Methodology and Implementation of Value-at-Risk Measures in Emerging Fixed-Income Markets with Infrequent Trading.

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Abstract

This paper deals with the issue of calculating daily Value-at-Risk (*VaR*) measures within an environment of thin trading. Our approach focuses on fixed income portfolios with low frequency of transactions in which the missing data problem makes *VaR* measures difficult to calculate. We propose and implement a methodology to calculate *VaR* measures with an incomplete panel of prices. The methodology is composed of three phases: Phase I, generates a complete panel of prices, using a term-structure dynamic model of interest rates. Phase II, calculates portfolio *VaR* measures with several alternative methods using the complete panel data generated in phase I. Phase III, shows how to back-test the *VaR* measures obtained in phase II using the original incomplete panel of prices. We provide an empirical implementation of the methodology for the Chilean fixed income market. The proposed methodology seems to provide reliable *VaR* measures for thinly traded markets addressing an important issue for financial risk management in emerging markets.

JEL: C51, C52, G11, G15

Keywords: Risk, Value-at-Risk, Fixed Income, Incomplete Panels, Term-Structure Dynamic Models, Extreme Value, GARCH, Kalman Filter.

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I Introduction

One important concern of financial institutions is measuring market risks. Moreover, regulatory agencies are requiring them to periodically report risk exposures in order to set up the required capital levels.[1] One of the risk measurement procedures which is becoming a the-facto international standard is the Value-at-Risk (*VaR*), initially proposed to measure only market risks, but later used also for credit and operational risk.

The *VaR* uses econometric techniques to measure the probable loss in value of an investment, within a time interval, under normal market conditions and for a given confidence level[2]. The risk is expressed in money units, which is a simple and easy to understand metric.

In many emerging markets there is an added difficulty for calculating this measure because of the missing data problem associated with thin trading. In these markets assets do not trade every day, thus price panels are incomplete and *VaR* calculations using the traditional methodologies are impossible to perform.

Previous research in this issue is scarce, including Chernobai, Menn, Trück y Rachev (2005) who addressed the problem of incomplete data, but only with an approach towards operational *VaR*. A similar approach is provided by Moscadelli, Chernobai and Rachev (2005). However, to our knowledge, there is no literature related with the evaluation of market risks in thinly traded fixed income markets, which is the focus of this paper.

In this paper we address the issue of how to compute and back-test a *VaR* market risk measurement in a thinly traded fixed income market. We also provide an empirical implementation of the proposed methodology for the Chilean fixed income market. The proposed methodology seems to provide reliable *VaR* measures for thinly traded markets addressing an important issue for financial risk management in emerging markets.

The paper is organized as follows. The next Chapter briefly explains the *VaR* concept. Chapter III outlines the proposed methodology. Chapter IV discusses the econometric approach. Chapter V presents the data. Chapter VI reports empirical results and their interpretation. Finally, chapter VII concludes.

II The Concept of Value-at-Risk.

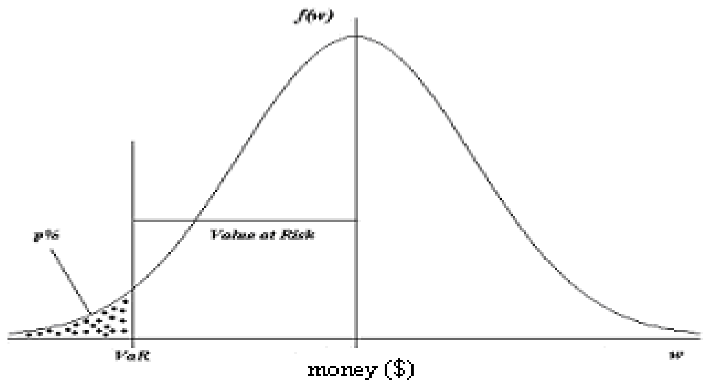
Value-at-Risk is a measure used to estimate how much the value of an asset could decrease over a certain time period for a given confidence level.

Let $w_{t+\Delta t,t}$ be the variation in value of an investment resulting from a price variation in time interval Δt, and $f(w_{t+\Delta t,t})$ the distribution function of the value variations for the investment (which is not necessarily known). The *VaR* of an asset (or portfolio of assets), is the quantity of money that could be lost from negative events which occurs with probability '*p*' or more (Figure 2.1).

[1] See Jackson, Maude and Perraudin (1997).

[2] A $VaR_{5\%}$= \$-100,000 on an investment is equivalent to saying that it is expected that 5% of the times there will be a loss of \$100,000 or more.

Figure 2.1: Distribution Function of the Variation in Value of an Investment and the *VaR* concept.



The *VaR* can be computed as:

$$P(w_{t+\Delta t,t} \leq VaR_{t+\Delta t,t}) = p \tag{1}$$

then,

$$\int_{-\infty}^{VaR_{t+\Delta t,t}} f(w_{t+\Delta t,t})dw = p \tag{2}$$

To compute the *VaR* we could follow different approaches. The first one, known as *parametric*, adjusts the historical returns of an investment to a known distribution, i.e. a Normal. Once the parameters of the assumed distribution are estimated the *VaR* can be computed using the assumed distribution.

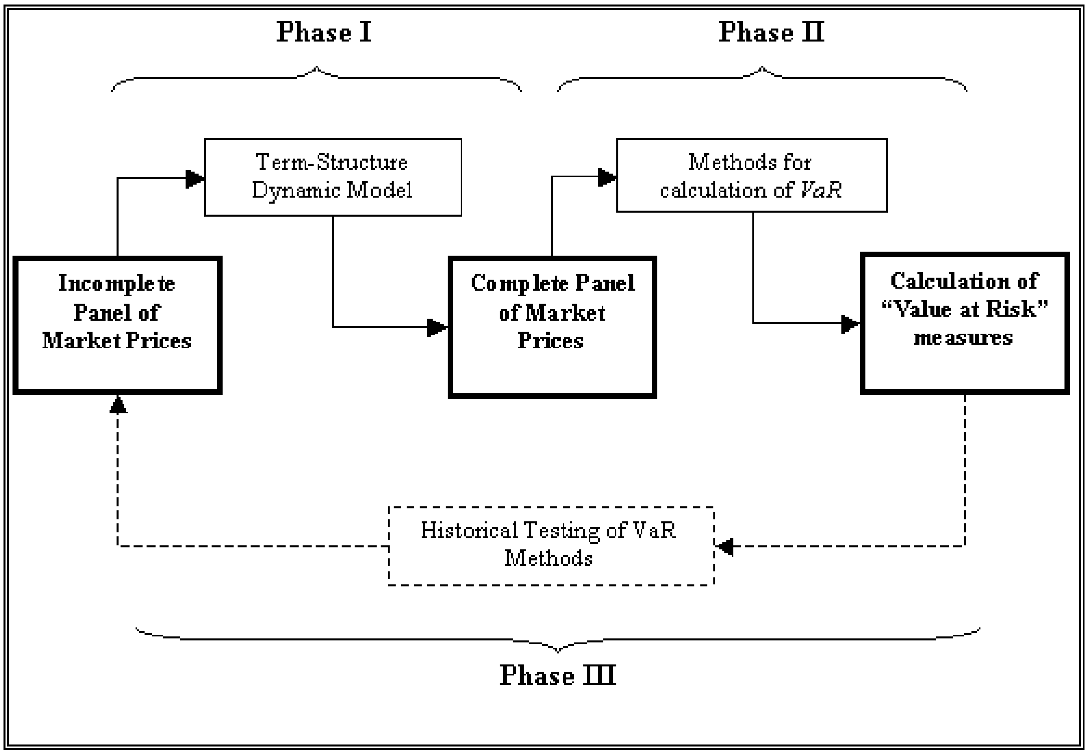
The second one is the historical simulation which is *non-parametric*, and does not assume any distribution of returns, thus no parameter estimations are necessary.

Finally, the Monte Carlo simulation uses elements of both of the preceding approaches. First, a stochastic process is estimated using historical data. Then the estimated process is simulated and the *VaR* is computed.

III The Methodology.

The following proposed methodology is one of the first attempts in the literature to calculate *VaR* measures for a thinly traded market. The methodology is composed of three phases. The first two phases are focused on calculating the *VaR* measures, and the last phase proposes a back-testing procedure to check the reliability of the proposed methodology (Figure 3.1).

Figure 3.1: Scheme of the Three-Phase methodology to deal with the problem of calculating and back-testing daily *VaR* measures in thinly traded fixed-income markets



3.1 Phase I (Generation of a Complete Panel Data of Prices).

In the case of a thinly traded market, we propose first to generate a complete data panel which later will be used to compute the *VaR* measures. Previous research in emerging fixed income markets (Cortazar, Schwartz and Naranjo, 2007) has shown that dynamic term-structure models are much better than static models (Nelson y Siegel, 1987, Svensson, 1994) for computing missing prices in thinly traded markets. This should be particularly true when we are concerned with obtaining reliable volatility estimates, which is the case when our goal is to compute *VaR* estimates.

We propose choosing a multifactor dynamic term-structure model and then calibrating it using the incomplete panel of market prices. It must be noted that Kalman Filter estimation procedures may be used with incomplete panel data and consistent volatility term structure estimates are obtained (Cortazar, Schwartz and Naranjo, 2007). Once the model is estimated we can then compute discount factors for all maturities and construct a complete panel of "model" prices, which we will consider the "fair" prices.

Then, after we have the complete panel, we have two options in order to calculate the *VaR* measures:

- Take the available prices for the instruments when they were traded and “fill” the holes for the days in which the instruments were not traded with the calculated “fair” prices. This panel would be a mixed one, with actual observed prices for days in which trade was observed, and calculated “fair” prices for days in which trade was absent.
- Or, alternatively, use a panel which includes only “fair” calculated prices.

We will later argue why in our in our implementation it is better to use the second panel.

3.2 Phase II (Estimation of “Value at Risk” measures).

Once we have a complete panel of prices, we are able to calculate the *VaR* measures for each asset individually, and for a portfolio of assets as a whole. In this phase, we will calculate daily *VaR* measures using different methods proposed in the literature. We outline the methods used in chapter IV.

3.3 Phase III (Back-Testing with Incomplete Panel Data)

Once we compute the *VaR* measures, we back-test them using historic data. We are interested in two issues. First, we want to find the coherence of the *VaR* measures obtained with the proposed methodology[3]. Second, we pretend to find which *VaR* calculation method provides a better measurement of market risk for the tested portfolio. The specific tests used are detailed in chapter IV.

Again, we have several options on how to perform our back-test considering market and/or calculated “fair” prices. We will later argue why we should combine both panel of prices, using all existing market prices, but updating some of them resorting to “calculated” returns in order to back-test *VaR* calculations when there are missing observations.

IV The Econometric Approach.

In order to implement our methodology several econometric approaches must be made at each procedure phase. First, we must choose a dynamic term-structure model and an estimation method to generate the complete panel of “fair” bond prices. Second, we need to choose the *VaR* estimation method to co the compute the daily *VaR* measures from the “fair” panel of prices. Finally, it is necessary to choose the type of back-testing technique used to asses the reliability of the *VaR* estimations.

We present in this section the econometric techniques that will be used in this study. However, the proposed methodology is open and could be implemented with many other econometric techniques.

4.1 Phase I: Generation of a Complete Panel Data of Prices with a Dynamic Term-Structure Model.

In order to estimate the yield curve needed to generate a complete panel data of “fair” prices, we choose the approach proposed by Cortazar, Schwartz and Naranjo (2007) to jointly estimate the current term structure and its dynamics for markets under infrequent trading. They use a three-factor generalized-

[3] A coherent measure expects that the percentage of times in which losses exceeds the calculated *VaR*, does not exceed the confidence level ‘*p*%’ under which these measures were calculated.

Vasicek model[4] and estimate the model using the Kalman filter with missing data. Using this model they are able to obtain an estimate of the current term structure even for days with an arbitrary low number of price observations. In what follows we provide a brief description of their approach.

First, three stochastic unobservable mean-reverting state variables, represented with the 3x1 vector $\mathbf{x}_t$, are defined. Let δ be a constant. Then, the instantaneous interest rate, $\breve{r}_t$, may be defined as:

$$\breve{r}_t = \mathbf{1}'\mathbf{x}_t + \delta \tag{3}$$

Let the vector of state variables $\mathbf{x}_t$, follow a multifactor Vasicek-type process, governed by the following stochastic differential equation:

$$d\mathbf{x}_t = -\mathbf{K}\mathbf{x}_t dt + \mathbf{\Sigma} d\mathbf{w}_t \tag{4}$$

where $\mathbf{K}$=diag(κ_i) and $\mathbf{\Sigma}$=diag(σ_i) are 3x3 diagonal matrices with entries that are strictly positives constants and different. Also, $d\mathbf{w}_t$ is a 3x1 vector of correlated Brownian motion increments such that:

$$(d\mathbf{w}_t)'(d\mathbf{w}_t) = \mathbf{\Omega} dt \tag{5}$$

where the (i,j) element of $\mathbf{\Omega}$ is $\rho_{ij} \in [1,-1]$, the instantaneous correlation of the state variables i and j. Under this specification, the state variables have the multivariate normal distribution and each of them reverts to 0, at a mean reversion rate given by k_i. Thus, according to equation (3) the instantaneous interest rate, $\breve{r}_t$, reverts to a long-run mean given by the constant δ.

Cortazar, Schwartz and Naranjo (2007) show that assuming a constant 3x1 vector of market price of risk, $\boldsymbol{\lambda}$, the price of any pure-discount bond is:

$$P(\mathbf{x}_t, \tau) = \exp\left(\mathbf{u}(\tau)'\mathbf{x}_t + v(\tau)\right) \tag{6}$$

where

$$u_i(\tau) = -\frac{1-\exp(-k_i\tau)}{k_i} \tag{7}$$

$$\begin{aligned} v(\tau) = {} & \sum_{i=1}^{N} \frac{\lambda_i}{k_i}\left(\tau - \frac{1-\exp(-k_i\tau)}{k_i}\right) - \delta\cdot\tau \\ & + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\sigma_i\sigma_j\rho_{ij}}{k_i k_j}\left(\tau - \frac{1-\exp(-k_i\tau)}{k_i} - \frac{1-\exp(-k_j\tau)}{k_j} + \frac{1-\exp(-(k_i+k_j)\tau)}{k_i+k_j}\right) \end{aligned} \tag{8}$$

and the equivalent annualized spot rate, is:

[4] A generalized Vasicek model is a dynamic multifactor mean-reverting Gaussian model of the instantaneous spot interest rate which extends the classic Vasicek (1977). In Vasicek (1977) the interest rate follows an Ornstein-Uhlembeck process and therefore is assumed to revert to a long-run mean.

$$\breve{R}(x_{t,}\tau) = -\frac{1}{\tau}\left(u(\tau)' x_{t,} + v(\tau)\right) \tag{9}$$

which is a linear function of the state variables. Therefore, under the generalized Vasicek model, spot rates also have the Gaussian distribution.

Then, they propose to estimate the chosen dynamic model of interest rates using the Kalman filter, a methodology which recursively calculates optimal estimates of the unobservable state variables contained in vector $\mathbf{x}_t$, given all the information available up to some moment in time. In addition, by using Maximum Likelihood methods, consistent estimates of model parameters may be obtained.

