The Comparisons of Information Content for Various Volatility Measures: Evidence from Individual Stock Options

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Abstract

Britten-Jones and Neuberger (2000), and Jiang and Tian (2005) derived a model-free implied volatility under the pure diffusion assumption and asset price processes with jumps, respectively. In this paper, we first extend their model-free implied volatility to the processes of both asset price and volatility with jumps. We then compare the forecasting abilities of different volatility estimates for individual options with the underlying assets of 304 U.S. firms during the period from January 4, 1999 to December 31, 2004. The volatility estimates include the model-free implied volatility, the Black-Scholes implied volatility, the realized volatility (calculated by high-frequency intraday data) and the conditional volatility under GJR model. For one-day-ahead estimation, 54% of firms indicate that the realized volatility measured by 5-minute interval returns outperforms other estimates. The Black-Scholes implied volatility has the best performance for 62% of firms when the forecast horizon agrees with the period form the closed day after expiration date to next expiration. The empirical results also show the forecasting performance of model-free implied volatility is worse than that of Black-Scholes implied volatility whether the estimation of one-day-ahead or monthly prediction. Overall, the results show that there is less volatility information contained in the model-free expectations than in the at-the-money implied volatilites.

Keywords: Implied volatility; Model-free volatility; Realized volatility;
High-frequency intraday data.

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1. Introduction

It is widely recognized that the option implied volatility is a good estimator to predict future volatility of stock index, for recent researches such as Christensen and Prabhala (1998), Fleming (1998), Lin, Strong and Xu (1998) and Blair, Poon and Taylor (2001). However, the strictest argument of the Black-Scholes implied volatility (Black and Scholes (1978)) is based on a constant volatility assumption. Therefore, the conditional volatility under time series models such as ARCH (Engle (1982)), GARCH (Bollerslev (1986)) and other ARCH specifications were also placed important on researches.

Alternative implied volatility called model-free implied volatility is constructed by Britten-Jones and Neuberger (2000), extending the work of Derman and Kani (1994), Dupire (1994, 1997), and Rubinstein (1994), who only considered the volatility as a deterministic instead of stochastic process. The advantage of model-free implied volatility is general because it is neither based on a deterministic volatility process nor a constant volatility assumption, and it does not require other option-pricing model besides option price alone.

Jiang and Tian (2005) generalized the model-free volatility to asset price process with jumps and examined the volatility forecast abilities of the model-free implied volatility, the Black-Scholes implied volatility and the historical volatility. They found the model-free implied volatility performs best for Standard & Poor's 500 index. In contrast, Taylor, Yadav and Zhang (2006) found the model-free implied volatility contains less volatility information than the Black-Scholes implied volatility for individual stocks during January 1996 to December 1999.

In this paper, we first extended the Britten-Jones and Neuberger (2000), and Jiang and Tian (2005) model free implied volatility models to asset price and volatility with jumps proposed by Duan, Ritchken and Sun (2006). We then compare the forecasting abilities of for different volatility measures, including the model-free implied volatility, the Black-Scholes implied volatility, the conditional volatility provided by GJR model, and the historical volatility measured by high-frequency intraday data for individual options with the underlying assets of 304 U.S. firms during the period from January 4, 1999 to December 31, 2004.

We also extend the curve-fitting method of Taylor, Yadav and Zhang (2006) by considering the no-arbitrage condition on the volatility curve construction. We also included more than double the amount of their samples and our sample period, which was from January 4, 1999 to December 31, 2004, is also longer than their study. In addition, we adopt high frequency data to calculate the realized volatility for our historical volatility proxy. The advantage of high-frequency data is it could hold higher information content than daily data for estimating the true volatility and it is an unconditional volatility, which means it is not based on specific volatility models, such as the time series model or the stochastic volatility model.

Our empirical results show that the realized volatility measured by 5-minute intraday returns outperforms other volatilities, for one-day-ahead estimation in ARCH including the conditional volatility under GJR, the model-free implied volatility, and the Black-Scholes implied volatility. However, the Black-Scholes at-the-money implied volatility has the leading forecasting ability across our 304 firms when the forecast horizon extends until the expiration date in OLS regression.

The remainder of this article proceeds as follows: the next section derived the model-free implied volatility formula under Merton (1979) jump process and GARCH jump process, respectively. Section 3 describes volatility constructions and our data. Section 4 presents empirical methodology and results. The final section contains the conclusions.

2. Model-Free Implied Volatility

2.1 Model-Free Implied Volatility with Jumps in the Return of Underlying Asset

Britten-Jones and Neuberger (2000) derived the model-free implied volatility under the diffusion assumption. Assume that the dynamics process of underlying stock price be

$$\frac{dS_t}{S_t} = \sigma_t dZ_t \quad , \tag{1}$$

Volatility σ_t is function of time and other parameters. dZ_t is a Wiener process. S_t is the underlying stock price at time t. Let T be a given maturity and a set of options with a continuum of strikes $K \geq 0$ and a continuum of maturities $t \geq 0$ and prices $C^S(T,K)$. Under risk-neutral, obtained that

$$E\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^\infty \frac{C^S(T,K) - \max(S_0 - K, 0)}{K^2} dK \tag{2}$$

First, we take jumps in the return of underlying asset into consideration in this section. We consider a continuous trading economy with trading interval $[0,T]$. Assume that the dynamic process of underlying stock price as follows

$$\frac{dS_t}{S_t} = \sigma_t dZ_t + (Y_t - 1)dN_t \tag{3}$$

The jump size Y_t are log normal distributed with parameters μ and δ and the Poisson process $\{N_t : t \in [0,T]\}$ with intensity λ.

Under this situation, the expected value of the implied volatility is

$$E\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right] = E\left[\int_0^T \left(\sigma_t^2 + \left(\delta^2 + k^2\right)\lambda\right)dt\right] \tag{4}$$

where $k = E[Y_t - 1]$.

In order to get the implied volatility with jump framework, we define a stock process with different volatility term.

$$\frac{dS_t^{B^N}}{S_t^{B^N}} = \sigma_t^{B^N} dZ_t \tag{5}$$

where

$$\left(\sigma_t^{B^N}\right)^2 = \sigma_t^2 + \left(\delta^2 + k^2\right)\frac{N_t}{T}$$

and N_t is a Poisson with mean λt, when given the number of jump variance is constant. μ and k are the parameters of Y_t. Given the times of jumps, n, the process of $S_t^{B^N}$ described in equation (1). Therefore we can use the method mentioned above to calculate the implied volatility under the jump process.

Proposition 1: The expected value of squared returns between the time 0 to the date T under the jump process is as follows

$$E\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right] = 2\sum_{n=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!} \int_0^{\infty} \frac{C^{B^n}(T,K) - \max(S_0^{B^n} - K, 0)}{K^2} dK \tag{6}$$

where

$$\frac{dS_t^{B^n}}{S_t^{B^n}} = \sigma_t^{B^n} dZ_t \tag{7}$$

$$\left(\sigma_t^{B^n}\right)^2 = \sigma_t^2 + \left(\delta^2 + k^2\right)\frac{n}{T}$$

and $C^{B^n}(T,K)$ be a set of option prices when given the number of jumps, n.

2.2 Model-Free Implied Volatility with Jumps in Return and Volatility

In the literatures, jumps could happen in both asset return and volatility. In this section, we use NGARCH(1,1) jump framework mentioned in Duan, Ritchken, and Sun (2005) to calculate the implied volatility.

According to Duan, Ritchken and Sun, assume that

$$\frac{m_t}{m_{t-1}} = e^{a+bJ_t} \tag{8}$$

where $J_t = X_t^{(0)} + \sum_{j=1}^{N_t} X_t^{(j)}$, $X_t^{(0)}$ be $N(0,1)$, and for j=1,2,…., $X_t^{(j)}$ is $N(\mu,\gamma^2)$. N_t is distributed as a Poisson random variable with λ. The asset price S_t is assumed to follow the process

$$\frac{S_t}{S_{t-1}} = e^{\alpha_t + \sqrt{h_t}\overline{J_t}} \tag{9}$$

where $\overline{J_t} = \overline{X}_t^{(0)} + \sum_{j=1}^{N_t} \overline{X}_t^{(j)}$, $\overline{X}_t^{(0)}$ be $N(0,1)$, for j=1,2,…, $\overline{X}_t^{(j)}$ is $N(\mu,\gamma^2)$.

For $t = 1,2,\cdots,T$, and $i = j$, $t = \tau$, $Corr(\overline{X}_t^{(i)}, \overline{X}_\tau^{(j)}) = \rho$, otherwise, $Corr(\overline{X}_t^{(i)}, \overline{X}_\tau^{(j)}) = 0$. The term of N_t is the same Poisson random variable as in the pricing kernel.

The local variance of the logarithmic returns from date t, viewed from date $t-1$ is

$$h_t Var(\overline{J}_t) = h_t(1 + \lambda \hat{\gamma}^2) \tag{10}$$

where $\hat{\gamma}^2 = \overline{\mu}^2 + \overline{\gamma}^2$. This assume h_t follows a simple NGARCH(1,1)

$$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2 h_{t-1}\left(\frac{\overline{J}_{t-1} - \lambda\overline{\mu}}{\sqrt{1+\lambda\hat{\gamma}^2}} - c\right)^2 \tag{11}$$

where β_0 is positive and β_1, β_2 are nonnegative to ensure that the local scaling process is positive. Assume that the single period continuously compounded interest rate is constant, say r. Thus the following restrictions must hold

$$E^P\left[\frac{m_t}{m_{t-1}} \mid F_{t-1}\right] = e^{-r} \tag{12}$$

$$E^P\left[\frac{m_t}{m_{t-1}}\frac{S_t}{S_{t-1}} \mid F_{t-1}\right] = 1 \tag{13}$$

Define a new measure Q by following

$$dQ = e^{rT} \frac{m_T}{m_0} dP \tag{14}$$

Then

$$\frac{S_t}{S_{t-1}} = e^{\tilde{\alpha} + \sqrt{h_t}\tilde{J}_t} \tag{15}$$

and

$$h_t = \beta_0 + \beta_1 h_{t-1} + \beta_2^* h_{t-1} \left(\frac{\tilde{J}_{t-1} - \tilde{\lambda}(\bar{\mu} + b\rho\gamma\bar{\gamma})}{\sqrt{1 + \tilde{\lambda}\tilde{\gamma}^2}} - c^* \right)^2 \tag{16}$$

where

$$\beta_2^* = \beta_2 \left(\frac{1 + \tilde{\lambda}\tilde{\gamma}^2}{1 + \lambda\hat{\gamma}^2} - \right)$$

$$c^* = \frac{c\sqrt{1 + \lambda\hat{\lambda}^2} + \lambda\bar{\mu} - \tilde{\lambda}(\bar{\mu} + b\rho\gamma\bar{\gamma}) - b\rho}{\sqrt{1 + \tilde{\lambda}\tilde{\lambda}^2}}$$

$$\tilde{\gamma}^2 = (\bar{\mu} + b\rho\gamma\bar{\gamma})^2 + \bar{\delta}^2$$

$$\tilde{J}_t = \tilde{X}_t^{(0)} + \sum_{j=1}^{N_t} \tilde{X}_t^{(j)}$$

$$\tilde{X}_t^{(0)} \sim N(0,1)$$

$$\tilde{X}_t^{(j)} \sim N(\bar{\mu} + b\rho\gamma\bar{\gamma}, \bar{\gamma}^2) \quad \text{for j=1,2,....}$$

N_t has a Poisson distribution with $\tilde{\lambda} = \lambda k$.

Under measure Q ,

$$h_t Var(\tilde{J}_t) = h_t(1 + \lambda\tilde{\gamma}^2)$$

where $\tilde{\lambda} = \lambda k$.

Above all, we consider the NGARCH(1,1)-Normal model. In order to consistent with condition in Britten-Jones and Neuberger (2000), we set r=0. The following

Lemma 2.2.1 and Proposition 2 could be obtained under these assumptions. And we can prove Proposition 2 by the similar transformation in section 2.1.

Lemma 2.2.1 : If there is no jump and r=0, the NGARCH(1,1) model can be expressed as

$$\frac{dS_t^G}{S_t^G} = \sqrt{h_t^G}\, dZ_t \tag{17}$$

where

$$h_t = \beta_0 + \beta_1 h_{t-1}^G + \beta_2^* h_{t-1}^G (\tilde{X}_t^{(0)} - c^*)^2$$

Proposition 2 : The expected value of squared returns between the time 0 to the date T under NGARCH(1,1) model is as follow

$$E\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^\infty \frac{C^G(T,K) - \max(S_0^G - K, 0)}{K^2} dK \tag{18}$$

where $C^G(T,K)$ be a set of option prices under NGARCH(1,1)-Normal model.

3. Data and Volatility Calculation

3.1 Model-Free Implied Volatility Formula

As mentioned earlier, Britten-Jones and Neuberger (2000) derived the model-free implied volatility under the diffusion assumption; the risk-neutral expectation of underlying variance can be replaced as[2]:

$$E^Q[V_{0,T}] = 2e^{rT}\left[\int_0^{F_{0,T}} \frac{P(K,T)}{K^2}dK + \int_{F_{0,T}}^{\infty} \frac{C(K,T)}{K^2}dK\right]. \tag{19}$$

Were $F_{0,T}$ is the underlying forward price and $P(K,T)$ and $C(K,T)$ denote the put and call option price with strike price K. Equation (1) means that the variance expectation can display the form of integration of all out-the-money option prices weighted by the square strikes. The new VIX is a typical application of model-free implied volatility.

In September 2003, the CBOE launched the new volatility index, and it was based on the model-free implied volatility. They calculate the new volatility index, VIX, using S&P 500 index options calculated from the following formula[3]:

$$\sigma^2{}_{MF} = \frac{2}{T}e^{rT}\sum_{i=1}^{M}\frac{\Delta K_i}{K_i^2}Q(K_i,T) - \frac{1}{T}\left[\frac{F_{0,T}}{K_0}-1\right]^2. \tag{20}$$

Where T is time to expiration, r is the risk-free interest to expiration, K_0 is the strike price used to decide $Q(K_i,T)$ is call or put option price, $Q(K_i,T)$ is call price with strike K_i if $K_i \geq K_0$ and it is put price otherwise, and ΔK_i is the interval between strike price, defined as $\frac{K_{i+1}-K_{i-1}}{2}$[4].

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2 See the proof in Appendix A.

