Hsiang-Tai Lee

Assistant Professor

Department of Banking and Finance,

National Chi Nan University, Taiwan 54561

(886) 49-2910960

sagerlee@ncnu.edu.tw

Optimal Hedging with a Regime Switching Gumbel-Clayton Copula GARCH Model

Abstract

The article develops a regime switching Gumbel-Clayton copula GARCH model (RSGC) for optimal futures hedging. There are three major contributions of RSGC. First, the dependence of spot and futures return series in RSGC is modeled using switching copula instead of assuming bivariate normality. Second, RSGC adopts an independent switching GARCH process to avoid the path dependency problem. Third, based on the assumption of independent switching in RSGC, a formula is derived for calculating the minimum variance hedge ratio. RSGC exhibits good out-of-sample hedging effectiveness based on corn and oats futures data traded in the Chicago Board of Trade.

I. Introduction

It is widely known that the time-varying variance-minimizing futures hedge is given by the ratio of the conditional covariance of the futures and spot returns to the conditional variance of the futures return. A considerable amount of studies have applied various Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models for estimating the time-varying minimum variance hedge ratio. Baillie and Myers (1991) estimate the time-varying minimum variance hedge ratio by using a GARCH model with diagonal vech specification. Kroner and Sultan (1993) and Park and Switzer (1995) apply the constant correlation GARCH (CC-GARCH) for estimating the optimal hedge ratios. Gagnon and Lypny (1995), and Brooks et al. (2002) apply BEKK (Baba-Engle-Kraft-Kroner) GARCH, and Byström (2003) applies orthogonal GARCH for estimating the time-varying minimum variance hedge ratios.

Recent studies recognize that the relationship between spot and futures returns may be characterized by regime shifts (Sarno and Valente, 2000, 2005a, 2005b) and adopt regime switching models to generate state-dependent dynamic hedging strategies, which are found to perform better than state-independent strategies (Alizadeh and Nomikos, 2004; Lee et al, 2006; and Lee and Yoder, 2007a, 2007b). Alizadeh and Nomikos (2004) and Lee et al (2006) estimate the minimum variance hedge ratio by treating it as a time-varying coefficient. Lee and Yoder (2007a and 2007b) estimate the minimum variance hedge ratio by estimating the conditional second moments with regime switching GARCH models.

Although these regime switching GARCH models have captured much of the observed behavior in the spot and futures return series, they possess some drawbacks.

First, previous regime switching GARCH hedging methods usually assume a joint normality between spot and futures return series. This may be misleading in the presence of non-normality in these returns series. Second, previous regime switching GARCH hedging models use Gary's recombining method for solving the dependency problem. However, the interpretation of the variance process in Gray's approach is problematical. Third, in measuring the hedging performance, previous regime switching hedging models derive the minimum variance hedge ratios by recombining the variances and covariance in each state. If Gray's recombining method is problematical, the minimum variance hedge ratio derived is also problematical.

This article develops a regime switching Gumbel-Clayton copula GARCH model (RSGC) for dealing these problems. Instead of assuming bivariate normality, RSGC models the dependence structure of spot and futures return series with switching Gumbel-Clayton copula. In additions, RSGC adopts an independent switching GARCH process to avoid the path dependency problem. Finally, a formula is derived for calculating the minimum variance hedge ratio for the hedging portfolio under independent switching assumption. The proposed model is applied to the corn and oats futures data traded in the Chicago Board of Trade. Results show that RSGC provides good out-of-sample hedging performance.

The remainder of the article is organized as follows. The proposed regime switching Gumbel-Clayton copula GARCH (RSGC) model is presented next. Section III derives the formula of the minimum variance hedge ratio for hedging portfolio and discusses the hedging performance measurements. This is followed by data description and empirical results. A conclusion ends the article.

II. Regime Switching Gumbel-Clayton Copula GARCH (RSGC)

The previous regime switching GARCH hedging methods (Lee and Yoder; 2007a and 2007b) possess some drawbacks. In this section, these drawbacks will be discussed and we will see how the proposed regime switching Gumbel-Clayton Copula GARCH (RSGC) deals these problems. RSGC can be specified in the following way. Let $r_{c,t}$ and $r_{f,t}$ be the returns on the spot and futures, respectively:

$$r_{c,t} = \mu_{c,s_t} + e_{c,t,s_t} \tag{1}$$

$$r_{f,t} = \mu_{f,s_t} + e_{f,t,s_t}, \tag{2}$$

where μ_{c,s_t} and μ_{f,s_t} are state-dependent mean returns to spot and futures returns, and e_{c,t,s_t} and e_{f,t,s_t} are state-dependent disturbances. The state variable $s_t = \{1, 2\}$ is assumed to follow a first-order, two-state Markov process and state transition probabilities are assumed to follow a logistic function such that

$$\Pr(s_t = 1 \mid s_{t-1} = 1) = P = \frac{\exp(p_0)}{1 + \exp(p_0)}, \tag{3}$$

$$\Pr(s_t = 2 \mid s_{t-1} = 2) = Q = \frac{\exp(q_0)}{1 + \exp(q_0)}, \tag{4}$$

where p_0 and q_0 are unconstrained constants that can be estimated along with other unknown parameters via maximum likelihood estimation. In the previous regime switching GARCH hedging methods (Lee and Yoder; 2007a and 2007b), the conditional variances at time t given s_t for spot and futures returns denoted as h_{c,t,s_t} and h_{f,t,s_t} are assumed to follow a state-dependent GARCH(1,1) process

$$h_{c,t,s_t} = \gamma_{c,s_t} + \alpha_{c,s_t} e_{c,t-1}^2 + \beta_{c,s_t} h_{c,t-1}, \tag{5}$$

$$h_{f,t,s_t} = \gamma_{f,s_t} + \alpha_{f,s_t} e^2_{f,t-1} + \beta_{f,s_t} h_{f,t-1}. \tag{6}$$

When a recursive process is subject to regime switching, the recursive nature of the process makes the model intractable due to the dependence of the conditional variances on the entire past history of the data (Cai, 1994; Hamilton and Susmel, 1994; Gray, 1995, 1996).

To solve this path-dependency problem, previous regime switching GARCH hedging models use Gray's (1996) recombining method to recombine the conditional variances as given by

$$h^2_{i,t} = p_{1t}\left(\mu^2_{i,1} + h^2_{i,t,1}\right) + \left(1 - p_{1t}\right)\left(\mu^2_{i,2} + h^2_{i,t,2}\right) - \left[p_{1t}\mu_{i,1} + \left(1 - p_{1t}\right)\mu_{i,2}\right]^2, \tag{8}$$

for $i = \{c, f\}$, where p_{1t} is the probability of being in regime 1 at time t, defined as

$$\begin{aligned} p_{1t} &= \Pr\left(s_t = 1 \mid \psi_{t-1}\right) \\ &= P\left[\frac{f_{1t-1}p_{1t-1}}{f_{1t-1}p_{1t-1} + f_{2t-1}\left(1 - p_{1t-1}\right)}\right] + \left(1 - Q\right)\left[\frac{f_{2t-1}\left(1 - p_{1t-1}\right)}{f_{1t-1}p_{1t-1} + f_{2t-1}\left(1 - p_{1t-1}\right)}\right], \end{aligned} \tag{9}$$

and P and Q are transition probabilities defined in (3) and (4).

