Modelling Higher Moments using a simplified Multivariate ARCD: An Application to International Equity and Currency Portfolio VaR

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Abstract. This paper proposes a new approach to estimate time varying conditional variance and covariance matrices allowing for the impact of higher moments in the framework of the Autoregressive Conditional Density (ARCD) model. The proposed method is based on the estimation of only univariate ARCD models and is numerically feasible and easier to estimate than existing complicated multivariate volatility processes that often suffer from unrealistic a priori restrictions and convergence problems. An empirical application of the new model is provided to forecast the VaR of aggregate equity portfolios for the US and UK and foreign exchange portfolio for EUR and GBP against USD and is compared to GARCH and BEKK models. Our results, using both statistical and economic criteria, suggest that the simplified multivariate version of ARCD performs at least well as the other two models indicating the higher moments' importance in volatility forecasting and VaR calculation.

Keywords : VaR, GARCH, BEKK, Simplified Multivariate ARCD.

EFMA Classification Codes: 370, 380, 450

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Introduction

Understanding and estimating time varying conditional variances and covariances is important for many issues in finance since there are many applications that rely on multivariate covariance models. It is essential, for optimal hedging, asset allocation, derivatives pricing and risk management, the accurate modelling and forecast of the assets returns co-movement. Bollerslev, Engle and Wooldridge (1988), Cecchetti (1988), Myers and Thompson (1989), Baillie and Myers (1991), Kroner and Sultan (1993), Ng and Kroner (1998), argue that financial prices are characterized by time varying variances and covariances, presenting a variety of multivariate GARCH models. Bollerslev, Engle and Wooldridge (1988) suggested the VEC model and the diagonal VEC (DVEC) model in which the variances depend only on their past squared errors and the covariances on their past cross-products of errors. Given the excessively large parameters needed to estimate the VEC model and the necessity to impose strong restrictions on the parameters Engle and Kroner (1995) proposed the BEKK parametrization avoiding unrealistic assumptions such as that the correlation between the conditional variances is constant (Constant correlation model by Bollerslev 1990), and guaranteeing that the time varying covariance matrix is positive definite. Additional models can be found in Engle, Ng and Rothschild (1990b) who proposed factor models (FGARCH), in Alexander and Chibumba (1997) who proposed the orthogonal GARCH models (O-GARCH), in Tse and Tsui (2002) and of Engle (2002) who suggested the Dynamic Conditional Correlation models (DCC). All the above models assume that asset returns are jointly normally distributed ignoring the fact of asymmetry in volatility and covariance, fat tailness and skewness. However, asymmetry and skewness in distribution, is found in many financial assets since their return distributions depart far away from normality. For instance, French, Schwert and Stambaugh (1987) rejected normality claiming significant conditional skewness in daily residuals of the SP500 returns, Hong (1988) found abnormally high kurtosis in daily NYSE stock returns, Harvey (1995) observed deviations form normality in emerging stock markets indices, Harvey and Siddique (1999) showed that conditional skewness is important and consistent with asymmetric variance in daily, weekly and monthly returns of selected markets.

Since there is well established stylized evidence that financial returns exhibit fat tails and skewness, a lot of studies focused on using of non normal distributions to better model this excess kurtosis and skewness. More specifically, in the univariate framework, a large variety of conditional densities has been employed to accommodate the asymmetry and fat tailness. Hansen (1994) was the first to propose a Skew-Student distribution which allows for conditional higher moments. Recently, Harvey and Siddique (2002a, 2002b), Jondeau and Rockinger (2006) and Yan (2005), Brooks, Burke and Persand (2005) among others, have discussed ways to jointly estimate time varying conditional variance and skewness, but their resulting formulation is difficult to be implemented, moreover in a multivariate extension.

More precisely, none of the popular multivariate models are compatible with the skewness and kurtosis of asset returns since they assume multivariate normality. A few studies exist on the higher moments modelling in multivariate approaches. Harvey, Ruiz and Shepard (1994) and Fiorentini, Sentana and Galzolari (2003) replace multivariate Gaussian density with student density by letting conditional innovations to follow a Student-t distribution. Sahu, Dey, and Branco (2001), and Bawens and Laurent (2005) propose a multivariate skew Student density with support on the full Euclidian space. Their main finding is that this density improves the quality of out of sample VaR forecasts. More recently, Hafner and Rombouts (2004) and Rombouts and Verbeek (2005) apply a multivariate semi parametric GARCH estimation technique to capture higher moments showing that in within sample portfolios' VaR the model's superiority and robustness is confirmed. Azzallini (1996) and De Luca, Genton and Loperfido (2006) propose the multivariate Skew-GARCH model including a parameter to control skewness. Lee and Long (2005) introduce copula-based multivariate GARCH, the C-MGARCH with uncorrelated dependent errors, arguing that in terms of in sample model selection and out of sample multivariate density forecast, the choice of copula functions is more important than the volatility models. The main drawback of the above models is that are rather complex, and suffer from a large parameters estimation and convergence problems.

In this paper, we propose an alternative, simplified multivariate model, the simplified Multivariate Autoregressive Conditional Density Model (S-ARCD) which is compatible with the skewness and kurtosis of the financial returns and is easy to be implemented increasing the computational efficiency. It is based on the Autoregressive Conditional Density Model (ARCD) proposed by Hansen (1994) and involves the estimation only of the univariate specification of the above model. The conditional variances are calculated by the simple univariate models, and the conditional covariance is then imputed from these variance estimates. We illustrate the S-ARCD to forecast the VaR of aggregate equity portfolios for the US and UK and foreign exchange portfolio for EUR and GBP against USD and is compared to the ad hoc multivariate version of GARCH (Wang, Yao, 2005) and BEKK models. Our results, using both statistical and economic criteria, suggest that the simplified multivariate version of ARCD performs at least well as the other two models indicating the higher moments' importance in volatility forecasting and VaR calculation.

The remainder of the paper is organized as follows: the next section introduces S-ARCD and briefly describes BEKK and the ad hoc multivariate version of GARCH (Wang, Yao, 2005) models. The third section describes the data and the empirical results on the VaR estimation. The next section compares the VaR performance of the alternative models. The final section concludes the paper.

1. Methodology

This section describes the three models under consideration: the ad hoc multivariate version of GARCH, BEKK and S-ARCD. The first two models are presented briefly since there is extensive description in the academic literature.