3 The source downloaded from the site: http://www.cboe.com/micro/vix/vixwhite.pdf

4 ΔK_i for the lowest strike is defined as the difference between the lowest strike and the next higher

$F_{0,T}$ is the forward index level derived form index prices and K_0 is first strike below $F_{0,T}$ in the definition of VIX. However, we use equation (20) to calculate the model-free implied volatility by approximating the underlying strike price K_i is continuous; therefore, we ignore the final term of equation (20) by setting $K_0 = F_{0,T}$ when calculating the model-free implied volatility.

3.2 Implementations of the model-free implied volatility

We require out-the-money option prices as many as possible in this paper so we can estimate the model free volatility expectation in equation (19). There are no adequate available stock options for us to directly calculate model-free implied volatility in practice. Hence, we apply the implementation method stated by Taylor, Yadav and Zhang (2006) to calculate the model-free volatility expectation. Their method of volatility curve fitting is based on Malz (1997).

Malz (1997) presented the volatility smile could be described as a quadratic function form an option's delta instead of directly taken from the exercise price, stated by Shimko (1993). The quadratic volatility function is stated as follows:

$$\hat{\sigma}(\Delta_i) = \phi_{atm} - 2\phi_{rr}(\Delta_i - 0.5) + 16\phi_{str}(\Delta_i - 0.5)^2, \tag{21}$$

where the constant, ϕ_{atm}, provides the basis of this volatility curve, ϕ_{rr} is the coefficient to indicate the skew of this volatility curve, and the second power coefficient, ϕ_{str}, shows the degree of curvature for this volatility curve.

The model-free implied is assumed in risk-neutral measure, delta Δ_i is defined as the first derivative of Black-Scholes call option price with respect to the underlying

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strike. Likewise, ΔK_i for the highest strike is the difference between the highest strike and the next lower strike.

forward price:

$$\Delta_i = \frac{\partial c\left(F_{0,T}, K_i, T\right)}{\partial F_{0,T}} = e^{-rT} N\left(d_1\left(K_i\right)\right), \tag{22}$$

with

$$d_1\left(K_i\right) = \frac{\ln(F_{0,T} / K_i) + 0.5\left(\sigma^*\right)^2 T}{\sigma^* \sqrt{T}}.$$

Here σ^* is defined as the implied volatility with the strike price nearest to the forward price $F_{0,T}$ as σ^*[5]. Taylor, Yadav and Zhang (2006) suggested to estimate the three parameters (ϕ_{atm} , ϕ_{rr} and ϕ_{str}) of volatility quadratic function by minimizing the following function:

$$\sum_{i=1}^{N} w_i \left(\sigma_i - \hat{\sigma}_i\left(\Delta_i; \phi_{atm}, \phi_{rr}, \phi_{str}\right)\right)^2, \tag{23}$$

where N is the number of observed strike price that could be use to calculate delta Δ_i that was given by equation (22), $w_i = \Delta_i\left(1 - \Delta_i\right)$ and its purpose is to reduce the impact of the away-the-money options. σ_i denotes the observed implied volatility corresponding the strike price K_i and $\hat{\sigma}_i\left(\Delta_i; \phi_{atm}, \phi_{rr}, \phi_{str}\right)$ is Malz's (1997) volatility quadratic function in equation (21).

At least three different available strike prices of options are required to estimate the parameters of quadratic volatility function by minimizing equation (23). The constraints are placed on so that the volatility curve is always positive[6]. In addition, we also consider the no-arbitrage condition where the out-the-money call (put) price decreases as strike price increases (decreases).

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[5] Taylor, Yadav, and Zhang (2006) followed Bliss and Panigirtzoglou (2002, 2004) look σ^* is a constant for a convenient one-to-one mapping between delta and the strike price.

[6] We assume $\phi_{str} > 0$ to ensure the volatility curve we fitted is a convex function, and $\phi_{rr}{}^2 - 16\phi_{atm}\phi_{str} < 0$ for volatility curve is always positive.

Afterwards we equally divide the 1000 parts covering the range from 0 to e^{-rT}, then find the corresponding volatility on the volatility quadratic function. Finally, we fitted and obtain the one-to-one mapping strike price from the strike price's inverse function of equation (22).

Now we can calculate the out-the-money option price which is required in equation (20) by Black-Scholes option pricing formula. We add 0.01 times of $F_{0,T}$ continuously to the maximum strike price until the least put price is less than 0.001 cents, and also extend the minimum strike price by the same increment, 0.01 times of $F_{0,T}$ as well, while least call price is greater than 0.001 cents. The extrapolation method is used to eliminate the truncated error caused by the integral beyond the strike price range between the minimum and the maximum strike price and we assume that the extended implied volatilities are equal to appropriate end-point volatility of the quadratic function.

3.3 Data Descriptions

In this study, the option data used is from Ivy DB database of OptionMetrics and the high-frequency stock price data is from the Trade And Quotation (TAQ) database. Our sample period goes from January 4, 1999 through December 31, 2004 and includes 1508 trading days. We use the CUSIP code to match the firms in Ivy DB database and in TAQ database.

The implied volatilities used to constrict volatility curves are obtained from the Ivy DB because the implied volatility provided by Ivy DB is considered the dividends and option exercised types. They set the theoretical option price equal to the midpoint of best closing bid price and best closing offer price, then they back out the implied volatility from Black-Scholes formula if the option is European; and from the

Cox-Ross-Rubinstein binomial tree model if the option is American. When constructing a volatility curve, each of the implied volatilities within the same trading day must map to a strike price for calculating corresponding delta. If it is available for both call and put option, the average value of the two implied volatilities is used.

The forward price $F_{0,T}$ is required for calculating the model-free implied volatility. It is the future value of spot stock price reduced by the present value of all the dividends before the maturity time, T, as follows:

$$F_{0,T} = \left(S_0 - \sum_i D_{0,i} \right) e^{rT} , \tag{24}$$

where S_0 is the underlying spot price, $D_{0,i}$ is the i-st dividend whose ex-date are in the interval [0, T]. Both spot price and dividend distribution are included in the Ivy DB. The sign r denotes the risk-free interest rate that corresponds to each option's expiration. It is obtained by linearly interpolating between the two closed zero-curve rates on the zero curve file provided by the Ivy DB, and each discount factor of dividends is obtained similarly.

The details for data filters from Ivy DB and TAQ database are described in the next two parts.

3.4 Construction of the model-free and Black-Scholes implied volatility

The stock options usually have the quotes with the maturities within 30 days, 60 days, 120 days and 180 days in the marketplace. If either the options have less than eight days to maturity or with missing values of implied volatility are excluded from our data because the former may have liquidity and market microstructure problems and the latter can not be used to construct the volatility curve.

We know that at least three strike prices and their corresponding implied volatilities are required to construct the volatility curve in the foregoing section. The data are picked for a certain day during our sample period if there are at least three available strike prices with the nearest maturity. Meanwhile, we switch to the second nearest maturity if available strikes are less than three with the nearest maturity. If the available strike prices are less than three with both the two nearest maturity, we treat our model-free implied volatility estimates and Black-Scholes implied volatility estimates as missing values for the certain day. We assume both model-free and Black-Scholes missing values are unchanged from the previous trading day[7].

Following the previously mentioned rule, we search the all the firms in NYSE, NASDAQ and AMEX from January 4, 1999 to December 31, 2004, from the Ivy DB. The firms must be included in the whole sample period and the missing values for every firm must be less than 2 percent. A total of 481 firms stayed, we continue to find their intraday transaction prices from TAQ database. The data also must cover our sample period, with less than 2% of missing data, the same exchange and with active trades[8]. Finally, a total of 304 firms are included in this study.

The model-free implied volatility is calculated every day by the method described in Section 2.2. The Black-Scholes implied volatility is defined as the implied volatility provided by Ivy DB database whose strike price is closest to the forward price $F_{0,T}$.

The explanatory variables σ_{MF} and σ_{BS} for ARCH specifications denote the daily calculation of the model-fee implied volatility and the Black-Scholes implied volatility respectively, but for OLS regression, they represent the monthly non-overlapping[9] forecasts. We extract the monthly non-overlapping forecasts from the daily model-free and the Black-Scholes series when the trading days of the

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[7] Tow firms in our sample have missing values in the first trading day, we use the nearest volatility estimates after to substitute them.

[8] See the details in Section 3. 2.

[9] Christensen and Prabhala (1998) and Christensen, Hansen and Prabhala (2001) argued against the overlapping problems.

options follows pervious maturity date. In other words, the explained variables used in ARCH specifications are one-day-ahead estimations, and they used in OLS regression employ a forecast horizon equal to the option's time to maturity; there are 1508 observations for the daily volatility variables for 304 firms and at most 71 observations for the monthly non-overlapping variables[10].

Figure 1 plots daily Microsoft's model-free implied and Black-Scholes implied volatility time series during our sample period. The two series have approximately the same tendency. The Black-Scholes volatility tends to be lower than the model-free volatility, it is because it only responds the near-the-money option's behavior, but the model-free responds the volatilities of all the out-the-money options.

3.5 Construction of the realized volatility

Unlike stock return, volatility is a latent variable. The general measure for realized volatility is the standard deviation over the relevant return horizon, but the measure is restricted to calculate daily volatility when using daily return. Therefore, we also use high-frequency data to estimate the true latent volatility by summing the intraday squared returns, whose advantage is it contains more information content than daily data.

An important issue for high-frequency data is the microstructure noise that always comes along and the higher the frequency is, the more noise it contains. Aït-Sahalia, Mykland, and Zhang (2005) demonstrated that the selection of optimal sampling interval for calculating realized volatility is dependant on the amount of microstructure noise relative to the volatility horizon. In other words, if a longer volatility horizon, such as monthly volatility, is taken for analysis, then the longer

[10] The monthly observations begin in the volatility forecast with the horizon form January 19, 1999 tough the expiration date, February 20, 1999, and end in the volatility forecast for the option trading in November 22, 2004 until expiration date December 18, 2004. There are 71 compete monthly observations in our sample period, but not all of firms have 71 monthly observations restricted by the option data with enough available strike prices.

sampling interval should be selected than the daily volatility. Hence, we select 30-minute individual stock returns to calculate monthly realized volatility and 5-minute individual stock returns to calculate daily realized volatility. This setting is consistent with Jiang and Tian (2005) for index volatility.

ABDE (2001) calculate daily realized volatility by using 5-minute returns of 30 DJIA firms; we employ their empirical method to calculate our daily realized volatility estimates. We extract 5-minute transaction prices for the target firms which have been picked for calculating implied volatility from TAQ database in the period form 9:30 EST to 16:05 EST every trading day, and the transaction record's exchange must be consistent with the option record's exchange[11]. The 5-minute prices are taken at or immediately before the 5-mimute ticks except the first price, that we use is the price at 9:30 EST or immediately after 9:30 EST. There are a total of eighty 5-minute prices for each trading day and we can use them to get 79 logarithmic difference returns. In order to ensure our stocks are liquid enough to extract 5-minute transaction prices, the stocks we picked are at least 158 trades per day at the beginning, the middle, and the end of our sample period; the three days are respectively January 4, 1999, January 2, 2002, and December 31, 2004.

The 30-minute returns are constructed similarly, and we also extract 10-minute, 15-minute, and 20-minute intraday returns that all span through the period from 9:30 EST to 16:05 EST. Table 1 presents the summary distributions of intraday returns for different time intervals. In Panel A, all of the intraday returns are not significant from zero, and their skewness is also close to zero, but they are extremely leptokurtic. Panel B provides their first to third order autocorrelations, and their autocorrelations are low and decrease as the time intervals rise, which implies that the longer time

[11] The transaction records for each firm have different price performances from different exchange, so we extract the transaction prices form the exchange where is primary exchange for our option data.

interval diminishes, the more microstructure noise there is.

After we construct intraday returns, the annualized realized volatility is measured by the following equation:

$$\sigma^{(1)}_{RE,t,T} = \sqrt{\frac{251}{\tau}\sum_{i=1}^{n} r_i^2}\ , \tag{25}$$

where r_i denotes the intraday stock return, τ is the volatility relevant horizon, and n is the number of intraday returns form t to T.

Alternative measure for annualized realized volatility based on standard deviation of daily returns is:

$$\sigma^{(2)}_{RE,t,T} = \sqrt{\frac{251}{\tau}\sum_{i=1}^{\tau} \left(r_i - \overline{r}_{t,T}\right)^2}\ , \tag{26}$$

where r_i is the stock daily return and $\overline{r}_{t,T}$ is the average return during this period τ.

The explanatory variable σ_{LRE} for ARCH specifications and OLS regression analysis in the next section denotes the lagged daily realized volatility that is the realized volatility at time t-1 calculated by equation (25) with 5-minute returns. Following Jiang and Tian (2005), we adopted it for historical volatility proxy by assuming the volatility process is a Markov process. It contains the nearest information for forecasting future volatility. The $\sigma^{(1)}_{RE}$ and $\sigma^{(2)}_{RE}$ in OLS regression represent the monthly realized volatility measured by equation (25) with 30-minute returns and (26) with daily returns provided by Ivy DB.

Table 2 reports summary statistics of the realized volatilities calculated by different time interval returns. The distinction between Panel A and Panel B is the volatility horizons. The volatilities of Panel A are daily volatilities calculated by equation (25), and the volatilities of Panel B are monthly estimates by summing the daily volatilities during the corresponding option maturity period. Both of them are annualized values. The distributions for different time intervals are similar, and the same tendency for both daily and monthly results is they both decrease as the time interval increases. This result implies the shorter time interval contains more information and more microstructure noises.

Table 3 provides the cross-sectional correlation matrices of daily and monthly realized volatility for different time intervals. We can see both the daily and monthly correlations are high, but the daily correlation for different time intervals is lower than the monthly correlation. That implies the time intervals influence the daily realized volatility estimation easier. Therefore, in the analysis of ARCH specifications in next section, we also report the results from the five different time intervals (5-, 10-, 15-, 20- and 30-minute).

4. Empirical methodology and Result

4.1 Descriptive Statistics

The summary statistics for all variables used in ARCH specifications and regression are reported in Table 4. Panel A provides summary statistics for the daily model-free implied volatility. The daily Black-Scholes at-the-money implied volatility and the difference between model-free and Black-Scholes implied volatility, the daily estimates are used in ARCH specification. Panel B presents the summary statistics for the monthly non-overlapping variables used in the OLS regression analysis, including the two kinds of realized volatility estimates are the model-free implied volatility, σ_{MF}, and the Black-Scholes implied volatility, σ_{BS}, and the lagged realized volatility σ_{LRE}.