While the model of Gray is attractive in that it combines Markov switching and GARCH effects and solves the estimation difficulties, its analytical intractability is a serious drawback (Hass, et. al., 2004). Consider a state-dependent GARCH(1,1) process,

$$h_t = \gamma + \alpha\, e^2_{t-1} + \beta\, h_{t-1}. \tag{10}$$

If $\beta < 1$, the variance h_t can be expressed as

$$h_t = \gamma\left(1 - \beta\right)^{-1} + \alpha \sum_{i=1}^{\infty} \beta^{i-1} e^2_{t-i}, \tag{11}$$

where α reflects the magnitude of a unit shock's immediate impact on the next period's variance, β is a parameter of inertia and indicates the memory in the variance, and the total impact of a unit shock to future variance is $\alpha(1-\beta)^{-1}$. In the regime switching GARCH model, the relationship between the pattern with which h_t responds to shocks and the parameters α, β, and γ is far from obvious if Gray's recombining method is used because the lagged variances is replaced with the recombined variances. Moreover, in Gray's model it is possible that the variance of one regime will still be affected by shocks even if α in that regime is zero.

Hass, et. al. (2004) suggests an independent switching GARCH process to solve these problems. The independent switching GARCH process is specified below:

$$h_{t,1} = \gamma_1 + \alpha_1 e_{t-1,1}^2 + \beta_1 h_{t-1,1}, \tag{12}$$

$$h_{t,2} = \gamma_2 + \alpha_2 e_{t-1,2}^2 + \beta_2 h_{t-1,2}, \tag{13}$$

where the subscripts 1 and 2 stands for regime 1 and regime 2. This specification preserves the economic significant of the variance dynamics in each regime and admits analytical analysis of the variance process.

Following the concept of Hass, et. al., this study also suggests an independent switching variance dynamics for the RSGC hedging model. The volatility equations of RSGC are modeled as

$$e_{c,t,s_t} \sim N\left(0, h_{c,t,s_t}\right), \tag{14}$$

$$e_{f,t,s_t} \sim N\left(0, h_{f,t,s_t}\right), \tag{15}$$

and the conditional variance vector is specified as

$$\mathbf{h}_t = \boldsymbol{\gamma}_t + \boldsymbol{\alpha}_t \mathbf{e}_t^2 + \boldsymbol{\beta}_t \mathbf{h}_{t-1}, \tag{16}$$

where

$$\mathbf{h}_t = \begin{bmatrix} h_{c,t,1} \\ h_{c,t,2} \\ h_{f,t,1} \\ h_{f,t,2} \end{bmatrix}, \ \boldsymbol{\gamma}_t = \begin{bmatrix} \gamma_{c,1} \\ \gamma_{c,2} \\ \gamma_{f,1} \\ \gamma_{f,2} \end{bmatrix}, \ \boldsymbol{\alpha}_t = \begin{bmatrix} \alpha_{c,1} & 0 \\ \alpha_{c,2} & 0 \\ 0 & \alpha_{f,1} \\ 0 & \alpha_{f,1} \end{bmatrix}, \ \mathbf{e}_t^2 = \begin{bmatrix} e_{c,t-1}^2 \\ e_{f,t-1}^2 \end{bmatrix}, \text{ and}$$

$$\boldsymbol{\beta}_t = \begin{bmatrix} \beta_{c,1} & 0 & 0 & 0 \\ 0 & \beta_{c,2} & 0 & 0 \\ 0 & 0 & \beta_{f,1} & 0 \\ 0 & 0 & 0 & \beta_{f,1} \end{bmatrix}. \tag{17}$$

Another problem of previously proposed regime switching GARCH hedging methods (Lee and Yoder; 2007a and 2007b) is that they impose a joint-normality restriction to the disturbance vector $\mathbf{e}_{t,s_t}$ as bellows:

$$\mathbf{e}_{t,s_t} \mid \psi_{t-1} = \begin{bmatrix} e_{c,t,s_t} \\ e_{f,t,s_t} \end{bmatrix} \mid \psi_{t-1} \sim \quad BN\left(0, \quad \mathbf{H}_{t,s_t}\right), \tag{18}$$

where ψ_{t-1} refers to the information available at time $t-1$, BN stands for bivariate normal, and $\mathbf{H}_{t,s_t}$ is a time-varying, state-dependent, 2×2 positive definite conditional covariance matrix.

This restriction ignores the potential important dimension of the dynamic futures hedging, the non-normal dependence. The proposed regime switching Gumbel-Clayton (RSGC) copula GARCH model captures the regime shifts as previous regime switching GARCH methods do and also captures non-normal dependence of spot and futures returns. Copulas are functions that join or couple multivariate distribution functions to their one-dimension marginal distribution functions. The most important result in copula theory is Sklar's theorem which says that it is possible to separate the univariate margins from the dependence structure represented by copula (Rodriguez, 2007)[1].

The marginal distributions are assumed to be mixture of normal distributions as defined in equations (14) and (15) and the dependence structure is modeled with the following switching Gumbel-Clayton copula

$$C(u_t, v_t \mid s_t, \psi_{t-1}) = \pi_{c,s_t} C_t^C (u_t, v_t; \delta_{t,s_t}^C \mid \psi_{t-1}) + (1 - \pi_{c,s_t}) \, C_t^G (u_t, v_t; \delta_{t,s_t}^G \mid \psi_{t-1}), \qquad (19)$$

where $\pi_{c,s_t} \in [0,1]$ is a state-dependent shifting parameter of the mixture Gumbel-Clayton copula and C_t^C and C_t^G are state-dependent versions of Clayton (1978) and Gumbel (1960) copulas, respectively. The Gumbel-Clayton copula describes situations of asymmetric tail dependence and the nested Gumbel and Clayton copulas exhibit the upper tail dependence and the lower tail dependence, respectively. Lower tail dependence increases as π_{c,s_t} goes from zero to one.

The state-dependent Clayton and Gumbel copula are defined as

$$C_t^C (u_t, v_t; \delta_t^C \mid \psi_{t-1}) = (1 + \delta_t^C) \, (u_t v_t)^{-1-\delta_t^C} \left(u_t^{-\delta_t^C} + v_t^{-\delta_t^C} - 1\right)^{\frac{-1-2\delta_t^C}{\delta_t^C}}, \qquad (20)$$

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[1] Sklar's Theorm: (Sklar, 1959): Let D be an n-dimensional distribution function with margins $F_1 \cdots F_n$. Then there exists an n-copula C such that for all x in $\overline{R}^n$,

$$D(x_1, x_2, \cdots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)).$$

If $F_1 \cdots F_n$ are all continuous, then C is uniquely determined on $\mathrm{Ran}\ \mathrm{F}_1 \times \ldots \times \mathrm{Ran}\ \mathrm{F}_n$. Conversely, if C is an n-copula and $F_1 \cdots F_n$ are distribution functions, the function D defined above is an n-dimensional distribution function with margins $F_1 \cdots F_n$.

