Liquidity and Optimal Market Transparency

Ariadna Dumitrescu*

ESADE Business School

October 2007

Abstract

In this paper we explore some of the consequences of greater market transparency for market performance in the presence of a strategic specialist. Although numerous studies have dealt with this issue, previous work has only considered either fully transparent or fully opaque markets. Our model allows for different levels of transparency, and therefore sheds light on how transparency affects market performance. We show that an intermediate level of transparency can improve market performance relative to the more extreme cases of full transparency or no transparency at all.

JEL Classification Numbers: D82, G12, G14.

Keywords: Market Liquidity, Market Transparency, Strategic Specialist.

.

*I am very grateful to Jordi Caballé, Javier Gil-Bazo, Timothy Kehoe, Roberto Pascual and participants at XIV Foro de Finanzas and XXXI Simposio de Análisis Económico, for their comments. Any errors are my own responsability.

Correspondence address: Ariadna Dumitrescu, ESADE Business School, Av. Pedralbes, 60-62, Barcelona, 08034, Spain. *Phone:* (34) 934 952 191, *e-mail:* ariadna.dumitrescu@esade.edu

1 Introduction

Market transparency has been a fundamental issue in recent academic and policy debate concerning the organization and structure of financial markets, since it is generally considered to play an important role in promoting market fairness and efficiency. A central attribute of financial markets, transparency represents the degree to which information regarding quotations for securities, the prices of transactions, the volume of those transactions and source of order flow is made publicly available. Although there is a continuum of degrees of transparency, the existent studies have focused mainly on the extreme cases, either an opaque market or a fully transparent market. In this paper we consider a situation where different levels of transparency are possible, and show that an intermediate level of market transparency might improve market performance.

By transparency we understand the disclosure of information to market participants. The timing of information disclosure gives two different dimensions to transparency: pre-trade and post-trade transparency. Pre-trade transparency refers to order-book information regarding the size and price of prospective trading interest, such as firm quotations in representative size, and resting limit orders, both at the best firm bid and ask quotations and away from such quotations. Pre-trade market transparency can be very difficult to achieve in markets where there are no mechanisms to consolidate the quotes and make them accessible to investors. However, markets like NASDAQ or the London Stock Exchange have an electronically connected platform which is publicly accessible.

Post-trade transparency refers to the quick dissemination of past trade price and volume of completed transactions for that security. Once again, the presence of an electronic platform would facilitate the transmission of public information to traders.

Financial markets exhibit different degree of transparency. For example, the automated limit-order book markets such as Paris Bourse CAC (Cotation Assistee en Continu), Toronto Stock Exchange CATS (Computer Assisted Trading System), and Spanish Stock Exchange SIBE (Sistema de Interconexión Bursátil Español) provide information on the entire limit order book. On the other hand, U.S. markets are generally less transparent (AMEX, CBOT and other regional exchanges). This might be due to the fact that they are usually floor-based markets where the orders are placed either by a specialist or by individual brokers.

The NYSE initiated a program in 2002 called OpenBook through which they started selling the information on the limit order book. This system allows traders to observe the depth in the book for all prices (previously, only the best bid and ask prices were observed). The specialist still has some private information on the individual orders that make up the book. This great variety of market structures divides both regulators and academics in the debate regarding the costs and benefits of transparency. Thus, the U.S. Security Exchange Commission and the U.K. Office of Fair Trading believe that transparency plays a vital role in market efficiency: "transparency [...] helps to link dispersed markets and improve the price discovery, fairness, competitiveness and attractiveness of U.S. markets."[1] On the other hand, the U.K. Securities and Investment Board opposes an increase in transparency since in their view there is a trade-off between liquidity and transparency.

This paper addresses for the first time the impact of different degrees of transparency on market performance. We consider a setup where there is a specialist who behaves strategically. Due to his position in the market, the specialist observes the component of order flow which is not sensitive to price movements. We also allow the value-informed traders to receive a signal about this component of the order flow, which permits us to model different degrees of transparency defined as the variance of this signal. We find that too much or too little transparency induces a great reduction in market liquidity (measured as the volume needed to move the price by one unit). The intuition for this result is the following. When transparency is too high, too much information is revealed to the market. The value-informed traders thus receive precise information about the price non-sensitive component of the order flow, which reduces dramatically the informational advantage of the specialist. Therefore, the specialist cannot hide anymore and his trading has a large impact on price. On the other hand, in a less transparent market, the price, which aggregates all the information, becomes very noisy. Since traders use the price as a signal either about the liquidation value of the asset or about supply, their trading decision is affected by this noise. For example, if value-informed traders increase their demand based on this noisy information, then the price increases by a given quantity. The specialist observes this increase in price and he associates it with good news about the liquidation value (he knows

[1] See Bloomfield and O'Hara (1999), quote from SEC Market Study, Chapter IV-1.

the supply, so the price increase cannot be due to a decrease in supply). As a result, he increases his demand and this has a further positive impact on price. When the information inferred from prices is very noisy and traders use this erroneous signal when placing their demands, we might have a situation where traders prefer not to trade and therefore, no equilibrium exists. The results of this paper provide support to Madhavan's (2000) view that some disclosure might be better for market performance than no disclosure at all. Consequently, our results have important implications on market design as they suggest that an intermediate level of transparency would improve market performance.

The literature concerned with the effects of market transparency is extensive, but the results are not conclusive as to whether market transparency improves market performance or not. On one hand, the theoretical models that study how transparency affects market performance reach mixed conclusions. While transparency can have a positive effect by reducing adverse selection and therefore the spread quoted by the dealer, it can also have a negative effect on an illiquid market (Admati and Pfleiderer (1991), Pagano and Röell (1996), Madhavan (1995, 1996) and Madhavan et al. (2005), Baruch (2005), Rindi (2007)). On the other hand, empirical studies suggest that in transparent markets, traders are reluctant to place limit orders since in this way they might extend a free option to others and face the risk of being "picked off." This literature is limited to a few studies, mainly because of the lack of detailed data. The studies are more concerned with one-time events that offer the possibility to evaluate the effects of a structural change in market transparency.[2] As in the theoretical literature, there is also a lack of agreement of the empirical evidence concerned with the effects of pre-trade transparency. On one hand, Madhavan et al. (2005), conclude that greater limit order book transparency can reduce liquidity. On the other hand, Boehmer et al. (2005) find that increasing transparency leads to more aggressive limit order submission and increase in market depth. Also, Hendershott and Jones (2005) show that a reduction in transparency worsens price discovery and increases trading costs.[3]

[2] Another strand of literature conducts experiments to overcome the lack of detailed data and the existence of only a few structural changes that provide "natural experiments." They give an interesting insight into several issues relating to the effects of different market microstructure and reach contradictory conclusions (see Bloomfield and O'Hara (1999), Flood et al. (1999), Bloomfield and O'Hara (2000)).

[3] Most of the empirical studies were concerned with equity markets, which were already quite transparent. Lately, several papers Bessembinder et al. (2006), Goldstein et al. (2007), Edwards et al.(2007) study the

Finally, several other authors study the impact on market liquidity of the disclosure of broker identifiers in limit order books. Theoretical studies in this strand tend to predict that anonymous trading systems will attract more informed trading (see, for example Röell (1990), Fishman and Longstaff (1992), Forster and George (1992)). A model proposed by Foucault et al. (2003) studies the decision of Euronext Paris to stop displaying the identities of limit orders traders and predicts that large traders in a transparent regime (where limit orders are non-anonymous) will post worse prices to reduce free riding by uninformed traders. Also, Simaan et al. (2003), examine pre-trade transparency (anonymity of quotes posted in dealer market) and found that spreads quoted by a dealer are tighter in a market where he is anonymous. Finally, Waisburd (2003) analyzes the post-trade anonymity also in Paris Bourse and obtains opposite results. Therefore, pre-trade and post-trade transparency have very different effects and in order to reconcile the great variety of results in the previous literature we have to be cautious about which dimension of transparency is considered in each study. The problem of deciding on the optimal degree of transparency becomes even more challenging if we consider the great differences in markets' architecture (see O'Hara (2007)).

Our model is related to the above mentioned anonymity models and Madhavan (1996). These models also study the effects of informing market participants on the composition of the order flow and find mixed results on the effect of transparency on liquidity. A different view is given by Baruch (2005) who shows that opening the limit order book improves liquidity and informational efficiency. All these models study the effects of pre-transparency considering the possibility that a component of the order flow or limit order book is observable by some market participants, as is the case of the NYSE' specialist system.

The present model differs from Madhavan (1996) and Baruch (2005) in two important ways. As in those papers, we consider that some market participants can observe a component of the order flow. However, the specialist in our model uses his information on the order flow strategically, taking into account the effect that trading on this information has both on price and the other traders' demand. Moreover, the specialist in our model can

impact of the transaction disclosure in a set of corporate bonds and obtain a positive effect on liquidity.

place limit orders. The specialist may have incentives to place limit orders because limit orders preserve his anonymity. As Simaan et al. (2003) point out, placing a limit order on an Electronic Communication Network (ECN) is different from posting a quote directly onto the quote montage because ECN limit orders are displayed anonymously, while a direct quote identifies the dealers. Posting a limit order instead of a quote reduces market transparency since the specialist's trade cannot be identified. However, when placing limit orders, the specialist reveals part of order flow information he has, thus improving the amount of disclosed information on the order flow. Moreover, since he submits limit orders and therefore conditions on price, he can infer noisy information about the orders of the other informed traders. Since he also receives information about the supply, he actually ends up having information about the entire order book.

The organization of the paper is the following. Section 2 presents the strategic specialist model with transparency. Section 3 presents the market indicators and analyzes the effect transparency has on market performance. Finally, Section 4 concludes. Proofs of the characterization of the equilibrium and calculations of market indicators are presented in the Appendix.

2 The Model

We consider an economy where several informed traders and a strategic specialist trade a risky asset.[4] In order to emphasize the role of the specialist, we consider a simpler setup where traders are risk neutral and behave strategically by taking into account the effect of their trading on prices. There are no uninformed traders. In order to emphasize the role played by the specialist, we model the noise by assuming a random supply and assume that the specialist receives perfect information. This approach has been used before by Gennotte and Leland (1990) who consider a model where speculators possess private and diverse information. This model extends the model in Dumitrescu (2007), which studies the role of a strategic specialist on market performance. However, my model differs from this one in that I study here how different levels of transparency affect market performance.

[4] This framework is similar to the one in Kyle (1989) to which we add a strategic specialist (see also Dumitrescu (2007)).

