a loss L, which is assumed to be independent of the default probability.[3] At any time $t_0 \leq \tilde{t} \leq t_1$, the CDS contract quote includes an accrued premium, equal to the spread pro rata temporis since t_0. Hence, the pricing equation at $\tilde{t}$ reads

$$L \sum_{i=1}^{T} e^{-r_i t_i} \boldsymbol{P} \left(\text{default in } (t_{i-1},\, t_i]\right) = s \left(\frac{\tilde{t} - t_0}{t_1 - T_0} + \sum_{i=1}^{T} e^{-r_i t_i} \boldsymbol{P} \left(\text{default after } t_i\right) \right), \tag{4}$$

where the interest rates r_t are assumed to be known and constant. For simplicity, we assume that all default events occur precisely at the times of the premium

[3] In many cases it is even assumed to be constant and equal to 40%. This assumption is used for example in the Bloomberg system's CDS valuation tools.

payments. These two assumptions do not have a strong influence on prices: interest rates enter both sides of the pricing equations in a similar way and the effect of changing interest rates is rather weak,[4] see for example Longstaff and Rajan (2007). Since in the case of a default event, the protection seller receives the spread premium accrued since the last (quarterly) payment date, the timing of a default event does not have a strong effect on the value of a contract.

As announced above, we set

$$\boldsymbol{P}\left(\text{default before } t_i\right) = 1 - e^{-\lambda t_i} \tag{5}$$

for the default probabilities in equation (4), and solve for λ for given spread s, interest rates $r(t_i)$, and loss given default L. Since the default probabilities are assumed to be equal for all CDS contracts in the iTraxx Index and CDO portfolio, it is unclear which of the 125 individual spreads to use. Candidates for a representative spread are the index spread, the average spread of all contracts in the index, and the duration weighted average of the spreads in the index. In the empirical part, we consider these three candidates. We refer to these spreads as the input spreads of the model.

2.2 CDO pricing

The CDO studied slices the portfolio loss on the iTraxx Index into tranches: a first tranche takes all the losses on the portfolio up to a level $K_1 = 3\%$, from this level on, a second tranche suffers all the losses up to a level $K_2 = 6\%$. Consequently, a third to sixth tranche take the subsequent losses to the levels of 9%, 12%, 22%, and 100%. Denoting the expected loss on such a tranche for a time horizon t by

[4] Consequently, the effect of stochastic interest rates and of daycount conventions is also weak.

$EL_{K_{j-1},K_j}(t)$, its pricing equation corresponds to equation (4):

$$\sum_{i=1}^{T} e^{-r_i t_i} \left(EL_{K_{j-1},K_j}(t_i) - EL_{K_{j-1},K_j}(t_{i-1}) \right) = \tag{6}$$
$$s \left(\frac{\tilde{t} - t_0}{t_1 - T_0} + \sum_{i=1}^{T} e^{-r_i t_i} \left(1 - EL_{K_{j-1},K_j}(t_{i-1}) \right) \right),$$

where default probabilities have been replaced by expected losses.[5] The expected relative loss on a CDO tranche can be calculated from the distribution in (3) by integration. It depends on the correlation as the only free parameter. Given the interest rates and the input default probabilities, this parameter is used to calibrate the model to market quotes for each tranche. The calibrated correlations are referred to as *implied correlations*. Regarding default timing and interest rates, we make the same assumptions as for the CDS valuation above.

2.3 Compound correlations and base correlations

The correlations backed out by direct application of the model to individual tranche prices are called "compound correlations." They show a pronounced *smile*: the correlation on the second loss tranche is lower than that of all other tranches as can be seen in Figure 1. Since this shape of the implied correlations is impractical for interpolations,[6] it has become standard to consider an alternative type of implied correlations called "base correlations." These were suggested by McGinty, Beinstein, Ahluwalia, and Watts (2004) and are discussed by e.g. St. Pierre, Rousseau, Zavattero, van Eyeren, and Aurora (2004) and Willemann and Isla (2007). Base correlations are calculated by iteratively calibrating expected losses to tranche prices and utilizing the interrelation between expected

[5] In the case of a CDS $EL(t) = L \cdot \boldsymbol{P}(\text{default before t})$.

[6] In order to price off-market tranches with attachment and detachment levels different from those of the standard tranches, market practitioners use interpolated implied correlations. In the presence of a smile interpolation is difficult, if not impossible.

losses on sequential tranches with an attachment level of 0:

$$EL^{\rho_{0,i+1},\rho_{0,i}}_{K_i,K_{i+1}}(t) = EL^{\rho_{0,i+1}}_{K_0,K_{i+1}}(t) - EL^{\rho_{0,i}(t)}_{K_0,K_i}. \tag{7}$$

Superscripts were introduced to indicate the dependence of senior tranche expected losses on junior tranche implied (base) correlations; note that the base correlation of a senior tranche depends on *all* base correlations of the more junior tranches. The expected loss on a tranche suffering losses from attachment level K_i up to detachment level K_{i+1} is equal to the difference between the loss on a tranche suffering losses from 0 to K_{i+1} (priced with base correlation $\rho_{0,i+1}$) and that on a tranche suffering losses from 0 to K_i (priced with base correlation $\rho_{0,i}$). As can be seen in Figure 2, base correlations increase with seniority and hence, interpolating is straightforward. McGinty, Beinstein, Ahluwalia, and Watts (2004) report that the Gaussian copula model occasionally cannot be calibrated to market prices of the 3-6% second loss tranche. Whereas this does not happen for base correlations. However, a similar problem has been found for the pricing of the 12-22% tranche with base correlations by St. Pierre, Rousseau, Zavattero, van Eyeren, and Aurora (2004). This is also the case in our empirical study, hence the gaps in the surfaces in Figures 1 and 2 . For the valuation of non-standard structures, correlation is seen as a function of the attachment-detachment level and interpolated to match attachment detachment levels. Even though, this procedure seems to be market practice, it is questionable because it does not guaranty arbitrage free pricing. For a given correlation structure, tranches with negative expected losses can be found.

2.4 Sensitivities and CDO hedging

In a given valuation framework, a CDO tranche's value's sensitivity to changes in the model's factors can be calculated. Using these, positions in a tranche can be hedged, i.e. immunized against small changes in these quantities. In the case of the iTraxx CDO, hedge portoflios can be formed from the CDS Index, the CDO tranches, and the individual underlying names. Since we use the LHGM, we do not consider hedges using individual CDS contracts. Also, we do not hedge in the iTraxx Index because it serves as an input to the model. We only use the CDO tranches to hedge each other.

Portfolios consisting of the CDO tranches only are frequently used to construct trading positions which are referred to as "correlation trades." An increase in correlation - if the default probability remains unchanged - decreases the expected loss on the first loss tranche, while that of the more senior tranches increases. This is because the default probability determines the expected loss on the entire portfolio whereas the correlation determines the shape of the loss distribution: high correlation leads to higher probabilities of high losses, reducing the value of higher tranches, whereas it reduces the probability of lower losses, increasing the value of lower tranches, and vice versa. Hence, long positions in the first loss tranche hedged against changes in the individual default probabilities (i.e. spread movements) using short positions in the more senior tranches are seen as "long correlation." Accordingly, short positions in the first loss tranche hedge by long positions in the more senior tranches are seen as "short correlation." Mostly, the first two or three tranches are used in these trades since for these the sensitivities towards all parameter changes are the highest. Consequently, these are the most liquidly traded of the five standard index tranches. The second loss tranche takes a special role: since it lies between the first loss piece and the senior tranches for which the effects of changing spreads and correlations is clear, it can at times behave more like one or the other, depending on the level of spread and correlation. See Felsenheimer, Gisdakis, and Zaiser (2006) or Rajan, McDermott, and Roy (2007) for a more detailed description of correlation and CDO trading strategies.

The sensitivities of the CDO tranches net present value (NPV) with respect to the input spread are referred to as *delta*, and *gamma*. More formally these are the first and second derivative with respect to the spread,

$$\Delta_{Tranche} = \frac{\partial Tranche}{\partial s} \tag{8}$$

$$\Gamma_{Tranche} = \frac{\partial^2 Tranche}{\partial^2 s}, \tag{9}$$

$Tranche$ is the tranche's NPV and the s is the spread. The tranches NPV is the difference of the left and right side of equation (4), at inception of a position it is 0.

The sensitivity to changes in correlation is called *rho*. Its role is similar to that of the volatility derivative "vega" in the Black-Scholes formula, we define

$$\mathcal{R}_{Tranche} = \frac{\partial Tranche}{\partial \rho_{Tranche}}, \tag{10}$$

where $\rho_{Tranche}$ is the tranche implied correlation. In the case of a default event, the value of all tranches is reduced: either a tranche is hit by the default event, or its protection cushion decreases. This effect is quantified as the *omega* or "default01" of a contract. Since the considered iTraxx index contains only investment grade companies, default events are very rare. In fact, there is no default event in the sample period. Thus, we do not study the effect of the omega here. Since the value of the CDS index is (by the absence of arbitrage) equal to the sum of the CDO tranches (including the value of a non-quoted 22-100% tranche), the sensitivities of the CDS index can be attributed to the different tranches. All the sensitivities are highest for the more junior tranches. The (relative) impact of default events decreases with subordination, while the (relative) effect of spread changes increases. This is further studied by Skarabot, Roy, and Ryu (2007).

We do not hedge changes in time decay or interest rates. Time decay is not a risk by itself since it is perfectly previsible. As argued above, interest rate effects are offsetting, they affect the premium and the protection side of the CDS contract in a similar way.Ehlers and Schönbucher (2006) suggest that there might me macroeconomic factors that drive both, CDS spread and interest rates. This effect might be important in periods of distress, such as in the subprime lending crisis in 2007[7]. However, by the same argument as above it affects both, the premium and the protection side of the contracts and thus, we ignore it for this study.

The ratio to hedge one tranche against the change in the driving spread in another tranche is given as the quotient of the tranche delta and the hedge instrument delta,

$$HR = \frac{\partial Trache1}{\partial Tranche2} = \frac{\frac{\partial Tranche1}{\partial s}}{\frac{\partial Tranche2}{\partial s}} = \frac{\Delta_{Tranche1}}{\Delta_{Tranche2}} \tag{11}$$

where $Tranche1$ denotes the NPV of a position of unit size in the hedged tranche, and $Tranche2$ is the NPV of a position of unit size in the tranche used for hedging; s is the input spread level. A delta-hedged CDO tranche position's NPV should not be affected by small changes in the index spread. This assumes, that all other quantities remain unchanged.

To calculate the Δ values numerically, we follow St. Pierre, Rousseau, Zavattero, van Eyeren, and Aurora (2004)[8] and Skarabot and Gaurav (2007): we approximate the delta by considering shifting the spread level up and down by 1 bp. The choice of 1 bp is arbitrary; we checked other values from 0.01 to 20 bp, leading to very similar values and results in the empirical study. For the shifted spreads, the sensitivity of the tranche position ($\partial Tranche1/\partial s$) and the hedge position ($\partial Tranche2/\partial s$) is approximated by the change in NPV that arises due to the spread level change holding all other parameters at current levels. For each scenario, the quotient of the approximated sensitivities approximates the delta from equation (11). The hedge ratio HR is obtained by averaging the two scenario

[7]Caused by distress in the subprime mortgage CDO market, on August 17, 2007 the US Federal Bank of Reserve lowered the discount rate by 50 bp to 5.75%.

[8]The authors describe the methodology for the general case of a finite heterogeneous portfolio model, i.e., not for LHP.

results:

$$HR \approx 0.5\frac{Tranche1\left(\rho,\lambda_{+1bp}\right)-Tranche1\left(\rho,\lambda_{0bp}\right)}{Tranche2\left(\rho,\lambda_{+1bp}\right)-Tranche2\left(\rho,\lambda_{0bp}\right)} + 0.5\frac{Tranche1\left(\rho,\lambda_{-1bp}\right)-Tranche1\left(\rho,\lambda_{0bp}\right)}{Tranche2\left(\rho,\lambda_{-1bp}\right)-Tranche2\left(\rho,\lambda_{0bp}\right)}, \tag{12}$$

where $\lambda_{\cdot}$ denotes the default intensities which result from the shifted spreads. Corresponding subscripts mark an increased, decreased, or unchanged index level. We obtain different sensitivities for the compound and the base correlations.[9] By construction, the deltas and hence the hedge ratios are are always positive in the compound correlation case. For base correlations negative deltas could be calculated. However, this never happened in our sample. The positive hedge ratios imply that long positions in a tranche should always be hedged in short positions in another and vice versa. All other sensitivities were calculated in a corresponding fashion. For the sensitivity towards correlation changes, we shifted the entire correlation curve in the base correlation case.

