A Theory of an Intermediary with Nonexclusive Contracts

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Abstract

When agents cannot observe contracts entered by others, agents may have the incentive to promise the same asset to multiple counterparties and subsequently default. I show how an intermediary with a very minimal role can achieve second best efficiency in this setting. The intermediary sets limits on the number of contracts that agents can report to it, but agents can also enter contracts secretly without reporting. I also show that in some cases, the intermediary must set position limits that are nonbinding in equilibrium, and that in some cases the intermediary must not make reported trades public.

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1 Introduction

Regulators and intermediaries put a lot of effort into assessing default risk. An implicit assumption is that an agent's total position is observed, so the only problem is to determine how risky the position is. However, this assumption should not be taken for granted, as agents may have the incentive to enter contracts secretly or to pledge the same asset to multiple counterparties.[1] This paper shows how a well-designed intermediary can induce agents to reveal all their trades to it *voluntarily.* The main result is that such an intermediary can achieve the second best (and therefore increase welfare) even if agents can enter contracts secretly and the intermediary can observe only contracts that agents *choose* to reveal to it.

The intermediary in this paper is an entity that sets limits on the number of contracts that agents can report to it. In addition to the main result I show that: (1) For some parameter values, the intermediary must set position limits that are not binding in equilibrium. For example, to implement an equilibrium in which every agent enters one contract, the intermediary must allow each agent to report, say, up to three contracts. (2) The intermediary is not a bulletin board. For some parameter values, the intermediary must not make reported trades public. (3) Without the intermediary we will see collateralized trade, and the gain from the intermediary increases when the fixed cost per trade decreases.

The basic setting is as follows: Agents invest their endowments in two-period projects. They can benefit from bilateral trade because the interim cash flows from their projects are negatively correlated. An agent cannot commit to pay out of interim or final cash flows, but if he defaults, it is possible to terminate his project, which cannot be transferred to others or pledged as collateral. With exclusive contracts, the threat of losing future cash flows may

[1] For example, according to the Wall Street Journal (August 25, 2005), "(hedge) funds sometimes move out of trades—"assign" them—without telling the bank that sold them the credit-derivative contract that their counterparty has changed." Another example is the Nigerian barge deal between Enron and Merrill Lynch, in which Enron allegedly arranged for Merrill Lynch to serve as a temporary buyer (of the barges) so as to make Enron appear more profitable than it was. According to a release by the Department of Justice (October 15, 2003), "Enron promised in a secret oral "handshake" side-deal that Merrill Lynch would receive a return on its investment plus an agreed-upon profit..."

induce agents to invest in their projects and deliver what they promised. However, when agents can enter multiple contracts secretly, they may choose to enter as many contracts as they can and subsequently default on all of them.

One solution is to require that agents put up cash as collateral; however, using collateral is costly because agents forgo investing in their positive NPV projects. Collateral reduces the incentive to default in two ways. First, an agent cannot default on the amount of cash posted as collateral. Second, an agent may not have enough collateral to enter the number of contracts needed to make default profitable. Thus, agents in this paper can credibly promise more than the amount of cash they post as collateral. In addition, the optimal amount of collateral decreases when agents have more future income to lose.

Another solution —which is the heart of this paper—is to create a central entity whose role is to make sure that agents do not enter contracts beyond their capacity to pay. If such an entity, which I call an intermediary, could observe all the transactions that agents make, it could achieve the same outcome that would be obtained with exclusive contracts. However, when agents can enter contracts secretly, monitoring every transaction that an agent can make may be too costly. I show that it is enough that the intermediary monitors only the transactions that agents *choose* to report to it. In equilibrium agents report all their trades to the intermediary even though they do not have to. This is true even if reporting involves a small fee.

The logic is as follows: If a pair of agents deviates by not reporting their trade (e.g., by entering a contact secretly), each faces the risk that the other agent will cheat by entering additional contracts and defaulting on all. In equilibrium, since all other agents report, the number of additional contracts that an agent can enter is limited by the position limit L: If the agents report the current contract, each one of them can enter a total of L contracts, but if they do not report, each one of them can enter a total of $L + 1$ contracts. To make sure that agents find it optimal to report, the position limit L must be such that an agent who can enter at most L contracts will prefer to enter only one contract and deliver, but an agent who can enter $L+1$ contracts will prefer to enter $L+1$ contracts and default. Depending on the parameters of the problem, the number L that satisfies the restriction above is sometimes more than one, which means that the position limits is nonbinding in equilibrium. Intuitively, position limits that are too low do not give an agent of a deviating pair enough scope to cheat on his counterparty, and this makes the initial joint deviation (of not reporting) desirable. Revealing information about previously reported trades is similar to imposing a postilion limit of one, and following the logic above may give too little scope to cheat, thereby making it desirable not to report. Agents who do not report their trade may attempt to prevent default by requiring more collateral, but when the fee for reporting a trade is small, the opportunity cost of collateral is higher than the fee.

The main theoretical contribution is to illustrate a minimal condition for an intermediary to be welfare improving. The intermediary sets limits on the number of contracts that agents can report to it *voluntarily*. The intermediary does not need to monitor everything that an agent can do; it only needs to keep track of what an agent chooses to tell. In addition, the intermediary cannot be replaced by a bulletin board. Unlike Diamond (1984), I do not rely on diversification, and unlike Townsend (1978), my intermediary arises when the fixed cost per trade is low rather than high.[2]

While the paper does not attempt to model any particular intermediary, the intermediary here has some features of a clearing house; the clearing house may be a part of a futures exchange or a separate entity; it can clear exchange-traded contracts as well as over-the-counter products, such as swaps.[3] Clearing houses around the world deploy a number of safeguards to protect their members and customers against the consequences of default by a clearing house participant. In addition to requiring collateral, the clearing house monitors the financial status of its members. On a daily basis (or sometimes more than once a day), the clearing house monitors and controls the positions of its members;

[2] See also Brusco and Jackson (1999), who show that a market maker can economize on the fixed costs of trading across periods.

[3] For example, the London Clearing House clears over-the-counter interest rate swaps without being involved in the matching process and bargaining process. The Chicago Mercantile Exchange has also launched clearing services for some over-the-counter products.

periodically, the clearing house monitors financial statements, internal controls, and other indicators of financial strength; some clearing houses (e.g., in Sydney and Hong Kong) also set capital-based position limits.[4] These safeguards, which reduce the amount of collateral that clearing house members need to post, are more effective when clearing house members do not enter contracts secretly.[5] In practice, the incentive to default may depend on activities in more than one market; indeed, clearing houses have recently moved toward more central clearing.[6]

The intermediary can also be interpreted as a regulator. For example, hedge funds are now required to register with the U.S. Securities and Exchange Commission (SEC). This means that hedge funds need to disclose their transactions to the SEC on a regular basis. Another example is a central bank that regulates banks. Central monitoring may reduce the cost of trading with one another because the risk of default is reduced, and it is more cost effective when firms make truthful reports. My theory suggests that this may be the case. However, to induce banks to report all their transactions voluntarily, the regulator may need to commit not to make these reports public. The theory also illustrates a connection between regulation and private-sector incentives to discipline. The regulator, who sets position limits, relies on firms in the private sector to discipline one another; that is, each firm makes sure that its trading partner reports the trade to the regulator. The theory implies that regulations that are too stringent may be counterproductive because they undermine private-sector incentives for agents to discipline one another.[7]

Other related literature. The existing literature has focused on the role of a clearing house in providing liquidity.[8] Bernanke (1990) distinguishes between two roles of a clearing house.

[4] Capital-based position limits, whose purpose is to make sure that members maintain positions within their financial capability, are different from speculative position limits. The latter are set by exchanges and regulators to prevent speculators from manipulating spot prices.

[5] Netting may also reduce collateral. However, some clearing houses (e.g., the Hong Kong Futures Exchange Clearing Corporation) calculate margin on a gross basis rather than a net basis.

[6] For example, in 2004, the Chicago Mercantile Exchange (CME) fully integrated the clearing of all trades of the Chicago Board of Trade in addition to those of the CME. The CME has also developed cross margin arrangements with other clearing houses, so that margins can be calculated based on the total position.

[7] While my model provides a novel rationale for regulatory secrecy, I do not present a full discussion of the costs and benefits of regulatory secrecy.

[8] There is also an extensive literature that studies the effects of different trading mechanisms on liquidity

First, a clearing house reduces the transactions cost of consummating agreed-upon trades; this is analogous to a bank that clears checks. Second, the clearing house standardizes contracts by setting terms and format but, most important, by guaranteeing performance to both sides of trade; this is analogous to an insurance company.[9] I focus on the role of a clearing house in monitoring the positions of its members, but I do not exclude other roles. In particular, the main results remain even if the intermediary guarantees performance in addition to setting position limits. Finally, while existing literature focuses on the role of an intermediary in reducing transaction costs, I start with markets in which transaction costs are already low. In the sense that my paper illustrates a negative aspect of liquidity (as measured by low transaction costs), the paper also relates to Myers and Rajan (1998).[10]

In a different framework, Bizer and DeMarzo (1992) and Parlour and Rajan (2001) study the effect of nonexclusivity on equilibrium interest rates and competition in credit markets.[11] Bizer and DeMarzo assume that contracts entered in the past are observable and have a priority; additional contracts impose a negative externality on existing contracts because the agent's hidden effort affects his future income. In their setting an intermediary cannot improve welfare. In contrast, I assume that previous contracts are not observable. Parlour and Rajan assume that intermediaries offer contracts simultaneously, and then a single borrower can accept any subset of these contracts. As in my paper, agents who strategically default do so on all the contracts they entered. In their model this can rule out entry even though competing lenders make positive profits. In my paper, this helps to sustain an equilibrium in which agents do not enter contracts secretly.

Paper outline. In Section 2, I present a simple model of trade between a pair of agents. The model serves as a benchmark showing the best outcome that can be achieved when contracts are exclusive. Section 3 extends the model for a continuum of agents who cannot

provision. Madhavan (2000) provides a survey.

[9] See also Telser and Higinbotham (1977) and Edwards (1983).

[10] In their model, greater asset liquidity reduces the firm's capacity to raise external finance because it reduces the firm's ability to commit to a specific course of action.

