The Effects of Price Limits on Informed Trading and Market Efficiency

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Shu-fan Hsieh and Tai Ma

Department of Finance, National Sun Yat-sen University
Kaohsiung, Taiwan

Abstract

This paper investigates the effect of price limits on strategic informed trading and market performances. Ex ante effects of rule-based price limits will result in informed traders' large scaling back. We show that the duration of information plays an important role in determining informed traders' strategic behavior. With long-lived information, the rule based stabilizing mechanism encourages stealthily informed trading, distorts the price dynamics and increases the trading costs of small liquidity traders. Additionally, market performance is worsened in terms of less liquidity and higher volatility. Our findings suggest that the ex ante effects of price limits on market performances may be contrary to what the stabilizing mechanism is intended to achieve, especially when information is long-lived.

1. Introduction

Since the October 1987 stock market crash, the question whether the market is adequately stabilized by circuit breakers has often been addressed in both academics and practitioners. While the trading halt rule is a well-known mechanism in NYSE, the price limit rule, adopted in the U.S. futures market, is another type of stabilizing mechanism. Many other markets around the world also have price limits rule, including Austria, Belgium, France, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Spain, Switzerland, Taiwan and Thailand.

The price limits rule, a rule-based mechanism imposed on individual stocks, is a set of boundaries within which the security prices are allowed to move. Since trading usually stops when limit-hits occur, price limits are in a way similar to trading halts. The effectiveness of the price limit and trading halts is, however, still an ongoing debate. Proponents justify the price limit rule by stating that it can protect the markets from extreme volatility and protect liquidity traders from trading losses that they will likely incur in a poorly functioning market. Additionally, price limit or trading halt can provide a time-out period to let valued traders enter the market and provide liquidity (Greenwald and Stein, 1988, 1991; Kodres and O’Brien, 1994; Ma, Rao and Sears, 1989a; Corwin and Lipson, 2000; Christie, Corwin and Harris, 2002; Westerhoff, 2003). Exchanges, such as Taiwan Stock Exchanges and Tokyo Stock Exchanges, claim that they adopt the price limits rule primarily to maintain a stable stock market. For example, Taiwan Stock Market has 7 % price limits, which is the narrowest among stock markets in Asia. Many Asian stock markets have wider price limits, such as 10% in Thailand, 15% in Korea, 30% in Malaysia and 5%~50% in Japan. If the price limits are able to reduce the volatility, Taiwan Stock Market is supposed to have the lowest volatility. We perform a simple test by using the index returns from January 2000 to October 2007. The standard deviation of index returns is used as the proxy of the volatility. According to the results summarized in Table 1, we find that the daily return volatility of Taiwan Stock Market is significantly higher than other markets, except the Korean Stock Market. However, since the price limits are usually implemented under a daily basis, the volatility of daily return may be biased. We also calculate the volatility of monthly returns and find that the volatility of the Taiwan stock market remains high and become statistically indifference to the volatility of the Korea market. Therefore, our results suggest that narrow price limits may not make the market less volatile.

Opponents criticize trading halts and price limits by finding evidences to show that these exogenous restrictions can neither reduce volatility nor improve information dissemination (Lee, Ready and Seguin, 1994; Lehmann, 1989; Miller 1989; Ma, Rao and Sears, 1989b; Kuhn, Kurserk and Locke, 1991; Kim and Rhee, 1997; Lauterbach and Ben-Zion, 1993; Cho, Russell, Tiao and Tsay, 2003; Chan, Kim and Rhee, 2005). Fama(1989) claims that if the price discovery process is interfered with, underlying volatility may increase as a consequence. Lehmann (1989) reason the abnormal volatility that limits prevent the immediate corrections in order imbalance. In this paper, we analyze the effect of price limits on volatility from a perspective which has received less attention, that is, the informed traders' strategic behavior under the price limit.

Table 1 Volatility in Asian stock markets

Country	Index	Price limit	Volatility		
			Daily	Weekly	Monthly
Taiwan	TAIEX	7%	0.0155	0.0354	0.0750
Japan	Nikkei 225	5%~50%	0.0138**	0.0276*	0.0509**
Thailand	SET Index	10%	0.0107**	0.0234**	0.0506**
Korea	KOSPI 200	15%	0.0181**	0.0393*	0.0770
Malaysia	KLSE Index	30%	0.0092**	0.0223**	0.0497**
Hong Kong	Hang-Seng Index	N/A	0.0133**	0.0298	0.0573*
Singapore	SEX All-Share Index	N/A	0.0145*	0.0312	0.0727

Note: We use the standard deviation of index returns as the proxy of volatility. The volatilities are computed from January 2000 to October 2007 in order to avoid the Asian financial crisis. Levene's test is performed to test the equality of variances between TAIEX and other indexes.

The bulk of the related empirical research focuses on the effect of price limits on market performance, such as volatility and trading volume. However, little, if any, literature addresses the effect of price limits on traders' behavior and analyzes its impacts on market performances from this viewpoint. Subrahmanyam (1994, 1995) provides a strategic model that illustrates the magnetic effect of trading halts. He shows that abnormal volatility occurs before trading halts because traders would advance their orders ahead of trading halts in case they can not trade when trading stops. In another paper of 1997, Subrahmanyam provides a theoretical model to illustrate the informed traders' strategic behaviors when the trading halt is imposed. Informed traders would scale back by trading small orders in response to the trading halt. He points out that the ex ante effects of trading halts widen the spread of small orders and hurt liquidity traders.[1] However, Subrahmanyam's model is a one-shot

[1] Kim and Sweeney (2002) also have similar arguments. They conjecture that informed trading does

model and only allows two types of order-size. Restricted by price limits or trading halts, informed traders may not be able to trade with very large quantity which is their first choice. Under this situation, they may choose to trade with second large quantity which may drive the price up to the limits or choose to trade much less aggressively. Their strategic behaviors should be determined by the expected profits and the characteristic of the private information. Furthermore, rule-based price limits affect market performances through traders' strategic behavior.

In this paper, we analyze the strategic informed trading behavior with price limits by extending the models of Easley and O'Hara (1987) and Subrahmanyam (1997) regarding the types of orders and trading periods. This paper is distinguished from Subrahmanyam (1997) by discussing the effects of the type of information on the informed traders' strategic behavior and the impact of price limits after the imposition of the price limit. Our findings suggest that the horizon of private information plays an important role in informed traders' behaviors. Informed traders strategically switch from trading with large quantities to smaller quantities when facing the imposition of price limits. When the price limits restrain informed traders from trading big quantities, they may evaluate alternative strategies, choosing to trade either aggressively or stealthily, i.e. trade as much as they can or trade with small orders, depending on the expected profits and the horizon of the information. We find informed traders do not necessarily trade as much as they can, i.e. driving the price up to the price limit. If the information is long-lived enough, ex ante effects of rule-based price limits will result in informed traders' large scaling back, i.e. trading with small quantities and, thus, the increase of the trading costs of small liquidity traders. Therefore, the characteristics of information should be taken into account when the price limits are to be adopted. This viewpoint is also raised in Kim and Sweeney (2000). In their paper, they argue that price limits may induce an informed trader to shift part or all of her profit-motivated trades until the next day. Specifically, if the current price is near, but the equilibrium price is substantially beyond, the limit, and if the information is long-lived enough, informed traders may delay their trade from one day until the next. However, Kim and Sweeney did not model the horizon of information formally. By the model we present here, we can further investigate the ex ante effects of price limits on price paths, market liquidity and volatility.