This analysis is performed allowing the value-informed traders to also receive information about supply.

There is a single security in the market that trades at market clearing price $\widetilde{p}$ and yields an exogenous liquidation value $\widetilde{v}$, which has a normal distribution with mean $\overline{v}$ and variance V_v. There are N value-informed traders, indexed $n = 1, ..., N$ and a specialist. Since in most of the cases, there is only one specialist trading in the stock, we assume here that there is only one specialist. The value-informed trader n observes a private signal $\widetilde{i}_n = \widetilde{v} + \widetilde{e}_n$. We assume that $\widetilde{e}_n$ is distributed $N(0, V_e)$ for all $n = 1, ..., N$. We suppose that for any $j \neq n$ $\widetilde{e}_j$ and $\widetilde{e}_n$ are uncorrelated and moreover, they are uncorrelated with all the other random variables in the model. The specialist observes a private signal about the random supply $\widetilde{S}$ which is normally distributed with mean 0 and variance $V_S > 0$. However, in order to understand the effect of transparency on market performance, we assume that the value-informed traders also receive a signal about the supply $\widetilde{s} = \widetilde{S} + \widetilde{\varepsilon}$, where $\widetilde{\varepsilon}$ is normal distributed with mean 0 and variance V_ε. I define transparency as the quality of this signal, the variance of $\varepsilon, V_\varepsilon$. Thus, the transparency in my model is reflected by the quality of disclosed information about the limit order book. When the information about the supply received by the value-informed traders is very precise, the market is highly transparent. The value-informed traders can easily infer information about the order book. They receive information about the price-insensitive component of the order flow and can infer from the price, the orders of the specialist and the average order of the value-informed traders (they actually infer the average signal the value-informed traders receive). Therefore, they can practically observe with some noise the order flow. Of course, the more precise the information about the supply (i.e. the more transparent the market), the greater their inference about the order flow. When the information about supply is very noisy, the market is less transparent, and their inference very poor.

Traders understand that prices partially reveal their private information when they place their orders. The link between information and prices via trades provides an explicit mechanism for information transmission between traders. The existence of private information means that a trader may have incentives to act strategically in order to maximize his profits. Therefore, given his private information, a trader maximizes his conditional expected profit taking into account the effect of his trading on prices and taking as given the strategies the other traders use to choose their demand schedules. When placing their orders, their demand schedules, the traders actually design strategies. The n^{th} value-informed trader has a strategy X_n which depends on his signal about liquidation value of the asset i_n, on his signal $\widetilde{s}$ about the supply and, since he submits limit orders, on the price. Similarly, the specialist has a strategy Y, which depends on the signal he receives $\widetilde{S}$ and the price p. Given a market clearing price p, the quantities traded by value-informed traders and specialist can be written $x_n = X_n(i_n, s, p)$, $n = 1, ..., N$ and $y = Y(S, p)$. In the above notations, a tilde distinguishes a random variable from its realization. Thus, x_n denotes a particular realization of $\widetilde{x}_n$.

The price of the asset is set such that the market clears. The traders submit their demand schedules to an auctioneer who aggregates all the schedules submitted, calculates the market clearing price and allocates quantities to satisfy traders' demand. Thus, the market clearing price $\widetilde{p}$ should satisfy with probability one

$$\sum_{n=1}^{N} X_n\left(\widetilde{i}_n, \widetilde{s}, \widetilde{p}\right) + Y\left(\widetilde{S}, \widetilde{p}\right) = \widetilde{S}.$$

Given their private information, traders maximize their conditional expected profits taking into account the effect of their trading on prices and taking as given the strategies other traders use to choose their demand schedules.

We look for a symmetric linear Bayesian Nash Equilibrium, that is, an equilibrium where the strategies X_n and Y are linear functions:

$$\begin{aligned} &X_n\left(\widetilde{i}_n, \widetilde{s}, \widetilde{p}\right) = \alpha^{PI} + \beta^{PI}\widetilde{i}_n + \delta^{PI}\widetilde{s} - \gamma^{PI}\widetilde{p}, \text{ for any } n = 1, ..., N \text{ and} \\ &Y\left(\widetilde{S}, \widetilde{p}\right) = \alpha^{SI} + \beta^{SI}\widetilde{S} - \gamma^{SI}\widetilde{p}, \end{aligned} \tag{1}$$

where $\alpha^{PI}, \beta^{PI}, \delta^{PI}, \gamma^{PI}, \alpha^{SI}, \beta^{SI}, \gamma^{SI}$ are constant coefficients determined in the equilibrium.

2.1 Characterization of the Equilibrium

The equilibrium has linear trading and pricing rules and is shown to be unique among all linear, symmetric Bayesian-Nash equilibria. As in most Kyle-type models, the linearities are not ex-ante imposed in the agents strategy sets: as long as the informed traders use linear trading strategies, the pricing rule will be linear and vice-versa.

Lemma 1 *In a symmetric linear equilibrium the optimal demand for the value-informed trader n and for the specialist are, respectively,*

$$x_n\left(\widetilde{i}_n, \widetilde{s}, \widetilde{p}\right) = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left[E\left(\widetilde{v}\left|\widetilde{p}, \widetilde{i}_n, \widetilde{s}\right.\right) - \widetilde{p}\right] \tag{2}$$

$$y(\widetilde{S}, \widetilde{p}) = N\gamma^{PI}\left[E\left(\widetilde{v}\left|\widetilde{p}, \widetilde{S}\right.\right) - \widetilde{p}\right] \tag{3}$$

with $\gamma^{PI} > 0$, *and* $(N-1)\gamma^{PI} + \gamma^{SI} > 0$.

We evaluate the expected value conditional on the information each trader receives and solve thus for the equilibrium.

Proposition 2 *There exists a linear symmetric equilibrium where agents' strategies are given by:*

$$\begin{aligned} x_n\left(\widetilde{i}_n, \widetilde{s}, \widetilde{p}\right) &= \alpha^{PI} + \beta^{PI}\widetilde{i}_n + \delta^{PI}\widetilde{s} - \gamma^{PI}\widetilde{p}, \text{ for any } n = 1, ..., N \text{ and} \\ y\left(\widetilde{S}, \widetilde{p}\right) &= \alpha^{SI} + \beta^{SI}\widetilde{S} - \gamma^{SI}\widetilde{p}, \end{aligned}$$

with $\alpha^{PI}, \beta^{PI}, \gamma^{PI}, \delta^{PI}, \alpha^{SI}, \beta^{SI}, \gamma^{SI}$ *solution of the system (18) defined in the Appendix.*

The equilibrium price is given by

$$p = \left(N\gamma^{PI} + \gamma^{SI}\right)^{-1}\left(N\alpha^{PI} + \alpha^{SI} + \beta^{PI}\sum_{n=1}^{N}\widetilde{i}_n + N\delta^{PI}\widetilde{s} + \left(\beta^{SI} - 1\right)\widetilde{S}\right). \tag{4}$$

Due to the analytical complexity of the problem, we solve it numerically. First, we determine the coefficients that measure the aggressiveness of trading on private information and then study several market indicators. We perform comparative statics with respect to two variables: the variance of the error of the signal V_ε and the variance of the supply shock V_S. Notice that the quality of the signal received by the value-informed trader, V_ε, is a measure of transparency. The lower the variance of the error V_ε, the better is the information about the supply and therefore, the greater the transparency.

Let us first analyze, how the coefficient β^{PI}, which represents the intensity of trading on private information concerning liquidation value, changes for different levels of transparency.

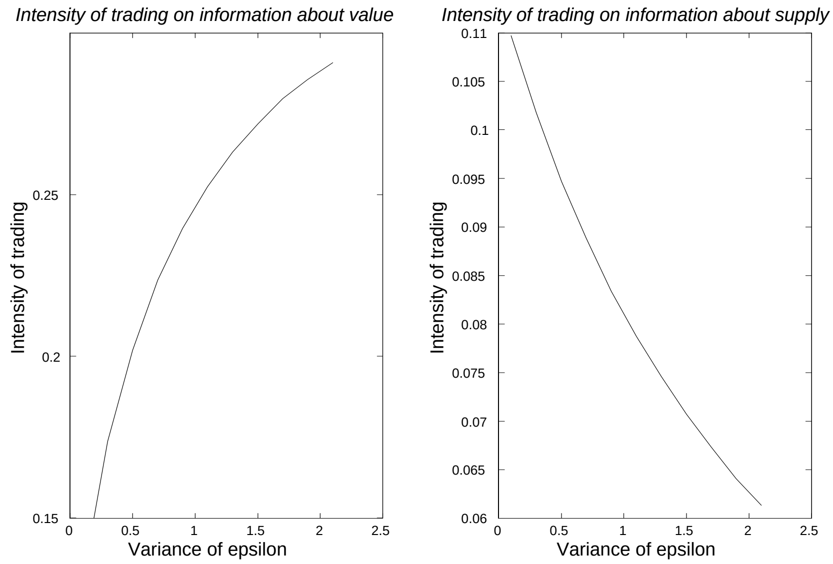

Figure 1: Intensity of trading on information. Comparative Statics with respect to the variance of supply signal V_ε. Parameters value: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_S = 1$.

As the variance of the error increase V_ε, the value-informed traders rely more on their private information and therefore trade more intensively on it (see Figure 1). Notice that for very low transparency (very noisy signal about the supply) there is no equilibrium[5]. Similarly, when we study the variation of the trading intensity on information about supply, δ^{PI}, we find that the higher the variance of the error the lower the intensity of trading (see Figure 1). As a result, the lower the transparency, the more intensively the value-informed trade on their private information on liquidation value and the less intensively on the information regarding supply. This is consistent with the view that in less transparent markets, the value-informed traders can hide better, making better use of their informational advantage.

On the other hand, when we look at how these trading intensities vary with respect to the variance of supply (see Figure 2), we find that the higher the variance of the supply V_S, the higher the trading intensity of trading both on the information about the supply and information about the liquidation value of the asset. This means that the value-informed

[5] This is similar to Kyle's (1989) condition $N > 2$, or Dumitrescu (2007), $N(N-2) > \frac{V_e}{V_v}$ and it tells us that too much noise in the market may lead to a situation in which there is no equilibrium. This is due to the fact that poor information about supply is aggregated in prices and then used further by all the traders who use the price as a signal.