The measurement equation, relating the vector of observable variables $\mathbf{z}_t$ with the vector of un observable state variables $\mathbf{x}_t$, is:

$$\mathbf{z}_t = \mathbf{H}_t \mathbf{x}_t + \mathbf{d}_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim N(\mathbf{0}, \mathbf{R}_t) \tag{10}$$

We must recall that the standard Kalman filter assumes a fixed number of observable variables at each time. However, Cortazar, Schwartz and Naranjo (2007) relax this assumption in order to allow for missing observations. Let m_t be the number of observations available at time *t*, $\mathbf{z}_t$ is a $m_t x1$ vector, $\mathbf{H}_t$ is a $m_t x3$ matrix, $\mathbf{x}_t$ is a 3x1 vector, $\mathbf{d}_t$ is a $m_t x1$ vector, and $\mathbf{v}_t$ is a $m_t x1$ vector of serially uncorrelated Gaussian disturbances with mean **0** and covariance matrix $\mathbf{R}_t$ with dimensions m_t x m_t.

The transition equation, which describes the dynamics of the state variables, may be written as:

$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{c}_t + \boldsymbol{\varepsilon}_t \qquad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \mathbf{Q}_t) \tag{11}$$

where $\mathbf{A}_t$ is a *3x3* matrix, $\mathbf{c}_t$ is a *3x1* vector, and $\boldsymbol{\varepsilon}_t$ is a *3x1* vector of uncorrelated Gaussian disturbances with mean **0** and covariance matrix $\mathbf{Q}_t$.

Once the state-space representation, defined by Equations (10) and (11) is obtained, an extended Kalman filter, which accounts for the missing data and nonlinearities arising from the use of coupon bonds, is applied to calibrate the model.

The calibrated model provides estimates of interest rates for all maturities for each day. Therefore, using this estimated yield curve, "fair" prices for every day *t* and all instruments in the portfolio may be computed.

4.2 Phase II: Estimation of "Value at Risk" measures using alternative Methods.

In order to calculate the daily *VaR* measures, we will compare the results of using each of the following methods:[5]

[5] Here, we only mention the different methods of *VaR* calculation. However, refer to Annex A for a detailed explanation of each method.

- Parametric methods in a world of multivariate Normal distributions (methods of the variance-covariance matrix):
  - Method of the sample variance and covariance, with a window of 250 days for the estimations.
  - Method of exponential decay, in the Risk-Metrics version, with a window of 250 days for the estimations.
  - GARCH(1,1) using a variance-covariance matrix decomposition, with a window of 250 days for the estimations.
- Parametric methods accounting for asymmetric and multi-kurtosis effects:
  - Use of t-student distribution, with a window of 250 days for the estimations.
  - "Extreme Value Theory", in a static version, with a window of 400 days for the estimations.
  - "Extreme Value Theory", in a dynamic version, with a window of 400 days for the estimations.
- Non parametric method of historical simulation, with a window of 250 days for the estimations
- Monte Carlo simulation method of a three-factor Vasicek model of interest rates (Cortazar, Schwartz and Naranjo (2007).

4.3 Phase III: Back-Testing with an Incomplete Panel.

In this section we show how to back-test the VaR measures obtained in Phase-II, when we have an incomplete panel of data. We start comparing alternative back-testing methods to later propose a procedure that makes full use of all available data.

4.3.1 The problem

Let's assume that an amount M is invested in a single asset at day *'t'*, and that during the following *'d'* days the asset is not traded. Thus, there are two consecutive prices for the asset at times *'t'* and *'t+d+1'*. The unresolved issue is how to use this multi-day price return to compute en empirical estimate of the confidence level 'p' under which a daily *VaR* measure was calculated.

One way of handling the incomplete panel data would be to back-test the estimated risk measures using a reduced sub-sample which includes only successive trading days. The problem with this approach is that if trading is thin this sub-sample could be very small and discarding available data.

Another back-testing procedure, which has the advantage of using all data transactions, is to compare each pair of consecutive prices with multi-day *VaR* measures assuming a process for asset returns like the following:

$$G = \ln(P) \tag{12}$$

$$dG = (u - \frac{\sigma^2}{2})dt + \sigma dz$$

$$dz = \zeta \sqrt{dt} \qquad \zeta : N(0,1)$$

(13)

(14)

where u y σ^2 are the mean and variance of returns, P is the price of the asset, t is the time, dz is a Brownian Motion, and ζ is distributed $N(0,1)$. Thus, the gain or loss in one day of investing in the asset ($w_{t+1,t}$) could be computed as:

$$w_{t+1,t} = \left(e^{(u-\frac{\sigma^2}{2})+\sigma\zeta} - 1 \right) M \tag{16}$$

To estimate *VaR*, the amount of money such that the probability of a loss exceeding that amount would be p, the inverse of the normal distribution *N(0,),* for a probability p, denoted as α, can be computed. Then:

$$VaR_{t+1,t} = \left(e^{(u-\frac{\sigma^2}{2})+\sigma\alpha} - 1 \right) M \tag{17}$$

Following this procedure for any Δt (not only one day) the *VaR,* for different time periods, can be computed as:

$$VaR_{t+\Delta t,t} = \left(e^{(u-\frac{\sigma^2}{2})\Delta t+\sigma\alpha\sqrt{\Delta t}} - 1 \right) M \tag{18}$$

This procedure presents, however, some drawbacks. First, empirical evidence suggests (Mandelbrot, 1963, and Fama, 1965) that daily logarithmic returns do not follow a normal distribution. Furthermore, recent studies (Dacorogna et.al.,1999) show that returns calculated over longer periods of time, depart even more heavily from normality. Therefore, this approach looses reliability as missing data increases.

Moreover, it seems somehow contradictory to use the normality assumption to handle missing data problems while methods like "Extreme Value" are later proposed on the grounds that this assumption is not consistent with the data.

4.3.2 The proposed back-testing procedure.

The proposed procedure does not discard any market data nor requires estimating multi-day VaR measures which are difficult to compute.

The basic idea is to use the initial market price at *'t'* together with our dynamic model, used previously to generate a complete panel of "fair" calculated prices, to obtain $\widetilde{P}_{t+d}$, an estimate of the market price for the asset on the day before the next observed market price P_{t+d+1}. Back-tests are then performed by comparing the one-day *VaR* measure with the difference between the observed price at *'t+d+1'* and the calculated "fair" price at *'t+d'*.

More precisely, let $VaR_{t+d+1,t+d}$ be the one-day Value-at-Risk measure under a probability level 'p' and $w_{t+d+1,t+d}$ the money won or lost for one day, then:

$$P\left(w_{t+d+1,t+d} \le VaR_{t+d+1,t+d}\right) = p \tag{19}$$

Should prices at all dates exist, the above expression could be computed as:

$$P\left(\left[e^{\ln\left(\frac{P_{t+d+1}}{P_{t+d}}\right)} - 1\right]M \le VaR_{t+d+1,t+d}\right) = p \tag{20}$$

or

$$P\left(\left[\frac{P_{t+d+1}}{P_{t+d}} - 1\right]M \le VaR_{t+d+1,t+d}\right) = p$$

In this example it is necessary to estimate P_{t+d} because on that date the asset was not traded. To do so, actual and "fair" prices are assumed to induce similar returns, thus $\widetilde{P}_{t+d}$, the estimate for the price *'t+d',* can be computed as:[6]

$$\ln\left(\frac{P_{t+d}}{P_t}\right) \approx \ln\left(\frac{\hat{P}_{t+d}}{\hat{P}_t}\right) \Rightarrow \widetilde{P}_{t+d} \approx P_t \cdot \frac{\hat{P}_{t+d}}{\hat{P}_t}$$

The computation of the one-day loss when no there was no trading at the initial date ($w_{t+d+1,t+d}$) boils down to updating our previous observed price (P_t) using the model returns during the whole time span ($\frac{\hat{P}_{t+d}}{\hat{P}_t}$). Thus:

$$P\left(\left[\frac{P_{t+d+1}}{P_t \cdot \frac{\hat{P}_{t+d}}{\hat{P}_t}} - 1\right]M \le VaR_{t+d+1,t+d}\right) = p \tag{23}$$

It is important to note that missing observations are not replaced directly with generated "fair" prices ($\hat{P}_{t+d}$), given that doing so could potentially introduce a bias in the testing procedure, as will be discussed in section 6.3.1. Annex B illustrates this procedure using a numerical example.

To test the different *VaR* methods a comparison of the estimates on the money won or lost '$\widetilde{w}_{t+d+1,t+d}$', with the '$VaR_{t+d+1,t+d}$' must be preformed. If the *VaR* is calculated for a 5% level, it is expected that 5% of the times an actual loss would exceed the value provided by the *VaR*. To test

[6] Because we are interested in calculating *VaR* measures, it is important that the new panel of returns closely replicates the actual ones. We empirically analyze this assumption in section VI.

deviations from model estimates, a Kupiec (1995) test for each *VaR* calculation is performed on the historic proportion of losses exceeding the *VaR*.

Let Y be the number of losses which exceeded the *VaR*. Then Y follows a Binomial distribution with parameters (N_m, p), being N_m the number of comparisons done between the actual outcomes and the calculated *VaR*, and p the expected percentage of losses exceeding the *VaR*:

$$\binom{N_m}{Y} \cdot p^Y \cdot (1-p)^{N_m - Y} \tag{26}$$

The Kupiec (1995) test-statistic is:

$$K = -2 \cdot \ln\left\{(1-p)^{N_m - Y} \cdot p^Y\right\} + 2 \cdot \ln\left\{\left(1 - \frac{Y}{N_m}\right)^{N_m - Y} \cdot \left(\frac{Y}{N_m}\right)^Y\right\} \tag{27}$$

where K is distributed as χ^2 with one degree of freedom. Under the null (H_0) the proportion of losses exceeding the *VaR* 'Y/N_m' is equal to p, meaning that the tested *VaR* method provides a reliable measure. Taking a significance level of 5% for the test, our critical value will be 3.84.

An additional measure which is reported for each alternative calculation method is the "average *VaR*" over the whole time period. This measure is particularly relevant for regulated institutions required to maintain a capital level dependent on their reported *VaR*. For these institutions a higher average *VaR* implies a higher cost of capital.

Another reported measure is the "Average excess over *VaR*", a slight variation of the "Conditional Value at Risk" (*CVaR*). Recall that the *CVaR* is the average loss conditional on losses greater than the calculated *VaR,* thus it exceeds our measure by an amount equal to the average VaR.

Results for the tests will be discussed in section VI and Annex C and D. The procedure is tested at the asset and not at the portfolio level because thin trading reduces the probability that all assets in a portfolio trade the same day.

V Data Description and the Testing Portfolio.

The available transaction data is divided into three groups. The first one is used to calibrate the term structure dynamic model outlined in Phase I, the second one is used to computing VaR estimates for the testing portfolio required for Phase II and the third data panel is a used to perform the back-test of Phase III.

5.1 Data for Phase I: Generation of a Complete Panel Data of Prices with a Dynamic Term-Structure Model.

The data consists of all daily transactions of pure-discount bonds and semi-annual amortizing coupon bonds issued by the Chilean Central Bank at the Santiago Stock Exchange from January 1997 to September 2002 (1430 trading days). Pure-discount bonds are usually denominated PRBC ("Pagare Reajustable Banco Central") bonds, and semi-annual amortizing coupon bonds are called PRC ("Pagare

Reajustable con Cupones") bonds. Both bonds are inflation-protected with payments brought to real terms using monthly inflation.[7]

Table 5.1.1 summarizes the data. It can be noted that pure-discount bonds have maturities of less than 1 year while coupon bonds have maturities ranging from 1 to 20 years. Trading frequency is defined as the number of days for which we have at least one transaction of a bond of a specific maturity over all available trading days. A trading frequency of 20% means that at least one bond with that maturity was traded an average of 50 days per year. Standard deviation of observed yields generally decreases as bond maturity increases, which is consistent with mean reversion in interest rates.

Table 5.1.1: Daily transactions of Chilean government inflation-protected pure discount and coupon bonds from January 1997 to September 2002

Maturity Range (years)	Number of Observations	Average Trading Frequency*	Average Yield**	Yield Standard Deviation**
Pure Discount Bonds (PRBCs)				
0-1	1303	91,18%	5,73%	2,35%
Coupon Bonds (PRCs)				
1-1.5	284	19,86%	6,65%	2,11%
1.5-2.5	457	31,96%	6,25%	1,81%
2.5-3.5	477	33,36%	6,22%	1,39%
3.5-4.5	737	51,54%	5,97%	1,56%
4.5-5.5	561	39,23%	6,36%	1,38%
5.5-6.5	605	42,31%	6,33%	1,19%
6.5-7.5	917	64,13%	6,12%	1,31%
7.5-8.5	1136	79,44%	5,98%	1,23%
8.5-9.5	506	35,38%	6,27%	1,11%
9.5-10.5	603	42,17%	6,45%	0,79%
10.5-11.5	317	22,17%	6,19%	1,04%
11.5-12.5	510	35,66%	6,23%	0,90%
12.5-13.5	311	21,75%	6,12%	0,91%
13.5-14.5	567	39,65%	6,16%	0,80%
14.5-15.5	349	24,41%	5,92%	0,97%
15.5-16.5	373	26,08%	6,04%	0,90%
16.5-17.5	316	22,10%	6,18%	0,78%
17.5-18.5	376	26,29%	6,19%	0,93%
18.5-19.5	609	42,59%	5,95%	0,98%
19.5-20	748	52,31%	6,01%	0,95%

* Trading frequency is defined as the number of days for which there is a transaction of a given bond over all available trading days.
** Continuous Compounding

[7] In practice this is done by expressing payments in another unit, the UF ("Unidad de Fomento"), which is updated every month using the previous month variation of the Chilean CPI.

5.2 Data for Phase II: Estimation of "Value at Risk" measures on a Testing Portfolio.

A portfolio including 20 PRC bonds with different maturities ranging from 1 to 20 years is created. This will be the testing portfolio in order to apply the proposed methodology and to calculate the *VaR* measures. The portfolio is constructed assuming that UF$10,000 is invested in each of the 20 bonds, for a total investment of UF$200,000. The portfolio is re-balanced daily so the UF$10,000 investment in each asset remains constant over time.

Daily transactions of PRC bonds from January 1997 to February 2003 (1517 trading days) are used. Table 5.2.1, which presents a sub-sample of the traded prices for the 20 assets on our testing portfolio, illustrates the missing data problem common in emerging markets.

Table 5.2.1: Sub-sample of daily traded bond prices on the testing portfolio between 03/20/2000 and 05/15/2000. Bonds prices have been standardized to 100. Black spaces represent days in which the instrument was not traded.