[11] See also Kahn and Mookherjee (1998), who study insurance contracts, Bisin and Rampini (2006), who study bankruptcy, and Bisin and Guaitoli (2004), who show that intermediaries can make positive profits by offering contracts that are not traded in equilibrium.

observe previous transactions of other agents. I show that the most efficient outcome without an intermediary involves collateral. I also calculate the optimal level of collateral and illustrate its dual role. In Section 4—which contains the main results—I introduce an intermediary. I explain the role of nonbinding position limits and show that in some cases the intermediary must not make reported trades public. I also discuss possible extensions. Section 5 concludes.

2 The two-agent economy benchmark

In this section I present a simple model of trade between a pair of agents, referred to as agent 1 and agent 2. The motive for trade is hedging, contracts may involve collateral, and collateral has an opportunity cost. In the next section, I extend this benchmark model to include a continuum of agents of two types, where type $i \in \{1, 2\}$ corresponds to agent i from this section. The outcome of the benchmark model in this section is the same as the outcome that would be obtained in the extended model if agents could commit to enter exclusive contracts. The objective throughout the paper is to maximize the unweighted sum of agents' expected utilities. This is a plausible assumption, since agents are identical ex ante and there is an equal mass of both types.

2.1 The model

There are two periods and one divisible good, called cash, or simply dollars. Both agents are risk neutral and obtain an expected utility of $E(c_0 + c_1 + c_2)$ from consuming c_0, c_1 and c_2 dollars at dates $0, 1$, and 2, respectively. They are protected by limited liability, so $c_t \geq 0$.

At date 0, each agent has one dollar and a two-period constant-returns-to-scale project in which he can invest at most one dollar.[12] If the project continues until maturity, it yields RI dollars at date 2 for every I dollars invested at date 0. Interim cash flows for the agents' projects are negatively correlated. When the project of one agent yields a positive cash

[12] Proposition 7, which says that the intermediary must not reveal information, relies on the assumption that it is not possible to invest more than one dollar. All other results hold even if it is possible to invest more than one dollar.

flow, the project of the other agent has a negative flow; it requires an additional investment that must be made in full for the project to continue. More specifically, there are two equal probability states, state 1 and state 2, one of which becomes publicly observable at date 1. The project of agent i ($i = 1, 2$) yields εI dollars in state i, but requires an additional investment of εI in the other state, denoted by $-i$; if this investment is not made, the project terminates at date 1 and yields no further cash flows. (See Figure 1.)

$$\begin{array}{ccccc} \text{date } 0 & & \text{date } 1 & & \text{date } 2 \\ & \overset{\textit{state } i}{\nearrow} & \varepsilon I & \longrightarrow & RI \\ -I & & & & \\ & \underset{\textit{state } -i}{\searrow} & -\varepsilon I & \longrightarrow & RI \end{array}$$

Figure 1: Project's cash flows for agent i if project operates to maturity.

In addition to investing, the two agents can store cash through a third party who can commit not to divert it. One can think of this as an escrow account. For simplicity, storage can take place only between date 0 and date 1, and the interest rate is normalized to be zero percent. It is assumed that $R > \varepsilon$, so it is efficient to make the additional investment at date 1 if cash is available. It is also assumed that $R > 1$, so in a world without frictions both projects have positive NPVs; the NPV is $(R - 1)I$.

Money placed in escrow is observable to both agents and can be contracted upon.[13] The state realized at date 1 is also observable. However, consumption, investments in projects, and projects' cash flows are private information. In particular, an agent can default even if he has enough cash to pay; he can claim that he invested nothing. If an agent defaults, the other agent can shut down his business so that the defaulting agent cannot continue his project. While it is observable whether the project/business operates, the level of investment is private information. In addition, a project can operate even if $I = 0$; for example, an agent can go to work and keep his business open but effectively do nothing. Projects' liquidation values are zero at each date. This can be motivated by assuming that

[13] Nothing would change if agents could also hide cash by storing privately.

projects require human capital that is inalienable.

It follows that agents cannot commit to pay at date 2. Therefore, once an agent realizes a negative cash flow, he cannot borrow against future cash flows.[14] However, the two agents can hedge at date 0 by entering a forward contract according to which the agent who realizes a positive cash flow transfers cash (at date 1) to the agent who realizes a negative flow. The fear of losing future cash flows if the project is shut down may induce the agents to pay. The two agents may also find it optimal not to hedge. To rule this out, I assume that $\varepsilon < 1$. This is a sufficient condition to ensure that hedging and bilateral trade are beneficial.[15]

2.2 Optimal contract

A natural contract is as follows: At date 0, agent i invests I_i dollars in his project and puts a_i dollars in escrow; the total amount stored is $s = a_1 + a_2$. At date 1, in state i, agent i obtains εI_i dollars from his project and transfers $b_i \leq \varepsilon I_i$ dollars to the other agent. The other agent also receives s.[16] A contract can also specify the probability $\lambda_i(\widehat{b}_i|b_i)$ that the project of agent i continues if he delivers $\widehat{b}_i$ instead of b_i. However, in our case, if an agent does not pay in full, it is optimal to shut down his project with probability one; in other words, $\lambda_i(\widehat{b}_i|b_i)$ equals one if $\widehat{b}_i \geq b_i$ and zero otherwise.[17] The set of feasible contracts is $\Psi \equiv \{(a_1, a_2, b_1, b_2, I_1, I_2) : 0 \leq I_i \leq 1 - a_i \text{ for } i = 1, 2;\ 0 \leq b_i \leq \varepsilon I_i \text{ for } i = 1, 2; \text{ and } s = a_1 + a_2 \geq 0\}$. Note that b_i is restricted to be nonnegative, but a_i is not; thus, date-0 transfers between the agents are not ruled out; for example, $a_1 < 0$ indicates a transfer from agent 2 to agent 1.

First best. The first-best contract is given by $a_1 = a_2 = 0$, $b_1 = b_2 = \varepsilon$, and $I_1 = I_2 = 1$.

[14] This is similar to Holmström and Tirole (1998), who assume that because of moral hazard, an agent facing a liquidity shock can borrow only against a fraction of his future income.

[15] To see that, note that in autarky an agent has two options. The first is to make sure that he has enough cash to continue his project in both states, that is, invest I and store $s = 1 - I$ so that $s = \varepsilon I$; in this case, $I = \frac{1}{1+\varepsilon}$, and the agent's utility is $s + RI = \frac{R+\varepsilon}{1+\varepsilon}$. The second option is to invest $I = 1$ and obtain a utility of $\frac{R+\varepsilon}{2}$; in this case the agent cannot continue his project when he realizes a negative shock. The first alternative is preferred if and only if $\varepsilon < 1$. But then the agents can do better by making sure that all the cash stored is transferred to the agent who needs it.

[16] This is without loss of generality. A contract in which $s_i > 0$ is transferred to agent i in state i, and only $s - s_i$ is transferred to agent $-i$ is suboptimal.

[17] It is assumed that it is possible to commit to this continuation/closure policy.

At date 0 each agent invests his entire endowment, and at date 1 the agent who obtains ε from his project transfers it to the other agent. The two agents can continue their projects in both states, and each agent obtains a utility R.

Second best. A second-best contract is a feasible contract that maximizes the (unweighted) sum of agents' utilities subject to the constraints that (1) each agent prefers the contract to autarky (participation), and (2) each agent invests and delivers according to what the contract says (incentive compatibility). Note that an agent can default only on the amount b_i.

The problem can be simplified by noting that after an agent invests, the decision to pay out of project cash flows is as follows: If $b_i \leq \varepsilon I_i$, the agent delivers the full amount b_i; otherwise, he delivers nothing. Defaulting when $b_i \leq \varepsilon I_i$ is suboptimal because the agent keeps b_i but loses $RI_i > \varepsilon I_i$. Making a partial payment is suboptimal because the agent still loses his project.

Since utilities are linear in I_i, an agent will either invest and deliver according to what the contract says, or invest nothing and subsequently default; in the second case, the agent consumes his entire endowment. The incentive constraint is therefore

$$U_i(\psi) \geq \overline{U}_i(\psi) \quad \text{for} \quad i = 1, 2, \tag{1}$$

where $U_i(\psi)$ is agent i's utility if he enters the contract ψ and follows it, and $\overline{U}_i(\psi)$ is agent i's utility if he invests nothing and subsequently defaults; in both cases it is assumed that the other agent follows the contract. The expressions for $U_i(\psi)$ and $\overline{U}_i(\psi)$ are derived in the appendix.

Proposition 1 (second best) *If $R \geq 1 + \frac{1}{2}\varepsilon$, the second-best contract equals the first-best contract. Otherwise, the second-best contract is given (uniquely) by $a_1 = a_2 = 1 - I$, $b_1 = b_2 = (2 + \varepsilon)I - 2$, and $I_1 = I_2 = I$, where $I = \frac{1}{2 + \frac{1}{2}\varepsilon - R} < 1$.*

The idea behind the proof is as follows: First, the optimal contract is symmetric and can be denoted by $\psi = (a, b, I)$; this follows from the symmetric nature of the problem.

Second, the contract must be designed so that no consumption takes place at date 0; it is better to invest more and consume later; therefore,

$$a + I = 1. \tag{2}$$

Third, since the contract is entered for hedging purposes, it should be designed so that each agent has enough cash to continue his project when he realizes a negative shock, that is,

$$s + b = \varepsilon I. \tag{3}$$

It follows that if everyone follows the contract, each agent obtains

$$U_i(\psi) = 1 + (R-1)I. \tag{4}$$

The first term is the initial endowment, and the second term is the project's NPV. It also follows that the highest utility that an agent can obtain if he deviates from what the contract says is

$$\overline{U}_i(\psi) = 1 + \frac{1}{2}b. \tag{5}$$

The first term is the initial endowment and the second term is the expected cash obtained from the other agent. The amount stored cancels out because the agent puts a at date 0 and obtains $2a$ at date 1 with probability half. Finally, one can use equations (2), (3), and $s = 2a$, to express a and b as a function of I, then use equations (1), (4), and (5) to find the optimal I.