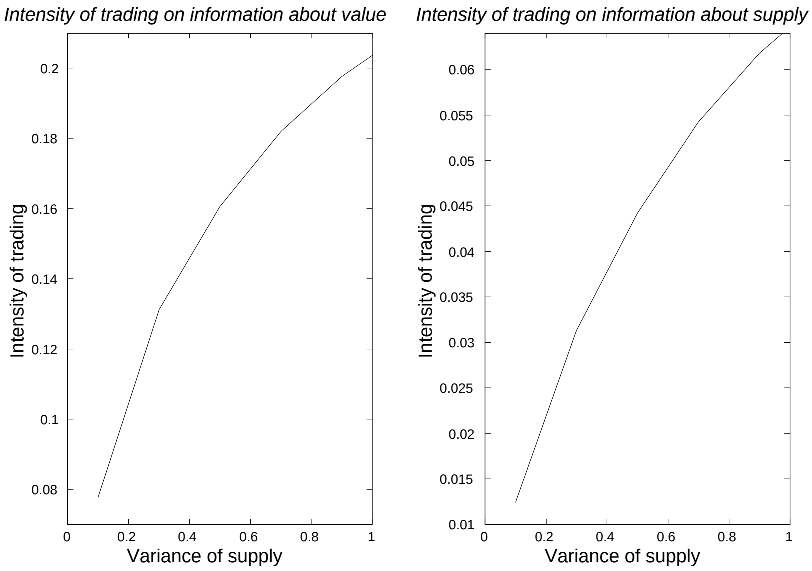

Figure 2: Intensity of trading on information. Comparative Statics with respect to the variance of supply V_S. Parameters value: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_\varepsilon = 1$.

traders understand that they are in the same situation as the specialist with respect to the noise of the supply, and therefore, increase their intensity of trading as more noise there is in the market.

3 Transparency and Market Performance

The focus of our work is on understanding how different degrees of transparency affect market performance. To this end, we study how transparency affects several market indicators: market liquidity, informativeness of prices, price volatility, and expected profits of value-informed traders. We first study market liquidity. Various measures of market liquidity are used in the literature: market depth, bid-ask spread and price movement after trade. We use market depth (as defined by Kyle (1985)) as a measure of liquidity, which represents the trading volume needed to move prices one unit,

$$\gamma \equiv N\gamma^{PI} + \gamma^{SI},$$

where γ^{PI}, γ^{SI} are the coefficients characterized by (18).

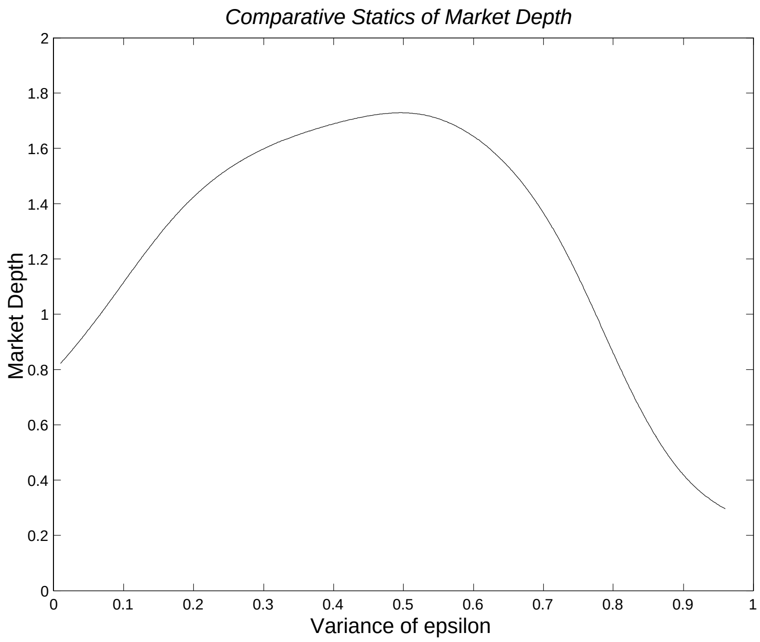

Figure 3: Market Depth. Comparative Statics with respect to the variance of supply V_ε. Parameters value: $N = 5$, $V_v = 0.2$, $V_e = 0.5$, $V_S = 1$.

We have found that market depth is highest for an intermediate level of transparency. Too much or too little transparency induces a marked reduction in market liquidity, as it can be seen in Figure 3. On the one hand, too little transparency (high variance of the noise V_ε) leads to low market depth. In this case, the specialist has private information about the supply and since the variance of the noise V_ε is high, he has a great informational advantage. Moreover, he can infer from price the assets' liquidation value. As we have already explained, the specialist can disentangle a signal from price which is informationally equivalent to the vector of signals received by the value-informed traders (it is equal to the average information of the value-informed traders). When the variance of the noise of their private signals about the supply, V_ε is high, the price is a very noisy signal about the liquidation value of the asset. Since the specialist uses this signal when choosing his trading strategy and this signal is erroneous, it alters both his strategy and the information revealed by him about supply. So, the value-informed traders, who infer from prices information about supply, fail to do so because the information about supply contained in prices is erroneous (it is based on the poor signal). As a result, when there is too much noise in the market, the propagation of this poor quality signal might lead to a situation where equilibrium does not exist.

On the other hand, too much transparency leads to too much information being revealed. Note that an increase in transparency in our model is equivalent to a decrease in the variance of the error of the signal about supply V_ε received by value-informed traders. As transparency increases the value-informed traders receive better information about supply. So, the increase in transparency can have a negative effect on market liquidity because the disclosure of information about supply puts the specialist in a worse position, reducing his informational advantage on the order flow and thus increasing the impact his trading will have on prices.

Consequently, transparency has two opposite effects on market liquidity and therefore, there is a trade-off which leads to an inverted U-shape relationship between market depth and transparency. This makes that the optimal level of transparency (the one that maximizes the market liquidity) to be an intermediary level of transparency.

I omit here the analysis with respect to the asymmetry of information about the liquidation value of the asset (the variance of liquidation value of the asset), V_v because the behavior is very similar to the results obtained in Dumitrescu (2007). Note that the relationship between market liquidity and the variance of liquidation value, V_v is also inverted U-shape. It has been proved analytically there that the market depth is inverted U-shape with respect to the variance of the liquidation value of the asset. However, in that case there was no exogenous disclosure of information about supply. However, since the traders submit limit orders, they can infer some of this information from price. Therefore, there is some endogenous disclosure of the private information. The same result applies here, and therefore we have the same relationship between the market liquidity and asymmetry of information about the liquidation value of the asset.

We also study how price volatility is affected by transparency. We measure price volatility as the variance of price.

Corollary 3 *The volatility of price is equal to*

$$Var\left(\widetilde{p}\right) = \frac{\left(\left(N\beta^{PI}\right)^2 V_v + N\left(\beta^{PI}\right)^2 V_e + \left(N\delta^{PI}\right)^2 V_\varepsilon + (1-\beta^{SI}-N\delta^{PI})^2 V_S\right)}{\left(N\gamma^{PI}+\gamma^{SI}\right)^2}.$$

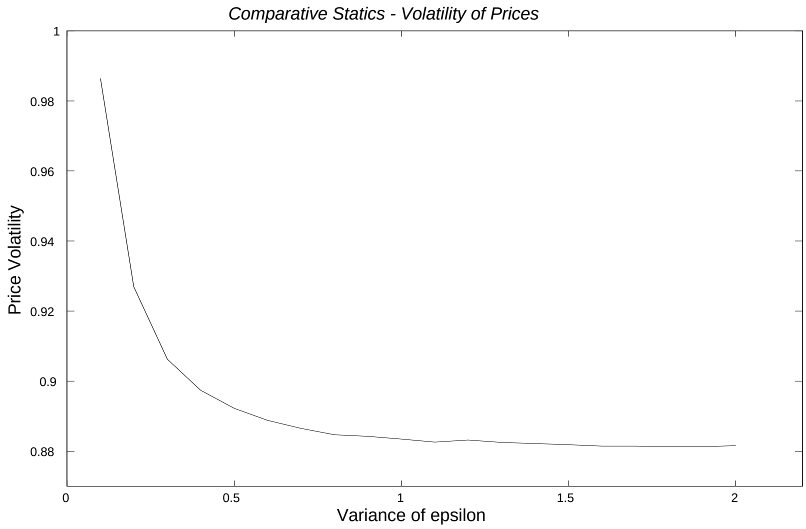

Figure 4: Comparative Statics with respect to the variance of supply signal V_ε. Parameters value: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_S = 1$.

The volatility of prices is affected in a similar way by changes in transparency, the results being driven by the same bidirectional flow of information. When the market transparency increases a lot, the information about supply revealed in prices is higher. Consequently, as the transparency increases (i.e. V_ε decreases), more information is aggregated in prices and therefore, the price volatility increases (see Figure 4). However, when the transparency decreases significantly, too much noise in the market produces market breakdown. Our result is similar to Madhavan (1996) in the sense that we obtain a negative relationship between market depth and price volatility, but the behavior of both market depth and price volatility is very different.

We study next the informativeness of prices, defined as the reduction in the variance of liquidation value upon observing the price.

Corollary 4 *The informativeness of price is equal to*

$$Var\left(\widetilde{v}\right) - Var\left(\widetilde{v}\middle|\,\widetilde{p}\right) = \frac{\left(N\beta^{PI}V_v\right)^2}{\left(\left(N\beta^{PI}\right)^2 V_v + N\left(\beta^{PI}\right)^2 V_e + \left(N\delta^{PI}\right)^2 V_\varepsilon + (1-\beta^{SI}-N\delta^{PI})^2 V_S\right)}.$$

As we can see in Figure 5, the price informativeness decreases when transparency increases. Despite more information being aggregated in price, the prices provide less infor-

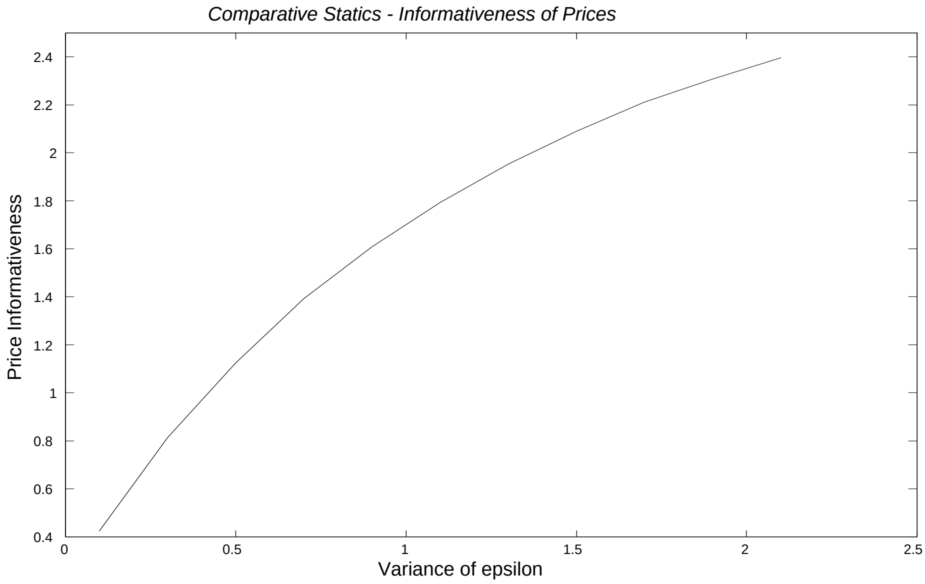

Figure 5: Informativeness of Prices. Comparative Statics with respect to the variance of supply signal V_ε. Parameters values: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_S = 1$.

mation on the liquidation value of the asset because there are two types of information that aggregates in prices. By losing his informational advantage on the information about supply, the specialist also loses the ability to induce the value-informed traders to reveal more of their information. Since they receive a good quality signal about the supply, they can strategically control the quantity of information they reveal. Note however, that the traders use the price in combination with their private information to better understand the information regarding the liquidation value of the assets or the information about supply. Thus, as we explained earlier, the specialist disentangle from price the information revealed about the asset value and the information about supply. Similarly, the value-informed traders are able to understand better the revealed information about supply and the average signal about the liquidation value of the asset.