We examine the effects of the strategically informed trading on the price dynamics and market performance. In the case with long-lived information and the imposition

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not take place during trading sessions when limit-hits occur because rational expectation prices cannot be realized. Informed traders may wait for subsequent trading sessions when price limits have been revised.

of price limits, strategically informed trading will induce wider spreads and higher volatility in the following trading period, contradicting to the objectives that price limits are supposed to achieve. Our results are in line with the empirically evidence that there is an increase in volatility after price limit hits. For example, Kim (1997), by examining the daily data of Tokyo Stock Exchange, finds that the price limit causes the volatility to spill over to subsequence trading days. He justifies the volatility spillover by stating that the price limit prevents immediate correlation in order imbalance. Here we provide another possible explanation: the abnormal volatility after the imposition of the price limit may be caused by the strategic informed trading. Because of informed traders trading small orders to react to the price limit restriction, the following volatility becomes bigger than usual as a consequence. Additionally, in markets with higher information asymmetry and more informed traders, the negative effects of the price limits rule on liquidity and volatility can be more severe.

2. Model Setting and Separating Equilibrium

We extend the models in Subrahmanyam (1997) and Easley and O'Hara (1987) by allowing three trade sizes and two rounds of trade. The economy consists of a mass of μ informed traders, $1-\mu$ uninformed traders, and one competitive, risk neutral market maker. In each trading period, a market maker executes exactly one order which can arise from either an uninformed trader or an informed trader. A risky asset represented by the random variable V is traded in the market. After trade is complete, the asset pays off either $\overline{V}$ or $\underline{V}$ with $\overline{V} > \underline{V}$. The probability that the payoff is high ($\overline{V}$) is δ, the probability that it is low ($\underline{V}$) is 1-δ, and thus the unconditional mean of the asset's value is $V^* = \delta\overline{V} + (1-\delta)\underline{V}$. The probability of informational event occurring and thus revealing $\overline{V}$ or $\underline{V}$ to informed traders is α. Before the event occurring, there is no adverse selection and the price is V^*. The state values of the asset in period 3 can be written by:

$$\text{V}=\begin{cases} \overline{V} & \text{Prob} = \alpha\delta \\ V^* & \text{Prob} = 1-\alpha \\ \underline{V} & \text{Prob} = \alpha(1-\delta) \end{cases}$$

Because of the different liquidity demands, uninformed traders wish to buy or sell with 3 different quantities: B^b, B^m or B^s (S^b, S^m or S^s), with $0 < B^s < B^m < B^b$ ($0 < S^s < S^m < S^b$), where a superscript denotes the size of orders: bid, medium and small. Transactions costs and risk aversion can be reasons to explain why uninformed traders prefer to trade big or median quantities. We define X_B^b, X_B^m *and* X_B^s (X_S^b, X_S^m *and* X_S^s) as the fraction of uninformed traders who want to trade B^b, B^m and B^s (S^b, S^m *and* S^s). Because liquidity traders are usually individual traders, it is reasonable to assume the possibility of trading small quantities is higher than median quantities, which in turn is greater than large quantities. There exist risk neutral informed traders who aim to maximize profits in the market. They mimic uninformed traders to trade the quantities among B^b, B^m and B^s (S^b, S^m *and* S^s) in order not to be identified by market maker.

A risky asset is traded in period 1 and 2, and pays off in period 3. Following Kaniel and Liu (2006), we assume the horizon of information is random. Before the initial trading date, informed traders learn the value of the asset. This private information will be revealed to the public at a random future time, implying a random horizon for the information. With probability γ, with $0 \le \gamma \le 1$, the information is short lived and will be revealed to the market by the end of the first trading period. If $\gamma = 1$, there is no informed trading at t=2. In addition, this model is a sequential model, so the equilibrium price in the second period reflects the information contained in the first and second trade. $a_{b,1}$, $a_{m,1}$ and $a_{s,1}$ denote the ask prices for bid, medium and small quantities at t=1, while $a_{b,2}(Q_1)$, $a_{m,s}(Q_1)$ and $a_{s,s}(Q_1)$ denote the ask prices at t=2 with Q_1 traded at t=1. Accordingly, an informed trader maximizes his expected profits taking into account his effects in both periods. Put differently, informed traders decide the strategies by considering the combined profits in both periods.

Easley and O’Hara (1987) present two equilibriums: separating equilibrium and pooling equilibrium. In a separating equilibrium informed traders trade only big quantities. In a pooling equilibrium there is a positive probability of the informed trading in both big and small orders. In our model, a trader arrives and wishes to trade for quantities B^b, B^m, B^s, S^b, S^m or S^s. In this market, three forms of equilibrium can occur: (1) a separating equilibrium with informed traders trading only big quantities, (2) a hybrid equilibrium in which there is a positive probability of the informed trading in both big and medium quantities but not trading in small quantities, (3) a pooling equilibrium in which there is a positive probability of the informed trading for one of there order quantities.[2] The following analyses focus on buy side. The results of sell side are similar.

We follow the equilibrium concept adopted by Subrahmanyam (1997) in which informed traders act as Stackelberg leaders and market makers behave as followers. Specifically, the market maker changes his pricing strategies in response to the deviation of informed traders. The informed trader knows ex ante that the market maker observes his action, so he takes the responses of the market maker into account while he chooses his strategies.

2.1 Separating Equilibrium

First consider a market in separating equilibrium. In the case with very short-lived information, i.e. $\gamma = 1$, an informed trader choose to trade with big quantities only at t=1. The market maker update $\delta_1(Q_1)$ and set the ask price by zero profit condition. The ask price if an informed trader trades B^b is given by

$$a_{1,b} = E[V \mid B^b] = V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^b} \tag{1}$$

Because of the market maker acting as Stackelberg follower, he sets the ask prices if the informed traders trade B^m and B^s given by

$$a_{1,m} = E[V \mid B^m] = V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^m} \tag{2}$$

$$a_{1,s} = E[V \mid B^s] = V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^s} \tag{3}$$

Therefore, for a trader informed of $\overline{V}$, we will have a separating equilibrium if and only if the profit to an informed trader is no lower trading B^b than trading B^m and B^s, or

$$B^b(\overline{V} - a_{1,b}) \geq B^m(\overline{V} - a_{1,m}) \quad \text{and} \quad B^b(\overline{V} - a_{1,b}) \geq B^s(\overline{V} - a_{1,s}) \tag{4}$$

In the case with longer lived information, informed trading happens in both trading periods. We start with the analysis at t=2. Given any order size at t = 1, we will have a separating equilibrium at t = 2 if and only if

$$B^b(\overline{V} - a_{b,2}(Q_1)) \geq B^k(\overline{V} - a_{k,2}(Q_1)), \qquad k = m, s \tag{5}$$

[2] We denote the second equilibrium as a hybrid one because it consists of both features of separating and pooling equilibrium.

A market maker determines the price at t=2 by incorporate information learned from previous trade and present trade into their pricing strategy. Again, he updates $\delta_2(Q_1, Q_2)$ and set the ask price by zero profit condition. The ask prices at t=2 are given by

$$a_{b,2}(B^b) = E[V \mid B^b, B^b]$$

$$= V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu[\mu + 2(1-\mu)X_B^b]}{\alpha\delta\mu^2 + 2\alpha\delta\mu(1-\mu)X_B^b + [1+\alpha(-2+\mu)\mu](X_B^b)^2} \quad (6)$$

$$a_{b,2}(B^m) = E[V \mid B^m, B^b]$$

$$= V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu[\mu + (1-\mu)(X_B^b + X_B^m)]}{\alpha\delta\mu^2 + \alpha\delta\mu(1-\mu)(X_B^b + X_B^m) + [1+\alpha(-2+\mu)\mu](X_B^b X_B^m)} \quad (7)$$

$$a_{b,2}(B^s) = E[V \mid B^s, B^b]$$

$$= V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu[\mu + (1-\mu)(X_B^b + X_B^s)]}{\alpha\delta\mu^2 + \alpha\delta\mu(1-\mu)(X_B^b + X_B^s) + [1+\alpha(-2+\mu)\mu](X_B^b X_B^s)} \quad (8)$$

The equation above denotes multiple conditions where Q_1 is an order received in the first order. We can easily prove that when the condition with $X = B_1^b$ is satisfied, the other two conditions will be satisfied simultaneously no matter what kind of equilibrium in the first period. Therefore, the condition can be simplified to

$$B^b(\overline{V} - a_{b,2}(B^b)) \geq B^k(\overline{V} - a_{k,2}(B^b)), \quad k = m, s \quad (9)$$

Then we back to the equilibrium at t=1. To determine the strategies of informed traders, the combined profits to informed traders should be considered. The market at t=1 is in a separating equilibrium with the informed trading for big quantities if the combined profit to an informed trader from a small order or a medium order is smaller than that from a big order, i.e.,

$$(\overline{V} - a_{1,b})B^b + (1-\gamma)[\overline{V} - a_{2,b}(B^b)]B^b > (\overline{V} - a_{1,k})B^k + (1-\gamma)[\overline{V} - a_{2,b}(B^k)]B^b, \quad k = m, s \quad (10)$$

Substituting $a_{1,b}, a_{1,s}$ and $a_{2,b}$ from (1), (3) and (6)-(8) leads us to the following proposition.