With this theorem he density of D can be expressed as the product of the copula density and the univariate marginal structure. This is can be seen by deriving both sides of the above equation to get the density of D

$$\frac{\partial^n D(x_1, x_2, \cdots, x_n)}{\partial x_1, \cdots, \partial x_n} = \frac{\partial^n C(F_1(x_1), \ldots, F_n(x_n))}{\partial x_1, \cdots, \partial x_n} \times f_1(x_1) \times \cdots \times f_n(x_n)$$

$$C_t^G\left(u_t, v_t; \delta_t^G \mid \psi_{t-1}\right) = \frac{\exp\left\{-\Lambda^{\frac{1}{\delta_t^G}}\right\}\left[\ln\left(u_t\right)\ln\left(v_t\right)\right]^{\delta_t^G - 1}\left\{\Lambda^{\frac{1}{\delta_t^G}} + \delta_t^G - 1\right\}}{u_t v_t \Lambda^{2-\frac{1}{\delta_t^G}}}, \quad (21)$$

where $\Lambda = \left[\left(-\ln(u_t)\right)^{\delta_t^G} + \left(-\ln(v_t)\right)^{\delta_t^G}\right]$, $u_t = F_c\left(r_{c,t} \mid s_t, \psi_{t-1}\right)$ and $v_t = F_f\left(r_{f,t} \mid s_t, \psi_{t-1}\right)$ are respectively the state-dependent conditional cumulative distribution functions of the spot and futures returns at time t, and δ_t^C and δ_t^G are parameters of Gumbel and Clayton copulas, respectively.

Patton (2006a and 2006b) introduced the concept of conditional copula to capture the time shifts in the dependence structure. This study also allows δ_t^C and δ_t^G to be time-varying by defining $\delta_t^C = \frac{2\tau_t}{1-\tau}$, and $\delta_t^G = \frac{1}{1-\tau_t}$ (Nelsen, 1999), where τ_t is the time-varying scaled invariant dependence measure Kendall's τ which is given by $\tau_t = \left(\frac{2}{\pi}\right)\sin^{-1}\left(\rho_t\right)$ (Lindskog et. al., 2001; and Lai et. al., 2007). The dependence process can be specified as

$$\rho_t = \left(1 - \theta_1 - \theta_2\right)\rho + \theta_1 \rho_{t-1} + \theta_2 \phi_{t-1}, \quad (22)$$

where parameters θ_1 and θ_2 are assumed nonnegative and $\theta_1 + \theta_2 \le 1$ (Tse and Tsui, 2002). The disturbance term ϕ_t is given by

$$\phi_{t-1} = \frac{\sum_{j=1}^{2} \varepsilon_{c,t-j}\varepsilon_{f,t-j}}{\sqrt{\left(\sum_{j=1}^{2} \varepsilon_{c,t-j}^2\right)\left(\sum_{j=1}^{2} \varepsilon_{f,t-j}^2\right)}}, \quad (23)$$

where $\varepsilon_{c,t-1} = \dfrac{e_{c,t-1}}{\sqrt{h_{c,t}}}$ and $\varepsilon_{f,t-1} = \dfrac{e_{f,t-1}}{\sqrt{h_{f,t}}}$ are standardized residuals of spot and futures returns.

The unknown parameters in RSGC are $\mathbf{\Theta} = \{p_0,\ q_0,\ \mu_{c,s_t},\ \mu_{f,s_t},\ \gamma_{c,s_t},\ \gamma_{f,s_t},\ \alpha_{c,s_t},\ \alpha_{f,s_t},\ \beta_{c,s_t},\ \beta_{f,s_t},\ \pi_{,s_t},\ \theta_1,\ \theta_2,\ \rho\ \}$ for $s_t = \{1,2\}$, which can be estimated by maximizing the following log-likelihood function[2]

$$L(\Theta) = \sum_{t=1}^{T} \log\left[q\left(r_{c,t}, r_{f,t} \mid \psi_{t-1}\right)\right], \tag{24}$$

where

$$q\left(r_{c,t}, r_{f,t} \mid \psi_{t-1}\right) = \sum_{i=1}^{2} q\left(r_{c,t}, r_{f,t}, s_t = i \mid \psi_{t-1}\right)$$

$$= \sum_{i=1}^{2} C\left(u_t, v_t \mid s_t = i, \psi_{t-1}\right) \times f\left(r_{c,t} \mid s_t = i, \psi_{t-1}\right) \times g\left(r_{f,t} \mid s_t = i, \psi_{t-1}\right) \times p\left(s_t = i \mid \psi_{t-1}\right),$$

with C the copula density and f and g the marginal densities of spot and futures returns, respectively.

III. Measuring Hedging Performance

Hedging performance is evaluated from both a risk-minimization and a utility standpoint. Let χ_t be the estimated optimal hedge ratios derived from various hedging strategies. The estimated time-varying minimum variance hedge ratio denoted as χ_t is given by

[2] When the copula and marginal parameters change simultaneously according to a Markov Switching process, the two-step approach cannot be used. All parameters must be estimated simultaneously (Rodriguez, 2007).

$$\chi_t = \frac{Cov\left(r_{c,t}, r_{f,t} \mid \psi_{t-1}\right)}{Var\left(r_{f,t} \mid \psi_{t-1}\right)}. \tag{25}$$

If the hedge ratio is subjected to regime shifts, the recombined covariance and variance are found first and the optimal hedge ratio is then calculated with equation (25). In the proposed regime switching Gumbel-Clayton GARCH model, recombining process is not required and the time-varying minimum variance hedger ratio is given by

$$\chi_t = \frac{p_{1t}^2 Cov\left(r_{c,1,t}, r_{f,1,t}\right) + \left(1 - p_{1t}\right)^2 Cov\left(r_{c,2,t}, r_{f,2,t}\right)}{p_{1t}^2 Var\left(r_{f,1,t}\right) + \left(1 - p_{1t}\right)^2 Var\left(r_{f,2,t}\right)}.^{3} \tag{26}$$

The return from the hedged portfolio can be expressed as $r_{p,t+1} = r_{c,t+1} - \chi_t r_{f,t+1}$. From a risk-minimization standpoint, a hedger chooses a hedging strategy to minimize the variance of the hedged portfolio return or equivalently to maximize the variance reduction of a hedging strategy compared to the unhedged position. To better understand the economic significance of portfolio variance reduction, the utility-based criterion is also used to investigate the hedging effectiveness of RSGC model. Consider a hedger with a mean-variance expected utility function:

[3] This can be easily proved as follows. Let r_p be the hedging portfolio return which is given by

$$\begin{aligned} r_p &= p_1\left(portfolio\ return\ in\ state\ 1\right) + \left(1 - p_1\right)\left(portfolio\ return\ in\ state\ 2\right) \\ &= p_1\left(r_{c,1} - \chi\ r_{f,1}\right) + \left(1 - p_1\right)\left(r_{c,2} - \chi\ r_{f,2}\right), \end{aligned}$$

where $r_{c,i}$ and $r_{f,i}$ are the returns spot and futures would have in the state i, respectively. We would like to choose χ such that the variance of the portfolio return is minimized.

$$\min_{\chi} Var\left(r_p\right) = \min_{\chi} \left\{ \begin{array}{l} p_1^2\left[Var\left(r_{c,1}\right) - 2\chi Cov\left(r_{c,1}, r_{f,1}\right) + \chi^2 Var\left(r_{f,1}\right)\right] \\ + \left(1 - p_1\right)^2\left[Var\left(r_{c,1}\right) - 2\chi Cov\left(r_{c,1}, r_{f,1}\right) + \chi^2 Var\left(r_{f,1}\right)\right] \\ + 2 p_1\left(1 - p_1\right)\ Cov\left(r_{c,1} - \chi\ r_{f,1},\quad r_{c,2} - \chi\ r_{f,2}\right) \end{array} \right\}$$

Deriving the above equation with respect to χ and using the assumption of independent switching gives equation (26).

$$E\left[U\left(r_{p,t}\right)|\psi_{t-1}\right] = E\left[r_{p,t} \mid \psi_{t-1}\right] - \xi \; Var\left(r_{p,t} \mid \psi_{t-1}\right), \tag{25}$$

where ξ is the degree of risk aversion and E stands for expectation operator. A dynamic hedging strategy is considered to be superior to a static ordinary least square (OLS) method if it has higher expected utility net of transaction costs.