Dates	Bond Maturities in Years																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
20-03-2000				96,90			99,65			99,26				98,85						
21-03-2000	100,22		100,13		99,98			99,64		99,48		99,24		99,14						98,79
22-03-2000	100,18			97,00		95,74						99,45	99,19			98,98	99,00	98,96		
23-03-2000				97,14				99,90		99,70								99,24		99,10
24-03-2000	100,19							99,97						99,67	99,65	99,63				
27-03-2000		100,26						99,90	99,93	99,97					99,65					99,41
28-03-2000												99,81								99,56
29-03-2000		100,35		97,18	100,23			100,05					99,80	99,84	99,77	99,76				
30-03-2000		100,42	100,58	97,29	100,13			100,16		100,06		100,02	100,13	99,96						99,87
31-03-2000		100,43		97,42				100,23		100,19			99,96					99,81		
03-04-2000				97,40			100,21	100,20	100,22	100,19	100,06		99,96	99,78				99,81	99,80	99,87
04-04-2000	100,16		100,37	97,42	100,55	96,19		100,20		100,10		100,17	99,96	99,96	99,96					
05-04-2000			100,39				100,18	100,12			100,06			99,84	99,83					99,95
06-04-2000			100,31	97,38	100,48		100,31				100,06			99,96	99,83	99,76		99,88		
07-04-2000	100,28	100,44	100,36	97,30	100,48			100,05	100,09					99,78	99,71	99,63	99,61	99,59	99,58	
10-04-2000			100,25	97,30	100,23	95,91	100,18	100,09							99,83					
11-04-2000				97,30						100,06		99,96						99,59		99,79
12-04-2000	100,21			97,26	100,43		100,21	100,09	99,97	100,24	99,97									
13-04-2000	100,33	100,55				95,72		100,20			100,06	100,07					99,68			
14-04-2000								100,27				99,91								
17-04-2000				97,30						99,97							99,54			
18-04-2000		100,49			100,11					99,88		99,76		99,61		99,50	99,34	99,45	99,21	99,64
19-04-2000				97,30		95,99	100,31	100,09									99,48	99,45		
20-04-2000		100,62		97,36					99,93	99,88										
24-04-2000							100,04			99,79				99,49		99,43				99,41
25-04-2000									99,69	99,70			99,52	99,37						99,18
26-04-2000			100,25							99,66		99,55		99,37						99,25
27-04-2000		100,34						99,83				99,60								
28-04-2000	100,25	100,69	100,52	97,38	100,58	96,30		99,83		99,61		99,55								
02-05-2000										99,53							99,20	99,31		
03-05-2000	100,28	100,24				95,63		99,64												99,18
04-05-2000		100,14			100,28	95,44		99,72		99,57										
05-05-2000		100,63	100,58	97,27				99,68		99,57			99,52			99,24				
08-05-2000								99,42		99,26										98,64
09-05-2000	100,19									99,17				98,85		98,66		98,67		
10-05-2000	100,31							99,21		98,95		98,83					98,60		98,47	98,41
11-05-2000	100,29	100,69				95,17		99,24		99,04							98,60			
12-05-2000				97,20				99,50		99,17							98,73			
15-05-2000														98,79						

Trading Prices

Table 5.2.2 describes the complete PRC bond sample for our testing portfolio. Column 1 shows the observed range of maturities. Then, column 2 classifies the PRC bonds with an approximate maturity taking into account the number of coupons remaining for each PRC until maturity. Column 3 provides the number of days, within our sample of 1517 observations, in which the bonds were traded. Finally, column 4 provides the percentage of days with respect to the total sample in which the PRCs were traded.

Table 5.2.2: Description of the complete sample for each bond of the testing portfolio. The sample consists of daily bond transactions between January 1997 and February 2003.

Matutity Range (years)	Aproximate Maturity	Number of Days Asset was Traded	Average Trading Frequency
1-1.5	1	285	18,79%
1.5-2.5	2	480	31,64%
2.5-3.5	3	491	32,37%
3.5-4.5	4	760	50,10%
4.5-5.5	5	585	38,56%
5.5-6.5	6	651	42,91%
6.5-7.5	7	988	65,13%
7.5-8.5	8	1221	80,49%
8.5-9.5	9	538	35,46%
9.5-10.5	10	620	40,87%
10.5-11.5	11	336	22,15%
11.5-12.5	12	523	34,48%
12.5-13.5	13	333	21,95%
13.5-14.5	14	590	38,89%
14.5-15.5	15	387	25,51%
15.5-16.5	16	419	27,62%
16.5-17.5	17	345	22,74%
17.5-18.5	18	422	27,82%
18.5-19.5	19	689	45,42%
19.5-20	20	815	53,72%

5.3 Data for Phase III: Back-testing with Incomplete Panel Data

Given that the VaR estimation methods require up to 400 historic observations, we are only able to back-test the procedures over a sub-sample of 1,116 trading days between August 1998 and February 2003. Table 5.3.1 describes the sub-sample. Columns 1 and 2 show the ranges of maturities and the approximate maturities of the instruments. Column 3 shows the number of days each bond is traded and Column 4 the average trading frequency. With this information it is possible to calculate the win or loss between day *‘t’* and the next day *‘t+d+1’* in which the PRC was traded (here *‘d’* is variable).

Table 5.3.1: Description of the sub-sample used to perform the “Back Test”. The sample consists of daily PRC bond transactions between August 1998 and February 2003.

Maturity Range (years)	Aproximate Maturity	Number of Days Asset was Traded	Average Trading Frequency
1-1.5	1	227	20,34%
1.5-2.5	2	345	30,91%
2.5-3.5	3	369	33,06%
3.5-4.5	4	634	56,81%
4.5-5.5	5	408	36,56%
5.5-6.5	6	432	38,71%
6.5-7.5	7	665	59,59%
7.5-8.5	8	895	80,20%
8.5-9.5	9	350	31,36%
9.5-10.5	10	384	34,41%
10.5-11.5	11	235	21,06%
11.5-12.5	12	375	33,60%
12.5-13.5	13	256	22,94%
13.5-14.5	14	458	41,04%
14.5-15.5	15	351	31,45%
15.5-16.5	16	359	32,17%
16.5-17.5	17	297	26,61%
17.5-18.5	18	347	31,09%
18.5-19.5	19	496	44,44%
19.5-20	20	588	52,69%

VI Empirical Results.

6.1 The complete panel of prices.

Table 6.1 presents the parameter estimates and standard errors of the three-factor generalized Vasicek term-structure dynamic model.

Table 6.1: Parameter estimates and standard errors from daily transactions of Chilean government inflation-protected pure discount and coupon bonds from January 1997 to September 2002.

Parameter	Parameter Estimate	Standard Error
$k1$	0,01820	0,00437
$k2$	0,97969	0,01478
$k3$	2,14709	0,05319
σ_1	0,01930	0,00021
σ_2	0,17974	0,00286
σ_3	0,21104	0,00417
ρ_{12}	-0,79976	0,01105
ρ_{13}	0,38726	0,01093
ρ_{23}	-0,81982	0,00208
δ	0,08044	0,03803
λ_1	0,00004	0,00001
λ_2	-0,01545	0,00404
λ_3	-0,02252	0,00793

The table displays estimates of the mean reversion parameters (k_1, k_2, k_3), the diffusion parameters ($\sigma_1, \sigma_2, \sigma_3$), the correlation coefficients of the state variables ($\rho_{12}, \rho_{13}, \rho_{23}$), the long-run mean of interest rates δ, and the market prices of risk ($\lambda_1, \lambda_2, \lambda_3$). Using these 13 constant parameter estimates, we estimate the state variables contained in $\mathbf{x}_t$, for each day t, using the recursive estimation technique of the extended Kalman filter. Using the calibrated model a complete panel of “fair” bond prices is computed.

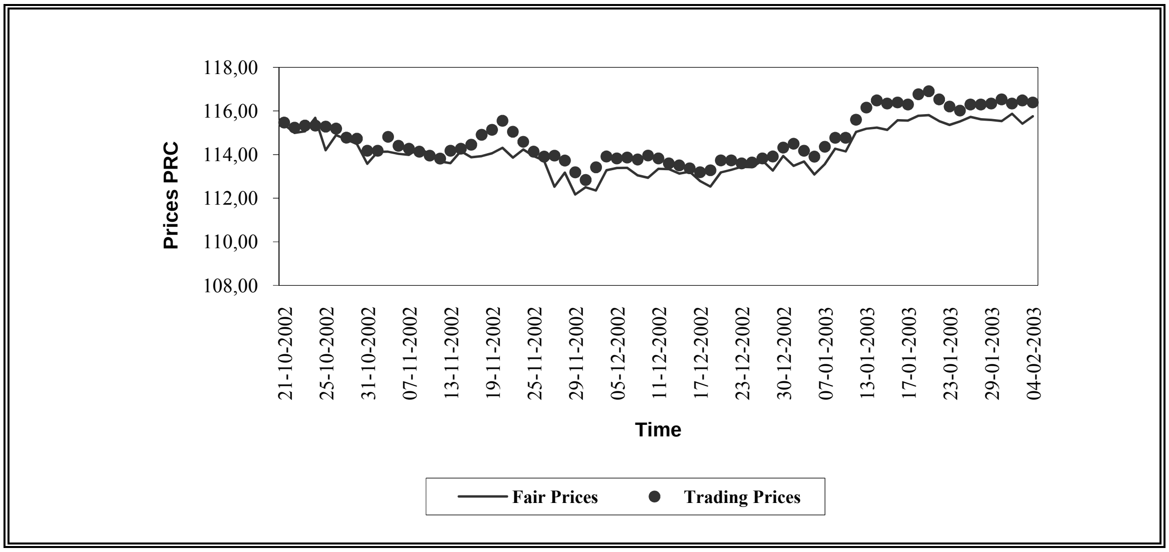
Table 6.2 presents a sub-sample of the complete panel of “fair” bond prices for the 20 assets of our testing portfolio between 03/20/2000 and 05/15/2000.

Table 6.2: Sub-sample of daily “fair” prices on the testing portfolio between 03/20/2000 and 05/15/2000. Bonds prices have been standardized to 100.

Dates	Bond Maturities in Years																				Generated "fair" Prices
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
20-03-2000	100,27	100,16	100,06	99,97	99,88	99,77	99,66	99,53	99,39	99,26	99,12	98,99	98,87	98,77	98,68	98,61	98,56	98,53	98,52	98,55	
21-03-2000	100,25	100,17	100,11	100,04	99,97	99,88	99,77	99,66	99,53	99,41	99,28	99,16	99,05	98,95	98,87	98,80	98,76	98,74	98,74	98,76	
22-03-2000	100,20	100,15	100,11	100,07	100,01	99,94	99,85	99,75	99,63	99,52	99,41	99,30	99,20	99,11	99,04	98,98	98,95	98,94	98,95	98,98	
23-03-2000	100,23	100,20	100,18	100,15	100,11	100,04	99,96	99,86	99,76	99,64	99,54	99,43	99,33	99,25	99,18	99,13	99,10	99,09	99,10	99,14	
24-03-2000	100,21	100,15	100,13	100,11	100,08	100,04	99,99	99,92	99,84	99,76	99,68	99,60	99,54	99,48	99,44	99,41	99,40	99,42	99,45	99,51	
27-03-2000	100,29	100,25	100,23	100,21	100,18	100,14	100,08	100,00	99,92	99,84	99,75	99,67	99,59	99,53	99,48	99,45	99,44	99,45	99,48	99,54	
28-03-2000	100,33	100,29	100,27	100,25	100,22	100,18	100,12	100,05	99,96	99,88	99,79	99,71	99,63	99,57	99,52	99,49	99,47	99,48	99,51	99,57	
29-03-2000	100,36	100,32	100,31	100,29	100,27	100,23	100,18	100,11	100,04	99,96	99,88	99,80	99,73	99,68	99,63	99,61	99,60	99,62	99,65	99,72	
30-03-2000	100,43	100,42	100,42	100,42	100,40	100,36	100,31	100,24	100,16	100,08	100,00	99,92	99,85	99,79	99,74	99,71	99,71	99,72	99,75	99,81	
31-03-2000	100,46	100,46	100,46	100,45	100,43	100,40	100,35	100,28	100,21	100,13	100,05	99,97	99,90	99,84	99,80	99,77	99,77	99,78	99,82	99,88	
03-04-2000	100,44	100,44	100,45	100,45	100,43	100,40	100,34	100,28	100,20	100,12	100,04	99,96	99,89	99,83	99,79	99,76	99,75	99,76	99,80	99,86	
04-04-2000	100,22	100,29	100,35	100,39	100,41	100,40	100,37	100,33	100,26	100,20	100,13	100,06	100,00	99,96	99,92	99,90	99,90	99,92	99,97	100,03	
05-04-2000	100,28	100,32	100,36	100,39	100,39	100,37	100,33	100,27	100,21	100,13	100,06	99,99	99,92	99,87	99,83	99,80	99,80	99,81	99,85	99,92	
06-04-2000	100,26	100,31	100,35	100,38	100,39	100,37	100,34	100,29	100,22	100,15	100,08	100,01	99,95	99,90	99,86	99,84	99,84	99,86	99,90	99,96	
07-04-2000	100,32	100,36	100,40	100,41	100,40	100,37	100,32	100,25	100,17	100,08	99,99	99,91	99,83	99,77	99,72	99,68	99,67	99,67	99,70	99,75	
10-04-2000	100,28	100,29	100,30	100,31	100,30	100,27	100,23	100,17	100,09	100,02	99,94	99,87	99,80	99,75	99,71	99,68	99,68	99,70	99,73	99,80	
11-04-2000	100,35	100,36	100,37	100,38	100,36	100,33	100,28	100,21	100,13	100,05	99,97	99,89	99,82	99,76	99,71	99,68	99,67	99,69	99,72	99,78	
12-04-2000	100,24	100,25	100,28	100,29	100,29	100,27	100,23	100,17	100,11	100,03	99,96	99,89	99,83	99,77	99,73	99,71	99,71	99,73	99,77	99,83	
13-04-2000	100,31	100,37	100,41	100,43	100,42	100,39	100,33	100,26	100,17	100,08	99,99	99,90	99,82	99,75	99,70	99,66	99,64	99,64	99,67	99,72	
14-04-2000	100,41	100,46	100,49	100,50	100,48	100,44	100,37	100,29	100,20	100,11	100,01	99,92	99,83	99,76	99,70	99,66	99,64	99,64	99,66	99,71	
17-04-2000	100,38	100,40	100,42	100,42	100,40	100,36	100,29	100,22	100,13	100,04	99,94	99,85	99,77	99,70	99,65	99,61	99,59	99,59	99,62	99,67	
18-04-2000	100,44	100,43	100,42	100,40	100,36	100,30	100,22	100,13	100,03	99,93	99,82	99,72	99,63	99,56	99,49	99,44	99,42	99,41	99,43	99,47	
19-04-2000	100,44	100,44	100,43	100,41	100,37	100,32	100,24	100,15	100,05	99,95	99,85	99,75	99,66	99,58	99,51	99,47	99,44	99,44	99,45	99,50	
20-04-2000	100,59	100,58	100,56	100,52	100,46	100,39	100,29	100,18	100,07	99,94	99,82	99,71	99,60	99,51	99,43	99,37	99,33	99,31	99,31	99,34	
24-04-2000	100,45	100,42	100,39	100,36	100,31	100,24	100,16	100,06	99,96	99,85	99,74	99,64	99,54	99,46	99,39	99,34	99,31	99,30	99,32	99,36	
25-04-2000	100,38	100,33	100,29	100,25	100,20	100,13	100,05	99,96	99,85	99,75	99,64	99,54	99,45	99,37	99,31	99,26	99,23	99,22	99,24	99,28	
26-04-2000	100,35	100,29	100,25	100,21	100,16	100,09	100,02	99,92	99,82	99,72	99,62	99,52	99,43	99,35	99,29	99,24	99,22	99,21	99,23	99,28	
27-04-2000	100,37	100,32	100,28	100,24	100,19	100,12	100,04	99,94	99,83	99,72	99,61	99,51	99,41	99,33	99,26	99,21	99,18	99,18	99,19	99,23	
28-04-2000	100,34	100,43	100,47	100,46	100,42	100,35	100,25	100,13	100,00	99,86	99,73	99,60	99,48	99,37	99,28	99,20	99,14	99,11	99,10	99,12	
02-05-2000	100,35	100,41	100,43	100,42	100,37	100,30	100,20	100,08	99,95	99,82	99,69	99,56	99,44	99,34	99,25	99,18	99,12	99,09	99,09	99,11	
03-05-2000	100,28	100,25	100,23	100,19	100,14	100,07	99,98	99,87	99,76	99,64	99,52	99,41	99,31	99,22	99,14	99,08	99,05	99,03	99,04	99,07	
04-05-2000	100,23	100,18	100,14	100,10	100,04	99,98	99,89	99,79	99,68	99,57	99,46	99,36	99,26	99,18	99,11	99,06	99,03	99,02	99,03	99,07	
05-05-2000	100,54	100,52	100,48	100,42	100,34	100,24	100,12	99,99	99,85	99,70	99,56	99,42	99,29	99,18	99,08	99,00	98,94	98,90	98,89	98,90	
08-05-2000	100,37	100,31	100,24	100,16	100,08	99,97	99,85	99,72	99,58	99,44	99,30	99,17	99,04	98,93	98,84	98,76	98,71	98,67	98,66	98,68	
09-05-2000	100,25	100,15	100,06	99,99	99,91	99,81	99,70	99,58	99,45	99,32	99,19	99,07	98,95	98,85	98,77	98,70	98,66	98,63	98,63	98,66	
10-05-2000	100,29	100,16	100,05	99,95	99,85	99,73	99,61	99,47	99,33	99,19	99,05	98,91	98,79	98,68	98,59	98,51	98,46	98,42	98,41	98,43	
11-05-2000	100,38	100,37	100,31	100,21	100,09	99,95	99,78	99,60	99,41	99,22	99,03	98,85	98,68	98,53	98,39	98,27	98,17	98,10	98,05	98,03	
12-05-2000	100,39	100,35	100,28	100,19	100,07	99,93	99,78	99,61	99,43	99,25	99,07	98,90	98,74	98,60	98,47	98,36	98,27	98,21	98,17	98,15	
15-05-2000	100,37	100,33	100,27	100,18	100,07	99,94	99,79	99,62	99,45	99,28	99,11	98,94	98,79	98,65	98,53	98,42	98,34	98,28	98,24	98,23	