***Proposition 1**: Consider the game where the informed trader acts as a Stackelberg leader and the market maker acts as a Stackelberg follower. In separating equilibriums without the price limit, informed traders prefer to trade large quantities* $\{B_1^b, B_2^b\}$ *in both trading periods if and only if*

$$\frac{B^b}{B^k} \geq 1 + \frac{\alpha\delta\mu[\mu(1-\alpha\mu) + (1-\mu(2-\alpha\mu))X_B^b]}{[1-\alpha\mu(2-\mu)][\alpha\delta\mu + (1-\alpha\mu)X_B^b]X_B^b}, \quad k = m, s \quad (11)$$

and

$$\frac{B^b}{B^k} \geq \frac{(1-\alpha\mu)X_B^k}{X_B^b(\alpha\delta\mu+(1-\alpha\mu)X_B^k)} * \tag{12}$$

$$\left\{ \begin{array}{l} \dfrac{1-\alpha\mu}{(\alpha\delta\mu+(1-\alpha\mu)X_B^b)} + \\ (1-\gamma) * \dfrac{\alpha\delta\mu(\mu+(1-\mu)X_B^b)}{[X_B^b(J+GX_B^k)+\alpha\delta\mu(\mu+(1-\mu)X_B^k)]} * \dfrac{G(X_B^b - X_B^k)}{(\alpha\delta\mu^2+2JX_B^b+G(X_B^b)^2)} \end{array} \right\}^{-1}$$

where

$G = (1+\alpha(-2+\mu)\mu);\ J = \alpha\delta\mu(1-\mu);\ k = m, s$

are satisfied.

2.2 Price Limit

The price limit implemented in many exchanges is a rule-based mechanism which does not allow the trade happening outside an exogenous bound. The sequential trade model of Easley and O'Hara (1987) characterizes the behavior of security prices when all agents act competitively. However, after price limits are imposed, the informed traders have an incentive to act strategically to maximize his profits.

The bounds of the price limit rule are usually determined by a fixed percentage of previous closing price, and they will be revised in the next trading period, which is usually the next trading day or trading session. We denoted the upper bound of the price as $\overline{a}_t, t = 1,2$. If the unconditional mean of asset value V^* is the equilibrium price before trading, $\overline{a}_1$ equals to $V^*(1+i)$, where i is the percentage of the price limit. And $\overline{a}_2$, computed as $a_1(1+i)$, varies with the trading price at t=1. For simplifying, we assume that the limit i is within some boundary so that $\overline{a}_1$ is smaller than $\overline{V}$ and $\overline{a}_2$ is not less than $\overline{V}$. In other word, the constraint of the price limit is binding at t=1 but not binding at t=2. This assumption allows us to analyze the market in which the constraint is widen and no longer binding. Trades are allowed at or within the bounds. In the model with only one trade in every period, price limits operate in a manner identical to market closure. If the equilibrium bid and ask prices without a limit cross the exogenous bounds, the market maker will refuse to execute this trade because he can not post a price outside the limit. Moreover, if the market is not open at t=1, the bounds of the price at t=2 is not changed and thus, the market at t=2 will not open either.

Now, we proceed to analyze the strategies of informed traders under the price limits. After the price limit is imposed, an informed trader considers strategically by taking into account the possibility of the price across the bound. For example, if he knows a big order pushes the price across the upper bound, he will switch to smaller quantities in response to the institution. Our model with three types of orders allows as analyzing informed traders' strategic behavior after the imposition of the price limit. Assume that a medium order drives the price up to the upper bound while a small order does not. An informed trader can switch to trade with either medium or small quantities to maximize his expected profits. The expected profits from trading with medium quantities is given by

$$\pi_{\{B^m,B^b\}} = (\overline{V} - a_{m,1})B^m + (1-\gamma)(\overline{V} - a_{b,2}(B^m))B^b \tag{13}$$

while the expected profits from trading with the small quantities is given by

$$\pi_{\{B^s,B^b\}} = (\overline{V} - a_{s,1})B^s + (1-\gamma)(\overline{V} - a_{b,2}(B^s))B^b \tag{14}$$

When $\pi_{\{B^m,B^b\}}$ is greater than $\pi_{\{B^s,B^b\}}$, informed traders will switch his trade size to medium in response to the price limit. Comparing with Subrahmanyam (1997), our model suggests that the informed traders will scale back in response to the price limit, but the magnitude of scaling back depends on the horizon of information. Intuitively, if the probability of long-lived information is very high, the informed traders may switch to trade small quantities after the price limit is imposed. However, if the information is very short-lived, an informed trader has a bad chance of making any profit in the second period. Thus he will trade as much as he can trade to ensure his profits. This argument can be stated formally as follows.

Proposition 2*: Consider the game where the informed trader acts as a Stackelberg leader and the market maker acts as a Stackelberg follower. Suppose that, without the price limit, (11) and (12) hold, so that the market is in separating equilibriums with the informed trading large quantities in two periods. Then the following statements hold.*

1. If $V^* + \dfrac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^b} < \overline{a}_1$*, the market never closes.*

2. If $V^* + \dfrac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^s} > \overline{a}_1$*, the market never opens.*

3. If
$$V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^s} \le \overline{a}_1 < V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^m} \tag{15}$$

then, after imposition of the price limit, the strategy traded by the informed trader *in equilibrium switches to trading with a small order first and with a large order sequentially, or* $\{B_1^s, B_2^b\}$. *Then the price impact for large orders and small orders will be changed.*

4. *If* $V^* + \dfrac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^m} \le \overline{a}_1 < V^* + \dfrac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^b}$ (16)

then, after imposition of the price limit, informed trading depends on the horizon of information:

(a) the strategy traded by the informed trader in equilibrium switches to trading with a medium order if

$$\gamma > 1 + \frac{[B_1^m(\overline{V} - a_{m,1}) - B_1^s(\overline{V} - a_{s,1})]}{B_2^b[(\overline{V} - a_{b,2}(B_1^m)) - (\overline{V} - a_{b,2}(B_1^s))]}$$

(b) the strategy traded by the informed trader in equilibrium switches to trading with a small order if

$$\gamma < 1 + \frac{[B_1^m(\overline{V} - a_{m,1}) - B_1^s(\overline{V} - a_{s,1})]}{B_2^b[(\overline{V} - a_{b,2}(B_1^m)) - (\overline{V} - a_{b,2}(B_1^s))]} \quad (17)$$

We give a numerical example to illustrate the effects of price limits. Consider the following parameter values: $\overline{V} = 10$, $\underline{V} = 0$, $\alpha = 1$, $\delta = 0.7$, $\mu = 0.4$, $X_B^s = 0.9$, $X_B^m = 0.2$, $X_B^b = 0.1$, $B_1^b = B_2^b = 500$, $B_1^m = 50$, $B_1^s = 10$. Then it is easy to find that $a_{b,1} = 9.47$, $a_{m,1} = 9.1$, $a_{s,1} = 8.02$, $a_{b,2}(B_1^b) = 9.93$, $a_{b,2}(B_1^m) = 9.87$, $a_{b,2}(B_1^s) = 9.69$ and RHS of (17) equals to 0.725. If $\overline{a}_1$ is greater than 9.47, then the market never closes, while if $\overline{a}_1$ is smaller than 8.02, the market never opens. If $\overline{a}_1$ is between 9.1 and 8.02, after imposition of the price limit, the quantity traded by the informed trader in equilibrium becomes B_1^s. If $\overline{a}_1$ is between 9.47 and 9.10, the informed trader switches to trade small quantity when γ is smaller than 0.725. For example, if γ is 0.5, the combined expected profit of medium quantity (76.84) is smaller than that of small quantity (97.54).