As shown in Table 4, either daily or monthly model-free implied volatilities are higher than Black-Scholes implied volatilities. In Panel B, realized volatility that used intraday returns is lower than the ones that used daily returns. It is because we ignored the overnight effect when estimating the realized volatility. Both the Black-Scholes and model-free implied volatilities are lager than realized volatility, which is known to be a downward biased measured of the risk-neutral expected variance for the Black-Scholes implied volatility and a positive bias by Jensen's inequality[12] for the model-free implied volatility.

Table 5 presents the cross-sectional mean and median values of the correlation matrices across 304 firms. The correlation matrix for each firm are calculated using the monthly non-overlapping volatilities. From the correlation matrix, we can find the Black-Scholes implied has the highest correlation and the lagged realized volatility has the lowest correlation, for both measurements of realized volatility despite the statistics, mean or median. Overall, the highest correlation is between the model-free and the Black-Scholes implied volatility.

4.2 ARCH specifications

4.2.1 Description of Model

This model is combined with the models below: Glosten, Jagannathan, and Runkle (1993), Blair, Poon and Taylor (2001) and Taylor, Yadav and Zhang (2006). The general specification is as bellow:

$$\begin{aligned} r_t &= \mu + \varepsilon_t + \theta\varepsilon_{t-1}, \\ \varepsilon_t &= h_t^{1/2} z_t, \qquad z_t \sim i.i.d.N(0,1), \\ h_t &= \frac{\omega + \alpha\varepsilon_{t-1}^2 + \alpha^- s_{t-1}\varepsilon_{t-1}^2}{1-\beta L} + \frac{\gamma\sigma_{MF,t-1}^2}{1-\beta_\gamma L} + \frac{\delta\sigma_{BS,t-1}^2}{1-\beta_\delta L} + \frac{\lambda\sigma_{RE,t-1}^2}{1-\beta_\lambda L}. \end{aligned} \tag{27}$$

Where the daily stock returns r_t is modeled by conditional mean μ, residuals ε_t,

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[12] The Jensen's inequality is as $E_0^Q\left[\sqrt{\int_0^T\left(\frac{dS_t}{S_t}\right)^2}\right] \le \sqrt{2\int_0^\infty \frac{C(K,T)-\max(0,S_0-K)}{K^2}dK}$.

and previous residuals ε_{t-1} with coefficient θ. The standard residuals z_t assume the following as a standard normal distribution. L is the lag operator to capture the autocorrelation of conditional variance h_t. The conditional variance is described as four different explanations of volatility:

(1) The GJR(1,1) model in Glosten, Jagannathan, and Runkle (1993):

Place restrictions on $\lambda = \beta_\lambda = \gamma = \beta_\gamma = \delta = \beta_\delta = 0$, and the conditional variances are as follows: $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \alpha^- s_{t-1}\varepsilon_{t-1}^2 + \beta h_{t-1}$,

where s_{t-1} is a dummy variable, it is 1 if $\varepsilon_{t-1} < 0$ and 0 otherwise.

(2) The volatility model only with explanations of model-free volatility alone:

Place restrictions on $\alpha = \alpha^- = \beta = \lambda = \beta_\lambda = \delta = \beta_\delta = 0$, and the conditional variances are as follows: $h_t = \left(1 - \beta_\gamma\right)\omega + \gamma\sigma_{MF,t-1}^2 + \beta_\gamma h_{t-1}$,

where $\sigma_{MF,t-1}$ is daily model-free implied volatility at period t-1, form dividing annualized volatility values by $\sqrt{251}$.

(3) The volatility model only with explanations of Black-Scholes volatility alone:

Place restrictions on $\alpha = \alpha^- = \beta = \lambda = \beta_\lambda = \gamma = \beta_\gamma = 0$, and the conditional variances are as follows, $h_t = \left(1 - \beta_\delta\right)\omega + \delta\sigma_{BS,t-1}^2 + \beta_\delta h_{t-1}$,

where $\sigma_{BS,t-1}$ is daily Black-Scholes implied volatility at period t-1, from dividing annualized volatility values by $\sqrt{251}$.

(4) The volatility model with explanations of lagged realized volatility alone:

Place restrictions on $\alpha = \alpha^- = \beta = \gamma = \beta_\gamma = \delta = \beta_\delta = 0$, and the conditional variances are as follows, $h_t = \left(1 - \beta_\lambda\right)\omega + \lambda\sigma_{RE,t-1}^2 + \beta_\lambda h_{t-1}$,

where lagged realized volatility, $\sigma_{LRE,t-1}$, is daily realized volatility at period t-1.

4.2.2 Parameter Estimation

The parameters are estimated by maximizing the quasi-log-likelihood function:

$$\log - LLF = \sum_{t=1}^{n} l_t(\Theta), \tag{28}$$

with

$$l_t(\Theta) = -\frac{1}{2}\left[\log(2\pi) + \log(h_t(\Theta)) + z_t^2(\Theta)\right].$$

Where Θ presents a set of the parameters in the ARCH specifications, this method of estimation assumes that the standardized returns z_t follow the standard normal distribution. We set the restrictions on the parameter to guarantee the positive conditional variances[13].

Table 6 presents the summary statistics of parameter estimates from the four ARCH specifications defined in equation (27). Mean, Med, L_q, and U_q are the mean, median, lower quartile and upper quartile respectively for each estimates across 304 firms. Panel A provides the estimates for GJR(1,1)-MA(1) model. Panel B and Panel C provide the values estimated by the model-free implied volatility and Black-Scholes implied volatility respectively. Panel D provides the values estimated by the realized volatilities form different time interval returns. Numbers in the last two rows for each panel are the percentages of estimates that are significantly different from zero at the 5% and 10% significant levels.

The first model is the GJR model, which describes the conditional volatility as asymmetry by α and α^- for positive and negative residuals respectively. The estimates of α and α^- are not significantly different from zero at 5% level for the

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[13] We follow Taylor et al. (2006), the constraints are $\omega > 0$, $\alpha \geq 0$, $a + a^- \geq 0$, $\beta \geq 0$, $\beta_\lambda \geq 0$, $\beta_\gamma \geq 0$, and $\beta_\delta \geq 0$.

mass of firms. This is probable that the asymmetric volatility model can not describe the volatility well. The median of the volatility persistence parameter $\alpha + 0.5\alpha^{-} + \beta$ equals to 0.996.

The second model only use the information proved by time series of model-free implied volatility. We use one-day-ahead model-free volatility, $\sigma_{MF,t-1}$, to calculate the conditional variances. For half of the firms, the estimates of γ are between 0.55 and 0.71; 67.7 percent of firms have estimates γ that are significantly different from zero at 5% level. In contrast, only 29.9 percent of firms have estimates β_{γ} are significantly different from zero at the same level. This suggests that the conditional variance calculated from the model-free implied volatility is provided mainly by $\sigma_{MF,t-1}$ and the information provided by older implied volatility is limited.

The third model uses only the information contained in the time series of Black-Scholes implied volatility, $\sigma_{BS,t-1}$, to calculate the conditional variances. For half of the firms, the estimates of δ are between 0.6 and 0.9 and 71 percent of δ are significantly different from zero at 5% level. Generally speaking, δ exceeds γ and β_{δ} is less than β_{γ}.

The fourth model uses only the lagged realized volatility with five different time interval returns: 5-minute, 10-minute, 15-minute, 20-minute and 30-minute. λ tends to increase and β_{λ} tends to decrease as time intervals rises. Around seventy percents of λ are significantly different from zero at 5% level for each time intervals but the numbers of β_{λ} are significantly different from zero at 5% increase with time intervals.

The weight in conditional variance for the second, third and fourth model are respectively defined by $\frac{\gamma}{1-\beta_\gamma}$, $\frac{\delta}{1-\beta_\delta}$ and $\frac{\lambda}{1-\beta_\lambda}$. These values indicate the degree of information content contained in each model. We can find $\frac{\lambda}{1-\beta_\lambda}$ is higher than $\frac{\gamma}{1-\beta_\gamma}$ and $\frac{\delta}{1-\beta_\delta}$ regardless of time intervals and $\frac{\gamma}{1-\beta_\gamma}$ is the least for the majority of 304 firms in Table 6.

Figure 2 respectively shows the relation between $\frac{\gamma}{1-\beta_\gamma}$ and $\frac{\delta}{1-\beta_\delta}$ in Panel A, the relation between $\frac{\gamma}{1-\beta_\gamma}$ and $\frac{\lambda}{1-\beta_\lambda}$ in Panel B and the relation between $\frac{\delta}{1-\beta_\delta}$ and $\frac{\lambda}{1-\beta_\lambda}$ in Panel B for 304 firms in our sample. The scatter diagram slants toward Black-Scholes axis in Panel A, only one point drops above the 45-degree line. The scatter diagram slants toward Realized axis with 268 firms in Panel B. Finally, there are 208 firms with the larger weight values for Realized volatility in Panel C. This result implies the lagged realized volatility contains the most information in all of volatility estimates.

4.2.3 Model Fitting

Here we use log-likelihood function values to judge the performance of each model. A higher log-likelihood value indicates a higher responsibility for the model fitting. The first column in Panel A of Table 8 shows the percentage of log-likelihood values order across all the firms. The percentage for each row presents how many of the firms including in the six probable outcomes.

First, we decide what explanatory variable is our historical volatility. The log-likelihood values for lagged realized volatility are close for each time intervals. However, 35.5% of firms have highest log-likelihood values for 5-minute interval and 21.1%, 17.4%, 11.5% and 14.5% for 10-, 15-, 20- and 30-minute interval respectively. Therefore, we take the 5-minute realized volatility to represent our lagged realized volatility. Then we compare the performance of this lagged realized volatility with the conditional volatility under GJR model; 88.5 percent of firms have larger log-likelihood value in lagged realized volatility model. Thus we use lagged realized volatility for our explanatory variable of historical volatility. L_{LRE}, L_{MF} and L_{BS} denote the log-likelihood value under the 5-minute lagged realized volatility model, the model-free implied volatility model and the Black-Scholes implied volatility model respectively.

More than a half the firms have a log-likelihood value, L_{LRE}, which is higher than L_{MF} and L_{BS} and only 14.8% and 31.3% of firms have highest L_{MF} and L_{BS} respectively. This result suggests the lagged realized volatility calculating by 5-minute returns is the superior estimate for predicting

4.3 OLS regression

4.3.1 Description of Model

Following Canina and Figlewski (1993), Christensen and Prabhala (1998), Jiang and Tian (2005) and Taylor et al. (2006), we apply univariate and encompassing regression to inspect the information content from different volatility explanatory variables. While the univariate regressions here is used to judge the model explanations provided by the single volatility explanatory variable we also use it to compare the forecasting abilities with different individual volatility explanatory variables. The encompassing regression addresses the relative importance of competing volatility expectation variables and implies the marginal contribution when new expectation variables are included in the model.

The regression model is as follows:

$$\sigma_{RE,t,T} = \alpha + \beta_{LRE}\sigma_{LRE,t,T} + \beta_{MF}\sigma_{MF,t,T} + \beta_{BS}\sigma_{BS,t,T} + \varepsilon_{t,T}, \tag{29}$$

where $\sigma_{RE,t,T}$ is some measure of the monthly realized volatility form time t to time T. The lagged realized volatility $\sigma_{LRE,t,T}$ is our explanatory variable for historical volatility, $\sigma_{MF,t,T}$, and $\sigma_{BS,t,T}$ are non-overlapping model-free implied volatility and Black-Scholes implied volatility respectively. All of the variables here are annualized values.

4.3.2 Parameter Estimation and Explanation Performance

Table 7 presents summary statistics for the parameter estimates, adjusted R-square, mean square errors (MSE) and Durbin-Watson statistics across 304 firms. Med, L_q, and U_q are the median, lower quartile and upper quartile respectively for each estimates across 304 firms. Numbers in the parentheses are the number of firms whose coefficient estimates are significantly different from zero at the 5% and 10% significant level.

The first part of Panel A is the univariate regression with explained variable, $\sigma_{RE}^{(1)}$, which is measured by 30-minute intraday returns. Almost all firms the estimates for β_{LRE}, β_{MF} and β_{MF} are significantly different from zeros. The median of them are 0.50, 0.78 and 0.86 respectively. From the median of adjusted R-square, the value of Black-Scholes is the highest (0.55) and the value of model-free is slightly lower (0.53) and the value of lagged realized volatility is only 0.39. This evidence refers the Black-Scholes implied volatility contains more information than one-day-ahead estimation of realized volatility.

We continue discussing the encompassing regression with two different variables in Panel A. When the lagged realized volatility variable, $\sigma_{LRE,t,T}$, is added into the univariate model of the model-free and the Black-Scholes, both models increase 5%

in responsibility. For the majority of firms, the coefficients of them are both significantly different from zeros. However, when the bivariate regression model includes $\sigma_{MF,t,T}$ and $\sigma_{BS,t,T}$ at the same time, only 15.8% of firms whose β_{MF} are significantly different form zeros and the increments of adjusted R-square from univariate model with $\sigma_{MF,t,T}$ and univariate model with $\sigma_{BS,t,T}$ are both small. This result can be explained by the high correlation between the model-free implied volatility and the Black-Scholes implied volatility in Table 3.

The last model includes all of these three explanatory variables, $\sigma_{LRE,t,T}$, $\sigma_{MF,t,T}$ and $\sigma_{BS,t,T}$. The median of adjusted R-square is 0.6 and only 38 firms whose incepts β_{MF} are significantly different form zeros; 118 firms whose incept β_{BS} are significantly different form zeros. This could be due to the two implied volatility contain similar information. The medians of β_{LRE}, β_{MF} and β_{BS} are 0.19, 0.12 and 0.47 which suggests the Black-Scholes volatility is the most informative one.

The explained variable of Panel B is monthly non-overlapping realized volatility, $\sigma_{RE}^{(2)}$, measured by daily returns. The result is similar to Panel A, but the adjusted R-square values and the firms that are significantly different from zero are all less than Panel A. The result is released by the lower correlation between $\sigma_{RE}^{(2)}$ and other variables.

Most of the Durbin-Watson statistics are not small enough to reject the null hypothesis that the regression residuals are correlated. Therefore, there is no autocorrelation problem in our analysis.

