

THE CAPM AND FAMA-FRENCH MODELS IN BRAZIL¹

Fernando Daniel Chague²
Rodrigo D. L. S. Bueno³
Fundação Getulio Vargas
R. Itapeva, 474 - 7.º andar - Bela Vista
01332-000 - São Paulo - S.P.

December 2007

¹The first author thanks CAPES and EESP-FGV for the financial support. The second author thanks GVPesquisa for the financial support.

²MA in Economics, graduate student at University of North Caroline, Chapell Hill.
Email: fchague@email.unc.edu

³Getulio Vargas Foundation, Sao Paulo and and CFC/FGV. rodrigo.bueno@fgv.br

Resumo

Abstract

This paper confronts the Capital Asset Pricing Model - CAPM - and the 3-Factor Fama-French - FF - model using both Brazilian and US stock market data for the same sample period (1999-2007). The US data will serve only as a benchmark for comparative purposes. We use two competing econometric methods, the Generalized Method of Moments (GMM) by Hansen (1982) and the Iterative Nonlinear Seemingly Unrelated Regression Estimation (ITNLSUR) by Burmeister and McElroy (1988). Both methods nest other options based on the procedure by Fama-MacBeth (1973). The estimations show that the FF model fits the Brazilian data better than CAPM, however it is imprecise compared with the US analog. We argue that this is a consequence of an absence of clear-cut anomalies in Brazilian data, specially those related to firm size. The tests on the efficiency of the models - nullity of intercepts and fitting of the cross-sectional regressions - presented mixed conclusions. The tests on intercept failed to reject the CAPM when Brazilian value-premium-wise portfolios were used, contrasting with US data, a very well documented conclusion. The ITNLSUR has estimated an economically reasonable and statistically significant market risk premium for Brazil around 6.5% per year without resorting to any particular data set aggregation. However, we could not find the same for the US data during identical period or even using a larger data set.

JEL Classification: G12, C52, C12

Key-words: CAPM, Fama-French, risk premium, ITNLSUR, GMM

EFM Code: 310

1 Introduction

This paper confronts the Capital Asset Pricing Model - CAPM - and the 3-Factor Fama-French - FF - model using both Brazilian and US stock market data for the same sample period (1999-2007). The focus is on the Brazilian data analysis, since the results for the US' are widely known. Therefore, the US data will serve only as a benchmark for comparative purposes. We use two competing econometric methods, the Generalized Method of Moments (GMM) by Hansen (1982) and the Iterative Nonlinear Seemingly Unrelated Regression Estimation (ITNLSUR) by Burmeister and McElroy (1988). Both methods nest other options like the Generalized Least Squares (GLS), the Ordinary Least Squares (OLS) and the Weigthed Least Squares (WLS) frameworks based on the procedure by Fama-MacBeth (1973). Although GMM method accounts for possible autocorrelated errors, it has a poor performance for small samples. The ITNLSUR is strongly consistent and asymptotically normally distributed and hence it is robust with respect to non-normal errors; however, while it assumes heteroskedasticity errors, it does not takes into account possible error autocorrelations.

The estimations show that the FF model fits the Brazilian data better than CAPM, however it is imprecise compared with the US analog. We argue that this is a consequence of an absence of clear-cut anomalies in Brazilian data, specially those related to firm size. The tests on the efficiency of the models - nullity of intercepts and fitting of the cross-sectional regressions - presented mixed conclusions. The tests on intercept failed to rejected the CAPM when Brazilian value-premium-wise portfolios were used, contrasting with US data, a very well documented conclusion. The ITNLSUR has estimated an economically reasonable and statistically significant market risk premium for Brazil around 6.5% per year without resorting to any particular data set aggregation. However, we could not find the same for the US data during identical period or even using a larger data set. *These findings seem to contradict the perception that market efficiency would unlikely prevail in an emerging market like Brazil but rather in economies like the US.*

The CAPM model of Sharpe (1964) and Lintner (1965) embodies successfully the agents' behavior features of Markowitz (1952), and establishes a theoretical relationship between asset returns that can be tested empirically. The empirical evidence, however, has pointed against the theory since its very beginning. Black, Jensen and Scholes (1972) and Blume and Friend (1975) found out a constant different from the risk free asset in the post-war period. Fama and MacBeth (1973) achieved the same conclusion, although the β factor presented some desirable characteristics.

By the end of 70's and early 80's, the empirical evidence against the CAPM be-

came very strong (see Basu, 1977 and Banz, 1981). Some assets grouped according to firms financial characteristics presented distinct returns not captured by the β s. For instance, portfolios containing firms with low market value or low market value relative to book value turned out to have higher returns than the CAPM predicted. Moreover the literature documented other "anomalies" due to portfolios grouped according to profits over price per share, momentum and calendar effect (January and Fridays have returns significantly different from other months or days).

In view of such empirical evidences, Fama and French (1992, 1993) proposed a multifactorial model capable of explaining such anomalies, even though those factors were included without much economic content. As a result from their efforts, they propose a multifactorial model that adds two other explanatory variables besides the CAPM factor: a premium for the firm size and a premium for the market value relative to the book value. The inclusion of those factors improved the model fitting to the US empirical data.

The CAPM and the FF models deliver some predictions that can be tested. The first is the null hypothesis that the intercepts of the time series regressions are zero; the second is the cross section return adjustments between fitted and actual data; the third is whether the risk premium coefficient is positive; and the fourth is whether the risk premium coefficient is equal to the time mean of the excess return.

To perform the tests, the market portfolio used will be the Bovespa stock market index - Ibovespa - which groups together the more liquid stocks in Brazil. For robustness, we also consider other benchmarks as market portfolios. The first is the portfolio based on the Morgan Stanley Capital International methodology - MSCI. The second portfolio is an aggregation, equally weighted, of all the stocks in our database. Finally, the last portfolio weights the stocks based on their market value. In addition, the models will have to price three different set of assets. Two sets are portfolios, each with 10 portfolios grouped together according to the following financial characteristics: market value (ME) and the ratio book value over market value (BE/ME). The third set consists of the 44 out of 60 more representative stocks in the Ibovespa, without any aggregation of returns. The motivation for using portfolios grouped as mentioned follows the literature on anomalies (see Fama and French, 1996), which inspired the FF model. The third set of assets offers an additional test, specially regarding the FF model, which was tailored to explain such anomalies.

Gibbons, Ross and Shanken (1989) and MacKinlay and Richardson (1991) suggest procedures for testing whether the intercepts of the portfolios are jointly null. As will be seen, the results for the Brazilian data are contradictory, depending on the proxy for the market portfolio and on the portfolios used as assets. However, it predominates the non rejection of the null, which is consistent with theory. The

estimates are important because both methodologies are applied with distinct market portfolios, which increases the support for eventually more solid remarks, specially when compared to Málaga and Securato (2004), for instance. By contrast, the US data produced important differences between the CAPM and FF models. The FF model reduced vigorously the CAPM pricing error, mainly when the entire sample collected between 1963:07 to 2006:08 is used. Of course, that's part of the reason why the FF model enjoys such a success.

The estimations for Brazil using portfolios aggregations revealed a strong value premium (market value/book value), but none size premium. Reflecting this finding, the FF model adjusts better than the CAPM where anomalies are relevant in terms of mean absolute pricing error.

From the cross section returns, we show that the Brazilian risk premium is positive, statistically significant around 6.5% per year using the ITNLSUR. The data used to estimate the risk premium consists of the 44 more liquid stocks composing the Ibovespa stock market index. This result is particularly important because it avoids using foreign data and arbitrary assumptions to calculate the equity cost of capital, given the widespread market practice of using the CAPM to make such an evaluation.¹

The work is divided in the following way. In the next section, we present the portfolio composition and data to be used in this work. In the section 3, we present the theory and econometric tests that are executed in the section 4. Also this last section develops the analysis of the results. Finally the last section concludes.

2 Data Set and Portfolios Construction

2.1 Data Set

This study uses a data set containing 172 stock prices of 123 firms, all of them traded in the São Paulo Stock Exchange - Bovespa. To avoid eventual distortions related to the change in the Brazilian exchange rate regime, the sample used spans from January 1999 to August 2006.

Financial data were obtained from both Economática data set and Bovespa website for completion and double checking². All prices are closing prices, adjusted for

¹In Brazil, there is an argument over the empirical content of CAPM. Some studies document anomalies and look for multifactorial models (see, for instance Costa, Leal and Lemgruber, 2000).

²Firms must send files as Standard Financial Balance Sheet (DFP) and Quarterly Information Sheet (ITR) to Bovespa. Those file must be open through software available in the same website: <http://www.bovespa.com.br>.

splits and dividends, and were included according to the following criteria:

1. Missing data for 3 or more consecutive months were excluded from the sample³;
2. Missing data were fulfilled by linear interpolation between the last and the following observed price⁴.

The return of asset $i = 1, 2, \dots, N$ at time $t = 1, 2, \dots, T$ is defined as:

$$R_{i,t} = \ln \left(\frac{P_{i,t}}{P_{i,t-1}} \right),$$

where P_{it} is the price of asset i at time t .

The market portfolio follows the convention of other researchers of taking the Ibovespa index. That index is composed with the more liquid stocks of Bovespa. However, for robustness purposes, other proxies for the market portfolio are also used and the results are reported in the appendix.

As usual in Brazil, the risk free rate was calculated from the Selic interest rate, which is the interest rate that government tracks⁵. Since it is given yearly, the risk free rate at month t , $R_{f,t}$ is:

$$R_{f,t} = \frac{\ln(1 + Selic_t)}{12}.$$

US results are used as benchmarks and comparative purposes. The portfolios for american firms are from CRSP website and Kenneth French's website⁶. The sample period starts in July 1963 and ends in August 2006. There are stocks traded in NYSE, AMEX and Nasdaq (together, they counted for more than 4,000 firms in 2006).

³There are two exceptions, though. The stocks PRGA4 and CMET4 left Bovespa off during the last 4 and 3 months before the sample end up, respectively. In that case, we kept the last price traded during such months.

⁴BRKM3 and PEFX5 were the only two stocks with more than 10% of months interpolated. Excluding those stocks, the remaining 42 stocks had at least one month interpolated. The interpolation covered on average 2,9 non consecutives months.

⁵Because of the Megainflationary period in Brazil, even short run bonds or notes were not liquid in Brazil. The status has been changed a lot and fast ultimately. We follow the convention of using Selic for comparative purposes. Bueno suggests that the results would not change had we changed the risk free proxy for future contracts of Selic maturing within 30 or 360 days.

⁶http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

2.2 Portfolio Construction

This work aims at testing asset pricing models with Brazilian data under several specifications, so as we consider three data sets. The first set consists of the more liquid stocks in Bovespa and, consequently, represents most of the volume traded in the exchange house. Indeed, out of 57 stocks composing Ibovespa in the last quarter of 2005, the criteria defined in the last section leave us with 44 stocks for 92 months. In this set, the stocks are not aggregated in any way at all, and it will serve as a "control" group allowing us to compare results with those of composed portfolios in ways to be described soon. Note that a desirable property of the asset pricing model is that it explains return differences regardless how assets are grouped.

The other two data sets follow studies that pointed out returns not explained by the theory, the so-called anomalies. We focused on the financial indexes that presented the most challenge to the CAPM model: size and ratio between book equity and size. Size is measured by the market equity value - ME - that is, the number of stocks, ordinary and preferred, times the market price⁷. The ratio book equity and size - BE/ME - is measured by the ratio of total assets minus total liabilities and size (as defined above).

The 123 firms were ranked yearly in 10 groups, either by ME or BE/ME, according to the balance sheet of December of each year. The return of each portfolio is the arithmetic mean of the stocks's returns, hence it is equally-weighted as are the portfolios from US data. Appendix provides more details about the portfolios composition⁸.

Table I shows the statistics from the two sets of portfolios compared with similarly composed US portfolios during the same period. The portfolios are ranked in ascending order in terms of expected return, "anomaly-wise". That is, portfolio 1 contains firms with the lowest BE/ME and the highest ME, inversely to portfolio 10.

Table I
Descriptive Statistics BE/ME and ME. Data from Bovespa, NYSE, AMEX and Nasdaq

Monthly returns in local currency, between January 1999 and August 2006. The portfolios are based on December 2005. ME corresponds to the average size of the firms in US\$ million. All portfolios are equally-weighted. D_x represents the x -th decile.

⁷Preferred stock in Brazil is not the same as in the US. It means that the owner are not allowed to vote and that dividends must go to owners of preferred stock at first.

⁸Exception is the portfolios in 1999, whose compositions are based on data of January.