The second best may not equal the first best because the incentive constraint limits the amount of cash that an agent can credibly promise. In particular, equations (1), (4), and (5) imply that

$$b \leq 2(R-1)I. \tag{6}$$

If $\varepsilon \leq 2(R-1)$, which is equivalent to $R \geq 1 + \frac{1}{2}\varepsilon$, the first best is achieved because an agent has the incentive to invest $I = 1$ and deliver $b = \varepsilon$. Otherwise, some of the demand for liquidity at date 1 must be satisfied from storage; thus, agents cannot invest their entire endowments.

Collateral. Denoting $k_i = a_i$ and $x_i = a_i + b_i$, a contract can be interpreted as follows: Agent i promises to pay x_i at date 1 if state i happens; he also posts k_i dollars as collateral. If state $-i$ happens, the agent receives his collateral back; otherwise, the collateral is transferred to the other agent. The agent can default only on the amount $x_i - k_i$. Note that since $a_i = 1 - I_i$, equation (4) can be rewritten as

$$U_i(\psi) = R - (R-1)k. \tag{7}$$

The first term represents the first-best utility, and the second represents the opportunity cost of collateral: By posting collateral, agents forgo investing in their positive NPV projects.

Example 1 (second best) Suppose $\varepsilon = 0.3$, $R = 1.39$, and initial endowments are scaled to be \$100. Since $R \geq 1 + \frac{1}{2}\varepsilon$, the second best equals the first best. At date 0 each agent invests \$100 in his project, and at date 1 agent i transfers \$30 to the other agent if state i happens. This contract is optimal because: (1) An agent who invests \$100 in his project is better off paying what he promised; otherwise, he keeps current cash flows (\$30) but loses future cash flows (\$139); (2) If an agent consumes his initial endowment rather than investing it, he obtains a utility of $100 + \frac{1}{2}(30) = 115$. But this is less than the utility of 139 that he obtains by following the contract.

3 Decentralized trade with nonexclusivity

In this section I extend the benchmark model from the previous section to include a continuum of agents, who cannot commit to enter exclusive contracts. The trading environment captures the idea that agents can find multiple trading partners easily, and that agents cannot observe contracts that other agents may enter in the future or may have entered in the past.

3.1 Trading environment

Trade takes place at date 0 during an infinite but countable number of rounds. Each round a continuum of agents arrives to trade for the first time with an equal mass of both types. Agents can stay for subsequent rounds, but once an agent leaves the trading process, he cannot come back.[18]

The sequence of events in each round is as follows: (1) Agents who are present are matched pairwise according to their types (types are observable); each pair includes one agent of each type, and if the mass of type-1 agents does not equal the mass of type-2 agents, some agents remain unmatched. (2) After being matched, the two agents negotiate a contract as described below. (3) Each agent decides whether to leave or for the next round.

Finally, after all rounds have ended, agents put money in escrow and make date-0 transfers simultaneously. Then each agent makes his individual date-0 investment and consumption decisions.

Contract negotiation is modeled as follows: The two agents offer contracts simultaneously. If they both offer the same contract, they enter that contract; otherwise, they do not enter a contract. The results in this paper are robust to other types of negotiation. For example, one can assume, that one agent offers a contract and the other agent accepts or rejects.

To capture the idea that every pair of agents enters the best contract for them (assuming other agents stick to their equilibrium strategies), I require that every contract entered be renegotiation proof; that is, a pair of agents will not replace the contract they agreed on if they are given another opportunity to negotiate. A formal definition is in the next subsection.

The main assumption is that

Assumption 1 *Agents cannot observe contracts that other pairs of agents enter (both in the past and in the future).*

There are a few interpretations to the assumption: (1) Trading is too fast for agents to

[18]The assumption of an infinite number of rounds gives us stationarity; that is, every agent faces the same future no matter when he shows up. The assumption of a continuum of agents captures the idea that the actions of a single agent cannot affect contract terms for other agents.

keep track of a counterparty's history of transactions. (2) Existing contracts are observable but not understood. An example is the complex derivative positions and off-balance-sheet transactions made by many hedge funds. (3) Agents can enter contracts secretly, as illustrated in Footnote 1.

I also assume that

Assumption 2 *Projects' assets cannot be posted as collateral. In other words, the right to terminate an agent's project cannot be promised exclusively.*

Trading cost. Entering a contract involves a small cost δ per agent. This cost is measured in utility terms and represents the time and effort involved in entering a contract. All the results hold if $\delta = 0$, but assuming $\delta > 0$ lets us derive some additional comparative statics (see Proposition 6).[19]

3.2 Equilibrium

Definition. I analyze the trading process above as an extensive-form game with imperfect information. The outcome of the game is the set of contracts entered, the amount that each agent invests, and the amount that each agent delivers. The payoff for each agent is his utility.[20] The information that each agent has is the sequence of contracts that he has entered, and a strategy specifies an action for each possible sequence. In particular, an agent needs to decide whether to stay or leave the trading game; if he stays, he needs to decide what contract to offer; if he leaves, he needs to decide how much to invest and how

[19] The cost δ must be low enough so that entering a contract is preferred to autarky; a sufficient condition is $\delta < \frac{R+\varepsilon}{1+\varepsilon} - \frac{R+\varepsilon}{2} = \frac{(R+\varepsilon)(1-\varepsilon)}{2(1+\varepsilon)}$. For technical reasons (to ensure that the mass of agents present in each round is finite for any given strategies), one can add the assumption that each agent can enter at most $\frac{1}{\delta}$ contracts, where $\delta > 0$; that is, each agent has one unit of "time." But for ease of exposition I assume that an agent can enter into an infinite number of contracts. Nonetheless, in equilibrium the number of contracts that an agent can enter is bounded.

[20] More specifically, consider a type-i agent who has entered the sequence of contracts $h \equiv (\psi^1, \psi^2, \ldots, \psi^n)$, where $\psi^j = (a_1^j, a_2^j, b_1^j, b_2^j, I_1^j, I_2^j)$. Suppose that he invests $\widehat{I}_i$ and delivers a total amount $\widehat{b}_i$, and suppose that his $j's$ counterparty delivers $\widehat{b}_{-i}^j$. The agent's utility is $U_i(\widehat{I}_i, \widehat{b}_i \mid \sum_{j=1}^n \widehat{\psi}^j) - n\delta$, where U_i is derived in the appendix, and $\widehat{\psi}^j$ denotes the contract ψ^j in which the element b_{-i}^j is replaced with $\widehat{b}_{-i}^j$. Contracts that were offered but not entered are not included because they do not affect payoffs.

much to deliver.[21]

To solve the game, I use the standard perfect Bayesian equilibrium (PBE) notion, focusing on symmetric equilibria, in which agents of the same type follow the same (pure) strategy.[22] I restrict attention to equilibria whose outcome is that every agent enters one contract and then leaves the trading game; I refer to the contract entered as the *equilibrium contract*.[23] In such equilibria, the only beliefs consistent with the equilibrium path are that "all agents present in the current round have just showed up to trade"; agents who appeared in previous rounds must have entered the equilibrium contract exactly once and then left. Given these beliefs, an agent need not worry about contracts that his counterparty might have entered in the past. However, an agent needs to worry about contracts that his counterparty may enter in the future. In particular, a counterparty may enter as many contracts as he can and subsequently default on all of them. The next example illustrates this.

Example 2 (strategic default) Suppose the equilibrium contract is as in example 1 (agents invest \$100 and promise \$30), and suppose that $\delta = 0$. If an agent enters the equilibrium contract exactly once, he obtains \$139. If he enters the equilibrium contract with n additional counterparties, he can obtain a utility of $100 + \frac{1}{2}(30)(n+1)$ by acting as follows: At date 0, he consumes his entire endowment of \$100. At date 1 when he needs to deliver, he defaults on all contracts and uses his limited liability to guarantee a payoff of zero; in the other state he obtains a total of $30(n+1)$ dollars from his counterparties. Requiring collateral puts a cap on n, and therefore reduces the gains from strategic default. For example, if agents require \$20 as collateral, an agent can enter at most five contracts.

To prevent the type of default above, the equilibrium contract (ψ) must satisfy the

[21] It is assumed that an agent does not know in what round he arrived to trade. It is also assumed that the same pair of agents cannot be matched more than once.

[22] Note that an agent need not form beliefs about the whole history of the game. It is enough to form beliefs about the sequence of contracts that agents present in the same round have entered.

[23] Equilibria in which agents enter the same contract are not renegotiation proof; see Lemma 2 in the appendix.

following incentive constraint for $i = 1, 2$ and for $n_i \in [0, \frac{1-a_i}{a_i}]$:

$$U_i(\psi) \geq \overline{U}_i(\psi + n_i\psi) - n_i\delta \text{ if } b_i > 0. \tag{8}$$

In this equation $n_i\psi$ denotes the aggregate contract $(n_ia_1, n_ia_2, n_ib_1, n_ib_2, n_iI_1, n_iI_2)$, and $\frac{1-a_i}{a_i}$ is the maximum number of additional contracts that an agent can enter given the collateral he needs to post; note choosing how much collateral to post is part of the contract. To avoid technical problems that may arise later, I do not require n_i to be an integer. A micro foundation for this is obtained if we assume that initial endowments are different across agents.[24] Equation (8) needs to hold only when $b_i > 0$; when $b_i = 0$, one need not worry about default because the agent does not promise anything.

Denote by $\widetilde{\Psi}$ the set of contracts $\psi \in \Psi$ that are preferred to autarky and satisfy equation (8) (note that equation (8) implies equation (1)). The next proposition says that ψ can be an equilibrium contract if and only if $\psi \in \widetilde{\Psi}$. Therefore, I refer to $\widetilde{\Psi}$ as the set of equilibrium contracts, and to $\psi \in \widetilde{\Psi}$ as an equilibrium contract.