It is clear that on days where we encounter a model failure for any of the tranches, no corresponding delta can be computed. We excluded these dates from the empirical study.

[9]Both, the current tranche's and the subsenior tranche's base correlations need to be kept constant in the base correlation approach to delta computation.

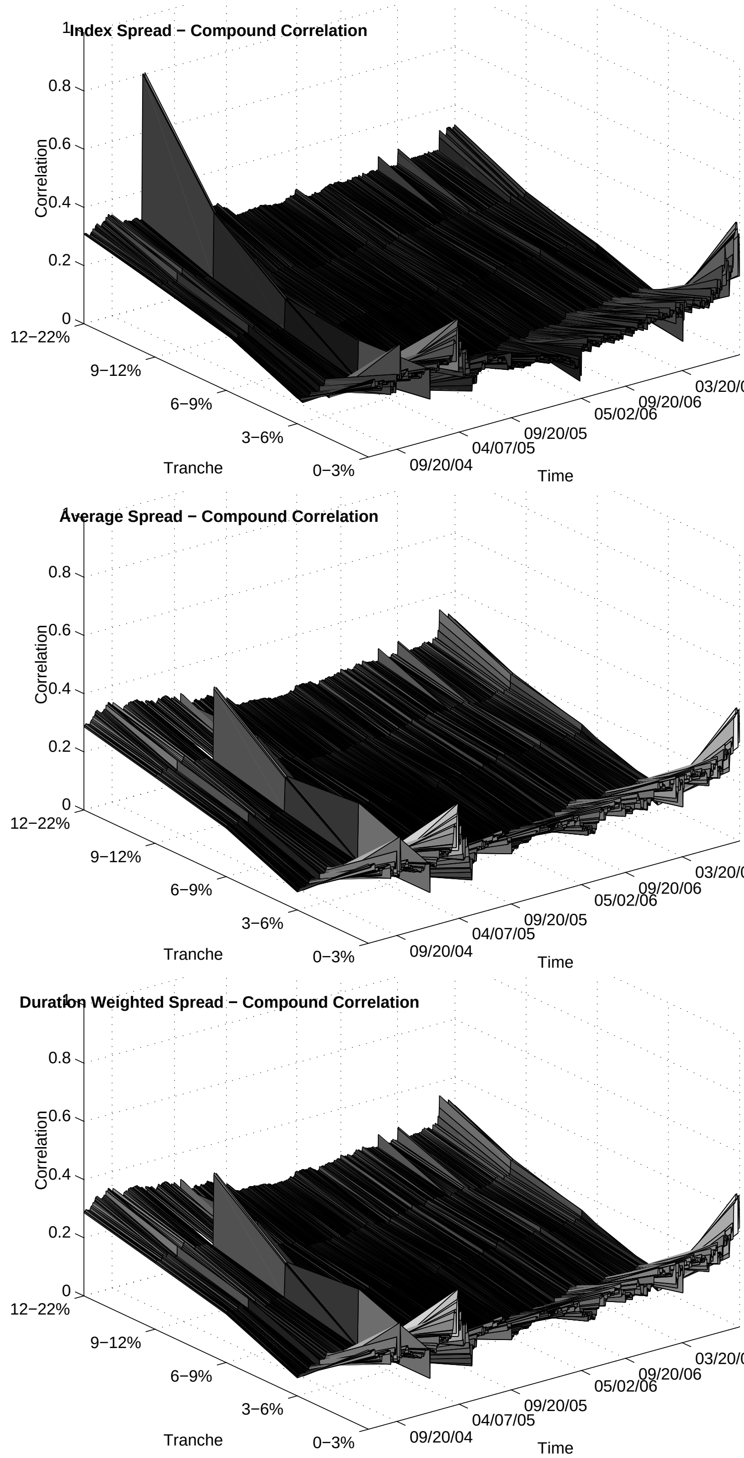

Figure 1: Implied compound correlations for all five tranches for the entire observation period. The smile shape of the correlations can be observed throughout the sample period. On several occasions, the model was not able to reproduce the spread on the second loss tranche.

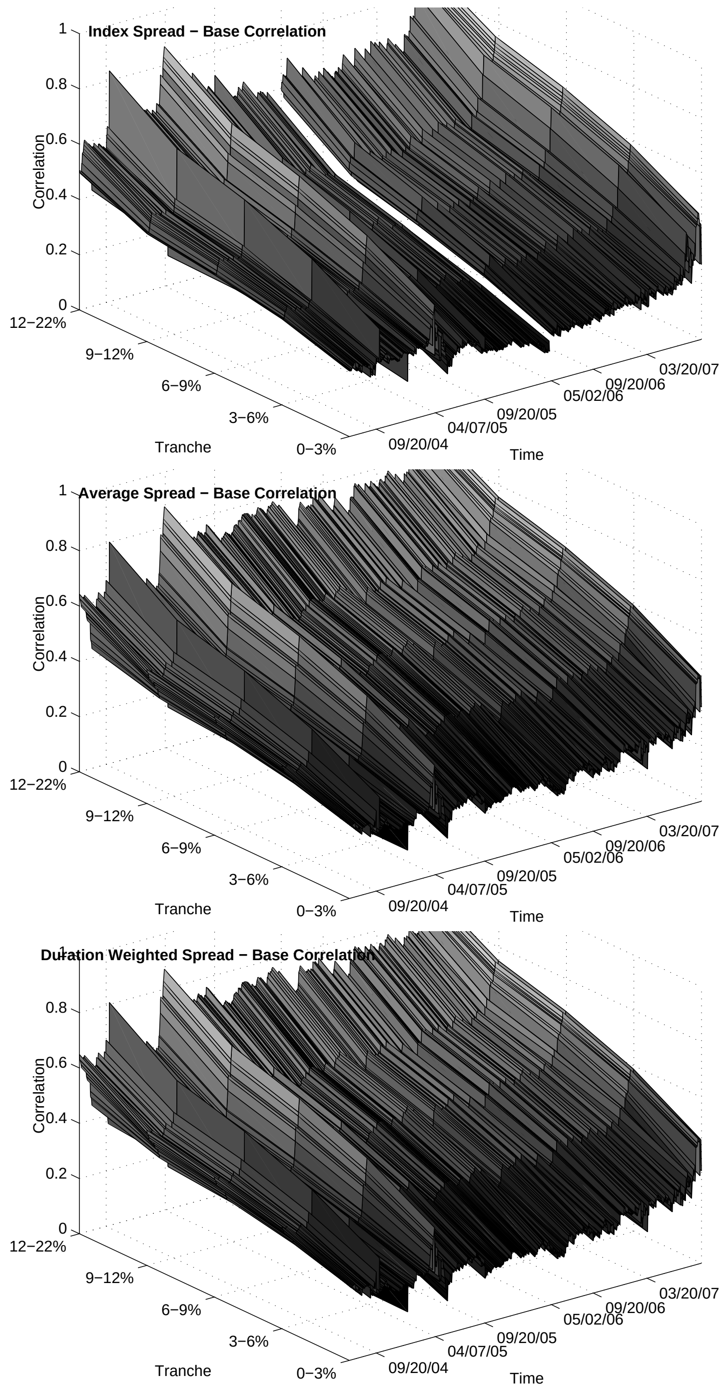

Figure 2: Implied base correlations for all five tranches for the entire observation period. The level of correlation is increasing with seniority. On several occasions, the model was not able to reproduce the spread on some of the more senior tranches and, consequently, also not on all more senior tranches.

3 CDS Index and CDO tranche spreads

We investigate daily mid-market closing quotes of the five-year Dow Jones iTraxx Europe Index, its components, and five tranches of the iTraxx CDO. The attachment-detachment levels of the tranches are 0%, 3%, 6%, 9%, 12%, and 22% percent,[10] respectively. The sample period stretches from June 22, 2004 to September 19, 2007. All data are retrieved from the Bloomberg system. The Index itself is a single-trnche CDO which suffers from the losses on an equally weighted portfolio of the 125 most liquid European CDS contracts on investment grade entities from a range of industry sectors.[11] New series of the iTraxx Index are introduced on the 20th of March and September of every year, with the exception of the first series which was launched on June 20, 2004. At each point in time, the most recently issued iTraxx Index and CDO are called the "on-the-run" series. This is the most liquid of the existing series.[12] The dataset reports the full on-the-run history of first seven Index series. For data quality reasons we delay the occurring rolls from the second to the third and from the fourth to the fifth series by several days in order to have quotes for all tranches for all days during the observation period.[13] Interest rates were taken from Datastream, daily quotes of the Euribor rates for monthly maturities were used throughout the study. Figure 3 shows the quoted spreads for the iTraxx Index, the average spread of the CDS in the on-the-run Index and the spreads of the five tranches. All tranches pay a

[10]The sixth tranche, which suffers from losses between 22% and 100% on the reference portfolio is typically not quoted but quoted implicitly: by the absence of arbitrage, its price is given by the prices of the first five tranches and the Index.

[11]The precise rules for the construction of the Index can be found on the Index provider homepage, www.Indexco.com.

[12]As reported by, see for example Felsenheimer, Gisdakis, and Zaiser (2006) or Rajan, McDermott, and Roy (2007), market participants tend to roll-over their trading positions around the Index roll dates.

[13]The roll from series 2 to 3 was delayed to April 7, 2005. That from series 4 to 5 to May 2, 2006.

running spread which is payed out quarterly. For the equity tranche, the running spread is fixed to 500 basis points per year and an additional upfront premium is quoted. For all other tranches the running spread is quoted. However, the accrued running premium since the last payment date is to be paid paid upfront in any transaction. The premium is quoted as a par spread, i.e. the spread that makes the contract value equal to zero in the valuation formula (6).

Figure 3 displays the observed quotes. The data quality of the first series of the iTraxx CDO, as well as that of the first three month of the second seems to be lower than for the rest of the sample. Frequently, the quotes do not move and if they do, it appears to be rather irregular. Without further study, we attribute this irregularity to a lack of liquidity. This explanation is supported by similar issues in the sample of the constituents quotes for the Index, where some quotes are even missing for the first two month of the first series. However, we include the data in our study and report all findings for each of the series separately. The entire sample has a length of 848 daily observations. All over, the data quality can be regarded as fairly high: quotes exist for all days in the sample period, dynamics are similar across all tranches and bid-ask spreads, which we also observed for all days, tranches, and individual quotes are small. The tranche data show a pronounced spike in December 2004. This spike even violates the no-arbitrage conditions between the third and fourth loss tranches; this is also violated by a second spike in tranche four in May 2005. On these dates, the model fails to fit the prices for the affected tranches and, hence, we exclude them from the study. The spreads of the different tranches are monotonically decreasing with the seniority of a tranche at any point in time. The equity tranche's running spread of 500 bp, for example, is much higher than that of any of the more senior tranches despite the additional up-front payment, which ranges from 5.85% to 49.53% during the observation period. Similarly, the spread on the 3-6% tranche ranges from 38.83 to 263.30 basis points, which compares to 10.18 to 123.78 basis points on the 6-9% tranche. The minimum and maximum spread were 4.22 and