BOVESPA										
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
BE/ME	0.18	0.33	0.43	0.52	0.63	0.74	0.92	1.21	1.66	4.01
Mean	1.36	1.89	2.37	2.57	1.80	2.53	2.62	2.49	2.46	2.87
Std. Dev.	(7.87)	(6.99)	(6.96)	(6.50)	(7.14)	(6.82)	(7.05)	(6.20)	(6.90)	(10.22)
ME	23,283	5,370	2,425	1,588	667	405	196	101	44	8
Mean	2.42	2.04	1.81	2.20	3.01	2.03	1.66	2.40	2.58	2.18
Std. Dev.	(7.90)	(8.15)	(7.65)	(7.26)	(6.61)	(6.15)	(5.53)	(6.32)	(6.96)	(10.64)
NYSE/AMEX/NASDAQ										
BE/ME	0.15	0.26	0.35	0.39	0.46	0.53	0.61	0.73	0.81	1.37
Mean	0.13	0.83	0.98	1.14	1.34	1.60	1.55	1.55	1.77	2.07
Std. Dev.	(9.76)	(7.52)	(6.39)	(5.67)	(5.07)	(4.89)	(4.56)	(4.51)	(5.11)	(6.10)
ME	55,576	12,044	6,035	3,562	2,383	1,640	1,187	815	468	116
Mean	0.10	0.61	0.68	0.89	0.61	0.64	0.63	0.87	0.98	1.66
Std. Dev.	(5.00)	(4.95)	(5.81)	(5.61)	(5.77)	(6.53)	(6.62)	(6.64)	(7.23)	(6.87)

It is vastly documented in the economic literature the existence of a return differential between the extreme portfolios classified by ME and BE/ME. For the US, such a differential is 1.94% per month ($D10 - D1$) in portfolios ordered by BE/ME (1.44% per month if one takes quintiles⁹). Similarly, the differential in portfolios ordered by ME is positive, about 1.56% per month (0.97% in quintiles).

Brazilian data follows the same pattern when the portfolios are ordered by BE/ME, but not when they are ordered by ME. In the first case, the difference is 1.51% per month (1.04% in quintiles). In the second case, the difference is practically null: -0.24% (0.15% in quintiles).

Another feature identified in US data is a high degree of autocorrelation, which is important to the choice of econometric methods to be used to estimate the model parameters. Table II presents the autocorrelation coefficients and the cross-correlation when the statistics overcomes two standard-deviations ($\pm 2/\sqrt{T}$) between the series. The series are well correlated in general with coefficients higher than 0.50. All series are cross-correlated, and one-lagged correlations are significant to roughly half of the series. Other lags are not significant at all.

Table II
Serial Correlation Between Portfolios BE/ME and ME

⁹The quintile calculation is: $(D10 + D9)/2 - (D1 + D2)/2$

Cross-correlation between monthly returns (panel T; symmetric matrix) and lagged 1 month (panel T-1). standard-deviation higher than $(2/\sqrt{T})$.

Portfolios BE/ME											Portfolios ME											
T	T										T	T										
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10		D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	
D1	1.00										D1	1.00										
D2	0.82	1.00									D2	0.90	1.00									
D3	0.80	0.85	1.00								D3	0.81	0.82	1.00								
D4	0.71	0.79	0.81	1.00							D4	0.75	0.76	0.76	1.00							
D5	0.66	0.72	0.71	0.79	1.00						D5	0.77	0.81	0.79	0.78	1.00						
D6	0.60	0.64	0.68	0.74	0.73	1.00					D6	0.67	0.72	0.71	0.77	0.74	1.00					
D7	0.70	0.70	0.76	0.78	0.74	0.72	1.00				D7	0.54	0.58	0.53	0.53	0.64	0.59	1.00				
D8	0.50	0.58	0.62	0.66	0.66	0.75	0.76	1.00			D8	0.53	0.64	0.63	0.61	0.66	0.63	0.55	1.00			
D9	0.58	0.64	0.63	0.68	0.67	0.70	0.71	0.72	1.00		D9	0.59	0.62	0.67	0.66	0.67	0.77	0.55	0.61	1.00		
D10	0.54	0.49	0.54	0.62	0.67	0.56	0.66	0.68	0.66	1.00	D10	0.52	0.52	0.55	0.58	0.60	0.57	0.54	0.53	0.60	1.00	
T	T - 1										T	T - 1										
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10		D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	
D1					0.25						D1											
D2	0.22		0.21		0.24						D2											
D3	0.21				0.23						D3	0.23		0.22	0.27							0.24
D4					0.25						D4	0.26	0.28	0.22	0.25	0.33	0.29	0.36	0.22			0.27
D5					0.26						D5	0.29	0.23	0.23	0.27	0.25	0.27	0.31				
D6	0.27	0.26	0.28		0.27						D6	0.31	0.32	0.35	0.33	0.39	0.37	0.43			0.32	0.34
D7	0.34	0.31	0.31	0.31	0.36	0.25	0.22		0.28	0.24	D7			0.22			0.28					
D8	0.41	0.35	0.37	0.32	0.36	0.33	0.26		0.27	0.27	D8	0.35	0.33	0.31	0.42	0.42	0.34	0.28	0.28	0.29	0.34	
D9	0.30	0.26	0.28	0.22	0.27	0.25		0.23	0.28	0.23	D9	0.35	0.34	0.34	0.38	0.34	0.35	0.31			0.23	0.37
D10		0.23	0.26	0.22	0.26						D10			0.22	0.22	0.29	0.30	0.31				

2.3 Market Portfolio and Factors

The Bovespa Index (IBOV) is the main proxy for market portfolio in Brazil. In order to make this paper comparable to others, we take IBOV for analysis in the body of the text, but leave in the appendix the results with other alternatives like the Morgan Stanley Capital International for Brazil (MSCI-Brazil) and two other alternative proxies. The first is made up of all stocks in our database equally weighted (BEW). In the second all stocks are weighted by the market value of the firms (BVW). Next table presents the basic statistics of these indexes in terms of returns.

Table III
Descriptive Statistics: Market Portfolios and Risk Free Rate

Percentual mean and standard-deviation. Correlation between January 1999 and August 2006 (92 months).

	Mean	Std-dev	$t(\text{mean})$	Sharpe	Correlation			
					IBOV	MSCI	BVW	BEW
SELIC	1.48	0.32	43.8					
IBOV	1.82	8.64	2.02	0.041	1			
MSCI	1.92	7.54	2.45	0.059	0.937	1		
BVW	3.03	7.25	4.01	0.214	0.958	0.966	1	
BEW	2.24	6.11	3.51	0.125	0.915	0.838	0.908	1

Because the CAPM is not able to explain premia and returns of portfolios based on ME and BE/ME, Fama and French (1992) suggest instead a multifactorial model. Accordingly, they add two other factors to the CAPM, named SMB and HML (SMB - *small minus big* and HML - *high minus low*). SMB corresponds to the differential returns between firms of big size and small size. HML corresponds to the differential returns between portfolios with high and low BE/ME. We, then, construct such indexes for Brazil following these authors (see appendix for details on the construction of these indexes).

It is desirable that the factors are not correlated. For example, Fama and French (1995) point that between July 1963 and December 1992, the correlation between them is -0.08 . Notwithstanding, possibly because of the short period of time available, the Brazilian series HML and SMB are highly correlated. Table IV shows a correlation between such series around -0.68 . However, for the same 92 months, Fama and French's data have a correlation of -0.55 . In addition, the unconditional mean of all factors are not statistically significant both for Brazilian and US data.

Tabela IV
Descriptive Statistics - 3 Factors - BOVESPA, NYSE, AMEX and Nasdaq

Percentual Mean and Standard-Deviation. Correlation between January 1999 and August 2006 (92 months). In panel Bovespa, the factor RM-RF is the monthly excess return between Ibovespa and Selic. In panel NYSE/AMEX/Nasdaq, RM-RF is the excess return between the value-weighted portfolio and the US T-Bill of one month.

	Mean	Std.-dev.	$t(M)$	Correlation		
				RM-RF	SMB	HML
BOVESPA						
$RM - RF$	0.35	8.55	0.39	1		
SMB	0.99	6.38	1.49	-0.40	1	
HML	-0.79	8.74	-0.87	0.08	-0.68	1
NYSE/AMEX/NASDAQ						
$RM - RF$	0.06	4.51	0.12	1		
SMB	0.57	4.58	1.18	0.26	1	
HML	0.60	4.22	1.37	-0.52	-0.55	1

3 Theory and Econometric Tests

The main prediction of the CAPM is that the expected return of a portfolio follows a linear relationship with its β :

$$E[R_{i,t} - R_{f,t}] = \lambda_m \beta_i \quad (1)$$

for every $t = 1, 2, \dots, T$, where

λ_m is the risk price or risk premium given by $E[R_{m,t} - R_{f,t}]$;

$R_{m,t}$ is the return of the market portfolio at time t ; and

β_i is the amount of risk in portfolio i .

The coefficient β_i may be estimated by ordinary least squares according to the following equation:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i (R_{m,t} - R_{f,t}) + \varepsilon_{i,t} \quad (2)$$

for every $i = 1, 2, \dots, N$.

The CAPM theory generates three main implications that may be tested, according to Campbell, Lo and MacKinlay (1997):

1. The intercept is zero in equation (2);
2. The risk premium is positive $\lambda_m > 0$;
3. There is a linear relationship between β s and expected return, following equation (1).

As a matter of fact, the empirical evidence regarding the US rejected the theory. Blume and Friend (1975) and Black, Jensen and Scholes (1972) report an intercept

different from zero, specially in the post-War period. Fama and MacBeth (1973)¹⁰ confirmed their results, although they have not rejected the other two implications of equation (1).

In order to take into account some of these stylized facts, Black (1972) relaxes the assumption about the existence of a risk free asset and introduces the zero-beta portfolio, which is a minimum variance portfolio uncorrelated with the market portfolio. On the other hand, Roll (1977) criticizes the proxy for the market portfolio by arguing that it contains only observed asset returns, rather than representing the returns of the entire economy. The market portfolio, hence, would not be observable, because any possible proxy one can think of cannot consider intangible assets for example, therefore the theory is not testable. In view of this situation, Ross (1976) proposes another model, named *arbitrage pricing theory* (APT), with assumptions less restrictive than the CAPM¹¹. In fact, by resuming portfolio choices to choices over mean-and-variances, the CAPM implicitly assumes a direct relationship between wealth and consumption, which is a very restrictive hypothesis considering there is a temporal dynamics between these variables. As a consequence, Merton (1973) introduces an intertemporal asset pricing model, heavily criticizing the static assumption of the CAPM. The models of Lucas (1978) and Breeden (1979) follow that line of representative agents and formulate the consumption CAPM theory, also known as CCAPM. In those models, the decisions about where allocating wealth result from an intertemporal maximization of the utility function, where the consumption is the controlled variable and the financial markets enters in order to smooth consumption over time.

Parallel to the theoretical development described, new empirical evidence came out, heavily questioning the CAPM. In particular, assets grouped together according the financial characteristics of firms as size, book value and dividends presented returns unexplained by β (Basu (1977), Banz (1981) and Rosenberg, Reid and Lanstein (1985)).

Such an evidence has motivated Fama and French (1992, 1993) to estimate a

¹⁰In that work, the authors introduce as estimation methodology known as 2-stage Fama and MacBeth methodology. The portfolios are ordered according to the β s magnitude, in order to reduce dispersion, which would make insignificant parameter λ_m . Thus, in the first stage, one estimates by OLS each β using time series data. In the second stage, the estimated β s become explanatory variables of the mean returns of the portfolios at each time t . The total risk premium is then obtained by averaging the risk premia of each time OLS estimation of λ_m , and the standard-deviation is calculated accordingly.

¹¹For instance, the CAPM demands homogeneous agents with quadratic preferences or multivariate normality of returns (as in Markowitz (1952)), and an efficient market portfolio. The APT assumes non arbitrage and a considerable number of assets.

multifactorial model by including some *ad hoc* factors tailored to capture observed anomalies. Thus the expected excess return between the portfolio (R_i) and the risk free rate (R_f) responds to three factors: the excess return of the market portfolio, the return spread between portfolios with big and small firms, *SMB*, and the return spread between firms with high and low BE/ME, *HML*:

$$E [R_{i,t} - R_{f,t}] = \lambda_m \beta_i + \lambda_s s_i + \lambda_h h_i \quad (3)$$

where λ_i , $i = m, s, h$, are the expected premia, respectively measured by $E [R_m - R_f]$, $E [SMB]$ and $E [HML]$. The coefficients (β_i, s_i, h_i) measure the portfolio i sensitivity to each factor and are obtained through the time series regression:

$$R_{i,t} - R_{f,t} = a_i + \beta_i (R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t + \varepsilon_{t,i} \quad (4)$$

In equilibrium, agents require that premium to offset the risk of owning such assets. Fama e French do not provide a theory based of primitive foundations to justify why using exactly those factors. However, Fama e French (1996) argue that firms with profits systematically low are facing financial difficulties tend to have a high BE/ME. Given those additional risks, returns must be higher in equilibrium. By contrast, consolidated firms have low BE/ME, hence a low return, independently of other risks. Similarly, the market value of a firm indicates a negative financial status or mirrors a natural risk of an incipient firm. In such case, returns ex-ante must reflect such additional risks.

This paper tests the implication of the CAPM and the three factors model of Fama and French, henceforth FF, for Brazil and the US. The tests focus on two basic implications of the models:

1. The intercepts are jointly null for all assets in the time series regressions represented by equations 2 and 4;
2. The cross-section adjustment between the expected return predicted by the model and the observed mean return, according to equations 1 e 3.

A by-product of the procedure is to estimate the risk premium for Brazil, spreadly used by market practitioners for evaluating cash flows of corporations. However, we do not test formally another possible implication regarding whether the risk premium is the expected excess return, $\lambda_m = E (R_{m,t} - R_f)$.