Proposition 2 *A PBE whose outcome is that every agent enters ψ and then leaves the trading game exists in the decentralized trading environment if and only if ψ is preferred to autarky and satisfies equation (8); that is, if and only if $\psi \in \widetilde{\Psi}$.*

The idea behind the proof is simple: An agent will enter ψ only if the contract is preferred to autarky and if the agent believes that his counterparty will follow the contract (equation (8)). Offering a contract different from ψ is suboptimal because everyone else offers ψ, and a contract is entered only if both agents offer the same contract.[25]

[24] In more detail, assume that instead of one economy, there are an infinite number of economies corresponding to the interval $(0, 1]$, and that agents in economy $\mu \in (0, 1]$ have an initial endowment of μ. The economy to which an agent belongs and an agent's endowment are private information, and the cost of being matched in economy μ is scaled to be $\mu\delta$. Assume for simplicity that when an agent first shows up to trade, he must trade in his original economy, but afterward an agent can switch back and forth among the different economies. The only restriction is that an agent with an endowment e can trade in economy v only if $v \leq e$; that is, an agent can say that he has less than what he has, but he cannot say that he has more. Then if $\mu\psi$ is the equilibrium contract in economy μ, it is possible to enter it n times, where n is not restricted to be an integer. If n_i is restricted to be an integer, an optimal contract may not exist in Proposition 4 below because the set of feasible contracts that satisfy equation (8) may be open (because n_i is not a continuous function of a_i).

[25] If one agent offers a contract and the other agent accepts or rejects, one can sustain the PBE by assuming

The next proposition identifies the cases when the second best (denoted by ψ_{sb}) can be achieved in the decentralized trading environment. This happens when the fixed cost of entering an additional contract is higher than the expected net cash obtained from an additional counterparty; that is, when $\delta \geq \frac{1}{2}b_{sb}$; note that if $\delta = 0$, the second best can never be achieved.

Proposition 3 *The second best can be achieved in the decentralized trading environment (that is, $\psi_{sb} \in \widetilde{\Psi}$) if and only if either (1) $R \geq 1 + \frac{1}{2}\varepsilon$ and $\delta \geq \frac{1}{2}\varepsilon$, or (2) $R < 1 + \frac{1}{2}\varepsilon$ and $\delta \geq \frac{R-1}{2+\frac{1}{2}\varepsilon - R}$.*

Renegotiation proof contracts. Consider the following extended game: In each round, we add the following events. After agents decide on a contract, one pair of agents is chosen randomly. The randomly chosen pair repeats the negotiation process, i.e., they both offer contracts simultaneously. If they both offer the same contract, they enter this contract instead of the initial contract; otherwise, they stick with the initial contract. (As before, one can assume a different renegotiation process without affecting the result, e.g., one agent offers a contract and the other agent accepts or rejects.)

Definition 1 *An equilibrium contract $\psi \in \widetilde{\Psi}$ is renegotiation proof (given the trading environment) if the extended game above does not have a PBE whose outcome is that the randomly chosen pair initially agrees on the contract ψ and then replaces it by the contract $\psi' \neq \psi$.*

A PBE is renegotiation proof if the equilibrium contract associated with the PBE is renegotiation proof. Note that renegotiation here does not occur because of the arrival of new information. Instead, it is away to capture the idea that each pair of agents enters the best contract for them given that all other agents stick to their equilibrium strategies.[26]

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that on seeing any offer other than ψ, an agent assumes that his counterparty will default. Since this is an out-of-equilibrium event, any beliefs may be assigned.

[26] The definition here is in the spirit of Laffont and Martimort (1997). They used the term *collusion proof* to model collusion between two firms with private information about their costs. In their setting a regulator offers a mechanism (a grand contract), then an uninformed third party offers a side contract.

Optimal contracts. As mentioned earlier, I focus on contracts that maximize the unweighted sum of agents' utilities. Therefore the problem is to find an equilibrium contract that maximizes $\sum_{i=1}^{2} U_i(\psi)$. The unique solution to this problem is referred to as *third best* and is denoted it by ψ_{tb}

Proposition 4 (third best) *(1) There is a unique contract that solves* $\max_{\psi \in \widetilde{\Psi}} \sum_{i=1}^{2} U_i(\psi)$. *If the conditions in Proposition 3 hold, this contract equals the second best. Otherwise, the contract is given by* $a_1 = a_2 = 1 - I$, $b_1 = b_2 = (2+\varepsilon)I - 2$, *and* $I_1 = I_2 = I$, *where* $I = \frac{1}{4(R-1)}\left(2(R-2) - \rho + \sqrt{8\rho + (2R-\rho)^2}\right)$ *and* $\rho = \varepsilon - 2\delta$. *(2) The contract above is renegotiation proof in the decentralized trading environment. (3) The contract above is the only equilibrium contract that is both symmetric and renegotiation proof in the decentralized trading environment.*

Corollary 1 *The third-best contract requires less collateral when the return on the project (R) increases and/or when the fixed cost per trade (δ) increases.*

Intuitively, when the cost per trade (δ) is higher and/or when agents have more future income to lose (higher R), strategic default becomes less desirable. Therefore, less collateral is needed to prevent default.

The dual role of collateral. Using the notation $k = a$ and $x = a + b$, and setting $\delta = 0$, for simplicity, equations (4), (5), and (8) imply that

$$x \leq k + \frac{2(R-1)I}{n}, \tag{9}$$

where $n = \frac{1}{k}$. When an agent posts k dollars as collateral, the amount of cash that he can credibly promise (x) increases by more than k. First, the agent cannot default on the amount of cash that he posted as collateral (first term in equation (9)). Second, the fact that the contract requires collateral limits the number of contracts (n) that the agent can enter. This makes the threat of losing future cash flows valuable in backing promises (second term).

All the results hold even if we require a weaker notion of renegotiation proof, namely, that for the same decision nodes, beliefs in the extended game are the same as the beliefs in the original game.

4 An equilibrium with an intermediary

In this section—which contains the main results—I show how an intermediary can increase welfare by implementing the second best. The intermediary has a minimal role. Before trading begins, the intermediary sets a contract $\psi \in \Psi$ and a position limit $L \in \{1, 2, \ldots\}$. Given ψ and L, agents play the game from the previous section, but now an agent who offers a contract also specifies whether he wants to report the contract to the intermediary; in other words, the decision to report is part of the contract. (From now on, I use the word contract to refer to $\psi \in \Psi$ as well as to the "extended contract" that includes the decision to report.) If an agent chooses to report, he must offer ψ. Otherwise, he can offer ψ or any other contract $\psi' \in \Psi$.[27]

Reporting a contract means reporting the identity of the two agents who enter the contract. It is assumed that agents cannot lie about their identity. The intermediary keeps a record of the number of contracts that each agent reports, and whenever a pair of agents report a contract, the intermediary updates the record of each one of them. Crucially, the intermediary does not observe contracts that agents may enter without reporting. In other words, the intermediary can keep track only of contracts that agents report to it voluntarily.

The position limit means that an agent can report at most L contracts. In particular, if an agent attempts to report more than L contracts, the intermediary does not register the contract; in other words, the intermediary does not update the records of the agents involved in the contract. (Note that in equilibrium no agent attempts to enter more than L contracts. In particular, an agent who learns that his counterparty has already reported

[27] The assumption that agents who report a contract must enter ψ is used in Proposition 7. Without this assumption, an agent who observes that his counterparty has already reported one contract or more has more flexibility in forming beliefs as to what these contracts are. Alternatively, one could assume that agents who report a contract can enter whatever contract they want, but the intermediary can verify (and make public) contracts that are reported to it.

When an agent's endowment can be less than one as in footnote 24, the position limits for each agent are scaled according to the endowment that the agent chooses to reveal to the intermediary. The intermediary cannot observe an agent's endowments, but agents have the incentive to reveal their endowments truthfully. In particular, an agent cannot reveal more than what he has, and if he reveals less, his position limit is lower.

L contracts does not enter an additional contract with him.)[28]

It is assumed that the intermediary makes public whether an agent has reached the limit or not, but that the intermediary does not reveal the exact number of contracts an agent has reported. Later, I relax this assumption. There are two equilibria that are renegotiation proof: one in which no agent reports, and one in which everyone reports. The first equilibrium is the same as in the previous section. In this section I focus on the second equilibrium.

Throughout, I assume that there is some fee associated with reporting, but let's consider first the case in which it is costless to report a trade. Then it is easy to verify that when the intermediary sets ψ_{sb} and $L = 1$, there is a PBE in which agents report all their trades to the intermediary, i.e., no pair of agents enters a contract without reporting. In particular, since all agents report and $L = 1$, an agent can enter at most one contract and nonexclusivity is not an issue. This PBE is renegotiation proof because a pair of agents cannot gain by entering ψ_{sb} without reporting.

But what if reporting a contract involves a small fee? In this case, a pair of agents may attempt to enter ψ_{sb} without reporting it to the intermediary, and the PBE above may not be renegotiation proof. The next example illustrates this. The fee per agent of reporting a contract is denoted by $\theta > 0$ and is assumed to be very small (it can be as small as one wants). It is assumed for simplicity that θ is in terms of utility, so incurring θ does not come instead of investing in the project, i.e., it has no opportunity cost.

Example 3 (nonbinding position limits) Consider Example 1 with $\delta = 0$. I show that the intermediary can implement the second best via a PBE that is renegotiation proof if $L = 2$, but not if $L = 1$.

Suppose first that $L = 1$, and suppose by contradiction that all agents enter and report the second-best contract (invest \$100 and promise \$30). What is the best response for a pair

[28] Formally, let ξ denote whether a contract is reported to the intermediary ($\xi = 1$) or not ($\xi = 0$). The contract space in the new setting is $\Psi_{\psi,L} = \{(\psi', \xi) : \psi' \in \Psi,\ \xi \in \{0,1\},\ \xi = 1 \Rightarrow \psi' = \psi\}$. The set of individual histories that are feasible is $H_{\psi,L} = \{(\psi'_i, \xi_i)_{i=1,\dots,n} : \forall\, i = 1, \dots, n,\ (\psi'_i, \xi_i) \in \Psi_{\psi,L},\ \sum_{i=1}^{n} a'_i \leq 1,$ $\sum_{i=1}^{n} \xi_i \leq L\}$.

of agents? If the agents enter the second-best contract and report it, each agent obtains a utility of $139 - \theta$. If they deviate by entering the second-best contract without reporting, they save on the fee and each agent obtains 139. The deviating agents need not worry about a counterparty's default because given that all other agents report, a counterparty who plans to default can enter at most one additional contract. But with a total of two contracts, he obtains a utility of only $100 + \frac{1}{2}(2 * 30) - \theta = 130 - \theta$, which is less than the utility he obtains if he enters only one contract and follows it.