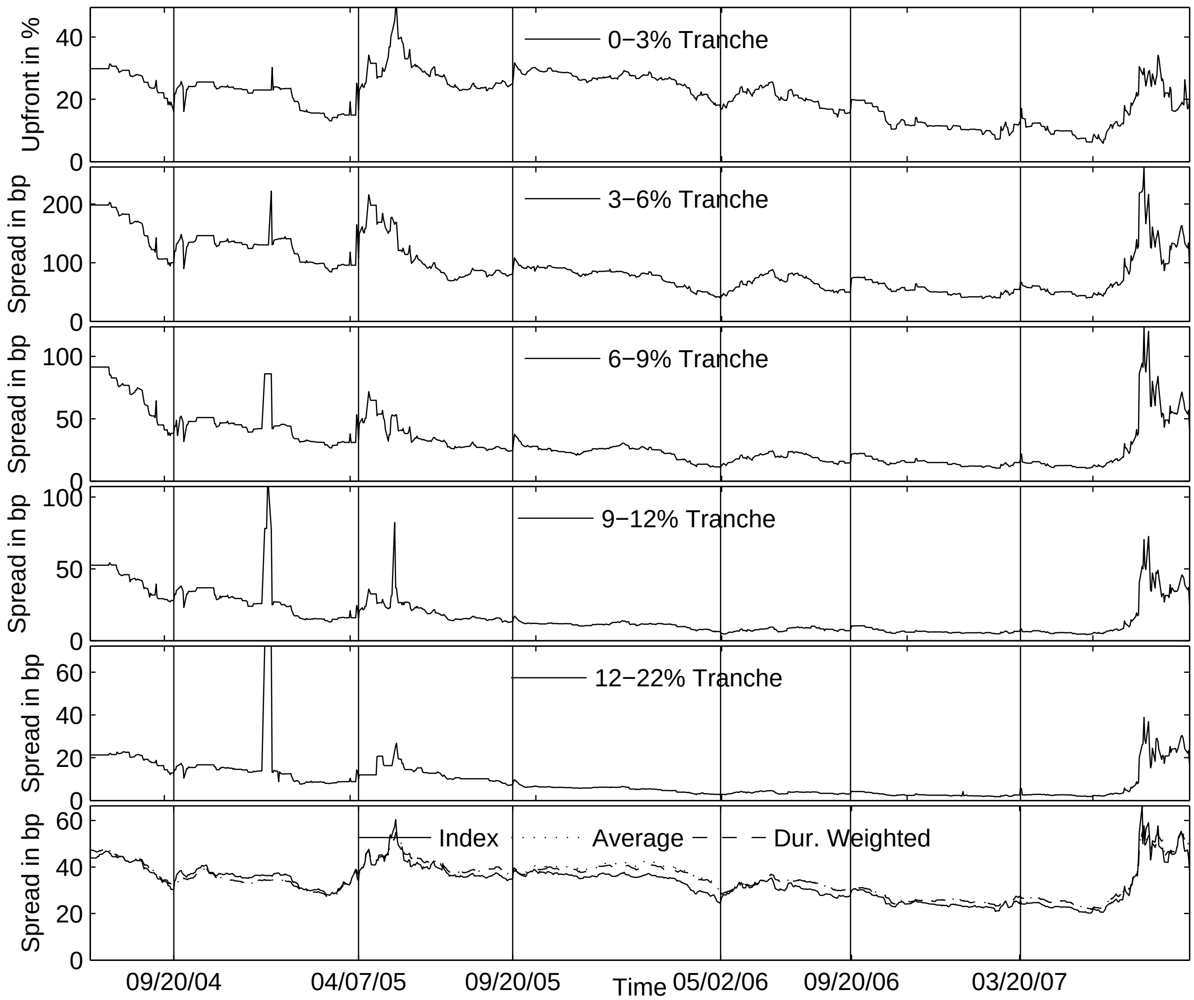

Figure 3: Tranche spreads of the iTraxx CDO, the iTraxx Index, and the average and duration-weighted average spread over the sample period. The vertical lines indicate the roll dates between the series. The spreads decline over the period and show very moderate dynamics. An exception are the spreads during of third series, the period of the GM and Ford downgrades and on the end of the sample period in the subprime crisis.

72.61 bp on the 9-12% tranche (we ignore the peaks in the second and third series for these values) and 1.68 and 38.86 bp on the 12-22% tranche (we ignore the peak in the second series), respectively. These values compare to an minimal

	Index	0-3%	3-6%	6-9%	9-12%	12-22%
Index	1.0000	0.2941	0.3851	0.4735	0.3028	0.2366
0-3%		1.0000	0.5619	0.4831	0.2300	0.0690
3-6%			1.0000	0.8326	0.4646	0.4839
6-9%				1.0000	0.7140	0.7263
9-12%					1.0000	0.7308
12-22%						1

Table 1: Correlations between changes (first differences) in the iTraxx Index and the CDO tranches. It can be seen that the correlations are highest close to the diagonal. Thus, neighboring tranches are expected to show similar spread dynamics. The first-loss tranche and the most senior tranche seem have the lowest correlation to the other tranches.

Index spread of 20.16 bp and a maximal spread of 66.33 bp. The differences in spreads reflect the differences in risk associated with the respective tranches. It is apparent from Figure 3 that all tranches (spreads and upfront payments) are to some extent positively correlated with the Index. This is intuitively clear, since a change in the default risk of the reference basket should generally be reflected by the pricing of all tranches. However, the degree to which a change in the basket spread is reflected in the tranche spreads differs across tranches and is not consistent over time, the tranches feature different sensitivities to changes in the basket credit quality. Market-wide changes in credit quality are reflected in price changes of all five tranches, while idiosyncratic changes' effect depends on the respective spread level as well as on the tranche size and subordination, see e.g. Fitch (2006) and Mina and Stern (2003). As an illustration, we show the correlations between the changes in spreads (upfront payment) between the tranches and the Index in Table 1. Summarizing, the correlations between the Index and the tranches are surprisingly weak, in particular in comparison to US data, for which Longstaff and Rajan (2007) find much higher correlations. The pattern of correlations is peculiar, with the equity tranche featuring the largest correlation, while it is very similar for the four more senior tranches. This pattern (roughly) corresponds to the correlations of US data where correlations are decreasing with increasing seniority (Longstaff and Rajan (2007)). To foreshadow the results of the later hedging study it is clear that hedging tranches in non-neighboring tranches will be much harder than hedgig in the next junior or senior tranches. We call tranches that share an attachment-detachment level neighboring. Given the low correlation, we expect the most-senior tranche to be the hardest to hedge.

The Auto Crisis One example of events that affect the CDO tranches differently is given by the "Auto Crisis," the period following the downgrades of Ford's and General Motors' debt junk status by S&P on May 5, 2005. During this period, the Index surged from 42.42 basis points to a high of 60.3 basis points on May 17, 2005. Although these US based companies' debt was not part of the iTraxx Europe, the downgrades triggered a large demand for long-protection positions in the Index and the equity tranche as traders who had sold protection became concerned with idiosyncratic risks and sought to unwind their outstanding short protection positions (see Hawkins (2005), Packer and Wooldridge (2005), or Felsenheimer, Brommundt, Gisdakis, and Zaiser (2006) for a discussion). Whereas the Index' surge was accompanied by a large increase of the equity tranche (the upfront payment rose from 28.96 to 49.12%), it can be seen that mezzanine and senior tranches did not follow the movements on the Index. Reportedly, many credit hedge funds were following "relative value arbitrage" strategies in the beginning of 2005 by which they were selling protection on the 0-3% equity tranche and buying protection on the 3-6% mezzanine tranche, such that their portfolio was neutral to market-wide changes in credit spreads. While these positions provided a positive running premium, they were heavily exposed to changes in individual firms credit quality, or to changes in implied correlations of the equity and mezzanine tranches ("long correlation" trades, see the previous section and Packer and Woolridge (2005)). The spread on the 3-6% mezzanine tranche remained virtually constant over the period in question (168.25 bp on May 5 and 168.83 bp on May 17) and the more senior tranches increased only very slightly. In Figure 4, left, we display the implied compound correlation of the equity and mezzanine tranche along with the tranche spreads and the Index spread. While the correlation on the second loss tranche decreases significantly during the period from 0.1 to practically 0, it remained rather stable around 0.2 for the first loss tranche. The decreasing correlation shows that the crisis is seen as a credit event affecting only a few companies. Frequently, this period of time is also referred to as the "correlation crisis."

The Subprime Crisis A completely different scenario is the subprime crisis in the summer of 2007. As concerns about the future growth and a possible recession in the United States increased, housing price growth slowed and prices even started to decline in some areas. This lead to an increased number of foreclosures on subprime mortgages, i.e. credit contracts with very low debtor quality. These mortgages were typically wrapped up in mortgage backed securities (MBOs) and sold to institutional investors, who were attracted by the comparably high yield for the given rating. Since house prices grew rapidly in the previous years, default and foreclosure rates on MBOs were at historic lows and, thus, the risk associated with these structures appears to have been underestimated. As housing prices started to decline and foreclosure rates went up, investors became concerned about their exposure to these risks and started to sell off their MBO positions, leading to a rapid decline in market prices. This in turn lead to an increase in risk aversion among investors. At some points, the market for subprime MBOs dried out almost completely, as no buyers could be found. As a consequence of this illiquidity, mortgage lenders were not able to sell off their mortgage portfolios putting them into distress and forcing several into filing for Chapter 11. The right panels of Figure 4 show the Index spread and the spreads

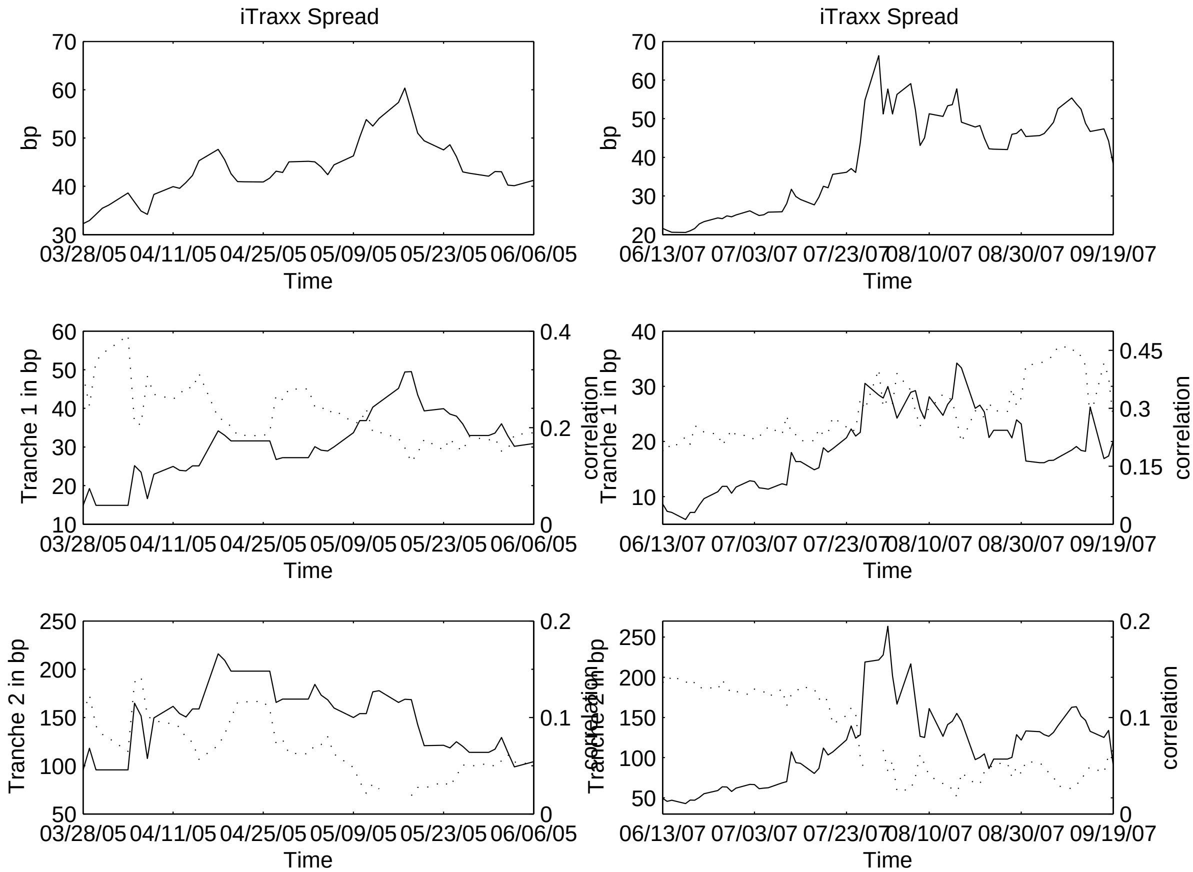

Figure 4: Tranche spreads, Index spreads, and implied compound correlations (dotted lines) during the two cirses. Whereas the implied correlation on the second loss tranche decreased significantly during the auto crisis, it remains practically unchanged during the subprime crisis. For the first loss tranche, however the correlation increases during the end of the sample period, indicating a higher risk of joint default events.

of the first and second tranche of the CDO along with their correlations. From July 16, 2007 to July 30, 2007, the Index spread surged from 27.69 to 66.33 bp. The quotes on the first loss tranche almost doubled from 14.87% to 28.39% during that period while the spread on the second loss tranche almost tripled from 80.50 to 221.50 bp, respectively. The spreads on the higher tranches increased by even higher factors, the 12-22% loss tranche's spread for example went from 4.04

to 26.08 bp. To the contrary, correlations appear to be practically unchanged during the crisis, indicating that the market wide increase in spreads was mostly driven by increased (risk neutral) default probabilities. We cannot detect extreme behavior of the implied compound correlations. While the compound correlation for the first loss tranche increases, that of the second tranche practically remains unchanged. This is also the case for the correlations of the higher tranches. Correlations only start to change only in August, as spread volatility increases.