Panel B of Table 8 presents the performance of each variable in univariate regression with explained variable, $\sigma_{RE}^{(1)}$. The results refer that the Black-Scholes is the best estimate for 61.8% of our firms to forecast the monthly realized volatility in the horizon relevant option's life. We also consider changing those volatility variables to their logarithms and their variances, the performances under variance and logarithm regression models are similar to Table 8 for each explanatory variable. The Black-Scholes implied volatility is best estimate for 63.8% of the 304 firms when the volatility variables change to their logarithm and for 59.2% of the 304 firms when the volatilities are replaced by their variances. The results are shown in Appendix B.

4.4 Comparisons of the performance

4.4.1 Comparison for groups defined by average available strike prices

We already know that σ_{LRE} performs best in one-day-ahead estimation and σ_{BS} performs best in monthly volatility forecast. We continue considering whether the performance under different groups is the same or not. Figure 2 shows the average available strikes for the 304 firms. Average strike numbers of most of firms are between 4 and 6. As a firm has more available strikes, the option prices are possibly more efficient so that the model-free implied volatility and Black-Scholes implied volatility should perform better than lagged realized volatility.

Hence, we allocate 304 firms into 3 groups by average number of available strike price, $\overline{N}_i$. Group 1 (with 40 firms; n=40) includes the firms with $\overline{N}_i$ between 3 and 4.Group 2 (n=101) includes the firms with $\overline{N}_i$ between 4 and 5. Group 3 (n=163) includes the firms with $\overline{N}_i$ higher than 5. The resorted results are showed in the second, third and fourth columns of Table 8.

After dividing all firms into three groups, we find the best performance of the lagged realized volatility decreases from 85% to 40%, the best performance of the model-free implied volatility increases from 5% to 18% and the best performance of the Black-Sholes implied volatility increases from 10% to 42% in Panel A. The null hypothesis that there is no relationship between the performance of ARCH model and these three groups is rejected by 3×3 contingency table test at the 5% significant level, with chi-square value 33.26. Similar tendencies for adjusted R-square after grouping is showed in Panel B. The contingency table test with chi-square value 27.29 tells us the tendencies are also significant.

4.4.2 Comparison for groups defined by intermediate delta options

In Malz's (1997) study, the 0.25-, 0.5- and 0.75-delat options anchor the volatility curve and their relation between the three parameters of the quadratic volatility. If the option delta is outside the range of 0.15 to 0.85, the price could be bias, so we use the number of intermediate delta options to group our firms.

The 304 firms are divided into 3 groups by average number of delta values within the interval [0.15, 0.85], that are denoted $\bar{D}_i$. Group 1 (n=85) contains the firms with $\bar{D}_i$ between 1 and 2.Group 2 (n=137) includes the firms with $\bar{D}_i$ between 2 and 3. Group 3 (n=82) includes the firms with $\bar{D}_i$ higher than 3.

The results are provided in the last three columns of Table 8. The tendencies for the log-likelihood values of three models are similar to the tendencies under the groups by strikes. The chi-square value is 64.81, which indicates the tendencies are more distinguishable. Although the chi-square statistic is statistically significant (28.16) in Panel B, the adjusted R-square for the model-free implied volatility does not increase. This refers the number of intermediate delta could not be the main reason to explain the adjusted R-square of model-free implied volatility is lower than the Black-Scholes implied volatility for the majority of our firms.

5. Conclusions

In this paper, we first generalized the model-free implied volatility model derived by Jiang and Tian (2005). We then use individual stock option data to compare the forecasting performance under various volatility measures. We find the realized volatility measured by 5-minute intraday returns outperforms GJR conditional volatility and the other two implied volatilities for one-day-ahead estimation. In addition, the ARCH model indirectly indicates the five-minute return is the optimal frequency for calculating the realized volatility. However, the realized volatility becomes a bad estimate for predicting the monthly volatility during the option maturity time. We find the Black-Scholes at-the-money implied volatility has the highest responsibility across our firms. In the regression analysis, the Black-Scholes implied volatility has more information for more than sixty percent of our firms.

Theoretically, the model-free implied volatility should be a better estimate than the Black-Scholes implied volatility. Jiang and Tian (2005) reported that the Model-free implied volatility contains the most information content. Carr and Wu (2006) found the new VIX could predict movement in future realized volatility and it contains all the information of GARCH volatilities, but Taylor, Yadav and Zhang (2006) demonstrated the Black-Scholes implied volatility performed best for monthly volatility forecasting, which is similar to our result.

The forecasting ability of model-free implied volatility is weaker than the Black-Schloes implied volatility for individual stocks no matter what the prediction horizons are. It brings us to a couple conclusions. First, the rare available strike prices for individual stocks would induce fitting error easily. Second, the method volatility curve fitting could be biased and we discussed the issue in Appendix C. Third, the option market for individual stocks is not informationally efficient because of illiquid trading. Finally, the out-the-money options are mispriced and the model-free implied volatility exactly extracts the error information form the out-the-money options.

References

Andersen, T.G., T. Bollerslev, F.X. Diebold, and H. Ebens, 2001, "The Distribution of Realized Stock Return Volatility," Journal of Financial Economics, 61, 43-76.

Aït-Sahalia, Y., P.A. Mykland, and L. Zhang, 2005, "How Often to Sample a Continuous-Time Process in the Presence of Market Microstructure Noise," The Review of Financial Studies, 18, 351-416.

Barndor-Nielsen, O.E. and N. Shephard, 2003, "Realized Power Variation and Stochastic Volatility Models," Bernouilli, 9, 243-265.

Blair, B.J., Poon, S.-H., Taylor, S.J., 2001, "Forecasting S&P 100 volatility: The incremental information content of implied volatilities and high frequency index returns," Journal of Econometrics, 105, 5-16.

Bliss, R. and Panigirtzoglou, N., 2002, "Testing the stability of implied probability density functions," Journal of Banking and Finance, 26, 381-422.

Bliss, R. and Panigirtzoglou, N., 2004, "Option-implied risk aversion estimates," The Journal of Finance, 6, 406-446.

Canina, L., and Figlewski, S., 1993, "The information content of implied volatility," Review of Financial Studies, 6, 659-681.

Carr, P. and Wu, L. 2006, "A Tale of Two Indices," The Journal of Derivatives; Spring 2006; 13-29.

Christensen, B.J., and Prabhala, N.R., 1998, "The relation between implied and realized volatility," Journal of Financial Economics, 59, 125-150.

Christensen, B.J., C.S. Hansen, and N.R. Prabhala, 2001, "The Telescoping Overlap Problem in Options Data," Working paper, University of Aarhus and University of Maryland.

Duan, J.C.,

Derman, E., and I. Kani, 1994, "Riding on a Smile," Risk, 7, 32-39.

Dupire, B., 1994, "Pricing with a Smile," Risk, 8, 76-81.

Dupire, B., 1997, "Pricing and Hedging with Smiles," In Dempster, M.A.H., and S.R. Pliska, eds.: Mathematics of Derivative Securities, Cambridge University Press, Cambridge, U.K.

Fleming J., 1998, "The quality of market forecasts implied by S&P 100 index option prices," Journal of Empirical Finance, 5, 317-345.

Glosten, L., R. Jagannathan, and D. Runkle(1993), "On the Relation Between the Expected Value and the Volatility on the Nominal Excess Returns on Stocks," Journal of Finance, 48, 1779-1801.

Hansen. P. R., and A. Lunde, 2005, "A Realized Variance for the Whole Day Based on Intermittent High-Frequency Data," Journal of Financial Economics, 3, 525-554.

Jiang, G., and Tian, Y., 2005, "The model-free implied volatility and its information content," Review of Financial Studies, 18, 1305-1342.

Lin Y., N. Strong and G. Xu, 1998, The encompassing performance of S&P 500 implied volatility forecasts, Working Paper, University of Manchester.

Malz, A.M., 1997, "Option-based estimates of the probability distribution of exchange rates and currency excess returns," Federal Reserve Bank of New York.

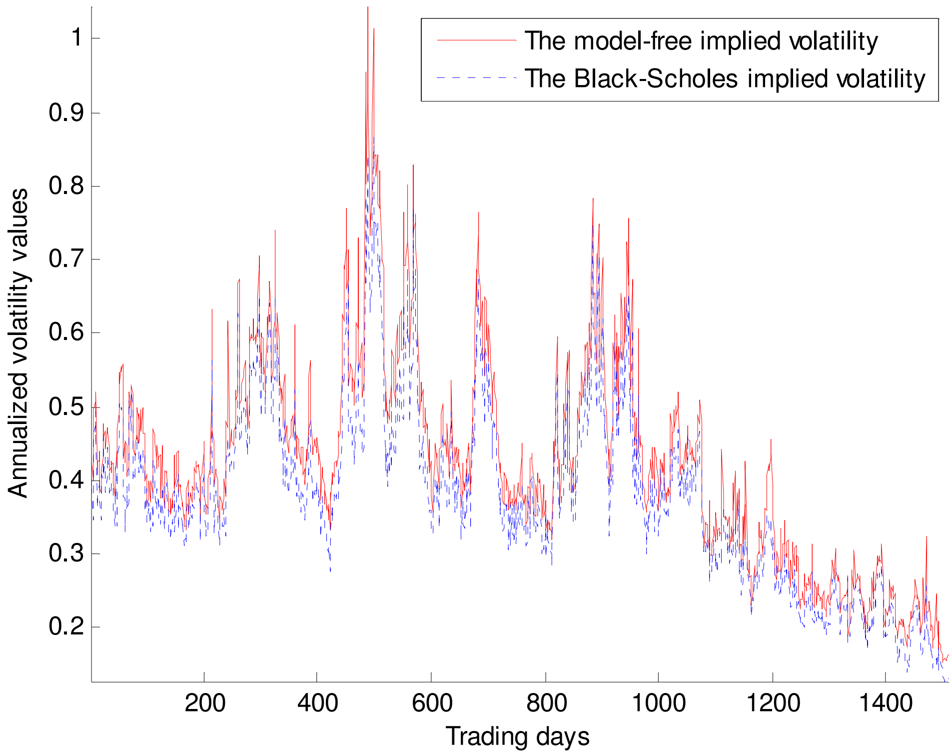
Malz, A.M., 1997, "Estimating the probability distribution of the future exchange rate from option prices," Journal of Derivatives; Winter 1997; 5, 2.

Rubinstein, M., 1994, "Implied Binomial Trees," Journal of Finance, 49, 771-818.

Shimko, D., 1993, "Bounds of Probability," Risk, 6, 33-37.

Taylor, S.J., Yadav, P.K., and Zhang, Y., 2006, "The information content of implied volatility expectations: Evidence from options written on individual stocks," Working paper, Lancaster University, U.K.

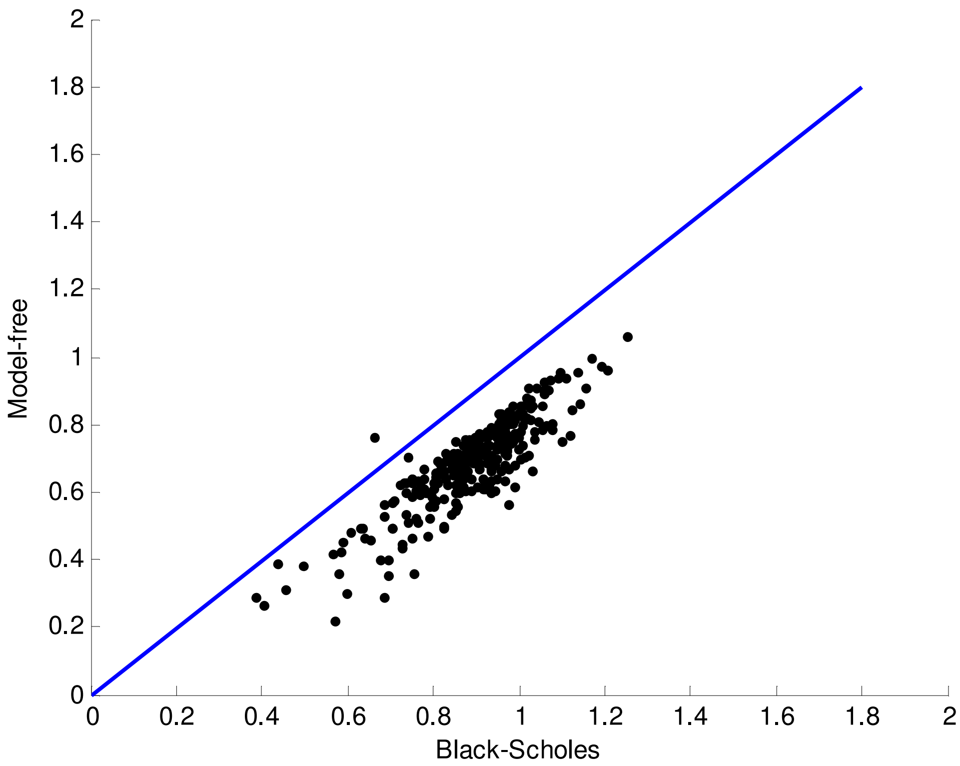
Figure 1 The model-free and Black-Scholes implied volatility time series plots for Microsoft



This figure plots the time series of daily model-free implied volatility and daily Black-Scholes implied volatility during the period from from January 4, 1999 to December 31, 2004 for Microsoft.