IV. Data Description and Empirical Results

Corn and Oats nearby futures contract traded in the Chicago Board of Trade (CBOT) are investigated in this study. Both spot and futures rates were collected from Datastream from January 1991 to December 2007. The spot and futures data are Wednesday's closing price. Estimation of all models was conducted using data from January 1991 to December 2006; the data of the most recent year are used for out-of-sample analysis. Summary statistics corn and oats data are shown in table I and parameter estimates from all alternative models are presented in table II. The parameters are estimated by maximizing the log-likelihood functions using GAUSS.

For Corn data, the β s are 0.865 and 0.914 for spot and return series in low volatility state (state one) and 0.778 and 0.731 for spot and return series in high volatility state (state two). This shows that volatility in the low volatility state has higher shock persistence than in the high volatility state. The weighting parameters π_1 and π_2 equal to 0.235 and 0.385 in low and high volatility states, respectively. This implies that the lower tail dependence increases in the high volatility state. For oats data, the volatility in the low volatility state also has higher shock persistence than in the high volatility state but the lower tail dependence decreases in the high volatility state.

Table III presents summary statistics regarding the effects of static and dynamic hedging strategies on out-of-sample hedging effectiveness. This study investigates the out-of-sample hedging effectiveness of futures hedging. For the hedger, what matters most is the hedging performance in the future not in the past. The performance of RSGC is compared with performances of OLS, the ordinary least square, CC, the constant correlation GARCH (Bollerslev, 1990), VC, the varying correlation GARCH (Tse and Tsui , 2002), and GC, the state-independent Gumbel-Clayton GARCH. Results show that RSGC hedging method exhibits good hedging performance for both corn and oats data in terms of variance reduction. For corn data, RSGC has the lowest variance which is equal to 2.727. The variance of the hedged portfolio with RSGC hedging is reduced by 2.22%, 0.65%, 0.6%, and 0.14% compared to OLS, CC, VC, and GC hedging. Oats data provide similar results. RSGC has the lowest variance which is equal to 10.472. The variance of the hedged portfolio with RSGC hedging is reduced by 3.83%, 1.28%, 5.65%, and 3.18% compared to OLS, CC, VC, and GC hedging. In general, time-varying hedging methods improve the hedging effectiveness compared to static OLS hedging expect VC model for the oats data. Allowing the dependence structure to be non-normal does not always improve the hedging performance. GC has better performance than CC in corn but worse performance than CC in oats. Allowing both non-normality and regime shifts in the RSGC, however, improves the effectiveness for both corn and oats data.

To better understand the economic significance of portfolio variance reduction, the utility based criterion is also used in comparing the performances of alternative models. Table III gives the utility gains of RSGC over other hedging strategies. Following other empirical studies in the literature (Switzer, 1995; Alizadeh and Nomikos, 2004, and Lee et al, 2006), the hedger is assumed to have an expected utility function given by equation (25) with a degree of risk aversion ξ equal to 4. As shown in table III, RSGC has the best hedging performance in terms of utility gain. RSGC provides utility gain of 2.419, 0.668, 0.613, and 0.169 compared to OLS, CC, VC, and GC hedging for corn data and provides utility gain of 3.045, 10.41, 4.360, and 2.449 compared to OLS, CC, VC, and GC hedging for oats data. The hedger's net benefit from using RSGC hedging over OLS hedging is 2.419 (241.9%) in corn and 3.045 (304.5) in oats net of transaction cost from dynamic rebalancing. Since the typical round trip transaction cost is around 0.02% to 0.04%, a mean-variance expected utility-maximizing hedger will benefit from hedging with RSGC even after taking account of these transaction costs.

To show the robustness of the superiority of the proposed RSGC hedging method, in addition to the most recent year hedging period, the most recent two year hedging period is also examined and the results are shown in table IV. Again, RSGC provides the highest variance reduction and utility gain compared to other hedging strategies investigated in this study.

Figures 1 to 4 show the hedge ratios, state-dependent volatility process, and regime probabilities for corn data. Figure 1 compares the hedge ratios of GC and RSGC. All these hedge ratios are time-varying and fluctuate around the static OLS hedge ratio which is equal to 0.88. Figure 2 shows the RSGC estimates of the state-dependent time-varying volatilities. The spot return series has an average volatility equals to 4.81 in the high volatility state and an average volatility equals to 2.50 in the low volatility state. The futures return series has average volatilities equal to 4.64 and 2.23 in the high and low volatility states, respectively. The state probabilities of being in the low volatility state estimated from RSGC are shown in figure 4. The state probability fluctuates between 0 and 1.

Analogous graphs for the oats data are shown in figures 5 to 8. Figure 5 compares the hedge ratios of GC and RSGC and Figure 6 shows the RSGC estimates of the state-dependent time-varying volatilities. The spot return series has an average volatility equals to 6.52 in the high volatility state and an average volatility equals to 2.41 in the low volatility state. The volatility of the volatility is lower in the low volatility state than that in the high volatility state. The volatility of the volatility for low volatility state is equal to 15.11% and the volatility of the volatility for high volatility state is 38.01%. The futures return series has average volatilities equal to 7.07 and 2.66 in the high and low volatility states, respectively. Again, the volatility of the volatility (13.77%) is lower in the low volatility state than the volatility of the volatility (31.69%) in the high volatility state. The state probabilities of being in the low volatility state estimated from RSGC are shown in figure 8.

V. CONCLUSIONS

This article investigates the effects of non-normal dependence structure and regime shifts on the hedging effectiveness by proposing a regime switching Gumbel-Clayton copula GARCH model (RSGC). RSGC solves problems of recently developed regime switching models. It specifies a non-normal dependence structure with a Gumbel-Clyton copula to avoid the restrictive joint normal assumption and adopts an independent switching GARCH process to avoid using Gray's recombining method which is problematical in interpreting and analyzing the variance process. Besides, the assumption of the independent switching allows us to derive a modified formula for calculating the minimum variance hedge ratio. Empirical results suggest that allowing a hedging strategy to possess both properties of regime shift and non-normal dependence structure improves the hedging effectiveness for both corn and oats data traded in the Chicago Board of Trade in terms of both criterion of variance reduction and utility gain.