3.1 Test of Intercepts

By assumption, the multifactorial model is able to capture the behavior of returns. Such an implication will be true if the intercepts are all null. A usual test for that implication was suggested by Gibbons, Ross and Shanken (1989). However, the test does not adjust for autocorrelated series, which is a feature often found in financial series. Alternatively, one may use the test by MacKinlay e Richardson (1991), based on the overidentification test using GMM estimation method.

In this work, we perform and compare both alternatives. Therefore, let us describe each one in what follows.

3.1.1 Gibbons, Ross e Shanken's (1989) Test

Using the OLS estimates of equation 4, one can test the null hypothesis that $\alpha_i = 0 \forall i$. If the disturbances are temporally independent and jointly normal, with mean zero, Gibbons, Ross and Shanken (1989) suggest using the following statistics:

$$GRS = \left(\frac{T}{N}\right) \left(\frac{T - N - K}{T - K - 1}\right) \left[\frac{\hat{a}' \hat{\Sigma}^{-1} \hat{a}}{1 + \hat{\mu}'_f \hat{\Sigma}_f^{-1} \hat{\mu}_f} \right] \sim F(N, T - N - K) \quad (5)$$

where

- \hat{a} is an $N \times 1$ vector of constants estimated by either equation 2 or 4;
- $\hat{\Sigma}$ is an $N \times N$ residual covariance matrix;
- $\hat{\mu}_f$ is a $K \times 1$ factors means¹²; and
- $\hat{\Sigma}_f$ is a $K \times K$ factor covariance matrix.

3.1.2 Overidentification Test

An advantage of the GMM estimation over the OLS model (used in the GRS test) is that GMM does not assume any hypothesis over the distribution of the series like *i.i.d.* (serial uncorrelatedness and homoskedasticity). However, in general, financial series do present at least autocorrelation. Therefore, MacKinlay and Richardson (1991) suggest two tests through GMM to overcome the difficulties of the GRS test. The first test consists of estimating $(\alpha, \beta)'$ by GMM and testing the null hypothesis that $\alpha_i = 0, \forall i$, where β' is the vector of factor coefficients.

The second test consists of imposing $\alpha_i = 0, \forall i$, estimating the model by GMM, and then testing whether the objective function at the minimum is statistically null.

¹² $K = 1$ in the CAPM model and $K = 3$ in the FF model.

That is, use a vector of ones as instrument, and, if the null is false, the objective function will fail in being statistically zero. We perform this second test in this work.

Therefore, let $r_{i,t} = R_{i,t} - R_{f,t}$, and define the population moments in the following way:

$$E[g(\mathbf{w}_t, \boldsymbol{\beta}_0)] = E \begin{bmatrix} \mathbf{r}_t - \boldsymbol{\beta}'_0 \mathbf{f}_t \\ (\mathbf{r}_t - \boldsymbol{\beta}'_0 \mathbf{f}_t) \otimes \mathbf{f}_t \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (6)$$

where

$\mathbf{r}_t = [r_{1,t}, r_{2,t}, \dots, r_{N,t}]'$ is an $N \times 1$ vector of excess returns;

\mathbf{f}_t is a $K \times 1$ vector of factors;

\otimes is the Kronecker operator.

By assumption data is ergodic, therefore the population moments have a sample analog given by:

$$g_T(\mathbf{w}, \boldsymbol{\beta}) = T^{-1} \sum_{t=1}^T g(\mathbf{w}_t, \boldsymbol{\beta}),$$

where \mathbf{w}_t is a vector of observed variables.

The GMM estimator gives us $\hat{\boldsymbol{\beta}}^{gmm} = \arg \min_{\boldsymbol{\beta}} \{Q_T(\boldsymbol{\beta})\}$, from:

$$Q_T(\boldsymbol{\beta}) = g_T(\mathbf{w}, \boldsymbol{\beta})' \mathbf{W}_T g_T(\mathbf{w}, \boldsymbol{\beta}), \quad (7)$$

where \mathbf{W}_T is a matrix that weights efficiently each moment.

There are more moments than parameters to be estimated. It is necessary to estimate NK coefficients (N portfolios and K parameters in each), and there are $N(K+1)$ moment conditions, every portfolio is multiplied by the factor plus the conditions that the errors are zero in each portfolio. Hence, the model is overidentified and, as such, the quantity $Q_T(\hat{\boldsymbol{\beta}}^{gmm})$ follows a χ^2 distribution with N degrees of freedom (Hansen (1982)). Consequently, it can be statistically tested. If we fail to reject the null that all moments are statistically zero, then the model is correctly identified and, consequently the intercepts are statistically jointly null.

3.2 Risk Premium

The popularity of the CAPM and FF models probably resides in the fact that they establish a straightforward relationship of return differentials through the pricing equations (1) and (3). A natural way of testing the models emerges from the

coefficients significance and sign of the cross-sectional pricing equation between excess returns and β_s .

However, since the β s are estimated in the first step, it is needed to adjust the variance of the estimated parameters, λ s, in the second step. That's why Fama and MacBeth (1973) suggested the OLS estimation in two stages. However, nowadays there are other techniques which make Fama and MacBeth's procedure obsolete. They are the GMM and the ITNLSUR.

Importantly, Shanken and Zhou (2007) recently have studied the small sample properties of cross-sectional expected return estimators like GMM, Maximum Likelihood, Fama and MacBeth's (1973) procedure and some variants as the Generalized Least Squares - GLS - and the Weighted Least Squares - WLS . They conclude that all estimators are identical for samples greater than 960 observations (30 years). GMM performs as well as the other methods for $N = 48$ and $T > 120$. However, GMM gets worse when $N = 25$.

Unfortunately, it seems that nobody has tested CAPM and the FF models using the ITNLSUR, which is a method that nests both the GLS and the WLS procedures, with asymptotic properties potentially better than those estimators'.

3.2.1 ITNLSUR

Burmeister and McElroy (1988)? proposed a new econometric model to estimate the cross-sectional expected returns. Their model focuses on the APT, but it can be used for testing the CAPM and the FF models. The method is an iterated nonlinear seemingly unrelated regression and meant to correct many problems related with the two stage methods (OLS, GLS, WLS). Among the deficiencies associated with the conventional methods are the unrobustness of the estimates under non normality, loss of estimation efficiency, non uniqueness of the second stage estimators due to the need of grouping assets in portfolios and problems with statistical inference.

This section presents briefly the model by Burmeister and McElroy (1988), in order to make it clear how it works. Thus, consider a multifactorial model with K factors and N assets. The ITNLSUR methods allows one to estimate simultaneously the NK β s and the K λ s, through the econometric model:

$$R_{i,t} - \lambda_{0,t} = \sum_{j=1}^K \beta_{ij} (\lambda_j + f_{j,t}) + \varepsilon_{i,t}, \quad (8)$$

where, in principle, $\lambda_{0,t}$ is observed and will stand for the risk free asset.

Notice in the model that β_{ij} multiples both $f_{j,t}$ (conventional first stage) and λ_j (conventional second stage) simultaneously. That's the way that Burmeister and

McElroy (1988) nest all models into one.

Also, it is assumed that

$$\begin{aligned} E_t [\varepsilon_{i,t}] &= 0; \\ E_t [\varepsilon_{i,t} \varepsilon_{j,s}] &= \begin{cases} \sigma_{ij}, & t = s \\ 0, & t \neq s \end{cases} ; \text{ and} \\ E_t [\varepsilon_{i,t} | f_{j,s}] &= 0. \end{aligned}$$

Observe that there is no assumption about the error distribution. And when the errors are multivariate normal, then the estimator is a maximum likelihood estimator.

Let $\rho_i \equiv R_{i,t} - \lambda_{0,t}$ be a $T \times 1$ vector representing the temporal excess returns and rewrite the model in the following way (8):

$$\rho_i = [\boldsymbol{\lambda}' \otimes \mathbf{1}_T + \mathbf{f}] \beta_i + \varepsilon_i = \mathbf{X}(\boldsymbol{\lambda}) \beta_i + \varepsilon_i \text{ for } i = 1, \dots, N,$$

where

$\boldsymbol{\lambda}$ is a $K \times 1$ vector of risk premia, depending on the factor loading;

β_i is a $K \times 1$ vector of β s of factors;

$\mathbf{1}_T$ is a $T \times 1$ vector of ones; and

$\mathbf{X}(\boldsymbol{\lambda})$ is a $T \times K$ matrix.

Stacking the N equalities above, one gets:

$$\boldsymbol{\rho} \equiv \begin{pmatrix} \rho_1 \\ \vdots \\ \rho_N \end{pmatrix} = \begin{bmatrix} \mathbf{X}(\boldsymbol{\lambda}) & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X}(\boldsymbol{\lambda}) \end{bmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_N \end{pmatrix}.$$

Rewriting more compactly, one gets:

$$\boldsymbol{\rho} = [I_N \otimes \mathbf{X}(\boldsymbol{\lambda})] \boldsymbol{\beta} + \boldsymbol{\varepsilon}, \tag{9}$$

where

$$\boldsymbol{\beta} = [\beta'_1, \beta'_2, \dots, \beta'_N]';$$

$$\boldsymbol{\varepsilon} = [\varepsilon'_1, \varepsilon'_2, \dots, \varepsilon'_N]';$$

The estimation process involves several steps. Firstly, the parameter $\hat{\boldsymbol{\theta}} = (\hat{a}_{ij}, \hat{\beta}_{ij})$, where $\hat{a}_{ij} = \beta_{ij} \boldsymbol{\lambda}$, is obtained by OLS conventionally, following equation (8). Then the residuals are used to estimate the covariance matrix given by $\hat{\Sigma} = [T^{-1} \hat{\varepsilon}'_i \hat{\varepsilon}_j]$. Then, the parameters $(\tilde{\boldsymbol{\beta}}, \boldsymbol{\lambda})$ are obtained iteratively, by minimizing the quadratic error of equation (9):

$$Q(\boldsymbol{\lambda}, \boldsymbol{\beta}; \hat{\Sigma}) = [\boldsymbol{\rho} - [I_N \otimes \mathbf{X}(\boldsymbol{\lambda})] \boldsymbol{\beta}]' (\hat{\Sigma}^{-1} \otimes I_N) [\boldsymbol{\rho} - [I_N \otimes \mathbf{X}(\boldsymbol{\lambda})] \boldsymbol{\beta}].$$

The process follows iteratively until convergence, using the estimated values $(\tilde{\boldsymbol{\beta}}^s, \boldsymbol{\lambda}^s)$ to generate $\hat{\Sigma}^s$ and to minimize $Q(\boldsymbol{\lambda}, \boldsymbol{\beta}; \hat{\Sigma}^s)$. Notice that the process is not more than quadratic residuals minimizations, which implies estimators strongly consistent and asymptotically normal, even when the errors are not normally distributed.

3.2.2 GMM

The ITNLSUR method assumes the errors are conditionally homoskedastic and serially uncorrelated. The GMM method relaxes those hypotheses and allows one to analyse in detail some economic implications. The main advantage is doubtless the possibility of expressing the information available to the agents through the moment conditions in order to make decisions. As such, the number of moments may be greater than the number of parameters, which makes GMM overidentified and reflects the information set of agents.

However, GMM has not been much used to estimate the cross-section returns in equation (1). According to Shanken and Zhou (2007), the reason for that is the difficulty in finding out numerical solutions for the problem, since there are a large number of parameters to estimate coupled with the nonlinearity of the model. Therefore, if the problem can be linearized in some way and conveniently weighted, then those difficulties may be overcome.

In line with this argument and following Harvey and Kirby (1995), Shanken and Zhou (2007) suggest estimating the model sequentially. To see how it works, define \mathbf{R}_t as the $N \times 1$ vector of portfolios or assets to which corresponds an $N \times 1$ vector of means, μ_r , that is:

$$E(\mathbf{R}_t) = \mu_r.$$

Similarly, let \mathbf{f} be a $K \times 1$ vector of factor to which correspond another $K \times 1$ vector of factor means, μ_f :

$$E(\mathbf{f}_t) = \mu_f.$$

Let Σ_f be a $K \times K$ factor covariance matrix, such that:

$$E[(\mathbf{f}_t - \mu_f)(\mathbf{f}_t - \mu_f)'] = \Sigma_f.$$

The matrix contains $\frac{K(K+1)}{2}$ distinct covariances. To obtain a vector of these covariances, apply the VECH operator, which stacks the columns of the lower portion of a covariance matrix, such that:

$$E \left(\text{vech} \left[(\mathbf{f}_t - \mu_f) (\mathbf{f}_t - \mu_f)' \right] \right) = \text{vech} (\boldsymbol{\Sigma}_f).$$

Now, it is necessary to obtain the covariance matrix between \mathbf{R}_t and \mathbf{f}_t , whose dimension is $N \times K$. Such covariance is given by

$$E \left[(\mathbf{R}_t - \mu_r) (\mathbf{f}_t - \mu_f)' \right] = \boldsymbol{\Sigma}_{rf}.$$

For ease of notation in what follows define $(\mathbf{R}_t - \mu_r) (\mathbf{f}_t - \mu_f)' \equiv \boldsymbol{\Sigma}_{rft}$, such that $E(\boldsymbol{\Sigma}_{rft}) = \boldsymbol{\Sigma}_{rf}$. Then, the last N moment conditions requires that the cross-sectional errors are jointly zero, such that:

$$E \left(\mathbf{R}_t - \gamma_0 \mathbf{1}_N - \boldsymbol{\Sigma}_{rft} \boldsymbol{\Sigma}_f^{-1} \gamma_k \right) = \mathbf{0},$$

where γ_k is a $K \times 1$ vector of parameters. If $K = 1$, then this could be the market premium, for example.