The effects of price limits are addressed in Proposition 2. There are four different cases. If the price limits are smaller than the price impact of small orders, the market always remains shut for all orders. If equation (15) is satisfied, after imposition of the price limit, then the price impact of big orders goes to zero and the price impact of small orders increase. Since the model is symmetry, the spread of small orders is the double of the difference between ask price and V^*. As suggested in Subrahmanyam

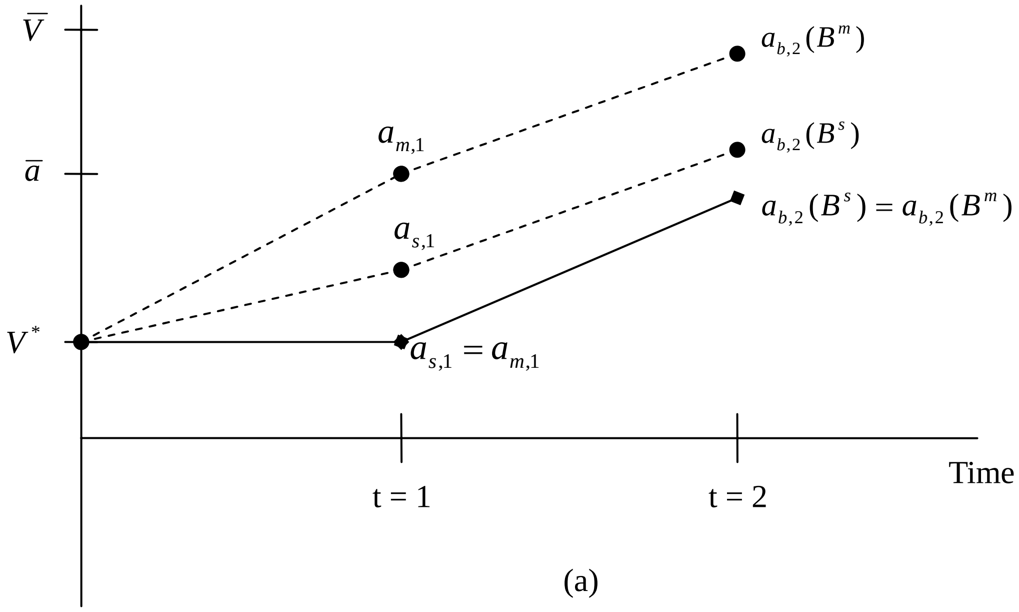
(1997), the price limit results in an increase in the spread for small quantities which is usually used by liquidity traders. In other word, the price limit hurt liquidity trades by increasing their trading cost, which is exactly the opposite of what the rule is supposed to accomplish. If the price limit is wider and satisfies equation (23), the informed trader switches to trade with either medium or small orders. If he considers only the expected profits in the first period, he may choose the medium quantity to acquire as many profits as they can. However, if he evaluates his alternative strategy by taking into account his effects in both periods, he may choose to trade with small orders. An informed trader’s strategy depends on the characteristic of information.

We denote RHS of (17) as γ^*, which is a crucial value below which informed traders switch to trade with small orders and above which they switch to trade with medium orders. γ^* is increasing with the expected profits of the second period, while γ^* is decreasing with the expected profits of the first period from trading with a medium order versus a small order. Accordingly, if so much information leaks after the first trade that trading in the second period is not profitable, or if it is more profitable to make as many profits as possible at t=1, a informed trader submits a medium order to maximize his profit. Therefore, the price limit i will increase the spread of medium orders and decrease the spread of big orders. Contrarily, if the amount of information that leaks after the first trade is not excessive, i.e. the information is long-lived, or if trades with medium quantities in the first period reveal so much information that trading tomorrow is not profitable, the informed trader will choose to trade with small quantity to make sure larger profits from the trade at t=2. In this case, the price impact of a small order is raised because of the limit.

Different from Subrahmanyam (1997), our model suggests that the horizon of information plays a role on informed trading. The price limit may encourage the stealthily informed trading. If the information is long-lived enough, the informed trader turn to trade with small orders and thus increase the trading costs of small liquidity traders. Therefore, the characteristics of information should be taken into account when the price limits are to be adopted.

3. The Effects of Price Limits on Market Performances

After the imposition of the price limit, informed trading strategies will be changed, which in turns change market performances in both trading periods. In this section the effects of price limits on price path and volatility are examined. Given the level of the price limit and the horizon of information, informed traders switch to trade with either medium or small quantities instead of big quantities. Since the market maker responses to strategic informed trading, the spreads and price path are changed. Consider a market in which informed traders separate at the bid quantity in both periods. They switch to trade with medium and small quantities under certain conditions stated in proposition 3. Figure 3 illustrate effects of the price limit on price paths and spreads. Panel (a) of Fig. 2 depicts price paths for a small/medium order and then a large order, while panel (b) depicts price paths for a small/medium order and then a small/medium order. The solid line represents the price path in normal case (separating equilibrium), while the dotted line represents the price path when informed traders switch to trading with small and medium quantities. Since the bid and ask side markets are symmetric in this model, only the ask side of market is analyzed here. In normal case, a small/medium order submitted in the first period contains no information and thus the spread equals to zero. However, after the price limits are imposed, because informed traders act strategically and the market maker act as Stackelberg follower, the spreads of small/medium quantities are widened in the first period. Price limits make small/medium orders contain more information than usual. This conforms to the argument in Kim and Sweeney (2000, unpublished paper): those stocks near but not at their limits may contain crucial information on the price-limit effects. They argue that because price limits may induce an informed trader to shift part or all of her profit-motivated trades until the next day, with price limit the stock which is close to limits may contain more information. Furthermore, given small orders in previous period, spreads of both big and small quantities in the second period are increased after the imposition of price limits. Put differently, the price limit widens the spreads in the following period and thus hurt the market liquidity.



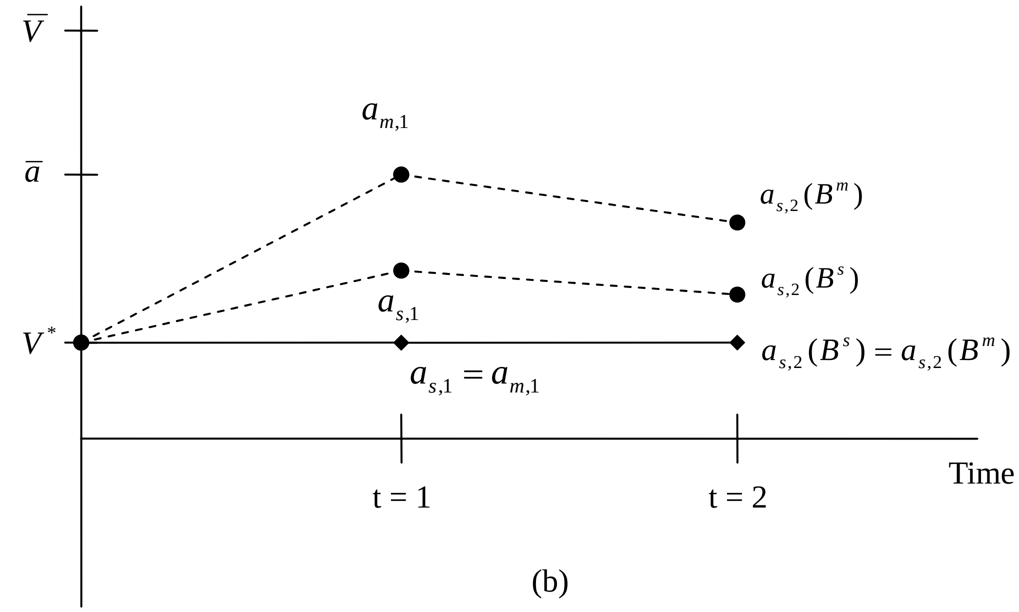


Figure 2. The figures depict price paths for (a) small/medium buy order, big buy order (b) small/medium buy order, small buy order. The time path of market maker quotes and transaction prices in a separating equilibrium with long-lived private information. The solid line represents the price path in normal case, while the dotted line represents the price path when informed traders switch to trading small quantities.