4 Calibration and Hedging

On each day of the sample period, we calibrate the pricing equation (6) to the observed quotes. This way, we back out a time series of implied correlation for each tranche. The calibration is done by changing the tranche correlation. We estimate the necessary default probabilities from the spread of the iTraxx index, the average spread of the CDS contracts in the portfolio, and from the duration-weighted average spread of the CDS contracts.

Intuitively, we expect the first two of these spreads to be very similar: the index is an equally weighted portfolio of the CDS contracts. However, while both the equally weighted portfolio and the index suffer the same loss in the case of a default event, the change in the premium after default is different: while the portfolio loses the specific spread of the defaulted contract, the index spread is reduced by the proportion of the defaulted contract on the index notional. Historically, these spreads are close to each other. The difference between these spreads fluctuates. During the observation period, the Index spread is up to 10.6 basis points (bp) higher and up to 6.7 bp lower than the average spread. Some authors (e.g. Choudry (2006)) argue that the index spread should be equal to the duration-weighted average spread of the CDS contracts. This would be the case for the yield-to-maturity (YTM) of a portfolio of bonds which is the duration weighted average of the YTM of its components. As for the average spread, however, the premium on the two portfolios change differently after a default event. In the extreme cases, the Index is 11.6 bp higher and 6.1 bp lower than the duration weighted average spread. The bottom panel of Figure 3 displays all three spreads, all three spreads are similar and comove closely. The duration weighted average spread and the average spread appear to be more closely related to each other than to the index spread. The difference between these two spreads is between 0.3 and -1.4 bp (duration weighted minus average spread).

Figures 1 and 2 display the implied compound correlations and base correlations for the three input spreads. The compound correlations show a pronounced "smile" throughout the sample period: the correlation level for the second loss tranche is much lower than that of any other tranche. Base correlation overcome this undesirable feature and increase in tranche seniority. By construction, base correlations for all tranches comove much stronger than compound correlations. Consequently, the latter seem to behave much more heterogeneously; compare for example the two most junior tranches. Since the correlations are backed out of the same data, however, all the plots have the same information content. Similar to the tranche spreads in Figure 3, the senior tranches' correlations appear to be much less volatile than those of the junior tranches.

For all implementations, on several days of the sample period no correlation could replicate market prices. These dates show up as gaps in the correlation surfaces. Of course it is problematic if a model cannot be fitted to market data. In this article, however, we ignore these difficulties and exclude these dates from the empirical study. As mentioned in the previous section, we suspect some of these dates to be caused by data errors in the sample.

4.1 Performance of the Delta Hedge

Using the hedge ratios from (12), we form portfolios that are hedged against the linear effect of spread changes by. The portfolios consist of a long position of unit size in one tranche and a position whose size is given by the appropriate hedge ratio of one of the other tranches. We construct all possible hedge portfolios using any two tranches. As indicated by the correlation matrix in Table 1, we expect the hedges in tranches that share an attachment-detachment level to perform better than that in the other tranches. We report the mean absolute hedging errors (MAHE) and the root mean squared errors (RMSE) for the entire sample period in Tables 2 to 6. Figure 5 and 6 apportion the MAHE among the different series for compound and base correlations using the iTraxx spread as an input. Tests could not reject the hypothesis that the observed hedge errors have a zero mean. As expected, most of the hedges are successful: the MAHE is lower than that of the unhedged position in the tranche. An exception are hedges in "distant" tranches, for example those involving the first and the fifth tranche. Generally, hedges involving the fifth tranche seem not to work very well. Given the low correlation between the respective spreads in Table 1, however, this is no surprise. The RMSE points into the same direction: portfolios consisting of neighboring tranches also have a lower hedge error standard deviation. Hedges in more distant tranches seem frequently to be able to reduce the MAHE, however, they fail to reduce the RMSE. The hedge errors decrease in magnitude with increasing tranche seniority. Also, hedge errors calculated using base correlations tend to be smaller than those for compound correlations. Comparing the MAHE and RMSE of the different input spreads, we cannot conclude that one of the three spreads is superior to the others.

For each implementation and tranche, the hedge errors are qualitatively similar for all the six series: a hedge that performed well in one series also performs well in the others; the ordering of the hedge portfolios by MAHE remains practically unchanged. This is revealed in Figures 5 and 6. Irrespective of the input spread, the hedge errors are largest in the third and seventh series, which are the on-the-run tranches during the Auto Crisis in 2005 and the Subprime Crisis 2007. The lowest hedge errors are found in the fourth and fifth series for all tranches.

4.2 Largest Hedge Errors and Serial Dependence

A closer look at the hedge errors reveals that the largest absolute hedge errors are 20 to 100 times higher than the MAHE or the RMSE. This difference is smaller for portfolios of neighboring tranches and largest for portfolios consisting of the first and second and fourth and fifth tranche. Thus, a couple of events contribute a huge proportion to the (average) hedge performance. The contribution of the 20

Spread	Corr.	Unh.	Tr. 2	Tr. 3	Tr. 4	Tr. 5
Average	**base MAHE**	65.60 (24.90%)	36.55 (31.12%)	42.03 (34.53%)	49.43 (34.25%)	55.80 (31.08%)
	base RMSE	139.86 (78.02%)	92.87 (88.56%)	116.87 (91.10%)	148.38 (92.61%)	137.00 (86.81%)
	comp. MAHE	65.60 (24.90%)	44.17 (32.45%)	42.79 (34.79%)	53.44 (39.63%)	53.84 (31.71%)
	comp. RMSE	139.86 (78.02%)	118.27 (89.41%)	122.74 (90.52%)	186.84 (95.10%)	134.56 (86.99%)
Index	**base MAHE**	65.63 (24.95%)	39.55 (35.85%)	42.50 (35.09%)	49.18 (33.86%)	54.27 (31.87%)
	base RMSE	139.98 (78.05%)	116.94 (92.78%)	118.25 (91.07%)	144.93 (92.10%)	135.70 (87.79%)
	comp. MAHE	65.63 (24.95%)	41.85 (33.06%)	40.92 (34.92%)	48.30 (34.82%)	64.12 (42.80%)
	comp RMSE	139.98 (78.05%)	112.29 (89.78%)	116.89 (91.22%)	156.83 (93.50%)	280.30 (97.11%)
Dur. Weig.	**base MAHE**	65.39 (24.98%)	36.57 (31.12%)	41.89 (34.52%)	49.52 (34.31%)	55.72 (31.25%)
	base RMSE	139.86 (78.00%)	92.90 (88.56%)	116.75 (91.11%)	148.82 (92.65%)	137.39 (86.93%)
	comp. MAHE	65.39 (24.98%)	43.91 (32.60%)	42.75 (34.85%)	60.32 (41.99%)	77.57 (35.36%)
	comp. RMSE	139.86 (78.00%)	117.86 (89.53%)	122.68 (90.51%)	236.26 (96.61%)	239.25 (92.62%)

Table 2: Measures for the hedge errors of Tranche 1. First Column: Unhedged Position. In brackets is the relative contribution of the 20 largest changes in value to the respective measure.

Spread	Corr.	Tr. 1	Unh.	Tr. 3	Tr. 4	Tr. 5
Average	**base MAHE**	10.88 (37.62%)	15.99 (30.98%)	7.45 (36.27%)	10.44 (40.39%)	12.79 (35.71%)
	base RMSE	32.17 (91.88%)	39.31 (85.30%)	26.05 (94.68%)	53.21 (98.20%)	39.42 (92.92%)
	comp. MAHE	23.74 (42.59%)	17.14 (33.14%)	11.02 (41.46%)	18.93 (49.39%)	20.15 (40.51%)
	comp. RMSE	78.47 (93.07%)	44.91 (87.45%)	38.06 (94.10%)	94.61 (98.28%)	63.91 (92.90%)
Index	**base MAHE**	12.11 (44.26%)	17.04 (33.28%)	7.28 (35.81%)	10.47 (39.74%)	12.60 (35.08%)
	base RMSE	44.55 (95.81%)	44.73 (87.46%)	25.23 (94.64%)	51.57 (98.00%)	38.69 (92.81%)
	comp. MAHE	21.24 (44.66%)	17.13 (33.48%)	10.91 (44.66%)	15.24 (41.55%)	23.63 (52.58%)
	comp RMSE	72.10 (93.73%)	45.09 (87.62%)	39.79 (95.57%)	61.45 (96.68%)	131.18 (98.64%)
Dur. Weig.	**base MAHE**	10.86 (37.64%)	15.92 (31.12%)	7.42 (36.33%)	10.46 (40.52%)	12.79 (35.88%)
	base RMSE	32.12 (91.88%)	39.32 (85.27%)	26.00 (94.69%)	53.39 (98.21%)	39.56 (92.97%)
	comp. MAHE	23.15 (42.65%)	17.02 (33.39%)	10.87 (41.38%)	18.19 (55.99%)	33.89 (46.67%)
	comp. RMSE	76.55 (93.08%)	44.92 (87.43%)	37.66 (94.16%)	114.03 (99.24%)	132.22 (95.81%)

Table 3: Measures for the hedge errors of Tranche 2. Second Column: Unhedged Position. In brackets is the relative contribution of the 20 largest changes in value to the respective measure.

Spread	Corr.	Tr. 1	Tr. 2	Unh.	Tr. 4	Tr. 5
Average	**base MAHE**	5.25 (44.05%)	2.92 (37.66%)	7.19 (36.86%)	3.73 (39.87%)	4.48 (36.11%)
	base RMSE	17.42 (93.56%)	10.12 (94.35%)	20.64 (89.04%)	18.30 (97.64%)	14.21 (92.78%)
	comp. MAHE	7.20 (46.95%)	3.50 (44.80%)	7.42 (37.90%)	4.67 (43.75%)	5.73 (36.49%)
	comp. RMSE	26.66 (94.35%)	13.44 (95.58%)	21.89 (89.76%)	22.62 (97.75%)	17.52 (91.15%)
Index	**base MAHE**	5.20 (45.06%)	2.82 (38.61%)	7.13 (37.18%)	3.76 (40.29%)	4.61 (36.62%)
	base RMSE	17.52 (93.35%)	10.01 (94.77%)	20.61 (89.19%)	18.40 (97.65%)	14.53 (92.50%)
	comp. MAHE	6.61 (48.71%)	3.46 (46.18%)	7.40 (38.33%)	3.72 (31.27%)	6.83 (48.14%)
	comp RMSE	25.49 (95.42%)	13.49 (96.15%)	21.94 (89.96%)	11.90 (92.24%)	35.69 (98.02%)
Dur. Weig.	**base MAHE**	5.21 (44.11%)	2.90 (37.80%)	7.13 (36.99%)	3.73 (39.81%)	4.53 (36.62%)
	base RMSE	17.34 (93.56%)	10.09 (94.39%)	20.59 (88.99%)	18.29 (97.64%)	14.39 (92.79%)
	comp. MAHE	7.12 (46.96%)	3.48 (44.73%)	7.36 (38.15%)	5.51 (52.99%)	9.87 (44.59%)
	comp. RMSE	26.42 (94.29%)	13.37 (95.58%)	21.88 (89.71%)	31.86 (98.92%)	36.89 (95.33%)

Table 4: Measures for the hedge errors of Tranche 3. Third Column: Unhedged Position. In brackets is the relative contribution of the 20 largest changes in value to the respective measure.

Spread	**Corr.**	**Tr. 1**	**Tr. 2**	**Tr. 3**	**Unh.**	**Tr. 5**
Average	**base MAHE**	3.79 (48.64%)	2.45 (44.87%)	2.31 (44.35%)	4.56 (43.62%)	2.89 (45.72%)
	base RMSE	16.32 (97.12%)	13.07 (98.45%)	12.71 (98.32%)	16.45 (94.26%)	13.53 (97.49%)
	comp. MAHE	4.97 (55.67%)	3.12 (53.88%)	2.55 (49.15%)	5.15 (48.28%)	3.17 (44.37%)
	comp. RMSE	23.69 (97.87%)	16.41 (98.52%)	13.88 (98.43%)	20.58 (95.88%)	13.77 (96.88%)
Index	**base MAHE**	3.67 (50.36%)	2.39 (46.29%)	2.28 (45.22%)	4.60 (43.91%)	2.90 (45.94%)
	base RMSE	16.26 (97.29%)	13.14 (98.56%)	12.76 (98.38%)	16.56 (94.28%)	13.55 (97.54%)
	comp. MAHE	4.13 (51.66%)	2.52 (45.24%)	1.96 (34.98%)	4.54 (44.37%)	3.20 (46.16%)
	comp RMSE	19.59 (97.56%)	11.26 (97.33%)	7.02 (94.14%)	16.85 (94.88%)	15.20 (97.55%)
Dur. Weig.	**base MAHE**	3.77 (48.83%)	2.44 (45.13%)	2.30 (44.38%)	4.54 (43.78%)	2.89 (45.69%)
	base RMSE	16.30 (97.15%)	13.06 (98.47%)	12.70 (98.33%)	16.44 (94.23%)	13.51 (97.49%)
	comp. MAHE	6.57 (50.88%)	3.39 (50.40%)	3.40 (46.12%)	5.12 (48.54%)	5.65 (39.45%)
	comp. RMSE	25.93 (97.02%)	17.03 (98.53%)	15.52 (97.80%)	20.57 (95.86%)	19.01 (93.55%)

Table 5: Measures for the hedge errors of Tranche 4. Fourth Column: Unhedged Position. In brackets is the relative contribution of the 20 largest changes in value to the respective measure.