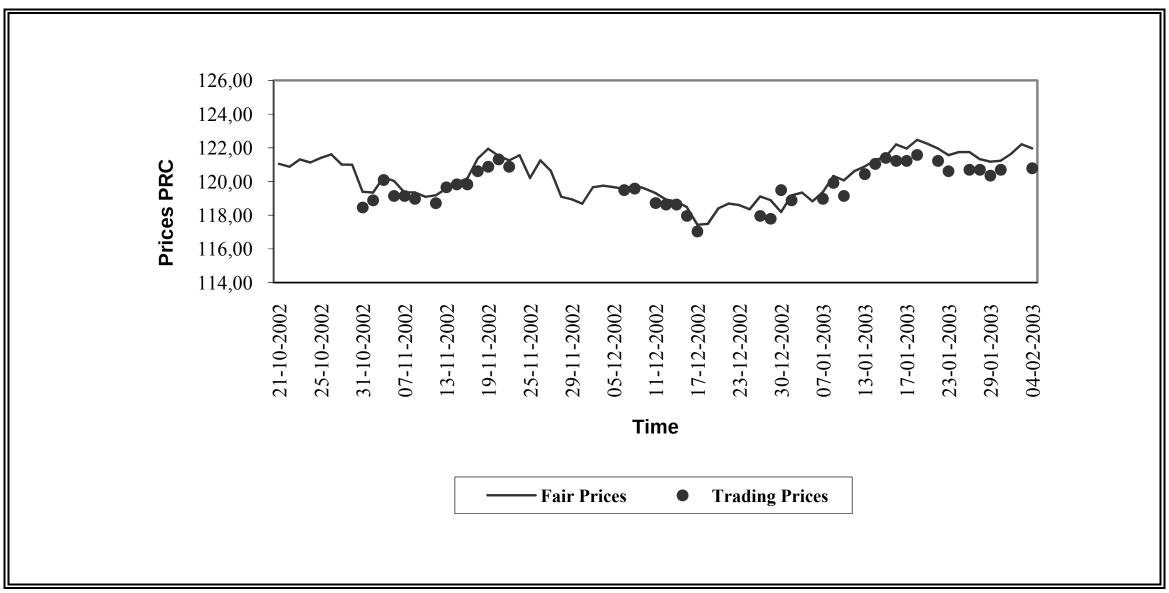
One issue that has not been discussed in detail yet is how close are “fair” prices and “fair” returns to actual transaction data. Figure 6.1 plots a sub-sample of the trading prices for the PRC bonds with 8 years maturity, along with the calculated “fair” prices obtained in Phase I of the methodology.

Figure 6.1: Daily trading prices and calculated “fair” prices for PRC bonds with 8 years maturity between October 2002 and February 2003



From the figure, we can appreciate that model prices for this instrument systematically underestimate the traded. The sign of the bias changes for other instruments, as 6.2 shows for the sixteen-year maturity bond.

Figure 6.2: Daily trading prices and calculated "fair" prices for PRCs with 16 years maturity between October 2002 and February 2003



The reason for this bias lies on the differences of liquidity among bonds, which systematically bias model estimates. This is why in a previous section we considered that we should not mix in the same price panel traded and "fair" prices.

The situation with returns, is different, however, with traded and “fair” returns being very similar. For example Figures 6.3 and 6.4 plot a sub-sample of daily returns obtained from the “fair” panel, along with daily real returns for days in which the instrument was traded in two consecutive days, both for the eight and the sixteen-year maturity bond. We can see that traded and “fair” returns are very similar.

Figure 6.3: Daily real returns and “fair” returns for PRCs with 8 years maturity between August 1998 and December 1999

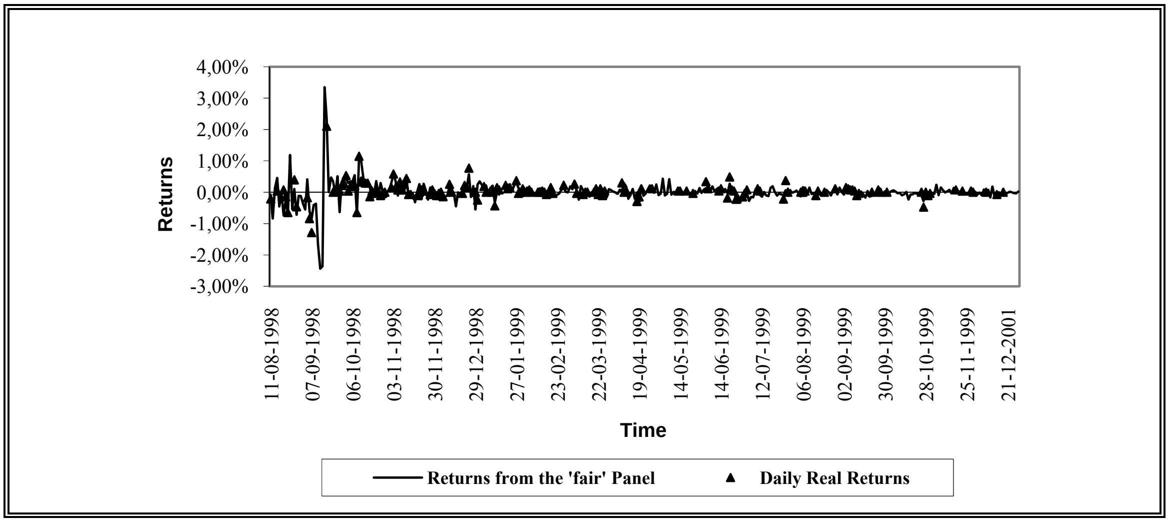
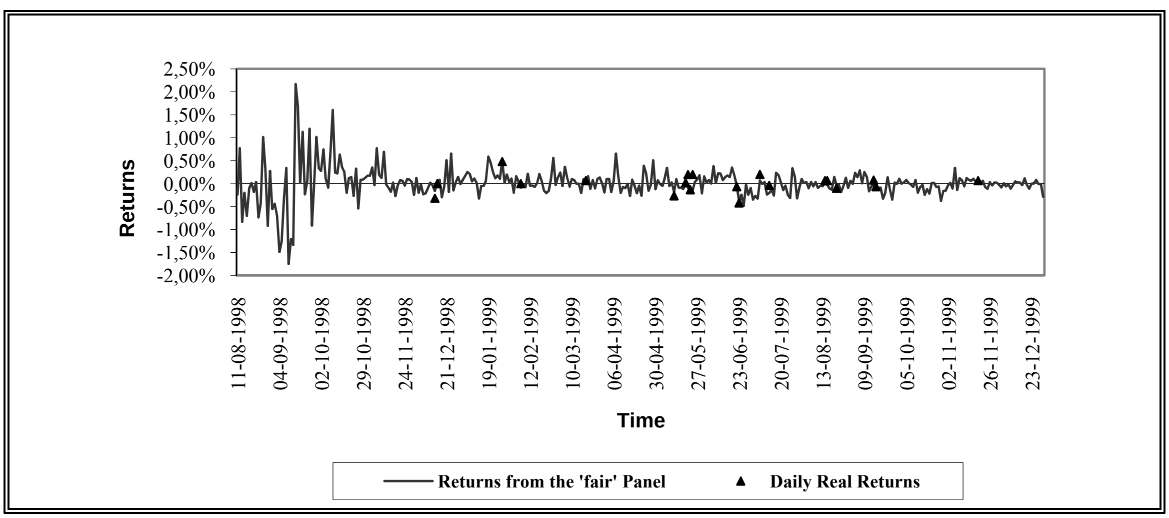


Figure 6.4: Daily real returns and “fair” returns for PRCs with 16 years maturity between August 1998 and December 1999



In addition, Table 6.3 provides the Mean Error (ME), the Absolute Mean Error (AME), the Square Root of the Mean Squared Error (RMSE), and the U Theil statistic (U) obtained form the differences between "fair" returns and real returns. For every pair wise consecutive days in which trading was observed for the whole sample between January 1997 and February 2003, and for all the instruments in the experimental portfolio.

From Table 6.3, we can appreciate the great approximation between "fair" and real returns with a very close to zero U Theil statistic. This means that the "fair" returns replicate very closely the traded market returns.

To make an addition validation, we now compare two-day "fair" and traded returns for the same two bonds. From Figures 6.5 and 6.6 we can see that "fair" returns replicate very closely the traded returns, which are more formally compared in Table 6.4. We can appreciate that the values for the U Theil statistic are again small and close to zero. This shows that "fair" returns replicate closely traded returns even for time periods greater than one day.

Table 6.3: Mean Error (ME), Absolute Mean Error (AME), Square Root of the Mean Squared Error (RMSE), and U Theil statistic (U) obtained form the differences between "fair" returns and traded returns. For every pair wise consecutive days in which trading was observed between January 1997 and February 2003.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ME	1.14E-06	-6.46E-06	-9.38E-06	-3.48E-06	3.45E-06	-5.26E-06	7.53E-07	9.17E-07	-1.02E-07	-3.08E-06	-1.21E-05	6.08E-06	-1.07E-05	-2.82E-06	-1.94E-05	-1.20E-05	-2.05E-06	-1.09E-06	-2.28E-06	3.20E-06
AME	3.15E-05	6.82E-05	8.84E-05	8.23E-05	1.13E-04	9.70E-05	1.02E-04	7.71E-05	9.52E-05	6.24E-05	1.02E-04	7.58E-05	9.39E-05	7.38E-05	9.03E-05	1.13E-04	9.49E-05	1.21E-04	1.25E-04	1.24E-04
RMSE	5.85E-05	1.25E-04	1.52E-04	1.42E-04	2.25E-04	1.67E-04	1.84E-04	1.17E-04	1.58E-04	1.00E-04	2.04E-04	1.10E-04	1.43E-04	1.12E-04	1.33E-04	2.04E-04	1.30E-04	1.74E-04	1.82E-04	1.80E-04
U	2.51E-02	2.87E-02	4.16E-02	4.72E-02	4.54E-02	4.92E-02	3.97E-02	5.85E-02	4.29E-02	4.51E-02	4.39E-02	3.38E-02	3.72E-02	3.41E-02	3.38E-02	4.24E-02	3.45E-02	3.82E-02	4.22E-02	4.03E-02

Table 6.4: Mean Error (ME), Absolute Mean Error (AME), Square Root of the Mean Squared Error (RMSE), and U Theil statistic (U) obtained form the differences between "fair" returns and traded returns. For every pair wise, not necessarily consecutive days, in which trading was observed between January 1997 and February 2003.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ME	1.18E-06	3.47E-07	-3.63E-07	1.32E-07	5.72E-08	-6.20E-07	-1.66E-07	-3.32E-07	-1.96E-06	-6.62E-08	1.61E-06	1.78E-06	2.32E-06	1.57E-06	1.04E-06	1.43E-06	9.65E-07	8.29E-07	4.96E-07	2.51E-07
AME	4.46E-05	8.20E-05	1.16E-04	1.09E-04	1.59E-04	1.40E-04	1.17E-04	8.88E-05	1.17E-04	9.24E-05	1.54E-04	1.05E-04	1.31E-04	1.04E-04	1.39E-04	1.53E-04	1.39E-04	1.58E-04	1.48E-04	1.52E-04
RMSE	8.38E-05	1.50E-04	2.05E-04	2.07E-04	2.86E-04	2.41E-04	2.01E-04	1.42E-04	1.91E-04	1.56E-04	2.80E-04	1.55E-04	1.97E-04	1.61E-04	2.17E-04	2.51E-04	2.05E-04	2.40E-04	2.12E-04	2.20E-04
U	3.60E-02	3.44E-02	5.61E-02	6.86E-02	5.77E-02	7.08E-02	4.36E-02	7.08E-02	5.16E-02	7.00E-02	6.03E-02	4.75E-02	5.13E-02	4.92E-02	5.52E-02	5.23E-02	5.43E-02	5.28E-02	4.92E-02	4.92E-02

Figure 6.5: Two-day traded and “fair” returns for 8 years maturity bonds between August 1998 and December 1999