Putting together all the moment conditions, one obtains:

$$\begin{aligned} E[g(\mathbf{w}_t, \boldsymbol{\theta}_0)] &= E \left[\begin{array}{c} g_1(\mathbf{w}_t, \boldsymbol{\theta}_0^1) \\ g_2(\mathbf{w}_t, \boldsymbol{\theta}_0^1, \boldsymbol{\theta}_0^2) \end{array} \right] = \\ &= E \left[\begin{array}{c} \mathbf{R}_t - \mu_r \\ \mathbf{f}_t - \mu_f \\ \text{vech} \left[(\mathbf{f}_t - \mu_f) (\mathbf{f}_t - \mu_f)' - \boldsymbol{\Sigma}_f \right] \\ \mathbf{R}_t - \gamma_0 \mathbf{1}_N - \boldsymbol{\Sigma}_{rft} \boldsymbol{\Sigma}_f^{-1} \gamma_k \end{array} \right] = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \end{aligned}$$

Notice the $N \times K$ matrix $\boldsymbol{\beta} \equiv \boldsymbol{\Sigma}_{rf} \boldsymbol{\Sigma}_f^{-1}$ corresponds to the matrix of β s of the first step of multifactorial model. In other words, it corresponds to the explanatory variables of the second step of Fama and MacBeth's procedure. Hence, there are $M = N + K + \frac{K(K+1)}{2} + N$ moments and $P = N + K + \frac{K(K+1)}{2} + K + 1$ parameters to estimate. The sequential estimation proposed by Shanken and Zhou (2007) proceeds in two steps. In the first step estimate the $P_1 = N + \frac{K(K+3)}{2}$ parameters using the first $N + \frac{K(K+3)}{2}$ moments, that is, obtain $\hat{\boldsymbol{\theta}}^1 = \left[\hat{\mu}_r', \hat{\mu}_f', \text{vech} \left(\hat{\boldsymbol{\Sigma}}_f \right)' \right]'$. Of course, in that case the weighting matrix is $W_{1T} = I_{N + \frac{K(K+3)}{2}}$. Then, use the estimated parameters

in the last set of moments to estimate the other $P_2 = K + 1$ parameters, that is, obtain $\hat{\boldsymbol{\theta}}^2 = (\hat{\gamma}_0, \hat{\gamma}'_k)'$. The advantage of this procedure is that all the estimations are linear, which makes it easier to find out the solutions.

In the second step, there is overidentification, provided that $N > P_2$, which is true either for the CAPM or for the FF models. However, in this case, it is very important how to choose the weighting matrix, W_{2T} . Following Ogaki (1993) and given W_{1T} is identity, Shanken and Zhou (2007) show that the optimal weighting matrix in the second step is:

$$W_{2T} = \left[\begin{pmatrix} -\Gamma_{21}\Gamma_{11}^{-1} & I \end{pmatrix} S_T \begin{pmatrix} -\Gamma_{21}\Gamma_{11}^{-1} \\ I \end{pmatrix} \right]^{-1},$$

where $\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix}$ is an $M \times P$ corresponding to the derivative of the moments with respect to the parameters and I a $P_2 \times P_2$ identity matrix. Note that Γ_{11} is a $P_1 \times P_1$ matrix and that $\Gamma_{22} = [-1_N \quad -\boldsymbol{\beta}_{N \times K}]$ is an $N \times P_2$ matrix.

For simplicity, but following Harvey and Kirby (1995), we assume that $\Gamma_{21} = \mathbf{0}$. As pointed out by Shanken and Zhou (2007), this assumption implies that the factor model disturbances have zero mean conditional on the factors.

4 Empirical Analyses

Empirical studies testing either the CAPM or FF models are scarce in Brasil compared with the US literature. One of the possible reasons is the low amount of data, mainly because the long megainflationary period before 1994 distorted the available data. Another possible reason is the low number of corporations with stocks traded, which makes the researchers disagree about which stocks taking into account. Given such a situation, we compare the results with US data by estimating the models using the same period and firms grouping. The intention is to establish a dialogue with the vast US literature, where the main results were extensively documented. By contrasting the results, we believe the conclusions would be more reliable.

4.1 Intercept Tests

This section tests whether the intercepts of equations (2) and (4) are jointly null, both for the Brazilian and the US samples, using portfolios based on the ME and BE/ME criteria.

For the Brazilian data and portfolios based on the BE/ME criterium, the statistics GRS and χ fail to reject the null of jointly intercepts equal to zero both in the CAPM and in the FF models. In the CAPM model, the intercepts are individually significant in portfolios D4 and D7 (5%) and D3, D6 and D8 (10%). However, by introducing the SMB and HML factors, only D4's intercept rejects the null at 10% of significance. In terms of adjusted R^2 , the FF model is marginally better than the CAPM, except in the upper extreme portfolios D8, D9 and D10 where the difference is clear.

For the portfolios based on the ME criterium, the statistics GRS and χ reject the null of jointly intercepts equal to zero in both CAPM and FF models. The rejection is due to the non zero intercepts of portfolios D1 and D5. In this data set case, the difference between models in terms of adjusted R^2 is larger on portfolios D6 through D10, where the FF models' adjustment coefficients are higher.

Although most of the intercepts are not significant across the portfolios, it is possible to observe an increasing pattern in the point estimates from the lowest to the highest BE/ME decile. Surprisingly is the fact of the β s being less than one in every portfolio in both models.

Table V
Intercept Tests and Time Series Regression of Portfolios BE/ME and ME (BOVESPA)

The Estimated Equation is $R_{i,t} - R_{f,t} = a_i + b_i (R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t + \varepsilon_{t,i}$. $t()$ is the t -statistics of the OLS regressions. $p(GRS)$ and $p(\chi)$ correspond to the p -values of the statistics GRS and χ described in the last section. The Market Portfolio is the Ibovespa and Selic is the risk free rate. Sample from January 1999 to August 2006 (92 months).

IBOV												
	Portfolios										GRS	χ
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	$p(GRS)$	$p(\chi)$
BE/ME												
a	-0.37	0.16	0.66	0.88	0.09	0.85	0.92	0.85	0.79	1.14	1.10	10.89
t(a)	-0.79	0.48	1.68	2.26	0.20	1.79	2.01	1.73	1.62	1.37	0.37	0.37
b	0.75	0.71	0.68	0.62	0.68	0.58	0.63	0.47	0.58	0.74		
t(b)	13.65	17.78	14.77	13.50	13.67	10.35	11.79	8.06	10.07	7.59		
R^2	0.67	0.78	0.70	0.67	0.67	0.54	0.60	0.41	0.52	0.38		
a	-0.62	0.01	0.35	0.75	0.14	0.60	0.57	0.48	0.50	0.91	1.19	9.17
t(a)	-1.45	0.03	0.99	1.91	0.32	1.34	1.40	1.29	1.34	1.31	0.31	0.52
b	0.79	0.73	0.75	0.66	0.69	0.69	0.77	0.64	0.73	0.92		
t(b)	14.06	17.83	16.39	12.81	12.40	11.87	14.23	13.11	14.71	10.10		
s	0.07	0.02	0.23	0.17	0.06	0.45	0.52	0.70	0.63	0.79		
t(s)	0.71	0.28	2.76	1.83	0.62	4.25	5.29	7.98	7.02	4.77		
h	-0.20	-0.16	-0.07	0.07	0.15	0.29	0.27	0.49	0.49	0.77		
t(h)	-2.98	-3.24	-1.33	1.15	2.15	4.03	4.08	8.22	8.14	6.91		
R^2	0.74	0.82	0.77	0.67	0.69	0.62	0.69	0.68	0.73	0.59		
ME												
a	0.64	0.25	0.08	0.51	1.32	0.38	0.06	0.77	0.94	0.48	2.08	21.92
t(a)	2.50	0.89	0.18	1.04	3.26	0.82	0.12	1.49	1.61	0.51	0.04	0.02
b	0.87	0.89	0.75	0.64	0.62	0.50	0.36	0.45	0.49	0.66		
t(b)	29.04	26.25	14.94	11.19	12.96	9.09	6.48	7.37	7.20	6.04		
R^2	0.90	0.88	0.71	0.58	0.65	0.47	0.31	0.37	0.36	0.28		
a	0.84	0.18	0.04	0.32	1.06	-0.09	-0.38	0.33	0.44	-0.06	2.71	19.99
t(a)	3.59	0.62	0.08	0.69	2.73	-0.23	-0.88	0.84	0.86	-0.07	0.01	0.03
b	0.80	0.92	0.78	0.73	0.71	0.65	0.50	0.64	0.67	0.90		
t(b)	25.98	24.02	13.94	12.18	13.88	12.57	8.92	12.10	9.90	8.00		
s	-0.27	0.13	0.15	0.39	0.34	0.59	0.50	0.75	0.68	0.94		
t(s)	-4.81	1.80	1.52	3.60	3.69	6.19	4.90	7.89	5.54	4.62		
h	-0.11	0.08	0.16	0.29	0.14	0.21	0.14	0.47	0.31	0.60		
t(h)	-3.05	1.62	2.29	4.01	2.26	3.31	2.01	7.36	3.74	4.40		
R^2	0.92	0.89	0.72	0.64	0.69	0.63	0.47	0.64	0.51	0.42		

It is important to stress, however, that the results may vary if the market portfolio proxy changes to MSCI, BVW or BEW. The results can be seen in the appendix. For example, if the proxy is the MSCI or BEW (equally weighted), the intercept tests always fail to reject the null. By contrast, the value-weighted proxy, BVW always rejects the null because of the intercepts of portfolios D1, D2 and D3, although these coefficients are negative¹³. See other references on these tests in Bonomo (2002) and Costa Jr., Leal and Lemgruber (2000).

The results with the US data and portfolios based on the BE/ME index show that the models are not able to capture the returns of portfolios; intercepts are significant in most regressions and particularly high in the CAPM model. As a consequence, the null is rejected in both tests. The FF model is consistently better than the CAPM in terms of adjusted R^2 , with improvements around 20 percentage points. One can also observe that the intercept point estimates increases from the lowest to the highest decile.

The estimations based on the ME index portfolios show similar results. The null of jointly zero intercepts is rejected in both tests. In fact, the null is rejected in

¹³Málaga and Securato (2004) reach similar results, although they do not perform all the exercises we do here.

the CAPM because of portfolios D4 and D10. If the model is the FF the model is not rejected to 5% using the GMM test. The increasing pattern of intercepts' point estimates is also present in the CAPM model but not in the FF.

Differently from the Brazilian case, the β s of the portfolios BE/ME and ME are all greater than one in the CAPM model, but are less than 1 in portfolios D5-D10 if estimation includes the HML and SMB factors.

Table VI
Intercept Tests and Time Series Regression of Portfolios BE/ME and ME (NYSE, AMEX and Nasdaq)

The Estimated Equation is $R_{i,t} - R_{f,t} = a_i + b_i (R_{m,t} - R_{f,t}) + s_i SMB_t + h_i HML_t + \varepsilon_{t,i}$. $t()$ is the t -statistics of the OLS regressions. $p(GRS)$ and $p(\chi)$ correspond to the p -values of the statistics GRS and χ described in the last section. The Market Portfolio is the value-weighted and T-Bill is the risk free rate. There are 92 monthly observations between January 1999 and August 2006.

VW												
	Portfolios										GRS	χ
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	$p(GRS)$	$p(\chi)$
BE/ME												
a	-0.23	0.48	0.64	0.82	1.02	1.29	1.24	1.24	1.45	1.76	4.19	37.90
t(a)	-0.40	1.16	1.85	2.58	3.29	4.23	3.96	3.85	3.93	3.74	0.00	0.00
b	1.82	1.44	1.23	1.08	0.93	0.89	0.78	0.75	0.83	0.94		
t(b)	13.90	15.20	15.59	15.09	13.24	12.88	11.03	10.28	9.96	8.85		
R^2	0.68	0.72	0.73	0.71	0.66	0.64	0.57	0.54	0.52	0.46		
a	-0.17	0.12	0.16	0.28	0.43	0.70	0.59	0.57	0.76	1.00	4.64	29.29
t(a)	-0.42	0.38	0.62	1.20	2.01	3.51	2.89	2.90	3.02	2.73	0.00	0.00
b	1.36	1.26	1.18	1.10	0.98	0.93	0.87	0.82	0.88	0.98		
t(b)	13.14	16.42	18.10	18.72	18.28	18.28	16.78	16.39	13.89	10.55		
s	0.58	0.66	0.61	0.57	0.59	0.59	0.60	0.65	0.71	0.79		
t(s)	5.59	8.64	9.41	9.72	11.00	11.76	11.72	13.10	11.23	8.66		
h	-0.60	0.01	0.23	0.36	0.43	0.41	0.51	0.50	0.49	0.51		
t(h)	-4.79	0.08	2.95	4.98	6.58	6.64	8.05	8.19	6.30	4.53		
R^2	0.85	0.86	0.86	0.86	0.85	0.86	0.83	0.84	0.80	0.70		
ME												
a	-0.22	0.29	0.34	0.56	0.28	0.31	0.29	0.53	0.65	1.34	2.81	27.66
t(a)	-1.02	1.43	1.61	2.40	1.04	0.95	0.81	1.39	1.40	2.54	0.00	0.00
b	1.03	1.03	1.23	1.16	1.16	1.30	1.27	1.25	1.29	1.06		
t(b)	21.31	22.49	25.57	22.00	18.91	17.75	15.54	14.39	12.39	8.85		
R^2	0.83	0.85	0.88	0.84	0.80	0.78	0.73	0.69	0.63	0.46		
a	-0.14	0.17	0.12	0.30	0.00	-0.08	-0.29	-0.12	-0.04	0.85	2.66	16.30
t(a)	-0.84	0.88	0.57	1.31	-0.01	-0.30	-1.20	-0.49	-0.12	2.03	0.01	0.09
b	1.15	1.14	1.31	1.23	1.19	1.25	1.25	1.22	1.21	0.89		
t(b)	27.41	23.60	24.93	21.43	17.98	18.67	20.07	19.92	15.88	8.32		
s	-0.26	-0.03	0.13	0.19	0.28	0.50	0.69	0.76	0.88	0.78		
t(s)	-6.27	-0.61	2.44	3.37	4.32	7.48	11.13	12.50	11.67	7.40		
h	0.10	0.22	0.24	0.25	0.21	0.18	0.33	0.37	0.31	0.09		
t(h)	2.03	3.70	3.78	3.57	2.58	2.18	4.37	4.92	3.38	0.68		
R^2	0.91	0.87	0.89	0.86	0.83	0.86	0.88	0.89	0.86	0.69		

4.2 Risk Premium

This section presents the estimates of equations (1) and (3) by GMM and ITNL-SUR methods. In the GMM case, the covariance function is estimated using the Parzen weighting function following Andrews (1991), who argues it is preferable over the Bartlett window, and, according to Ogaki (1993), it is computationally more convenient than the quadratic option.