1.1 The ad hoc multivariate of GARCH

Wang and Yao (2005) first proposed an ad hoc multivariate method using univariate GARCH models in order to allow the return covariance matrix to vary over time. More precisely, using the popular GARCH(1,1) specification, the conditional variances of two return series $y_{1,t}$, $y_{2,t}$,can be modelled respectively as:

$$h_{1,t} = c_{11} + a_1\varepsilon^2_{1,t\text{-}1} + \beta_1 h_{1,t\text{-}1} \tag{1a}$$

$$h_{2,t} = c_{22} + a_1\varepsilon^2_{2,t\text{-}1} + \beta_2 h_{2,t\text{-}1} \tag{1b}$$

where $\varepsilon^{2}_{1,t-1}$ and $\varepsilon^{2}_{2,t-1}$ are the lagged squared residuals from the conditional mean equations for the spot and futures returns respectively. Then, to model the conditional covariance, the following steps are implemented. Firstly, a new series is constructed as: $x_{12,t} = \frac{(y_{1,t} + y_{2,t})}{2}$. Secondly, the conditional variance of the new series $h_{12,t}$ is estimated from another univariate GARCH(1,1) as:

$$h_{12,t} = c_{12} + a_{12}\varepsilon^{2}_{12,t\text{-}1} + \beta_{12}h_{12,t\text{-}1} \tag{1c}$$

Finally, the time varying conditional covariance of returns is given by the equation:

$$\sigma_{12,t} = 2h_{12,t} - \frac{(h_{1,t} + h_{2,t})}{2} \tag{2}$$

1.2 Multivariate GARCH

Engle and Kroner (1995), among others, have relaxed this assumption by proposing a multivariate GARCH process. Using a BEKK representation, the conditional variance matrix is the following:

$$\mathbf{H_t} = CC' + \mathbf{A}\varepsilon_{t-1}\varepsilon_{t-1}'\mathbf{A}' + \mathbf{B}\mathbf{H_{t\text{-}1}}\mathbf{B}' \tag{3}$$

where **C, A, B** are $n \times n$ matrices, with **C** being upper triangular, symmetric and positive definite. The conditional variance matrix $\mathbf{H_{t\text{-}1}}$ is positive definite since the second and third terms in the above equation (5) are expressed in quadratic forms. This means that no other constraints for the matrices **A** and **B** are necessary. For the case of the bivariate GARCH(1,1), the BEKK model is estimated in a restricted form with **C** as a 2×2 lower triangular matrix, and, **A, B** being 2×2 diagonal matrices. This can be expressed by the following equations:

$$\begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix} = \begin{pmatrix} c_{11} & 0 \\ c_{22} & c_{12} \end{pmatrix}\begin{pmatrix} c_{11} & c_{22} \\ 0 & c_{12} \end{pmatrix} + \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix}\begin{pmatrix} \varepsilon^{2}_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon^{2}_{2,t-1} \end{pmatrix}\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} + \\ + \begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix}\begin{pmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{pmatrix}\begin{pmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{pmatrix} \tag{4a}$$

or,

$$h_{11,t} = c_{11}^2 + a_1^2 \varepsilon_{1,t-1}^2 + \beta_1^2 h_{11,t-1}$$
$$h_{22,t} = c_{22}^2 + c_{12}^2 + a_2^2 \varepsilon_{2,t-1}^2 + \beta_2^2 h_{22,t-1} \tag{4b}$$
$$h_{12,t} = c_{11} c_{22} + a_1 a_2 \varepsilon_{1,t-1} \varepsilon_{2,t-1} + \beta_1 \beta_2 h_{12,t-1}$$

where c_{11}, c_{22}, c_{12}, are constants and $\varepsilon_{1,t-1}^2$ $\varepsilon_{2,t-1}^2$ are the lagged residuals from the conditional mean equation for the spot and futures returns respectively.

1.3 The Simplified Multivariate Autoregressive Conditional Density Model

Let $y_{1,t}$ and $y_{2,t}$ two univariate discrete time real-valued stochastic processes, (i.e. the rate of return of an asset or market portfolio) and I_t is the information set at time t, which encompasses $y_{i,t}$ and all the past realizations of the process $y_{i,t}$ where i=1,2. Then, the conditional mean returns are denoted as: $\mu_{1.t} = E(y_{1,t} | I_{t-1})$, $\mu_{2.t} = E(y_{2,t} | I_{t-1})$ and the conditional covariance matrix of $y_{1,t}$ and $y_{2,t}$ is given by:

$$Var(y_{1,t}, y_{2,t} | I_{t-1}) = \sum_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} \tag{5}$$

The goal is to model the elements of the conditional covariance matrix taking into consideration the time varying skewness and kurtosis. Our approach is based on Hansen (1994) Autoregressive Conditional Density Model (ARCD), who proposed a generalization of the Student-t distribution to capture asymmetry and fat tailness, involving only univariate modeling. Although alternative skewed Student-t distributions have been considered in the literature (eg., Jondeau and Rockinger, 2003, 2006; Harvey and Siddique, 1999, 2002a, 2002b), we selected this specification because it has a clear computational advantage over competing models (e.g., see Harvey and Siddique, 1999; Brooks and Persand, 2005) and the variation in the shape parameters may be smaller and easier to manage numerically. Also, only few parameters are estimated in each model and, generally, it is easier to implement than other multivariate models such as BEKK, Vech or stochastic variance models. An alternative approach, the simplified multivariate GARCH, has been presented by Harris, Stoja and Tucker (2007) in order to estimate the minimum-variance hedge ratio for the FTSE 100 index portfolio. In this paper, the proposed method involves two steps and is based on Wang and Yao (2005) method. Firstly, the conditional variances are estimated using the following univariate form of the ARCD model's distribution density function:

$$f(z|\eta,\lambda)=\left\{bd(1+\ (\frac{1}{\eta-2}(\frac{bz+k}{1-\lambda})^2)^{\frac{-(\eta+1)}{2}} \qquad \text{for } z\ <\frac{-k}{b}\right.$$

and (6)

$$f(z|\eta,\ \lambda)=\left\{bd(1+\ (\frac{1}{\eta-2}(\frac{bz+k}{1+\lambda})^2)^{\frac{-(\eta+1)}{2}} \qquad \text{for } z\ \succ\frac{-k}{b}\right.$$

where

$2<\eta<\infty$, $-1<\lambda<1$, η_t are the degrees of freedom, λ_t is the skewness, while k, b, d are constant parameters defined by the following equations:

$$k\ =4\lambda d(\frac{\eta-2}{\eta-1}),\quad b^2=1+3\lambda^2-k^2,\quad d\ =\frac{\Gamma(\frac{\eta+1)}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\frac{\eta}{2)}} \tag{7}$$

More specifically, the degrees of freedom η_t and skewness λ_t are specified as following:

$$\eta_t\ =\gamma_0+\gamma_1\varepsilon_{t-1}+\gamma_2\varepsilon_{t-1}^2 \tag{8a}$$

$$\lambda_t=\lambda_0+\ \lambda_1\varepsilon_{t-1}+\ \lambda_2\varepsilon_{t-1}^2 \tag{8b}$$

Jondeau and Rockinger (2003) have presented the exact formulas for the calculation of the kurtosis and skewness. The conditional log-likelihood of the full ARCD model is calculated as:

$$LLK\ =\sum_{t=\max(p,q)+1}^{T}\left\{\log f(z_t;\eta_t)-\frac{1}{2}\log h_t\right\} \tag{9}$$