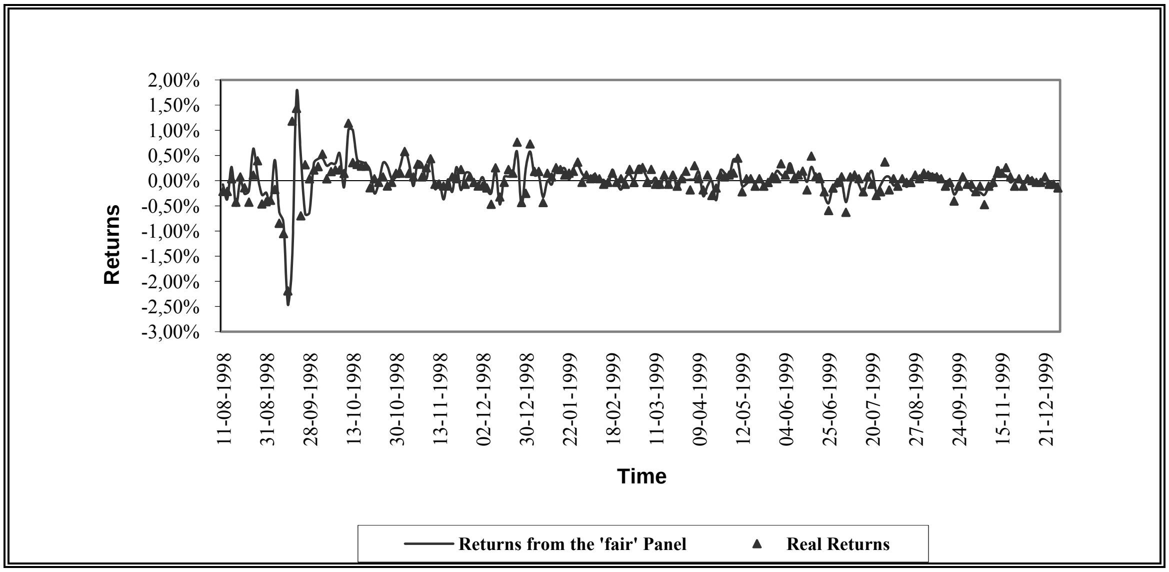
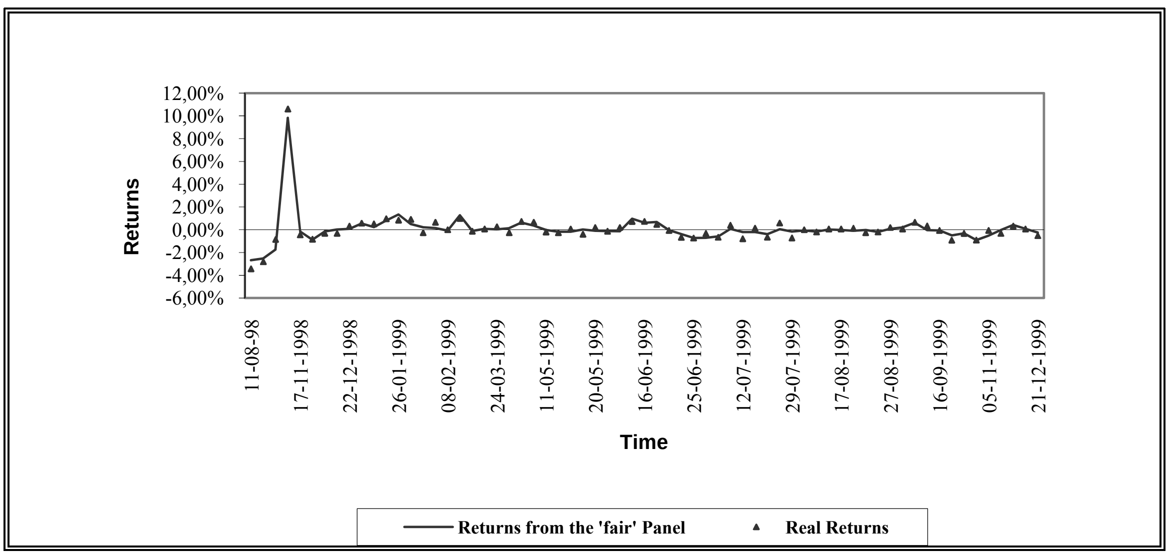


Figure 6.6: Two-day traded and “fair” returns for 16 years maturity bonds between August 1998 and December 1999



Therefore, if we denote by $r_{t+d,t}$ the traded return between 'd' days of a bond and $\psi_{t+d,t}$ the "fair" return between the same time period, the evidence suggests that:

$$r_{t+d,t} \sim \psi_{t+d,t} \tag{34}$$

$$\ln\left(\frac{P_{t+d}}{P_t}\right) \sim \ln\left(\frac{\hat{P}_{t+d}}{\hat{P}_t}\right) \tag{35}$$

This means that the "fair" returns calculated with the "fair" panel, derived from the generalized Vasicek three-factor dynamic term-structure model, replicate very closely the actual observed returns traded in the Santiago Stock Exchange. This observation is very important, given that it is a necessary condition to obtain reliable *VaR* measures. In addition, the consistency of the proposed historical testing ("Back-Test") relies on this assumption that appears to be fulfilled empirically.

In short, despite the fact that the calculated "fair" prices are biased estimates of actual trading prices we have observed that the "fair" returns closely replicate trading returns, Therefore, for the purpose of *VaR* calculations, no bias is foreseen arising from using the "fair" returns.

6.2 Back-testing the *VaR* Measures.

In this section point, we calculate the *VaR* measures using the "fair" panel of bond returns. We calculate the *VaR* measures with the alterntaive estimation methods outlined in section 4.2. Tables 6.5 and 6.6 show the "Back-Test" summary indicators (the average percentage loss in excess of the *VaR*, the Kupiec *K* statistic, the Kupiec test result, the average *VaR*, the average loss in excess of the *VaR*, and the maximum excess over the *VaR*) for a 5% and 1% confidence level, respectively.

A "perfect" *VaR* measure would exhibit exactly a *p*% average excess over the *VaR* for a *VaR* calculated with a *p*% confidence level. In Table 6.6, we can appreciate that the "best" *VaR* measures are provided by the GARCH(1,1) and the Monte Carlo calculation methods. They offer an average percentage excess over the *VaR* of 4.84% and 5.17% respectively. These values are very close to the 5% confidence level. Also the Risk Metrics method offers a value of 5.57%, which even though is not as good as the GARCH (1,1) or the Monte Carlo has the advantage that its calculation is very simple.

On the other hand, methods like the static version of the Extreme Value Theory and the Historical Simulation offer relatively poor *VaR* measures, with average percentage excesses over the *VaR* of 7.48% and 7.10% respectively.

We can also see in Table 6.5 that the Kupiec Test at the 5% level only rejects the null hypothesis for the static version of the Extreme Value Theory (EVT). Therefore, with the exception of the static EVT, we can say that for the other procedures the average percentage of losses in excess of the *VaR* are statistically indistinguishable from 5%.

Table 6.5 also shows the average excess over the *VaR*. This measure allows us to observe which method adjusts better to market fluctuations. Therefore, in this metric the "best" method is the lowest. We can see that all methods show a similar value for this measure, being the Risk Metrics method slightly better.

Banks or other financial institutions are concerned with having low VaR numbers , so supervisory agencies impose low capital requirements. Table 6.5 shows that the lowest average *VaR* is provided by the static EVT. However, we must recall that this method was rejected by the Kupiec Test.

Therefore, from the available methods which were not rejected by the test, we can observe that the dynamic EVT offers the lower level of average *VaR*.

Table 6.5: "Back-Test" summary indicators of the *V*aR measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the VaR calculations. Sample between August 1998 and February 2003.

Indicators	**Summary Indicators**							
	Var-Cov Matrix	**RiskMetrics**	**GARCH(1,1)**	**t - Student**	**Hist. Sim.**	**Static EVT**	**Dynamic EVT**	**Monte Carlo**
% excess over VaR	6.27%	5.57%	4.84%	6.26%	7.10%	7.48%	5.81%	5.17%
***K* (Kupiec Test)**	1.34	0.28	0.02	1.31	3.49	4.78	0.56	0.02
Reject H_0						X		
Average VaR	-47.99	-42.71	-40.81	-48.12	-40.02	-31.61	-36.61	-37.56
Average Excess over VaR	-25.72	-18.42	-20.62	-25.73	-25.91	-27.80	-20.38	-20.23
Maximum Excess over VaR	-197.01	-141.75	-112.32	-196.97	-201.89	-215.80	-113.27	-115.97

Table 6.6: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations. Sample between August 1998 and February 2003.

Indicators	**Summary Indicators**							
	Var-Cov Matrix	**RiskMetrics**	**GARCH(1,1)**	**t - Student**	**Hist. Sim.**	**Static EVT**	**Dynamic EVT**	**Monte Carlo**
% excess over VaR	3.21%	2.65%	2.14%	3.21%	2.35%	2.86%	1.71%	2.47%
***K* (Kupiec Test)**	13.23	7.98	4.18	13.23	5.65	9.83	1.79	6.57
Reject H_0	X	X	X	X	X	X		X
Average VaR	-68.87	-47.67	-56.90	-71.52	-78.65	-80.12	-70.93	-70.91
Average Excess over VaR	-28.31	-14.90	-20.06	-28.19	-25.42	-26.86	-16.76	-21.00
Maximum Excess over VaR	-173.97	-96.16	-86.53	-173.89	-120.79	-157.09	-65.38	-89.81

In short, for *VaR* calculations at the 5% confidence level, it appears that the GARCH (1,1), Monte Carlo, Risk Metrics, and Dynamic EVT methods erform the best. However, the Risk Metrics method offers better conditions for extreme market fluctuations as the average excess over the *VaR* is the lowest. Also, the dynamic EVT might be preferred in order to report *VaR* measures for the purpose of capital requirements, as the method offers the least average *VaR* measure.

Table 6.6 repeats the previous exercise, but now at the 1% confidence level. We can appreciate that the only method which is not rejected by the Kupiec test is the dynamic EVT. This means that for all of the other methods the average percentage of excesses over the *VaR* is significantly different from 1%. The poor performance of the alternative *VaR* methods at the 1% level evidences the relevance of the adequate tail modeling of the returns distributions for the 1% level. However, not only tails are important, because if that was the case, the static EVT or the Historical Simulation would also offer good results. Therefore, another important characteristic is the adequate adjustment of the model to the time varying volatility of returns. In that way, we can observe that although the GARCH (1,1) was rejected, it performed better than methods such as the static EVT and Historical Simulation which exhibited average percentage of excesses over the *VaR* of 2.86% and 2.35% respectively (the GARCH method exhibited an excess of 2.14%).

It is worth to note that the poor performance of the alternative *VaR* methods at the 1% confidence level has been already documented in several studies for both emerging and developed markets. Fernandez (2003) found that, for a Chilean proxy of zero coupon bonds, the dynamic EVT method performed the best. Delfines and Gutierrez (2002) analyzed *VaR* measures for different Argentinean assets such as Brady bonds and Global Government bonds. Their findings suggest similar percentages of excesses over the *VaR* as the ones reported here.

Kiesel et. al. (2000) provide an analysis for emerging markets using Brady bonds (Mexico, Venezuela, Morocco, and Poland, among others). Their findings suggest similar measures as the ones reported here for the 5% and 1% confidence levels. Finally, Bao et. al. (2003) evaluate *VaR* models for Asian emerging markets (Korea, Indonesia, Malaysia, Taiwan, and Thailand). They focus on stock indexes and not on fixed income instruments. However, their calculations and conclusions are again very close from ours for both 5% and 1% confidence levels.

Although the previous studies did not address the problem of incomplete panels of prices, they are useful regarding the conclusions obtained from the alternative *VaR* methods at different confidence levels. Their results are similar to ours. In addition, our historical testing provides coherent values and adjusted to reality. This suggests that our estimations, obtained with the proposed methodology, track closely what is actually happening in the market. However, further testing of the proposed methodology with different econometric approaches and applied to different markets is needed. We have only provided an analysis for the Chilean particular case, but our results are encouraging and open the horizon for further research. The study offers an alternative approach for financial risk management in low-transaction fixed income markets.

VII Conclusions

The estimation of daily risk measures has become a crucial issue for financial institutions and regulatory agencies. It is important for implementing and evaluating risk management strategies and regulations in the financial sector. Among the alternative approaches, the Value-at-Risk (*VaR*) has become a the-facto international standard endorsed by many international entities including the Basle Committee (1996a, 1996b).

Much research has been done on how to implement *VaR* measures in developed markets, but very little in emerging markets with thin trading. In this paper we argue that the absence of prices makes computing *VaR* measures very difficult and we show how to deal with this issue.

We propose a methodology to calculate and test daily *VaR* measures in thinly traded markets. The methodology is composed of three phases: Phase I, generates a complete panel of prices, using a term-structure dynamic model of interest rates. Phase II, calculates portfolio *VaR* measures with several alternative methods using the complete panel data generated in phase I. Phase III, shows how to back-test the *VaR* measures obtained in phase II using the original incomplete panel of prices. We provide an empirical implementation of the methodology for the Chilean fixed income market.

Our results show that for the calculation of *VaR* measures for a 5% confidence level only one method was rejected by the Kupiec test (the static EVT). It appears that methods such as the GARCH (1,1), Monte Carlo, Risk Metrics, and Dynamic EVT perform the best. However, the Risk Metrics method offers better conditions for extreme market fluctuations as the average excess over the *VaR* is the lowest. Also, the dynamic EVT might be preferred in order to report *VaR* measures for the purpose of capital requirements, as the method offers the least average *VaR* measure.

For the *VaR* calculations with a 1% confidence level, the only method which is not rejected by the Kupiec test is the dynamic EVT. The poor performance of the alternative *VaR* methods at the 1% level evidences the relevance of the adequate left tail modeling of the return distributions. However, not only tails are important, because if that was the case, the static EVT or the Historical Simulation would also offer good results. Another important characteristic is the adequate adjustment of the model to the heteroskedasticity of returns. We observe that although the GARCH (1,1) was rejected, it performed better than methods such as the static EVT or the Historical Simulation.

The proposed methodology is broad and flexible, and could be implemented with different innovations in term-structure dynamic modeling or historical testing analysis. Therefore, it could be applied in any economy with low frequency fixed income markets. It would be interesting to observe future research work using alternative term-structure dynamic models or using data of different fixed income markets.

This study is one of the firsts dealing with the problem of calculating daily *VaR* measures with a panel of incomplete data and may provide a basis for further research in emerging markets where thin trading is a serious issue.

References

BAO, Y., LEE, T. and SALTOGLU B. (2003) Evaluating predictive performance of value-at-risk models in emerging markets: a reality check. *Working paper*, University of California, Riverside.

BASLE COMMITTEE ON BANKING SUPERVISION (1996a) *Amendment to the Capital Accord to Incorporate Market Risks*. Basle Committee on Banking Supervision, Basle.

BASLE COMMITTEE ON BANKING SUPERVISION (1996b) *Supervisory Framework for the Use of Backtesting in Conjunction with the Internal Models*. Basle Committee on Banking Supervision, Basle.