Spread	**Corr.**	**Tr. 1**	**Tr. 2**	**Tr. 3**	**Tr. 4**	**Unh.**
Average	**base MAHE**	2.19 (41.54%)	1.54 (35.21%)	1.36 (34.37%)	1.46 (41.58%)	2.04 (43.10%)
	base RMSE	7.01 (92.11%)	4.38 (89.88%)	3.86 (89.27%)	5.70 (95.59%)	6.55 (90.45%)
	comp. MAHE	2.24 (43.73%)	1.48 (41.46%)	1.37 (35.83%)	1.42 (41.25%)	2.06 (41.96%)
	comp. RMSE	7.68 (92.68%)	4.73 (91.71%)	3.98 (87.99%)	5.39 (95.02%)	6.52 (90.07%)
Index	**base MAHE**	2.07 (43.54%)	1.46 (36.97%)	1.38 (36.09%)	1.45 (43.02%)	2.06 (43.01%)
	base RMSE	6.88 (92.45%)	4.32 (90.75%)	4.00 (89.28%)	5.94 (96.12%)	6.57 (90.53%)
	comp. MAHE	2.88 (58.31%)	1.96 (57.08%)	1.78 (51.63%)	1.53 (46.50%)	2.65 (55.80%)
	comp RMSE	17.74 (98.76%)	12.60 (98.97%)	10.70 (98.48%)	7.58 (97.58%)	15.34 (98.33%)
Dur. Weig.	**base MAHE**	2.17 (41.75%)	1.52 (35.44%)	1.38 (35.19%)	1.45 (41.54%)	2.02 (43.33%)
	base RMSE	6.98 (92.11%)	4.36 (89.94%)	3.95 (89.50%)	5.67 (95.55%)	6.54 (90.36%)
	comp. MAHE	2.29 (48.81%)	1.52 (41.72%)	1.39 (37.31%)	1.35 (37.34%)	2.04 (42.32%)
	comp. RMSE	8.79 (95.68%)	4.78 (91.38%)	4.08 (89.91%)	4.06 (90.26%)	6.52 (90.00%)

Table 6: Measures for the hedge errors of Tranche 5. Fifth Column: Unhedged Position. In brackets is the relative contribution of the 20 largest changes in value to the respective measure.

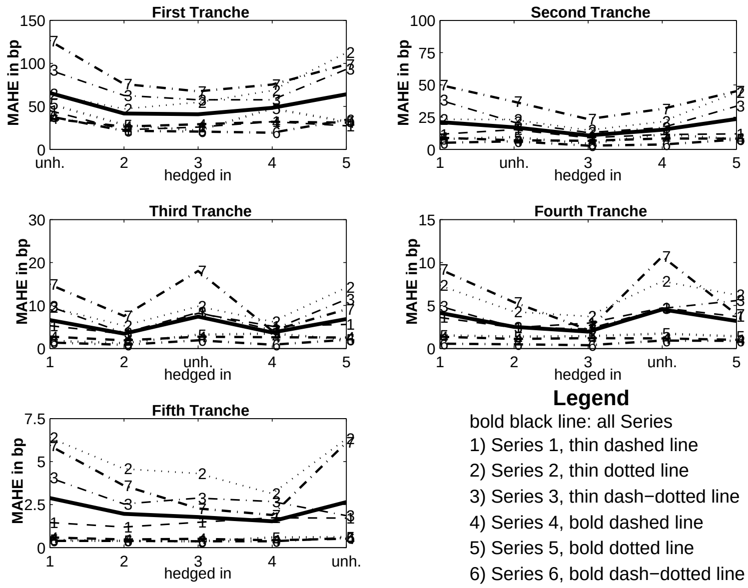

Figure 5: Mean absolute hedging errors (MAHE) for hedge portoflios of the five tranches of the iTraxx index, constructed using the index spread and compound correlations. The numbers on the y-axis refer to the tranche used to hedge respective tranche. The solid line shows the error for the entire sample, the thin lines for the respective series.

hedge errors of both signs to the measures is given in Tables 2 to 6 in brackets. For the MAHE, these observations contribute between 25 and 53% of the hedge error, whereas the RMSE seems almost completely determined by these observations, the minimum contribution is 78%, whereas the maximum is 98% of the RMSE. Figure 7 pictures the changes in (unhedged) tranche values in basispoints for the implementation using index spreads. Inspecting the graphs clearly reveals the higher risk associated with the tranches which suffer from earlier losses. Volatility seems to change over time: during the second and third series, and during the seventh series, large changes occured. During the first and forth to sixth series, tranche values hardly changed. Remarkably, the largest changes in value of both signs on the unhedged portfolios tend to occur on (around) the same dates for all tranches. Second, large changes in value on a tranche seem to be frequently

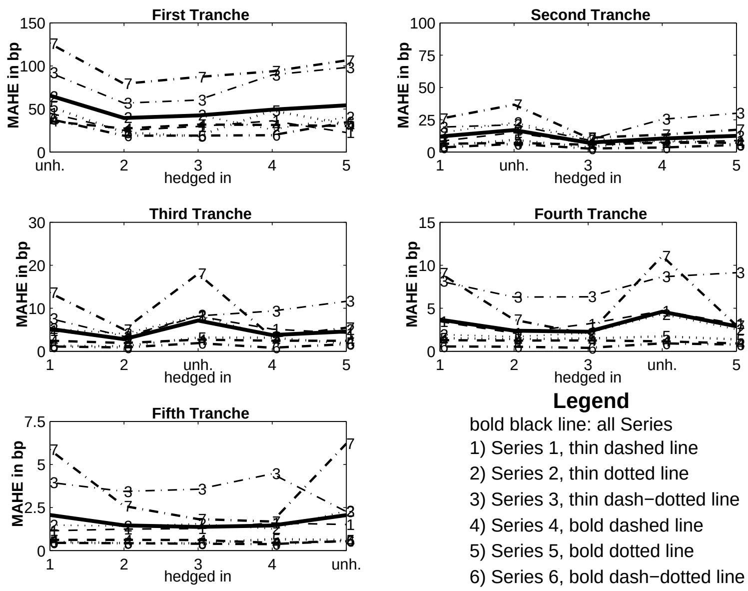

Figure 6: Mean absolute hedging errors (MAHE) calculated using the index spread and base correlations.

followed or preceded by large changes of the opposite sign. Whereas some of the earlier large value changes might be attributed to lower data quality during the first part of the sample period, we clearly recognize the higher volatility of the hedge errors during the Auto and Subprime crises.

Motivated by the former observations, we check whether there is autocorrelation in the value changes of the unhedged positions. We found practically no autocorrelation for all tranches (values around ± 0.1) with the exception of the second tranche, where we found an autocorrelation of the daily changes of about -0.2 for all compound correlation cases and for the index spread implementation in the base correlation case. For the fourth tranche, the autocorrelation was about -0.3 for the base correlation cases and -0.2 for the average and duration weighted average spread in the compound correlation case. Thus, we cannot conclude that there is autocorrelation in the value changes. However, we check for possible effects by extending the holding period of the hedge portfolio to 2 and 5 days for the best performing hedges of the one-day holding period. The errors increase substantially when doubling the rebalancing period to two days. Average increases add about 50% to the MAHE but increase the RMSE by a factor of 3 to 5.[14] The further extension to five days leads to practically no further increase in the MAHE and RMSE. This indicates that there might be some serial dependence in the hedge errors. The relative increase in the MAHE and RMSE for the not so well performing hedges is stronger, MAHE can almost double whereas RMSE can be up to 10 times larger. The MAHE and RMSE are calculated using overlapping hedge-periods. If we use non-overlapping hedge-periods, the numbers practically do not change.

4.3 Decomposing the Change in Value

Motivated by the results from the first section, we decompose the changes in tranche value in three components: the first and second order absolute effect of spread changes (delta and gamma) and the effect of correlation changes. Specifically, these effects are $||\Delta\,(\text{spread}_t - \text{spread}_{t-1})\,||$, $||\Gamma/2\,(\text{spread}_t - \text{spread}_{t-1})^2\,||$, and $||\mathcal{R}\,(\rho_t - \rho_{t-1})\,||$. In order to have a meaningful comparison, we normalize by the sum of these three effects and display their relative contribution to the change in value in Figures 8 and 9 for all five tranches for the model implementation using the average spread as the driver of the default probabilities. Note the different scale on the ordinate. For the two implementations using the index spread and the duration weighted spread, the results are very similar. The predominant driver of the changes in tranche value is the delta effect, the linear contribution of the changes in the driving spread. For the first tranche it accounts for practically the entire change in tranche value. For the other tranches its contribution is also very high. On several occasions, however, it drops to much smaller levels. The second order effect of the spread change is much smaller for all tranches. Throughout

[14] If the hedge errors variance is constant over time and hedge errors of subsequent days are independent then the increase in MAHE should be 41% ($\sqrt{2}-1$) for the step from one to two days. It should be 2 for the increase in RMSE.

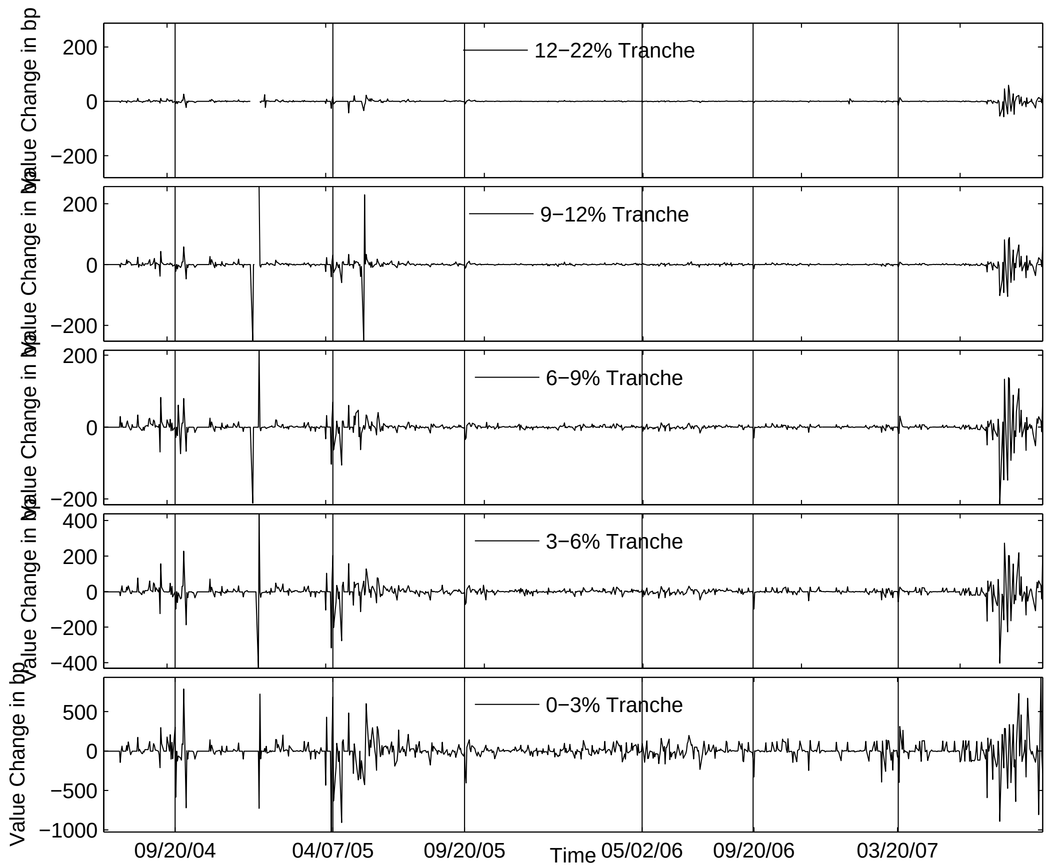

Figure 7: Value changes in an unhedged position in the respective CDO tranches in basis points from the Index spread implementation. One can clearly see the higher risk associated with more junior tranches. Moreover, it is obvious that there are times of higher and lower volatility. Remarkably, the largest changes in tranche value of both signs are followed or preceded by very large changes of the opposite sign. Vertical lines again indicate the index roll dates.