The ARCD shape parameters η_t can be estimated by standard iterative methods by assuming arbitrary reasonable initial value η_1. It is advisable to compute robust standard errors since they generate asymptotically valid confidence intervals for the pseudo–true parameter values which minimize the information distance between the true probability and the quasi-likelihood measure. In this manner, we can achieve the maximum possible accuracy in our results. Finally, the Nyblom L-statistic, for testing the constancy of the estimated parameters, takes the form:

$$Lk = \frac{1}{n}\sum_{t=1}^{n}\frac{G_{it}^{2}}{V_{ii}} \tag{10}$$

where G_{it} are the cumulative scores and V_{ii} is the ith diagonal element of the estimated variance. The L-statistic is used to test the stationarity of the parameters of the distribution function and can be considered as the LM test of the null hypothesis that the parameters are stable. Asymptotic critical values for the Nyblom test and an extensive analysis have been presented in Hansen (1990). The conditional variances are given by:

$$h_{1,t} = c_{11} + a_{1}(\varepsilon_{1,t\text{-}1}^{2} - h_{1,t-1}) + \beta_{1}h_{1,t-1} \tag{11a}$$

$$h_{2,t} = c_{22} + a_{2}(\varepsilon_{2,t\text{-}1}^{2} - h_{2,t-1}) + \beta_{2}h_{2,t-1} \tag{11b}$$

Secondly, the conditional covariance $\sigma_{ij,t}$ is estimated following the method proposed by Wang and Yao (2005) constructing a new series with the general form of:

$\omega_{12,t} = \dfrac{(y_{1,t} + y_{2,t})}{2}$ for the i<j elements of the equation (5) where i,j =1,2. The conditional variance of the above new series is estimated as $h_{12,t} = Var(\omega_{12,t} \mid I_{t-1})$ using the univariate version of the ARCD model described above using the following equation:

$$h_{12,t} = c_{12} + a_{12}(\varepsilon_{12,t\text{-}1}^{2} - h_{12,t-1}) + \beta_{12}h_{12,t-1} \tag{11c}$$

Then, for all $1 \le i < j \le 2$ the conditional covariance is calculated using the following equation:

$$\sigma_{12,t} = 2h_{12,t} - \frac{(h_{1,t} + h_{2,t})}{2}. \tag{12}$$

The above identity has been proposed by Wang and Yao (2005) in order to derive estimators of the covariance matrix when there is no multivariate extension of the underlying univariate model. Overall, the simplified ARCD model involves the estimation of only univariate ARCD models. Therefore it is easier and computationally simpler to be implemented than Vech and BEKK models avoiding overparametrization since only a few parameters are estimated in each model, and the maximum likelihood converges more efficiently. Also, there are no restrictions for the coefficients of the conditional covariance unlike diagonal Vech, BEKK and constant correlation models' covariance matrix elements.

However, the above flexible structure of the S-ARCD model comes with some disadvantage. The resulting estimate of the conditional correlation matrix is not necessarily positive semidefinite since there is no possibility to set any restrictions in the conditional covariance coefficients. There several procedures to encounter this problem. Harris, Stoja and Tucker (2007) propose three simple techniques to ensure the estimated correlation matrix positive semi –definiteness[1].

3. Data Description and Empirical Analysis

The data used in the present study includes daily closing prices for the UK Financial Times Stock Exchange Index (FTSE), and the US Dow Jones Index (DJ) and daily spot prices of two exchange series, the EUR and the GBP against USD. The sample covers the period from 3 January 2000 until 30 June 2007 for all four data series, a total of 1952 observations for both the exchange rates and 1861 for the FTSE and Dow Jones indices respectively. The last 100 observations for each series are left for *ex ante* (out of sample) portfolio VaR estimation. Descriptive statistics of logarithmic returns of all series data under study are provided in Table 1.

There is strong evidence that all series are non-normally distributed with high peaks and fat tails. For all series, there is negative asymmetry in the distribution. The Ljung-Box portmanteau test for all series except FTSE shows no significant autocorrelation while the ARCH-LM test for serial correlation in squared returns reveals volatility clustering in all series and more significantly in equity indices.

Table 1. Descriptive Statistics of spot and futures index returns

	Obs.	Mean	St. Dev.	Skewness	Kurtosis	JB	$Q_1(10)$	ARCH(4)
Equity Indices								

[1] A detailed analysis can be found in Harris, Stoya and Tucker paper: A simplified approach to modelling the co-movement of asset returns published in The Journal of Futures Markets (2007).

FTSE	1,861	-0.003	1.123	-0.231	6.385	920.55	37.71	153.8
DJ	1,861	0.009	1.075	-0.079	7.103	1321.95	12.99	77.51
FX Spot against USD								
EUR	1,952	0.014	0.618	-0.027	3.776	49.22	8.66	38.84
GBP	1,952	0.011	0.510	-0.021	3.564	26.08	2.93	9.40

Jarque-Bera (JB) is asymptotically distributed as a Chi-squared with 2 degrees of freedom under the null hypothesis of normality. The $Q_1(k)$ represents the Ljung-Box portmanteau test of the return series. ARCH(4) statistic tests the null hypothesis that the first four partial autocorrelation of squared returns are zero.

For the empirical implementation of the simplified ARCD the conditional variances are estimated using the simple version of ARCD model applying equations 11a and 11b for each data series separately. We then construct the new series $r_{F+D} = \frac{(r_{FTSE} + r_{DJ})}{2}$ for the equity indicess FTSE and DJ so as to estimate the conditional covariance by equation 12. The conditional variance of the new series r_{F+D} is estimated applying another univariate version of ARCD.

The above procedure is implemented, also, for the foreign exchange series currencies EUR and GBP. The new series, from the currencies EUR and GBP against USD, $r_{E+G} = \frac{(r_{EUR} + r_{GBP})}{2}$ is constructed and used for the conditional covariance calculation. The estimated parameters of the simplified multivariate ARCD S-ARCD model for the DJ and FTSE indices and the EUR and GBP currencies are presented in Table 2. The simplified ARCD model estimation results are presented in Table 2. We report the conditional variance, degrees of freedom, skewness and Nyblom test values. For the conditional variances, the conditional degrees of freedom and the conditional skewness, the Nyblom L test indicates that the parameters are all stable since the test statistic is less that the 1% level critical value of 0.75. This is also confirmed by the joint Nyblom test which is smaller than the 1% critical value of 2.8. Overall, the coefficients for the conditional degrees of freedom and the conditional skewness seem to be highly significant implying that the simplified ARCD model is well specified and fits the data capturing the higher moments' time variation.

A log likelihood ratio ratio (LR) statistic is applied to test the null hypothesis that the series follow the normal distribution against the alternative of time varying higher moments. Since the normal distribution (of GARCH model) is nested to the skewed

Student-t distribution of the simplified ARCD model, the LR statistic is calculated by the following formula: LR= -2[|LGARCH| – |LARCD|) where LGARCH and LARCD are the absolute values of the maximum values of the log likelihood functions under the normal distribution and the skewed Student-t distribution respectively. As shown in Table 2, all LR values are greater than their critical value of 9.21 at 1% significance level, strongly rejecting the null hypothesis of time invariant shape parameters which GARCH assumes, implying that the empirical distribution of data returns do not follow a normal distribution.

Overall, the simplified ARCD model seems to fit the data better compared to the ad hoc GARCH(1,1) model since the Nyblom Joint test statistic for the stability of the parameters of GARCH(1,1) model is rejected at 1% significance level, evidence that a further dynamic specification is needed.