The choice of the bandwidth is more critical than the choice of the weighting function (see Hall (2005)). For this reason, we estimate the model with 3 distinct bandwidths: none, 1 and 4. If the series are uncorrelated, then the bandwidth should be zero. Hall (2005) suggests setting it equal to $T^{\frac{1}{3}} \simeq 4$ because it minimizes the mean square error of the covariance estimator. Finally, we also set the the bandwidth equal to 1 because the analysis of the series in Table II has shown that only first-lag correlations are statistically significant.

The tables show the parameters estimated by GMM at the first stage, using the identity as weighting matrix - GMM1 - and the inverse of the long run covariance matrix as weighting matrix, iterated until convergence - GMM IT. Monte Carlo simulations show that when the sample is small, GMM estimates are better if the objective function is iterated until convergence (Ogaki (1993)). The argument for presenting GMM1 is as follows. When one groups portfolios according to financial characteristics, the weights are the same for every asset. If the weighting matrix is optimal, the economic content of the covariance matrix is lost (see Yogo, 2006, Cochrane, 2005). Therefore, by comparing the parameters of the GMM1 we are able to compare results from distinct, economic based, specifications. On the other hand, estimates obtained in GMM IT have nicer statistic properties. The estimates based on the identity weighting matrix and one iteration are similar to those obtained by OLS. The estimates based on the identity weighting matrix itered until convergence are similar to those obtained by GLS¹⁴.

The portfolios used in the estimations are those defined by the BE/ME and ME criteria. Moreover, we repeat the exercise using 44 stocks without any kind of grouping (44IBOV). In doing that, we avoid the criticism of Lewellen, Nagel and Shanken (2006). They argue that the the β s are correlated with the other indexes.

Before going on, one should choose to include or not a constant, λ_0 , in the model (1) and (3) (λ_0 stands for the exceptional return beyond the excess return of the asset with respect to the risk free rate). This turned out to be crucial in our estimates since it changed sensitively the results. Such a phenomenon is also present in the US data

¹⁴The estimate in two stages by OLS, GLS and WLS as suggested by Shanken and Zhou (2007) are available under request.

and is often disregarded in studies. As a matter of fact, Lewellen, Nagel e Shanken (2006) show that recent successful articles in explaining the cross-sectional variability of returns, such as Lettau and Ludvigson (2001) and Yogo (2006), present huge estimates for λ_0 , contradicting theory. Following economic arguments, we impose the intercept to be null; the appendix presents the estimation under the less restrictive specification for portfolios ME using Brazilian data. As can be seen, the inclusion of the constant lead to negative risk premium and high positive intercepts estimates.

Table VII presents the estimates for the 3 sets of portfolios (panels BE/ME, ME and 44IBOV) for Brazilian data. The CAPM model presented a positive market risk premium for all variants, except in the case 44IBOV with 4 lags, where the model failed to converge. In the cases the coefficient was significantly different from zero (ITNLSUR for ME and 44IBOV portfolios), the market risk premium in annual terms is about 6.5% a very plausible and generally agreed figure in the private market.

When the SMB and HML factors are included in the model, the risk premium is still significant in the ITNLSUR method (ME and 44IBOV) at the same magnitude. Using the GMM method, the point estimates hardly resemble the time series means, possibly only on the ME portfolios. The estimates of the SMB premium, λ_{smb} , approximates the time series mean around 0, 99% in the portfolios BE/ME according to the GMM method. The HML premium, λ_{hml} , is negative and approximates the time series mean around -0, 79% in portfolios ME. However, GMM delivers non significant coefficients and, in this sense, agree with time series means estimates.

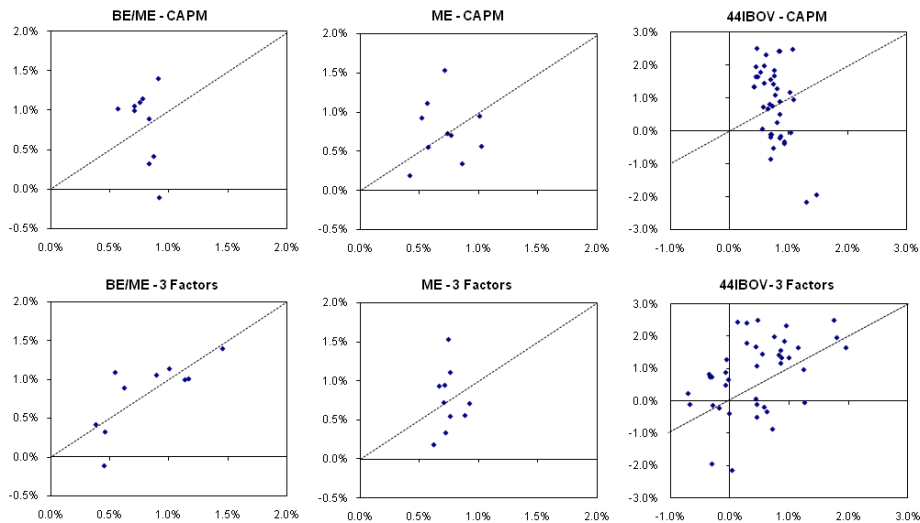
Table VII
Cross-Sectional Regressions.

Econometric Model: $E[R_{i,t} - R_{f,t}] = \lambda_m \beta_i + \lambda_s s_i + \lambda_h h_i$. In the GMM1, $W_T = I$. The weighting covariance matrix is based on Parzen Spectral Density. The GMM IT model iterated until convergence of the objective function. Bandwidth: 0, 1 and 4 lags. χ and $p(\chi)$ are the value and the p-value, respectively, of the GMM objective function. MAE is the mean absolute error between the predicted value of the model and the observed value. January 1999 to August 2006 (92 months).

<i>Method</i>	$\hat{\lambda}_m$	$t(\hat{\lambda}_m)$	$\hat{\lambda}_{mb}$	$t(\hat{\lambda}_{mb})$	$\hat{\lambda}_{mml}$	$t(\hat{\lambda}_{mml})$	χ	$p(\chi)$	<i>MAE</i>
Time Series	0.35	0.39	0.99	1.49	-0.79	-0.87			
BE/ME									
GMM 1 <i>0 lag</i>	1.23	1.22							0.43
GMM IT <i>0 lag</i>	1.15	1.18					10.90	0.28	0.45
GMM IT <i>1 lag</i>	1.09	1.07					10.88	0.28	0.46
GMM IT <i>4 lags</i>	0.98	0.96					13.01	0.16	0.49
ITNLSUR	0.43	1.39							0.63
GMM 1 <i>0 lag</i>	0.53	0.53	1.05	0.44	0.19	0.10			0.22
GMM IT <i>0 lag</i>	0.47	0.47	1.07	0.82	0.12	0.08	6.45	0.49	0.22
GMM IT <i>1 lag</i>	0.60	0.58	1.03	0.80	0.14	0.10	6.19	0.52	0.22
GMM IT <i>4 lags</i>	1.31	1.25	0.73	0.61	0.34	0.26	6.02	0.54	0.50
ITNLSUR	0.04	0.10	0.39	0.33	0.66	0.57			0.51
ME									
GMM 1 <i>0 lag</i>	1.15	1.14							0.32
GMM IT <i>0 lag</i>	1.46	1.51					17.74	0.04	0.41
GMM IT <i>1 lag</i>	1.25	1.28					17.20	0.05	0.34
GMM IT <i>4 lags</i>	0.55	0.60					19.85	0.02	0.43
ITNLSUR	0.56	2.77							0.43
GMM 1 <i>0 lag</i>	0.96	1.02	0.44	0.41	-0.58	-0.29			0.32
GMM IT <i>0 lag</i>	0.93	1.00	0.17	0.17	-0.45	-0.25	15.39	0.03	0.31
GMM IT <i>1 lag</i>	0.84	0.89	0.19	0.18	-0.62	-0.33	15.09	0.03	0.34
GMM IT <i>4 lags</i>	0.43	0.46	0.58	0.64	-1.25	-0.74	18.39	0.01	0.51
ITNLSUR	0.62	2.86	-0.54	-0.70	-0.21	-0.14			0.59
44IBOV									
GMM 1 <i>0 lag</i>	0.86	0.90							1.15
GMM IT <i>0 lag</i>	1.26	1.38					77.37	0.00	1.17
GMM IT <i>1 lag</i>	1.09	1.22					84.53	0.00	1.15
GMM IT <i>4 lags</i>	-1.98	-4.88					569.61	0.00	2.75
ITNLSUR	0.51	5.13							1.20
GMM 1 <i>0 lag</i>	0.94	1.01	-1.28	-1.32	-0.73	-0.61			1.12
GMM IT <i>0 lag</i>	0.87	0.95	0.40	0.50	-1.76	-1.65	69.16	0.00	1.68
GMM IT <i>1 lag</i>	0.74	0.82	0.21	0.28	-1.45	-1.39	77.42	0.00	1.54
GMM IT <i>4 lags</i>	-1.10	-2.41	2.25	6.31	-1.40	-2.67	516.68	0.00	2.38
ITNLSUR	0.56	5.30	-0.30	-0.66	-1.52	-2.78			1.04

When comparing both models in terms of mean absolute error (MAE), the greatest improvement occurs in the BE/ME portfolios, where the error is reduced from 0.43% with the CAPM specification to 0.22% with FF. There is only a tiny improvement in the 44IBOV portfolios, decreasing from 1.15% to 1.12%, although the reduction is greater in the ITNLSUR estimation.

In the ME portfolios, both the CAPM and the FF models had similar performance in terms of MAE. Figure 1 plots GMM1 estimates versus actual time series means for the indexes portfolios, and ITNLSUR estimates for the 44IBOV data set. In any case, the χ statistics has rejected the models as well specified, but that is not a surprise as we shall see with the US data.



(BOVESPA) Average Monthly Returns (y-axis) \times Predicted Monthly Return (x-axis)

Since Brazilian studies use short time series and a few firms, it is very difficult to compare the reliability of the results with those from the US, for instance. Therefore, we take the same period and grouping for the US data and compare the results with a longer sample. The portfolios are defined according the BE/ME criterium. The first sample, as usual, starts at January 1999 and ends at August 2006, hence it includes 92 months. The longer sample ends at the same date, but starts at July 1963, hence it includes 518 months.

Table VIII show the results with the US data. The small sample compares to Brazil, except that ITNLSUR model does not deliver a significant market risk premium. Indeed, in both CAPM and FF models, the estimates of the market risk premium are negative. Only the GMM IT with 1 and 4 lags for the FF model shows significant and similar coefficients, although the market risk premium is very negative. The MAE reduced a lot using the 3FF model, although the χ test still rejects the null.

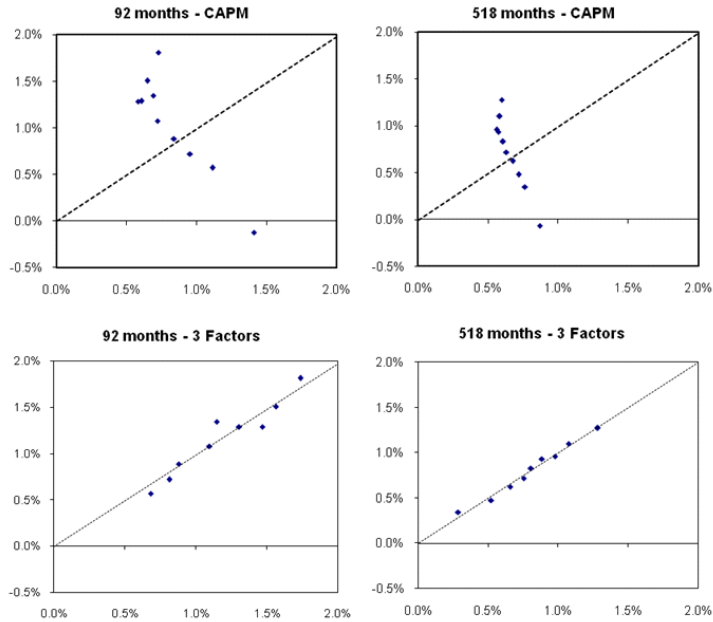
Table VIII
Cross-Sectional Regressions (NYSE/AMEX/Nasdaq) with 2 Samples:
92 and 518 Months.