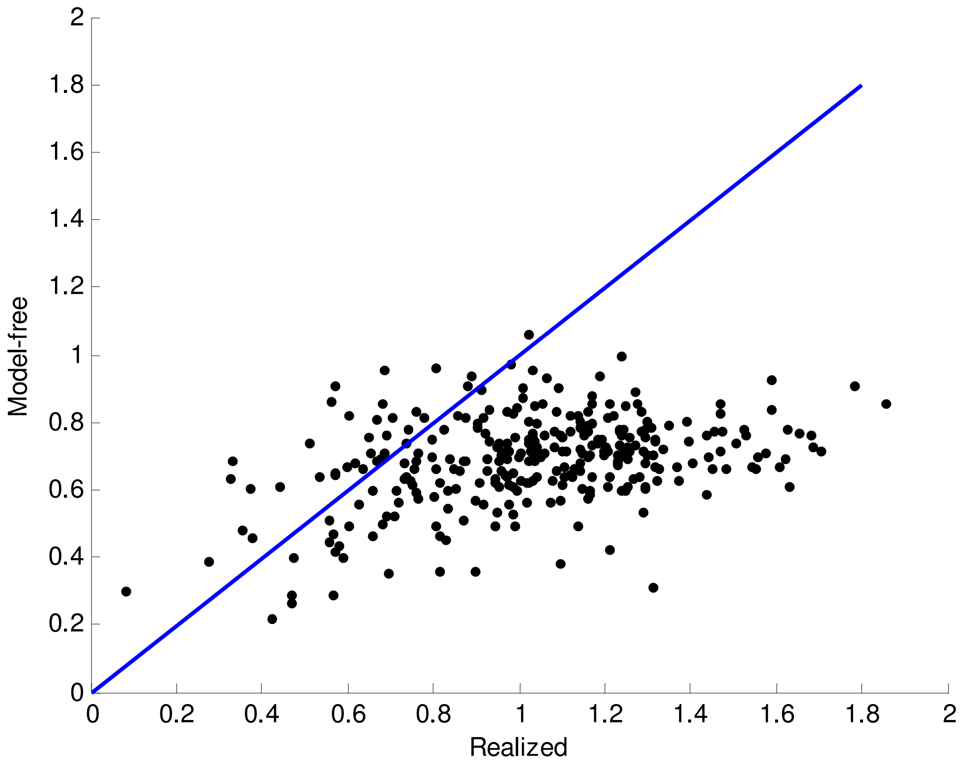
Figure 2 Comparison of the estimated weight values of model-free ($\frac{\gamma}{1-\beta_\gamma}$), Black-scholes ($\frac{\delta}{1-\beta_\delta}$) and realized ($\frac{\lambda}{1-\beta_\lambda}$) for 304 firms

Panel A



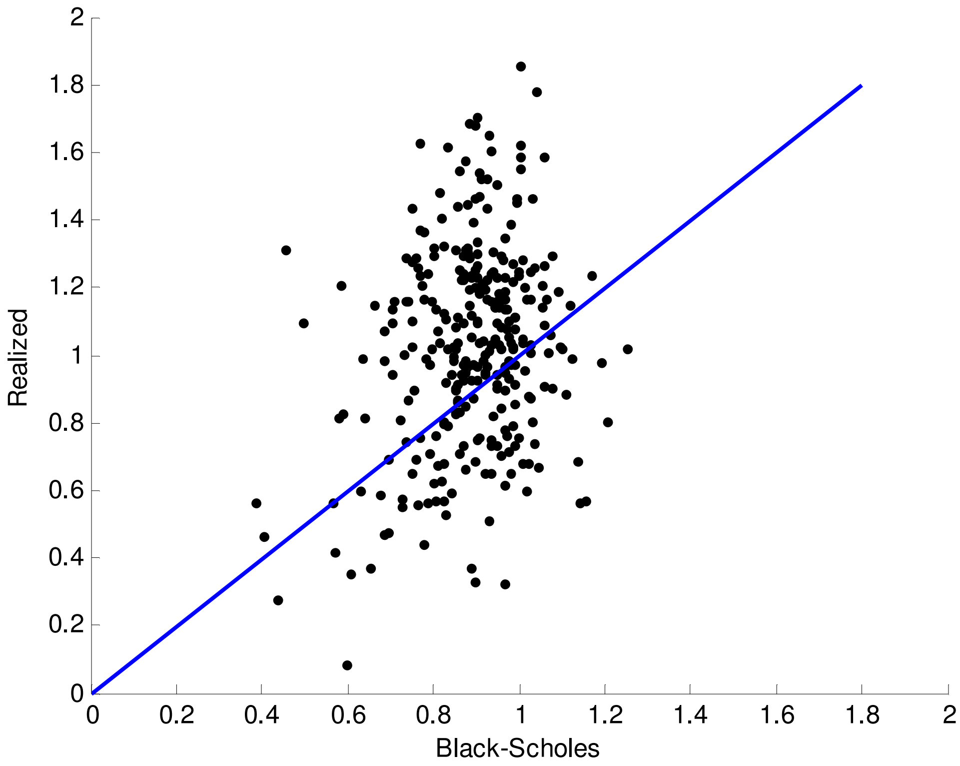
This scatter diagram shows the relation between $\frac{\delta}{1-\beta_\delta}$ and $\frac{\gamma}{1-\beta_\gamma}$ for each firm. The x-axis presents the weight of information content provided by one-day-ahead Black-Scholes implied volatility. The y-axis presents the weight of information content provided by one-day-ahead model-free implied volatility. $\frac{\gamma}{1-\beta_\gamma}$, $\frac{\delta}{1-\beta_\delta}$ and $\frac{\lambda}{1-\beta_\lambda}$ are estimated by the ARCH specification models using the information provided by the model-free implied volatility only, the Black-Scholes implied volatility only and the 5-minute realized volatility only respectively. The straight line is the 45-degree line.

Panel B



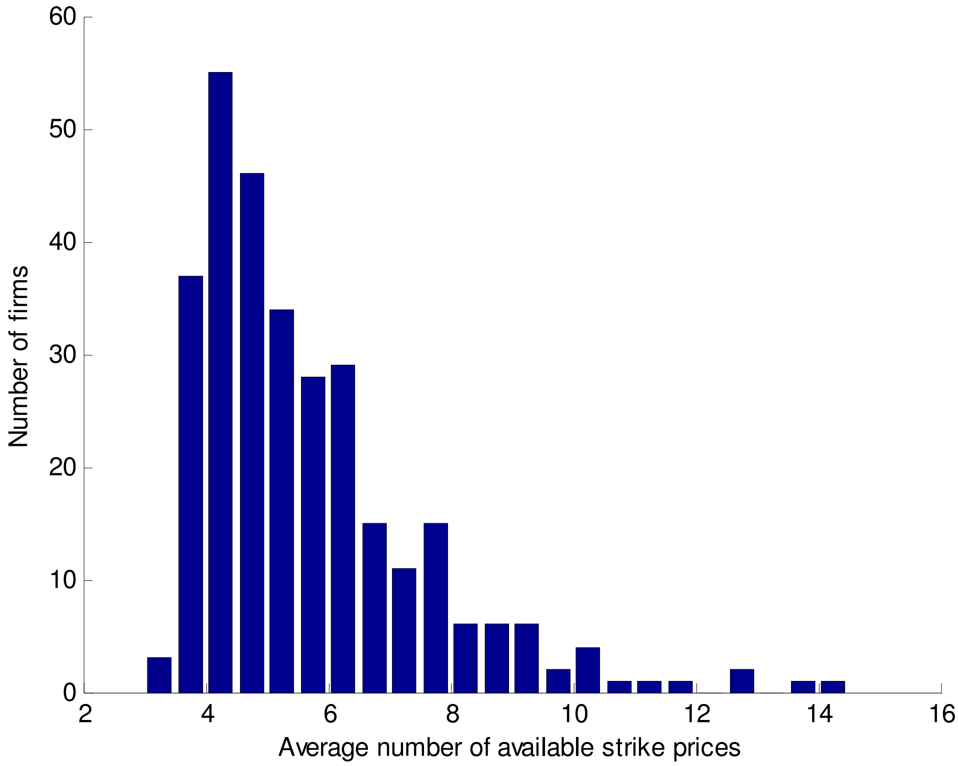
The plot shows the relation between $\frac{\lambda}{1-\beta_\lambda}$ and $\frac{\gamma}{1-\beta_\gamma}$ for 304 firms. The x-axis presents the weight of information content provided by one-day-ahead realized volatility. The y-axis presents the weight of information content provided by one-day-ahead model-free implied volatility. The straight line is the 45-degree line.

Panel C



The plot shows the relation between $\frac{\delta}{1-\beta_\delta}$ and $\frac{\lambda}{1-\beta_\lambda}$ for 304 firms. The x-axis presents the weight of information content provided by one-day-ahead Black-Scholes implied volatility. The y-axis presents the weight of information content provided by one-day-ahead realized volatility. The straight line is the 45-degree line.

Figure 3 The average number of daily available strike prices for 304 firms



This histogram shows the distribution of average number of daily available strike prices for 304 firms.

Table 1 Summary distributions of intraday returns for different time intervals

Panel A: summary distributions for different time intervals intraday returns

	Mean ($\times10^5$)				Std. Dev.				Skew.				Kurt.			
	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q
5-minute Ret.	0.137	0.270	-0.236	0.819	0.003	0.002	0.002	0.004	0.065	0.055	-0.030	0.167	25.256	19.485	13.852	33.102
10-minute Ret.	0.110	0.280	-0.974	1.499	0.004	0.003	0.003	0.005	0.062	0.099	-0.053	0.208	22.313	15.428	11.960	23.336
15-minute Ret.	0.165	0.420	-1.461	2.249	0.005	0.004	0.004	0.006	0.036	0.086	-0.061	0.214	19.286	14.393	11.092	19.177
20-minute Ret.	-0.531	-0.058	-3.054	2.529	0.006	0.005	0.004	0.007	0.017	0.083	-0.072	0.209	19.460	13.330	10.927	19.311
30-minute Ret.	0.330	0.839	-2.923	4.498	0.007	0.006	0.005	0.008	0.000	0.080	-0.094	0.207	17.227	12.278	10.184	17.586

Panel B: summary statistics for first- to third-order autocorrelation for different time intervals intraday returns

	First-order autocorrelation					Second-order autocorrelation					Third-order autocorrelation				
	Min	L_q	Med	U_q	Max	Min	L_q	Med	U_q	Max	Min	L_q	Med	U_q	Max
5-minute Ret.	-0.183	-0.087	-0.036	0.002	0.063	-0.060	-0.016	-0.006	0.001	0.020	-0.033	-0.007	-0.001	0.006	0.019
10-minute Ret.	-0.170	-0.059	-0.028	0.000	0.053	-0.041	-0.010	0.000	0.008	0.032	-0.022	0.000	0.005	0.011	0.032
15-minute Ret.	-0.168	-0.043	-0.020	0.001	0.068	-0.042	-0.004	0.005	0.014	0.037	-0.024	-0.005	0.002	0.009	0.075
20-minute Ret.	-0.172	-0.034	-0.009	0.014	0.075	-0.040	-0.004	0.005	0.013	0.083	-0.031	-0.007	0.001	0.008	0.032
30-minute Ret.	-0.159	-0.026	-0.003	0.017	0.082	-0.038	-0.006	0.001	0.010	0.078	-0.033	-0.010	-0.002	0.004	0.040

Panel A presents summary descriptive statistics, mean, standard deviation, skewness and kurtosis, for a cross-section of 304 firms. Panel B provides the summary statistics for first-, second- and third-order autocorrelation cross 304 firms. Mean, Med, L_q, U_q, Max and Min are respectively the mean, median, lower quartile, upper quartile, maximum and minimum for each statistic of intraday returns across 304 firms. The 5-minute Ret. denotes the intraday returns measured by five-minute interval prices, and so are the others.

Table 2 Summary statistics of realized volatility estimates for different time intervals

	Mean				Std. Dev.				Max				Min			
	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q
Panel A : summary statistics for daily realized volatility for different time intervals																
$\sigma_{RE}^{(5m)}$	0.384	0.323	0.281	0.457	0.184	0.150	0.121	0.223	1.865	1.472	1.075	2.602	0.089	0.082	0.069	0.107
$\sigma_{RE}^{(10m)}$	0.368	0.313	0.271	0.436	0.182	0.151	0.124	0.217	1.872	1.513	1.127	2.416	0.078	0.072	0.060	0.093
$\sigma_{RE}^{(15m)}$	0.360	0.308	0.268	0.422	0.184	0.155	0.128	0.219	1.896	1.547	1.169	2.408	0.071	0.064	0.053	0.085
$\sigma_{RE}^{(20m)}$	0.350	0.302	0.263	0.402	0.185	0.156	0.131	0.220	1.939	1.614	1.182	2.475	0.063	0.059	0.046	0.077
$\sigma_{RE}^{(30m)}$	0.349	0.301	0.260	0.399	0.192	0.160	0.136	0.228	2.012	1.677	1.245	2.436	0.054	0.049	0.039	0.066
Panel B : summary statistics for monthly realized volatility for different time intervals																
$\sigma_{RE}^{(5m)}$	0.402	0.335	0.292	0.479	0.145	0.119	0.095	0.174	0.821	0.678	0.551	0.994	0.186	0.160	0.134	0.224
$\sigma_{RE}^{(10m)}$	0.388	0.327	0.286	0.456	0.139	0.116	0.095	0.166	0.795	0.660	0.545	0.960	0.180	0.157	0.133	0.217
$\sigma_{RE}^{(15m)}$	0.382	0.326	0.283	0.444	0.137	0.115	0.095	0.164	0.785	0.654	0.539	0.954	0.177	0.156	0.132	0.213
$\sigma_{RE}^{(20m)}$	0.373	0.320	0.279	0.434	0.135	0.114	0.094	0.162	0.777	0.653	0.532	0.957	0.172	0.154	0.126	0.204
$\sigma_{RE}^{(30m)}$	0.375	0.323	0.281	0.437	0.136	0.114	0.094	0.165	0.787	0.656	0.541	0.954	0.171	0.152	0.127	0.206

The table presents summary statistics, mean, standard deviation, maximum and minimum, for a cross-section of 304 firms. Med, L_q, and U_q are the median, lower quartile and upper quartile respectively for each statistics of realized volatilities across 304 firms. Panel A provides daily realized volatilities for different time intervals and Panel B provides monthly non-overlapping realized volatilities for different time intervals. $\sigma_{RE}^{(5m)}$ denotes the realized volatility measured by five-minute returns, and so are the others.

Table 3 Summary statistics of the correlation matrices

	Mean					Med				
	$\sigma_{RE}^{(5m)}$	$\sigma_{RE}^{(10m)}$	$\sigma_{RE}^{(15m)}$	$\sigma_{RE}^{(20m)}$	$\sigma_{RE}^{(30m)}$	$\sigma_{RE}^{(5m)}$	$\sigma_{RE}^{(10m)}$	$\sigma_{RE}^{(15m)}$	$\sigma_{RE}^{(20m)}$	$\sigma_{RE}^{(30m)}$
Panel A: correlation matrix of daily realized volatility for different time intervals										
$\sigma_{RE}^{(5m)}$	1.000					1.000				
$\sigma_{RE}^{(10m)}$	0.954	1.000				0.959	1.000			
$\sigma_{RE}^{(15m)}$	0.921	0.941	1.000			0.924	0.943	1.000		
$\sigma_{RE}^{(20m)}$	0.893	0.938	0.928	1.000		0.896	0.940	0.930	1.000	
$\sigma_{RE}^{(30m)}$	0.853	0.898	0.928	0.918	1.000	0.856	0.899	0.929	0.919	1.000
Panel B: correlation matrix of monthly realized volatility for different time intervals										
$\sigma_{RE}^{(5m)}$	1.000					1.000				
$\sigma_{RE}^{(10m)}$	0.989	1.000				0.992	1.000			
$\sigma_{RE}^{(15m)}$	0.981	0.990	1.000			0.984	0.992	1.000		
$\sigma_{RE}^{(20m)}$	0.973	0.988	0.988	1.000		0.978	0.990	0.990	1.000	
$\sigma_{RE}^{(30m)}$	0.961	0.978	0.986	0.985	1.000	0.967	0.981	0.988	0.988	1.000

The table presents the cross-sectional mean and median of the correlation matrices for 304 firms in our sample. The correlation matrix for each firm are calculated using the daily realized volatility in Panel A and the monthly non-overlapping realized volatilities in Panel B. $\sigma_{RE}^{(5m)}$ denotes the realized volatility measured by five-minute returns, and so are the others.