Table 2. S-ARCD estimates for FTSE, DJ, (F+D), EUR, GBP, (E+G)

S-ARCD	FTSE	DJ	(F+D)	EUR	GBP	(E+G)
Conditional Variance						
c_{11} , c_{22} , c_{12}	0.0125	0.0065	0.0084	0.0010	0.0021	0.0030
	(0.0033)	(0.0033)	(0.0032)	(0.014)	(0.0014)	(0.0008)
$\varepsilon^2_{1,t-1}-h_{1,t-1}, \varepsilon^2_{2,t-1}-h_{2,t-1}, \varepsilon^2_{12,t-1}-h_{12,t-1}$	0.1110	0.0809	0.1004	0.0248	0.0330	0.0274
	(0.0137)	(0.0137)	(0.0176)	(0.0061)	(0.0061)	(0.0073)
$h_{1,t-1}, h_{2,t-1}, h_{12,t-1}$	0.9934	1.0018	0.9970	1.0002	0.9924	0.9992
	(0.0056)	(0.0055)	(0.0073)	(0.0416)	(0.0416)	(0.0032)
Nyblom L_σ test						
c_{11} , c_{22} , c_{12}	0.1922	0.2446	0.2682	0.2827	0.1935	0.3552
$\varepsilon^2_{1,t-1}-h_{1,t-1}, \varepsilon^2_{2,t-1}-h_{2,t-1}, \varepsilon^2_{12,t-1}-h_{12,t-1}$	0.0275	0.0294	0.0329	0.0314	0.0845	0.1479
$h_{1,t-1}, h_{2,t-1}, h_{12,t-1}$	0.4510	0.4646	0.5500	0.1588	0.1202	0.1669
Conditional Degrees of Freedom						
γ_0	1.1540	-1.0340	0.5543	1.5386	-0.5457	32.4942
	(0.3926)	(0.3926)	(0.6614)	(1.3801)	(0.3801)	(0.8161)
$\varepsilon_{t-1}(\gamma_1)$	-2.3983	-2.2531	-2.1126	6.0572	-0.6676	11.4998
	(0.5764)	(1.0053)	(0.8126)	(2.2752)	(0.4100)	(0.5764)
$\varepsilon^2_{t-1}(\gamma_2)$	0.3063	0.4199	0.2927	1.9576	0.3059	-9.5452
	(0.1538)	(0.0358)	(0.0206)	(0.9169)	(0.0537)	(0.1538)
Nyblom L_σ test						
γ_0	0.2278	0.9729	0.8485	0.1806	0.4026	0.3000
$\varepsilon_{t-1}(\gamma_1)$	0.0482	0.5098	0.2242	0.0781	0.0762	0.4246
$\varepsilon^2_{t-1}(\gamma_2)$	0.0580	0.1986	0.2162	0.0304	0.0527	1.1531
Conditional Skewness						

λ_0	-0.3660	-0.1161	-0.1904	-0.0811	-0.0685	-0.0257
	(0.0797)	(0.0754)	(0.0830)	(0.0424)	(0.0424)	(0.0033)
$\varepsilon_{t-1}(\lambda_1)$	0.0122	0.2370	-0.1904	0.2693	0.1239	0.2142
	(0.0067)	(0.0682)	(0.0521)	(0.1025)	(0.0125)	(0.0154)
$\varepsilon^2_{t-1}(\lambda_2)$	0.0287	-0.0565	0.0352	-0.1376	0.2513	0.3062
	(0.0041)	(0.0409)	(0.0031)	(0.0809)	(0.0809)	(0.0661)
Nyblom L_σ test						
λ_0	0.1361	0.4993	0.1172	0.1195	0.2403	0.2250
$\varepsilon_{t-1}(\lambda_1)$	0.0676	0.4063	0.1393	0.0514	0.0622	0.0731
$\varepsilon^2_{t-1}(\lambda_2)$	0.0267	0.0798	0.0567	0.1275	0.1376	0.1602
Log Likelihood	2486.19	2498.18	2215.48	1782.38	1489.80	1461.46
LR test	25.72	18.82	46.78	72.84	83.72	27.38
Nyblom Joint Test	1.68	2.46	1.94	1.75	1.61	1.87

Numbers in brackets under the parameter estimates give the standard errors values. Nyblom L statistic has been introduced by Nyblom (1989) and modified by Hansen (1990) for testing the constancy of the estimated parameters. It takes the form: $L_i = 1/n*(\Sigma(G_{it}^2/\tilde{V}_{ii})$ where G_{it}^2 are the cumulative scores, $\tilde{V}_{ii}$ is the i_{th} diagonal element of the estimate variance $\tilde{V}$ and can be considered as the LM test of the null hypothesis that all parameters are stable. The asymptotic critical values for the Nyblom test have been presented in Hansen (1990). For the Nyblom test, the 1% critical value is equal to 0.75, and for the Nyblom Joint test, the 1% critical value is equal to 2.8. LR statistic is the likelihood ratio test of the null hypothesis of normal distribution against the alternative that the data returns follow a skewed student t distribution. The LR statistic is asymptotically distributed as a Chi-squared with 2 degrees of freedom and its critical value at 1% significance level is 9.21.

Also, the ad hoc multivariate version of GARCH (Wang, Yao, 2005) and BEKK models are estimated for all data. Tables 3 and 4 present the parameters for the ad hoc multivariate version of GARCH and BEKK models respectively. For both GARCH(1,1) and Bivariate GARCH(1,1) models, all coefficients are positive and statistically significant. Especially, for the simple univariate GARCH(1,1), the near-unity sum of the coefficients suggests very high persistence in the conditional variances.

Table 3. Ad-Hoc GARCH(1,1) estimates for DJ, FTSE, (F+D), EUR, GBP, (E+G)

Ad Hoc GARCH(1,1)	FTSE	DJ	(F+D)	EUR	GBP	(E+G)
$c_{11,t}$, c_{22} , c_{12}	0.0132	0.0114	0.0098	-0.0001	0.5073	0.0011
	(0.0038)	(0.0021)	(0.0021)	(0.0005)	(0.0187)	(0.0008)
$\varepsilon^2_{1,t-1}, \varepsilon^2_{2,t-1}, \varepsilon^2_{12,t-1}$	0.1010	0.0758	0.0839	0.0216	-0.0505	0.0297
	(0.0132)	(0.0087)	(0.0096)	(0.0043)	(0.0042)	(0.0058)
$h_{1,t-1}\ h_{2,t-1}, h_{12,t-1}$	0.8872	0.9141	0.9035	0.9786	-0.9615	0.9665
	(0.0138)	(0.0103)	(0.0110)	(0.0045)	(0.0440)	(0.0073)
LL	-2473.33	-2488.77	-2192.09	-1745.96	-1447.94	-1447.77
Nyblom Joint Test	4.15	3.21	3.98	3.83	3.65	3.76

Standard errors appear in brackets. For the Nyblom Joint test, the 1% critical value is equal to 2.8.