Econometric Model: $E[R_{i,t} - R_{f,t}] = \lambda_m \beta_i + \lambda_s s_i + \lambda_h h_i$. In the GMM1, $W_T = I$. The weighting covariance matrix is based on Parzen Spectral Density. The GMM IT model iterated until convergence of the objective function. Bandwidth: 0, 1 and 4 lags. χ and $p(\chi)$ are the value and the

p -value, respectively, of the GMM objective function. MAE is the mean absolute error between the predicted value of the model and the observed value.

BE/ME									
Method	λ_m	$t(\lambda_m)$	λ_{smb}	$t(\lambda_{smb})$	λ_{hml}	$t(\lambda_{hml})$	χ	$P(\chi)$	MAE
92 months									
<i>Time series</i>	0.06	0.13	0.56	1.19	0.60	1.37			
GMM 1 <i>0 lag</i>	0.78	1.29							0.67
GMM IT <i>0 lag</i>	-0.09	-0.17					47.43	0.00	
GMM IT <i>1 lag</i>	-0.40	-0.78					47.86	0.00	
GMM IT <i>4 lags</i>	-1.53	-3.76					66.60	0.00	
ITNLSUR	-0.15	-0.63							1.19
GMM 1 <i>0 lag</i>	-0.89	-1.36	2.69	2.53	0.85	1.55			0.08
GMM IT <i>0 lag</i>	-0.77	-1.31	2.21	2.71	0.98	1.90	16.25	0.02	
GMM IT <i>1 lag</i>	-0.81	-1.40	2.06	2.48	1.10	2.09	16.35	0.02	
GMM IT <i>4 lags</i>	-0.84	1.75	2.16	3.04	1.19	2.49	20.93	0.00	
ITNLSUR	-1.26	-2.49	2.53	2.66	0.16	0.45			0.66
518 months									
<i>Série Temporal</i>	0.37	1.89	0.20	1.41	0.41	3.20			
GMM 1 <i>0 lag</i>	0.60	2.33							0.39
GMM IT <i>0 lag</i>	0.18	0.79					98.21	0.00	
GMM IT <i>1 lag</i>	0.16	0.70					94.82	0.00	
GMM IT <i>8 lags</i>	0.03	0.12					78.26	0.00	
ITNLSUR	-0.14	-1.59							0.88
GMM 1 <i>0 lag</i>	-0.04	-0.14	0.50	1.60	1.11	7.08			0.03
GMM IT <i>0 lag</i>	0.02	0.07	0.44	1.44	1.13	7.25	10.83	0.15	
GMM IT <i>1 lag</i>	0.03	0.09	0.42	1.31	1.11	7.14	10.95	0.14	
GMM IT <i>8 lags</i>	0.03	0.09	0.36	1.04	1.20	6.45	11.61	0.11	
ITNLSUR	-0.36	-1.66	0.25	0.84	0.73	7.34			0.65

The larger sample shows more robust results. The FF model is not rejected and the ITNLSUR does not reduce the MAE with a larger sample. Interestingly, now only the HML coefficients are significantly different from zero, although still far from the time series mean. However, the fit using the larger sample is much better as Figure 2 shows. This result is responsible for the great success the FF model had in fitting the anomalies in the US data.



(NYSE/AMEX/Nasdaq) Average Monthly Returns (y-axis) \times Predicted Monthly Return (x-axis)

5 Conclusions

The intercept tests produce similar conclusions with respect to the CAPM and the 3 Factors Fama-French models. When the market portfolio proxy is the Ibovespa for Brazil, the intercepts with portfolios selected according to the BE/ME criterium are jointly null in both models, whereas under the ME criterium, the models are both rejected. However, other proxies for the market portfolio may change the results. If the proxies is either the MSCI Brazil or a portfolios with equal weights for the assets, then the models are not rejected at all.

Notwithstanding, the point estimates increases from the lowest to the highest portfolios as in the US case when portfolios selected according to the BE/ME criterium. Such an observation is independent of the proxy for the market portfolio. Possibly this reflects the presence of a value-premium in the Brazilian data, as the descriptive statistics already showed. The phenomenon does not occur to portfolios chosen by the ME criterium.

The cross-section regressions reflect this value premium; the inclusion of the two additional factors in the Fama-French model improves explanatory power over the

CAPM model on the BE/ME portfolios: mean absolute error - MAE - falls from 0.43% to 0.22% per month. On the other hand, both models perform similarly on the ME portfolios, around 0.32% of monthly MAE. In the case of the 44IBOV data set, the multifactorial model is marginally superior to the CAPM (1.15% to 1.12%).

The US data are more conclusive, and justifies the success of the 3 factor model relatively to the CAPM (MAE from 0.67% to 0.08% in the BE/ME portfolios). With US data, anomalies are sharper than Brazilian data, what really makes the CAPM performance in the US to be unsatisfactory. Indeed, in the more robust results with the larger sample, the 3 Factor model was not rejected whereas the CAPM was. In the reduced sample, the same qualitative conclusion is reached, although the significance of the parameters worsens.

The risk premium, in general, was not statistically significant with Brazilian data. However, when the premium was significant, it was positive; the ITNLSUR point estimate indicates a premium of 6.5% per year, in line with private market usually agreed expectations, whereas the time series mean pointed to 4.25% per year although not statistically significant. The factor premia SMB and HML were both statistically non significant, even when accounting for serial correlation. Using the US data, the premia SMB and HML were significant with the small and large samples, respectively, and the market premia was zero.

This paper aimed to contribute to the Brazilian empirical asset-pricing literature. Firstly, by comparing the performance of two of the most studied asset pricing models, the CAPM and Factors Fama-French models, using Brazilian data, under a variety of data sets. As we have seen, the success of the multifactorial model over the CAPM in the US data is not matched in the Brazilian data. Despite finding evidence of a value-premium anomaly, the size anomaly is not as sharp as in the US data in the 92 month period. On the other hand, we found that the reduced size of our sample may have limited the robustness of the results. In fact, the FF model is rejected by the χ statistic in the short sample but not in the large sample. Secondly, two econometric methods were compared: GMM e ITNLSUR. We found the choice of the econometric tool to be important. In particular, we were able measure the Brazilian risk premium to be positive and significative using the ITNLSUR method. To the best of our knowledge nobody had reached to the same result before. That viabilizes the use of domestic data to calculate such a premium. Importantly the results come from ungrouped assets, hence the arbitrariness of chosing the portfolios is mitigated.

6 References

7 Bibliografia

- ANDREWS, D. W. K. (1991): “Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation,” *Econometrica*, 59(3), 817–858.
- BANZ, R. W. (1981): “The Relationship between Return and the Market Value of Common Stocks,” *Journal of Financial and Quantitative Analysis*, 14, 421–441.
- BASU, S. (1977): “Investment Performance of Common Stocks in Relation to their Price-Earnings Ratios: A Test of the Efficient Market Hypothesis,” *Journal of Finance*, 32, 663–682.
- BLACK, F. (1972): “Capital Market Equilibrium with Restricted Borrowing,” *Journal of Business*, 75(3), 444–455.
- BLACK, F., M. JENSEN, AND M. SCHOLES (1972): “The Capital Asset Pricing Model: Some Empirical Tests,” in *Studies in the Theory of Capital Markets*, ed. by M. C. Jensen, pp. 79–121. Praeger, New York.
- BLUME, M., AND I. FRIEND (1975): “The Asset Structure of Individual Portfolios with Some Implications for Utility Functions,” *Journal of Finance*, 30, 585–604.
- BONOMO, M. (2002): *Financas Aplicadas Ao Brasil*. Editora FGV, RJ.
- BREEDEN, D. T. (1979): “An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities,” *Journal of Financial Economics*, 7, 265–296.
- BURMEISTER, E., AND M. B. MCELROY (1988): “Arbitrage Pricing Theory as a Restricted Nonlinear Multivariate Regression Model,” *Journal of Business and Economic Statistics*, 6(1), 29–42.
- CAMPBELL, J. Y., A. LO, AND C. MACKINLAY (1997): *The Econometrics of Financial Markets*. Princeton University Press, Princeton.
- COSTA JR., NEWTON, R. L., AND E. LEMGRUBER (2000): *Mercado de Capitais: Analise Empirica No Brasil*. Editora Atlas, SP.
- FAMA, E. F., AND K. R. FRENCH (1992): “The Cross-Section of Expected Stock Returns,” *Journal of Finance*, 47(2), 427–465.

- (1993): “Common risk factors in the returns on stocks and bonds,” *Journal of Financial Economics*, 33, 3–56.
- (1995): “Size and Book-to-Market Factors in Earnings and Returns,” *Journal of Finance*, 50, 131–156.
- (1996): “Multifactor Explanations of Asset Pricing Anomalies,” *Journal of Finance*, 51(1), 55–84.
- FAMA, E. F., AND J. MACBETH (1973): “Risk, Return and Equilibrium: Empirical Tests,” *Journal of Political Economy*, 81, 607–636.
- GIBBONS, M. R., S. A. ROSS, AND J. SHANKEN (1989): “A Test of the Efficiency of a Given Portfolio,” *Econometrica*, 57(5), 1121–1152.
- HALL, A. R. (2005): *Generalized Method of Moments*. Oxford University Press, New York, NY.
- HANSEN, L. P. (1982): “Large Sample Properties of Generalized Method of Moments Estimators,” *Econometrica*, 50(4), 1029–1054.
- HARVEY, C., AND C. KIRBY (1995): “Analytic Tests of Factor Pricing Models,” Working Paper.
- LEWELLEN, JONATHAN, S. N., AND J. SHANKEN (2006): “A Skeptical Appraisal of Asset-Pricing Tests,” NBER Working Paper.
- LINTNER, J. (1965): “The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets,” *Review of Economics and Statistics*, 47, 13–37.
- LUCAS, JR., R. E. (1978): “Asset Prices in an Exchange Economy,” *Econometrica*, 46, 1426–14.
- MACKINLAY, A. C., AND M. P. RICHARDSON (1991): “Using Generalized Method of Moments to Test Mean-Variance Efficiency,” *Journal of Finance*, 46(2), 511–27.
- MALAGA, F. K., AND J. R. SECURATO (2004): “Aplicação Do Modelo de Três Fatores de Fama e French No Mercado Acionário Brasileiro - Um Estudo Empírico Do Período 1995-2003,” in *EnAnpad*. Anpad.
- MARKOWITZ, H. M. (1952): “Portfolio Selection,” *Journal of Finance*, 7(1), 77–91.

- MERTON, R. C. (1973): “An Intertemporal Capital Asset Pricing Model,” *Econometrica*, 41(5), 867–887.
- OGAKI, M. (1993): *Generalized Method of Moments: Econometric Applications*, 17, pp. 455–485, Handbook of Statistics, Vol 11. Elsevier Science Publishers.
- ROLL, R. W. (1977): “A Critique of the Asset Pricing Theory’s Tests,” *Journal of Financial Economics*, 4, 129–176.
- ROSENBERG, B., K. REID, AND R. LANSTEIN (1985): “Persuasive evidence of market inefficiency,” *Journal of Portfolio Management*, 11, 9–17.
- ROSS, S. A. (1976): “The Arbitrage Theory of Capital Asset Pricing,” *Journal of Economic Theory*, 13, 341–360.
- SHANKEN, J., AND G. ZHOU (2007): “Estimating and Testing Beta Pricing Models: Alternative Methods and their Performance in Simulations,” *Journal of Financial Economics*, 84, 40–86.
- SHARPE, W. F. (1964): “Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk,” *Journal of Finance*, 19, 425–442.
- YOGO, M. (2006): “A Consumption-Based Explanations of Expected Stock Returns,” *Journal of Finance*, 61(2), 539–580.

8 Appendix

The appendix division follows the body text. Details regarding each section follows the text section titles.

8.1 Data Set and Portfolio Construction

8.1.1 Portfolios

Composition of ME Portfolios The firms were ordered according to their market value (ME), and grouped in 10 portfolios (*D1* the largest corporations, *D10* the smallest corporations). Firms with more than one stock were kept in the same group with the same weight. The returns were equally weighted. Next table presents the composition of the portfolios in 2006, based on information released in December 2005.

Table A1
Portfolios Composition ME in December 2005

The data set contains 123 firms, whose stocks were traded in Bovespa between January 1999 and August 2006.