Table I

Summary Statistics for Spot and Futures Returns of Corn and Oats Data

	CORN			
	In-Sample		Out-of-Sample	
	Spot	Futures	Spot	Futures
Mean	0.0005	0.0006	0.0040	0.0039
SD	0.0337	0.0327	0.0526	0.0478
Min	-0.1483	-0.1531	-0.1465	-0.1371
Max	0.1598	0.1380	0.1265	0.1245
Skewness	-0.1021	0.2628	-0.3462	-0.3720
Kurtosis	2.1380	1.5364	0.4410	0.7341

	OATS			
	In-Sample		Out-of-Sample	
	Spot	Futures	Spot	Futures
Mean	0.0011	0.0011	0.0023	0.0032
SD	0.0424	0.0446	0.0443	0.0390
Min	-0.2287	-0.1489	-0.1146	-0.0797
Max	0.2058	0.1612	0.1475	0.0938
Skewness	-0.0908	0.1656	0.4654	0.0853
Kurtosis	3.7877	1.2624	1.7071	-0.4141

Note: Returns are calculated as the differences in the logarithm of price multiplied by 100. The in-sample data period is from January 1991 to December 2006 and the out-of-sample data period is from January 2007 to December 2007.

Table II
Estimates of Unknown Parameters of Alternative Models for Corn and Oats Data
Data period is from January 1991 to December 2006

	CORN				OATS			
	CC	VC	GC	RSGC	CC	VC	GC	RSGC
p_0				0.751 (0.285)**				1.219 (0.428)**
q_0				-1.200 (0.440)**				0.046 (0.300)
μ_{c1}	0.279 (0.100)**	-0.020 (0.039)	0.037 (0.035)	-0.215 (0.125)*	0.124 (0.137)	0.449 (0.004)**	0.451 (0.010)**	0.004 (0.025)
μ_{c2}				0.845 (0.334) **				0.056 (0.186)
μ_{f1}	0.286 (0.102)**	-0.104 (0.000)**	-0.104 (0.000)**	-0.538 (0.123)**	0.151 (0.110)	0.446 (0.066)**	0.448 (0.019)**	-0.053 (0.060)
μ_{f2}				1.542 (0.453)**				0.323 (0.211)
γ_{c1}	0.837 (0.194)**	1.086 (0.252)**	1.093 (0.310)**	0.106 (0.143)	16.340 (0.934)**	16.343 (1.296)**	10.607 (2.782)**	0.318 (0.048)**
γ_{c2}				2.553 (1.461)*				40.251 (5.962)**
γ_{f1}	0.786 (0.233)**	0.950 (0.271)**	1.229 (0.378)**	0.106 (0.123)	6.977 (1.777)**	6.818 (1.951)**	11.465 (4.872)**	0.318 (0.060)**
γ_{f2}				3.732 (2.060)*				16.409 (25.817)
α_{c1}	0.130 (0.022)**	0.134 (0.024)**	0.077 (0.018)**	0.044 (0.012)**	0.089 (0.027)**	0.101 (0.033)**	0.061 (0.024)**	0.002 (0.001)
α_{c2}				0.222 (0.063)**				0.134 (0.132)
α_{f1}	0.075 (0.015)**	0.075 (0.016)**	0.053 (0.017)**	0.029 (0.009)**	0.117 (0.036)**	0.105 (0.034)**	0.094 (0.050)*	0.005 (0.003)
α_{f2}				0.160 (0.072)*				0.093 (0.171)
β_{c1}	0.801 (0.030)**	0.774 (0.038)**	0.834 (0.036)**	0.865 (0.025)**	0.000 (0.021)	0.026 (0.049)	0.339 (0.164)**	0.986 (0.004)**
β_{c2}				0.778 (0.065)**				0.000 (0.084)
β_{f1}	0.852 (0.031)**	0.836 (0.035)**	0.833 (0.045)**	0.914 (0.020)**	0.533 (0.106)**	0.578 (0.108)**	0.368 (0.259)	0.940 (0.051)**
β_{f2}				0.731 (0.096)**				0.636 (0.515)
θ_1		0.000 (0.004)	0.187 (0.366)	0.345 (0.287)		0.731 (0.030)**	0.728 (0.041)**	0.674 (0.097)**
θ_2		0.200 (0.048)**	0.076 (0.029)**	0.085 (0.035)**		0.195 (0.026)**	0.167 (0.036)**	0.125 (0.048)**
ρ		0.859 (0.011)**	0.931 (0.006)**	0.943 (0.012)**		0.695 (0.058)**	0.950 (0.015)**	1.000 (0.058)**
π_1			0.445 (0.043)**	0.235 (0.077)**			0.507 (0.046)**	0.597 (0.075)**
π_2				0.385 (0.106)**				0.354 (0.090)**
LL	-3755.01	-3731.19	-3597.75	-3515.73	-4577.20	-4504.44	-4372.24	-4238.79

Note. Figures in parentheses are standard errors and LL stands for log-likelihood.
*Significant at the 5% level; **Significant at the 1% level.

Table III
Out-of-Sample Hedging Effectiveness for Corn and Oats Data
Hedging period is from January 2007 to December 2007

	CORN			
	Variance of Hedged Portfolio Return	Improvement of RSGC Over Other Hedging Strategies[1]	Expected Weekly Utility[2]	Utility Gain of RSGC Over Other Hedging Strategies[3]
Unhedged	27.653			
OLS	2.886	2.22%	-11.51	2.419
CC	2.452	0.65%	-9.76	0.668
VC	2.440	0.60%	-9.70	0.613
GC	2.312	0.14%	-9.26	0.169
RSGC	2.272		-9.09	
	OATS			
	Variance of Hedged Portfolio Return	Improvement of RSGC Over Other Hedging Strategies	Expected Weekly Utility	Utility Gain of RSGC Over Other Hedging Strategies
Unhedged	19.766			
OLS	11.229	3.83%	-44.92	3.045
CC	10.725	1.28%	-42.92	1.041
VC	11.590	5.65%	-46.24	4.360
GC	11.101	3.18%	-44.32	2.449
RSGC	10.472		-41.88	

Note. 1. Improvement of RSGC over other hedging strategies is defined as the difference of the percentage variance reduction of RSGC and the percentage variance reduction of alternative hedging strategies
2. Expected weekly utility is calculated based on equation (25)
3. Utility gains of RSGC over other hedging strategies are defined as the differences of the expected utilities of RSGC and the expected utility of alternative dynamic models.

Table IV
Out-of-Sample Hedging Effectiveness for Corn and Oats Data
Hedging period is from January 2006 to December 2007

	CORN			
	Variance of Hedged Portfolio Return	Improvement of RSGC Over Other Hedging Strategies[1]	Expected Weekly Utility[2]	Utility Gain of RSGC Over Other Hedging Strategies[3]
Unhedged	23.628			
OLS	3.094	1.57%	-12.27	1.424
CC	2.774	0.22%	-10.97	0.131
VC	2.864	0.60%	-11.33	0.486
GC	2.759	0.16%	-11.02	0.179
RSGC	2.722		-10.84	

	OATS			
	Variance of Hedged Portfolio Return	Improvement of RSGC Over Other Hedging Strategies	Expected Weekly Utility	Utility Gain of RSGC Over Other Hedging Strategies
Unhedged	15.409			
OLS	7.645	2.09%	-30.52	1.218
CC	7.439	0.75%	-29.71	0.414
VC	8.054	4.74%	-32.20	2.899
GC	7.873	3.57%	-31.50	2.202
RSGC	7.323		-29.30	

Note.
1. Improvement of RSGC over other hedging strategies is defined as the difference of the percentage variance reduction of RSGC and the percentage variance reduction of alternative hedging strategies
2. Expected weekly utility is calculated based on equation (25)
3. Utility gains of RSGC over other hedging strategies are defined as the differences of the expected utilities of RSGC and the expected utility of alternative dynamic models.