Now suppose the intermediary sets $L = 2$. In this case entering the second-best contract without reporting induces a counterparty to default because he can enter two additional contracts (having a total of three) and obtain $100 + \frac{1}{2}(3 * 30) - 2\theta = 145 - 2\theta$. This is more than what he gets if he enters only one contract (139). The pair of agents who deviate by not reporting can prevent default by requiring more collateral; however, when θ is small, not reporting and requiring more collateral is more costly than reporting the second best and paying θ.

Proposition 5 *Let*

$$L^* = \begin{cases} 1 & if \quad R < 1 + \frac{1}{2}\varepsilon \\ \lfloor \frac{R-1-(\delta+\theta)}{\frac{1}{2}\varepsilon-(\delta+\theta)} \rfloor & if \quad R \geq 1 + \frac{1}{2}\varepsilon, \end{cases} \tag{10}$$

and suppose that the intermediary reveals only whether an agent has reached the position limit or not. Then: (1) There is a PBE in which all agents enter the second-best contract and report it to the intermediary if and only if the intermediary sets $L \leq L^*$. (2) A PBE in which all agents enter the second-best contract and report it to the intermediary is renegotiation proof if and only if $L = L^*$. (3) L^* increases in R.

The idea behind the proof is as follows: The number L^* is the unique integer such that if an agent could enter the second-best contract at most L^* times, he would prefer to enter it only once, but if he could enter L^*+1 contracts, he would do so and default on all contracts. Clearly, the position limit cannot be more than L^*. Suppose the position limit is L^* and consider a PBE in which all agents enter and report ψ_{sb}. This PBE is renegotiation proof because if a pair of agents attempts to enter the second-best contract without reporting, each one of them has the incentive to enter additional contracts and default on all. In particular, an agent can enter L^* additional contracts (and report all) for a total of $L^* + 1$; but since entering $L^* + 1$ contracts is preferred to entering one contract, the agent will do so and default. If instead the two agents report their contract, each one can then enter only $L^* - 1$ additional contracts for a total of L^* contracts; but since entering one contract is preferred to entering L^* contracts, both agents will stick with one contract. The argument above assumes that a deviating pair enters ψ_{sb}. In the formal proof, I also consider the case where the deviating pair attempts to enter a contract $\psi \neq \psi_{sb}$ without reporting, showing that this is suboptimal. Now suppose the intermediary sets a limit of $L^* - 1$. Agents who enter ψ_{sb} without reporting need not worry about a counterparty's strategic default because a counterparty can enter at most $L^* - 1$ additional contracts for a total of L^* contracts. Therefore, the PBE is not renegotiation proof.

Intuitively, position limits cannot be too high and they cannot be too low. Position limits that are too high induce agents to strategically default by allowing them to enter too many contracts. Position limits that are too low make it too hard to default; in this case a pair of agents who do not report need not worry about a counterparty's default, and a PBE in which all agents report is not renegotiation proof. The third part in the proposition says that when agents have more future income to lose (R), position limits should be set higher. Higher R reduces the gains for strategic default, and therefore position limits should not be as stringent.

Corollary 2 *To implement the second best (via a PBE that is renegotiation proof) when* $L^* \geq 2$, *the intermediary must set position limits that are nonbinding in equilibrium.*

Gains from the intermediary. Since the intermediary can implement the second best, and without the intermediary we obtain the third best, it follows that when the conditions in Proposition 3 do not hold, the intermediary can increase welfare. The gain from the intermediary is $U_i(\psi_{sb}) - \theta - U_i(\psi_{tb}) = (R-1)(I_{sb} - I_{tb}) - \theta$. Since I_{sb} does not depend on δ, but I_{tb} does, it follows from Proposition 4 that when δ decreases, the gain increases.

This result holds also when $\theta = 0$, i.e., if there is no fee in reporting to the intermediary.

Proposition 6 *The gain from the intermediary increases when the fixed cost per trade (δ) decreases.*

4.1 Should the intermediary make reported trades public?

An important issue is whether the intermediary should reveal the information it has, in particular, whether the intermediary should reveal the number of contracts that each agents has already reported. The next proposition shows that the answer may be no. The proposition says that if the intermediary reveals the number of contracts an agent has entered, not only whether he reached the limit or not, there are cases in which the intermediary cannot implement the second best via a PBE that is renegotiation proof. The proposition therefore implies that the intermediary cannot be replaced by a bulletin board.[29]

Proposition 7 *If the intermediary reveals the exact number of contracts that an agent has reported, there is a nonempty set of parameters for which the intermediary cannot implement the second best via a PBE that is renegotiation proof.*

The intuition is similar to the one in Proposition 5. The intermediary wants to rule out deviations in which a pair of agents enters a contract without reporting. But a pair of agents will do so only if none of them expects the other agent to cheat by entering additional contracts and defaulting on all of them. Therefore, to rule out deviations in which a pair of agents enters a contract without reporting, the intermediary must make it easy enough for any member of a deviating pair to cheat on his counterparty. In particular, the counterparty must be able to enter enough contracts without being detected. Position limits that are high enough (as in Proposition 5) are necessary for a counterparty to be able to enter enough contracts. Not revealing information about the number of contracts that an agent has already entered ensures that a counterparty can indeed enter contracts according

[29] I focus on the case in which the intermediary either makes information public or keeps it to itself. Alternatively, one can assume that the intermediary reveals information only to agents who choose to report to it. Such a formulation changes the trading game slightly but has no effect on the results.

to his position limit without being detected. If the counterparty were to be detected, no one would trade with him; or alternatively the counterparty would need to collateralize all his promise, so that he cannot gain from a strategic default.

4.2 Remarks

(1) *Multiple intermediaries.* While the paper shows that one intermediary can implement the second best, it does not rule out the possibility of more than one intermediary. In particular, a nonbinding position limit ($L^* \geq 2$) can be implemented by N intermediaries, where intermediary j sets a position limit l_j, and $\sum_{j=1}^{N} l_j = L^*$.

To see how it works, adjust the trading environment as follows: Assume that there are L^* locations. Each location can have its own intermediary, and each intermediary can observe only the contracts reported to it. Each agent shows up for trade in one of the locations where an intermediary exists; the location is chosen randomly. Initially, an agent must trade in the location where he showed up, but if an agent decides to stay for more rounds, he can switch back and forth among the different locations. The trading process (matching plus bargaining) in each location is as in the previous section. In particular, agents can either report their trade to the intermediary in the location where they trade, or they can enter a contract without reporting.

Nonbinding position limits mean that if there is no cost to set up an intermediary, there is more than one way to implement the second best. For example, we can have one intermediary that sets a position limit L^*, or we can have L^* intermediaries where each intermediary sets a position limit of one. In the second case, there is a PBE (that is renegotiation proof) in which each agent enters one contract and reports it to the intermediary in the location where he showed up.

Proposition 7 implies that for this to work, we need to assume that an agent who trades in a given location can observe only whether his counterparty has reached the position limit within the intermediary in this location but he cannot observe whether a counterparty has reached the limit in any of the other locations. (This is because observing whether a counterparty has reached the limit in each location is like observing the exact number of contracts that the counterparty has reported.) Such a restriction would follow, for example, if an agent in a given location needs to pay an extra fee to see if an agent reported a contract to an intermediary in another location, which is a reasonable assumption. In equilibrium, however, these fees are not paid because an agent believes that his counterparty has just showed up for trade.

(2) *What if the intermediary guarantees payments?* The paper focuses on one role of an intermediary, but the results remain even if the intermediary has other roles. In particular, the results hold even if the intermediary guarantees payments by becoming a central counterparty for every trade, which is one of the roles of a clearing house. Proposition 7 needs more attention, however. If an agent were sure that the intermediary will pay him even if his counterparty defaults, an agent would be willing to enter a contract with a counterparty even if the agent knew that his counterparty has already reported some contracts. But to make such a guarantee credible, the intermediary needs to set some cash aside as collateral, which has an opportunity cost. Not revealing information about the number of contracts that an agent has already reported allows the intermediary to save on collateral. Also note that the intermediary needs to precommit to a policy that is suboptimal ex post: It needs to allow an agent to enter up to L^* contracts, even though once it sees an agent approach the position limit, it knows that the agent will default for sure.

(3) *The role of reputation in eliminating the risk of default.* In practice, reputation can play a role in preventing default even without an intermediary. But if the cost of losing reputation is finite, and the potential gain from default is unbounded, an agent may choose to strategically default. While this is not a formal model of reputation, one can interpret R as reputation. Losing one's reputation is like losing the project's future cash flows. In the decentralized trading environment, contracts need not be fully backed by collateral because of the threat of losing $R/$ reputation. In addition, an agent who plans to default does not

invest in R, which is analogous to an agent who does not invest in reputation.

(4) *The intermediary may also require collateral.* Proposition 5 shows that an intermediary can implement the second best. When the second best equals the first best, the intermediary does not require collateral. Otherwise, Proposition 1 implies that agents must post collateral even if they report their contract to the intermediary. The amount of collateral is less, however, than the amount of collateral that agents would post if they chose not to report their trade, or in a decentralized trading environment without an intermediary.

5 Conclusion

The paper shows how an intermediary can implement an equilibrium in which agents report (reveal) all their trades to it voluntarily. This holds even though pairs of agents can enter contracts secretly, and even though there is some small fee associated with reporting a trade to the intermediary.

The intermediary increases welfare because without it agents need to use collateral (which has opportunity cost) to make sure that a counterparty does not have the incentive to strategically default by entering too many contracts and defaulting on all. The intermediary is a cost-effective mechanism to prevent this type of strategic default because the intermediary needs to monitor only the contracts that agents choose to reveal to it, but it does not need to spend resources trying to monitor contracts that agent might enter secretly.

I show that in some cases, the intermediary must set position limits that are nonbinding in equilibrium, and that in some cases, the intermediary must not make reported trades public. Setting high enough limits as well as not revealing too much information may be necessary to ensure that no pair of agents have the incentive to deviate by not reporting their trade (i.e., by entering a contract secretly). Both tools give enough scope for a member of such a deviating pair to cheat on his counterparty by entering additional contracts, and this makes the original deviation of not revealing a trade to the intermediary suboptimal.