BEDER, T. (1995) *VAR*: Seductive but dangerous. *Financial Analysts Journal, Vol. 51*, N°5, 12-24.

BOLLERSLEV, T. (1986) Generalized autoregressive conditional heteroscadicity. *Journal of Econometrics, Vol. 31*, N°1, 307-327.

CHERNOBAI A., MENN C., TRÜCK S. and RACHEV S. (2005) Estimation of Operational Value-at-Risk in the Presence of Minimum Collection Thresholds. Working Paper, University of California, Santa Barbara.

COLES, S.(2001) An *introduction to statistical modeling of extreme values*. Springer-Verlag, London.

CORTAZAR, G., SCHWARTZ, E.S. and NARANJO, L. (2007) Term Structure Estimation in Markets with Infrequent Trading *International Journal of Finance and Economics* (forthcoming)

DACOROGNA M.M., MÜLLER U.A., PICTET O.V. and DE VRIES C.G. (1999) Extremal Forex Returns in Extremely Large Data Sets. *Extremes, Vol. 4*, N°1, 105–127.

DELFINER, M., and GUTIÉRREZ, M. (2002) Aplicación de la teoría de valores extremos al gerenciamiento del riesgo. *Working Paper*, Universidad del CEMA.

EMBRECHTS, P., KLÜPPELBERG, C. and MIKOSCH, T. (1997) *Modelling extremal events for insurance and finance*. Springer-Verlag, Berlin.

FAMA E. (1965) The Behavior of Stock Market Prices. *Journal of Business, Vol. 38*, N°1, 34–105.

FERNÁNDEZ, V. (2003) Extreme Value Theory and Value at Risk. *Revista de Analisis Economico*, Vol.18 N°1, 57-85.

JACKSON, P. (1995) Risk measurement and capital requirements for banks. *Bank of England Quarterly Bulletin, Vol. 35*, N°2, 177 - 184.

KIESEL R., PERRAUDIN W. and TAYLOR A. (2000), An extremes analysis of VaRs for emerging market benchmark bonds.*Working Paper*, Birbeck College.

KUPIEC, P. (1995) Techniques for Verifying the Accuracy of Risk Measurement Models. *Journal of Derivatives, Vol. 3*, N°2, 73-84.

LUCAS, A.(1997) The Effect of Fat Tails on Optimal Asset Allocations and Downside Risk. *Working Paper, Faculteit der Economische Wetenschappen en Econometrie*, Vrije Universiteit Amsterdam.

MANDELBROT, B.(1963) The Variation of Certain Speculative Prices. *Journal of Business, Vol. 36*, N°1, 394-419.

MCNEIL, A., FREY, R.(2000) Estimation of tail-related risk measures for heteroscedastic financial times series: An extreme value approach. *Journal of Empirical Finance, Vol. 7*, N°4, 271–300.

MORGAN , J.P. (1996) RiskMetrics. *Technical Document*, Morgan Guaranty Trust Company, New York.

MOSCADELLI M., CHERNOBAI A., RACHEV S. (2005) Treatment of incomplete data in the field of operational risk: the effects on parameter estimates, EL and UL figures. *Working Paper*, University of California, Santa Barbara,

NELSON, C.R. , SIEGEL, A.F. (1987) Parsimonious modeling of yield curves. *Journal of Business, Vol. 60*, N°4, 473-489.

SINGH, M.(1997) Value at Risk using principal components analysis. *Journal of Portfolio Management, Vol. 24*, N°1, 101-112.

SVENSSON, L.E.O.(1994) Estimating and interpreting forward interest rates: Sweden 1992-1994. *Working Paper*, National Bureau of Economic Research.

VASICEK, A.(1977) An equilibrium characterization of the term structure. *Journal of Financial Economics*, Vol. 5, N°2, 177-188.

WILSON, T. (1993) Infinite Wisdom. *Risk*, Vol. 6, N°6, 37-46.

ANNEX A: The *VaR* Methods.

Parametric methods in a world of multi-normal distributions (methods of the variance-covariance matrix):

The Method of Variance-Covariance is grounded on the assumption of a Multivariate Normal Distribution of returns.

For the case of one asset, from Itô's lemma, we can derive from the logarithmic returns, the *VaR* for one day:

$$VaR_{t+1,t} = \left(e^{(u-\frac{\sigma^2}{2})+\sigma\alpha} - 1 \right) M \tag{A.1}$$

where u y σ^2 are the mean and variance of returns, α is the inverse of a Normal (0,1) for a p% probability, M is the amount of money invested

To calculate 'u', it could be estimated as the sample measure of the historical returns. Therefore, what could vary is the form of estimating 'σ', existing for those different options shown later.

To calculate the portfolio *VaR*, this is obtained calculating the variance of that portfolio, and replacing it in (A.1). The portfolio variance is obtained using the Variance-Covariance Matrix as follows:

$$\hat{\sigma}^2_{port.} = \boldsymbol{\omega}' \sum \boldsymbol{\omega} \tag{A.2}$$

being **ω** y **ω'** the weighted vector of the different elements of the portfolio and its transpose respectively, and $\sum$ is the Variance-Covariance Matrix. If we want to estimate the portfolio mean, it could be calculated as the weighted average of the measures of each asset. Using the relative weight of each portfolio element.

Then, to estimate the standard deviations (and also the covariances), we can proceed with the following methods:

Method of the sample variance and covariance

$$\hat{\sigma}^2 = \frac{1}{(T-1)} \cdot \sum_{h=1}^{T} (r_h - \hat{u})^2 \tag{A.3}$$

being T the number of observations, r_h the logarithmic returns, and $\hat{u}$ the sample mean.

$$\hat{\sigma}^2_{i,j} = \frac{1}{(T-1)} \cdot \sum_{h=1}^{T} (r_{h,i} - \hat{u}_i)(r_{h,j} - \hat{u}_j) \tag{A.4}$$

Method of exponential decay in the Risk-Metrics versions

Following J.P. Morgan (1996),

$$\begin{aligned} \sigma_t^2 &= \lambda_{RM} \cdot \sigma_{t-1}^2 + (1-\lambda_{RM}) \cdot r_{t-1}^2 \\ \lambda_{RM} &= [day = 0.94; month = 0.97] \end{aligned} \tag{A.5}$$

$$\begin{aligned} \sigma_{i,j,t}^2 &= \lambda_{RM} \cdot \sigma_{i,j,t-1}^2 + (1-\lambda_{RM}) \cdot r_{i,t-1} r_{j,t-1} \\ \lambda_{RM} &= [day = 0.94; month = 0.97] \end{aligned} \tag{A.6}$$

then we used the ecuation (A.2) and (A.1).

GARCH(1,1)

Following Bollerslev (1986) for logarithmic returns:

$$r_t = (u - \frac{\sigma^2}{2}) + \sigma_t \cdot Z_t \tag{A.7}$$

$$\varepsilon_t = \sigma_t \cdot Z_t \tag{A.8}$$

$$\sigma_t^2 = \eta_0 + \eta_1 \cdot \varepsilon_{t-1}^2 + \eta_2 \cdot \sigma_{t-1}^2 \tag{A.9}$$

If we see J.P. Morgan (1996), to calculate a portfolio *VaR* we could use the variance-covariance matrix decomposition as follows:

$$\hat{\sigma}^2{}_{port.} = [\omega_1 \quad \dots \quad \dots \quad \omega_n]_{1xn} \cdot \begin{bmatrix} \sigma_1 & 0 & \dots & 0 & 0 \\ 0 & \sigma_2 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \sigma_n \end{bmatrix}_{nxn} \cdot \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \dots & \rho_{1n} \\ \rho_{21} & 1 & \rho_{23} & \dots & \dots \\ \rho_{31} & \rho_{32} & 1 & & \dots \\ \dots & & & & \dots \\ \rho_{n1} & \dots & \dots & \dots & 1 \end{bmatrix}_{nxn} \cdot \begin{bmatrix} \sigma_1 & 0 & \dots & 0 & 0 \\ 0 & \sigma_2 & & & 0 \\ \dots & & & \dots & \dots \\ 0 & & & \dots & \dots \\ 0 & & \dots & \dots & \sigma_n \end{bmatrix}_{nxn} \cdot \begin{bmatrix} \omega_1 \\ \dots \\ \dots \\ \omega_n \end{bmatrix}_{nx1} \tag{A.10}$$

where the ρ is the Pearson correlation and σ is calculated with (A.9). Then we replace this in equations (A.1) and (A.2).

Parametric methods accounting for asymmetric and multi-kurtosis effects:

These try to model with different distributions, the behavior of logarithmic portfolio returns. Of course, they are not anymore in a world of multinormal distributions. Therefore, we can no longer use the Variance-Covariance Matrix as in the previous examples in order to calculate the portfolio *VaR*. [8]

For that reason, one possibility is to use a simple approximation to model portfolios where the normality assumption no longer holds, this idea was taken from J.P. Morgan (1996). This consists in: If $r_{1,t}$, $r_{2,t}$, ..., $r_{c,t}$ are the returns of the *c* asset of a portfolio for time *t*, and that each asset has a weight ω_1 , ω_2 , ..., ω_c , respectively, then the portfolio return is:

$$r_{portf,t} = \sum_{i=1}^{c} \omega_i \cdot r_{i,t} \tag{A.11}$$

If the process is repeated for times *t-1, t-2,..., t-k* , where *k* is the size of the time window, then, a series for the portfolio will be maintained, which has implicitly included the correlations for each asset, and to which all the techniques could be applied as it was a single asset.[9]

T - Student

We used the t-Student distribution following the work of Wilson (1993) and Lucas (1997), they propose the possibility of substituting the Normal distribution with a t-Student. The latter has the advantage of adjusting to fat tails better that the former, depending on the degrees of freedom, attaining greater flexibility in the left tail.

"Extreme Value Theory" in its static version

This methodology, pretends to only model the left tail of returns distributions, such modeling is given by the data that is under the threshold 'μ' – look Embrechts, Klüpperberg y Mikosch (1997)- by a transformation of the Generalized Distribution of Pareto (GDP):

$$G_{\xi,\beta(\mu)}(y) = \begin{cases} 1 - \left(1 + \dfrac{\xi y}{\beta}\right)^{-1/\xi}, \xi \neq 0 \\ 1 - \exp\left(\dfrac{-y}{\beta}\right), \xi = 0 \end{cases} \tag{A.12}$$

where ξ and β are the parameters of the GDP.

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8 This is because the correlation matrix, starts loosing reliability because it is only defined for Multinormal variables.

9 Recall that "many bonds" are portfolios by their own, composed by different flows or coupons.

Following the studies outlined above, and adding the work of Coles (2001) and McNeil and Frey (2000), we can represent the *VaR* as:

$$VaR_p^{(EVT)} = \left[\mu + \frac{\beta}{\xi}\left(\left(\frac{n}{k_\mu}(1-p)\right)^{-\xi} - 1\right)\right]M \tag{A.13}$$

where k_μ is the number of observations in excess of the threshold 'μ'. And n is the sample size used for the calculation.

<u>"Extreme Value Theory" in its dynamic version</u>

The static version of the "Extreme Value Theory", like other methods such as sample variance and covariance, and historical simulation; assign the same weight to recent and past data. They do not account for time varying volatility or heteroskedasticity.

In that way, McNeil and Frey (2000) developed a dynamic "Extreme Value Theory" with the purpose that the estimations could be adjusted quickly to market changes. They estimate returns using a GARCH model and maximum likelihood estimation assuming Normal Distribution of error terms.

Then, they take the residuals $\hat{Z}_t$ of the preceding estimation, and with the residuals taken from the left tail, they adjust them to a Generalized Distribution of Pareto (GDP).

Being '*F(Z)*' the distribution of the obtained residuals, and if we recall equation (A.1), then the inverse of *F(Z)* corresponds to the α of that formula, therefore, this values could be used to estimate the *VaR* for a *p%* in that equation.

Then, with the adjusted distribution of residuals, we estimate the inverse function, that will be denoted by $INV(F(Z))_p$, for a p confidence level, then this is replaced in equation (A.1) instead of α as follows:

$$VaR_p^{EVT\ Dinamic} = \left(e^{(u-\frac{\sigma^2}{2})+\sigma\cdot INV(F(Z))_p} - 1\right)M \tag{A.14}$$

Non parametric method of historical simulation:

These methods do not assume a distribution for returns. They take a window of historical data to perform their estimations.

If we assume that an investment is done in one asset, and we take a series of historical returns for the investment, for example in the last 250 days, and it is multiplied times the positions that are being actually taken for the asset. We could then elaborate a histogram with the outcomes of the investment. Then it is enough taking the percentile p%, and that value will deliver the *VaR* .

For the case of one portfolio, it is enough to add in a contemporaneous form, the same series of investment outcomes mentioned for the case of an asset, but using all the assets conforming the portfolio.

After this, we can again create the histogram, but now of the portfolio as a whole, and it is enough to take the percentile p% to calculate the *VaR* for the portfolio as a whole.

Monte Carlo simulation method, using the stochastic structure developed in Cortázar, Schwartz and Naranjo (2007) with a dynamic three-factor Vasicek term-structure model:

Following Beder (1995), J.P. Morgan (1996) and Singh (1997), to model the risk using the Monte Carlo technique. The first thing to do is to specify some stochastic process describing the financial variables in study. Here, we take advantage of the assumed process that the term-structure of rates follow, a three factor Vasicek in this case.

Then, having the assumed process, the 'parameters' of that equation must be found. Here we have already estimated them in Phase I of the proposed methodology. Once the task is completed, we simulate the paths of the desired variables. For example, we simulate the prices using "shocks" in the stochastic process assumed (for that we simulate forward a vector of state variables $\mathbf{x}_t$).

With these simulations we can generate a series of returns and therefore a database of investment outcomes. With this information, we can generate a histogram and taking the percentile p%, we obtain the *VaR* measure.

ANNEX B: A demonstrative Example of the "Back Test" Analysis.

To understand the "BackTest" procedure proposed in the document, we provide here a demonstrative example. In this case we use the GARCH (1,1) method. We use a sub-sample ranging from June 8, 2000 through June 20, 2000 for a PRC bond with 8 years maturity.

If we observe Table C.1, the only days in which the daily money won or lost could be calculated, is between t=8 (06/20/2000) and t=7 (06/16/2000)[10], given that only here were two consecutive days in which the asset was traded. Therefore, this is the only time interval in which we are able to perform a comparison with the daily *VaR*.

What we intend to do with the proposed testing procedure, is to capture in some way, relevant information from dates in which not necessarily were two consecutive daily trading prices observed.

For example, day t=5 (06/14/2000) is the first one after t=1 (06/08/2000) in which a transaction happened for a PRC with 8 years maturity. Therefore, what we intend to do from these two market prices

[10] The fact that between these days there are more that one day is explained because by that time there was a weekend and two holidays in Chile and we are only considering working days.

is to obtain some information for the testing procedure. We can clearly observe that in *t*=5, we are unable to calculate the daily money won or lost, given that there was not a market price in *t*=4.