Finally, we study the expected volume traded both by value-informed and specialist traders. Since the expected demand for both type of traders is zero, the expected trading volume depends only on the variance of the demand.

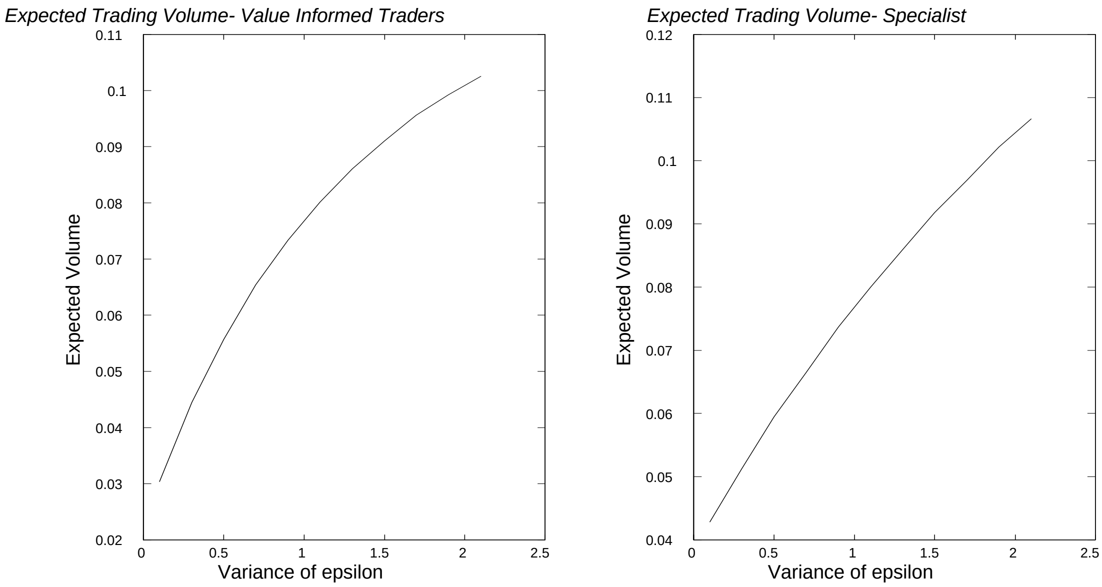

Figure 6: Expected Trading Volume. Comparative Statics with respect to the variance of supply signal V_ε. Parameters value: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_S = 1$.

Corollary 5 *The trading volume of value-informed traders and specialist are*

$$E(|x_n|) = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \left(\left(\beta^{PI}\right)^2 (V_v + V_e) + \left(\delta^{PI}\right)^2 (V_S + V_\varepsilon) + \left(\gamma^{PI}\right)^2 Var(\widetilde{p})\right)$$
$$E(|y|) = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \left(\left(\beta^{SI}\right)^2 V_S + \left(\gamma^{SI}\right)^2 Var(\widetilde{p})\right).$$

As the market approaches full transparency, the volume of trading both for the value-informed traders and the specialist decreases mainly because the asymmetry of information is greatly reduced (see Figure 6). This is in line with the stylized facts from the empirical literature, which show that the lower the asymmetry of information, the lower the volume of trading of informed traders. This applies also to the case of the specialist. Boehmer at el. (2005) show that in the case in which the OpenBook is introduced, the trading participation of the specialist declines. However, at the other extreme, as the market becomes almost opaque, the signals received by the value-informed traders about supply are very noisy and, as we have explained before, too much noise in the market leads to a situation where the equilibrium does not exist.

We also study how the market indicators are affected when the noise in the market is

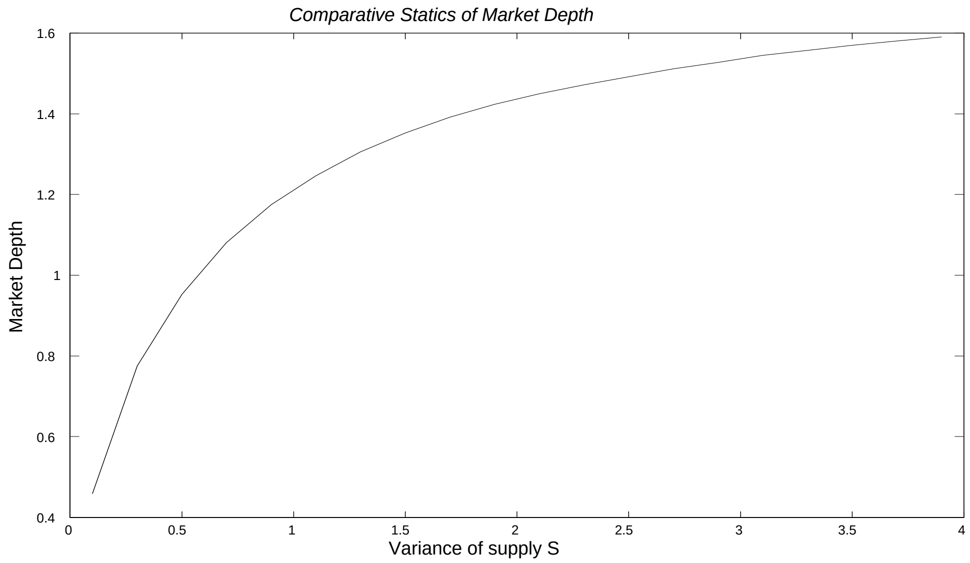

Figure 7: Comparative Statics with respect to the variance of supply signal V_S. Parameters value: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_\varepsilon = 0.5$.

introduced in a different way, namely through the variance of the price-insensitive component of supply, V_S. As we can see in Figure 7, market liquidity increases as the variance of the supply increases and this can be explained, as in the most Kyle-type setups, by the fact that the existence of more noise in the market gives traders greater opportunities to hide their activities. An interesting result is the one concerned with the behavior of price volatility. We obtain a U-shape relationship between volatility and the variance of supply V_S (see Figure 8). This result is very different from the other Kyle-type of models and the results in Dumitrescu (2007), where the volatility of prices does not depend on the noise in supply. Therefore, this behavior it is not due to the presence of the strategic specialist, but to the release of some information about supply to the value-informed traders. Looking at the way value-informed traders use their information about the supply, we see that the greater the noise in supply the more intensively they trade on this information, hence the greater is the amount of information revealed by prices (see also Figure 9). However, when the variance of the noise is very small, the specialists loses his informational advantage and therefore trade less on their information about supply. However, in this case they will rely more on the information they infer from price and trade more aggressively on this

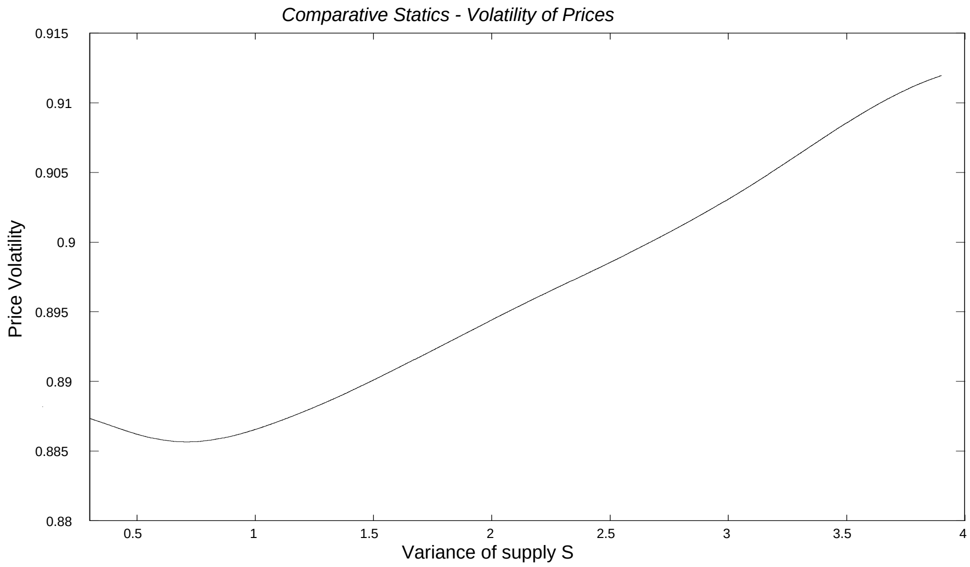

Figure 8: Comparative Statics with respect to the variance of supply signal V_S. Parameters value: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_\varepsilon = 0.5$.

information.

As expected, we find that the greater the noise in supply, the greater the volume of trading. Once again these results are due to the fact that when there is a lot of noise in the market, traders can hide better and trade more intensively on their information (see Figure 10).

These results have important policy implications. Our findings suggest that disclosing some information about the price-inelastic component of the order flow, can be beneficial for market performance. The first reason why we obtain results different from the ones in the previous literature is the fact that our model permits us to study a continuous of transparency levels.