Figures

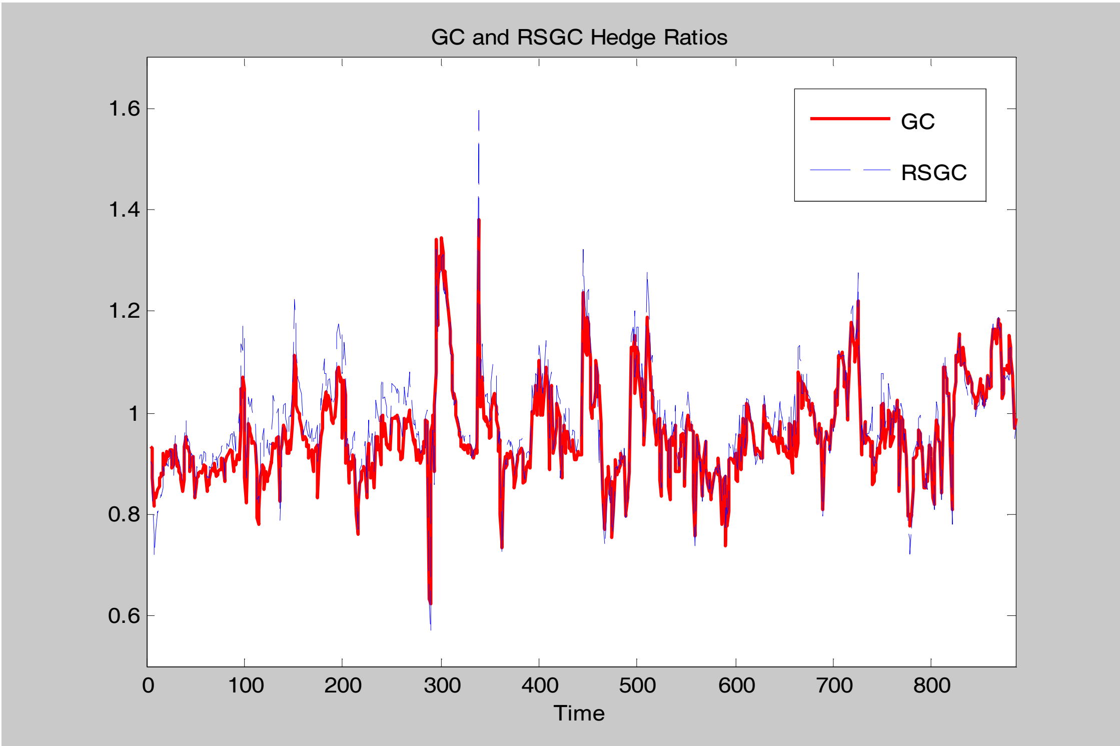

Figure 1
GC and RSGC Hedge Ratios of Corn

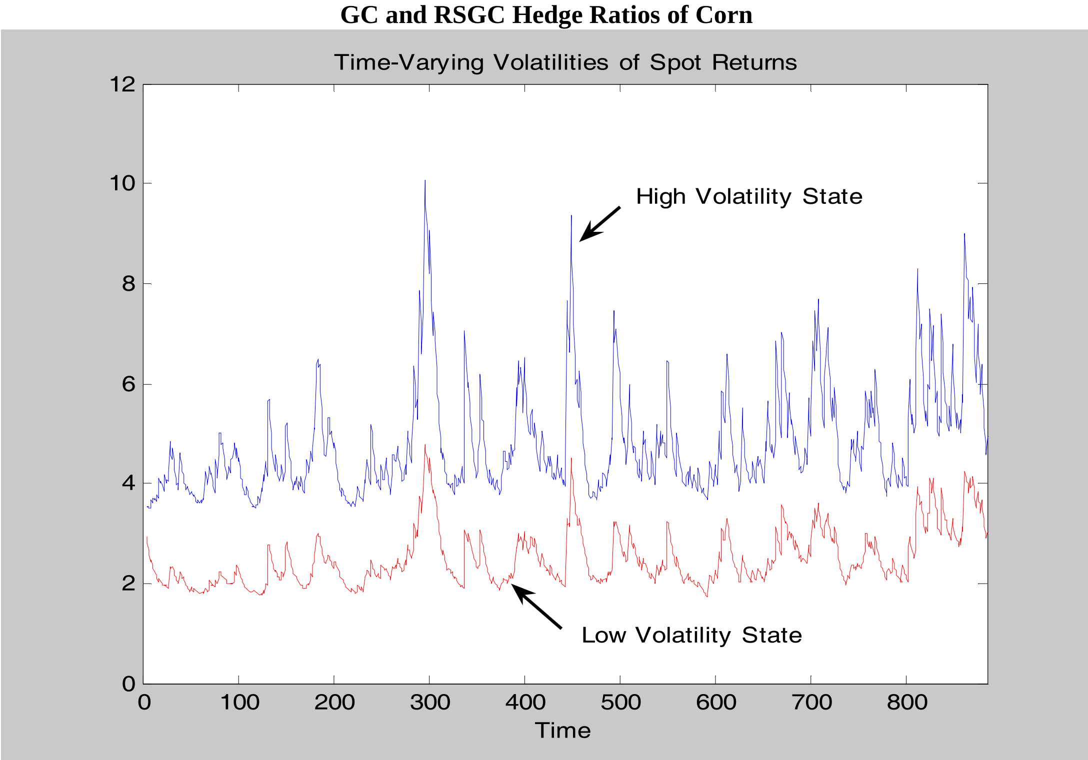

Figure 2
Time-Varying Volatilities of Spot Return of Corn

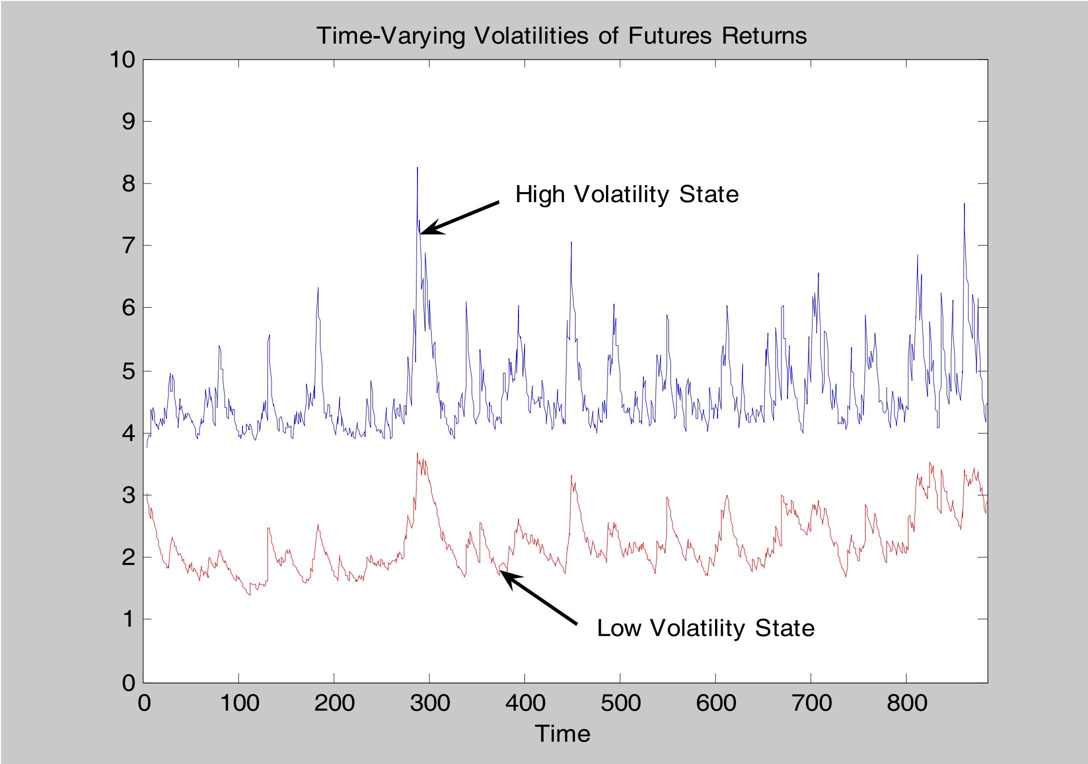

Figure 3
Time-Varying Volatilities of Futures Return of Corn