Table 4 Summary statistics of volatility estimates

	Mean				Std Dev				Max				Min			
	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q	Mean	Med	L_q	U_q
Panel A : summary statistics for daily measures of model-free volatility and B-S implied volatility																
σ_{MF}	0.492	0.448	0.380	0.581	0.148	0.133	0.107	0.183	1.293	1.127	0.913	1.553	0.234	0.212	0.179	0.281
σ_{BS}	0.434	0.388	0.326	0.510	0.129	0.115	0.089	0.160	0.980	0.848	0.695	1.178	0.201	0.186	0.149	0.246
$\sigma_{MF} - \sigma_{BS}$	0.058	0.053	0.044	0.069	0.049	0.047	0.037	0.057	0.496	0.441	0.335	0.608	-0.043	-0.032	-0.047	-0.021
Panel B : summary statistics for monthly non-overlapping volatility estimates																
$\sigma_{RE}^{(1)}$	0.375	0.323	0.281	0.437	0.136	0.114	0.094	0.165	0.787	0.656	0.541	0.954	0.171	0.152	0.127	0.206
$\sigma_{RE}^{(2)}$	0.401	0.352	0.298	0.472	0.172	0.149	0.122	0.214	0.997	0.867	0.709	1.233	0.150	0.140	0.108	0.180
σ_{MF}	0.487	0.441	0.374	0.578	0.141	0.129	0.099	0.173	0.914	0.839	0.664	1.107	0.265	0.245	0.203	0.319
σ_{BS}	0.437	0.393	0.329	0.516	0.126	0.111	0.088	0.155	0.777	0.701	0.572	0.938	0.234	0.213	0.174	0.285
σ_{LRE}	0.384	0.322	0.277	0.470	0.188	0.154	0.125	0.239	1.120	0.923	0.722	1.428	0.139	0.125	0.102	0.163

The table presents summary statistics, mean, standard deviation, maximum and minimum, for a cross-section of 304 firms. Mean, Med, L_q, and U_q are the mean, median, lower quartile and upper quartile respectively for each statistics across 304 firms selected in our sample. σ_{MF}, σ_{BS}, and $\sigma_{MF} - \sigma_{BS}$ are the daily model-free implied volatility, the daily Black-Scholes implied volatility, and the difference between these two implied volatilities in Panel A. $\sigma_{RE}^{(1)}$, $\sigma_{RE}^{(2)}$, σ_{MF}, σ_{BS} and σ_{LRE} are the monthly non-overlapping volatilities for the realized volatility measured by intraday data, the realized volatility measured by daily data, the model-free implied volatility, the Black-Scholes implied volatility and the lagged realized volatility respectively in Panel B.

Table 5 Summary statistics of the correlation matrices

	Mean					Med				
	$\sigma_{RE}^{(1)}$	$\sigma_{RE}^{(2)}$	σ_{MF}	σ_{BS}	σ_{LRE}	$\sigma_{RE}^{(1)}$	$\sigma_{RE}^{(2)}$	σ_{MF}	σ_{BS}	σ_{LRE}
$\sigma_{RE}^{(1)}$	1.000					1.000				
$\sigma_{RE}^{(2)}$	0.832	1.000				0.855	1.000			
σ_{MF}	0.708	0.610	1.000			0.727	0.624	1.000		
σ_{BS}	0.724	0.621	0.946	1.000		0.744	0.629	0.959	1.000	
σ_{LRE}	0.619	0.519	0.626	0.636	1.000	0.632	0.535	0.642	0.661	1.000

The table presents the cross-sectional mean and median of the correlation matrices across 304 firms. The correlation matrix for each firm are calculated using the monthly non-overlapping volatilities. $\sigma_{RE}^{(1)}$, $\sigma_{RE}^{(2)}$, σ_{MF}, σ_{BS} and σ_{LRE} denote these monthly non-overlapping volatilities for the realized volatility measured by intraday data, the realized volatility measured by daily data, the model-free implied volatility, the Black-Scholes implied volatility and the lagged realized volatility respectively.

Table 6 Summary statistics of ARCH parameter estimates across 304 firms

Parameters	$\mu\times10^3$	θ	$\omega\times10^5$	α	α^-	β	γ	β_γ	δ	β_δ	$\alpha+0.5\alpha^-+\beta$	$\frac{\gamma}{1-\beta_\gamma}$	$\frac{\delta}{1-\beta_\delta}$	Log-LLF
Panel A : estimates of GJR(1,1)-MA(1) model														
Med	0.613	-0.010	0.524	0.024	0.063	0.936					0.996			3562.661
L_q	0.335	-0.046	0.164	0.012	0.036	0.881					0.981			3141.519
U_q	0.938	0.019	1.910	0.051	0.101	0.957					0.999			3830.701
At 5%	12.17%	16.12%	11.84%	25.33%	42.11%	93.42%								
At 10%	24.67%	26.64%	22.37%	37.83%	57.57%	93.42%								
Panel B : estimates of ARCH specification that use model-free volatility only														
Med	0.513	-0.008	0.000				0.554	0.114				0.704		3580.954
L_q	0.246	-0.038	0.000				0.304	0.000				0.625		3163.395
U_q	0.790	0.023	0.199				0.714	0.516				0.775		3827.611
At 5%	7.57%	16.45%	5.59%				67.76%	29.93%						
At 10%	13.82%	24.01%	7.24%				76.32%	32.24%						
Panel C : estimates of ARCH specification that use B-S volatility only														
Med	0.519	-0.009	0.000						0.806	0.015			0.900	3580.899
L_q	0.276	-0.039	0.000						0.598	0.000			0.822	3157.200
U_q	0.785	0.021	1.066						0.906	0.293			0.973	3842.263
At 5%	7.57%	14.80%	3.95%						71.05%	18.09%				
At 10%	13.82%	24.01%	6.58%						78.29%	20.07%				

The table presents summary statistics for parameter estimates, the persistence ($\alpha+0.5\alpha^-+\beta$) and log-likelihood function of each model across 304 firms. Mean, Med, L_q, and U_q are the mean, median, lower quartile and upper quartile respectively for each estimates across 304 firms. Panel A provides the estimates for GJR(1,1)-MA(1) model; Panel B and Panel C provide the values estimated by the model-free implied volatility and Black-Scholes implied volatility respectively.

Parameters		$\mu\times10^3$	θ	$\omega\times10^5$	λ	β_λ	$\frac{\lambda}{1-\beta_\lambda}$	Log-LLF		$\mu\times10^3$	θ	$\omega\times10^5$	λ	β_λ	$\frac{\lambda}{1-\beta_\lambda}$	Log-LLF
Panel D : estimates of ARCH specification that use lag realized volatility from different time intervals																
	Med	0.507	-0.006	5.865	0.373	0.629	1.035	3582.988		0.525	-0.007	6.121	0.260	0.754	1.158	3572.717
	L_q	0.223	-0.037	1.667	0.190	0.460	0.832	3162.487		0.255	-0.033	1.837	0.145	0.646	0.978	3139.903
$\sigma_{RE}^{(5m)}$	U_q	0.806	0.022	13.278	0.564	0.778	1.233	3857.484	$\sigma_{RE}^{(20m)}$	0.826	0.022	13.147	0.417	0.870	1.304	3844.834
	At 5%	8.22%	16.78%	39.80%	69.74%	75.99%				8.88%	17.11%	36.84%	71.38 %	92.11%		
	At 10%	15.79%	24.34%	49.34 %	81.91%	81.58 %				16.45 %	23.03 %	46.38 %	82.24%	93.42%		
	Med	0.500	-0.005	5.462	0.324	0.696	1.093	3586.830		0.529	-0.005	5.486	0.225	0.786	1.134	3560.852
	L_q	0.245	-0.036	1.329	0.178	0.563	0.914	3161.302		0.262	-0.034	1.984	0.137	0.703	0.976	3137.262
$\sigma_{RE}^{(10m)}$	U_q	0.802	0.024	12.648	0.476	0.820	1.255	3854.975	$\sigma_{RE}^{(30m)}$	0.853	0.023	12.660	0.341	0.873	1.270	3843.761
	At 5%	8.88%	17.11 %	39.14%	68.42%	84.54%				8.22%	17.11 %	35.53%	69.74 %	93.75%		
	At 10%	16.45 %	25.00%	48.03 %	81.91%	88.16 %				15.79 %	22.37%	43.75 %	80.59%	97.04%		
	Med	0.510	-0.004	5.585	0.286	0.739	1.119	3585.219								
	L_q	0.251	-0.034	1.717	0.160	0.615	0.958	3154.368								
$\sigma_{RE}^{(15m)}$	U_q	0.815	0.024	12.461	0.432	0.849	1.260	3853.077								
	At 5%	8.22%	16.78 %	34.87%	72.37 %	89.47%										
	At 10%	15.13 %	24.67%	43.09%	81.58%	91.45%										

Panel D provides the values estimated by the realized volatilities for different time intervals. Numbers in the last two rows for each panel are the percentages of estimates that are significantly different from zero at the 5% and 10% significant levels.

The general specification of ARCH model is as follows:

$r_t = \mu + \varepsilon_t + \theta\varepsilon_{t-1}$, $\varepsilon_t = h_t^{1/2} z_t$, $z_t \sim i.i.d.N(0,1)$, $h_t = \frac{\omega + \alpha\varepsilon_{t-1}^2 + \alpha^- s_{t-1}\varepsilon_{t-1}^2}{1-\beta L} + \frac{\gamma\sigma_{MF,t-1}^2}{1-\beta_\gamma L} + \frac{\delta\sigma_{BS,t-1}^2}{1-\beta_\delta L} + \frac{\lambda\sigma_{RE,t-1}^2}{1-\beta_\lambda L}$, s_t is 1 if ε_t is negative, otherwise it is zero. σ_{MF} and σ_{BS} denote the model-free implied volatility and the Black-Scholes implied volatility and the lagged realized volatility respectively. $\sigma_{RE}^{(5m)}$ denotes the realized volatility measured by five-minute returns, and so are the others.

Table 7 Summary statistics of estimates for univariate regression across 304 firms

α			β_{LRE}			β_{MF}			β_{BS}			Ad. R square			MSE			Durbin-Watson		
Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q
Panel A: Realized volatility is calculated using high-frequency data																				
0.179	0.137	0.243	0.464	0.361	0.576							0.389	0.281	0.487	0.008	0.005	0.017	1.512	1.351	1.716
	(302/303)			(304/304)																
0.029	-0.008	0.074				0.706	0.583	0.793				0.526	0.404	0.620	0.006	0.004	0.013	1.613	1.401	1.803
	(80/104)						(302/303)													
0.019	-0.011	0.057							0.797	0.687	0.896	0.551	0.444	0.640	0.006	0.004	0.011	1.637	1.388	1.834
	(58/79)									(303/303)										
0.034	0.001	0.064	0.206	0.115	0.287	0.525	0.405	0.652				0.582	0.483	0.661	0.005	0.004	0.011	1.797	1.647	1.917
	(74/93)			(201/222)			(299/300)													
0.024	-0.005	0.054	0.191	0.090	0.271				0.613	0.506	0.745	0.595	0.498	0.674	0.005	0.004	0.011	1.806	1.638	1.931
	(58/79)			(183/208)						(298/300)										
0.016	-0.018	0.054				0.154	-0.076	0.396	0.606	0.348	0.885	0.558	0.454	0.640	0.006	0.004	0.011	1.648	1.437	1.837
	(50/75)						(48/75)			(156/186)										
0.022	-0.010	0.051	0.188	0.092	0.267	0.116	-0.115	0.305	0.469	0.230	0.763	0.604	0.507	0.679	0.005	0.004	0.011	1.810	1.655	1.903
	(53/79)			(182/203)			(38/57)			(118/144)										

α			β_{LRE}			β_{MF}			β_{BS}			Ad. R square			MSE			Durbin-Watson		
Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q	Med	L_q	U_q
Panel B: Realized volatility is calculated using daily returns																				
0.193	0.141	0.277	0.501	0.359	0.637							0.283	0.167	0.384	0.016	0.011	0.033	1.735	1.550	1.878
	(296/298)			(296/299)																
0.024	-0.021	0.078				0.777	0.618	0.890				0.382	0.265	0.501	0.013	0.009	0.026	1.797	1.671	1.895
	(46/69)						(301/302)													
0.016	-0.021	0.062							0.864	0.749	0.989	0.388	0.298	0.517	0.013	0.009	0.026	1.816	1.663	1.909
	(30/51)									(303/303)										
0.026	-0.010	0.075	0.192	0.083	0.312	0.589	0.424	0.753				0.420	0.316	0.540	0.013	0.008	0.026	1.822	1.709	1.913
	(44/67)			(127/157)			(275/285)													
0.020	-0.019	0.064	0.176	0.065	0.303				0.700	0.500	0.857	0.424	0.327	0.542	0.012	0.008	0.025	1.836	1.706	1.929
	(37/57)			(120/142)						(276/289)										
0.010	-0.026	0.059				0.234	-0.069	0.511	0.617	0.246	0.998	0.397	0.301	0.532	0.013	0.009	0.026	1.814	1.678	1.907
	(31/49)						(46/64)			(95/126)										
0.015	-0.018	0.060	0.171	0.058	0.298	0.154	-0.124	0.441	0.530	0.151	0.846	0.433	0.326	0.550	0.012	0.008	0.025	1.833	1.695	1.931
	(32/54)			(117/136)			(37/53)			(74/96)										

The table presents summary statistics for the coefficient estimates, adjusted R-square, mean square error and Durbin-Watson statistics, across 304 firms. Mean, Med, L_q, and U_q are the mean, median, lower quartile and upper quartile respectively for each estimates across 304 firms. Numbers in the parentheses are the number of firms whose coefficient estimates are significantly different from zero at the 5% and 10% significant level. The explained variable in Panel A is the monthly non-overlapping realized volatility, $\sigma_{RE}^{(1)}$, which is measured by 30-minute intraday returns. The explained variable of Panel B is monthly non-overlapping realized volatility, $\sigma_{RE}^{(2)}$, measured by daily returns.