The paper uses a simple framework, but the main results and intuition apply in richer settings. In particular, while the paper shows that one intermediary can implement the second best, it does not rule out the possibility of more than one intermediary. In addition, the main results hold even if in addition to its role in monitoring trades, the intermediary guarantees payments by becoming a central counterparty to every trade.

Appendix

Deriving utilities. Denote by $U_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi)$ the utility for agent i if he invests $\widehat{I}_i \in [0, 1-a_i]$ and delivers $\widehat{b}_i \in [0, \varepsilon \widehat{I}_i]$, given that he entered the contract $\psi = (a_1, a_2, b_1, b_2, I_1, I_2) \in \Psi$, and given that his counterparty, $-i$, follows the contract. Note that $U_i(\psi) = U_i(I_i, b_i \,|\, \psi)$ and $\overline{U}_i(\psi) = U_i(0, 0 \,|\, \psi)$.

The amount that agent i consumes at date 0 follows from his budget constraint and is given by $1 - a_i - \widehat{I}_i$. The total amount consumed at dates 1 and 2 depends on the state. In state i, when the agent realizes a positive cash flow, he consumes $\varepsilon \widehat{I}_i - \widehat{b}_i + \lambda_i(\widehat{b}_i | b_i) R \widehat{I}_i$. In state $-i$, when he realizes a negative cash flow, he can continue his project only if he has enough cash. If $s + b_{-i} \geq \varepsilon \widehat{I}_i$, he consumes $s + b_{-i} + (R - \varepsilon)\widehat{I}_i$; otherwise, he consumes $s + b_{-i}$. The agent's utility is, therefore,

$$U_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi) \equiv 1 - a_i - \widehat{I}_i + \frac{1}{2}[\varepsilon \widehat{I}_i - \widehat{b}_i + \lambda_i(\widehat{b}_i | b_i) R \widehat{I}_i] + \frac{1}{2}[s + b_{-i} + \beta_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi)(R - \varepsilon)\widehat{I}_i], \quad (11)$$

where

$$\beta_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi) = \begin{cases} 1 & \text{if } s + b_{-i} \geq \varepsilon \widehat{I}_i \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Proof of Proposition 1. We can assume, without loss of generality, that $\beta_1 = \beta_2 = 1$. Thus,

$$\begin{aligned} \sum_{i=1}^{2} U_i(\psi) &= 1 - a_1 - I_1 + \frac{1}{2}[\varepsilon I_1 - b_1 + R I_1] + \frac{1}{2}[s + b_2 + (R - \varepsilon) I_1] \quad (13) \\ &\quad +1 - a_2 - I_2 + \frac{1}{2}[\varepsilon I_2 - b_2 + R I_2] + \frac{1}{2}[s + b_1 + (R - \varepsilon) I_2] \\ &= 2 - a_1 - a_2 - I_1 - I_2 + R I_1 + R I_2 + s \\ &= 2 + (R - 1)(I_1 + I_2), \end{aligned}$$

and

$$
\begin{aligned}
& U_i(\psi) - \overline{U}_i(\psi) && (14)\\
= \; & 1 - a_i - I_i + \frac{1}{2}[\varepsilon I_i - b_i + RI_i] + \frac{1}{2}[s + b_{-i} + (R-\varepsilon)I_i] \\
& -[1 - a_i + \frac{1}{2}(s + b_{-i})] \\
= \; & (R-1)I_i - \frac{1}{2}b_i.
\end{aligned}
$$

The problem becomes:

$$\max_{\psi \in \Psi}(I_1 + I_2) \tag{15}$$

subject to

$$(R-1)I_i \geq \frac{1}{2}b_i,\ i = 1,2 \text{ (incentive)}, \tag{16}$$

$$s + b_{-i} \geq \varepsilon I_i, \quad i = 1,2 \;\; (\beta_i = 1), \tag{17}$$

and the participation constraint for each agent.

Consider the set of feasible contracts Ψ. Note that $b_i \leq \varepsilon I_i$ for $i = 1,2$ together with equation (17) imply that $s \geq 0$; and $b_i \geq 0$ together with equation (16) imply that $I_i \geq 0$. In addition, we must have $a_i + I_i = 1$; otherwise (if $a_i + I_i < 1$), we can obtain a better solution by increasing I_i and a_i by Δ and $\varepsilon\Delta$, respectively, where Δ is small enough. Thus, the relevant constraints in Ψ are

$$b_i \geq 0, \quad i = 1,2 \tag{18}$$

$$b_i \leq \varepsilon I_i, \quad i = 1,2 \tag{19}$$

and

$$a_i = 1 - I_i, \quad i = 1,2. \tag{20}$$

This is a linear programming problem. When $R \geq 1 + \frac{1}{2}\varepsilon$, equation (19) implies equation (16) and the solution is obtained by solving equations (17), (19), and (20) with equalities. When $R < 1 + \frac{1}{2}\varepsilon$, equation (16) implies equation (19) and the solution is obtained by solving equations (16), (17), and (20) with equalities. One can verify that the solution in each case is the one given in the proposition and that it satisfies equation (18) and the participation constraint. Q.E.D.

Proof of Proposition 2. If: Suppose that $\psi' \in \widetilde{\Psi}$. For $\psi \in \Psi$, let

$$(I_i^*(\psi), b_i^*(\psi)) \equiv \arg \max_{\widehat{I}_i \in [0,1-a_i],\, \widehat{b}_i \in [0,\varepsilon \widehat{I}_i]} U_i(\widehat{I}_i, \widehat{b}_i \,|\, \psi), \tag{21}$$

and $F_i(\psi) \equiv U_i(I_i^*(\psi), b_i^*(\psi) \,|\, \psi)$. For a sequence of contracts $h = (\psi^1, \psi^2, \ldots \psi^n)$, let $\psi(h) \equiv \sum_{j=1}^n \psi^j$, $a_i(h) \equiv \sum_{j=1}^n a_i^j$, and

$$n_i^*(h) \equiv \arg \max_{n' \in (0, \frac{1-a_i(h)}{a'})} [F_i(\psi(h) + n'\psi') - n'\delta]. \tag{22}$$

Consider the following strategy for an agent of type i: If $h = \emptyset$ or $n_i^*(h) > 0$, stay and offer ψ'; otherwise, leave the trading process and choose $I_i^*(\psi(h)), b_i^*(\psi(h))$. (Note that $0 < n_i^*(h) < 1$ means that the agent switches to another economy as described in footnote 24.) Since $\psi' \in \widetilde{\Psi}$, the equilibrium path is such that each agent enters ψ' and leaves the trading game. The only beliefs consistent with the equilibrium path is that "my counterparty has just showed up for trade." Given these beliefs and since $\psi' \in \widetilde{\Psi}$, the agent believes that his counterparty will follow the contract. In addition, the contract is preferred to autarky. Offering $\psi \neq \psi'$ is suboptimal because all other agents offer ψ' and a contract is entered only if both agents offer the same contract. The decision of whether to stay and how much to invest and deliver are optimal because of equations (21) and (22).

In the alternative negotiation game, in which one agent offers a contract and the other agent accepts or rejects, one can sustain the equilibrium above by assuming that on seeing any offer other than ψ' (in which $b_{-i} > 0$), an agent assumes that his counterparty will default (i.e., his counterparty has already promised more than he has). Since this is an out-of-equilibrium event, any beliefs may be assigned.

Only if: Suppose there exists a PBE in which agents enter ψ'. An agent will enter ψ' only if he believes that his counterparty will follow the contract (equation (8)) and if the contract is weakly preferred to autarky. Thus, $\psi' \in \widetilde{\Psi}$.

Q.E.D.

Proof of Proposition 3: The second best can be achieved when equation (1) implies equation (8). Since

$$\begin{aligned} & \overline{U}_i(\psi + n_i\psi) - n_i\delta \\ = \;& 1 + \frac{1}{2}b + \frac{1}{2}n_i b - n_i\delta \\ = \;& 1 + \frac{1}{2}b + \frac{1}{2}n_i(b - 2\delta), \end{aligned} \tag{23}$$

equation (1) implies equation (8) if and only if $b - 2\delta \leq 0$; that is when $\delta \geq \frac{1}{2}b$. If $R \geq 1 + \frac{1}{2}\varepsilon$, it follows from Proposition 1 that $\frac{1}{2}b = \frac{1}{2}\varepsilon$. If $R < 1 + \frac{1}{2}\varepsilon$, it follows from Proposition 1 that

$$\begin{aligned} \frac{1}{2}b &= \frac{1}{2}[(2+\varepsilon)\frac{1}{2 + \frac{1}{2}\varepsilon - R} - 2] \\ &= \frac{1}{2}\frac{2 + \varepsilon - 2(2 + \frac{1}{2}\varepsilon - R)}{2 + \frac{1}{2}\varepsilon - R} \\ &= \frac{R - 1}{2 + \frac{1}{2}\varepsilon - R}. \end{aligned} \tag{24}$$

Q.E.D.

Lemma 1 *A contract $\psi \in \widetilde{\Psi}$ is renegotiation proof in the decentralized trading environment if and only if there does not exist a contract $\psi' \in \Psi$ that satisfies the following constraints for $i = 1, 2$:*

$$U_i(\psi') \geq U_i(\psi), \text{ with strict inequality for at least one } i \tag{25}$$

$$U_i(\psi') \geq \overline{U}_i(\psi' + n_i\psi) - n_i\delta \text{ for } n_i \in [0, \frac{1 - a_i'}{a_i}]. \tag{26}$$

(Equation (26) needs to hold only if $b_i' > 0$.)

Proof of Lemma 1. Denote by $F(\psi)$ the set of contracts $\psi' \in \Psi$ that satisfy equations (25) and (26). We need to show that $\psi \in \widetilde{\Psi}$ is renegotiation proof if and only if the set $F(\psi)$ is empty.