Table 4. BEKK(1,1) estimates for (FTSE, DJ), (EUR, GBP)

BEKK(1,1)	(FTSE, DJ)	(EUR,GBP)
c_{11}	0.0398	0.0161
	(0.0194)	(0.0120)
β_1	0.9733	0.9875
	(0.0030)	(0.0017)
a_1	0.0.2135	0.1501
	(0.0117)	(0.0103)
c_{12}	0.0357	0.0200
	(0.0188)	(0.0105)
c_{22}	0.0647	0.0393
	(0.0151)	(0.0120)
β_2	0.9477	0.9828
	(0.0062)	(0.0029)
a_2	0.2988	0.1638
	(0.0177)	(0.0117)
LL	-4801.50	-2456.70

Standard errors appear in brackets.

In the following figures 1a, 1b, 1c the fitted conditional covariance is plotted for the three multivariate models. From a first view, for both portfolios, the EUR-GBP currencies portfolio and the Dow-FTSE indices portfolio, the magnitudes and patterns of the time varying conditional covariance obtained from S-ARCD are similar to these captured from the other two multivariate models ad hoc GARCH(1,1) and BEKK(1,1). As a result, a more sophisticated evaluation approach must be developed in order to examine the performance of the three multivariate models.

Figure 1a: Time varying Conditional Covariances for S-ARCD model.

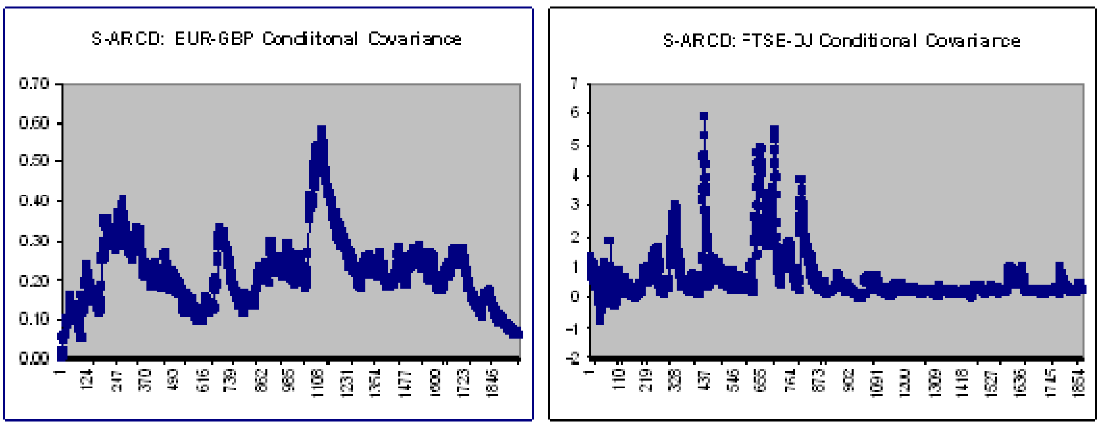


Figure 1b: Time varying Conditional Covariances for ad hoc GARCH(1,1) model.

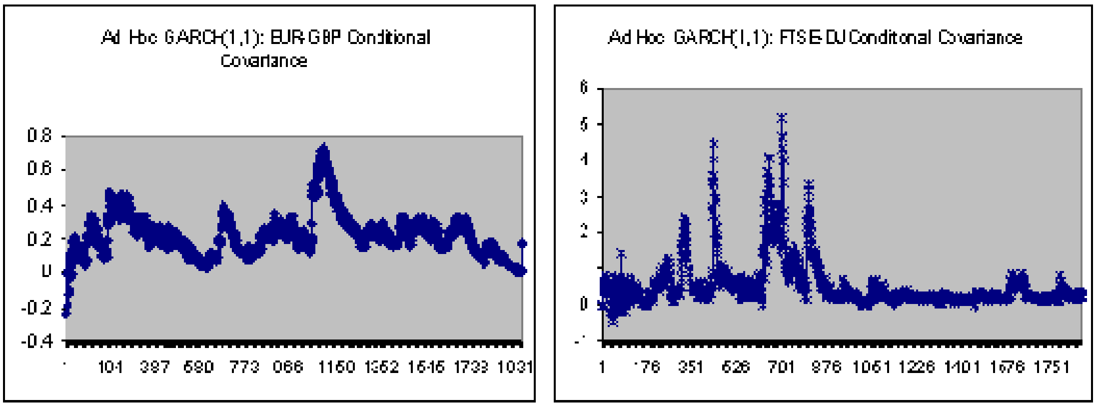
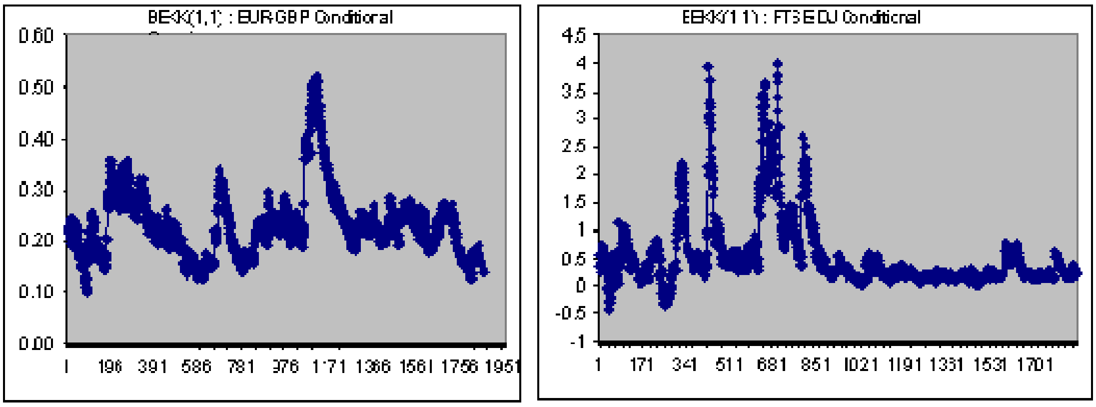


Figure 1c: Time varying Conditional Covariances for BEKK(1,1) model.



4. Evaluation

Two approaches are employed in order to evaluate the performance of the S-ARCD model against the other two multivariate models ad-hoc GARCH(1,1) and BEKK(1,1) models. Firstly, a statistical method is implemented using a regression model and secondly an economic approach is used to estimate the volatility for the VaR calculation of the aggregate equity portfolios and the foreign exchange portfolios.