Notice that the conclusions of our model stem from two important facts. First, the presence of a strategic specialist who trades on the information about supply has an important effect on the revelation of information as both the specialist and the value-informed traders will reveal more of the information they own. Thus, all market participants can infer from price (in addition to the private information they own) a part of the information the other traders own. Second, the value-informed traders receive some noisy information

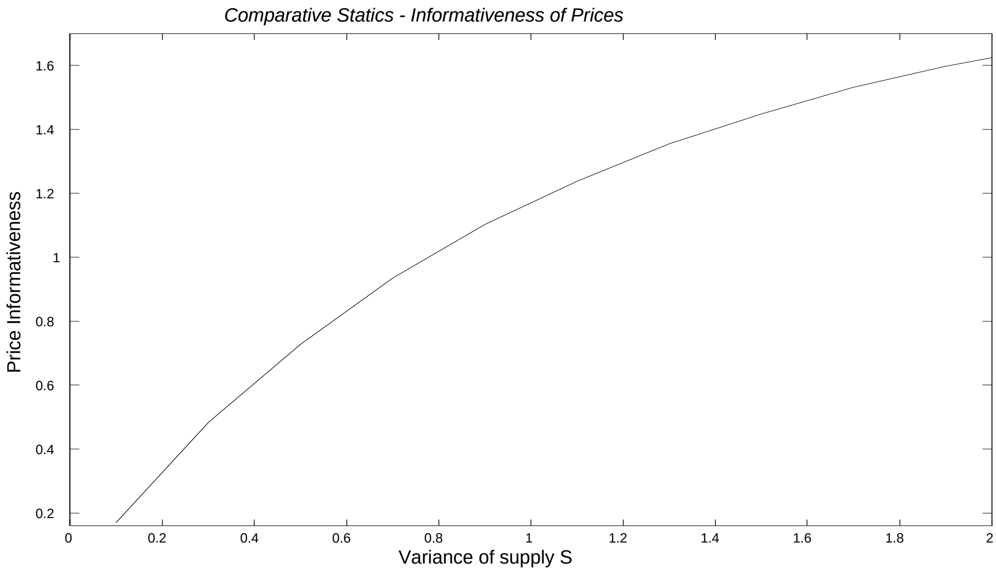

Figure 9: Comparative Statics with respect to the variance of supply signal V_S. Parameters value: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_\varepsilon = 0.5$.

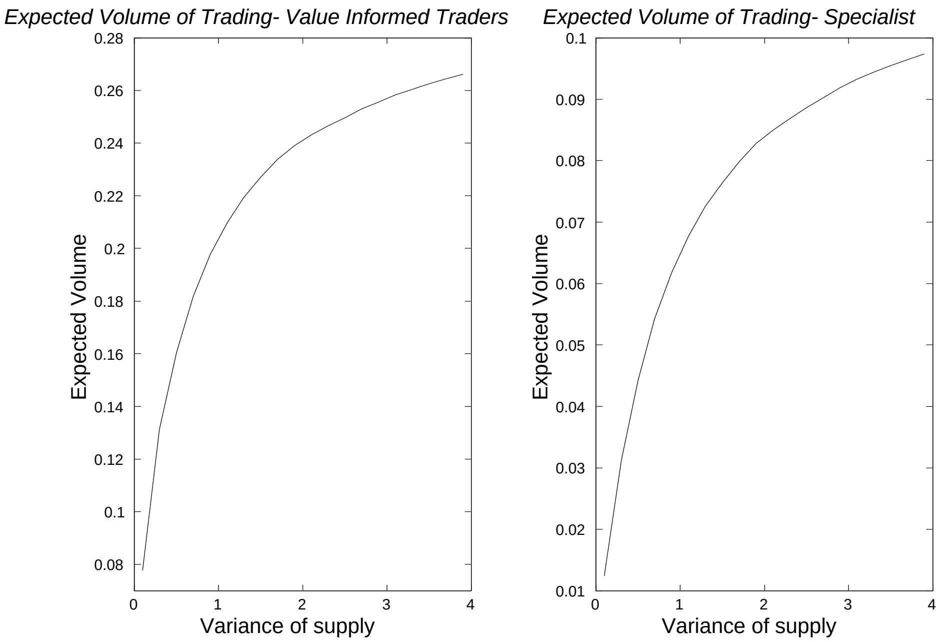

Figure 10: Comparative Statics with respect to the variance of supply signal V_S. Parameters value: $N = 5$, $V_v = 1$, $V_e = 0.5$, $V_\varepsilon = 0.5$.

about the price-insensitive component of supply. Notice that in Madhavan (1996), where the entire amount of supply was revealed to all market participants, the transparency had very different effects. In order to see Madhavan's case in our analysis, we have to look at values for which the variance of the signal V_ε is very small (transparency is high). This resembles the case of full transparency considered by Madhavan (1996). Note that for these values we obtain exactly the opposite results: transparency increases volatility and reduces market liquidity. However, these results are not due to the fact that the value-informed traders receive noisy information and not the entire price-insensitive component of supply. The presence of the specialist who behaves strategically and his role as liquidity provider is very important. Madhavan (1996) considers imperfect competition in liquidity provision but does not consider the presence of the specialist who also behaves strategically. On the other hand, Baruch (2005) considers liquidity suppliers who have market power over the information about the liquidation value of the asset and behave strategically and also considers the existence of a specialist. However, in their model the specialist is a follower whose role is to clear the market. In our setup, the specialist makes use of his informational advantage when choosing the quantity to trade in order to maximize his profits, but he also reveals some of the information he owns through his trading.

4 Conclusions

The implications of different degrees of transparency of financial markets are at the heart of an important debate in market design. However, the previous theoretical literature considers only two cases: a fully transparent market or a fully opaque market. As a result, the policy implications of these models exclude the possibility that some disclosure can be more beneficial than full disclosure or no disclosure at all.

The performance of financial markets depends on the trade-off between transparency (thought to promote competition, fairness and investor protection) and opacity (in the interest of encouraging ongoing participation of liquidity providers). In this paper, we explore some of the consequences of different levels of market transparency on market performance when the specialist behaves strategically. The fact that the strategic specialist and the value-informed traders submit limit orders and the possibility for all traders to observe a noisy signal about the price insensitive component of the order flow, permit us to consider a setup where we have different degrees of transparency. Varying the quality of the signal received by the value-informed traders gives us different degrees of transparency. Thus, we may have all possible ranges from a fully transparent market (when the quality is very good) or an opaque market (when the quality is very poor).

We perform our analysis in this setup and find that the relationships between market transparency and market liquidity is non-monothonical. More precisely, we find an inverted U-shape relationship between market liquidity and market transparency. Our analysis completes the previous studies which consider the effects of market transparency either in a fully transparent market or a fully opaque market. The results of our analysis suggest that an intermediate level of transparency leads to maximum market liquidity. Therefore, in limit order markets where traders behave strategically, some disclosure of information may increase liquidity. Thus, the prediction of our model is in line with the common belief that some disclosure can improve the market performance.

5 Appendix

Proof of Lemma 1. Let us first determine the optimal demand for the value-informed traders. The value-informed trader n considers the other players' strategies as given by (1). As a result, he is facing the following residual demand:

$$p = \frac{\alpha - \alpha^{PI} + \beta^{PI} \sum_{j \neq n} \widetilde{i_j} + (N-1)\,\delta^{PI} \widetilde{s} - (1-\beta^{SI})\widetilde{S}}{(N-1)\gamma^{PI} + \gamma^{SI}} + \frac{x_n}{(N-1)\gamma^{PI} + \gamma^{SI}}, \tag{5}$$

and he solves the following maximization problem:

$$\max_{x_n \in \mathbb{R}} E\left((\widetilde{v} - \widetilde{p})\, x_n \,\middle|\, \widetilde{p}, \widetilde{i_n}, \widetilde{s} \right) \Leftrightarrow$$

$$\max_{x_n \in \mathbb{R}} E\left(\left(\widetilde{v} - \frac{\alpha - \alpha^{PI} + \beta^{PI} \sum_{j \neq n} \widetilde{i_j} + (N-1)\,\delta^{PI} \widetilde{s} - (1-\beta^{SI})\widetilde{S} - x_n}{(N-1)\gamma^{PI} + \gamma^{SI}} \right) x_n \,\middle|\, \widetilde{i_n}, \widetilde{s}, \widetilde{p} \right).$$

The first order condition for this problem is

$$E\left(\widetilde{v}|\widetilde{p},\widetilde{i_n},\widetilde{s}\right)-E\left(\left.\frac{\alpha-\alpha^{PI}+\beta^{PI}\sum_{j\neq n}\widetilde{i_j}+(N-1)\,\delta^{PI}\widetilde{s}-(1-\beta^{SI})\widetilde{S}}{(N-1)\gamma^{PI}+\gamma^{SI}}\right|\widetilde{i_n},\widetilde{s},\widetilde{p}\right)$$
$$-\frac{2x_n}{(N-1)\gamma^{PI}+\gamma^{SI}}=0. \tag{6}$$

Using (5) we can write further (6) as

$$E\left(\widetilde{v}\left|\widetilde{p},\widetilde{i_n},\widetilde{s}\right.\right)-p-\frac{x_n}{(N-1)\gamma^{PI}+\gamma^{SI}}=0,$$

and from here we find that the optimal demand of value-informed trader n is:

$$x_n=((N-1)\gamma^{PI}+\gamma^{SI})\left(E\left(\widetilde{v}\left|\widetilde{i_n},\widetilde{s},\widetilde{p}\right.\right)-p\right).$$

The second order sufficient condition for this maximization problem is

$$-\frac{2}{(N-1)\gamma^{PI}+\gamma^{SI}}<0\Leftrightarrow(N-1)\gamma^{PI}+\gamma^{SI}>0.$$

Similarly, the specialist takes as given the strategies of the value-informed traders and in conformity with (1). The residual demand faced by him is therefore

$$p=\frac{N\alpha^{PI}+N\beta^{PI}\widetilde{v}+\beta^{PI}\sum_{n=1}^{N}\widetilde{e_n}+N\delta^{PI}\widetilde{s}-\widetilde{S}}{N\gamma^{PI}}+\frac{y}{N\gamma^{PI}}. \tag{7}$$

The specialist solves the following maximization problem:

$$\max_{y\in\mathbb{R}}E\left((\widetilde{v}-\widetilde{p})\,y\left|\widetilde{S},\widetilde{p}\right.\right)\Leftrightarrow$$
$$\max_{y\in\mathbb{R}}E\left(\left.\left(\widetilde{v}-\frac{N\alpha^{PI}+N\beta^{PI}\widetilde{v}+\beta^{PI}\sum_{n=1}^{N}\widetilde{e_n}+N\delta^{PI}\widetilde{s}-\widetilde{S}}{N\gamma^{PI}}+\frac{y}{N\gamma^{PI}}\right)y\right|\widetilde{S},\widetilde{p}\right).$$