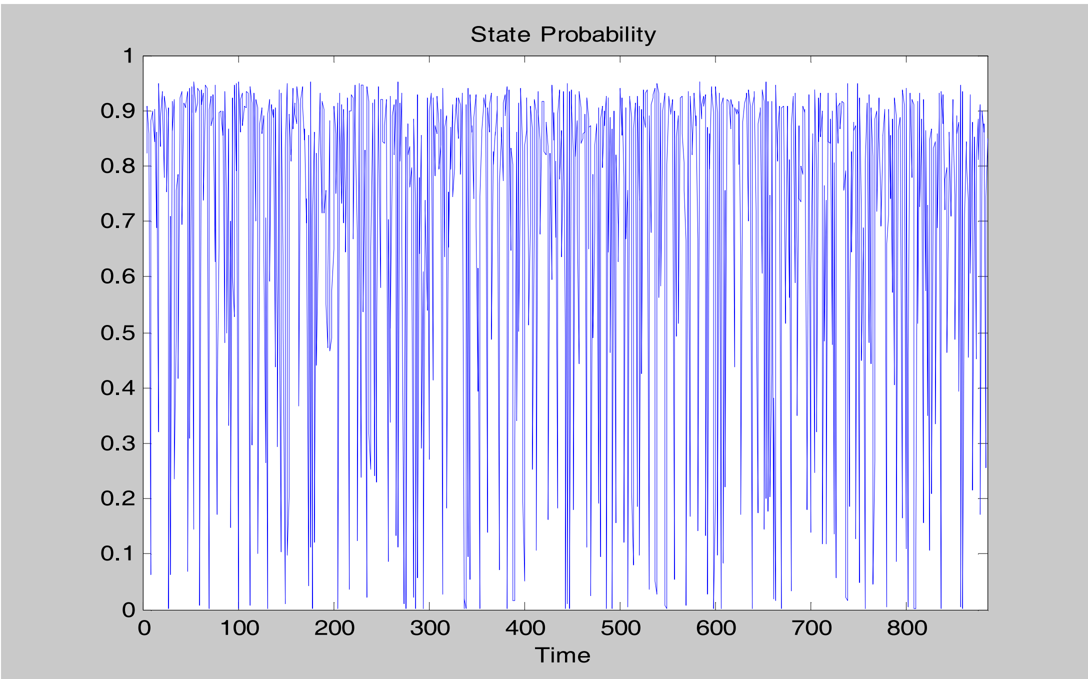

Figure 4
Probability of Low Volatility State for Corn

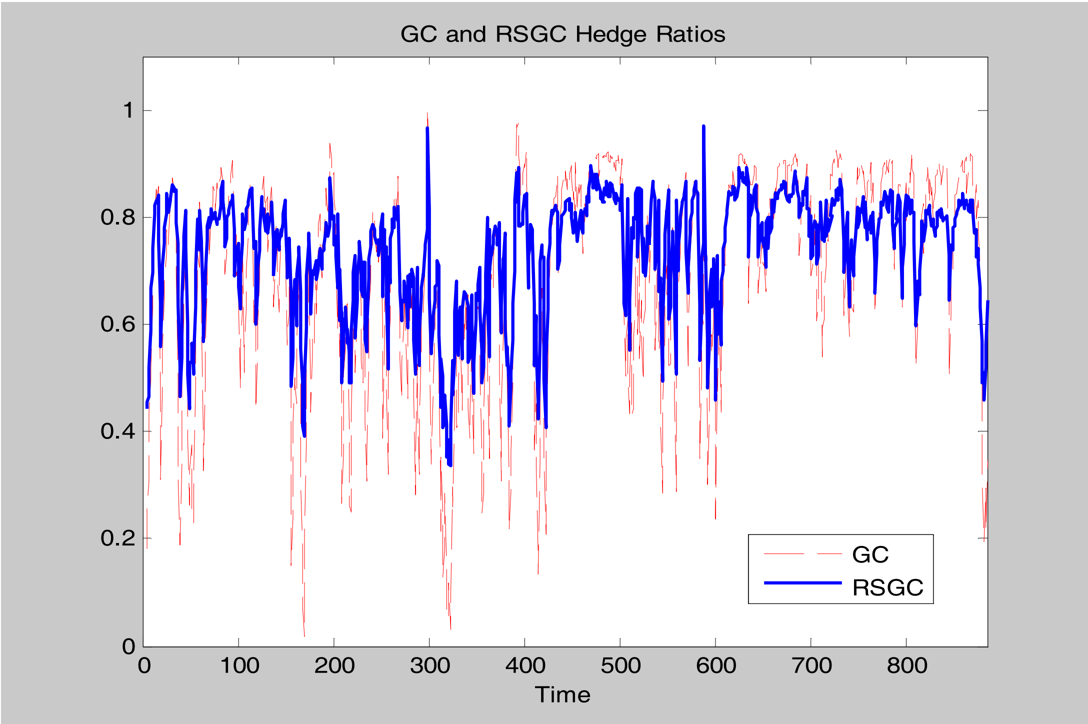

Figure 5
GC and RSGC Hedge Ratios of Oats

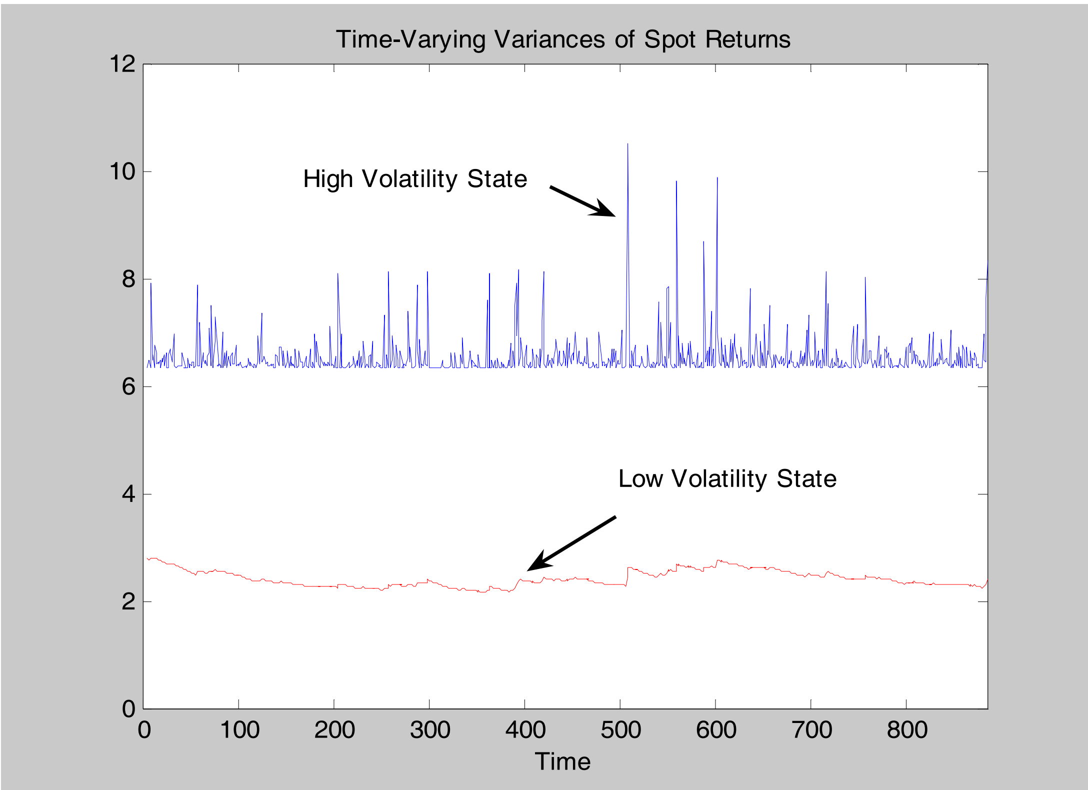

Figure 6
Time-Varying Volatilities of Spot Return of Oats