the sample it is less than 5% of the total change in value. The effect of changing correlations is complementary to that of the the delta: it is practically inexistent for the first tranche. For the other tranches, however, its contribution spikes on several occasions. On the dates of these spikes, the delta hedged portfolio typically drastically underperformed the unhedged portfolio. If also the change in correlation had been hedged, it would in most cases have compensated for a large

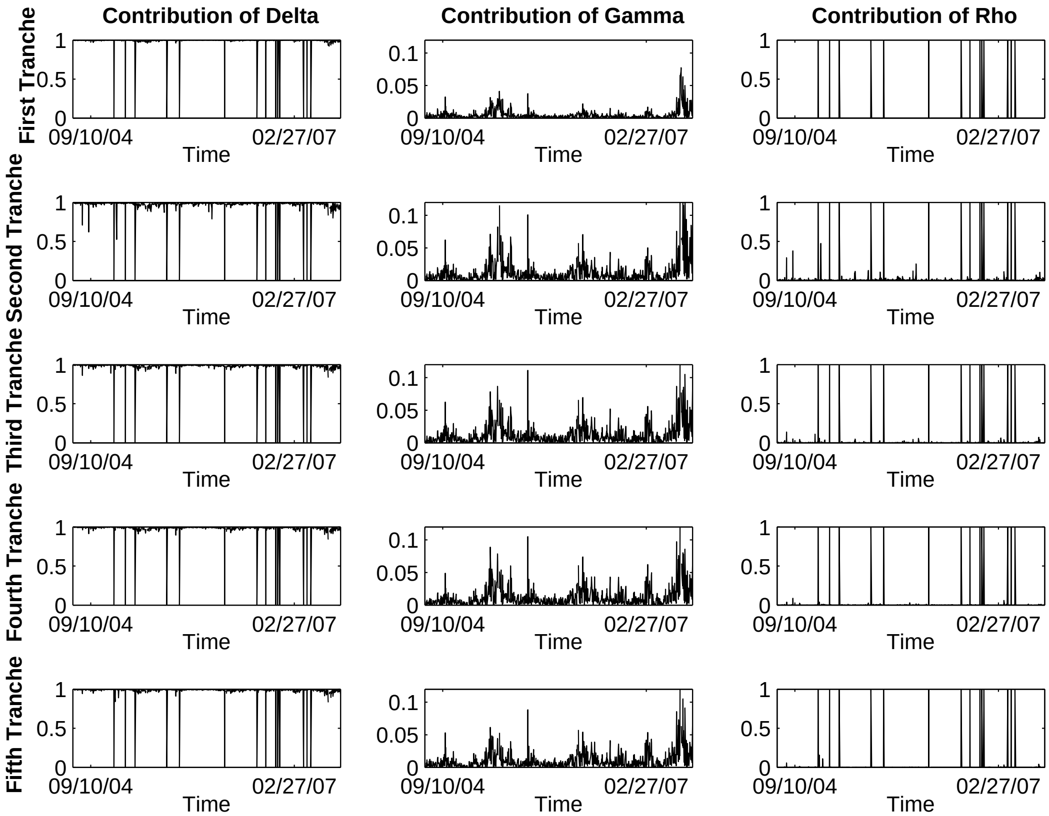

Figure 8: Relative contribution of the delta, gamma, and rho effects to the change in tranche value for compound correlations implemented with the Index spread. The main driver of value change is the linear component in the spread (delta), while the second order effect is much smaller. The effect of correlation changes seems to be small. This is due to the fact that correlations are usually small. However, large changes in spreads typically coincide with large changes in correlations.

part for the hedge error. Motivated by this finding, we added a position to the hedge portfolios in order to match the sensitivity to correlation and to the second order effect of changes in the spread. Since this portfolio could be formed from any two of the index tranches, there are twelve possible hedge portfolios for each tranche (if we form portfolios using only two tranches). We decided to display

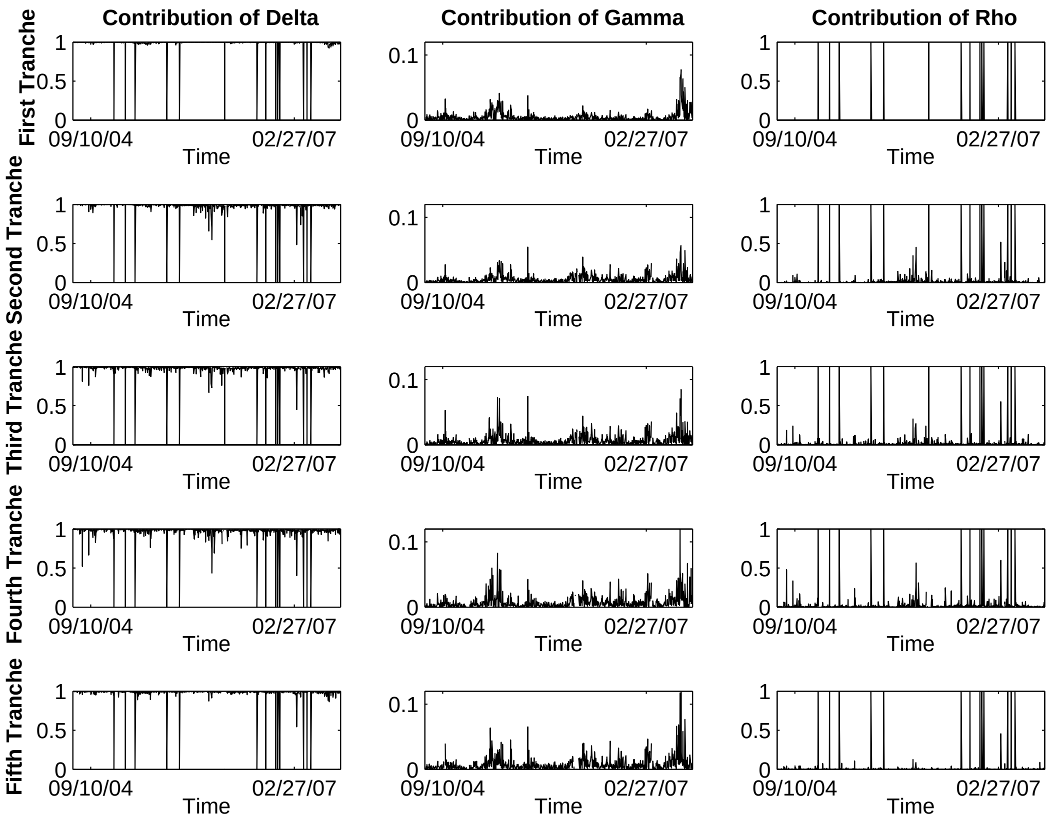

Figure 9: Relative contribution of the delta, gamma, and rho effects to the change in tranche value for base correlations implemented with the Index spread. The main driver of value change is the linear component in the spread (delta), while the second order effect is much smaller. The effect of correlation changes seems to be small. This is due to the fact that correlations are usually small. However, large changes in spreads typically coincide with large changes in correlations.

only the results for the hedge portfolio formed from the two tranches for which the delta hedge performed best for most implementations; these are also the portfolios which have the gamma and rho which is closest to that of the hedged tranche.[15] While this portfolio construction is clearly very naive, it could help to reduce the

[15]The delta hedged portfolios consist of the original and the second, third, second, third and forth tranche for the first to fifth tranche, respectively. To these a position in the third, first, fourth, second, and third tranche was added.

Spread	Corr.	Tr. 1	Tr. 2	Tr. 3	Tr. 4	Tr. 5
Average	**base MAHE**	640.18	93.31	170.34	165.83	99.43
	base RMSE	4668.72	304.81	1235.03	1434.96	635.32
	comp. MAHE	64.07	126.89	170.88	32.44	7.94
	comp. RMSE	161.89	390.70	4206.01	108.56	26.92
Index	**base MAHE**	605.01	90.22	192.14	119.25	89.48
	base RMSE	6074.67	282.42	2027.38	698.50	978.37
	comp. MAHE	72.16	112.41	27.75	37.67	8.83
	comp RMSE	322.91	353.91	154.42	175.83	35.44
Dur. Weig.	**base MAHE**	527.95	95.10	101.87	115.89	46.29
	base RMSE	1778.32	287.79	454.16	447.64	227.54
	comp. MAHE	65.08	123.29	247.47	113.15	5.63
	comp. RMSE	165.76	370.48	6508.95	503.51	19.34

Table 7: Hedge errors of the portfolios hedged against delta and gamma. Both, the MAHE and RMSE increased significantly. The hedge portfolios were constructed from the two tranches that performed best in delta hedging the respective tranche.

risk when hedging equity options. Tables 7 and 8 report the resulting MAHE and RMSE. The errors increased drastically for most implementations and models. When hedging all three effects, delta, gamma, and rho, the performance is even worse. Closer inspection can trace the error back to two sources. First, the gamma and rho differ drastically between tranches which leads to hedge portfolios including long and short positions in both tranches much larger in size than for the simple delta hedged portfolio. Consequently, the hedge errors of these portfolios have a much larger variance increasing MAHE and boosting RMSE. Considering hedges which are constructed to achieve minimal variance as motivated by Ross (1997) could eventually overcome this difficulty. However, the simple model considered here does not allow for such calculations. The second source of error

Spread	**Corr.**	**Tr. 1**	**Tr. 2**	**Tr. 3**	**Tr. 4**	**Tr. 5**
Average	**base MAHE**	520.13	5764.80	64.99	30.66	9.59
	base RMSE	7931.39	135363.37	386.03	217.27	48.05
	comp. MAHE	51.44	3142.77	8.30	25.90	3.20
	comp. RMSE	150.73	18270.27	42.72	146.15	11.38
Index	**base MAHE**	212.05	530.49	342.44	31.64	8.12
	base RMSE	2486.38	2981.75	8426.76	244.37	56.06
	comp. MAHE	49.65	7198.44	28.28	38.79	3.60
	comp RMSE	129.97	116282.74	608.69	441.80	19.14
Dur. Weig.	**base MAHE**	2129.50	650.83	98.36	45.67	11.74
	base RMSE	55760.13	5409.53	907.45	319.08	92.90
	comp. MAHE	58.30	13276.09	8.74	157.91	2.40
	comp. RMSE	260.08	292960.86	56.25	1672.15	9.87

Table 8: Hedge errors of the portfolios hedged against delta and gamma. Both, the MAHE and RMSE increased significantly. The hedge portfolios were constructed from the two tranches that performed best in delta hedging the respective tranche.

affects the sensitivity to correlation changes: a hedge against correlation changes only works if there is a common factor driving the correlations for all tranches. Only the common changes, that are parallel shifts, in the correlations would be hedged. If a portfolio is hedged against these, any non-parallel changes can have a much higher effect on the portfolio value than in the unhedged case. This is what happened in our sample. We performed a principal component analysis for the changes in all correlations displayed. A single component explained 63.22% (76.71%, 76.71%) and of the total variation of the compound correlations and 98.15% (97.70%, 97.76%) of the variation of the base correlations for the implementation with the Index (average, duration weighted average) spread. For the CDO with 10 years to maturity and using weekly data Willemann and Isla (2007) find the first principal component to explain 87% of the total correlation variation. The difference in the explained variance between the two correlation schemes is obvious: by construction, base correlations of higher tranches depend on that of the more junior tranches. This does not imply that base correlations are to be preferred to hedge correlation risk. Non-parallel changes have different effects in both schemes. However, we conclude that it should be possible to hedge against parallel changes in correlation in general. In the naive way investigated above, however, it is not possible. Moreover, if correlation is assumed to be changing over time, it should also be modeled as dynamic in the model setup. Several extensions of the Gaussian copula models can be seen as having random correlation coefficients: most considered alternatives to the normal distribution are mixture distributions, i.e. the product of a normally distributed random variable with one of a different distribution. Andersen and Sidenius (2005b) and Burtschell, Gregory, and Laurent (2005) discuss several of these extensions. However, all these models describe correlation as essentially static and not as a dynamic quantity that evolves over time.