Only if: Suppose $\psi \in \widetilde{\Psi}$ is renegotiation proof and suppose by contradiction that $F(\psi)$ contains the contract ψ'. (Note that this implies $\psi' \neq \psi$.) I will show that there exists a PBE whose outcome is that a pair of agents initially chooses ψ and then replaces it by ψ', which contradicts the fact that ψ is renegotiation proof. The equilibrium strategies are as in Proposition 2 with the following addition: An agent who is part of a pair that was chosen to renegotiate offers ψ'. The agent's beliefs are as in Proposition 2 and the agent does not change his beliefs after being chosen. (If ψ was agreed upon initially, not changing beliefs is the only belief consistent with the equilibrium path. If ψ was not agreed upon initially, we can assign any beliefs, including the original ones, because this is an out-of-equilibrium event.) These strategies and beliefs are a PBE because $\psi' \in F(\psi)$. Also note that since the probability that a pair of agents is chosen to renegotiate is zero, the possibility of renegotiation does not affect agents' decisions in the first stage of negotiation.

If renegotiation is such that one agent offers a contract and the other accepts or rejects, then strategies regarding renegotiation is to offer ψ', accept ψ', and reject any other offer. As in Proposition 2, the PBE can be sustained if an agent who receives an out-of-equilibrium offer believes that his counterparty will default for sure. In contrast, if an agent is offered ψ' (after ψ was chosen initially), which is on the equilibrium path in the extended game, the only beliefs consistent with the equilibrium path is not to revise one's beliefs.

If: Suppose that the set $F(\psi)$ is empty and suppose by contradiction that ψ is not renegotiation proof. Then there exists a PBE whose outcome is that a pair of agents initially chooses ψ and then replaces it by $\psi' \neq \psi$. The only beliefs consistent with the equilibrium path are that initially an agent believes that all agents present in that round have just showed up for trade, and after being chosen to renegotiate (and given that ψ was initially agreed upon) the agent does not revise his beliefs on his counterparty (and believes that all other agents entered ψ in the existing round). To obtain this equilibrium path it must be the case that in the renegotiation stage, an agent offers ψ' if ψ was chosen initially. But this strategy is optimal given the beliefs above only if $\psi' \in F(\psi)$. This contradicts the fact that $F(\psi)$ is empty. (Similar logic works also if the game is such that one agent offers a contract and the other agent accepts or rejects.)

Q.E.D.

Proof of Proposition 4.

Part (1). Note that

$$\begin{aligned} & U_i(\psi) - \overline{U}_i(\psi + n_i\psi) \qquad (27) \\ = \; & 1 - a_i - I_i + \frac{1}{2}[\varepsilon I_i - b_i + RI_i] + \frac{1}{2}[s + b_{-i} + (R-\varepsilon)I_i] \\ & -[1 - a_i - n_i a_i + \frac{1}{2}(s + b_{-i}) + \frac{1}{2}n_i(s + b_{-i})] \\ = \; & (R-1)I_i - \frac{1}{2}b_i - \frac{1}{2}n_i(a_{-i} + b_{-i} - a_i). \end{aligned}$$

Thus, equation 8 becomes

$$(R-1)I_i \geq \frac{1}{2}b_i + \frac{1}{2}n_i(a_{-i} + b_{-i} - a_i) - n_i\delta, \qquad (28)$$

for $n_i \in [0, \frac{1-a_i}{a_i}]$ and $i = 1, 2$. Equation (28) is binding when either $n_i = 0$ or $n_i = \frac{1-a_i}{a_i}$. The first case happens if the conditions in Proposition 3 hold, in which case we obtain the second best. I now focus on the case in which $n_i = \frac{1-a_i}{a_i}$ is binding, which happens if

$$a_{-i} + b_{-i} - a_i - 2\delta > 0. \qquad (29)$$

Note that $\psi \in \widetilde{\Psi}$ implies that

$$\sum_{i=1}^{2}(R-1)I_i \geq \sum_{i=1}^{2}[\frac{1}{2}b_i + \frac{1}{2}n_i(a_{-i} + b_{-i} - a_i) - n_i\delta], \text{ where } n_i = \frac{1-a_i}{a_i} \qquad (30)$$

$$\sum_{i=1}^{2}(a_1 + a_2 + b_{-i}) \geq \sum_{i=1}^{2}\varepsilon I_i \qquad (31)$$

and

$$a_i + I_i \leq 1, \; i = 1, 2. \qquad (32)$$

Therefore, to show that the contract proposed in the proposition is the unique solution to $\max_{\psi\in\widetilde{\Psi}} \sum_{i=1}^{2} U_i(\psi)$, it is enough to show that (i) it belongs to $\widetilde{\Psi}$, and (ii) it is the unique solution to $\max_{(a_1,a_2,b_1,b_2,I_1,I_2)}(I_1+I_2)$ subject to equations (30), (31), and (32). Part (ii) follows from the Kunn-Tucker theorem (as explained below). Part (i) then follows easily.

To use the Kunn-Tucker theorem, let $f(\psi)=I_1+I_2$ and consider the problem $\max_\psi f(\psi)$ subject to $g_i(\psi)\leq 0$, $i=1,\ldots,4$, where

$$g_1(\psi)=\sum_{i=1}^{2}[b_i+(\frac{1}{a_i}-1)(a_{-i}+b_{-i}-a_i-2\delta)]-2(R-1)\sum_{i=1}^{2}I_i, \tag{33}$$

$$g_2(\psi)=\varepsilon\sum_{i=1}^{2}I_i-\sum_{i=1}^{2}(a_i+a_{-i}+b_{-i}) \tag{34}$$

and

$$g_{i+2}(\psi)=a_i+I_i-1,\; i=1,2. \tag{35}$$

Denote the Lagrange multiplier of $g_i(\psi)$ by λ_i, and let $L(\psi)=f(\psi)+\sum_{j=1}^{4}g_i(\psi)$. Then $\lambda_i\leq 0$, and

$$\frac{\delta L}{\delta I_i}=1-2(R-1)\lambda_1+\varepsilon\lambda_2+\lambda_{2+i}=0,\; i=1,2. \tag{36}$$

$$\frac{\delta L}{\delta b_i}=\lambda_1+(\frac{1}{a_{-i}}-1)\lambda_1-\lambda_2=0 \tag{37}$$

$$\frac{\delta L}{\delta a_i}=-\lambda_1\frac{1}{a_i^2}(a_{-i}+b_{-i}-a_i-2\delta)+(\frac{1}{a_{-i}}-1)\lambda_1-(\frac{1}{a_i}-1)\lambda_1-2\lambda_2+\lambda_{2+i}=0 \tag{38}$$

Equation (36) implies that $\lambda_3=\lambda_4$, and equation (37) implies that $a_1=a_2\equiv a$. Equation (38) becomes

$$\frac{\delta L}{\delta a_i}=-\lambda_1\frac{1}{a^2}(b_{-i}-2\delta)-2\lambda_2+\lambda_3=0, \tag{39}$$

which implies that $b_1=b_2\equiv b$. Consider λ_2. If $\lambda_2=0$, then equation (37) implies $\lambda_1=0$, and equation (39) implies $\lambda_3=0$; but this contradicts equation (36). Therefore, $\lambda_2<0$. Then (37) implies $\lambda_1<0$ and $a_i>0$, and equations (29) and (39) imply $\lambda_3<0$. Therefore, equations (30), (31), and (32) are binding. Using simple algebra, it is then easy to verify that the contract proposed in the proposition is the unique solution. [In more detail, equation (32) implies that $I_1=I_2\equiv I$ and $a=1-I$, and equation (31) implies that $b=(2+\varepsilon)I-2$. Equation (30) then implies

$$(R-1)I=\frac{1}{2}[(2+\varepsilon)I-2]+\frac{1}{2}(\frac{I}{1-I})[(2+\varepsilon)I-2-2\delta]. \tag{40}$$

Equation (40) is equivalent to

$$2(R-1)I^2 + 2(2-R)I + \rho I - 2 = 0 \tag{41}$$

where $\rho \equiv \varepsilon - 2\delta$. The positive solution to this equation is

$$I = \frac{1}{4\,(R-1)} \left(2(R-2) - \rho + \sqrt{8\rho + (2R-\rho)^2} \right). \tag{42}$$

It is easy to verify that this solution satisfies $\psi \in \widetilde{\Psi}$.]

Part 2. Denote the contract in the proposition by ψ^*. We need to show that ψ^* is renegotiation proof. According to Lemma 1 above, it is enough to show that ψ^* is a solution to the following problem: $\max_{\psi \in \Psi} \sum_{i=1}^{2} U_i(\psi)$ subject to

$$U_i(\psi) \geq \overline{U}_i(\psi + n_i\psi^*) - n_i\delta \text{ for } \; n_i \in [0, \frac{1-a_i}{a_i^*}], \text{ if } b_i > 0. \tag{43}$$

Similar to the derivation of equation (28), and using the fact that ψ^* is given by (a^*, b^*, I^*), equation (43) becomes

$$(R-1)I_i \geq \frac{1}{2}b_i + \frac{1}{2}(\frac{1-a_i}{a^*})(b^* - 2\delta). \tag{44}$$

Therefore, we obtain a linear programming problem. Since ψ^* satisfies equation (28), it also satisfies equation (44). To show that ψ^* is a solution to the linear programming problem, it is enough to show that ψ^* is the unique solution to $\max_{\psi} \sum_{i=1}^{2} U_i(\psi)$ subject to equations (44), (17), and (20). This follows because ψ^* is the unique contract that satisfies equations (44), (17), and (20) with equalities. [In more detail, equation (20) implies that $a_i = 1 - I_i$, and equation (17) implies that $b_i = I_i + (1+\varepsilon)I_{-i} - 2$. Equation (44) then implies that

$$[2(R-1) - m^* - 1]I_i = (1+\varepsilon)I_{-i} - 1, \;\; i = 1, 2 \tag{45}$$

where $m^* = \frac{b^*-2\delta}{a^*}$. Equation (45) implies that $I_1 = I_2 = I$, and it follows that

$$[(2 + m^* + \varepsilon - 2(R-1)]I = 1. \tag{46}$$

Since we know from equation (40) that I^* satisfies (46), I^* is the unique solution, and it follows that ψ^* is the unique solution.]