D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
PETR	TNLP	BRKM	SBSP	ACES	GLOB	AVPL	RHDS	FJTA	IENG
VALE	TMAR	BRTP	SDIA	LIGT	LEVE	PEFX	BOBR	IGBR	INEP
BBDC	CMET	GOAU	GUAR	DURA	CNFB	ITEC	FBRA	HGTX	TELB
AMBV	EMBR	VIVO	NETC	TMCP	CLSC	EBCO	CRIV	MTSA	TRFO
ITAU	CSNA	EBTP	BFIT	PTIP	PQUN	ROMI	FLCL	TNCP	TEKA
BBAS	CMIG	VCPA	KLBN	TMGC	RSID	BRIV	BRGE	VAGV	MWET
UBBR	BESP	BRTO	PRGA	ALPA	MYPK	FRAS	ETER	PNVL	BCAL
ITSA	USIM	TCSL	SUZB	CTNM	COCE	DXTG	RPAD	MNDL	LIXC
TLPP	ARCZ	LAME	CGAS	CESP	CEPE	ASTA	LIPR	PLAS	ESTR
ELET	TBLE	CPSL	FFTL	UNIP	ELEK	PLTO	EMAE	MGEL	SULT
ARCE	CRUZ	WEGE	CEEB	PMAM	RIPI	ILMD	SGAS	CGRA	JBDU
GGBR	PCAR	CPL	CBEE	RAPT	BSCT	FESA	PNOR	BDLL	MNPR
				DPPI	POMO	MAGS			

BE/ME The firms were ordered according to the size of the BE/ME index, from the lowest to the highest and grouped in 10 portfolios (*D1* to *D10*). Firms with more than one stock were kept in the same group with the same weight. We have excluded firms with negative book equity, they were grouped in portfolio VX. Next table shows the composition of the portfolios in 2006, based on information released in December 2005.

Tabela A2
Portfolios Composition BE/ME in December 2005

The data set contains 123 firms, whose stocks were traded in Bovespa between January 1999 and August 2006.

VX	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10
BCAL	BSCT	AMBV	ARCZ	ALPA	ARCE	BRKM	AVPL	ACES	ASTA	BDLL
BOBR	CMET	BBDC	BESP	BBAS	CNFB	CEEB	BRTO	CEPE	BRGE	CESP
ESTR	CPSL	CGAS	BFIT	CBEE	DURA	COCE	BRTP	CGRA	BRIV	ELET
MWET	CRUZ	EMBR	CMIG	CSNA	DXTG	LIGT	CLSC	CRIV	CPL	EMAE
PMAM	GUAR	FFTL	ELEK	GLOB	GOAU	POMO	ETER	CTNM	EBCO	FJTA
TEKA	HGTX	FRAS	GGBR	LIPR	ILMD	PQUN	FBRA	DPPI	FESA	IENG
VAGV	LAME	ITAU	ITSA	MTSA	ITEC	ROMI	IGBR	EBTP	JBDU	INEP
	MYPK	LEVE	PCAR	PEFX	KLBN	TMAR	PTIP	FLCL	PLTO	LIXC
	NETC	PRGA	PETR	RIPI	TMCP	USIM	RHDS	MAGS	PNOR	MGEL
	RSID	RAPT	PLAS	SDIA	VIVO	VCPA	SUZB	MNDL	RPAD	MNPR
	TBLE	UBBR	TLPP	TCSL			TRFO	PNVL	SBSP	SULT
	VALE	WEGE	TNLP	TMGC			UNIP	SGAS	TNCP	TELB

8.1.2 Factors

Fama and French - SMB and HML The 123 firms were split into two groups: *B* (big) and *S* (small). The first group contained the 61 largest firms in terms of market value, and the other, the remaining 62. The firms were again split into 3 groups according to the size of the index BE/ME: H (high), M (medium) and L (low). The extreme groups counted each for 30% of the firms, and the medium group for 40% of the firms. The market value was calculated by multiplying the free float stocks times the price stocks.

The intersection of these two groups makes 6 sets of stocks: *HB*, *HS*, *MB*, *MS*, *LB* and *LS*, which weighted by the market value of each firm generate 6 portfolios. The composition of these stocks is rebalanced yearly on the basis of the Balance sheet corresponding to December of the previous year, except the initial portfolio based on January 1999. Firms with negative equity were excluded. The next table shows the last composition of firms in each group followed by a descriptive statistics.

Table A3
Fama French Portfolios for SMB and HML in December 2005

The data set contains 123 firms, whose stocks were traded in Bovespa between January 1999 and August 2006.

HB	HS	MB	MS	LB	LS
ACES	ASTA	ALPA	AVPL	AMBV	BSCT
CESP	BDLL	ARCE	CLSC	ARCZ	ELEK
CPLP	BRGE	BBAS	CNFB	BBDC	FRAS
CTNM	BRIV	BRKM	COCE	BESP	HGTX
EBTP	CEPE	BRTO	DXTG	BFIT	LEVE
ELET	CGRA	BRTP	ETER	CGAS	MYPK
SBSP	CRIV	CBEE	FBRA	CMET	PLAS
	DPPI	CEEB	IGBR	CMIG	RSID
	EBCO	CSNA	ILMD	CPSL	
	EMAE	DURA	ITEC	CRUZ	
	FESA	GLOB	LIPR	EMBR	
	FJTA	GOAU	MTSA	FFTL	
	FLCL	KLBN	PEFX	GGBR	
	IENG	LIGT	PNVL	GUAR	
	INEP	PTIP	POMO	ITAU	
	JBDU	SDIA	PQUN	ITSA	
	LIXC	SUZB	RHDS	LAME	
	MAGS	TCSL	RIPI	NETC	
	MGEL	TMAR	ROMI	PCAR	
	MNDL	TMCP	TRFO	PETR	
	MNPR	TMGC		PRGA	
	PLTO	TNLP		RAPT	
	PNOR	UNIP		TBLE	
	RPAD	USIM		TLPP	
	SGAS	VCPA		UBBR	
	SULT	VIVO		VALE	
	TELB			WEGE	
	TNCP				

Table A4
Descriptive Statistics - Fama French Six Portfolios: BOVESPA and NYSE/AMEX/Nasdaq between January 1999 and August 2006

*Percentual Mean and Standard-Deviation. Market value (ME) in US\$ million in December 2005
Portfolio returns equally weighted according to the market value.*

Portfolios	HB	HS	MB	MS	LB	LS
BOVESPA						
Mean	2.02	3.63	3.49	3.12	2.76	4.48
Std. dev.	11.52	7.18	8.99	7.20	7.32	11.61
Mean ME	2,534	122	8,293	150	5,808	281
Firms	7	28	26	20	27	8
NYSE/AMEX/NASDAQ						
Mean	0.57	1.37	0.65	1.29	0.06	0.32
Std. dev.	4.46	5.07	4.06	5.10	4.59	8.10
Mean ME	11,295	307	14,090	463	16,753	462
Firms	174	935	311	1,207	394	1,213

The Brazilian portfolio mean returns present a pattern different from the US data. Portfolios with low BE/ME (L) had a superior performance to portfolios with high BE/ME (H) given the size of the firm, against expected. On the other side, portfolios with small firms (S) had higher returns than big firms (B) given BE/ME, except in the medium range. In the US case, one can observe that small firms with high BE/ME had a superior mean return, as documented in the anomalies' literature.

In order to assess whether different aggregations of firms would result in different results, we change slightly the usual procedure for constructing these series. In the first change, instead of weighting firms according to its size, we equally weight them (EW). And second, we keep the value-weighted procedure but we define new sections' thresholds for the BE/ME index: top 15%, 30%, 70% and 85% (bottom 15%). With this we expect to differentiate more the firms, getting the ones in the extreme side (VW15).

The statistics for EW show that $HB > LB$ and $HS > LS$, in line with the US data. However, on the other side of the spectrum, only $HS > HB$, and slightly, while $MB > MS$ and $LB > LS$. The statistics for VW15, also does not resemble the US' data, for instance we have $H1S > L1S$ but $H1B < L1B$. On what follows and in the main part of the paper, we will adopt the standard procedure.

Table A5
Descriptive Statistics - Fama French Six Portfolios Variations

Returns were equally weighted (EW) or according to market value (VW). The two extremes of 30% of the BE/ME index were divided into two sections of 15% (VW15). Sample from January 1999 to August 2006 (92 months).

BOVESPA										
EW	HB		HS		MB	MS	LB		LS	
Mean	2.31		2.68		2.80	1.74	1.88		1.69	
Std. Dev.	9.74		6.52		7.34	5.49	7.09		9.19	
VW	HB		HS		MB	MS	LB		LS	
Mean	2.02		3.63		3.49	3.12	2.76		4.48	
Std. Dev.	11.52		7.18		8.99	7.20	7.32		11.61	
VW15	H1B	H2B	H1S	H2S			L2B	L1B	L2S	L1S
Mean	1.90	2.93	3.87	3.43			2.44	2.90	3.78	2.78
Std. Dev.	14.00	10.08	11.05	5.94			7.68	7.93	11.32	15.74

In face of the empirical evidence of the anomalies, Fama and French (1992, 1993) suggest a multifactorial model that effectively summarizes the facts seen in the previous table. They introduce the so-called SMB and HML factors (SMB - *small minus big* e HML - *high minus low*):

$$SMB_t = (HS_t + MS_t + LS_t) / 3 - (HB_t + MB_t + LB_t) / 3$$

$$HML_t = (HS_t + HB_t) / 2 - (LS_t + L1S_t) / 2$$

In the following two graphics we see the time series of the 12 month average log-return of the factors as well as the market portfolio returns in excess of the risk free rate for Brazil and US.

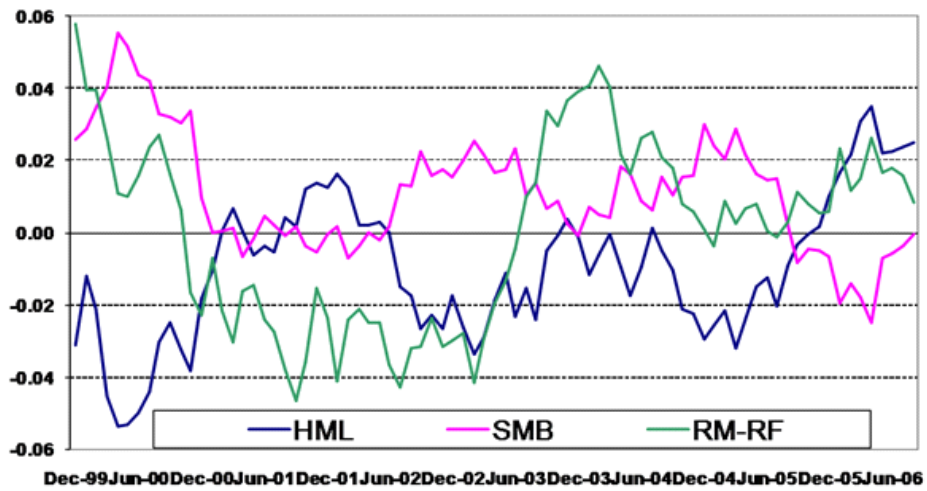


Figure A1: (Bovespa) 12-month moving average of log-returns

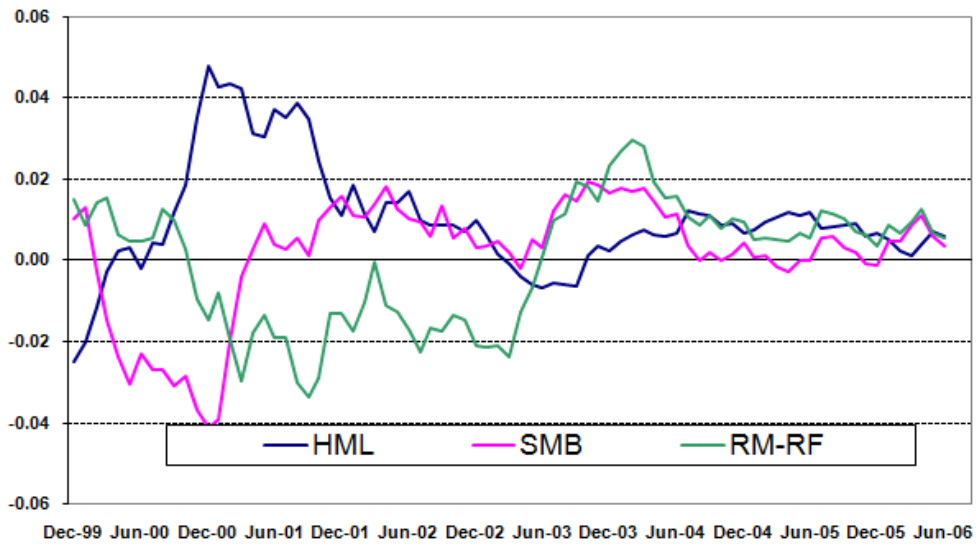


Figure A2: (NYSE, AMEX, Nasdaq) 12-month moving average of log-returns

8.2 Empirical Analysis

8.2.1 Intercept Tests

All the analysis were repeated using other proxies for the market portfolio as MSCI, BEW e BVW. The statistics GRS and χ differ.

Table A6
Intercept Tests and Time Series Regression of Portfolios BE/ME and ME (MSCI, BEW, BVW)

The estimated equation is $R_{i,t} - R_{f,t} = a_i + b_i(R_{m,t} - R_{f,t}) + s_iSMB_t + h_iHML_t + \varepsilon_{t,i}$. $t()$ is the t -statistics of the OLS regressions. $p(GRS)$ and $p(\chi)$ correspond to the p -values of the statistics GRS and χ described in the last section. The market portfolio is the Ibovespa and Selic is the risk free rate. Sample from January 1999 to August 2006 (92 months).