5 Adjusting Hedge Ratios

The latter results from the previous section are disappointing: the LHGM is not able to provide advice on how to hedge against changes in correlation or second order effects of the spreads. While the second order effect is responsible for only a small proportion of the change in the present value of a tranche position, the correlation change has a large impact on several occasions. Figure 7 reveals that the days of the largest changes in correlation also coincide with the largest changes in the driving spread (Index, average, duration weighted). Motivated by this finding, we investigate the interrelation between correlation changes and spread changes. This was also suggested by St. Pierre, Rousseau, Zavattero, van Eyeren, and Aurora (2004) and also Calamaro and Meli (2004). We estimate the coefficient in a simple linear regression between spread changes and correlation changes

$$\rho(t) - \rho(t-1) = \beta\left(\text{spread}(t) - \text{spread}(t-1)\right). \tag{13}$$

Table 9 lists the slope coefficients from a regression using the entire sample period and the subsamples of the individual index series of the base correlation implementation using the duration weighted spread. While the explanatory power of the regressions (measured by the R^2) is very small, about 10% in all cases, all coefficients are highly significant. We also ran the regression including a non-zero intercept. When estimated, the intercept is not significantly different from zero. Thus, we assume a zero intercept. A non-zero intercept would need more careful considerations since it could lead to arbitrage opportunities. Also, correlation has to stay in the interval [0, 1]. We reran the regressions for the individual series. The coefficients for these regressions were not significant, however. In Figures 10 and 11, we report the hedge errors (MAHE) for the simple delta-hedged portfolios for the Index spread implementation. We use the adjusted hedge ratios

$$HR_{adj} = \frac{\partial Tranche1}{\partial Tranche2} = \frac{\frac{\partial Tranche1}{\partial s} + \frac{\partial \rho(s)}{\partial s}\frac{\partial Tranche1}{\partial \rho}}{\frac{\partial Tranche2}{\partial s} + \frac{\partial \rho(s)}{\partial s}\frac{\partial Tranche1}{\partial \rho}}, \tag{14}$$

Series	**Tr. 1**	**Tr. 2**	**Tr. 3**	**Tr. 4**	**Tr. 5**
All	0.00047	0.00066	0.00012	-0.000159	-0.00060
Series 1	0.00480	0.00654	0.00794	0.00995	0.01864
Series 2	0.01250	0.01798	0.01825	0.02009	0.02584
Series 3	-0.00251	-0.00114	-0.00039	-0.00006	0.00029
Series 4	0.00176	0.00377	0.00444	0.00482	0.00455
Series 5	-0.00028	0.00168	0.00276	0.00328	0.00487
Series 6	-0.00745	-0.00934	-0.01054	-0.01145	-0.01224
Series 7	0.00004	-0.00074	-0.00183	-0.00259	-0.00403

Table 9: Slope coefficients for the regressions in equation (13) for the base correlations using average spreads for all tranches. In the top row, the results for the entire sample period are displayed, below we show those for the sub-samples of the respective series. In the regression changes in the duration weighted spread and base correlations were used. The slope coefficient was highly significant for the entire sample period. For the sub samples, it was not significant on the 5% level for any of the regressions.

where the second summand is comes from the relation $\rho(s + ds) = \rho(s) + \beta\, ds$. The figures correspond to Figures 5 and 6. The hedge ratios were adjusted using the regression results from the individual series regressions. Given the small slope coefficients, it is not surprising that the order of magnitude of the hedge errors did not change much. However, the adjusted hedge errors are a little bit smaller when the the adjusted hedge ratios for the individual series. This can be seen particular well for the base correlations/Index spread case. In Tables 10 to 14, we show the MAHE and RMSE resulting from the regression using the entire sample. In brackets behind the MAHE we report the test statistic for an adjusted t-test, the brackets behind the RMSE contain the p-value of a variance ration test. We test whether the hedge error is smaller for the adjusted hedge ratio. The t-statistic can be interpreted as follows: a value between ± 1.96 indicates that the hedge errors are not significantly different on the 95% confidence level, and a statistic of above 1.64 indicates that the hypothesis that the hedge error using the adjusted ratio is smaller cannot be rejected. We can interpret the statistics even further: a statistic below -1.64 means that the opposite hypothesis cannot be rejected, i.e. the adjusted portfolio performs worse than the unadjusted. While for the first two tranches, the improvement seems not to be statistically significant, it becomes controversial for the higher tranches. There the difference between adjusted and unadjusted hedge ratios are frequently found to be significantly different but often they don not improve but become worse. Comparing the results for base correlations and compound correlations, we cannot find strong differences to the unadjusted case. The results seem to be significant in more cases for base correlations which also seem to outperform compound correlations by a small amount in the hedge performance.

Spread	Corr.	Unh.	Tr. 2	Tr. 3	Tr. 4	Tr. 5
Average	**base MAHE**	65.60 (-)	36.58 (-5.13)	41.91 (0.43)	49.24 (0.47)	55.66 (0.84)
	base RMSE	139.86 (-)	92.95 (0.98)	115.32 (0.70)	146.69 (0.75)	136.04 (0.83)
	comp. MAHE	65.60 (-)	43.88 (-2.24)	42.72 (-3.95)	53.43 (-28.03)	53.80 (4.32)
	comp. RMSE	139.86 (-)	117.95 (0.94)	122.76 (0.99)	186.77 (0.99)	134.37 (0.96)
Index	**base MAHE**	65.63 (-)	39.66 (-2.27)	42.59 (-2.23)	49.08 (-1.41)	54.23 (-0.46)
	base RMSE	139.98 (-)	117.33 (0.93)	118.63 (0.94)	144.74 (0.97)	135.17 (0.78)
	comp. MAHE	65.63 (-)	41.66 (-3.47)	40.81 (-2.40)	48.38 (-1.87)	64.16 (-2.43)
	comp. RMSE	139.98 (-)	111.91 (0.93)	116.85 (1.00)	156.97 (0.98)	280.26 (1.00)
Dur. Weig.	**base MAHE**	65.39 (-)	36.57 (-16.48)	41.83 (0.82)	49.44 (0.83)	55.64 (1.31)
	base RMSE	139.86 (-)	92.92 (0.99)	116.18 (0.89)	148.14 (0.90)	136.89 (0.91)
	comp. MAHE	65.39 (-)	43.63 (-1.49)	42.69 (-3.39)	60.27 (-2.23)	77.08 (-0.75)
	comp. RMSE	139.86 (-)	117.55 (0.94)	122.70 (1.00)	235.46 (0.92)	236.79 (0.77)

Table 10: Hedge Errors of Tranche 1 using the adjusted hedge ratios for the entire sample. In brackets behind the MAHE are the t-statistics comparing the differences of the hedge errors of the adjusted and unadjusted portfolios. The brackets behind the RMSE contain the p-values of a variance ratio test between the adjusted and unadjusted hedge error variance. Again, the first column gives the results for an unhedged Position.

Spread	Corr.	Tr. 1	Unh.	Tr. 3	Tr. 4	Tr. 5
Average	**base MAHE**	10.87 (15.61)	15.99 (-)	7.40 (0.85)	10.32 (0.92)	12.72 (1.95)
	base RMSE	32.13 (0.97)	39.31 (-)	25.35 (0.45)	52.29 (0.62)	39.02 (0.77)
	comp. MAHE	23.26 (0.36)	17.14 (-)	10.87 (3.47)	18.63 (1.67)	19.77 (1.03)
	comp. RMSE	77.28 (0.66)	44.91 (-)	37.86 (0.88)	93.85 (0.82)	63.11 (0.71)
Index	**base MAHE**	12.09 (7.98)	17.04 (-)	7.25 (-12.11)	10.36 (-4.01)	12.52 (-1.46)
	base RMSE	44.40 (0.92)	44.73 (-)	25.12 (0.90)	50.95 (0.73)	37.98 (0.57)
	comp. MAHE	20.96 (1.72)	17.13 (-)	10.93 (1.10)	15.26 (1.30)	23.51 (0.91)
	comp. RMSE	71.43 (0.79)	45.09 (-)	39.95 (0.91)	61.66 (0.92)	131.19 (1.00)
Dur. Weig.	**base MAHE**	10.86 (63.30)	15.92 (-)	7.40 (1.48)	10.41 (1.75)	12.75 (3.43)
	base RMSE	32.11 (0.99)	39.32 (-)	25.75 (0.78)	53.04 (0.85)	39.37 (0.89)
	comp. MAHE	22.70 (0.13)	17.02 (-)	10.72 (2.48)	18.00 (-0.50)	33.21 (-0.59)
	comp. RMSE	75.46 (0.68)	44.92 (-)	37.46 (0.87)	112.75 (0.74)	129.65 (0.57)

Table 11: Hedge Errors of Tranche 2 using the adjusted hedge ratios for the entire sample. In brackets behind the MAHE are the t-statistics comparing the differences of the hedge errors of the adjusted and unadjusted portfolios. The brackets behind the RMSE contain the p-values of a variance ratio test between the adjusted and unadjusted hedge error variance. Again, the second column gives the results for an unhedged Position.

Spread	Corr.	Tr. 1	Tr. 2	Unh.	Tr. 4	Tr. 5
Average	**base MAHE**	5.34 (-0.01)	2.98 (-1.05)	7.19 (-)	3.74 (18.51)	4.50 (9.88)
	base RMSE	17.56 (0.81)	10.10 (0.99)	20.64 (-)	18.41 (0.86)	14.32 (0.83)
	comp. MAHE	7.16 (6.75)	3.48 (-19.17)	7.42 (-)	4.64 (12.00)	5.69 (8.19)
	comp. RMSE	26.58 (0.93)	13.42 (0.97)	21.89 (-)	22.48 (0.86)	17.41 (0.85)
Index	**base MAHE**	5.22 (20.76)	2.83 (32.44)	7.13 (-)	3.74 (-13.53)	4.57 (-5.62)
	base RMSE	17.50 (0.97)	10.01 (1.00)	20.61 (-)	18.26 (0.84)	14.33 (0.68)
	comp. MAHE	6.55 (6.55)	3.46 (-21.21)	7.40 (-)	3.72 (5.24)	6.79 (3.23)
	comp. RMSE	25.39 (0.91)	13.51 (0.97)	21.94 (-)	11.87 (0.94)	35.47 (0.86)
Dur. Weig.	**base MAHE**	5.24 (3.28)	2.92 (0.50)	7.13 (-)	3.73 (113.39)	4.54 (62.33)
	base RMSE	17.39 (0.93)	10.09 (0.99)	20.59 (-)	18.32 (0.96)	14.41 (0.97)
	comp. MAHE	7.09 (5.32)	3.46 (-13.77)	7.36 (-)	5.49 (-4.31)	9.75 (-3.13)
	comp. RMSE	26.36 (0.94)	13.36 (0.98)	21.88 (-)	31.59 (0.80)	36.36 (0.67)

Table 12: Hedge Errors of Tranche 3 using the adjusted hedge ratios for the entire sample. In brackets behind the MAHE are the t-statistics comparing the differences of the hedge errors of the adjusted and unadjusted portfolios. The brackets behind the RMSE contain the p-values of a variance ratio test between the adjusted and unadjusted hedge error variance. Again, the third column gives the results for an unhedged Position.

Spread	Corr.	Tr. 1	Tr. 2	Tr. 3	Unh.	Tr. 5
Average	**base MAHE**	3.84 (-3.05)	2.48 (-3.33)	2.30 (-26.17)	4.56 (-)	2.90 (46.37)
	base RMSE	16.38 (0.91)	13.08 (0.97)	12.70 (0.98)	16.45 (-)	13.53 (0.98)
	comp. MAHE	4.97 (186.68)	3.11 (-23.27)	2.55 (-35.93)	5.15 (-)	3.16 (50.26)
	comp. RMSE	23.69 (1.00)	16.40 (0.99)	13.88 (0.99)	20.58 (-)	13.76 (0.98)
Index	**base MAHE**	3.70 (25.50)	2.41 (24.01)	2.29 (37.36)	4.60 (-)	2.90 (-15.57)
	base RMSE	16.25 (1.00)	13.15 (0.99)	12.77 (0.98)	16.56 (-)	13.53 (0.96)
	comp. MAHE	4.11 (14.03)	2.52 (-44.93)	1.95 (-64.10)	4.54 (-)	3.18 (9.00)
	comp. RMSE	19.58 (0.99)	11.26 (0.99)	7.02 (0.98)	16.85 (-)	15.11 (0.87)
Dur. Weig.	**base MAHE**	3.79 (-2.96)	2.45 (-4.19)	2.30 (-220.21)	4.54 (-)	2.89 (142.22)
	base RMSE	16.32 (0.96)	13.07 (0.99)	12.70 (1.00)	16.44 (-)	13.51 (1.00)
	comp. MAHE	6.58 (16.19)	3.41 (-10.52)	3.43 (-16.24)	5.12 (-)	5.63 (-17.96)
	comp. RMSE	25.95 (0.98)	17.04 (0.98)	15.54 (0.98)	20.57 (-)	18.95 (0.93)

Table 13: Hedge Errors of Tranche 4 using the adjusted hedge ratios for the entire sample. In brackets behind the MAHE are the t-statistics comparing the differences of the hedge errors of the adjusted and unadjusted portfolios. The brackets behind the RMSE contain the p-values of a variance ratio test between the adjusted and unadjusted hedge error variance. Again, the fourth column gives the results for an unhedged Position.