The regression model is specified as follow:

$$\sigma_{RE,t,T} = \alpha + \beta_{LRE}\sigma_{LRE,t,T} + \beta_{MF}\sigma_{MF,t,T} + \beta_{BS}\sigma_{BS,t,T} + \varepsilon_{t,T}$$

Table 8 Frequency counts for the variables that best describe volatility of stock returns

	All firms	Group by average available strike number			Group by intermediate Delta		
		Group 1	Group 2	Group 3	Group 1	Group 2	Group 3
	n=304	n=40	n=101	n=163	n=85	n=137	n=82
Panel A : Frequency counts for the ARCH specifications that maximize the likelihoods of observed stock returns							
σ_{LRE} performs best	**53.95%**	**85.00%**	**64.36%**	**39.88%**	**88.24%**	**48.18%**	**28.05%**
$L_{LRE} > L_{MF} > L_{BS}$	15.46%	25.00%	19.80%	10.43%	28.24%	12.41%	7.32%
$L_{LRE} > L_{BS} > L_{MF}$	38.49%	60.00%	44.55%	29.45%	60.00%	35.77%	20.73%
σ_{MF} performs best	**14.80%**	**5.00%**	**12.87%**	**18.40%**	**5.88%**	**15.33%**	**23.17%**
$L_{MF} > L_{LRE} > L_{BS}$	1.97%	0.00%	0.99%	3.07%	0.00%	2.92%	2.44%
$L_{MF} > L_{BS} > L_{LRE}$	12.83%	5.00%	11.88%	15.34%	5.88%	12.41%	20.73%
σ_{BS} performs best	**31.25%**	**10.00%**	**22.77%**	**41.72%**	**5.88%**	**36.50%**	**48.78%**
$L_{BS} > L_{LRE} > L_{MF}$	8.55%	5.00%	6.93%	10.43%	3.53%	9.49%	12.20%
$L_{BS} > L_{MF} > L_{LRE}$	22.70%	5.00%	15.84%	31.29%	2.35%	27.01%	36.59%
Panel B : Frequency counts for the univariate regression model that has the highest adjusted R squared							
σ_{LRE} performs best	**12.50%**	**32.50%**	**17.82%**	**4.29%**	**27.06%**	**10.22%**	**1.22%**
$R^2_{LRE} > R^2_{MF} > R^2_{BS}$	3.95%	15.00%	4.95%	0.61%	9.41%	2.92%	0.00%
$R^2_{LRE} > R^2_{BS} > R^2_{MF}$	8.55%	17.50%	12.87%	3.68%	17.65%	7.30%	1.22%
σ_{MF} performs best	**25.66%**	**20.00%**	**23.76%**	**28.22%**	**24.71%**	**27.01%**	**24.39%**
$R^2_{MF} > R^2_{LRE} > R^2_{BS}$	2.96%	7.50%	3.96%	1.23%	4.71%	3.65%	0.00%
$R^2_{MF} > R^2_{BS} > R^2_{LRE}$	22.70%	12.50%	19.80%	26.99%	20.00%	23.36%	24.39%
σ_{BS} performs best	**61.84%**	**47.50%**	**58.42%**	**67.48%**	**48.24%**	**62.77%**	**74.39%**
$R^2_{BS} > R^2_{LRE} > R^2_{MF}$	3.62%	2.50%	4.95%	3.07%	5.88%	2.19%	3.66%
$R^2_{BS} > R^2_{MF} > R^2_{LRE}$	58.22%	45.00%	53.47%	64.42%	42.35%	60.58%	70.73%

Panel A presents the performance of each variable in ARCH specification model. L_{LRE} , L_{MF} and L_{BS} denote the log-likelihood values of lagged realized volatility, the model-free implied volatility and the Black-Scholes implied volatility respectively. Panel B presents the performance of each variable in univariate regression with explained variable, $\sigma^{(1)}_{RE}$. R^2_{LRE}, R^2_{MF} and R^2_{BS} denote the adjusted R-square of lagged realized volatility, the model-free implied volatility and the Black-Scholes implied volatility respectively. All firms are separated into three group by average strike $\bar{N}_i$: Group 1 is $3 \le \bar{N}_i < 4$; Group 2 is $4 \le \bar{N}_i < 5$; Group 3 is $\bar{N}_i \ge 5$. Alternative, all firms are divided into three group by average the delta values between 0.15 and 0.85, $\bar{D}_i$: Group 1 is $1 \le \bar{D}_i < 2$; Group 2 is $2 \le \bar{D}_i < 3$; Group 3 is $\bar{D}_i \ge 3$.

Appendix

Appendix A. Proof of Proposition 1

Lemma A.1 :

$$E\left[\int_0^T \left(\frac{dS_t^{B^N}}{S_t^{B^N}}\right)^2\right] = 2\sum_{n=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^n}{n!}\int_0^{\infty}\frac{C^{B^n}(T,K)-\max(S_0^{B^n}-K,0)}{K^2}dK$$

where

$$\frac{dS_t^{B^N}}{S_t^{B^N}} = \sigma_t^{B^N} dZ_t$$

$$\left(\sigma_t^{B^N}\right)^2 = \sigma_t^2 + \left(\delta^2 + k^2\right)\frac{N_t}{T}$$

$$\frac{dS_t^{B^n}}{S_t^{B^n}} = \sigma_t^{B^n} dZ_t$$

$$\left(\sigma_t^{B^n}\right)^2 = \sigma_t^2 + \left(\delta^2 + k^2\right)\frac{n}{T}$$

N_t ~Poisson(λ, t)

$C^{B^n}(T,K)$: a set of option prices when given the number of jumps

Proof:

Define

$$\frac{dS_t^{B^N}}{S_t^{B^N}} = \sigma_t^{B^N} dZ_t \tag{A.1}$$

where $\left(\sigma_t^{B^N}\right)^2 = \sigma_t^2 + \left(\delta^2 + k^2\right)\frac{N_t}{T}$.

Then

$$\begin{aligned} E\left[\int_0^T \left(\sigma_t^{B^N}\right)^2 dt\right] &= E\left[E\left[\int_0^T \left(\sigma_t^{B^N}\right)^2 dt \mid N_t = n\right]\right] \\ &= \sum_{n=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^n}{n!}E\left[\int_0^T \left(\sigma_t^{B^n}\right)^2 dt\right] \end{aligned} \tag{A.2}$$

where

$$\frac{dS_t^{B^n}}{S_t^{B^n}} = \sigma_t^{B^n} dZ_t$$

$$\left(\sigma_t^{B^n}\right)^2 = \sigma_t^2 + \left(\delta^2 + k^2\right)\frac{n}{T}$$

From equation (A.1), we get

$$\left(\frac{dS_t^{B^N}}{S_t^{B^N}}\right)^2 = \left(\sigma_t^{B^N}\right)^2 dt \tag{A.3}$$

Then

$$E\left[\int_0^T \left(\frac{dS_t^{B^N}}{S_t^{B^N}}\right)^2\right] = E\left[\int_0^T \left(\sigma_t^{B^N}\right)^2 dt\right] \tag{A.4}$$

By (A.2),

$$E\left[\int_0^T \left(\frac{dS_t^{B^N}}{S_t^{B^N}}\right)^2\right] = \sum_{n=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!} E\left[\int_0^T \left(\sigma_t^{B^n}\right)^2 dt\right] \tag{A.5}$$

We can use the method mentioned in Britten-Jones and Neuberger (2000).

$$E\left[\int_0^T \left(\frac{dS_t^{B^N}}{S_t^{B^N}}\right)^2\right] = 2\sum_{n=0}^{\infty} \frac{e^{-\lambda T}(\lambda T)^n}{n!} \int_0^{\infty} \frac{C^{B^n}(T,K) - \max(S_0^{B^n} - K, 0)}{K^2} dK \tag{A.6}$$

Lemma A.2 : $E\left[\int_0^T \left(\frac{dS_t^{B^N}}{S_t^{B^N}}\right)^2\right] = E\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right]$

Proof : Under jump process,

$$\frac{dS_t}{S_t} = \sigma_t dZ_t + (Y_t - 1)dN_t \tag{A.7}$$

$$\begin{aligned}\frac{dS_t}{S_t} &= \sigma_t^2 (dZ_t)^2 + 2\sigma_t (Y_t - 1) dZ_t dN_t + (Y_t - 1)^2 (dN_t)^2 \\ &\approx \sigma_t^2 dt + (Y_t - 1)^2 \lambda dt\end{aligned} \tag{A.8}$$

Then

$$E\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right] = E\left[\int_0^T \left(\sigma_t^2 + \left(\delta^2 + k^2\right)\lambda\right) dt\right] \tag{A.9}$$

From (A.4),

$$E\left[\int_0^T \left(\frac{dS_t^{B^N}}{S_t^{B^N}}\right)^2\right] = E\left[\int_0^T \left(\sigma_t^{B^N}\right)^2 dt\right]$$

$$= E\left[\int_0^T \left(\sigma_t^2 + \left(\delta^2 + k^2\right)\frac{N_t}{T}\right)dt\right]$$

$$= E\left[\int_0^T \left(\sigma_t^2 + \left(\delta^2 + k^2\right)\lambda\right)dt\right]$$

$$= E\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right]$$

Proof of Proposition 1 :

By Lemma A.1 and Lemma A.2, we can obtain

$$E\left[\int_0^T \left(\frac{dS_t}{S_t}\right)^2\right] = 2\sum_{n=0}^{\infty}\frac{e^{-\lambda T}(\lambda T)^n}{n!}\int_0^{\infty}\frac{C^{B^n}(T,K) - \max(S_0^{B^n} - K,0)}{K^2}dK$$

Appendix B. Proof of Lemma 2.2.1 and Proposition 2

Lemma B.1 : If there is no jump, the dynamics of the asset price can be expressed as

$$\frac{S_t}{S_{t-1}} = e^{-\frac{h_t}{2}+\sqrt{h_t}\,\overline{X}_t^{(0)}} \tag{B.1}$$

Proof : If there is no jump, then the dynamics of pricing kernel is

$$\frac{m_t}{m_{t-1}} = e^{a+bX_t^{(0)}}, \qquad X_t^{(0)} \sim N(0,1)$$

And the asset price process is

$$\frac{S_t}{S_{t-1}} = e^{\alpha_t+\sqrt{h_t}\,\overline{X}_t^{(0)}}, \qquad \overline{X}_t^{(0)} \sim N(0,1)$$

From equation (12),

$$E^P\left[\frac{m_t}{m_{t-1}} \mid F_{t-1}\right] = E^P\left[e^{a+bX_t^{(0)}} \mid F_{t-1}\right]$$

$$= e^{a+\frac{b^2}{2}}$$

$$= 1$$

Then $a + \dfrac{b^2}{2} = 0$.

And

$$E^P\left[\frac{m_t}{m_{t-1}}\frac{S_t}{S_{t-1}} \mid F_{t-1}\right] = E^P\left[e^{a+\alpha_t+bX_t^{(0)}+\sqrt{h_t}\,\overline{X_t^{(0)}}} \mid F_{t-1}\right]$$

$$= e^{a+\alpha_t+\frac{b^2+\sqrt{h_t}}{2}}$$

$$= 1$$

Thus $a + \alpha_t + \dfrac{b^2 + \sqrt{h_t}}{2} = 0$.

Then we have

$$\alpha_t = -\frac{h_t}{2}$$

Proof of Lemma2.2.1 : If there is no jump, the NGARCH(1,1) model is

$$\ln\frac{S_t^G}{S_{t-1}^G} = -\frac{h_t^G}{2} + \sqrt{h_t^G}\,\overline{X}_t^{(0)} \qquad \text{(B.2)}$$

where

$$h_t^G = \beta_0 + \beta_1 h_{t-1}^G + \beta_2^* h_{t-1}^G(\overline{X}_t^{(0)} - c^*)^2 \qquad \text{(B.3)}$$

$$\overline{X}_t^{(0)} \sim N(0,1)$$

Then

$$\ln S_{i\Delta t}^G - \ln S_{(i-1)\Delta t}^G = -\frac{h_{i\Delta t}^G}{2}\Delta t + \sqrt{h_t^G}\,\overline{X}_t^{(0)}\sqrt{\Delta t}$$

First, we have the following results for the conditional mean return

$$\frac{\frac{h_{i\Delta t}^G}{2}\Delta t}{\Delta t} \to \frac{h_t^G}{2}$$

Next, by the Donsker's Theorem, $\sum_{i=1}^{[nt/T]} \overline{X}_{\Delta t}^{(0)}\Delta t$ converges weakly to the standard

Brownian motion Z_t. Applying Theorem 5.4 of Kurtz and Protter (1991) yield a weak converge to (S_t^G, h_t^G). Thus, the limiting model under measure Q is

$$d\ln S_t^G = -\frac{h_t^G}{2} + \sqrt{h_t^G}\, dZ_t \tag{B.4}$$

We assume that

$$\frac{dS_t^G}{S_t^G} = a + bdZ_t$$

We can calculate $d\ln S_t^G$ by Ito lemma,

$$\begin{aligned} d\ln S_t^G &= \frac{\partial \ln S_t^G}{dS_t^G} + \frac{1}{2}\frac{\partial^2 \ln S_t^G}{(dS_t^G)^2} \\ &= (a - \frac{1}{2}b^2)dt - \frac{1}{2}(adt + bdZ_t)^2 \\ &= (a - \frac{1}{2}b^2)dt + bdZ_t \end{aligned} \tag{B.5}$$

Compared to equation (B.4), we can find that $b = \sqrt{h_t^G}$ and $a = 0$. Then there have

$$\frac{dS_t^G}{S_t^G} = \sqrt{h_t^G}\, dZ_t \tag{B.6}$$

Lemma B.2 : Under NGARCH(1,1)-Normal framework, the implied volatility is as follow

$$E\left[\int_0^T \left(\frac{dS_t^G}{S_t^G}\right)^2\right] = 2\int_0^\infty \frac{C^G(T,K) - \max(S_0^G - K, 0)}{K^2} dK \tag{B.7}$$

where $C^G(T,K)$ be a set of option prices under NGARCH(1,1)-Normal model.