MSCI												
	Portfolios										GRS $p(GRS)$	χ $p(\chi)$
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10		
BE/ME												
a	-0.49	0.05	0.57	0.79	-0.01	0.79	0.85	0.81	0.75	1.06	0.97	9.99
t(a)	-0.98	0.15	1.27	1.88	-0.01	1.48	1.61	1.49	1.31	1.19	0.48	0.44
b	0.84	0.80	0.73	0.68	0.73	0.60	0.65	0.45	0.54	0.75		
t(b)	12.49	15.87	12.22	12.03	11.32	8.46	9.10	6.10	7.08	6.20		
R^2	0.63	0.73	0.62	0.61	0.58	0.44	0.47	0.28	0.35	0.29		
a	-0.91	-0.26	0.08	0.48	-0.10	0.33	0.30	0.23	0.28	0.53	1.00	7.89
t(a)	-1.98	-0.76	0.19	1.20	-0.21	0.71	0.66	0.60	0.63	0.76	0.45	0.64
b	0.97	0.90	0.92	0.84	0.83	0.86	0.94	0.80	0.85	1.16		
t(b)	12.76	15.69	14.01	12.54	10.74	11.23	12.62	12.39	11.68	9.97		
s	0.33	0.26	0.47	0.41	0.27	0.69	0.76	0.92	0.83	1.12		
t(s)	2.75	2.88	4.50	3.87	2.20	5.64	6.48	9.07	7.14	6.05		
h	-0.04	-0.01	0.07	0.22	0.28	0.43	0.42	0.62	0.62	0.97		
t(h)	-0.55	-0.22	1.11	3.18	3.51	5.50	5.55	9.47	8.29	8.12		
R^2	0.70	0.78	0.71	0.66	0.63	0.59	0.64	0.65	0.63	0.59		
ME												
a	0.50	0.14	-0.01	0.42	1.24	0.31	0.03	0.76	0.89	0.41	1.63	16.31
t(a)	1.87	0.33	-0.03	0.79	2.70	0.63	0.05	1.30	1.43	0.41	0.11	0.09
b	1.00	0.94	0.78	0.68	0.65	0.54	0.35	0.39	0.48	0.67		
t(b)	27.92	16.93	11.24	9.53	10.51	8.20	5.03	4.91	5.76	5.05		
R^2	0.90	0.76	0.58	0.50	0.55	0.42	0.21	0.20	0.26	0.21		
a	0.53	-0.12	-0.20	0.01	0.80	-0.40	-0.54	0.15	0.19	-0.45	1.94	14.55
t(a)	1.97	-0.30	-0.40	0.03	1.91	-1.11	-1.19	0.34	0.35	-0.52	0.05	0.15
b	1.00	1.09	0.91	0.93	0.87	0.87	0.60	0.73	0.83	1.16		
t(b)	22.18	16.61	10.99	12.11	12.61	14.54	7.93	9.54	9.44	8.13		
s	0.00	0.39	0.36	0.66	0.57	0.86	0.65	0.91	0.91	1.29		
t(s)	0.07	3.73	2.75	5.43	5.24	9.08	5.40	7.50	6.50	5.71		
h	0.05	0.24	0.29	0.46	0.28	0.37	0.23	0.58	0.45	0.81		
t(h)	1.16	3.64	3.45	5.80	4.02	6.08	2.99	7.37	4.97	5.56		
R^2	0.90	0.79	0.62	0.64	0.65	0.70	0.41	0.53	0.49	0.43		

BVW												
	Portfolios										GRS	χ
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	$\rho(GRS)$	$p(\chi)$
BE/ME												
a	-1.49	-0.93	-0.36	-0.11	-0.93	-0.01	-0.02	0.18	-0.06	0.05	1.82	22.56
t(a)	-3.04	-2.80	-0.90	-0.33	-2.09	-0.01	-0.04	0.36	-0.11	0.06	0.07	0.01
b	0.89	0.86	0.81	0.78	0.80	0.68	0.75	0.53	0.67	0.87		
t(b)	13.20	18.94	14.58	16.15	13.18	9.97	11.39	7.49	9.60	7.29		
R ²	0.66	0.80	0.70	0.74	0.66	0.52	0.59	0.38	0.50	0.36		
a	-1.83	-1.18	-0.85	-0.43	-0.95	-0.52	-0.66	-0.54	-0.68	-0.60	2.65	24.58
t(a)	-3.92	-3.75	-2.29	-1.24	-2.14	-1.12	-1.57	-1.40	-1.79	-0.85	0.01	0.01
b	0.95	0.91	0.92	0.87	0.84	0.85	0.95	0.78	0.90	1.15		
t(b)	13.14	18.77	16.16	16.39	12.25	12.04	14.60	13.19	15.47	10.51		
s	0.13	0.10	0.30	0.28	0.13	0.52	0.60	0.77	0.71	0.89		
t(s)	1.21	1.36	3.48	3.51	1.22	4.83	6.06	8.50	7.99	5.38		
h	-0.15	-0.10	-0.02	0.14	0.20	0.34	0.33	0.54	0.55	0.84		
t(h)	-2.06	-2.10	-0.30	2.68	2.84	4.75	5.01	8.91	9.29	7.63		
R ²	0.71	0.83	0.77	0.77	0.68	0.62	0.70	0.68	0.75	0.61		
ME												
a	-0.71	-1.03	-1.02	-0.49	0.39	-0.39	-0.48	0.19	0.17	-0.53	2.80	22.08
t(a)	-3.38	-2.81	-2.23	-1.01	0.93	-0.85	-0.98	0.33	0.30	-0.55	0.01	0.01
b	1.06	1.02	0.87	0.78	0.74	0.61	0.43	0.48	0.60	0.79		
t(b)	36.98	20.39	13.91	11.83	12.99	9.56	6.32	6.20	7.47	6.03		
R ²	0.94	0.82	0.68	0.60	0.65	0.50	0.30	0.29	0.38	0.28		
a	-0.48	-1.19	-1.18	-0.95	-0.10	-1.22	-1.19	-0.59	-0.71	-1.58	2.94	24.31
t(a)	-2.35	-3.19	-2.56	-2.17	-0.24	-3.25	-2.67	-1.29	-1.41	-1.82	0.00	0.01
b	1.00	1.08	0.94	0.94	0.88	0.84	0.62	0.73	0.86	1.14		
t(b)	32.14	18.78	13.29	14.06	14.61	14.63	9.05	10.47	11.13	8.55		
s	-0.18	0.18	0.22	0.50	0.42	0.68	0.55	0.78	0.78	1.06		
t(s)	-3.70	2.05	2.03	4.90	4.59	7.77	5.30	7.30	6.60	5.23		
h	-0.05	0.13	0.21	0.37	0.20	0.27	0.18	0.50	0.37	0.69		
t(h)	-1.46	2.23	2.93	5.38	3.26	4.69	2.56	7.08	4.76	5.06		
R ²	0.95	0.83	0.70	0.70	0.71	0.70	0.47	0.57	0.57	0.45		

BEW												
	Portfolios										GRS	χ
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	$\rho(GRS)$	$p(\chi)$
BE/ME												
a	-0.92	-0.33	0.13	0.37	-0.45	0.35	0.37	0.39	0.30	0.41	0.99	9.95
t(a)	-1.97	-0.88	0.38	1.19	-1.22	0.88	1.06	1.01	0.71	0.60	0.46	0.45
b	1.07	0.98	1.00	0.95	1.02	0.93	1.02	0.82	0.91	1.30		
t(b)	13.81	15.62	17.02	18.68	16.52	14.04	17.66	13.03	13.31	11.42		
R ²	0.68	0.73	0.76	0.79	0.75	0.68	0.77	0.65	0.66	0.59		
a	-0.92	-0.25	0.05	0.45	-0.17	0.32	0.26	0.21	0.26	0.47	1.21	10.32
t(a)	-2.40	-0.82	0.18	1.45	-0.49	0.78	0.74	0.67	0.69	0.80	0.30	0.41
b	1.05	0.95	1.01	0.93	0.96	0.94	1.04	0.87	0.93	1.30		
t(b)	16.40	18.88	19.89	18.10	16.44	13.87	17.71	16.35	14.88	13.07		
s	-0.32	-0.35	-0.14	-0.14	-0.27	0.11	0.14	0.39	0.26	0.35		
t(s)	-3.88	-5.45	-2.22	-2.18	-3.58	1.27	1.89	5.79	3.23	2.77		
h	-0.41	-0.36	-0.27	-0.10	-0.03	0.10	0.07	0.32	0.29	0.53		
t(h)	-7.01	-7.79	-5.92	-2.10	-0.62	1.69	1.25	6.57	5.16	5.80		
R ²	0.79	0.84	0.83	0.80	0.79	0.69	0.78	0.77	0.73	0.70		
ME												
a	0.07	-0.36	-0.51	-0.05	0.79	-0.11	-0.30	0.34	0.42	-0.24	1.39	14.97
t(a)	0.18	-1.01	-1.36	-0.14	2.60	-0.32	-0.72	0.77	0.92	-0.31	0.20	0.13
b	1.15	1.21	1.11	1.03	0.97	0.87	0.64	0.77	0.91	1.25		
t(b)	17.92	20.50	18.00	16.33	19.29	15.51	9.23	10.62	12.14	9.58		
R ²	0.78	0.82	0.78	0.74	0.80	0.72	0.48	0.55	0.62	0.50		
a	0.56	-0.11	-0.29	-0.01	0.75	-0.40	-0.60	0.11	0.10	-0.53	2.01	15.28
t(a)	2.62	-0.32	-0.79	-0.02	2.42	-1.29	-1.52	0.27	0.25	-0.70	0.04	0.12
b	1.04	1.15	1.06	1.02	0.98	0.93	0.70	0.83	0.97	1.32		
t(b)	28.79	21.26	17.53	15.94	18.81	18.03	10.59	12.81	13.74	10.41		
s	-0.68	-0.35	-0.23	0.04	0.00	0.28	0.26	0.43	0.37	0.53		
t(s)	-14.68	-5.04	-2.94	0.51	0.00	4.19	3.08	5.21	4.09	3.28		
h	-0.33	-0.17	-0.05	0.10	-0.04	0.04	0.01	0.30	0.14	0.37		
t(h)	-9.98	-3.46	-0.88	1.76	-0.93	0.86	0.12	5.09	2.10	3.22		
R ²	0.93	0.86	0.80	0.75	0.80	0.78	0.55	0.66	0.67	0.55		

8.2.2 Risk Premium

Next table presents the cross-section equations estimate results when the intercept is included in the portfolios BE/ME. The intercept is the exceptional return over the excess return of the portfolios.

Table A7
Cross-Sectional Regressions. January 1999 to August 2006 (92 months).

Econometric Model: $E[R_{i,t} - R_{f,t}] = \lambda_m \beta_i + \lambda_s s_i + \lambda_h h_i$. In the GMM1, $W_T = I$. The weighting covariance matrix is based on Parzen Spectral Density. The GMM IT model iterated until convergence of the objective function. Bandwidth: 0, 1 and 4 lags. χ and $p(\chi)$ are the value and the p-value, respectively, of the GMM objective function. MAE is the mean absolute error between the predicted value of the model and the observed value.

		BE/ME									
<i>Method</i>	λ_0	$t(\lambda_0)$	λ_m	$t(\lambda_m)$	λ_{smb}	$t(\lambda_{smb})$	λ_{hml}	$t(\lambda_{hml})$	χ	$p(\chi)$	
GMM 1 0 lag			1.23	1.22							
GMM IT 0 lag			1.15	1.18					10.90	0.28	
GMM IT 1 lag			1.09	1.07					10.88	0.28	
GMM IT 4 lags			0.98	0.96					13.01	0.16	
ITNLSUR			0.43	1.39							
GMM 1 0 lag	2.27	1.51	-2.27	-0.96							
GMM IT 0 lag	1.02	0.90	-0.99	-0.53					9.18	0.33	
GMM IT 1 lag	0.92	0.87	-0.70	-0.39					9.33	0.32	
GMM IT 4 lags	0.88	1.00	-0.17	-0.12					11.11	0.20	
ITNLSUR	1.47	1.32	-1.71	-1.05							
GMM 1 0 lag			0.53	0.53	1.05	0.44	0.19	0.10			
GMM IT 0 lag			0.47	0.47	1.07	0.82	0.12	0.08	6.45	0.49	
GMM IT 1 lag			0.60	0.58	1.03	0.80	0.14	0.10	6.19	0.52	
GMM IT 4 lags			1.31	1.25	0.73	0.61	0.34	0.26	6.02	0.54	
ITNLSUR			0.04	0.10	0.39	0.33	0.66	0.57			
GMM 1 0 lag	0.76	0.45	-0.48	-0.19	0.97	0.72	0.30	0.20			
GMM IT 0 lag	-0.11	-0.08	0.62	0.30	1.07	0.81	0.13	0.09	6.37	0.38	
GMM IT 1 lag	-0.20	-0.15	0.86	0.43	1.04	0.80	0.14	0.10	6.10	0.41	
GMM IT 4 lags	-0.30	-0.28	1.70	0.98	0.77	0.64	0.29	0.23	5.87	0.44	
ITNLSUR	-0.73	-0.49	1.02	0.50	0.38	0.31	0.69	0.57			