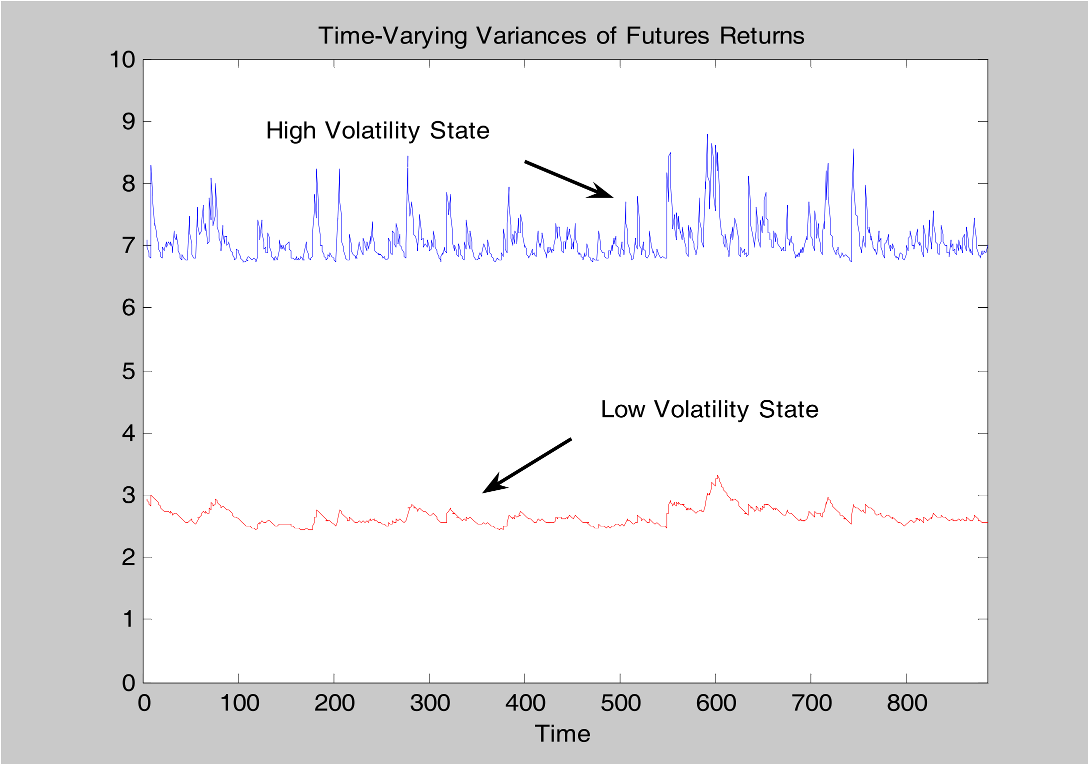

Figure 7
Time-Varying Volatilities of Futures Return of Oats

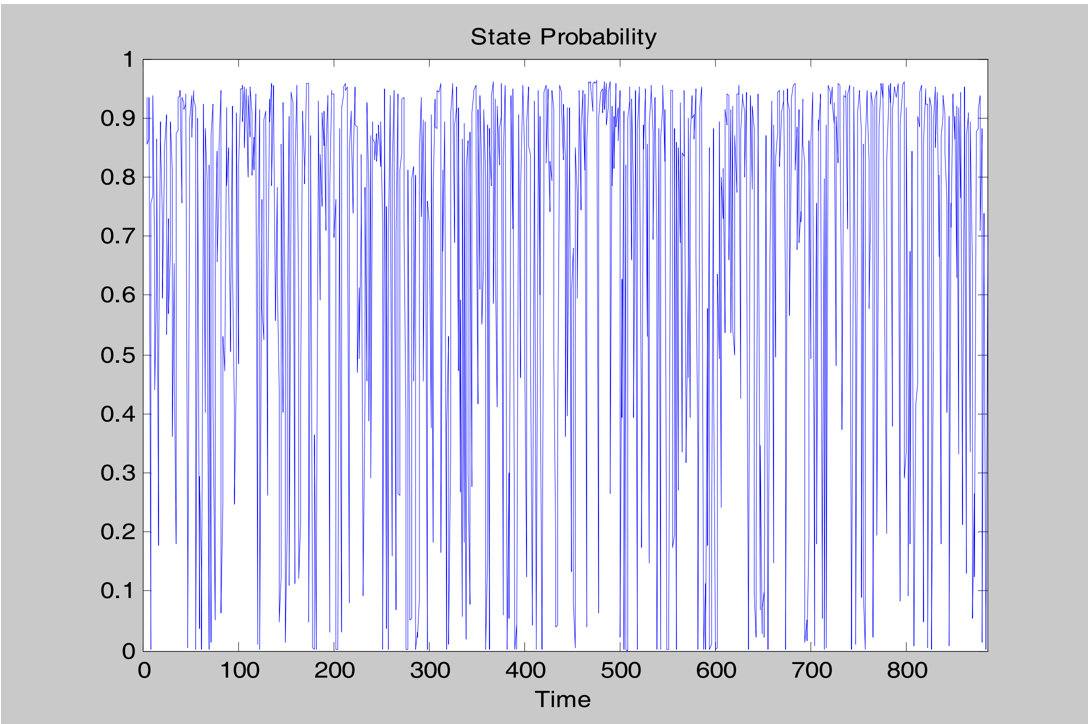

Figure 8
Probability of Low Volatility State for Oats

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