For the statistical evaluation, the Harris, Stoja and Tucker (2007) proposed a regression so as to test the conditional unbiasedness of the estimated conditional covariance matrix. We compare the estimated conditional variances and covariance $h_{1,t}, h_{2,t}, \sigma_{1,2,t}$ versus to the realized conditional variances and covariance which are the squares and the cross products of the residuals estimated by the relative multivariate model for each series $\hat{\varepsilon}_{1,t}, \hat{\varepsilon}_{2,t}$, respectively, using the following Ordinary Least Squares (OLS) regressions:

$$\begin{aligned}
&\hat{\varepsilon}_{1,t}^2 = \theta_{1,0} + \theta_{1,1}\sigma_{1,t}^2 + v_{1,t} \\
&\hat{\varepsilon}_{2,t}^2 = \theta_{2,0} + \theta_{2,1}\sigma_{2,t}^2 + v_{2,t} \qquad (13) \\
&\hat{\varepsilon}_{1,t}\hat{\varepsilon}_{2,t} = \theta_{12,0} + \theta_{12,1}\sigma_{12,t} + v_{12,t}
\end{aligned}$$

The above regressions are tested using an F-statistic. When the null hypothesis of a zero intercept and a slope coefficient equal to one is not rejected then the multivariate model under testing is well specified and correctly defined (Andersen and Bollerslev 1998, Harris, Stoja and Tucker, 2007).

For the economic approach, we compute out of sample one day period VaR forecasts using the variance covariance approach[2] (VCV) for both equities and foreign exchange portfolios, since the VCV approach considers and reveals directly the volatility and correlation effect in the Value at risk (VaR) estimation. The volatility is updated as in Hull and White (1988) procedure in order to capture the volatility clustering. The more accurate and efficient variance covariance estimation (VCV) is the one which gives the lower level of capital to cover against unexpected portfolio's losses and also the smaller average deviation between the estimated VaR and the actual return. Brooks and Persand (2003) showed that the forecasted portfolio's VaR based on the VCV approach is calculated as:

$$VaR_{p,t+1}^{i}(T,a) = \left[F_{t+1,T}^{i}\right]^{-1}\left(\frac{a}{100}\right)\sqrt{h_{p,t+1,T}^{i}} \qquad (14)$$

where i=S-ARCD, BEKK, ad-hoc GARCH(1,1), T is the forecast horizon, here equal to 1 day period, α is the desired confidence level, $\left[F_{t,T}^{i}\right]^{-1}$ is the inverse of the cumulative distribution function and $h_{p,t,T}^{i}$ is the portfolio's forecasted conditional variance which is given by the following type:

$$h_{p,t+1} = a^2 h_{1,t+1} + b^2 h_{2,t+1} + 2ab\sigma_{12,t+1}$$

(14a)

[2] We restrict our attention to the variance covariance approach but also and other methodologies such as historical simulation or parametric Riskmetrics approach could be applied as well.

where $h_{1,t+1}$, $h_{2,t+1}$ are the forecasted conditional variances estimated form the three models for the indices and foreign currencies respectively and $\sigma_{12,t+1}$ is the forecasted conditional covariance of the two indices or the currencies estimated form the respective model, with a and b being the proportion invested in each asset. In this study the weights a and b are each equal to 0.5, while the cumulative distribution function is the normal distribution and the significance confidence level is chosen as 5% and 1% which corresponds to a value of δ equal to 1.65 and 2.33 for the normal distribution respectively. In order to compare the VaR forecasts accuracy estimated by the three models the following measures for VaR evaluation are performed:

Unconditional Coverage

Kupiec (1995) proposed an unconditional test (LR^{un}) so as to test the proportion of times VaR is exceeded in a given sample and under the correct VaR model with the null hypothesis that expected violation frequency is equal to the desired significance level. The LR^{un} follows an asymptotic chi-square distribution with one degree of freedom $\chi^2(1)$ and computes the appropriate likelihood ratio statistic as:

$$LR^{un} = -2\ln\left[(1-p)^{T-N} p^{N}\right] + 2\ln\left[(1-N/T)^{T-N}(N/T)^{N}\right]$$

(15)

where T is the sample size, N is the number of failures or violations, and p is the desired significance.

Conditional Coverage

Christofferson (1998) developed a test statistic (LR^{ind}) to account for unconditional coverage and also for serial independence of VaR estimates. This is very useful since we can conclude if a model rejection is due the unconditional coverage failure or clustering of the exceptions or both. For testing the independence of the VaR violations, the statistic is asymptotically χ^2 distributed with one degree of freedom and is derived as:

$$LR^{ind} = 2\left[v_{00}\ln(\pi_{00}/(1-\pi)) + v_{01}\ln((1-\pi_{00})/\pi) + v_{10}\ln(\pi_{10}/(1-\pi)) + v_{11}\ln(1-\pi_{10})/\pi)\right] \quad (16)$$

where v_{ij} is the number of observations of I_t with value i followed by j, $\pi_{00} = v_{00}/(v_{00}+v_{01})$, $\pi_{10} = v_{10}/(v_{10}+v_{11})$, $\pi = (v_{01}+v_{11})/(v_{00}+v_{01}+v_{10}+v_{11})$. The indicator I_t is constructed as: $I_t = \begin{cases} 1, & \text{if exceedence occurs} \\ 0, & \text{if no exceedence occurs} \end{cases}$.

The joint test for conditional coverage capturing both unconditional coverage and the independence is simply given by the sum of the above individual tests and follows a χ^2 distribution with two degrees of freedom:

$$LR^{cc} = LR^{un} + LR^{ind} \quad (16a).$$

Root Mean Square Error (RMSE)

In VaR models evaluation, the root mean square error is a frequently used measure of the difference between the VaR estimated values and the actually observed portfolios' returns. The model with the smaller RMSE is considered as the most accurate VaR forecasting model. It is defined as:

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=0}^{T}(r_{t+1} - VaR^{i}{}_{p,t+1})^2} \quad (17)$$

Standard Deviation of Capital Employed

The Economic Capital or Capital Employed is considered as the amount to be set aside in order to cover most of the potential losses at a predetermined level. Its standard deviation is calculated as:

$$SD(VaR) = \sqrt{\frac{1}{T}\sum_{t=0}^{T}(VaR^{i}{}_{p,t+1} - \overline{VaR^{i}_{p}})^2} \quad (18)$$

where $\overline{VaR^{i}_{p}}$ is the average estimated portfolio VaR given by the following type:

$\overline{VaR^{i}_{p}} = \frac{1}{T}\sum_{t=0}^{T} VaR^{i}_{p,t+1}$. The lower the standard deviation of the capital employed, the most accurate is the model used for the VaR calculation since the uncertainty of the compulsory capital used to cover the unexpected portfolio's losses is reduced.

5. Results

Summary statistics for the estimated covariances $\hat{\sigma}_{F,D,t}$, $\hat{\sigma}_{E,G,t}$ from the three models are reported in Tables 5a and 5b, where $\hat{\sigma}_{F,D,t}$ is the estimated covariance for the FTSE and Dow equity indices portfolio and $\hat{\sigma}_{E,G,t}$ is the estimated covariance for the EUR and GBP currencies portfolio. For the equity indices, the S-ARCD model has the highest standard deviation, while for the exchange rate series the BEKK(1,1) model is the one which gives the lowest level of volatility.