The first order condition for this problem is

$$E\left(\widetilde{v}|\widetilde{p},\widetilde{S}\right)-E\left(\left.\frac{N\alpha^{PI}+N\beta^{PI}\widetilde{v}+\beta^{PI}\sum_{n=1}^{N}\widetilde{e_j}+N\delta^{PI}\widetilde{s}-\widetilde{S}}{N\gamma^{PI}}\right|\widetilde{S},\widetilde{p}\right)-\frac{2y}{N\gamma^{PI}}=0. \tag{8}$$

Using (7) we can write further (8) as

$$E\left(\widetilde{v}\left|\widetilde{S},\widetilde{p}\right.\right)-p-\frac{y}{N\gamma^{PI}}=0,$$

and from here we find the optimal demand of the specialist

$$y=N\gamma^{PI}\left(E\left(\widetilde{v}\left|\widetilde{S},\widetilde{p}\right.\right)-p\right).$$

The second order sufficient condition for this maximization problem is

$$-\frac{2}{N\gamma^{PI}}<0\Leftrightarrow N\gamma^{PI}>0.$$

Since $N\geq 1$ it results $\gamma^{PI}>0$. ■

Proof of Proposition 2. We look for a symmetric linear equilibrium. Therefore, we use the linear strategies defined in (1) and we can write the market clearing condition

$$\sum_{n=1}^{N}\widetilde{x_n}+\widetilde{y}=\widetilde{S}$$

as it follows:

$$N\alpha^{PI}+\beta^{PI}\sum_{n=1}^{N}\widetilde{i_n}+N\delta^{PI}\widetilde{s}-N\gamma^{PI}\widetilde{p}+\alpha^{SI}+\beta^{SI}\widetilde{S}-\gamma^{SI}\widetilde{p}=\widetilde{S}. \quad (9)$$

We define $\gamma\equiv N\gamma^{PI}+\gamma^{SI}$ and $\alpha\equiv N\alpha^{PI}+\alpha^{SI}$. In order to prove the proposition we first prove the following three lemmas.

Lemma A.1 *In a symmetric linear equilibrium for any $n=1,...,N$ we have*

$$\begin{aligned} E\left(\widetilde{v}\left|\widetilde{p}=p,\widetilde{i_n}=i_n,\widetilde{s}=s\right.\right) &= \overline{v}\left(1-A\left(N-1\right)\beta^{PI}-B\right)-A\alpha+(B-A\beta^{PI})\widetilde{i_n} \\ &\quad +\left(C-A\left(N\delta^{PI}+\beta^{SI}-1\right)\right)\widetilde{s}+A\gamma\widetilde{p}. \end{aligned}$$

Proof of Lemma A.1. We can rewrite the market clearing condition (9) as

$$\widetilde{p}\gamma-\alpha-\beta^{PI}\widetilde{i_n}-\left(N\delta^{PI}+\beta^{SI}-1\right)\widetilde{s}=\left(N-1\right)\beta^{PI}\widetilde{v}+\beta^{PI}\sum_{j\neq n}\widetilde{e_j}+(1-\beta^{SI})\widetilde{\varepsilon}. \quad (10)$$

From here it results that $\left(\widetilde{p},\widetilde{i_n},\widetilde{s}\right)$ is informationally equivalent to $\left(\widetilde{h_n},\widetilde{i_n},\widetilde{s}\right)$ where by definition $\widetilde{h_n}\equiv\left(N-1\right)\beta^{PI}\widetilde{v}+\beta^{PI}\sum_{j\neq n}\widetilde{e_j}+(1-\beta^{SI})\widetilde{\varepsilon}$. Consequently, we have that

$E\left(\widetilde{v}\middle|\widetilde{p}=p,\widetilde{i_n}=i_n,\widetilde{s}=s\right)=E\left(\widetilde{v}\middle|\widetilde{h_n}=h_n,\widetilde{i_n}=i_n,\widetilde{s}=s\right)$. Applying the projection theorem for normally distributed random variables we obtain that

$$E\left(\widetilde{v}\middle|\widetilde{h_n}=h_n,\widetilde{i_n}=i_n,\widetilde{s}=s\right)=\overline{v}+\begin{pmatrix} A & B & C \end{pmatrix}\begin{pmatrix} \widetilde{h_n}-E\left(\widetilde{h_n}\right) \\ \widetilde{i_n}-E\left(\widetilde{i_n}\right) \\ \widetilde{s}-E\left(\widetilde{s}\right) \end{pmatrix}, \tag{11}$$

where $\begin{pmatrix} A & B & C \end{pmatrix}=cov\left(\widetilde{v},\left(\widetilde{h_n},\widetilde{i_n},\widetilde{s}\right)\right)\left(var\left(\widetilde{h_n},\widetilde{i_n},\widetilde{s}\right)\right)^{-1}$, when $\left(var\left(\widetilde{h_n},\widetilde{i_n},\widetilde{s}\right)\right)^{-1}$ exists.

We compute $cov\left(\widetilde{v},\widetilde{h_n}\right)=cov\left(\widetilde{v},\ (N-1)\,\beta^{PI}\widetilde{v}+\ \beta^{PI}\sum_{j\neq n}\widetilde{e_j}+(1-\beta^{SI})\widetilde{\varepsilon}\right)=(N-1)\beta^{PI}V_v$. Hence, we have that

$$cov\left(\widetilde{v},\left(\widetilde{h_n},\widetilde{i_n},\widetilde{s}\right)\right)=\left(cov\left(\widetilde{v},\widetilde{h_n}\right),\ cov\left(\widetilde{v},\widetilde{i_n}\right),\ cov\left(\widetilde{v},\widetilde{s}\right)\right)=\left((N-1)\beta^{PI}V_v,\ V_v,0\right).$$

Then we calculate the variance matrix. We calculate firstly

$$\begin{aligned} var\left(\widetilde{h_n}\right) &= var\left((N-1)\,\beta^{PI}\widetilde{v}+\beta^{PI}\sum_{j\neq n}\widetilde{e_j}-(1-\beta^{SI})\widetilde{\varepsilon}\right)= \\ &= (\beta^{PI})^2(N-1)\left((N-1)V_v+V_e\right)+\left(1-\beta^{SI}\right)^2V_\varepsilon. \end{aligned}$$

In order to simplify the notation we define $q\equiv(N-1)\left((N-1)V_v+V_e\right)$. Next we see that $cov\left(\widetilde{h_n},\widetilde{i_n}\right)=(N-1)\beta^{PI}V_v$, $cov\left(\widetilde{h_n},\widetilde{s}\right)=(1-\beta^{SI})V_\varepsilon$ and $cov\left(\widetilde{i_n},\widetilde{s}\right)=0$. Consequently, we can write the variance matrix as

$$var\left(\widetilde{h_n},\widetilde{i_n},\widetilde{s}\right)=\begin{pmatrix} (\beta^{PI})^2q+(1-\beta^{SI})^2V_\varepsilon & (N-1)\beta^{PI}V_v & (1-\beta^{SI})V_\varepsilon \\ (N-1)\beta^{PI}V_v & V_v+V_e & 0 \\ (1-\beta^{SI})V_\varepsilon & 0 & V_S+V_\varepsilon \end{pmatrix}.$$

The determinant of the variance matrix is

$$M=(\beta^{PI})^2\,(N-1)\,(NV_v+V_e)\,V_e\,(V_S+V_\varepsilon)+(1-\beta^{SI})^2V_SV_\varepsilon\,(V_v+V_e).$$

and this is always higher than zero.

Since $M \neq 0$, the inverse of the variance matrix exists and equals

$$\left(var\left(\widetilde{h_n}, \widetilde{i_n}, \widetilde{s}\right)\right)^{-1}$$

$$= \frac{1}{M}\begin{pmatrix} var\left(\widetilde{i_n}\right) var\left(\widetilde{s}\right) & -cov\left(\widetilde{h_n},\widetilde{i_n}\right) var\left(\widetilde{s}\right) & -cov\left(\widetilde{h_n}, \widetilde{s}\right) var\left(\widetilde{i_n}\right) \\ -cov\left(\widetilde{h_n},\widetilde{i_n}\right) var\left(\widetilde{s}\right) & \begin{array}{c} var\left(\widetilde{h_n}\right) var\left(\widetilde{s}\right) - \\ -cov\left(\widetilde{h_n}, \widetilde{s}\right)^2 \end{array} & cov\left(\widetilde{h_n}, \widetilde{i_n}\right) cov\left(\widetilde{h_n}, \widetilde{s}\right) \\ -cov\left(\widetilde{h_n}, \widetilde{s}\right) var\left(\widetilde{i_n}\right) & cov\left(\widetilde{h_n}, \widetilde{i_n}\right) cov\left(\widetilde{h_n}, \widetilde{s}\right) & \begin{array}{c} var\left(\widetilde{h_n}\right) var\left(\widetilde{i_n}\right) - \\ -cov\left(\widetilde{h_n}, \widetilde{i_n}\right)^2 \end{array} \end{pmatrix}.$$

Once we have calculated $cov\left(\widetilde{v}, \left(\widetilde{h_n}, \widetilde{i_n}, \widetilde{s}\right)\right)$ and $\left(var\left(\widetilde{h_n}, \widetilde{i_n}, \widetilde{s}\right)\right)^{-1}$ we can proceed and identify A, B and C. It results that

$$\begin{aligned} A &= M^{-1}(N-1)\beta^{PI} V_v V_e \left(V_S + V_\varepsilon\right) \text{ and} \\ B &= M^{-1}\left[(\beta^{PI})^2 (N-1) V_v V_e \left(V_S + V_\varepsilon\right) + \left(1-\beta^{SI}\right)^2 V_v V_S V_\varepsilon\right] \\ C &= M^{-1}\left[(N-1)\beta^{PI} V_v V_e V_\varepsilon \left(\beta^{SI} - 1\right)\right]. \end{aligned} \tag{12}$$