Spread	Corr.	Tr. 1	Tr. 2	Tr. 3	Tr. 4	Unh.
Average	**base MAHE**	2.22 (-16.82)	1.55 (-12.21)	1.36 (-72.16)	1.46 (-45.38)	2.04 (-)
	base RMSE	7.07 (0.80)	4.40 (0.85)	3.86 (0.96)	5.68 (0.93)	6.55 (-)
	comp. MAHE	2.24 (-14.00)	1.47 (-35.99)	1.37 (-42.20)	1.42 (-87.98)	2.06 (-)
	comp. RMSE	7.69 (0.97)	4.72 (0.94)	3.98 (1.00)	5.40 (0.97)	6.52 (-)
Index	**base MAHE**	2.08 (20.67)	1.47 (21.89)	1.38 (33.51)	1.45 (40.39)	2.06 (-)
	base RMSE	6.86 (0.94)	4.33 (0.98)	4.01 (0.98)	5.93 (0.97)	6.57 (-)
	comp. MAHE	2.88 (34.66)	1.96 (-78.63)	1.78 (-66.58)	1.53 (-62.56)	2.65 (-)
	comp. RMSE	17.76 (0.98)	12.60 (1.00)	10.68 (0.97)	7.56 (0.95)	15.34 (-)
Dur. Weig.	**base MAHE**	2.18 (-28.21)	1.53 (-22.19)	1.38 (-170.54)	1.45 (-215.39)	2.02 (-)
	base RMSE	7.01 (0.90)	4.38 (0.93)	3.94 (1.00)	5.67 (1.00)	6.54 (-)
	comp. MAHE	2.29 (-20.89)	1.52 (-30.55)	1.40 (-44.20)	1.35 (96.39)	2.04 (-)
	comp. RMSE	8.80 (0.96)	4.77 (0.98)	4.08 (0.97)	4.06 (1.00)	6.52 (-)

Table 14: Hedge Errors of Tranche 5 using the adjusted hedge ratios for the entire sample. In brackets behind the MAHE are the t-statistics comparing the differences of the hedge errors of the adjusted and unadjusted portfolios. The brackets behind the RMSE contain the p-values of a variance ratio test between the adjusted and unadjusted hedge error variance. Again, the fifth column gives the results for an unhedged Position.

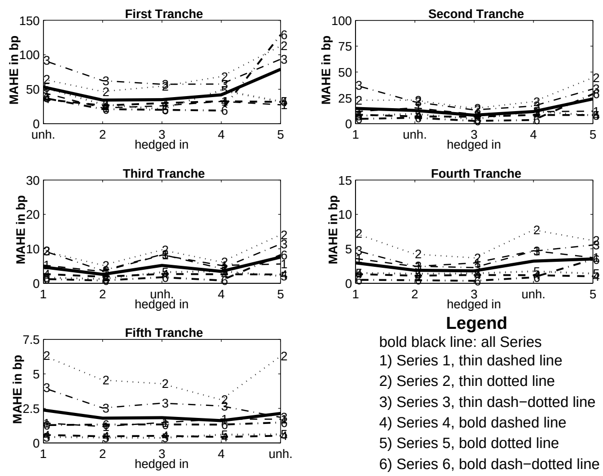

Figure 10: Mean absolute hedging errors (MAHE) for hedge portfolios of the five tranches of the iTraxx Index, constructed using the Index spread, compound correlations and adjusted hedge ratios. The numbers on the y-axis refer to the tranche used to hedge respective tranche. The solid line shows the error for the entire sample, the thin lines for the respective series.

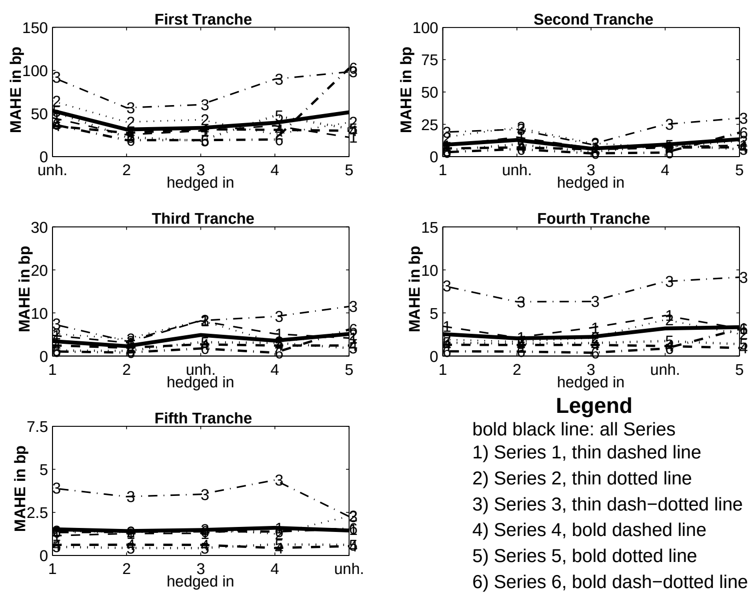

Figure 11: Mean absolute hedging errors (MAHE) calculated using the Index spread, base correlations and adjusted hedge ratios.

6 Conclusions

In an empirical study, we found the Gaussian One-Factor Copula Model to provide hedge ratios which could substantially reduce the risk on portfolios of CDO tranches. This result was robust across six implementations of the model using three input spreads and two correlation regimes, base and compound correlations. We found base correlation implementations to perform better than those using compound correlations. As expected, hedges in tranches that share an attachment-detachment level performed better than those in other tranches.

Extending the holding period of the hedge portfolios from one to two and five trading days, we found some evidence for serial dependence in the hedge errors. Moreover, extreme value changes tend to occur simultaneously for all tranches and to cluster in time. However, we were not able to utilize these two observations dependence to reduce the hedge errors.

Simple hedges against the risk of changing correlations, and second order effects of spread changes were constructed. The model failed dramatically on this task. Thus, we conclude that hedging these risks is not possible using the simple model. The reason for this is the essentially static model setup. In order to improve the models ability to describe hedging schedules, the inclusion of jumps and dynamic correlations seem to be crucial.

Motivated by our findings on serial dependence, we performed an regression between spread and correlation changes. This proved a weak but significant interrelation. Adjusting the hedge ratios for this interrelation reduced hedge errors some cases. However, the reduction was very small and in most cases not significant.

References

Andersen, L., and J. Sidenius (2005a): "CDO Pricing with Factor Models: Survey and Comments," *Journal of Credit Risk*, 1(3), 71 – 88.

——— (2005b): "Extensions to the Gaussian Copula: Random Recovery and Random Factor Loadings," *Journal of Credit Risk*, 1(1), 29–70.

Bluhm, C., L. Overbeck, and C. Wagner (2003): *Credit Risk Modeling*, Chapman & Hall/CRC Financial Mathematics Series. Chapman & Hall/CRC, Boca Raton.

Burtschell, X., J. Gregory, and J. P. Laurent (2005): "A Comparative Analysis of CDO Pricing Models," Working Paper.

Bystrom (2006): "CreditGrades and the iTraxx CDS Index Market," *Financial Analysts Journal*, Vol. 62(6), 65–76.

Calamaro, J.-P., and J. Meli (2004): "Trading Tranched Index Products: our Deltas Explained," Deutsche Bank, Working Paper.

Choudry, M. (2006): *The Credit Default Swap Basis*. Bloomberg Press.

Collin-Dufresne, P., R. Goldstein, and J. S. Martin (2001): "The Determinants of Credit Spread Changes," *The Journal of Finance*, 56(6).

Ehlers, P., and P. Schönbucher (2006): "Pricing Interest Rate-Sensitive Credit Portfolio Derivatives," *Swiss Finance Institute Research Paper No. 06-39*.

Embrechts, P., R. Frey, and A. McNeil (2005): *Quantitative Risk Management: Concepts, Techniques, and Tools*, Princeton Series in Finance. Princeton University Press, Princeton, New Jersey.

EMBRECHTS, P., A. MCNEIL, AND D. STRAUMANN (2002): *Correlation and Dependence in Risk management: Properties and Pitfalls*pp. 176–223. Cambridge University Press.

EOM, Y. H., J. HELWEGE, AND J.-Z. HUANG (2004): "Structural Models of Corporate Bond Pricing: an Empirical Analysis," *Review of Financial Studies*, 17, 499–544.

ERICSSON, J., J. RENEBY, AND H. WANG (2006): "Can Structural Models Price Default Risk? Evidence from Bond and Credit Derivative Markets," Working Paper.

FELSENHEIMER, J., B. BROMMUNDT, P. GISDAKIS, AND M. ZAISER (2006): "Recent Developments in Credit Markets," *Financial Markets and Portfolio Management*, 20(2), 221–234.

FELSENHEIMER, J., P. GISDAKIS, AND M. ZAISER (2006): *Active Credit Portfoilo Management*. John Wiley & Sons.

FINGER, C. (2005): "Issues in the Pricing of CDOs," *Journal of Credit Risk*, 1(1), 113 – 123.

FITCH (2006): "Understanding and Hedging Risks in Synthetic CDO Tranches," Discussion paper, Fitch Ratings.

HAWKINS, L. (2005): "S&P cuts rating on GM and Ford to junk status," *The Wall Street Journal*.

HULL, J., AND A. WHITE (2005): "The Perfect Copula," University of Toronto, Working Paper.

KALEMANOVA, A., B. SCHMID, AND R. WERNER (2007): "The Normal Inverse Gaussian Distribution for Synthetic CDO Pricing," *The Journal of Derivatives*, pp. 80–94.

Longstaff, F. A., and A. Rajan (2007): "An Empirical Analysis of the Pricing of Collateralized Debt Obligations," *The Journal of Finance*, forthcoming.

McGinty, L., E. Beinstein, R. Ahluwalia, and M. Watts (2004): "Introducing Base Correlations," JPMorgan, Credit Derivatives Strategy.

Mina, J., and E. Stern (2003): "Examples and Applications of Closed-form CDO Pricing," *RiskMetrics Journal*, pp. 5–34.

Moosbrucker, T. (2006): "Explaining the Correlation Smile Using Variance Gamma Distributions," *The Journal of Fixed Income*.

O'Kane, D., and L. Schloegl (2005): "A Note on the Large Homogeneous Portfolio Approximation with the Student-t Copula," *Finance and Stochastics*, 9(4), 577–584.

Packer, F., and P. Woolridge (2005): "Overview: Repricing in Credit Markets," *BIS Quarterly Review*.

Rajan, A., G. McDermott, and R. Roy (2007): *The Structured Credit Handbook*. John Wiley & Sons.

Ross, S. (1997): "Hedging Long-Run Commitments: Exercises in Incomplete Market Pricing," *Economic Notes*, (26), 385–419.

Skarabot, J., and B. Gaurav (2007): *Credit Derivatives Indexes*chap. 4, pp. 79 –109, Wiley Finance. John Wiley & Sons, Hoboken, New Jersey.

Skarabot, J., R. Roy, and J.-H. Ryu (2007): *Single Tranche CDOs*chap. 6, pp. 149–195, Wiley Finance. John Wiley & Sons, Hoboken, New Jersey.

St. Pierre, M., E. Rousseau, J. Zavattero, O. van Eyeren, and A. Aurora (2004): "Valuing and Hedging Synthetic CDO Tranches Using Base Correlations," Discussion paper, Bear Stearns.

Vasicek, O. A. (1977): "An Equilibrium Characterization of the Term Structure," *Journal of Financial Economics*, 5, 177–188.

Willemann, S., and L. Isla (2007): "Structured Credit Strategist - Correlation Modelling - from Vanilla to Exotic," Discussion paper, Barclays Capital.

Yu, F. (2005): "Accounting Transparency and the Term Structure of Credit Spreads," *Journal of Financial Economics*, 75, 53–84.