Table 5a. Descriptive Statistics of the estimated covariance $\hat{\sigma}_{F,D,t}$

$\hat{\sigma}_{F,D,t}$	Mean	St. Dev.	Skewness	Kurtosis	Min	Max
S-ARCD	0.594	0.577	3.089	11.407	-0.402	5.859
Ad-Hoc GARCH(1,1)	0.534	0.651	2.956	10.542	-0.480	5.145
BEKK(1,1)	0.527	0.615	2.629	7.797	-0.442	4.007

Table 5b. Descriptive Statistics of the estimated covariance $\hat{\sigma}_{E,G,t}$

$\hat{\sigma}_{E,G,t}$	Mean	St. Dev.	Skewness	Kurtosis	Min	Max
S-ARCD	0.218	0.090	0.939	1.876	0.000	0.582
Ad-Hoc GARCH(1,1)	0.220	0.126	1.072	2.644	-0.251	0.752
BEKK(1,1)	0.219	0.073	1.126	3.089	0.080	0.526

In Tables 6a, 6b the descriptive statistics of the conditional correlations, estimated by the three multivariate models, are presented. The BEKK(1,1) model estimates correlation with the lowest variability, while the S-ARCD follows. The Ad-Hoc GARCH(1,1) model gives the most variable multivariate correlation for both data series since the estimated standard deviations are the highest. Obviously, the fitted correlation process for all the three models remains between -1< $\hat{\rho}_{F,D,t}$ <1 and 1< $\hat{\rho}_{E,G,t}$ <1, meaning that the resulting estimated correlation matrix satisfies the condition for positive semi-definiteness for both equity indices and foreign currency data.

Table 6a. Descriptive Statistics of the estimated correlation $\hat{\rho}_{F,D,t}$

$\hat{\rho}_{F,D,t}$	Mean	St. Dev.	Skewness	Kurtosis	Min	Max
S-ARCD	0.459	0.161	-0.589	0.541	-0.301	0.960
Ad-Hoc GARCH(1,1)	0.456	0.177	0.109	3.969	-0.497	0.999
BEKK(1,1)	0.439	0.155	-0.962	2.264	-0.226	0.813

Table 6b. Descriptive Statistics of the estimated correlation $\hat{\rho}_{E,G,t}$

$\hat{\rho}_{E,G,t}$	Mean	St. Dev.	Skewness	Kurtosis	Min	Max

S-ARCD	0.697	0.134	-0.963	1.387	-0.098	0.915
Ad-Hoc GARCH(1,1)	0.691	0.242	-0.767	1.815	-0.976	0.999
BEKK(1,1)	0.705	0.104	-1.005	1.177	0.245	0.886

In Tables 7a, 7b the correlation matrix between the three models is presented. More precisely, for the equity indices data the models S-ARCD and Ad-hoc GARCH(1,1) have the highest correlation reflecting the fact that are based on the same theory framework, but the S-ARCD has higher correlation with BEKK(1,1) than the Ad-hoc model. Indeed, for the currency series, the highest correlation is between S-ARCD and BEKK(1,1) imposing that time varying higher moments such as skewness and kurtosis play important role in the estimation of multivariate variance covariance matrix and must not ignoring them such as in the case of Ad-hoc GARCH(1,1) model which has the lowest correlation with BEKK(1,1) model for both cases.

Table 7a. Correlation matrix for the estimated correlation $\hat{\rho}_{F,D,t}$

$\hat{\rho}_{F,D,t}$	S-ARCD	Ad-Hoc GARCH(1,1)	BEKK(1,1)
S-ARCD	1.000		
Ad-Hoc GARCH(1,1)	0.902	1.000	
BEKK(1,1)	0.643	0.583	1.000

Table 7b. Correlation matrix for the estimated correlation $\hat{\rho}_{E,G,t}$

$\hat{\rho}_{E,G,t}$	S-ARCD	Ad-Hoc GARCH(1,1)	BEKK(1,1)
S-ARCD	1.000		
Ad-Hoc GARCH(1,1)	0.587	1.000	
BEKK(1,1)	0.926	0.432	1.000

The results of the regressions for the statistical evaluation are presented in the following Tables 8a and 8b. The null hypothesis of the unconditional unbiasedness for all the covariance matrix elements is examined by the regressions (13) using an F-statistic in order to confirm if a multivariate model is well and correctly specified. For the equity indices, and their estimated variances $\sigma^2_{F,t}$, $\sigma^2_{D,t}$, the null hypothesis of unconditional unbiasedness is accepted for all multivariate models, while for their estimated covariance, all the multivariate models reject the unconditional unbiasedness, with the BEKK(1,1) model to have the weakest rejection. For the currency data, both S-ARCD and BEKK(1,1) models accept the unconditional unbiasedness for all the three elements of the conditional variance covariance matrix, while the ad-hoc GARCH(1,1) model reject again the null hypothesis. Overall, the BEKK(1,1) and the S-ARCD models are the ones with the best statistical evaluation performance since the unconditional unbiasedness condition is rejected only in one from the six cases.

Table 8a. Unconditional Unbiasedness testing Regressions for the equity indices

$\hat{\varepsilon}^2_{F,t} = \theta_{F,0} + \theta_{F,1}\sigma^2_{F,t} + v_{F,t}$	S-ARCD	Ad-Hoc GARCH(1,1)	BEKK(1,1)
$\hat{\theta}_{F,0}$	0.083	0.065	0.147
$\hat{\theta}_{F,1}$	0.953	0.960	0.891
F-statistic	2.839	0.520	1.368
$\hat{\varepsilon}^2_{D,t} = \theta_{D,0} + \theta_{D,1}\sigma^2_{D,t} + v_{D,t}$			
$\hat{\theta}_{D,0}$	0.121	0.119	0.099
$\hat{\theta}_{D,1}$	0.885	0.898	0.937
F-statistic	2.610	1.588	1.067
$\hat{\varepsilon}_{F,t}\hat{\varepsilon}_{D,t} = \theta_{FD,0} + \theta_{FD,1}\sigma_{FD,t} + v_{FD,t}$			
$\hat{\theta}_{FD,0}$	0.166	0.172	0.133
$\hat{\theta}_{FD,1}$	0.640	0.698	0.762
F-statistic	7.388	8.498	4.014

The F statistic tests the null hypothesis that $\hat{\theta}_{i,0}$ =0 and $\hat{\theta}_{i,1}$ =1 where i=F, D, FD and has an F(2,1861) distribution with critical value 3.00 at the 5% significance level.

Table 8b. Unconditional Unbiasedness testing Regressions for the currency series

$\hat{\varepsilon}^2_{E,t} = \theta_{E,0} + \theta_{E,1}\sigma^2_{E,t} + v_{E,t}$	S-ARCD	Ad-Hoc GARCH(1,1)	BEKK(1,1)
$\hat{\theta}_{E,0}$	0.083	0.081	0.028

$\hat{\theta}_{E,1}$	0.782	0.794	0.933
F-statistic	2.916	2.674	0.248
$\hat{\varepsilon}^2_{G,t} = \theta_{G,0} + \theta_{G,1}\sigma^2_{G,t} + v_{G,t}$			
$\hat{\theta}_{G,0}$	0.046	0.172	0.015
$\hat{\theta}_{G,1}$	0.816	0.331	0.926
F-statistic	1.512	1.072	0.307
$\hat{\varepsilon}_{E,t}\hat{\varepsilon}_{G,t} = \theta_{EG,0} + \theta_{EG,1}\sigma_{EG,t} + v_{EG,t}$			
$\hat{\theta}_{EG,0}$	-0.047	0.107	0.034
$\hat{\theta}_{EG,1}$	1.213	0.490	0.834
F-statistic	2.200	23.644	0.848

The F statistic tests the null hypothesis that $\hat{\theta}_{i,0}$ =0 and $\hat{\theta}_{i,1}$ =1 where i=E, G, EG and has an F(2,1952) distribution with critical value 3.00 at the 5% significance level.