Since $\widetilde{h_n} \equiv (N-1)\beta^{PI}\widetilde{v} + \beta^{PI}\sum_{j \neq n}\widetilde{e_j} + (1-\beta^{SI})\widetilde{\varepsilon}$ we have $E\left(\widetilde{h_n}\right) = (N-1)\beta^{PI}\overline{v}$. In addition, we assumed that $E\left(\widetilde{i_n}\right) = E\left(\widetilde{v} + \widetilde{e_n}\right) = \overline{v}$. Using the above values for expectations and the formula (10) for $\widetilde{h_n}$ the expression (11) can be written as

$$\begin{aligned} E\left(\widetilde{v}\,\middle|\,\widetilde{h_n} = h_n, \widetilde{i_n} = i_n, \widetilde{s} = s\right) &= \overline{v} + A\left(\widetilde{h_n} - (N-1)\beta^{PI}\overline{v}\right) + B\left(\widetilde{i_n} - \overline{v}\right) + C\widetilde{s} = \\ &\overline{v}\left(1 - A(N-1)\beta^{PI} - B\right) - A\alpha + (B - A\beta^{PI})\widetilde{i_n} \\ &+ \left(C - A\left(N\delta^{PI} + \beta^{SI} - 1\right)\right)\widetilde{s} + A\gamma\widetilde{p}, \end{aligned} \tag{13}$$

where A, B and C are given by (12). ■

Lemma A.2 *In a symmetric linear equilibrium we have*

$$E(\widetilde{v}|\widetilde{p} = p, \widetilde{S} = S) = \overline{v}\left(1 - DN\beta^{PI}\right) - D\alpha + (1 - \beta^{SI} - N\delta^{PI})D\widetilde{S} + D\gamma\widetilde{p}.$$

Proof of Lemma A.2. We write again the market clearing condition (9) this time in order to find a pair informationally equivalent to $\left(\widetilde{p}, \widetilde{S}\right)$.

$$\widetilde{p}\gamma - \alpha + (1 - \beta^{SI} - N\delta^{PI})\widetilde{S} = \beta^{PI}\sum_{n=1}^{N}\widetilde{i_n} + N\delta^{PI}\widetilde{\varepsilon}. \tag{14}$$

We define $\theta \equiv \beta^{PI} \sum_{n=1}^{N} \widetilde{i_n} + N\delta^{PI}\widetilde{\varepsilon}$. From here it results that $\left(\widetilde{\theta}, \widetilde{S}\right)$ is informationally equivalent to $\left(\widetilde{p}, \widetilde{S}\right)$. Consequently, $E\left(\widetilde{v}\,\middle|\,\widetilde{p} = p, \widetilde{S} = S\right) = E\left(\widetilde{v}\,\middle|\,\widetilde{\theta} = \theta, \widetilde{S} = S\right)$. Applying again the projection theorem for normally distributed random variables we obtain that

$$E\left(\widetilde{v}\,\middle|\,\widetilde{\theta} = \theta, \widetilde{S} = S\right) = \overline{v} + \begin{pmatrix} D & E \end{pmatrix} \begin{pmatrix} \widetilde{\theta} - E\left(\widetilde{\theta}\right) \\ \widetilde{S} - E\left(\widetilde{S}\right) \end{pmatrix}, \tag{15}$$

where $\begin{pmatrix} D & E \end{pmatrix} = cov\left(\widetilde{v}, \left(\widetilde{\theta}, \widetilde{S}\right)\right)\left(var\left(\widetilde{\theta}, \widetilde{S}\right)\right)^{-1}$.

Let us calculate $cov\left(\widetilde{v}, \left(\widetilde{\theta}, \widetilde{S}\right)\right)$. First we compute the covariance of $\widetilde{v}$ and $\widetilde{\theta}$ $cov\left(\widetilde{v}, \widetilde{\theta}\right) = cov\left(\widetilde{v},\ N\beta^{PI}\widetilde{v} + \beta^{PI} \sum_{n=1}^{N} \widetilde{e_n}\right) = N\beta^{PI}V_v$. Since $\widetilde{v}$ and $\widetilde{S}$ are independent random variables, it results that $cov\left(\widetilde{v}, \left(\widetilde{\theta}, \widetilde{S}\right)\right) = \begin{pmatrix} N\beta^{PI}V_v, & 0 \end{pmatrix}$. Similarly we calculate the variance-covariance matrix. First, we calculate

$$\begin{aligned} cov\left(\widetilde{\theta}, \widetilde{\theta}\right) &= cov\left(N\beta^{PI}\widetilde{v} + \beta^{PI}\sum_{n=1}^{N}\widetilde{e_n} + N\delta^{PI}\widetilde{\varepsilon},\ N\beta^{PI}\widetilde{v} + \beta^{PI}\sum_{n=1}^{N}\widetilde{e_n} + N\delta^{PI}\widetilde{\varepsilon}\right) \\ &= (\beta^{PI})^2 N\left(NV_v + V_e\right) + \left(N\delta^{PI}\right)^2 V_\varepsilon. \end{aligned}$$

Then notice that $cov\left(\widetilde{\theta}, \widetilde{S}\right) = cov\left(N\beta^{PI}\widetilde{v} + \beta^{PI} \sum_{n=1}^{N} \widetilde{e_n}, \widetilde{S}\right) = 0$. It results that

$$var\left(\widetilde{\theta}, \widetilde{S}\right) = \begin{pmatrix} (\beta^{PI})^2 N\left(NV_v + V_e\right) + \left(N\delta^{PI}\right)^2 V_\varepsilon & 0 \\ 0 & V_S \end{pmatrix}.$$

The variance matrix is nonsingular and its inverse is

$$\left(var\left(\widetilde{\theta}, \widetilde{S}\right)\right)^{-1} = \begin{pmatrix} ((\beta^{PI})^2 N\left(NV_v + V_e\right) + \left(N\delta^{PI}\right)^2 V_\varepsilon)^{-1} & 0 \\ 0 & (V_S)^{-1} \end{pmatrix},$$

and consequently,

$$D = N\beta^{PI}V_v\left((\beta^{PI})^2 N\left(NV_v + V_e\right) + \left(N\delta^{PI}\right)^2 V_\varepsilon\right)^{-1} \text{ and } E = 0. \tag{16}$$

Since $E\left(\widetilde{i_n}\right) = \overline{v}$, and $\widetilde{\theta} \equiv N\beta^{PI}\widetilde{v} + \beta^{PI} \sum_{n=1}^{N} \widetilde{e_n} + N\delta^{PI}\widetilde{\varepsilon}$ we have that $E\left(\widetilde{\theta}\right) = N\beta^{PI}\overline{v}$. In addition, we assumed that $E\left(\widetilde{S}\right) = 0$. Using the above values for expectations, the fact that $E = 0$ and the formula (14) for $\widetilde{\theta}$, the expression (11) can be written as

$$\begin{aligned} E\left(\widetilde{v}\,\middle|\,\widetilde{\theta} = \theta, \widetilde{S} = S\right) &= \overline{v} + D\left(\widetilde{\theta} - N\beta^{PI}\overline{v}\right) + E\widetilde{S} \\ &= \overline{v}\left(1 - DN\beta^{PI}\right) - D\alpha + (1 - \beta^{SI} - N\delta^{PI})D\widetilde{S} + D\gamma\widetilde{p}, \end{aligned} \tag{17}$$

where D is given by formula (16). ■

Lemma A.3 *The coefficients* $\alpha^{PI}, \beta^{PI}, \delta^{PI}, \gamma^{PI}, \alpha^{SI}, \beta^{SI}, \gamma^{SI}$ *are the solution of the following system of equations:*

$$\begin{cases} \alpha^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(\overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A\alpha\right) \\ \beta^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(B - A\beta^{PI}\right) \\ \delta^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(C + A\left(1 - \beta^{SI} - N\delta^{PI}\right)\right) \\ \gamma^{PI} = \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(1 - A\gamma\right) \\ \alpha^{SI} = N\gamma^{PI}(\overline{v}\left(1 - DN\beta^{PI}\right) - D\alpha) \\ \beta^{SI} = N\gamma^{PI} D\left(1 - \beta^{SI} - N\delta^{PI}\right) \\ \gamma^{SI} = N\gamma^{PI}\left(1 - D\gamma\right) \\ M = (\beta^{PI})^2\left(N-1\right)\left(NV_v + V_e\right)V_e\left(V_S + V_\varepsilon\right) + (1-\beta^{SI})^2 V_S V_\varepsilon\left(V_v + V_e\right) \\ A = M^{-1}(N-1)\beta^{PI} V_v V_e\left(V_S + V_\varepsilon\right) \\ B = M^{-1}\left((\beta^{PI})^2\left(N-1\right)V_v V_e\left(V_S + V_\varepsilon\right) + \left(1 - \beta^{SI}\right)^2 V_v V_S V_\varepsilon\right) \\ C = M^{-1}\left(N-1\right)\beta^{PI} V_v V_e V_\varepsilon\left(\beta^{SI} - 1\right) \\ D = N\beta^{PI} V_v\left((\beta^{PI})^2 N\left(NV_v + V_e\right) + \left(N\delta^{PI}\right)^2 V_\varepsilon\right)^{-1}. \end{cases} \tag{18}$$

Proof of Lemma A.3. In Lemma A.1 and Lemma A.2 we have established the expressions for $E\left(\widetilde{v}\middle|\widetilde{p} = p, \widetilde{i_n} = i_n, \widetilde{s} = s\right)$ and $E\left(\widetilde{v}\middle|\widetilde{p} = p, \widetilde{S} = S\right)$. We will use them now to find the expressions for the strategies for the value-informed agents and for the specialist.

First, since $E\left(\widetilde{v}\middle|\widetilde{p} = p, \widetilde{i_n} = i_n, \widetilde{s} = s\right) = E\left(\widetilde{v}\middle|\widetilde{h_n} = h_n, \widetilde{i_n} = i_n, \widetilde{s} = s\right)$ we plug (13) in (2) and we obtain that

$$\begin{aligned} x_n\left(\widetilde{p}, \widetilde{i_n}, \widetilde{s}\right) &= \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(\overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A\alpha\right. \\ &\quad \left. + (B - A\beta^{PI})\widetilde{i_n} + \left(C - A\left(N\delta^{PI} + \beta^{SI} - 1\right)\right)\widetilde{s} + (A\gamma - 1)\widetilde{p}\right). \end{aligned}$$

We identify the coefficients in the definition of the strategy of the value-informed trader

n (2) and we get the following equations:

$$\begin{aligned}
\alpha^{PI} &= ((N-1)\gamma^{PI} + \gamma^{SI}) \left(\overline{v}\left(1 - A\left(N-1\right)\beta^{PI} - B\right) - A\alpha\right) \\
\beta^{PI} &= ((N-1)\gamma^{PI} + \gamma^{SI})(B - A\beta^{PI}) \\
\delta^{PI} &= \left((N-1)\gamma^{PI} + \gamma^{SI}\right)\left(C + A\left(1 - \beta^{SI} - N\delta^{PI}\right)\right) \\
\gamma^{PI} &= ((N-1)\gamma^{PI} + \gamma^{SI})(1 - A\gamma),
\end{aligned} \tag{19}$$

where A, B and C are given by (12).