Part 3. Consider a symmetric contract $\psi = (a, b, I) \in \widetilde{\Psi}$ that is also renegotiation proof. Without loss of generality, $a = 1 - I$, and $b = (2 + \varepsilon)I - 2$. Part 1 of this proposition implies that $I \leq I_{tb}$. We need to show that we must have $I = I_{tb}$. Suppose not. Consider the problem $\max_{\psi' \in \Psi} \sum_{i=1}^{2} U_i(\psi')$ subject to equations (44), (17) and (5). Following same steps as in part (2), we obtain that the unique solution, denoted by ψ'^*, is symmetric and satisfies

$$[(2 + m(I) + \varepsilon - 2(R - 1)]I'^* = 1 \tag{47}$$

where $m(I) = \frac{b - 2\delta}{a}$. Since $I = I_{tb}$ implies that $I^{*\prime} = I_{tb}$, it follows that $I < I_{tb}$ implies that $I^{*\prime} > I_{tb}$. Therefore, ψ'^* satisfies equations (25) and (26). But then Lemma 1 implies that ψ is not renegotiation proof, which is a contradiction.

Q.E.D.

Proof of Corollary 1.

If $\psi_{tb} = \psi_{sb}$, the result follows from Proposition 1. Otherwise, the proof of Proposition 4 (in particular, equation (41)) implies that I_{tb} is the solution to $H(I, R, \delta) = 0$, where

$$H(I, R, \delta) = 2(R - 1)I^2 + 2(2 - R)I + \rho I - 2. \tag{48}$$

Note that

$$\frac{\partial H}{\partial I} = 4(R - 1)I + 2(2 - R) + \rho, \tag{49}$$

and since equation (42) implies that

$$4(R - 1)I = 2(R - 2) - \rho + \sqrt{8\rho + (2R - \rho)^2}, \tag{50}$$

we obtain

$$\frac{\partial H}{\partial I} = \sqrt{8\rho + (2R - \rho)^2} > 0. \tag{51}$$

In addition, $\frac{\partial H}{\partial R} = 2I(I - 2) < 0$, and $\frac{\partial H}{\partial \delta} < 0$. Therefore, $\frac{\partial I}{\partial R} = -\frac{\partial H/\partial R}{\partial H/\partial I} > 0$, and $\frac{\partial I}{\partial \delta} = -\frac{\partial H/\partial \delta}{\partial H/\partial I} > 0$. The result regarding collateral follows because the amount of collateral, a, satisfies $a = 1 - I$. Q.E.D.

Proof of Proposition 5. (1) Suppose the intermediary sets (L, ψ_{sb}). Consider a PBE in which all agents enter and report ψ_{sb}. After reporting one contract, an agent can enter at most $L-1$ additional contracts (and report all). To prevent this type of strategic default, L must satisfy the following incentive constraint

$$U_i(\psi_{sb}) - \theta \geq \overline{U}_i(L\psi_{sb}) - (L-1)\delta - L\theta. \tag{52}$$

This is equivalent to

$$U_i(\psi_{sb}) \geq \overline{U}_i(L\psi_{sb}) - (L-1)\delta - (L-1)\theta. \tag{53}$$

Using a similar logic as in Proposition 2, one can show that a PBE in which agents enter and report ψ_{sb} exists if and only if L satisfies equation (53); and using Proposition 1 and equations (4) and (5), one can show that L satisfies (53) if and only if $L \leq L^*$. [In more detail, when $R \geq 1 + \frac{1}{2}\varepsilon$, it follows that $I_{sb} = 1$, and L satisfies (53) if and only if

$$R \geq 1 + \frac{1}{2}L\varepsilon - (L-1)(\delta + \theta) \tag{54}$$

This is equivalent to

$$L \leq \frac{R - 1 - (\delta + \theta)}{\frac{1}{2}\varepsilon - (\delta + \theta)}. \tag{55}$$

When $R < 1 + \frac{1}{2}\varepsilon$, it follows that $I_{sb} = \frac{1}{2 + \frac{1}{2}\varepsilon - R}$,

$$\begin{aligned} b_{sb} &= (2+\varepsilon)I_{sb} - 2 \\ &= \frac{2+\varepsilon}{2 + \frac{1}{2}\varepsilon - R} - 2 = \frac{2 + \varepsilon - 2(2 + \frac{1}{2}\varepsilon - R)}{2 + \frac{1}{2}\varepsilon - R} \\ &= \frac{2(R-1)}{2 + \frac{1}{2}\varepsilon - R}, \end{aligned} \tag{56}$$

and L satisfies (53) if and only if

$$1 + (R-1)\frac{1}{2 + \frac{1}{2}\varepsilon - R} \geq 1 + \frac{1}{2}L[\frac{2(R-1)}{2 + \frac{1}{2}\varepsilon - R}] - (L-1)(\delta + \theta). \tag{57}$$

This is equivalent to

$$\frac{R-1}{2 + \frac{1}{2}\varepsilon - R} \geq \frac{L(R-1)}{2 + \frac{1}{2}\varepsilon - R} - (L-1)(\delta + \theta). \tag{58}$$

which holds only if $L = 1$.]

(2) Suppose the intermediary sets (L, ψ_{sb}). Consider a PBE in which all agents enter and report ψ_{sb}. Using a logic logic to the one in Lemma 1, one can show that entering ψ_{sb} and reporting it to the intermediary is renegotiation proof if and only if there does not exist a contract $\psi \in \Psi$ that satisfies the following two equations:

$$U_i(\psi) \geq U_i(\psi_{sb}) - \theta \tag{59}$$

$$U_i(\psi) \geq \overline{U}_i(\psi + L\psi_{sb}) - L\delta - L\theta. \tag{60}$$

Equation (59) says that by entering ψ without reporting each agent obtains a higher utility than the one obtained by entering ψ_{sb} and reporting. Equation (60) makes sure that if a pair of agents does not report, no agent has the incentive to enter additional contracts and default on all; since the first contract is not reported and all other agents report, an agent can enter at most L additional contracts, all of which are reported to the intermediary.

Suppose $L \leq L^* - 1$. Since equation (53) holds if and only if $L \leq L^*$, it follows that it holds for $L + 1$; in other words,

$$U_i(\psi_{sb}) \geq \overline{U}_i((L+1)\psi_{sb}) - L\delta - L\theta. \tag{61}$$

Thus, ψ_{sb} satisfies equations (59) and (60), and the PBE is not renegotiation proof.

Suppose now that $L = L^*$. Then since equation (53) holds if and only if $L \leq L^*$, it follows that it does not hold for $L + 1$; in other words,

$$U_i(\psi_{sb}) < \overline{U}_i((L+1)\psi_{sb}) - L\delta - L\theta. \tag{62}$$

Thus, ψ_{sb} does not satisfy equation (60), and the contract ψ^* that solves $\max_\psi \sum_{i=1}^2 U_i(\psi)$ subject to equations (59) and (60) is such that $U_i(\psi^*) < U_i(\psi_{sb})$. When θ is small enough, $U_i(\psi^*) < U_i(\psi_{sb}) - \theta$, and there does not exist a contract that satisfies (59) and (60). This means that the PBE is renegotiation proof.

(3) This follows from the definition of L^*.

Q.E.D.

Proof of Proposition 7. Consider the case $R > 1 + \varepsilon$. Then $L^* = 2$, and the second best equals the first best; that is, each agent invests one dollar and promises ε out of his project's cash flows. To show that for these parameters, a PBE in which all agents report ψ_{sb} to the intermediary is not renegotiation proof, it is enough to show that the extended game has a PBE whose outcome is that the randomly chosen pair of agents first decides on entering ψ_{sb} and reporting it to the intermediary, and then replaces it with entering ψ_{sb} without reporting.

Note that since an agent knows the number of contracts that his counterparty has already reported, an agent's strategy depends on that information. In equilibrium, every agent reports ψ_{sb} and then leaves the trading game; therefore, finding out that a counterparty has already reported a contract is an out-of-equilibrium event. Consider the following strategies and beliefs: An agent who learns that his counterparty has reported a contract believes that his counterparty has promised at least ε out of his project cash flows; the agent's action is to offer not to enter a contract. This strategy is optimal because the agent can stay for the next round and be matched with a counterparty who has a clean history, i.e., a counterparty who has not reported any contracts.

Now consider a pair of agent who is given another opportunity to negotiate. Suppose they enter ψ_{sb} without reporting. Given the equilibrium strategies, each member of this pair can deviate by entering at most one contract, not two as in the proof of Proposition 5. But since $L^* = 2$, an agent cannot gain by doing so. Therefore, entering ψ_{sb} without reporting is optimal.

If we add the restriction $\delta < \theta$, the proof works also if we change the negotiation game to one agent offers a contract and the other agent accepts or rejects. If an agent who finds out that his couterparty. has reported a contract believes that his counterparty has promised at least ε out of project cash flows, it makes sense to accept a contract only if a counterparty fully backs his promise with collateral; in this case, the contract is entered without reporting. But then a counterparty cannot gain by entering such a contract. In particular, since by entering a contract without reporting, an agent saves the fee θ, the counterparty can gain at most θ. But given the fixed cost per trade, the net gain is negative.

Finally, note that the proof works even if we apply a weaker definition as in footnote 26. In particular, since agents who report a contract must enter ψ_{sb} and since an agent cannot invest more than one dollar, the only possible beliefs are that an agent who sees that his counterparty reported a contract, believes that his counterparty has promised at least ε out of project cash flows, and therefore, the counterparty cannot promise more without default (unless everything is backed by collateral).

Q.E.D.

Lemma 2 *A PBE in which agents enter the same contract more than once is not renegotiation proof.*

Proof: Suppose the contract ψ is entered m times; without loss of generality, $a_i = 1 - I_i$, $b_{-i} = \varepsilon I_i - s$, and $I_1 = I_2 = I$. Each agent obtains $U(m\psi) - (m-1)\delta = R - (R-1)ma - (m-1)\delta$. In addition, ψ must satisfy the incentive constraint:

$$U(m\psi) - (m-1)\delta \geq \overline{U}(m\psi + n\psi) - (m-1)\delta - n\delta, \tag{63}$$

where $n = \frac{1-ma}{a}$. Then a pair of agents can do better by entering the contract $m\psi$. The incentive constraint is still satisfied, but the agents save $(m-1)\delta$. Q.E.D.

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