A rolling window of 100 observations is used for the out of sample estimation of each model. The model which has the greater percentage of VaR exceedences in the out of sample period the highest RMSE and the highest standard deviation of the capital employed is ranked as the worst model, while the most accurate is the one with the lower percentage of exceedences and also with the lowest RMSE and standard deviation of the capital employed. Tables 9a and 9b and tables 10a and 10b report the estimated five measures for the out of sample VaR evaluation of both equity and currency portfolios at the 95% and 99% levels respectively.

Table 9a. Out of sample VaR evaluation measures for the equity indices portfolio at 95% confidence level.

FTSE-DJ Portfolio 95% conf. level	LR^{un}	LR^{ind}	LR^{cc}	RMSE	SD(VaR)

S-ARCD	0.0000	0.2526	0.2526	2.3234	0.5817
Ad-Hoc GARCH(1,1)	0.1984	0.3832	0.5816	2.4596	0.6613
BEKK(1,1)	1.6158	1.1981	2.8139	2.4080	0.5967

The LR^{un} statistic tests the null hypothesis that the proportion of VaR exceedences is equal to the nominal significance level and has an chi squared distribution with critical value 3.84 at the 5% significance level. The LR^{ind} tests the null hypothesis that the VaR exceedences are serially uncorrelated and has an chi squared distribution with critical value 3.84 at the 5% significance level. The LR^{cc} tests the null hypothesis of both unconditional coverage and that the VaR exceedences are serially uncorrelated and has an chi squared distribution with critical value 5.99 at the 5%.

Table 9b. Out of sample VaR evaluation measures for the equity indices portfolio at 99% confidence level.

FTSE-DJ Portfolio 99% conf. level	LR^{un}	LR^{ind}	LR^{cc}	RMSE	SD(VaR)
S-ARCD	0.7827	0.1216	0.9043	3.0523	1.0295
Ad-Hoc GARCH(1,1)	0.7827	0.1216	0.9043	4.3285	4.0149
BEKK(1,1)	0.7827	0.1216	0.9043	3.1134	2.7171

The LR^{un} statistic tests the null hypothesis that the proportion of VaR exceedences is equal to the nominal significance level and has an chi squared distribution with critical value 6.63 at the 1% significance level. The LR^{ind} tests the null hypothesis that the VaR exceedences are serially uncorrelated and has an chi squared distribution with critical value 6.63 at the 1% significance level. The LR^{cc} tests the null hypothesis of both unconditional coverage and that the VaR exceedences are serially uncorrelated and has an chi squared distribution with critical value 9.21 at the 1%.

Table 10a. Out of sample VaR evaluation measures for the foreign currencies portfolio at 95% confidence level.

EUR-GBP Portfolio 95% conf. level	LR^{un}	LR^{ind}	LR^{cc}	RMSE	SD(VaR)
S-ARCD	0.7530	0.2584	1.0114	1.0051	0.1442
Ad-Hoc GARCH(1,1)	2.4286	3.55E-15	2.4286	1.4887	0.2144
BEKK(1,1)	0.2253	0.1251	0.3504	1.1385	0.1961

The LR^{un} statistic tests the null hypothesis that the proportion of VaR exceedences is equal to the nominal significance level and has an chi squared distribution with critical value 3.84 at the 5% significance level. The LR^{ind} tests the null hypothesis that the VaR exceedences are serially uncorrelated and has an chi squared distribution with critical value 3.84 at the 5% significance level. The LR^{cc} tests the null hypothesis of both unconditional coverage and that the VaR exceedences are serially uncorrelated and has an chi squared distribution with critical value 5.99 at the 5%.

Table 10b. Out of sample VaR evaluation measures for the foreign currencies portfolio at 99% confidence level.

EUR-GBP Portfolio 99% conf. level	LR^{un}	LR^{ind}	LR^{cc}	RMSE	SD(VaR)
S-ARCD	5.1821	-4.44E-15	5.1821	1.2762	0.2036
Ad-Hoc GARCH(1,1)	0.0000	1.77E-15	0.0000	1.9994	0.3027
BEKK(1,1)	0.7827	3.55E-15	0.7827	1.4809	0.2769

The LR^{un} statistic tests the null hypothesis that the proportion of VaR exceedences is equal to the nominal significance level and has an chi squared distribution with critical value 6.63 at the 1% significance level. The LR^{ind} tests the null hypothesis that the VaR exceedences are serially uncorrelated and has an chi squared distribution with critical value 6.63 at the 1% significance level. The LR^{cc} tests the null hypothesis of both unconditional coverage and that the VaR exceedences are serially uncorrelated and has an chi squared distribution with critical value 9.21 at the 1%.

All models provide correct unconditional and conditional coverage close to the significance levels since their LR^{un}, LR^{ind}, *and* LR^{cc} *values* do not violate both the 5% and 1% tolerance levels, and therefore their VaR forecasts are adequate. More precisely, the S-ARCD model in the equity indices portfolio achieves the required coverage in the out of sample period, meaning that there is no waste capital, while the ad-hoc GARCH(1,1) model is the one in the foreign currency portfolio. Summarizing the results for the unconditional and conditional coverage, the VaR predictions obtained from the three models are all within the percentage exceedences threshold. Since, all models have a good unconditional and conditional coverage performance, the examination of the root mean square deviation and the standard deviation of the capital employed is required. For both equity and foreign currency portfolios, the S-ARCD model provides the lower RMSE at 5% and 1% levels reflecting its enhanced efficiency in the presence of leptokurtosis since it captures time varying skewness and kurtosis, while the BEKK follows. Considering the capital employed, the S-ARCD model produces again the lowest standard deviation of the capital employed allowing the uncertainty, over the required capital reserved to cover against unexpected adverse price movements, to be highly reduced. This is very important for risk managers and investors since their capital can be allocated in other more profitable assets.

5. Conclusions

This paper proposes a simple and effective multivariate version of ARCD model the S-ARCD to model conditional covariance processes which allows for time variation in higher moments and is consistent with both asymmetries and fat-tails that are typically observed in financial data. Empirical results using the equity indices FTSE, DJ and the foreign currencies EUR and GBP against USD suggest that a significant ARCD model can be estimated for all series. Moreover, VaR estimates via the S-ARCD offer superior out-of-sample performance compared to the BEKK and ad-hoc GARCH(1,1) models respectively implying that a practical and computationally easier estimation of the conditional covariance approach can be obtained considering also the time variation of the higher moments.

To the best of our knowledge that for the first time ARCD model is used for conditional covariance estimation. On the basis of our results and the flexibility that the S-ARCD offers, we believe that further empirical research is justified.

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