Volatility will also be influenced by price limits. Fama (1989) suggests that the interference of process of price discovery may result in the increase of volatility. The volatility spillover argument indicates that rather than reducing volatility, the price limit may cause volatility to spread out to several days after because it prevent the large one-day price changes and may transfer transactions to subsequent days. Kim and Rhee (1997) test the argument by examining the samples from the Tokyo Stock Exchange and they find abnormal volatility one day after limit-hit. Here we proceed to examine effects of the price limit on price volatility by comparing the expected price changes in the cases with and without price limits. Only ask side price changes are analyzed as before. Since there are three types of orders in our model, nine combinations of orders need to be examined. In the second period, the volatility is given by,

$$Volatility = E[|\Delta a_2|] = E[|a_2 - a_1|]$$

$$= \begin{pmatrix} |a_{2,b}(B^b) - a_{1,b}| * \Pr\{B^b, B^b\} + |a_{2,b}(B^m) - a_{1,m}| * \Pr\{B^m, B^b\} + |a_{2,b}(B^s) - a_{1,s}| * \Pr\{B^s, B^b\} \\ + |a_{2,m}(B^b) - a_{1,b}| * \Pr\{B^b, B^m\} + |a_{2,m}(B^m) - a_{1,m}| * \Pr\{B^m, B^m\} + |a_{2,m}(B^s) - a_{1,s}| * \Pr\{B^s, B^m\} \\ + |a_{2,s}(B^b) - a_{1,b}| * \Pr\{B^b, B^s\} + |a_{2,s}(B^m) - a_{1,m}| * \Pr\{B^m, B^s\} + |a_{2,s}(B^s) - a_{1,s}| * \Pr\{B^s, B^s\} \end{pmatrix} \quad (18)$$

After imposition of price limits, informed traders change to trade with either small or medium quantities, and thus the volatility is changed as well. We calculate the volatility in two cases, with and without price limits' imposition. In the case without the imposition of the price limit, there are two sub-cases: one is a transfer to medium quantity, and the other is a transfer to small quantity. Since the results are similar, here we present only the result of small quantity. $DVolatility_k$ represents the difference of two volatilities, which is determined by market parameters and is given by

$$DVolatility_k = Volatility^{\substack{\text{with} \\ \text{limit}}} - Volatility_k^{\substack{\text{without} \\ \text{limit}}} = f(\alpha, \delta, \mu, X_B^b, X_B^m, X_B^s) \quad (19)$$

where $k = s, m$ and $0 \le \alpha, \delta, \mu, X_B^b, X_B^m, X_B^s \le 1$. Specifically, $DVolatility_m$ ($DVolatility_s$) denotes the difference of the volatilities of the case in which informed traders switch to trade with medium (small) quantities. By numeric analyses, we find that $DVolatility_k$ is always positive and $DVolatility_s$ is greater than $DVolatility_m$. Our results suggest that the imposition of price limit rules will increase the price volatility. This result confirms the findings of Lehmann (1989), Kuhn, Kurserk, and Locke (1991), and Kim and Rhee (1997) that the price limit may be ineffective to reduce the volatility. Moreover, our finding suggests another explanation for the increasing volatility: abnormal volatility results from the ex ante effects of the price limit on informed traders' trading behaviors. Because informed traders trading small orders to correspond to the restriction and market maker responses to their first-moved actions, the volatility in the following period becomes bigger than it usual does. Furthermore, if the information is long-lived, the increase of volatility is even worse.

Additionally, we also investigate how information asymmetry is related the volatility caused by price limits. In our model, the distribution of private information are controlled by parameters, α and δ. Figure 3 depicts the relation between α, δ and $DVolatility_s$. The result of $DVolatility_m$ are similar. The highest volatility occurs when α approaches 1 and δ approaches 0.5. Recall $(\overline{V}-\underline{V})^2\alpha\delta(1-\delta)$ is variance of private information. Given $\overline{V}$ and $\underline{V}$, the case with $\alpha=1$ and $\delta=0.5$ has the largest variance of private information. The variance of private information can be a measure of information asymmetry. Therefore, these results indicate that information asymmetry aggravates the volatility caused by the price limit. Moreover, as shown in figure 4, $DVolatility_s$ increases in μ monotonically. This result suggests that the higher fraction of informed trader, the higher price volatility caused by imposition of price limits.

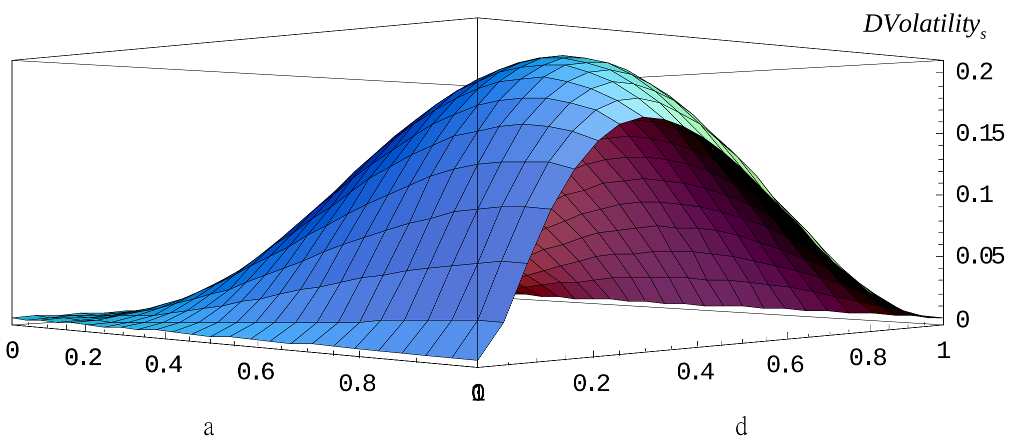


Figure 3. The figures depicts the relations between $DVolatility_s$ and α, where $DVolatility_s$ represents the increase of price volatility because of the imposition of price limit rules, α is the probability of informational event occurring and δ is the probability that the payoff is high. We draw the relation between three of them by setting $\overline{V}=10$, $\underline{V}=0$, $\mu=0.5$, $X_B^b=0.1$, $X_B^m=0.3$, $X_B^s=0.6$.