Proof : If the stock prices follow NGARCH(1,1)-Normal model and $r = 0$, then

$$\frac{dS_t^G}{S_t^G} = \sqrt{h_t^G}\, dZ_t \tag{B.8}$$

Thus

$$d\ln S_t^G = -\frac{h_t^G}{2} + \sqrt{h_t^G}\, dZ_t$$

And

$$E\left[\int_0^T d\ln S_t^G\right] = E\left[-\frac{1}{2}\int_0^T h_t^G dt\right]$$

$$E\left[\ln S_T^G - \ln S_0^G\right] = -\frac{1}{2} E\left[\int_0^T h_t^G dt\right]$$

From (B.8),

$$\left(\frac{dS_t^G}{S_t^G}\right)^2 = h_t^G dt \tag{B.9}$$

Hence

$$\begin{aligned} E\left[\int_0^T \left(\frac{dS_t^G}{S_t^G}\right)^2\right] &= E\left[\int_0^T h_t^G dt\right] \\ &= 2E\left[\ln S_0^G - \ln S_T^G\right] \\ &= 2\int_0^\infty \frac{C^G(T,K) - \max(S_0^G - K, 0)}{K^2} dK \end{aligned} \tag{B.10}$$

Lemma B.3 : If $h_0^G = h_0$, then $E\left[h_0^G\right] = E\left[h_0\right]$

Proof : Chang(   ), p.33.

Proof of Proposition 2 : By lemma B.3,

$$\begin{aligned} E\left[\int_0^T h_t dt\right] &= E\left[\int_0^T h_t^G dt\right] \\ &= E\left[\int_0^T \left(\frac{dS_t^G}{S_t^G}\right)^2\right] \\ &= 2\int_0^\infty \frac{C^G(T,K) - \max(S_0^G - K, 0)}{K^2} dK \end{aligned} \tag{B.11}$$

Appendix C. Proof of equation (19)

Britten-Jones and Neuberger (2000) derived the model-free implied volatility as integral spreads of options with a risk-neutral underlying, for different time to maturity:

$$E_0^Q\left[\int_{t_1}^{t_2} \left(\frac{dS_t}{S_t}\right)^2\right] = 2\int_0^\infty \frac{C(K,t_2) - C(K,t_1)}{K^2} dK\ . \tag{C.1}$$

The no-arbitrage argument implies that exists a forward measure F, so Jiang and Tian (2005) considered the forward asset:

$$E_0^F\left[\int_0^T\left(\frac{dF_t}{F_t}\right)^2\right]=2\int_0^\infty\frac{C^F\left(K,T\right)-\max\left(0,F_0-K\right)}{K^2}dK\,. \tag{C.2}$$

Set forward price at time t as $F_t=S_t/B\left(t,T\right)$ and forward option price at time t as $C^F\left(K,T\right)=C\left(K,T\right)/B\left(t,T\right)$. S_t, here is the asset price eliminates the present values of all future dividends paid prior to the option maturity. Thus, equation (C.2) can be replaced as:

$$\begin{aligned}
E_0^F\left[\int_0^T\left(\frac{dS_t}{S_t}\right)^2\right]&=2\int_0^\infty\frac{C\left(K,T\right)/B\left(0,T\right)-\max\left(0,S_0/B\left(0,T\right)-K\right)}{K^2}dK\\
&=\frac{2}{B\left(0,T\right)}\left[\begin{array}{l}\int_0^{S_0/B(0,T)}\frac{C\left(K,T\right)-\max\left(0,S_0-K\times B\left(0,T\right)\right)}{K^2}dK\\+\int_{S_0/B(0,T)}^\infty\frac{C\left(K,T\right)-\max\left(0,S_0-K\times B\left(0,T\right)\right)}{K^2}dK\end{array}\right]\\
&=\frac{2}{B\left(0,T\right)}\left[\int_0^{S_0/B(0,T)}\frac{C\left(K,T\right)-\left(S_0-K\times B\left(0,T\right)\right)}{K^2}dK+\int_{S_0/B(0,T)}^\infty\frac{C\left(K,T\right)}{K^2}dK\right]\\
&=\frac{2}{B\left(0,T\right)}\left[\int_0^{S_0/B(0,T)}\frac{P\left(K,T\right)}{K^2}dK+\int_{S_0/B(0,T)}^\infty\frac{C\left(K,T\right)}{K^2}dK\right]\\
&=\frac{2}{B\left(0,T\right)}\left[\int_0^{F_{0,T}}\frac{P\left(K,T\right)}{K^2}dK+\int_{F_{0,T}}^\infty\frac{C\left(K,T\right)}{K^2}dK\right].
\end{aligned}$$

The proof above uses the put-call parity $C\left(K,T\right)+K\times B\left(0,T\right)=P\left(K,T\right)+S_0$. Finally, assume the bond price as $B\left(0,T\right)=e^{-rT}$, equation (1) has been proved.

Appendix B. Performances under variance and logarithm regression models

Table A.1

	All firms	Group by average available strike number			Group by intermediate Delta		
		Group 1	Group 2	Group 3	Group 1	Group 2	Group 3
	n=304	n=40	n=101	n=163	n=85	n=137	n=82
Panel A : Frequency counts for the univariate regression model that has the highest adjusted R squared for replacing volatility variables by their logarithms							
$\log(\sigma_{LRE})$ **performs best**	**11.51%**	**42.50%**	**14.85%**	**1.84%**	**28.24%**	**8.03%**	**0.00%**
$R^2_{LRE} > R^2_{BS} > R^2_{MF}$	6.91%	20.00%	10.89%	1.23%	16.47%	5.11%	0.00%
$\log(\sigma_{MF})$ **performs best**	**24.67%**	**17.50%**	**22.77%**	**27.61%**	**22.35%**	**25.55%**	**25.61%**
$R^2_{MF} > R^2_{BS} > R^2_{LRE}$	22.70%	17.50%	18.81%	26.38%	21.18%	21.90%	25.61%
$\log(\sigma_{BS})$ **performs best**	**63.82%**	**40.00%**	**62.38%**	**70.55%**	**49.41%**	**66.42%**	**74.39%**
$R^2_{BS} > R^2_{MF} > R^2_{LRE}$	59.21%	30.00%	58.42%	66.87%	44.71%	61.31%	70.73%
Panel B : Frequency counts for the univariate regression model that has the highest adjusted R squared for replacing volatility by their variances							
σ_{LRE} **performs best**	**14.47%**	**30.00%**	**20.79%**	**6.75%**	**29.41%**	**11.68%**	**3.66%**
$R^2_{LRE} > R^2_{BS} > R^2_{MF}$	9.54%	17.50%	14.85%	4.29%	18.82%	8.03%	2.44%
σ_{MF} **performs best**	**26.32%**	**27.50%**	**25.74%**	**26.38%**	**27.06%**	**27.74%**	**23.17%**
$R^2_{MF} > R^2_{BS} > R^2_{LRE}$	22.70%	17.50%	20.79%	25.15%	21.18%	23.36%	23.17%
σ_{BS} **performs best**	**59.21%**	**42.50%**	**53.47%**	**66.87%**	**43.53%**	**60.58%**	**73.17%**
$R^2_{BS} > R^2_{MF} > R^2_{LRE}$	53.62%	37.50%	45.54%	62.58%	35.29%	56.20%	68.29%

Panel A presents the performance of each univariate regression mode as follows:

$$\log(\sigma_{RE,t,T}) = \alpha + \beta_{LRE} \log(\sigma_{LRE,t,T}) + \varepsilon_{t,T},$$
$$\log(\sigma_{RE,t,T}) = \alpha + \beta_{MF} \log(\sigma_{MF,t,T}) + \varepsilon_{t,T},$$
$$\log(\sigma_{RE,t,T}) = \alpha + \beta_{BS} \log(\sigma_{BS,t,T}) + \varepsilon_{t,T}.$$

Panel B presents the performance of each univariate regression mode as follows:

$$\sigma^2_{RE,t,T} = \alpha + \beta_{LRE}\sigma^2_{LRE,t,T} + \varepsilon_{t,T},$$
$$\sigma^2_{RE,t,T} = \alpha + \beta_{MF}\sigma^2_{MF,t,T} + \varepsilon_{t,T},$$
$$\sigma^2_{RE,t,T} = \alpha + \beta_{BS}\sigma^2_{BS,t,T} + \varepsilon_{t,T}.$$

All firms are separated into three group by average strike $\bar{N}_i$: Group 1 is $3 \le \bar{N}_i < 4$; Group 2 is $4 \le \bar{N}_i < 5$; Group 3 is $\bar{N}_i \ge 5$. Alternative, all firms are divided into three group by average the delta values between 0.15 and 0.85, $\bar{D}_i$: Group 1 is $1 \le \bar{D}_i < 2$; Group 2 is $2 \le \bar{D}_i < 3$; Group 3 is $\bar{D}_i \ge 3$.

Appendix C. Performance of different curve fitting methods

Table A.2

	All firms	Group by average available strike number			Group by intermediate Delta		
		Group 1	Group 2	Group 3	Group 1	Group 2	Group 3
	n=304	n=40	n=101	n=163	n=85	n=137	n=82
Panel A : Frequency counts for the ARCH specifications that maximize the likelihoods of observed stock returns							
$L_{noarb} > L_{quadratic}$	89.14%	92.50%	88.12%	88.96%	84.71%	92.70%	87.80%
$L_{quadratic} > L_{noarb}$	10.86%	7.50%	11.88%	11.04%	15.29%	7.30%	12.20%
$L_{noarb} > L_{cubic}$	89.80%	90.00%	87.13%	91.41%	89.41%	89.78%	90.24%
$L_{cubic} > L_{noarb}$	10.20%	10.00%	12.87%	8.59%	10.59%	10.22%	9.76%
$L_{quadratic} > L_{cubic}$	78.29%	70.00%	72.28%	84.05%	76.47%	78.10%	80.49%
$L_{cubic} > L_{quadratic}$	21.71%	30.00%	27.72%	15.95%	23.53%	21.90%	19.51%
Panel B : Frequency counts for the univariate regression model that has the highest adjusted R squared							
$R^2_{noarb} > R^2_{quadratic}$	80.59%	92.50%	83.17%	76.07%	91.76%	75.18%	78.05%
$R^2_{quadratic} > R^2_{noarb}$	19.41%	7.50%	16.83%	23.93%	8.24%	24.82%	21.95%
$R^2_{noarb} > R^2_{cubic}$	78.29%	72.50%	75.25%	81.60%	82.35%	77.37%	75.61%
$R^2_{cubic} > R^2_{noarb}$	21.71%	27.50%	24.75%	18.40%	17.65%	22.63%	24.39%
$R^2_{quadratic} > R^2_{cubic}$	59.54%	37.50%	49.50%	71.17%	48.24%	62.04%	67.07%
$R^2_{cubic} > R^2_{quadratic}$	40.46%	62.50%	50.50%	28.83%	51.76%	37.96%	32.93%

Here, we discuss three curve fitting method and compare their performance in Table A.2 and Table A.3. The first method used in this paper is the quadratic function fitting under no-arbitrage condition. The second method used in Taylor, Yadav and Zhang (2006) is quadratic function fitting doesn't condition on no-arbitrage. The third method is the cubic splines, which Jiang and Tian (2005) used. We compare their performance in pair in Table A.2.

In Panel A, we can clearly find our method, L_{noarb}, outperform both $L_{quadratic}$ (quadratic function fitting) and L_{cubic} (cubic splines fitting). There are obvious tendency under different group only between quadratic function method and cubic splines method. When the observations increase, the forecasting ability of quadratic function method also increases. This result indicates the quadratic function method is a not fitting method for the limited observations. The result is similar in Panel B.

Table A.3

	All firms	Group by average available strike number			Group by intermediate Delta		
		Group1	Group2	Group3	Group1	Group2	Group3
	n=304	n=40	n=101	n=163	n=85	n=137	n=82
Panel A : Frequency counts for the univariate regression model that has the highest adjusted R squared for replacing volatility variables by their logarithms							
σ_{noabr} **performs best**	**80.92%**	**82.50%**	**78.22%**	**82.21%**	**77.65%**	**83.21%**	**80.49%**
$L_{nobar} > L_{quadratic} > L_{cubic}$	68.42%	62.50%	62.38%	73.62%	63.53%	70.80%	69.51%
$\sigma_{quadratic}$ **performs best**	**9.87%**	**7.50%**	**9.90%**	**10.43%**	**12.94%**	**7.30%**	**10.98%**
$L_{quadratic} > L_{nobar} > L_{cubic}$	8.88%	7.50%	8.91%	9.20%	11.76%	6.57%	9.76%
σ_{cubic} **performs best**	**9.21%**	**10.00%**	**11.88%**	**7.36%**	**9.41%**	**9.49%**	**8.54%**
$L_{cubic} > L_{nobar} > L_{quadratic}$	8.22%	10.00%	9.90%	6.75%	7.06%	9.49%	7.32%
Panel B : Frequency counts for the univariate regression model that has the highest adjusted R squared for replacing volatility by their variances							
σ_{noabr} **performs best**	**62.83%**	**67.50%**	**63.37%**	**61.35%**	**75.29%**	**57.66%**	**58.54%**
$R^2_{nobar} > R^2_{quadratic} > R^2_{cubic}$	43.42%	32.50%	36.63%	50.31%	41.18%	41.61%	48.78%
$\sigma_{quadratic}$ **performs best**	**16.12%**	**5.00%**	**12.87%**	**20.86%**	**7.06%**	**20.44%**	**18.29%**
$R^2_{quadratic} > R^2_{nobar} > R^2_{cubic}$	15.46%	5.00%	11.88%	20.25%	7.06%	19.71%	17.07%
σ_{cubic} **performs best**	**21.05%**	**27.50%**	**23.76%**	**17.79%**	**17.65%**	**21.90%**	**23.17%**
$R^2_{cubic} > R^2_{nobar} > R^2_{quadratic}$	17.76%	25.00%	19.80%	14.72%	16.47%	17.52%	19.51%

Table A.3 presents the performance across these three methods. Although our method has the best performances whether Panel A or Panel B, there are still about forty percent of firms have a better fit for other methods in Panel B. In this paper, we diminish curve fitting error as far as possible by using the quadratic function fitting under no-arbitrage condition.