Table C.1: Sub-Sample of PRC bonds with 8 years maturity between 06/08/2000 and 06/20/2000

Date	Time (t)	Trading prices of PRC bonds with 8 years maturity	Daily money won or lost when investing UF$10.000 in the PRC bond with 8 years maturity.	Daily VaR 5% calculated with the "fair" panel when investing UF$10.000 in the PRC bond with 8 years maturity (GARCH Method)	PRC bonds 8 years maturity prices obtained from term-structure dynamic model.
08-06-2000	1	99.68		-14.68	99.86
09-06-2000	2			-15.56	99.98
12-06-2000	3			-16.48	99.87
13-06-2000	4			-15.58	99.88
14-06-2000	5	99.86	?	-15.58	99.97
15-06-2000	6			-15.39	100.02
16-06-2000	7	99.86	?	-15.43	100.10
20-06-2000	8	99.90	4.01	-14.56	100.11

$$\text{Money won or lost}_{5-4} = \left(\frac{P_5 - P_4}{P_4}\right)\cdot 10000$$

$$= \left(\frac{99.86 - P_4}{P_4}\right)\cdot 10000$$

$$= ?$$

$$VaR_{5-4} = -15.58$$

Taking advantage of the empirical observation that the returns for horizons greater than one day, calculated with the market prices and calculated with the "fair" prices are not significantly different:

$$\ln\left(\frac{P_{t+d}}{P_t}\right) \approx \ln\left(\frac{\hat{P}_{t+d}}{\hat{P}_t}\right) \Rightarrow \frac{P_{t+d}}{P_t} \approx \frac{\hat{P}_{t+d}}{\hat{P}_t}$$

$$ \tag{C.1} $$

Then, for the price P_4 we can find the following relation:

$$\frac{P_4}{P_1} \approx \frac{\hat{P}_4}{\hat{P}_1} \tag{C.2}$$

$$\breve{P}_4 \approx \frac{\hat{P}_4}{\hat{P}_1} \cdot P_1 \tag{C.3}$$

$$\breve{P}_4 \approx \frac{99.88}{99.86} \cdot 99.68 = 99.70 \tag{C.4}$$

If we observe, this price is obtained from a real trading price, in this case P_1. Therefore, if we replace this value in Table C.1, now we could perform a comparison between "the money won or lost" in t=5, with the daily *VaR* calculated for t=5. (See Table C.2)

We must note that in making a comparison between $\widetilde{P}_4$ (which is a transformation of P_1) and P_5 we are taking two market prices in our analysis (P_1 and P_5). Therefore, our testing procedure relies on actual trading prices directly.

Table C.2: Example of the testing procedure proposed.

Date	Time (t)	Trading prices of PRC bonds with 8 years maturity	Daily money won or lost when investing UF$10.000 in the PRC bond with 8 years maturity.	Daily VaR 5% calculated with the "fair" panel when investing UF$10.000 in the PRC bond with 8 years maturity (GARCH	PRC bonds 8 years maturity prices obtained from term-structure dynamic model.

$$\text{Money won or lost}_{5-4} = \left(\frac{P_5 - P_4}{P_4}\right)\cdot 10000$$

$$= \left(\frac{99.86 - 99.70}{99.70}\right)\cdot 10000$$

$$= 16.05 \qquad VaR_{5-4} = -15.58$$

We can observe that in this case, given that in t=5 existed a won and not a loss. The investment outcome was greater than the daily *VaR* calculated for that period.

ANNEX C: Measures for the Historical Testing (confidence level of 5%).

The next tables show the measures for the historical testing calculated for each individual asset, and for each *VaR* calculation method (Section 4.3).

The sub-sample used to perform the “Back Test”. The sample consists of daily PRCs transactions between August 1998 and February 2003 (1116 days), with confidence level of 5% for the *VaR* calculations

We must note, however, that the weakness of this procedure is that it can not be tested for the portfolio. This is apparent because is very difficult that all of the assets conforming a portfolio have been traded during the same day. Therefore, we would only be able to perform individual tests for each asset.

Given that the chosen measures for the historical testing will be calculated for each individual asset, and for each *VaR* calculation method. We need a global measure in order to compare the alternative *VaR* calculation methods. Therefore, we have created “summary indicators”.

The summary indicators are calculated simultaneously with all of the bonds in line (as it was a single bond). We did not use the average because the percentage of days in which ‘$w_{t+d+1,t}$’ could be calculated (the outcome in monetary units of an investment) is not the same along the sample.

Table C.1: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - Method of the sample variance and covariance. Sample between August 1998 and February 2003.

Indicators	**Bonds Differentiated by Maturities in Years**																				**Summary Indicators**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
% excess over VaR	8.37%	6.69%	5.69%	4.42%	5.88%	5.57%	6.33%	6.48%	5.71%	6.53%	8.51%	5.33%	7.42%	6.77%	5.41%	6.13%	6.40%	5.19%	7.06%	7.48%	6.27%
***K* (Kupiec Test)**	4.55	1.88	0.36	0.47	0.63	0.28	2.28	3.79	0.36	1.73	5.08	0.09	2.77	2.73	0.12	0.90	1.13	0.03	3.94	6.67	1.34
Reject H_0	X										X								X	X	
Average VaR	-41.16	-46.78	-57.05	-53.91	-54.02	-44.36	-43.77	-43.41	-40.74	-35.45	-35.42	-34.85	-40.55	-42.12	-44.35	-43.97	-50.47	-49.82	-55.99	-59.89	-47.99
Average Excess over VaR	-23.79	-28.24	-21.82	-22.68	-38.60	-27.85	-26.59	-18.57	-23.04	-18.12	-20.54	-11.68	-24.89	-16.66	-37.46	-20.54	-32.00	-41.39	-33.69	-32.56	-25.72
Maximum Excess over VaR	-64.42	-77.58	-64.21	-76.88	-177.13	-177.41	-197.01	-115.00	-122.74	-127.38	-66.21	-31.59	-129.27	-82.20	-126.31	-102.42	-119.45	-106.29	-126.41	-141.26	-197.01

Table C.2: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - "RiskMetrics". Sample between August 1998 and February 2003.

Indicators	**Bonds Differentiated by Maturities in Years**																				**Summary Indicators**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
% excess over VaR	10.13%	6.69%	5.42%	5.52%	6.37%	5.10%	5.27%	5.14%	4.57%	5.74%	7.23%	5.07%	7.81%	5.68%	4.84%	4.74%	5.05%	5.76%	5.04%	4.76%	5.57%
***K* (Kupiec Test)**	9.83	1.88	0.13	0.35	1.49	0.01	0.10	0.04	0.14	0.43	2.18	0.00	3.67	0.42	0.02	0.05	0.00	0.41	0.00	0.07	0.28
Reject H_0	X																				
Average VaR	-40.73	-47.65	-51.44	-44.81	-44.44	-44.54	-37.81	-38.92	-34.38	-37.06	-34.54	-36.34	-35.15	-41.73	-41.42	-39.58	-47.60	-48.38	-50.57	-59.71	-42.71
Average Excess over VaR	-13.15	-16.50	-12.81	-12.14	-22.72	-19.44	-19.78	-16.09	-18.18	-15.81	-15.98	-9.36	-16.80	-15.90	-26.05	-20.97	-22.75	-25.84	-24.37	-27.75	-18.42
Maximum Excess over VaR	-41.93	-52.94	-40.62	-61.76	-114.70	-105.33	-141.75	-79.34	-88.00	-92.48	-44.73	-25.47	-86.43	-64.44	-73.99	-71.53	-56.99	-74.47	-80.15	-91.76	-141.75

Table C.3: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - GARCH(1,1). Sample between August 1998 and February 2003.

Indicators	**Bonds Differentiated by Maturities in Years**																				**Summary Indicators**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
% excess over VaR	7.49%	6.10%	5.42%	4.26%	5.39%	4.41%	4.67%	4.92%	4.57%	4.44%	6.38%	4.00%	5.86%	5.68%	3.70%	3.90%	3.37%	4.32%	5.04%	4.76%	4.84%
***K* (Kupiec Test)**	2.59	0.83	0.13	0.77	0.13	0.33	0.16	0.01	0.14	0.26	0.87	0.85	0.38	0.42	1.36	0.99	1.87	0.35	0.00	0.07	0.02
Reject H_0																					
Average VaR	-40.94	-42.77	-49.33	-43.41	-42.83	-44.62	-38.95	-35.54	-32.42	-33.93	-33.53	-36.52	-33.53	-37.17	-38.23	-39.89	-41.66	-51.04	-50.62	-50.53	-40.81
Average Excess over VaR	-18.29	-24.76	-16.00	-19.46	-26.92	-19.20	-20.18	-18.39	-16.98	-14.66	-17.74	-9.54	-20.86	-12.01	-29.17	-19.44	-29.13	-31.73	-24.37	-28.69	-20.62
Maximum Excess over VaR	-47.33	-56.98	-57.27	-68.75	-112.32	-105.40	-80.62	-86.15	-94.94	-101.06	-48.83	-26.46	-84.64	-79.05	-84.36	-87.72	-65.92	-89.48	-88.80	-95.75	-112.32

Table C.4: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - t de Student. Sample between August 1998 and February 2003.

Indicators	**Bonds Differentiated by Maturities in Years**																				**Summary Indicators**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
% excess over VaR	8.37%	6.69%	5.69%	4.42%	5.88%	5.57%	6.33%	6.48%	5.71%	6.53%	8.51%	5.33%	7.42%	6.55%	5.41%	6.13%	6.40%	5.19%	7.06%	7.48%	6.26%
***K* (Kupiec Test)**	4.55	1.88	0.36	0.47	0.63	0.28	2.28	3.79	0.36	1.73	5.08	0.09	2.77	2.12	0.12	0.90	1.13	0.03	3.94	6.67	1.31
Reject H $_0$	X										X								X	X	
Average VaR	-57.53	-55.02	-59.39	-51.78	-52.82	-45.72	-48.34	-47.26	-41.52	-36.91	-35.31	-35.47	-40.15	-40.66	-45.81	-47.53	-48.70	-46.38	-49.41	-57.51	-48.12
Average Excess over VaR	-23.77	-28.21	-21.61	-22.66	-38.58	-27.82	-26.46	-18.54	-23.02	-18.10	-20.52	-11.67	-24.87	-17.20	-37.43	-20.52	-31.98	-41.37	-33.66	-32.53	-25.73
Maximum Excess over VaR	-64.40	-77.54	-64.16	-76.83	-177.08	-177.37	-196.97	-114.97	-122.71	-127.36	-66.19	-31.56	-129.25	-82.18	-126.29	-102.39	-119.42	-106.26	-126.37	-141.22	-196.97

Table C.5: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - Method of historical simulation. Sample between August 1998 and February 2003.

Indicators	**Bonds Differentiated by Maturities in Years**																				**Summary Indicators**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
% excess over VaR	8.37%	6.98%	6.78%	5.21%	6.62%	6.50%	6.93%	6.93%	6.29%	6.79%	8.51%	6.13%	8.20%	8.95%	7.12%	7.52%	6.73%	5.48%	7.86%	9.18%	7.10%
***K* (Kupiec Test)**	4.55	2.54	2.21	0.06	2.05	1.87	4.67	6.28	1.13	2.34	5.08	0.95	4.67	12.32	2.96	4.19	1.70	0.16	7.34	17.56	3.49
Reject H $_0$	X						X	X			X		X	X		X			X	X	
Average VaR	-42.98	-44.58	-44.93	-45.97	-41.09	-42.04	-42.51	-35.92	-34.53	-30.41	-27.59	-28.75	-29.58	-32.27	-38.52	-38.91	-41.35	-42.68	-51.08	-53.94	-40.02
Average Excess over VaR	-25.49	-32.56	-22.88	-22.15	-38.80	-27.72	-27.17	-19.31	-23.01	-19.15	-24.49	-12.51	-25.00	-15.23	-30.15	-19.43	-33.64	-42.88	-34.81	-31.11	-25.91
Maximum Excess over VaR	-68.78	-106.35	-84.39	-93.20	-196.05	-191.90	-201.89	-112.21	-116.50	-124.44	-64.83	-35.80	-128.37	-85.26	-123.62	-99.57	-118.36	-109.24	-129.17	-143.63	-201.89

Table C.6: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - “Extreme Value Theory” in its static version. Sample between August 1998 and February 2003.

Indicators	**Bonds Differentiated by Maturities in Years**																				**Summary Indicators**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
% excess over VaR	7.49%	8.14%	8.13%	5.68%	7.11%	7.19%	8.13%	6.82%	6.57%	5.48%	9.79%	6.40%	9.77%	9.61%	6.27%	7.24%	7.07%	5.48%	8.67%	9.52%	7.48%
***K* (Kupiec Test)**	2.59	6.07	6.45	0.59	3.39	3.87	11.65	5.61	1.66	0.18	8.97	1.43	9.69	16.31	1.10	3.36	2.39	0.16	11.64	20.26	4.78
Reject H $_0$		X	X			X	X	X			X		X	X					X	X	X
Average VaR	-30.30	-26.08	-29.67	-34.47	-36.10	-32.45	-33.61	-31.25	-32.11	-29.84	-25.74	-23.53	-24.37	-32.38	-30.30	-36.53	-34.85	-38.36	-36.67	-44.90	-31.61
Average Excess over VaR	-31.62	-33.42	-24.09	-24.74	-41.93	-28.40	-28.13	-22.86	-25.17	-24.51	-23.75	-13.48	-23.42	-15.22	-36.40	-21.59	-35.87	-43.09	-34.86	-33.05	-27.80
Maximum Excess over VaR	-70.58	-115.52	-103.17	-109.90	-209.58	-205.43	-215.80	-128.25	-133.65	-136.80	-74.49	-37.17	-138.87	-86.52	-136.52	-109.86	-133.32	-121.99	-134.46	-147.78	-215.80

Table C.7: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations - "Extreme Value Theory" in its dynamic version. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	8.81%	7.27%	6.78%	4.57%	6.13%	4.41%	5.42%	5.81%	5.43%	5.74%	8.09%	5.07%	7.42%	6.11%	3.99%	5.01%	4.38%	5.48%	6.65%	6.46%	5.81%
***K* (Kupiec Test)**	5.71	3.30	2.21	0.25	1.02	0.33	0.24	1.18	0.13	0.43	4.00	0.00	2.77	1.12	0.81	0.00	0.25	0.16	2.60	2.43	0.56
Reject H_0	X										X										
Average VaR	-33.48	-37.39	-40.93	-42.57	-39.25	-35.37	-39.36	-31.95	-30.40	-29.01	-27.43	-28.83	-33.38	-31.72	-34.58	-40.03	-37.81	-42.77	-42.22	-42.67	-36.61
Average Excess over VaR	-18.35	-28.93	-17.16	-21.98	-28.48	-22.53	-21.11	-17.50	-16.15	-13.01	-16.67	-9.28	-18.52	-13.81	-29.54	-17.69	-25.29	-29.17	-23.20	-25.65	-20.38
Maximum Excess over VaR	-50.46	-86.87	-80.18	-79.99	-113.27	-106.38	-96.07	-96.40	-100.26	-105.26	-49.54	-32.01	-83.50	-84.08	-88.56	-91.91	-66.99	-95.00	-94.98	-97.43	-113.27

Table C.8: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 5% for the *VaR* calculations Monte Carlo simulation method, using the stochastic structure developed in Cortázar, Schwartz and Naranjo (2003) with a three-factor Vasicek model. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	7.60%	6.54%	5.97%	4.43%	5.94%	4.88%	5.03%	5.09%	4.89%	4.47%	6.96%	4.57%	6.37%	5.73%	3.71%	4.15%	3.61%	4.47%	5.58%	5.30%	5.17%
***K* (Kupiec Test)**	2.81	1.57	0.69	0.45	0.71	0.01	0.00	0.02	0.01	0.23	1.70	0.15	0.93	0.49	1.36	0.58	1.32	0.21	0.34	0.11	0.02
Reject H_0																					
Average VaR	-32.90	-34.36	-38.10	-42.93	-37.13	-35.19	-39.57	-34.64	-30.45	-32.06	-30.58	-30.29	-33.85	-32.77	-32.88	-36.35	-39.26	-41.13	-50.11	-44.84	-37.56
Average Excess over VaR	-21.14	-25.14	-14.33	-17.99	-26.44	-16.69	-18.82	-18.64	-18.48	-13.74	-17.97	-11.44	-20.13	-12.83	-28.17	-21.68	-28.29	-29.26	-21.85	-27.92	-20.23
Maximum Excess over VaR	-46.24	-62.59	-57.60	-72.36	-115.97	-110.94	-81.15	-88.55	-94.88	-105.81	-51.89	-25.50	-85.84	-78.11	-83.86	-82.47	-66.28	-89.31	-88.71	-97.01	-115.97

ANNEX D: Measures for the Historical Testing (confidence level of 1%).