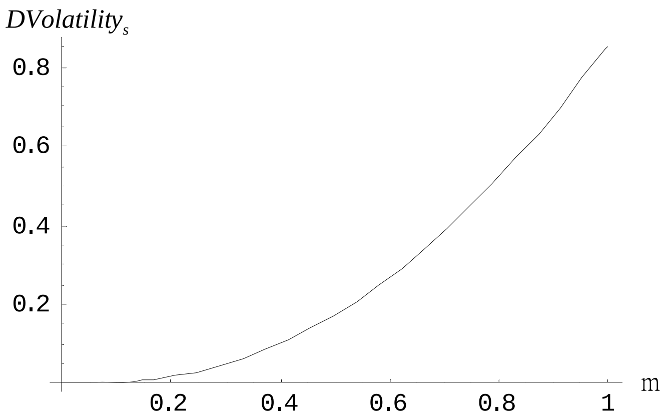


Figure 4. The figures depicts the relations between $DVolatility_s$, μ, where $DVolatility_s$ represents the increase of price volatility because of the imposition of price limit rules and μ is the fraction of informed traders. We draw the relation between $DVolatility_s$ and μ by setting $\overline{V}=10$, $\underline{V}=0$, $\alpha=0.5$, $\delta=0.5$, $X_B^b=0.1$, $X_B^m=0.3$, $X_B^s=0.6$.

Many exchanges adopting price limits aim to reduce price volatility. However, according to our findings, the imposition of price limits will induce the wider spreads and higher volatility in the following period. These effect results from the strategic informed trading which caused by price limit rules. Therefore, the ex ante effects of strategically informed trading prevent the price limit rule from achieving its goals. In additional to the possible failure of this institution, our results provide the following empirical implications: (1) the probability of limit-hits is related to the duration of information; (2) under long-lived information, less aggressively informed trading cause the higher volatility spillover in the following period; (3) with the price limit, those stocks near but not at their limits may be more volatile in the next period than those at their limits. In many empirical studies of the efficiency of price limits, the stocks near but not at their limits are chosen as the matching samples. However, according to the analyses here, this procedure may be confused because it is hard to identify whether these samples contain crucial information of price limits.

4. Other Equilibriums

4.1 Hybrid Equilibrium

In our model, there may exist other equilibriums. One of them is a hybrid equilibrium in which the informed trader is indifferent between trading big and medium orders and does not trade small orders in both trading periods. We also start with the market at t=2. Given a trade at t=1, there exist a hybrid equilibrium at t=2 if

$$B^b(\overline{V}-a^h_{b,2}(X)) = B^m(\overline{V}-a^h_{m,2}(X)) \geq B^s(\overline{V}-a_{s,2}(X)) \tag{20}$$

where $a^h_{b,2}(Q_1), a^h_{m,2}(Q_1)$ and $a^h_{s,2}(Q_1)$ represent the ask prices at t = 2.

If the market at t=1 is also in hybrid equilibrium, the similar condition is required, i.e.

$$B^b(\overline{V}-a^h_{b,1}) = B^m(\overline{V}-a^h_{m,1}) \geq B^s(\overline{V}-a_{s,1}) \tag{21}$$

where $a^h_{b,1}, a^h_{m,1}$ and $a_{s,1}$ represent the ask prices at t = 1

Let ψ_t, t = 1, 2, be the probability that a trader informed of $\overline{V}$ trades the large quantities. Using the Bayes' theorem and zero profit condition, ask prices in the first period are given by,

$$a^h_{b,1} = E[V \mid B^b] = V^* + \frac{(\overline{V}-\underline{V})(1-\delta)\alpha\delta\mu\psi_1}{\alpha\delta\mu\psi_1 + (1-\alpha\mu)X^b_B} \tag{22}$$

$$a^h_{m,1} = E[V \mid B^m] = V^* + \frac{(\overline{V}-\underline{V})(1-\delta)\alpha\delta\mu(1-\psi_1)}{\alpha\delta\mu(1-\psi_1) + (1-\alpha\mu)X^m_B} \tag{23}$$

while the ask prices in the second period depending on the order received in the previous period. For example, $a^h_{b,2}(B^b)$ and $a^h_{b,2}(B^m)$ are given by,

$$a^h_{b,2}(B^b) = E[V \mid B^b, B^b]$$
$$= V^* + \frac{(\overline{V}-\underline{V})(1-\delta)\alpha\delta\mu[\mu\psi_1\psi_2 + (1-\mu)(\psi_1+\psi_2)X^b_B]}{\alpha\delta\mu^2\psi_1\psi_2 + \alpha\delta\mu(1-\mu)(\psi_1+\psi_2)X^b_B + [1+\alpha(-2+\mu)\mu](X^b_B)^2} \tag{24}$$

$$a^h_{b,2}(B^m) = E[V \mid B^m, B^b]$$
$$= V^* + \frac{(\overline{V}-\underline{V})(1-\delta)\alpha\delta\mu[\mu(1-\psi_1)\psi_2 + (1-\mu)((1-\psi_1)+\psi_2)X^b_B]}{\alpha\delta\mu^2(1-\psi_1)\psi_2 + \alpha\delta\mu(1-\mu)((1-\psi_1)+\psi_2)X^b_B + [1+\alpha(-2+\mu)\mu](X^b_B)^2} \tag{25}$$

$$a^h_{b,2}(B^s) = E[V \mid B^s, B^b]$$
$$= V^* + \frac{(\overline{V}-\underline{V})(1-\delta)\alpha\delta\mu[(1-\mu)X^b_B + (\mu+(1-\mu)X^s_B)\psi_2)]}{\alpha\delta\mu(1-\mu)X^b_B + \alpha\delta\mu(\mu+(1-\mu)X^s_B)\psi_2 + [1+\alpha(-2+\mu)\mu]X^s_B} \tag{26}$$

To ensure the informed trading with big or medium but not small quantities, the deviation for trading with small quantities at t=1 should be less profitable, for instance,

$$(\overline{V}-a^h_{1,b})B^b + (1-\gamma)[\overline{V}-a^h_{2,b}(B^b)]B^b > (\overline{V}-a_{1,s})B^s + (1-\gamma)[\overline{V}-a^h_{2,b}(B^s)]B^b \tag{27}$$

The hybrid equilibrium will exist if it is possible to choose ψ_t, t = 1, 2, between 0 and 1 such that (20), (21) and (27) are satisfied. This can be formalized in the following proposition.

***Proposition 3**: Consider the game where the informed trader acts as a Stackelberg leader and the market maker acts as a Stackelberg follower. There exist hybrid equilibriums without the price limit if one can choose ψ_t, t = 1, 2, between 0 and 1 such that (28), (29) and zero-profit condition for the market maker are satisfied. It is possible to do so if*

$$\frac{B^b}{B^m} < \frac{\overline{V} - a_{m,2}(Q_1)}{\overline{V} - a_{b,2}(Q_1)}, \frac{B^b}{B^s} > \frac{\overline{V} - a_{s,2}(Q_1)}{\overline{V} - a^h_{b,2}(Q_1)} \qquad (28)$$

and

$$\frac{B^b}{B^s} > \frac{\overline{V} - a_{s,1}}{(\overline{V} - a^h_{1,b}) - [a^h_{2,b}(B^b) - a^h_{2,b}(B^s)]} \qquad (29)$$

are satisfied. In this situation, the economy will be in hybrid equilibriums without the price limit, with bid-ask spreads being given by (22)-(26).

In this case, the informed trader's strategic trading, after the price limit i is imposed, can be stated as follows.

***Proposition 4**: Consider the game where the informed trader acts as a Stackelberg leader and the market maker acts as a Stackelberg follower. Suppose that, without the price limit, (28) and (29) hold, ψ_t, t = 1, 2, between 0 and 1 can be chosen so that the market is in hybrid equilibriums and the informed traders are indifferent between trading big and medium orders. Then the following statements hold.*

1. If $a^h_{b,1} < \overline{a}_1$, i.e., $V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu\psi_1}{\alpha\delta\mu\psi_1 + (1-\alpha\mu)X^b_B} < \overline{a}_1$, the price limit is not binding.