Second, since $E\left(\widetilde{v}\left|\widetilde{p} = p, \widetilde{S} = S\right.\right) = E\left(\widetilde{v}\left|\widetilde{\theta} = \theta, \widetilde{S} = S\right.\right)$ we plug (17) in (3) and we obtain in a similar manner

$$y\left(\widetilde{p}, \widetilde{S}\right) = N\gamma^{PI}\left(\overline{v}\left(1 - DN\beta^{PI}\right) - D\alpha + (1 - \beta^{SI} - N\beta^{PI})D\widetilde{S} + (D\gamma - 1)\widetilde{p}\right).$$

We identify the coefficients in the definition of the strategy of the specialist (1) and we get the following equations:

$$\begin{aligned}
\alpha^{SI} &= N\gamma^{PI}(\overline{v}\left(1 - DN\beta^{PI}\right) - D\alpha) \\
\beta^{SI} &= N\gamma^{PI}(1 - \beta^{SI} - N\beta^{PI})D \\
\gamma^{SI} &= N\gamma^{PI}(1 - D\gamma),
\end{aligned} \tag{20}$$

where D is given by (16).

Putting together the equations (12), (19), (16) and (20) we obtain that α^{PI}, β^{PI}, $\gamma^{PI}, \alpha^{SI}, \beta^{SI}, \gamma^{SI}$ are the solution of the above system of equations. ■

Using Lemma A.1, A.2 we obtain that the equilibrium is characterized by the strategies defined by (1) and the price defined by (4), where the coefficients are the solution of the system (18) obtained in Lemma A.3 ■

Proof of Corollary 3. From the market clearing condition we have obtained that the equilibrium price is

$$\widetilde{p} = \left(N\gamma^{PI} + \gamma^{SI}\right)^{-1}\left(\alpha + \beta^{PI}\sum_{n=1}^{N}\widetilde{i_n} + N\delta^{PI}\widetilde{\varepsilon} - (1 - \beta^{SI} - N\delta^{PI})\widetilde{S}\right).$$

We have seen that the equilibrium price is given by (4). As a result, we can compute the volatility as the variance of price

$$Var(\widetilde{p}) = \left(N\gamma^{PI} + \gamma^{SI}\right)^{-2} Var\left(\alpha + \beta^{PI}\sum_{n=1}^{N}\widetilde{i_n} + N\delta^{PI}\widetilde{\varepsilon} - (1-\beta^{SI} - N\delta^{PI})\widetilde{S}\right)$$

$$= \left(N\gamma^{PI} + \gamma^{SI}\right)^{-2} Var\left(N\beta^{PI}\widetilde{v} + \beta^{PI}\sum_{n=1}^{N}\widetilde{e}_n + N\delta^{PI}\widetilde{\varepsilon} - (1-\beta^{SI} - N\delta^{PI})\widetilde{S}\right) =$$

$$\left(N\gamma^{PI} + \gamma^{SI}\right)^{-2}\left(\left(N\beta^{PI}\right)^2 V_v + N\left(\beta^{PI}\right)^2 V_e + \left(N\delta^{PI}\right)^2 V_\varepsilon + (1-\beta^{SI} - N\delta^{PI})^2 V_S\right).$$

■

Proof of Corollary 4. We define the informativeness of prices the reduction in the variance of the liquidation value of the asset after observing price, $Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p})$. Due to the normality assumptions we have that

$$Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p}) = (Var(\widetilde{p}))^{-1}(Cov(\widetilde{v},\widetilde{p}))^2.$$

We calculate the covariance

$$Cov(\widetilde{v},\widetilde{p}) = \left(N\gamma^{PI} + \gamma^{SI}\right)^{-1} N\beta^{PI} V_v,$$

and together with the formula for variance $Var(\widetilde{p})$ we obtained before, we plug them in above to obtain

$$Var(\widetilde{v}) - Var(\widetilde{v}|\,\widetilde{p}) = \frac{\left(N\beta^{PI}V_v\right)^2}{\left(\left(N\beta^{PI}\right)^2 V_v + N\left(\beta^{PI}\right)^2 V_e + \left(N\delta^{PI}\right)^2 V_\varepsilon + (1-\beta^{SI} - N\delta^{PI})^2 V_S\right)}.$$

■

Proof of Corollary 5. Since the demand of the value-informed agent x_n can be written as the sum of normal variables it results that x_n is also a normal variable. The mean of x_n is $\mu_n = 0$ while the variance $\sigma^2_{x_n}$ is

$$\sigma^2_{x_n} = Var(x_n) = Var\left(\alpha^{PI} + \beta^{PI}\widetilde{v} + \beta^{PI}\widetilde{e_n} + \delta^{PI}\widetilde{s} - \gamma^{PI}p\right)$$
$$= \left(\beta^{PI}\right)^2 (V_v + V_e) + \left(\delta^{PI}\right)^2 (V_S + V_\varepsilon) + \left(\gamma^{PI}\right)^2 Var(p).$$

Then, since x_n is $\mathcal{N}(\mu_n, \sigma^2_{x_n})$ it results that the expected volume of trade

$$E(|x_n|) = \int_{-\infty}^{\infty} |x_n| \frac{1}{\sigma_{x_n}\sqrt{2\pi}} \exp\left(-\frac{(x_n - \mu_n)^2}{2\sigma^2_{x_n}}\right) dx_n = 2\mu_n + \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \sigma^2_{x_n} = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \sigma^2_{x_n}. \tag{21}$$

Similarly, the quantity demanded by the specialist is a normal variable with mean $\mu_y = 0$ and variance

$$\sigma_y^2 = Var(y) = Var\left(\alpha^{SI} + \beta^{SI} S - \gamma^{SI} p\right) = \left(\beta^{SI}\right)^2 V_S + \left(\gamma^{SI}\right)^2 Var(p). \tag{22}$$

Then since y is $\mathcal{N}(\mu_y, \sigma_y^2)$ it results that the expected volume of trade of the specialist is

$$E\left(|y|\right) = \int_{-\infty}^{\infty} |y| \frac{1}{\sigma_y \sqrt{2\pi}} \exp\left(-\frac{\left(y-\mu_y\right)^2}{2\sigma_y^2}\right) dy = 2\mu_y + \sqrt{\frac{2}{\pi}} \sigma_y^2 = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} \sigma_y^2. \tag{23}$$

■

References

[1] Admati, A. and P. Pfleiderer, 1991, Sunshine Trading and Financial Market Equilibrium, Review of Financial Studies 4 (3), 443-482.

[2] Baruch, S., 2005, Who Benefits from an Open Limit Order Book?, Journal of Business 78 (4), 1267-1306.

[3] Bessembinder, H., Maxwell,W. and K. Venkataraman, 2006, Market Transparency, Liquidity Externalities, and Institutional Trading Costs in Corporate Bonds, Journal of Financial Economics 82 (2), 251-288.

[4] Bloomfield R. and O'Hara, M., 1999, Market Transparency: Who Wins and Who Loses?, Review of Financial Studies, 12 (1), 5-35.

[5] Bloomfield R. and O'Hara, M., 2000, Can Transparent Markets Survive?, Journal of Financial Economics, Vol. 55 (3), 425-459.

[6] Boehmer, E., G. Saar and L. Yu, 2005, Lifting the Veil: An Analysis of Pre-trade Transparency at the NYSE, Journal of Finance 60 (2), 783-815.

[7] Dumitrescu, A. (2007), Strategic Specialist and Market Liquidity, EFA 2007 Ljubljana Meetings Paper Available at SSRN: http://ssrn.com/abstract=942016

[8] Edwards, A., Harris, L. and M. Piwowar, 2007, Corporate Bond Market Transaction Costs and Transparency, Journal of Finance 62 (3), 1421–1451.

[9] Fishman, M.J., and F. A. Longstaff, 1992, Dual Trading in Futures Markets, Journal of Finance 47, 643-671.

[10] Flood, M., Huisman, R., Koedijk, K. and R. Mahieu, 1999, Quote Disclosure and Price Discovery in Multiple-dealer Financial Markets, Review of Financial Studies 12, 37–59.

[11] Forster, M., and T. George, 1992, Anonymity in Securities Markets, Journal of Financial Intermediation 2, 168–206.

[12] Foucault, T., S. Moinas, and E. Theissen, 2003, Does Anonymity Matter in Electronic Limit Order Markets?, Forthcoming in Review of Financial Studies.

[13] Gennotte, G. and H. Leland, 1990, Market Liquidity, Hedging and Crashes, American Economic Review 80, 999-1021.

[14] Goldstein, M., Hotchkiss, E. and E. Sirri, 2007, Transparency and Liquidity: a Controlled Experiment on Corporate Bonds, Review of Financial Studies 20, 235-273.

[15] Kyle, A.S., 1985, Continuous Auctions and Insider Trading, Econometrica 53, 1315-1336.

[16] Kyle, A.S., 1989, Informed Speculators with Imperfect Competition, The Review of Economic Studies 56, 317-356.

[17] Madhavan, A., 1995, Consolidation, Fragmentation, and the Disclosure of Trading Information, Review of Financial Studies 8, 579–603.

[18] Madhavan, A., 1996, Security Prices and Market Transparency, Journal of Financial Intermediation 5, 255–283.

[19] Madhavan, A., 2000, Market Microstructure: A Survey, Journal of Financial Markets 3 (3), 205-258.

[20] Madhavan, A., Porter, D. and D. Weaver, 2005, Should Securities Markets Be Transparent?, Journal of Financial Markets 8 (3), 265-287.

[21] O'Hara, M., 2007, Optimal Microstructures, forthcoming European Financial Management.

[22] Pagano, M., and A. Röell, 1996, Transparency and Liquidity: A Comparison of Auction and Dealer Markets with Informed Trading, Journal of Finance 51, 579–611.

[23] Rindi, B., 2007, Informed Traders as Liquidity Providers: Transparency, Liquidity and Price Formation, forthcoming Review of Finance.

[24] Röell, A., 1990, Dual Capacity Trading and the Quality of the Market, Journal of Financial Intermediation 1, 105-124.

[25] Simaan, Y, Weaver, D. and D. Whitcomb, 2003, Market Maker Quotation Behavior and Pre-trade Transparency, Journal of Finance 58 (3), 1247-1267.

[26] Waisburd, A. (2003), Anonymity and Liquidity: Evidence from the Paris Bourse, Working Paper.