The next tables show the measures for the historical testing calculated for each individual asset, and for each *VaR* calculation method (Section 4.3).

The sub-sample used to perform the “Back Test”. The sample consists of daily PRCs transactions between August 1998 and February 2003 (1116 days), with confidence level of 1% for the *VaR* calculations

We must note, however, that the weakness of this procedure is that it can not be tested for the portfolio. This is apparent because is very difficult that all of the assets conforming a portfolio have been traded during the same day. Therefore, we would only be able to perform individual tests for each asset.

Given that the chosen measures for the historical testing will be calculated for each individual asset, and for each *VaR* calculation method. We need a global measure in order to compare the alternative *VaR* calculation methods. Therefore, we have created “summary indicators”.

The summary indicators are calculated simultaneously with all of the bonds in line (as it was a single bond). We did not use the average because the percentage of days in which ‘$w_{t+d+1,t}$’ could be calculated (the outcome in monetary units of an investment) is not the same along the sample.

Table D.1: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - Method of the sample variance and covariance. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	5.29%	4.36%	2.98%	2.68%	4.17%	2.32%	3.31%	3.13%	3.71%	2.87%	3.83%	2.40%	3.91%	2.62%	3.13%	2.23%	3.37%	3.46%	3.23%	3.23%	3.21%
***K* (Kupiec Test)**	20.93	21.52	9.56	12.40	23.10	5.54	22.39	26.18	15.38	9.03	11.06	5.33	12.59	8.40	10.31	4.06	10.39	12.93	15.65	18.63	13.23
Reject H_0	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Average VaR	-64.27	-76.76	-74.55	-72.71	-69.08	-66.11	-59.09	-62.49	-54.19	-58.94	-50.28	-53.40	-54.77	-56.50	-62.28	-65.62	-62.45	-67.10	-72.38	-76.71	-68.87
Average Excess over VaR	-22.57	-23.02	-15.92	-16.37	-33.00	-42.05	-30.61	-19.35	-19.16	-24.71	-23.80	-10.01	-27.31	-21.24	-44.91	-32.19	-38.80	-38.29	-41.22	-43.26	-28.31
Maximum Excess over VaR	-52.99	-63.42	-34.78	-48.37	-147.98	-150.37	-173.97	-95.56	-105.26	-111.67	-52.17	-21.59	-114.89	-64.92	-109.22	-85.33	-98.28	-84.65	-99.47	-112.51	-173.97

Table D.2: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - “RiskMetrics”. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	5.29%	2.91%	2.71%	1.89%	2.94%	2.09%	2.26%	2.57%	2.57%	3.39%	3.40%	2.13%	3.13%	3.06%	2.56%	2.23%	2.69%	2.88%	2.22%	2.55%	2.65%
***K* (Kupiec Test)**	20.93	8.37	7.43	4.04	10.21	3.93	7.85	15.54	6.09	13.69	8.44	3.67	7.47	12.64	6.06	4.06	5.88	8.23	5.52	10.00	7.98
Reject H_0	X	X	X	X	X	X	X	X	X	X	X		X	X	X	X	X	X	X	X	X
Average VaR	-41.52	-51.14	-50.26	-47.89	-46.61	-45.82	-48.27	-38.81	-37.30	-41.43	-36.23	-36.09	-41.69	-37.47	-40.54	-49.03	-46.79	-51.23	-58.33	-53.45	-47.67
Average Excess over VaR	-8.48	-9.66	-6.67	-11.55	-20.68	-19.07	-19.48	-13.95	-13.67	-11.61	-11.58	-7.76	-18.61	-11.84	-23.15	-20.10	-14.95	-21.77	-16.60	-17.96	-14.90
Maximum Excess over VaR	-24.36	-29.47	-19.11	-43.26	-89.23	-80.86	-96.16	-53.44	-56.29	-62.45	-34.15	-13.47	-54.49	-39.87	-42.86	-44.27	-31.79	-43.09	-41.13	-42.84	-96.16

Table D.3: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - GARCH(1,1). Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	3.52%	3.78%	2.17%	2.21%	2.70%	1.62%	1.96%	2.12%	2.00%	1.57%	2.98%	1.33%	2.34%	1.75%	1.71%	1.39%	2.02%	2.31%	2.42%	2.04%	2.14%
***K* (Kupiec Test)**	8.84	15.76	3.81	6.95	8.10	1.43	4.82	8.62	2.74	1.06	6.07	0.38	3.39	2.11	1.47	0.50	2.41	4.36	7.23	4.94	4.18
Reject H_0	X	X		X	X		X	X			X							X	X	X	X
Average VaR	-52.03	-66.64	-63.31	-70.18	-65.13	-60.69	-58.23	-48.01	-47.84	-47.79	-48.68	-45.82	-56.42	-58.53	-52.55	-55.21	-69.15	-66.67	-74.85	-83.67	-56.90
Average Excess over VaR	-19.14	-16.10	-10.71	-14.98	-25.80	-27.08	-18.89	-20.70	-19.01	-20.29	-16.10	-8.59	-29.74	-19.52	-33.34	-25.56	-16.01	-27.50	-15.01	-24.27	-20.06
Maximum Excess over VaR	-32.53	-35.32	-32.17	-53.05	-86.53	-81.66	-46.09	-60.66	-64.63	-73.23	-39.68	-11.63	-52.96	-61.50	-66.34	-68.28	-37.82	-65.41	-61.94	-72.26	-86.53

Table D.4: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - t de Student. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	5.29%	4.36%	2.98%	2.68%	4.17%	2.32%	3.31%	3.13%	3.71%	2.87%	3.83%	2.40%	3.91%	2.62%	3.13%	2.23%	3.37%	3.46%	3.23%	3.23%	3.21%
K (Kupiec Test)	20.93	21.52	9.56	12.40	23.10	5.54	22.39	26.18	15.38	9.03	11.06	5.33	12.59	8.40	10.31	4.06	10.39	12.93	15.65	18.63	13.23
Reject H_0	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Average VaR	-89.81	-83.15	-79.66	-74.74	-75.93	-67.15	-64.01	-60.79	-61.18	-56.38	-53.41	-60.56	-59.15	-54.47	-57.62	-69.62	-72.76	-77.55	-81.84	-81.10	-71.52
Average Excess over VaR	-22.53	-22.96	-14.93	-16.31	-32.93	-41.94	-30.35	-19.27	-19.12	-24.67	-23.75	-9.97	-27.26	-21.19	-44.85	-32.14	-38.74	-38.22	-41.14	-43.18	-28.19
Maximum Excess over VaR	-52.95	-63.37	-34.68	-48.26	-147.87	-150.27	-173.89	-95.49	-105.20	-111.61	-52.12	-21.56	-114.84	-64.86	-109.16	-85.26	-98.21	-84.58	-99.37	-112.40	-173.89

Table D.5: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - Method of historical simulation. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	3.08%	2.03%	1.63%	1.42%	2.70%	1.62%	1.96%	1.90%	2.57%	2.09%	2.98%	1.60%	3.52%	2.84%	2.85%	2.51%	3.03%	3.46%	2.62%	2.89%	2.35%
K (Kupiec Test)	6.41	2.87	1.23	1.00	8.10	1.43	4.82	5.79	6.09	3.50	6.07	1.15	9.91	10.44	8.08	5.81	8.02	12.93	9.10	14.07	5.65
Reject H_0	X				X		X	X	X		X		X	X	X	X	X	X	X	X	X
Average VaR	-89.69	-97.77	-98.53	-96.88	-89.06	-87.83	-77.72	-80.82	-80.64	-83.54	-82.97	-70.02	-73.30	-75.58	-73.82	-71.29	-71.06	-87.82	-93.28	-105.80	-78.65
Average Excess over VaR	-18.74	-18.37	-10.11	-10.87	-25.51	-31.40	-30.75	-19.26	-18.61	-26.36	-24.14	-12.77	-23.56	-19.36	-34.81	-26.70	-25.36	-33.90	-36.96	-36.53	-25.42
Maximum Excess over VaR	-40.16	-48.10	-18.88	-24.86	-90.10	-90.91	-120.79	-82.01	-83.84	-86.00	-39.48	-22.84	-70.52	-65.10	-83.38	-71.11	-47.45	-68.42	-78.48	-94.55	-120.79

Table D.6: "Back-Test" summary indicators of the *VaR* measures calculated with the "fair" panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - "Extreme Value Theory" in its static version. Sample between August 1998 and February 2003.

Indicators	Bonds Differentiated by Maturities in Years																				Summary Indicators
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
% excess over VaR	4.85%	3.49%	2.17%	1.74%	3.43%	2.09%	2.86%	2.79%	3.43%	2.35%	2.98%	2.40%	3.52%	2.18%	2.85%	2.23%	3.70%	3.46%	3.43%	3.23%	2.86%
K (Kupiec Test)	17.60	13.12	3.81	2.84	14.93	3.93	15.49	19.55	12.78	5.12	6.07	5.33	9.91	4.84	8.08	4.06	12.97	12.93	18.10	18.63	9.83
Reject H_0	X	X			X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
Average VaR	-97.67	-117.03	-113.46	-121.01	-95.63	-93.32	-88.88	-72.87	-74.46	-64.26	-66.80	-72.61	-67.16	-71.98	-73.50	-85.93	-86.14	-101.16	-95.42	-111.44	-80.12
Average Excess over VaR	-19.23	-22.97	-12.44	-18.58	-33.02	-38.96	-30.44	-18.20	-18.44	-26.74	-27.63	-9.65	-26.83	-22.56	-39.27	-25.61	-30.14	-32.44	-34.30	-39.52	-26.86
Maximum Excess over VaR	-51.38	-61.83	-24.04	-38.23	-125.87	-128.62	-157.09	-84.09	-94.81	-101.86	-44.65	-22.08	-101.23	-62.32	-91.70	-73.46	-80.78	-71.34	-92.28	-105.03	-157.09

Table D.7: “Back-Test” summary indicators of the *VaR* measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the *VaR* calculations - “Extreme Value Theory” in its dynamic version. Sample between August 1998 and February 2003.

Indicators	**Bonds Differentiated by Maturities in Years**																				**Summary Indicators**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
% excess over VaR	3.08%	2.33%	1.08%	1.26%	1.72%	1.39%	1.36%	1.68%	1.71%	1.31%	2.98%	1.33%	1.95%	1.97%	1.71%	1.11%	1.35%	2.02%	2.22%	2.04%	1.71%
***K* (Kupiec Test)**	6.41	4.46	0.03	0.41	1.74	0.60	0.76	3.43	1.49	0.33	6.07	0.38	1.84	3.36	1.47	0.05	0.33	2.80	5.52	4.94	1.79
Reject H_0	X	X									X								X	X	
Average VaR	-75.38	-91.85	-97.08	-86.78	-90.80	-80.92	-71.66	-64.76	-56.77	-53.80	-51.99	-56.96	-62.29	-67.30	-65.40	-71.10	-77.32	-84.47	-80.26	-96.24	-70.93
Average Excess over VaR	-11.99	-11.24	-9.79	-14.49	-21.89	-17.76	-15.10	-18.35	-15.75	-19.51	-11.48	-3.99	-26.90	-16.36	-21.76	-22.63	-13.83	-26.42	-13.41	-20.59	-16.76
Maximum Excess over VaR	-29.72	-27.66	-21.04	-41.14	-62.84	-60.67	-33.66	-46.64	-39.14	-56.35	-39.91	-9.24	-55.49	-60.92	-64.13	-65.38	-35.74	-62.80	-61.39	-57.33	-65.38

Table D.8: “Back-Test” summary indicators of the VaR measures calculated with the “fair” panel, against the market data from the Santiago Stock Exchange. Confidence level of 1% for the VaR calculations - Monte Carlo simulation method, using the stochastic structure developed in Cortázar, Schwartz and Naranjo (2003) with a three-factor Vasicek model. Sample between August 1998 and February 2003.

Indicators	**Bonds Differentiated by Maturities in Years**																				**Summary Indicators**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	
% excess over VaR	4.00%	4.37%	2.53%	2.70%	3.07%	2.12%	1.98%	2.19%	2.34%	1.98%	3.34%	1.58%	2.65%	1.76%	2.28%	1.77%	2.40%	2.63%	2.95%	2.48%	2.47%
***K* (Kupiec Test)**	11.76	21.61	6.16	12.66	11.40	4.15	5.04	9.63	4.61	2.88	8.06	1.08	4.85	2.20	4.24	1.74	4.24	6.45	12.52	9.22	6.57
Reject H_0	X	X	X	X	X	X	X	X	X		X		X		X		X	X	X	X	X
Average VaR	-73.44	-86.98	-93.28	-90.66	-84.09	-76.73	-70.44	-60.88	-59.16	-55.99	-50.11	-52.19	-55.00	-61.45	-65.26	-68.62	-68.06	-75.05	-82.62	-88.12	-70.91
Average Excess over VaR	-20.50	-16.70	-10.78	-17.41	-26.32	-28.95	-19.36	-20.80	-22.15	-18.98	-15.79	-11.26	-29.19	-20.98	-34.25	-25.33	-16.66	-30.04	-15.64	-24.70	-21.00
Maximum Excess over VaR	-38.54	-35.41	-38.11	-55.84	-87.97	-89.81	-51.09	-63.26	-68.22	-79.57	-49.34	-16.74	-59.47	-67.41	-67.04	-73.02	-39.57	-72.79	-68.78	-72.42	-89.81