2. If $V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X^s_B} > \overline{a}_1$, the market never opens.

3. If

$$V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X^s_B} < \overline{a}_1 < V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X^m_B} \qquad (30)$$

then, after imposition of the price limit, the strategy traded by the informed trader *in equilibrium switches to trading with a small order and the equilibrium ask price is given by (3).*

4. If

$$\overline{a}_1 < V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu\psi_1}{\alpha\delta\mu\psi_1 + (1-\alpha\mu)X_B^b} \tag{31}$$

and

$$V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^m} < \overline{a}_1 < V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^b} \tag{32}$$

then, after imposition of the price limit,

(a) the strategy traded by the informed trader in equilibrium switches to trading with a medium order if

$$\gamma > 1 + \frac{[B_1^m(\overline{V} - a_{m,1}) - B_1^s(\overline{V} - a_{s,1})]}{B_2^b[(\overline{V} - a_{b,2}^h(B_1^m)) - (\overline{V} - a_{b,2}^h(B_1^s))]}$$

(b) the strategy traded by the informed trader in equilibrium switches to trading with a small order if

$$\gamma < 1 + \frac{[B_1^m(\overline{V} - a_{m,1}) - B_1^s(\overline{V} - a_{s,1})]}{B_2^b[(\overline{V} - a_{b,2}^h(B_1^m)) - (\overline{V} - a_{b,2}^h(B_1^s))]} \tag{33}$$

Informed traders' trading depends on the level of the price limit. There are four cases stated in proposition 5. If the limit is smaller than the price move caused by separating at the small quantity, the market never opens. If the limit is greater than the big quantity price move with hybrid, the price limit is not binding. As stated in the third case of proposition 4, the informed traders switches from hybrid to separating equilibrium by trading small quantities if the limit lies between the price moves caused by separating at the small quantity and at the medium quantity. Finally, The informed traders switches from hybrid to separating by trading either medium or small quantities if (Ⅰ) the large quantity price move with hybrid exceeds the price limit and (Ⅱ) the price limit is smaller than the price move caused by separating at the large quantity and greater than that caused by separating at the medium quantity, i.e. (31) and (32) holds. In this case, the horizon of information plays a role. When the private information is long-lived enough, informed traders trade with small quantities in order to acquire more expected profits in the second period. Conversely, if it is very likely that the information will be revealed after the first trading, informed traders trade with medium quantities to ensure their profits. Given the leading actions of informed traders, a market maker acting as a Stackelberg follower changes the quotes to response and thus the price paths are changed.

Numerical Example

We give a numerical example to illustrate the effects of price limits. Consider the following parameter values: $\overline{V}=10$, $\underline{V}=0$, $\alpha=1$, $\delta=0.7$, $\mu=0.4$, $X_B^s=0.9$, $X_B^m=0.2$, $X_B^b=0.1$, $B_1^b=B_2^b=200$, $B_1^m=B_2^m=50$, $B_1^s=B_2^s=10$. Under these parameter values, (28) and (29) hold, the unique equilibrium values of ψ_1 and ψ_2 can be found and hybrid equilibriums exist. Then it is easy to find that $a_{b,1}=9.47$, $a_{m,1}=9.10$, $a_{s,1}=8.02$, $a_{b,1}^h=9.41$, and $a_{m,1}^h=7.65$. If the upper price limit is greater than 9.41, the restriction is not binding; if it is smaller than 8.02, the market will never open. Given the limit lying between 9.10 and 8.02, informed traders switch to trade with small quantities and the price impact of a small order increases from 0 to 1.02 (=8.02-7). Finally, if the limit lies between 9.41 and 9.10, informed traders shrank their trading. In this situation, it is easy to verify that RHS of (33) equals to 0.387. Thus, if the information is short-lived, i.e. γ is bigger than 0.387, informed traders switch to trade with medium quantities and the price impact of a medium order increases from 0.65(=7.65-7) to 2.10(=9.10-7). Conversely, if the information is very long-lived, i.e. γ is smaller than 0.387, informed traders switch to trade with small quantities and the price impact of a small order increases from 0 to 1.02(=8.02-7). For example, if γ is 0.2, the combined expected profit of medium quantity (68.82) is smaller than that of small quantity (76.54).The costs of both small and medium orders are increased due to the imposition of the price limit.

Due to the strategically informed trading, the price path and the expected volatility are changed as well. The expected volatility is measure by the same procedure as (18). The expected volatility without price limit is 0.631. If the information is short-lived, informed traders switch to trading medium quantities to response to price limit rules, and the expected volatility increases by 22 percent (from 0.631 to 0.771). On the other hand, if the information is long-lived, the expected volatility increases by 28 percent (from 0.631 to 0.807) because of informed trading with small quantities. Therefore, we acquire similar results: The ex ante effects of the price limit on strategic informed trading result in the wider spreads and higher volatility in the following period, i.e. the subsequent trading days or trading sessions. Moreover, the long-lived information exacerbates the price volatility more than short-lived information. In other words, because of the ex ante effects of strategically informed trading, the price limit rule not only fails to achieve its goals but also worsens the market performances.

4.2 Pooling Equilibrium

There may exist pooling equilibriums in which the informed trader is indifferent among trading big, medium or small quantities, so that

$$B^b(\overline{V} - a_{b,1}^p) = B^m(\overline{V} - a_{m,1}^p) = B^s(\overline{V} - a_{s,1}^p) \tag{34}$$

$$B^b(\overline{V} - a_{b,2}^p(Q_1)) = B^m(\overline{V} - a_{m,2}^p(Q_1)) = B^s(\overline{V} - a_{s,2}^p(Q_1)) \tag{35}$$

where $a_{b,1}^p, a_{m,1}^p$ and $a_{s,1}^p$ represent the ask prices at t = 1 for a large, medium and small order, respectively, while $a_{b,2}^h(Q_1), a_{m,2}^h(Q_1)$ and $a_{s,2}^h(Q_1)$ represent the ask prices at t = 2.

The ask prices in the second period depend on the order received in the previous period. Given any order size at t = 1, we have a pooling equilibrium at t = 2 if

$$B^b(\overline{V} - a_{b,2}(Q_1)) < B^k(\overline{V} - a_{k,2}(Q_1)), k = m, s \tag{36}$$

We can easily prove that when the condition with $X = B_1^s$ is satisfied, the other two conditions will be satisfied simultaneously no matter what kind of equilibrium in the first period. Therefore, the condition can be simplified to

$$\frac{B_2^b}{B_2^s} < \frac{(\overline{V} - a_{s,2}(B_1^s))}{(\overline{V} - a_{b,2}(B_1^s))} \tag{37}$$

Let $\psi_t^{'}$ and $\psi_t^{''}$, t = 1, 2, be the probability that a trader informed of $\overline{V}$ trades the big and medium quantities. Suppose one can find $\psi_1^{'}$ and $\psi_1^{''}$ such that (34) and (35) is satisfied, where $0 \le \psi_1^{'} \le 1$, $0 \le \psi_1^{''} \le 1$, and $0 \le \psi_1^{'} + \psi_1^{''} \le 1$. Using the Bayes' theorem and zero profit condition, ask prices at t=1 are given by,

$$\begin{aligned}
a_{b,1}^p &= E[V \mid B^b] = V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu\psi_1^{'}}{\alpha\delta\mu\psi_1^{'} + (1-\alpha\mu)X_B^b} \\
a_{m,1}^p &= E[V \mid B^m] = V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu\psi_1^{''}}{\alpha\delta\mu(1-\psi_1^{''}) + (1-\alpha\mu)X_B^m} \\
a_{s,1}^p &= E[V \mid B^s] = V^* + \frac{(\overline{V} - \underline{V})(1-\delta)\alpha\delta\mu(1-\psi_1^{'} - \psi_1^{''})}{\alpha\delta\mu(1-\psi_1^{'} - \psi_1^{''}) + (1-\alpha\mu)X_B^s}
\end{aligned} \tag{38}$$

The ask prices at t=2 are given by,

$$a^p_{b,2}(B^b) = E[V \mid B^b, B^b]$$

$$= V^* + \frac{(\bar{V} - \underline{V})(1-\delta)\alpha\delta\mu\,[\mu\psi_1^{'}\psi_2^{'} + (1-\mu)(\psi_1^{'} + \psi_2^{'})X_B^b]}{\alpha\delta\mu^2\psi_1^{'}\psi_2^{'} + \alpha\delta\mu(1-\mu)(\psi_1^{'} + \psi_2^{'})X_B^b + [1+\alpha(-2+\mu)\mu](X_B^b)^2}$$

$$a^p_{b,2}(B^m) = E[V \mid B^m, B^b]$$

$$= V^* + \frac{(\bar{V} - \underline{V})(1-\delta)\alpha\delta\mu\,[((1-\mu)X_B^m - \mu\psi_1^{''})\psi_2^{'} + (1-\mu)\psi_1^{''}X_B^b]}{\alpha\delta\mu((1-\mu)X_B^m + \mu\psi_1^{''})\psi_2^{'} + \alpha\delta\mu(1-\mu)\psi_1^{''}X_B^b + (1+\alpha(-2+\mu)\mu)X_B^m X_B^b} \quad (39)$$

$$a^p_{b,2}(B^s) = E[V \mid B^s, B^b]$$

$$= V^* + \frac{(\bar{V} - \underline{V})(1-\delta)\alpha\delta\mu\,[(1-\mu)(1-\psi_1^{'} - \psi_1^{''})X_B^b + ((1-\mu)X_B^s + \mu(1-\psi_1^{'} - \psi_1^{''}))\psi_2^{'}]}{X_B^b\left[(1+\alpha(-2+\mu)\mu)X_B^s + \alpha\delta\mu(1-\mu)(1-\psi_1^{'} - \psi_1^{''})\right] + \alpha\delta\mu\psi_2^{'}\left[(1-\mu)X_B^s + \mu(1-\psi_1^{'} - \psi_1^{''})\right]}$$

This can be formalized in the following proposition.

Proposition 5*: Consider the game where the informed trader acts as a Stackelberg leader and the market maker acts as a Stackelberg follower. There exist pooling equilibriums without the price limit if one can choose* $\psi_t^{'}$ and $\psi_t^{''}$, t = 1, 2, *between 0 and 1 such that (34), (35) and zero-profit condition for the market maker are satisfied. It is possible to do so if*

$$\frac{B_1^b}{B_1^s} < 1 + \frac{\alpha\delta\mu}{X_B^b(1-\alpha\mu)} \quad (40)$$

$$\frac{B_2^b}{B_2^s} < \frac{\alpha\delta\mu\left[(1-\mu)X_B^s + \mu(1-\psi_1^{'} - \psi_1^{''})\right]}{X_B^b\left[(1+\alpha(-2+\mu)\mu)X_B^s + \alpha\delta\mu(1-\mu)(1-\psi_1^{'} - \psi_1^{''})\right]} \quad (41)$$

are satisfied. In this situation, the economy will be in pooling equilibriums without the price limit, with bid-ask spreads being given by (38)-(39).

The price limit i imposed in the market affects the strategies of informed traders. The analyses are summarized in the following proposition.

Proposition 6*: Consider the game where the informed trader acts as a Stackelberg leader and the market maker acts as a Stackelberg follower. Suppose that, without the price limit, (40) and (41) hold,* ψ_t*, t = 1, 2, between 0 and 1 can be chosen so that the market is in pooling equilibriums and the informed traders are indifferent among* *trading with big, medium or small quantities. Then the following statements hold.*

1. If $a_{b,1}^{p} < \bar{a}_1$, *i.e.,* $V^* + \dfrac{(\bar{V} - \underline{V})(1-\delta)\alpha\delta\mu\psi_1^{'}}{\alpha\delta\mu\psi_1^{'} + (1-\alpha\mu)X_B^b} < \bar{a}_1$, *the price limit is not binding.*

2. If $V^* + \dfrac{(\bar{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^s} > \bar{a}_1$, *the market never opens.*

3. If

$$V^* + \frac{(\bar{V} - \underline{V})(1-\delta)\alpha\delta\mu}{\alpha\delta\mu + (1-\alpha\mu)X_B^s} < \bar{a}_1 < V^* + \frac{(\bar{V} - \underline{V})(1-\delta)\alpha\delta\mu\psi_1^{'}}{\alpha\delta\mu\psi_1^{'} + (1-\alpha\mu)X_B^b} \qquad (42)$$

then, after imposition of the price limit, the strategy traded by the informed trader in equilibrium switches to trading with a small order and the equilibrium ask price is given by (3).

Informed traders' trading depends on the level of the price limit. There are three cases stated in proposition 7. Similarly, if the limit is smaller than the price move caused by separating at the small quantity, the market never opens. If the limit is over than the big quantity price move with pooling, the price limit is not binding. However, if the limit lies between the price moves caused by separating at the small quantity and by pooling at the big quantity, informed traders switch from pooling to separate equilibrium by trading small quantities. As stated in proposition 3 of Easley and O'Hara (1987), the market will be in a pooling equilibrium if the market is sufficiently narrow or shallow. In a thin market, since informed traders switch to trade with small quantities, the price limit may increase the costs of small orders no what information horizon is. Put differently, the price limit worsens liquidity traders' trading cost more in a thin market.

Numerical Example

We give a numerical example to illustrate the effects of price limits. Consider the following parameter values: $\bar{V} = 10$, $\underline{V} = 0$, $\alpha = 1$, $\delta = 0.7$, $\mu = 0.4$, $X_B^s = 0.9$, $X_B^m = 0.2$, $X_B^b = 0.1$, $B_1^b = B_2^b = 30$, $B_1^m = B_2^m = 20$, $B_1^s = B_2^s = 10$. Under these parameter values, (34) and (35) hold, and pooling equilibriums exist. The difference of order sizes in pooling equilibrium is smaller than that in separating and hybrid equilibriums. The unique equilibrium value of $\psi_1^{'}$ and $\psi_1^{''}$ between 0 and 1 can be found as 0.45 and 0.46 so that (34) and (35) hold. Then it is easy to compute that $a_{b,1} = 9.47$, $a_{m,1} = 9.10$, $a_{s,1} = 8.02$, $a_{b,1}^p = 9.04$, $a_{m,1}^p = 8.56$, and $a_{s,1}^p = 7.12$. If the upper price limit is greater than 9.04, the constraint of the price limit is not binding; if it is smaller than 8.02, the market will never open. Given the limit lying between 8.02 and 9.04, informed traders switch to trade with small quantities and the price impact of a small order increases from 0.12(=7.12-7) to 1.02 (=8.02-7). The costs of small orders are aggravated by about 8 times due to the imposition of the price limit. Similarly, we compute the expected volatility by the same measure as (18). The expected volatility without price limit is 0.347. If informed traders switch to trading small quantities to response to price limit rules, the expected volatility increases by over 200 percent (from 0.347 to 0.748).

Conclusively, our results suggest that the price limit has the same effect when any of equilibriums appears in the absence of the price limit. The ex ante effects of the price limit on strategic informed trading result in the distorted price path and higher volatility in the following period. When the price limit is imposed, it is not necessarily that informed traders trade as much as they can trade. They may act less aggressively by trading small quantity if the information is long-lived. Given the more intense liquidity trading on the small side of the market, the price limit hurts liquidity traders more badly in a market with long-lived information. Additionally, those stocks near but not at their limits may contain more information and may be more volatile in the next period than those at their limits.

5. Conclusion

Price limits rules are adopted in many markets as a stabilization mechanism. Proponents argue that the price limit rules can stabilize the market and protect uninformed traders, while critics claim that these rules only block trading and hurt price discovery. We contribute to this issue by focusing on informed traders' strategies and taking the horizon of information into account. Ex ante effects of rule-based price limits will result in informed traders' large scaling back, i.e. trading small quantities rather than trading as much as they can. The strategically informed behavior may lead to undesirable market qualities. A policy implication of our results is that the characteristic of information should be included into the consideration when the price limit is adopted. Alternatively, a different stabilizing mechanism may be designed for stocks having different investor's structure.

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